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Yangjian Cai
Qiang Lin
Olga Korotkova
University of Miami, o.korotkova@miami.edu

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Ghost imaging with twisted Gaussian Schell-model beam

Yangjian Cai1*, Qiang Lin2 and Olga Korotkova3
1. School of Physical Science and Technology, Suzhou University, Suzhou 215006, China
2. Institute of Optics, Department of Physics, Zhejiang University, Hangzhou 310027, China
3. Department of Physics, University of Miami, Coral Gables, Florida 33146, USA
*Corresponding author: yangjian_cai@yahoo.com.cn

Abstract: Based on the classical optical coherence theory, ghost imaging with twisted Gaussian Schell-model (GSM) beams is analyzed. It is found that the twist phase of the GSM beam has strong influence on ghost imaging. As the absolute value of the twist factor increases, the ghost image disappears gradually, but its visibility increases. This phenomenon is caused by the fact that the twist phase enhances the transverse spatial coherence of the twisted GSM beam on propagation.

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References and links
1. Introduction

Conventionally partially coherent beams are characterized by a second-order correlation function of the electric field which depends on two spatial arguments and the oscillation frequency and being called the cross-spectral density [1]. Gaussian Schell-model (GSM) beams are partially coherent beams whose spectral density, i.e. the cross-spectral density evaluated at a single argument and spectral degree of coherence, i.e. a suitably normalized version of the cross-spectral density have Gaussian shapes [2-9]. By scattering a coherent laser beam from a rotating grounded glass, then transforming the spectral density distribution of the scattered light into Gaussian profile with a Gaussian amplitude filter, a GSM beam can be generated [6, 7]. GSM beams can also be generated with specially synthesized rough surfaces, spatial light modulators and synthetic acousto-optic holograms (c.f. [9]).

A more general partially coherent beam can possess a twist phase, which differs in many respects from the customary quadratic phase factor. Simon and Mukunda first introduced the partially coherent twisted Gaussian Schell-model (GSM) beam [10] opening up “a new dimension” in the area of partially coherent fields [10, 11]. Unlike the usual phase curvature, the twist phase is bounded in strength due to the fact that the cross-spectral density function must be non-negative definite and, moreover, it is absent in a coherent Gaussian beam [10]. The twist phase has an intrinsic chiral or handedness property and is responsible for the rotation of the beam spot on propagation [10-12]. An essential aspect of the twist phase is that it is intrinsically two-dimensional, and it cannot be separated into a sum of one-dimensional contributions [10-12]. Experimental observation of the twisted GSM beam has been reported in Ref. [11]. Later the studies relating to superposition [13], coherent-mode decomposition [14, 15] and the analysis of the transfer of radiance [16] of the twisted GSM beams have been carried out.

The conventional method for treating the propagation of twisted GSM beams is Wigner-distribution function [10, 17]. Lin and Cai have introduced a convenient alternative tensor method for treating propagation of GSM and twisted GSM beams [18]. Paraxial propagation of the twisted GSM beams through free space, paraxial optical systems and some random linear media, have been then studied in details [16-23]. Dependence of the orbital angular momentum of a partially coherent beam on its twist phase was revealed in Ref. [24]. More recently, the influence of the twist phase on the second-harmonic generation by a partially coherent beam has been investigated [25].

Ghost imaging, also known as quantum or correlated, or coincidence or two-photon imaging, has been studied extensively in recent years [26-56]. It provides a technique for nonlocal image production of an object through the measurement of the fourth-order
correlation function, i.e., intensity correlation function, calculated for two optical paths with the object placed in one of them (see Fig.1). Ghost imaging was first demonstrated with entangled photon pairs generated by spontaneous parametric down conversion in 1995 [26]. Since then many relating experimental results were obtained and, soon, ghost imaging has found wide applications in quantum metrology, lithography and holography [17-33]. Recently, both theoretical and experimental results have shown that ghost imaging can be realized with classical momentum-correlated laser pulses [34, 35] and partially coherent light (i.e., pseudo-thermal light) [36-56]. In particular, it has been found that ghost imaging with partially coherent light is highly dependent on coherence properties of the source [36-56]. The visibility (typical quality criterion) of the image is generally higher in the quantum case than in the classical, however, due to the fact that ghost imaging with classical partially coherent light is much simpler realizable, it has its own advantage for practical applications [6, 7].

