11-1-2009

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Chengliang Zhao

Yangjian Cai

Olga Korotkova

University of Miami, o.korotkova@miami.edu

Recommended Citation

Zhao, Chengliang; Cai, Yangjian; and Korotkova, Olga, "Radiation Force of Scalar and Electromagnetic Twisted Gaussian Schell-Model Beams" (2009). Physics Articles and Papers. 8.
http://scholarlyrepository.miami.edu/physics_articles/8

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Radiation force of scalar and electromagnetic twisted Gaussian Schell-model beams

Chengliang Zhao¹, Yangjian Cai¹*, and Olga Korotkova²

¹School of Physical Science and Technology, Soochow University, Suzhou 215006, China
²Department of Physics, University of Miami, Coral Gables, Florida 33146, USA
*yangjian_cai@yahoo.com.cn

Abstract: Radiation force of a focused scalar twisted Gaussian Schell-model (TGSM) beam on a Rayleigh dielectric sphere is investigated. It is found that the twist phase affects the radiation force and by raising the absolute value of the twist factor it is possible to increase both transverse and longitudinal trapping ranges at the real focus where the maximum on-axis intensity is located. Numerical calculations of radiation forces induced by a focused electromagnetic TGSM beam on a Rayleigh dielectric sphere are carried out. It is found that radiation force is closely related to the twist phase, degree of polarization and correlation factors of the initial beam. The trapping stability is also discussed.

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OCIS codes: (030.1640) Coherence; (030.1679) Coherence optical effects; (260.5430) Polarization; (140.7010) Laser trapping.

References and links

1. Introduction

A Gaussian Schell-model (GSM) beam is a stochastic beam whose spectral degree of coherence and the intensity distribution are Gaussian functions [1–6]. Beams of this class can be generated by scattering a coherent laser beam with a rotating grounded glass and then by transforming the intensity distribution of the scattered light into profile with a Gaussian amplitude [2–4]. GSM beams can also be generated with the help of specially synthesized rough surfaces, spatial light modulators and synthetic acousto-optic holograms [5]. They have found wide applications in imaging [6, 7], free space optical communications [8], nonlinear optics [9], etc [10]. As an extension of a conventional GSM beam, Simon and Mukunda introduced a twisted GSM beam [11], whose phase, called twist phase, differs in many respects from the ordinary quadratic phase factor [11–15]. The twist phase is bounded in strength due to the positive semi-definiteness requirement on the cross-spectral density function and disappears in the limit of a coherent Gaussian beam. It has intrinsically a two-dimensional spatial dependence, i.e. it cannot be separated into a sum of one-dimensional contributions. The twist phase rotates the beam spot on propagation due to its intrinsic chiral (handedness) property and increases the beam divergence on propagation. The first experimental observation of a twisted GSM beam was reported by Friberg et al. [15]. Their realization by superposition, the coherent-mode decomposition, the orbital angular momentum and their propagation effects have been later extensively studied [16–30]. The conventional method for treating the propagation of GSM beam and twisted GSM beam is Wigner distribution function [22, 31]. Lin and Cai have recently introduced a convenient alternative tensor method for treating propagation of GSM and twisted GSM beams [24]. The tensor method has proved to be reliable for studying the passage of such beams through paraxial optical systems, fractional Fourier transform optical systems, dispersive media and apertured optical systems [4, 25–28]. More recently, Cai and associates applied the tensor method for the analysis of the effects of the twist phase on the second harmonic generation and on the ghost imaging [29, 30].

Polarization state is another important property of the beam arising from its vectorial nature. Conventionally, coherence and polarization states of light were studied separately [10,32], until James found, in 1994, that the degree of polarization of a stochastic electromagnetic beam may change on propagation in vacuum, such changes being dependent on the coherence properties of the source of the beam [33]. Since then such correlation-induced polarization modification of stochastic electromagnetic beams have been explored in depth [34–46] in general and in particular for the important class of beams called...
electromagnetic Gaussian Schell-model [EGSM] beams, introduced by Gori et al. [36]. Cai et al. later applied the tensor method for the analysis of the evolution of the EGSM beams in resonators and in radar systems operating through the turbulent atmosphere [47–49]. More recently, Cai and Korotkova introduced twisted EGSM beams, and studied their propagation in free space [50].