In any of the previous theoretical investigations on ghost imaging with partially coherent light, the twist phase of the cross-spectral density of the light source has not been taken into consideration. Therefore, since in the absence of the twist phase the two-dimensional ghost imaging problem can be regarded as a product of two one-dimensional ghost imaging problems, only one-dimensional ghost imaging has been previously studied.

If the partially coherent light source possesses a twist phase, two-dimensional ghost imaging problem cannot longer be regarded as a product of two one-dimensional ghost imaging problems, and one, therefore, has to study ghost imaging in the two-dimensional setting. In this paper, we study the ghost imaging with partially coherent twisted GSM beam in two-dimensions with the help of the tensor method. Thus for the first time, the twist phase of the cross-spectral density of the light source is taken into account for ghost imaging. The influence of the twist phase on ghost imaging is studied in detail. It is found that the ghost image disappears gradually but its visibility increases as the absolute value of the twist factor increases. This phenomenon is caused by the fact that the twist phase enhances the transverse spatial coherence of the twisted GSM beam on propagation.

2. Second-order correlation function of partially coherent twisted GSM beam

We begin by a brief review of the statistical description of the twisted GSM beams using the second-order coherence theory in the space-frequency domain [1] and the tensor method for treatment of interaction of the beam with paraxial optical systems.

According to Refs. [13-15], the cross-spectral density of a twisted GSM beam in source plane \( z = 0 \) is expressed as follows

\[
W(r, r') = \langle U(r) U^*(r') \rangle = \exp \left[ \frac{r^2 + r'^2}{4\sigma_{\mu}^2} - \frac{(r_r - r'_r)^2}{2\sigma_{\sigma}^2} \right] - \frac{i k \mu_0}{2} (r_r - r'_r) J (r_r + r'_r),
\]

where frequency dependence was omitted for conciseness. In Eq. (1) \( U \) is a scalar field, \( k = 2\pi / \lambda \) is the wave number with \( \lambda \) being the wavelength of light field, \( r \) and \( r' \) are two arbitrary position vectors in the source plane, \( \sigma_{\mu} \) and \( \sigma_{\sigma} \) denote the transverse beam width and spectral coherence width, respectively; \( \mu_0 \) is a scalar real-valued twist factor with the dimension of an inverse distance, limited by the double inequality \( 0 \leq \mu_0 \leq \left[ k^2 \sigma_{\sigma}^4 \right]^{-1} \) due to the non-negative definiteness requirement of Eq. (1). We note that in the coherent limit, \( \sigma_{\sigma} \to \infty \), the twist factor \( \mu_0 \) disappears. \( J \) is an anti-symmetric matrix, viz.,

\[
J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.
\]

Under condition \( \mu_0 = 0 \) the cross-spectral density in Eq. (1) reduces to the one of the conventional GSM beam [1-9]. Due to the existence of the term
(r_x - r_y)^T J (r_x + r_y) = x_1 y_2 - x_2 y_1 in the right side of Eq. (1), the two-dimensional cross-spectral density cannot be split in a product of two one-dimensional cross-spectral densities. Thus, we must treat the propagation of the twisted GSM beams using full two-dimensional versions of their cross-spectral densities. We note here that one should not confuse the twist factor of the GSM beam with its wave-front radius of curvature (i.e., the usual quadratic phase factor): while the former is present in Eq. (1) the later is absent. We also should point out that the modulus of the spectral degree of coherence \( \sigma_{\phi 0} \) of a twisted GSM beam in the source plane is determined solely by parameter \( \sigma_{\phi 0} \), and is independent of parameters \( \sigma_{\phi 0} \) and \( \mu_i \).

Equation (1) can alternatively be expressed in the following tensor form [18]

\[
W(\hat{r}) = \exp \left( \frac{-i}{2} \hat{r}^T M_0^{-1} \hat{r} \right),
\]

where \( \hat{r} = (r_x^T \ r_y^T) = (x_i \ y_i \ x_j \ y_j) \), and \( M_0^{-1} \) is a 4\times4 matrix, called partially coherent complex curvature tensor, takes the form

\[
M_0^{-1} = \left[ \begin{array}{cccc}
-\frac{i}{2k\sigma_{\phi 0}^2} - \frac{i}{k\sigma_{\phi 0}^2} & \frac{i}{k\sigma_{\phi 0}^2} - \mu_i J \\
\frac{i}{k\sigma_{\phi 0}^2} + \mu_i J^T & -\frac{i}{2k\sigma_{\phi 0}^2} - \frac{i}{k\sigma_{\phi 0}^2}
\end{array} \right],
\]

\( I \) being a 2\times2 unit matrix. Following Ref. [18], after propagation through a general astigmatic ABCD optical system, the cross-spectral density of the twisted GSM beam in the output plane is expressed as follows

\[
W(\hat{u}) = \frac{1}{\det((\hat{A} + \hat{B}M_0^{-1}))^{1/2}} \exp \left( \frac{-i}{2} \hat{u}^T M^{-1} \hat{u} \right),
\]

where \( \det \) stands for the determinant of a matrix, \( \hat{u} = (u_i^T \ u_j^T) \) with \( u_i \) and \( u_j \) being two arbitrary position vectors in the output plane, \( M^{-1} \) is the partially coherent complex curvature tensor in the output plane, and is related with \( M_0^{-1} \) through the following tensor ABCD law

\[
M^{-1} = (\hat{C} + \hat{D}M_0^{-1})(\hat{A} + \hat{B}M_0^{-1})^{-1},
\]

Here matrices \( \hat{A}, \hat{B}, \hat{C} \) and \( \hat{D} \) have the forms

\[
\hat{A} = \left[ \begin{array}{cc}
A & 0I \\
0I & A
\end{array} \right], \quad \hat{B} = \left[ \begin{array}{cc}
B & 0I \\
0I & -B^T
\end{array} \right], \quad \hat{C} = \left[ \begin{array}{cc}
C & 0I \\
0I & -C^T
\end{array} \right], \quad \hat{D} = \left[ \begin{array}{cc}
D & 0I \\
0I & D^T
\end{array} \right],
\]

with \( A, B, C \) and \( D \) being the 2\times2 sub-matrices of the astigmatic optical system. Symbol “*” in Eq. (7) which denotes the complex conjugate is required for a general optical system in the presence of loss or gain, although it does not appear in Eq. (13) of Ref. [18].

3. Ghost imaging with partially coherent twisted GSM beam

We will now use the twisted GSM beam as the illumination field in the typical lensless ghost imaging setup shown in Fig. 1 [45, 46, 55, 56]. A beam generated by a twisted GSM source is split into two beams which are sent to paths 1 and 2. Along path 1 the beam propagates first to an object with transmission function \( H(\chi_i) \) and then to single-photon detector \( D_1 \) (located
at \( u_1 = 0 \). Along path 2 the beam propagates to bucket detector \( D_2 \). We assume that the
distance from the light source to the object is the same as the distance from the light source to
detector \( D_2 \) [45, 46, 55, 56]. Finally, the output signals from detectors \( D_1 \) and \( D_2 \) are sent to
an electronic coincidence circuit in order to measure the coincidence counting rate [i.e., the
fourth-order correlation \( G^{(4)}(u_1, u_2) \)] of two detected signals.