The first optical trapping experiment dates back to early 1970s when Ashkin used radiation pressure of a laser beam for trapping micro-sized particles [51]. Since then, numerous theoretical and experimental papers have been published on manipulation of particles, and the radiation force has been extensively studied for various laser beams [52–64]. It has been found that coherence, polarization and spatial profile of the laser beam all affect the radiation force. However, to our knowledge, the effects of coherence and polarization of the beams on the radiation force were studied separately [59–62], while the effect of twist phase has not been explored so far. Since recent investigations have shown that coherence, polarization and twist phase are very intimately related [33–50, 63, 64] we find it quite interesting to explore the effects of source coherence, polarization and twist phase on the radiation force simultaneously. This paper is aimed to investigate the radiation force produced by focused scalar and electromagnetic twisted GSM beams on a Rayleigh particle.

2. Focusing properties of scalar and electromagnetic twisted Gaussian Schell-model beams

The second-order statistical properties of a scalar partially coherent beam in the source plane $z = 0$ can be characterized by the cross-spectral density (CSD) function $W(\mathbf{r}_1, \mathbf{r}_2; 0)$ with $\mathbf{r}_1$ and $\mathbf{r}_2$ being two arbitrary position vectors in that plane [10]. Using the tensor method the CSD of a scalar twisted GSM beam in the plane $z = 0$ can be expressed in the form [11, 24, 30]

$$W(\mathbf{r}_1, \mathbf{r}_2; 0) = W(\hat{\mathbf{r}}; 0) = G_o \exp\left(-\frac{ik}{2} \hat{\mathbf{r}}^T \overline{\mathbf{M}}^{-1}_o \hat{\mathbf{r}}\right),$$

(1)

where $G_o = Q / 2\pi \sigma^2$ is a normalization factor. Here we have assumed the power of the scalar twisted GSM beam to be $Q$, $k = 2\pi / \lambda$ is the wave number with $\lambda$ being the wavelength of light field. $\hat{\mathbf{r}}^T = (x_1, y_1, x_2, y_2)^T$, where $T$ standing for matrix transpose. Further, $\overline{\mathbf{M}}^{-1}_o$ is a $4 \times 4$ matrix, called partially coherent complex curvature tensor, of the form

$$\overline{\mathbf{M}}^{-1}_o = \begin{pmatrix} \frac{-i}{2k\sigma^2} & \frac{i}{\kappa \delta^2} & \frac{i}{k \delta^2} & 1 + \mu_o \overline{\mathbf{J}}^T \overline{\mathbf{M}}^{-1}_o \overline{\mathbf{J}} \end{pmatrix},$$

(2)

In Eq. (2) $\mathbf{I}$ being a $2 \times 2$ unit matrix, $\sigma$ and $\delta$ denote the transverse beam width and spectral coherence width, respectively, $\mu_o$ is a scalar real-valued twist factor with the dimension of an inverse distance, limited by the double inequality $0 \leq \mu_o^2 \leq \left(\frac{k^2 \delta^2}{\sigma^2}\right)^{-1}$ due to the non-negativity requirement of Eq. (1). This inequality implies that in the coherent limit, i.e. as $\delta \rightarrow \infty$, the twist factor $\mu_o$ disappears. $\overline{\mathbf{J}}$ is an anti-symmetric matrix, viz.,

$$\overline{\mathbf{J}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$
Under condition $\mu_0 = 0$ the cross-spectral density in Eq. (1) reduces to the one of the
conventional GSM beam [1–10]. The spectral density (intensity distribution) of a scalar
twisted GSM beam in the source plane $z = 0$ is given by relation $I(r;0) = W(r,r;0)$. Although
the intensity of a scalar GSM beam for $z = 0$ is independent of the twist phase, its
propagation and focusing properties are closely related to the twist phase as shown later.

After propagation through a general astigmatic ABCD optical system, the cross-spectral
density of the scalar twisted GSM beam in the output plane is expressed as follows [24]

$$W(\vec{u};z) = \frac{G_0}{\det(\hat{A} + \hat{B}M_{0}^{-1})^{1/2}} \exp\left( -\frac{ik}{2} \vec{u}^T M_1^{-1} \vec{u} \right),$$

where $\det$ stands for the determinant of a matrix, $\vec{u}^T = (u_x^T u_y^T)$ with $u_x$ and $u_y$ being two
arbitrary position vectors in the output plane transverse to direction of propagation, $M_1^{-1}$ is
the partially coherent complex curvature tensor in the output plane, related with $M_0^{-1}$ via the
tensor ABCD law

$$M_1^{-1} = (\hat{C} + \hat{D}M_0^{-1})(\hat{A} + \hat{B}M_0^{-1})^{-1},$$

where matrices $\hat{A}, \hat{B}, \hat{C}$ and $\hat{D}$ have the forms

$$\hat{A} = \begin{bmatrix} A & 0 \\ 0 & A^T \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} B & 0 \\ 0 & -B^T \end{bmatrix}, \quad \hat{C} = \begin{bmatrix} C & 0 \\ 0 & -C^T \end{bmatrix}, \quad \hat{D} = \begin{bmatrix} D & 0 \\ 0 & D^T \end{bmatrix},$$

with $A, B, C$ and $D$ being the $2 \times 2$ sub-matrices of the astigmatic optical system. Symbol $^{\ast \ast}$
in Eq. (6) denotes the complex conjugate. The spectral density (intensity) of a scalar twisted
GSM beam at the output plane is given by expression $I(u,z) = W(u,u;z)$.

The second-order statistical properties of a twisted EGSM beam can be characterized by
the $2 \times 2$ cross-spectral density matrix $W(r_x,r_y;0)$ specified at any two points with position
vectors $r_x$ and $r_y$ in the source plane with elements [33–50]

$$W_{\alpha\beta}(r_x,r_y;0) = W_{\alpha\beta}(r_x,0) = \langle E_\alpha^* (r_x;0) E_\beta (r_y;0) \rangle = A_\alpha A_\beta B_{\alpha\beta} \exp\left[ -\frac{ik}{2} r_x^T M_{\alpha\beta}^{-1} r_y \right], (\alpha = x, y; \beta = x, y)$$

where $A_\alpha$ is the square root of the spectral density of electric field component $E_\alpha$, $B_{\alpha\beta} = |B_{\alpha\beta}| \exp(i\phi)$ is the correlation coefficient between the $E_\alpha$ and $E_\beta$ field components, satisfying the relation $B_{\alpha\beta} = B_{\beta\alpha}^*$, $M_{\alpha\beta}^{-1}$ is a $4 \times 4$ matrix of the form [50]

$$M_{\alpha\beta}^{-1} = \begin{bmatrix}
\frac{1}{ik} & \left( \frac{1}{2\sigma_{\alpha\beta}^2} + \frac{1}{\delta_{\alpha\beta}^2} \right) \\
\frac{i}{k\delta_{\alpha\beta}^2} + \mu_{\alpha\beta} & \frac{1}{ik} + \mu_{\alpha\beta}
\end{bmatrix}$$

where $\sigma_{\alpha\beta}$ and $\delta_{\alpha\beta}$ denote the widths of the spectral density and correlation coefficient,
respectively. $\mu_{\alpha\beta}$ represents the twist factor and is limited by $\mu_{\alpha\beta}^2 \leq 1 / \left( k^2 \delta_{\alpha\beta}^4 \right)$ if $\alpha = \beta$ due to the non-negativity requirement of the cross-spectral density [Eq. (7)] [11,50].
$A, B, \sigma, \sigma', \sigma''$ and $\mu$ are independent of position but, in general, depend on the frequency. We note here that if off-diagonal elements are to be included in calculation the realizability condition of the source should relate the on- and off-diagonal twist factors and the coefficient $B$ (and, perhaps, some other source parameters). But these conditions are not known so far and, moreover, are not needed in this work since only the effect of the twist phases of diagonal matrix elements is important on the focusing and trapping ability of the beam, since the later depend only on the beam intensity.

The spectral density of a twisted EGSM beam at point $r$ of the source plane is defined by the expression $[33–50]$:

$$I(r;0) = \text{Tr}(r;0) = A_0 \exp \left[ -\frac{i k}{2} \hat{r}^T M_{0xx}^{-1} \hat{r} \right] + A_i \exp \left[ -\frac{i k}{2} \hat{r}^T M_{0xy}^{-1} \hat{r} \right]. \quad (9)$$

Assuming that $A_i = \eta A_0$, and that the total transmitted power of the twisted EGSM beam is $Q$, we find that

$$Q = \frac{A_i \sigma (1+\eta) (\sigma_{xx}^2 + \sigma_{yy}^2)}{2}. \quad (10)$$

The degree of polarization of an electromagnetic beam at point $r$ is defined by the expression $[33–50]$:

$$P_p(r;0) = \sqrt{1 - \frac{4 \text{Det}W(r,r;0)}{[\text{Tr}W(r,r;0)]^2}}, \quad (11)$$

where $\text{Tr}$ denotes the trace of the matrix.