With the help of the moment theorem for the complex Gaussian random fields in
classical optical coherence theory, the fourth-order correlation (i.e., intensity correlation)
function of the fields incident onto detectors \( D_1 \) and \( D_2 \) can be written as [1, 29, 35-53]

\[
G^{(4)}(u_1, u_2) = \langle I(u_1) I(u_2) \rangle = \left| W(u_1, u_2) \right|^2, \quad (8)
\]

where \( \langle I(u_1) I(u_2) \rangle \) and \( \langle I(u_1) \rangle \) denote the spectral densities of the beams on the surfaces
of detectors \( D_1 \) and \( D_2 \), respectively and, as before, \( W(u_1, u_2) \) is the mutual cross-spectral
density of fields at two detectors [57]. The term \( \left| W(u_1, u_2) \right|^2 \) denotes, in fact, the intensity
fluctuation correlation \( \langle I(u_1) I(u_2) \rangle - \langle I(u_1) \rangle \langle I(u_2) \rangle \), which contains the image of
the object. The term \( \langle I(u_1) \rangle \langle I(u_2) \rangle \) represents the background component and should be,
therefore, subtracted from the intensity correlation term.

Within the validity of the paraxial approximation \( \langle I(u_1) \rangle \) and \( \langle I(u_2) \rangle \) at the two detectors are given by the expressions

\[
W(u_1, u_2) = \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} \langle U(r_1) U^*(r_2) \rangle h_i(u_i, r_i) h_i^*(u_2, r_2) dr_1 dr_2,
\]

\[
\langle I(u_1) \rangle = \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} \langle U(r_1) U^*(r_2) \rangle h_i(u_i, r_i) h_i^*(u_2, r_2) \langle U(r_1) U^*(r_2) \rangle dr_1 dr_2, \quad i = 1, 2 \quad (10)
\]

where \( \langle U(r_1) U^*(r_2) \rangle \) is the second-order correlation function of the twisted GSM field in the
source plane given by Eq. (1) or Eq. (3); \( h_i(u_i, r_i) \) and \( h_i^*(u_2, r_2) \) are the response functions of the
two optical paths having the forms

\[
h_i(u_i, r_i) = \frac{1}{\lambda^2 z_i} \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} \exp \left[ -\frac{i k}{2 z_i} (r_i - v_i)^2 - \frac{i k}{2 z_i} (v_i - u_i)^2 \right] H(v_i) dv_i, \quad (11)
\]

\[
h_i^*(u_2, r_2) = \frac{i}{\lambda z_i} \exp \left[ -\frac{i k}{2 z_i} (r_2 - u_2)^2 \right]. \quad (12)
\]

Substituting from Eqs. (3), (11) and (12) into Eq. (9), we obtain (after some operation)
the formula

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**Fig. 1. Scheme for lensless ghost imaging with a twisted GSM beam**
\[ W(\mathbf{u}_1, 0, \mathbf{u}_2) = \frac{i}{\lambda^2 z_i^2 \left| \text{Det}(\mathbf{B}) \right|^{1/2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp \left[ -\frac{ik}{2} \hat{\mathbf{r}}^T \mathbf{M}_1^{-1} \hat{\mathbf{r}} + \frac{ik}{2} \left( \hat{\mathbf{r}}^T \hat{\mathbf{B}}^{-1} \hat{\mathbf{r}} - 2\hat{\mathbf{r}}^T \hat{\mathbf{B}}^{-1} \hat{\mathbf{v}}_a + \hat{\mathbf{v}}_a^T \hat{\mathbf{B}}^{-1} \hat{\mathbf{v}}_a \right) \right] \]

\times \exp \left[ -\frac{ik}{2z_i} \hat{\mathbf{v}}_a^T \hat{\mathbf{v}}_a \right] H(\mathbf{v}_1) d\hat{\mathbf{r}} d\mathbf{v}_1, \quad (13) \]

where \( \hat{\mathbf{v}}_a = (\mathbf{v}_1 - \mathbf{u}_2) \), \( d\hat{\mathbf{r}} = d\mathbf{r}_1 d\mathbf{r}_2 = dx_1 dy_1 dx_2 dy_2 \), and

\[ \hat{\mathbf{B}} = \begin{pmatrix} \mathbf{I} & 0 \mathbf{I} \\ 0 \mathbf{I} & -\mathbf{I} \end{pmatrix}. \quad (14) \]

After vector integration over \( \hat{\mathbf{r}} \), Eq. (13) reduces to

\[ W(\mathbf{u}_1, 0, \mathbf{u}_2) = \frac{i}{\lambda^2 z_i \left| \text{Det}(\mathbf{I} + \mathbf{B} \mathbf{M}_1^{-1}) \right|^{1/2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp \left[ -\frac{ik}{2} \hat{\mathbf{v}}_a^T \mathbf{M}^{-1} \hat{\mathbf{v}}_a - \frac{ik}{2z_i} \hat{\mathbf{v}}_a^T \hat{\mathbf{v}}_a \right] H(\mathbf{v}_1) d\mathbf{v}_1, \]

where \( \mathbf{I} \) denotes the 4x4 unit matrix, and

\[ \mathbf{M}^{-1} = \left( \mathbf{M}_0 + \mathbf{B} \right)^{-1} = \begin{pmatrix} \mathbf{M}_0^{-1} & \mathbf{M}_0^{-1} \\ \mathbf{M}_0^{-1} \mathbf{M}_0^{-1} & \mathbf{M}_0^{-1} \mathbf{M}_0^{-1} \end{pmatrix}. \quad (16) \]

and the first part of the exponential term under the integral sign, on the right side of Eq. (15), can be expanded as

\[ \exp \left[ -\frac{ik}{2z_i} \hat{\mathbf{v}}_a^T \hat{\mathbf{v}}_a \right] = \exp \left[ -\frac{ik}{2} \left( \mathbf{v}_1^T \mathbf{M}^{-1} \mathbf{v}_1 - i\mathbf{v}_1^T \mathbf{M}_0^{-1} \mathbf{u}_2 - \frac{ik}{2} \mathbf{u}_2^T \mathbf{M}_0^{-1} \mathbf{u}_2 \right) \right]. \quad (17) \]

After performing several algebraic operations, we find that Eq. (15) becomes

\[ W(\mathbf{u}_1, 0, \mathbf{u}_2) = \frac{i}{\lambda z_i \left| \text{Det}(\mathbf{B}_0) \right|^{1/2} \left| \text{Det}(\mathbf{I} + \mathbf{B} \mathbf{M}_0^{-1}) \right|^{1/2}} \times \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp \left[ -\frac{ik}{2} \mathbf{v}_1^T \mathbf{M}_1^{-1} \mathbf{v}_1 \right] H(\mathbf{v}_1) d\mathbf{v}_1, \]

with \( \mathbf{B}_0 = z_i \mathbf{I} \).

Fig. 2. 3D-distribution and corresponding contour graph of the square of the transmission

function \( \langle |H(\mathbf{v}_1)|^2 \rangle \) of a soft circular aperture versus \( \mathbf{v}_1 \), with \( a_0 = 0.1 \mu m \) and \( N = 30 \)

For the convenience of calculations and analysis, we assume that the object in path one is a soft circular aperture, whose transmission function can be expressed as a finite sum of Gaussian modes

\[ H(\mathbf{v}_1) \approx \frac{1}{C_0} \sum_{n=1}^{N} \frac{(-1)^{n+1} N!}{n!} \exp \left( \frac{-n \mathbf{v}_1^T \mathbf{v}_1}{a_0^2} \right), \quad (19) \]

where \( C_0 = \sum_{n=0}^{N} \frac{(-1)^{n+1} N!}{n!} \) is a normalization factor. Note that there are many different models for describing the transmission function of a soft circular aperture, such as super-Gaussian

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model [58], flattened-Gaussian model [59], flat-topped model [60] and flat-topped multi-Gaussian model [61]. We have chosen the flat-topped model in this paper just for mathematical convenience. Figure 1 shows the 3D-distribution and the corresponding contour graph of the square of the transmission function \( |H(v_i)|^2 \) of a soft circular aperture versus \( v_{11} \) and \( v_{12} \) with \( a_i = 0.1 \) mm and \( N = 30 \). It may be readily demonstrated, at least numerically, that the width of the aperture increases with increase in \( a_i \). We note that in the ghost imaging experiments, there is no special restriction on the transmission function of the object [26-56]. In other words, the ghost imaging method works similarly for all objects with soft boundaries, for sufficiently large number \( N \).