For the conciseness of the analysis, in this paper we will only consider the twisted EGSM beams that are generated by sources whose cross-spectral density matrices are diagonal, i.e., of the form

$$\hat{W} (r_1, r_2; 0) = \left[ \begin{array}{cc} W_{xx}^{(0)} (r_1, r_2; 0) & 0 \\ 0 & W_{yy}^{(0)} (r_1, r_2; 0) \end{array} \right]. \quad (12)$$

Furthermore, we will assume that $\sigma_{xx} = \sigma_{yy}$, exploring the effect of difference between correlations $\delta_i$ and $\delta_j$ alone. For such restricted type of beams the degree of polarization across the source is given by the formula

$$P_p(r;0) = \frac{1 - \eta}{1 + \eta}. \quad (13)$$

By applying the tensor ABCD law for partially coherent beams [24], after paraxial propagation through a general astigmatic ABCD optical system, we can express the elements of the cross-spectral density matrix of a twisted EGSM beam as follows

$$W_{u_0} (u_1, u_2; z) = W_{u_0} (\tilde{u}; z) = A_0 A_B B \left[ \text{Det} \left( \tilde{A} + \tilde{B} M_{u_0}^{-1} \right) \right]^{-1/2} \exp \left[ -\frac{i k}{2} \tilde{u}^T M_{u_0}^{-1} \tilde{u} \right], \quad (14)$$

where $M_{u_0}^{-1}$ and $M_{u_0}^{-1}$ are related by the following tensor ABCD law [24]

$$M_{u_0}^{-1} = \left( \tilde{C} + \tilde{D} M_{u_0}^{-1} \right) \left( \tilde{A} + \tilde{B} M_{u_0}^{-1} \right)^{-1}. \quad (15)$$
\( \mathbf{A}, \mathbf{B}, \mathbf{C} \) and \( \mathbf{D} \) is given by Eq. (6). The spectral density of a twisted EGSM beam at the output plane is given by \( I(u; z) = \text{Tr} \tilde{W}(u; u; z) \). Equations (4)-(6), (14) and (15) can be used conveniently to study the paraxial propagation properties of scalar and electromagnetic twisted GSM beams through a general astigmatic ABCD optical system.

In the rest of this section we will study the focusing properties of scalar and electromagnetic twisted GSM beams passing through a thin lens. The power \( Q \) of all considered beams in this paper at the input plane is set to be 1W and the wavelength is set to be \( \lambda = 632.8 \text{nm} \). The schematic diagram of a focusing optical system is shown in Fig. 1, where a thin lens with focal length \( f \) is located at the input plane \( (z=0) \), and the output plane is located at \( z \). Then the transfer matrix between the input and output planes can be expressed as follows

\[
\begin{pmatrix}
A & B \\
C & D
\end{pmatrix}
= 
\begin{pmatrix}
I & zI \\
0I & I
\end{pmatrix}
\begin{pmatrix}
I & 0I \\
(-1/f)I & I
\end{pmatrix}
= 
\begin{pmatrix}
(1-z/f)I & zI \\
(-1/f)I & I
\end{pmatrix}
\] (16)

![Fig. 1. Schematic diagram of a focusing optical system](image)

Fig. 1. Schematic diagram of a focusing optical system

The inset shows scaled down intensity distribution for the case \( |\mu| \leq 1 \). (b) Focus shift \( f - z_{\text{max}} \) of a scalar twisted GSM beam behind the thin lens versus the normalized initial twist factor \( \mu / \gamma \).
Substituting from Eq. (16) into Eqs. (4) - (6), we calculate in Fig. 2(a) the intensity distribution of a scalar twisted GSM beam at the real focal plane for different absolute values of the initial twist factor normalized with \( \gamma = \left[ k^2 \delta^4 \right]^{1/2} = 0.1 \text{m}^{-1} \). Here the real focal plane is defined as the plane transverse to direction of propagation \( z \), where the maximum on-axis intensity is located. We calculate in Fig. 2(b) the focus shift \( f - z_{\text{max}} \) of a scalar twisted GSM beam behind the thin lens versus the normalized initial twist factor \( |\mu_0 / \gamma| \). \( z_{\text{max}} \) is the position of the real focal plane. We have chosen the other parameters to be \( \sigma = 10 \text{mm}, \delta = 1 \text{mm} \) and \( f = 5 \text{mm} \). One finds from Fig. 2(a) that the twist phase has strong influence on the focusing properties of a scalar twisted GSM beam. As the absolute value of the twist factor increases, the focused beam spot becomes larger and the maximum intensity decreases. Thus the twist phase is expected to affect the radiation force, mainly determined by the focused intensity, as will be shown later. As was shown in [15] (see Eq. (8)) as the absolute value of the twist factor increases, the effective degree of coherence decreases, the focused beam spot becomes larger and the maximum intensity decreases. Thus the twist phase is expected to affect the radiation forces (mainly determined by the focused intensity) induced by the focused beam as shown later.