Equation (19) can alternatively be expressed in the following tensor form as

\[
H(v_i) = \frac{1}{C_0} \sum_{n=1}^{N} (-1)^{n+1} \binom{N}{n} \exp \left( -\frac{ik}{2} v_i \mathbf{H}_n v_i \right),
\]

(20)

where

\[
\mathbf{H}_n = \frac{2n}{ika_i} \mathbf{I}.
\]

(21)

Substituting from Eq. (20) into Eq. (18), we obtain, after integration over \( v_1 \), the expression

\[
W(u_i = 0, u_z) = \frac{1}{C_0} \sum_{n=1}^{N} (-1)^{n+1} \binom{N}{n} \frac{1}{\left[ \det (\mathbf{B}_n \mathbf{M}_n^{-1} + \mathbf{B}_n \mathbf{H}_n + \mathbf{I}) \det (\mathbf{I} + \tilde{\mathbf{B}} \mathbf{M}_n^{-1}) \right]^{\frac{N}{2}}} \times \exp \left( \frac{ik}{2} u_z \mathbf{M}_n^{-1} u_z + \frac{ik}{2} u_i \left( \mathbf{M}_n^{-1} \right)^T \left( \mathbf{M}_n^{-1} \right)^{-1} \mathbf{M}_n^{-1} u_z \right).
\]

(22)

In a similar way, we obtain the following expressions for the intensities of beams at two detectors

\[
< I(u_i = 0) > = \frac{1}{C_0} \sum_{n=1}^{N} (-1)^{n+1} \binom{N}{n} \frac{1}{N} \left[ \det (\mathbf{B}_n \mathbf{M}_n^{-1} + \mathbf{B}_n \mathbf{H}_n + \mathbf{I}) \det (\mathbf{I} + \tilde{\mathbf{B}} \mathbf{M}_n^{-1}) \right]^{\frac{N}{2}} \exp \left( -\frac{ik}{2} \tilde{u}_2 \mathbf{M}_n^{-1} \tilde{u}_2 \right),
\]

(23)

\[
< I(u_z) > = \frac{1}{\left[ \det (\mathbf{I} + \tilde{\mathbf{B}} \mathbf{M}_n^{-1}) \right]^{\frac{N}{2}}} \exp \left( -\frac{ik}{2} \tilde{u}_2 \mathbf{M}_n^{-1} \tilde{u}_2 \right),
\]

(24)

where \( \tilde{u}_2 = (u_2^T, u_z^T) \), \( \tilde{\mathbf{B}} \) is given by Eq. (14), \( \mathbf{M}^{-1} \) is given by Eq. (16), \( \tilde{\mathbf{B}} \) and \( \mathbf{H}_m \) are expressed as follows

\[
\tilde{\mathbf{B}}_i = \begin{pmatrix} z_i \mathbf{I} & 0 \mathbf{I} \\ 0 \mathbf{I} & -z_i \mathbf{I} \end{pmatrix}, \quad \mathbf{H}_m = \frac{2}{ika_i} \begin{pmatrix} n \mathbf{I} & 0 \mathbf{I} \\ 0 \mathbf{I} & m \mathbf{I} \end{pmatrix}.
\]

(25)

4. Numerical example

In this section, we calculate numerically the ghost image formed with a typical twisted GSM beam using the formulae derived in section 3.
Fig. 3. Normalized ghost image $|W(u_{1}=0,u_{2})|^{2}/|W(u_{1}=0,u_{2}=0)|^{2}$ of a soft circular aperture formed with twisted GSM beam for different values of the initial transverse coherence width $\sigma_{0}$.

Fig. 4. Normalized ghost image $|W(u_{1}=0,u_{2})|^{2}/|W(u_{1}=0,u_{2}=0)|^{2}$ of a soft circular aperture formed with twisted GSM beam for different values of the initial twist factor $\mu_{0}$. 
Figure 3 shows the normalized ghost image $\left| W(u_i = 0, u_j = 0) \right|^2 / \left| W(u_i = 0, u_j = 0) \right|^2$ of a soft circular aperture formed with twisted GSM beam for different values of the initial transverse coherence width $\sigma_{g0}$ with $\lambda = 632.8 \text{nm}$, $\sigma_{i0} = 10 \text{mm}$, $\mu_0 = 0$, $z = 200 \text{mm}$, $z_i = 20 \text{mm}$, $a_i = 0.1 \text{mm}$ and $N = 30$. One finds from Fig. 3 that the ghost image is closely related to transverse coherence width $\sigma_{g0}$. When the value of $\sigma_{g0}$ is very small (see Fig. 3(a)), a perfect ghost image of the object is formed. As the value of $\sigma_{g0}$ increases, the image of the object gradually disappears (see Fig. 3(b)-(d)). Our results are consistent with those reported for usual GSM beam in [40, 43, 52]. This phenomenon can be explained by the fact that as the initial transverse coherence increases, the intensity fluctuation correlation between two points or two detectors after propagation also decreases, which leads to the disappearance of the ghost image. Figure 4 shows the normalized ghost image $\left| W(u_i = 0, u_j) \right|^2 / \left| W(u_i = 0, u_j = 0) \right|^2$ of a soft circular aperture formed with a twisted GSM beam for different values of the initial twist factor $\mu_0$ with fixed $\sigma_{g0} = 0.0001 \text{mm}$, at distance $z = 200 \text{mm}$ from the source. It is clear from Fig. 4 that the twist phase has strong influence on the ghost image. A good quality of the image of the object is formed for small values of the twist factor (see Fig. 4(a) and (b)). As the absolute value of the twist factor increases gradually, the ghost image of the object disappears gradually, although the initial transverse coherence length is chosen to be very small. Figure 5 shows the normalized ghost image $\left| W(u_i = 0, u_j) \right|^2 / \left| W(u_i = 0, u_j = 0) \right|^2$ of a soft circular aperture formed with the twisted GSM beam for different values of $z$ with $\sigma_{g0} = 0.0001 \text{mm}$. The other parameters for calculation of Figs. 4 and 5 are the same as in Fig. 2. We can find from Fig. 5 that the ghost image is also closely related to $z$. In our case, when the distance $z < 500 \text{mm}$, a ghost image with good quality can be obtained (see Fig. 5(a) and (b)). As the distance $z$ increases, the image of the object disappears gradually (see Fig. 5(c) and (d)).
In order to fully understand the effect of the initial twist phase and of the distance \( z \) on ghost image we will now analyze their dependence on the state of coherence of the twisted GSM beam on propagation. Applying Eqs. (5) - (7), the cross-spectral density \( W \) of the twisted GSM beam after propagation at distance \( z \) in free space can be expressed in the form

\[
W(\hat{u}) = \frac{1}{\text{Det}(I + \hat{B}M^{-1})} \exp \left( -\frac{ik}{2} \hat{u} \hat{M}^{-1} \hat{u} \right),
\]

where \( M^{-1} \) is the partially coherent complex curvature tensor at distance \( z \)

\[
M^{-1} = (M_0 + \hat{B})^{-1} = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix},
\]

with \( \hat{B} \) is given by Eq. (14). Comparing Eqs. (3), (4) and (26), we obtain, after some computations, the expression for the transverse coherence length \( \sigma_{gc} \) on propagation of the form

\[
\sigma_{gc} = \left( \frac{i}{km_{11}} \right)^{1/2} = \left( \frac{i}{km_{22}} \right)^{1/2} = \left( \frac{i}{km_{33}} \right)^{1/2} = \left( \frac{i}{km_{44}} \right)^{1/2} = \left( \frac{\lambda^2 z^2}{4\pi^2 \sigma_{g0}^2} + \sigma_{g0}^2 + \sigma_{g0}^2 \mu_0 z^2 + \frac{\lambda^2 z^2 \sigma_{g0}^2}{16\pi^2 \sigma_{g0}^2} \right)^{1/2}.
\]

It is now clearly seen that the transverse coherence width \( \sigma_{gc} \) of the twisted GSM beam on propagation is closely related to propagation distance \( z \) and twist factor \( \mu_0 \). Of course, parameter \( \sigma_{gc} \) reduces to its source value \( \sigma_{g0} \) when \( z = 0 \). Figure 6 shows the dependence of the transverse coherence width \( \sigma_{gc} \) of the twisted GSM beam on propagation distance \( z \) for different values of the initial twist factor \( \mu_0 \) with \( \lambda = 632.8 \text{nm} \) and \( \sigma_{g0} = 10 \text{mm} \). It is clear from Fig. 6 that the transverse coherence length \( \sigma_{gc} \) of the twisted GSM beam increases practically linearly with the increase of the propagation distance (note, however, that in Eq. (28) this dependence is not strictly linear) and it does so more rapidly as the absolute value of the initial twist factor \( \mu_0 \) increases.

Fig. 6. Dependence of the transverse coherence length \( \sigma_{gc} \) of the twisted GSM beam on the propagation distance \( z \) for different values of the initial twist factor \( \mu_0 \)
Now we can better understand the effect of the initial twist phase and of the distance $z$ on the ghost image: although the initial spatial coherence width ($\sigma_{x0}$) of the twisted GSM beam is independent of the initial twist phase, the spatial coherence width ($\sigma_{xz}$) of the twisted GSM beam on propagation depends on the initial twist phase. Moreover, since the spatial coherence width of the twisted GSM beam always increases on propagation, doing so more rapidly with increasing values of the initial twist factor, the intensity fluctuation correlation between two detectors must decrease, forcing the ghost image to disappear. Such a conclusion is numerically demonstrated by Figs. 4 and 5.

At this point we stress on the fact that in the preceding numerical calculations, we have not taken the term $<I(u_1 = 0)><I(u_2)>$ into consideration. Experimentally, one can measure both $G^{(2)}(u_1 = 0,u_2)$ (i.e., $<I(u_1 = 0)I(u_2)>$) and $<I(u_1 = 0)><I(u_2)>$. Quantity $W(u_1 = 0,u_2)$ can be obtained by subtracting $<I(u_1 = 0)><I(u_2)>$ from $G^{(2)}(u_1 = 0,u_2)$.

If the product $<I(u_1 = 0)><I(u_2)>$ is not subtracted, then the twist phase will influence the visibility of the ghost image, which is defined by the formula

$$V = \frac{W(u_1 = 0,u_2)}{G^{(2)}(u_1 = 0,u_2)}$$ (29)

Thus we have shown that the visibility of the ghost image enhances with the increase of the initial transverse coherence width $\sigma_{x0}$ of the beam, which is in the agreement with results obtained previously in Refs. [40] and [43]. Moreover, the visibility of the ghost image also enhances as the value of the initial twist factor or of the distance $z$ increases, because the presence of the twist factor results in the increase in the transverse coherence width of a twisted GSM beam on propagation in free space.

5. Conclusion

In conclusion, we have investigated ghost imaging with a partially coherent twisted GSM beam with the help of a tensor method. We have found that the ghost image formed with a twisted GSM beam is also closely dependent on the twist factor, in addition to the dependence on the imaging distance and on the initial coherence length. The image disappears gradually as either the absolute value of the twist factor or the imaging distance increases, but the visibility of the image enhances as the values of these parameters increase. This is caused by the fact that the transverse coherence width of a twisted GSM beam grows on propagation in free space, and that the non-zero initial twist phase stimulates its increase on propagation. Thus, in any practical situations involving the use of twisted GSM beams in ghost imaging it is necessary to take both the twist phase and the imaging distance into consideration, in addition to the initial transverse coherence width.

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