It is important to note that the beam waist (minimum spot size and maximum intensity) is not located in the focal plane (i.e., at \( z = f \)), but rather closer to the lens due to the focus shift [see Fig. 2(b)]. The focus shift is dependent on the intensity and coherence widths, as well as on the twist parameter which decreases the effective coherence width (see, for instance, Ref. 6).

Substituting from Eq. (16) into Eqs. (14) and (15), we calculate in Fig. 3 the intensity distribution of a twisted EGSM beam at the real focal plane for different absolute values of the normalized initial twist factors \( |\mu_\sigma / \gamma_\sigma| \) and \( |\mu_\gamma / \gamma_\gamma| \) with
\[
\gamma_\sigma = \left[ k^2 \delta_\sigma^4 \right]^{1/2} = 0.1 \text{m}^{-1} \quad \text{and} \quad \gamma_\gamma = \left[ k^2 \delta_\gamma^4 \right]^{1/2} = 0.025 \text{m}^{-1}
\]
respectively.

Substituting from Eq. (16) into Eqs. (14) and (15), we calculate in Fig. 3 the intensity distribution of a twisted EGSM beam at the real focal plane for different absolute values of the normalized initial twist factors \( |\mu_\sigma / \gamma_\sigma| \) and \( |\mu_\gamma / \gamma_\gamma| \) with
\[
\sigma_\sigma = \sigma_\gamma = 10 \text{mm}, \quad \delta_\sigma = \delta_\gamma = 1 \text{mm}, \quad \delta_\gamma = 2 \text{mm}, \quad P_0 = 0.333, \quad \gamma_\sigma = \left[ k^2 \delta_\sigma^4 \right]^{1/2} = 0.1 \text{m}^{-1} \quad \text{and} \quad \gamma_\gamma = \left[ k^2 \delta_\gamma^4 \right]^{1/2} = 0.025 \text{m}^{-1}\). Figure 4 shows the intensity distribution of a twisted EGSM beam at the real focal plane for different values of the initial degree of polarization \( P_0 \) with \( \sigma_\sigma = \sigma_\gamma = 10 \text{mm}, \delta_\sigma = 1 \text{mm}, \delta_\gamma = 2 \text{mm} \). Figure 5 shows the intensity distribution of a twisted EGSM beam at the real focal plane for different values of the initial correlation

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Fig. 3. Intensity distribution of a twisted EGSM beam at the real focal plane for different absolute values of the initial twist factors \( \mu_\sigma \) and \( \mu_\gamma \) normalized with
\[
\gamma_\sigma = \left[ k^2 \delta_\sigma^4 \right]^{1/2} = 0.1 \text{m}^{-1} \quad \text{and} \quad \gamma_\gamma = \left[ k^2 \delta_\gamma^4 \right]^{1/2} = 0.025 \text{m}^{-1}
\]
coefficients $\delta_x$ and $\delta_y$ with $\sigma_x = \sigma_y = 10\,mm$. In calculation of Figs. 4 and 5, we have chosen the twist factors to be $\mu_x / \gamma_x = 0.04$, $\mu_y / \gamma_y = 0.1$ with $\gamma_x = \left[k^2 \delta_x^4 \right]^{1/2} = 0.1\,m^{-1}$ and $\gamma_y = \left[k^2 \delta_y^4 \right]^{1/2} = 0.025\,m^{-1}$. One finds from Figs. 2 and 3 that the influence of the twist factors on the focused intensities of scalar and electromagnetic twisted GSM beams is similar, i.e., the width of the focused beam spot increases with the increase of the absolute value of the twist factors. From Fig. 4, it is clear that the focused intensity is also determined by the initial degree of polarization of the electromagnetic twisted GSM beam. A partially polarized twisted EGSM beam whose cross-spectral density matrices only have on-diagonal elements can be focused more tightly than a completely polarized one. If the off-diagonal elements are included, we need to consider the state of polarization, then this conclusion may not always be valid, and we leave this for future study. From Fig. 5, we find that the initial correlation coefficients $\delta_x$ and $\delta_y$ also affect the focusing properties of a twisted EGSM beam, and the twisted EGSM beam with larger values of correlation coefficients can be focused more tightly.

**Fig. 4.** Intensity distribution of a twisted EGSM beam at the real focal plane for different values of the initial degree of polarization

**Fig. 5.** Intensity distribution of an electromagnetic twisted GSM beam at the real focal plane for different values of the initial correlation coefficients $\delta_x$ and $\delta_y$. In Fig. 5 The inset shows the scaled down intensity distribution for the case $\delta_x = 1\,mm$ and $\delta_y = 2\,mm$. 
3. Radiation force induced by focused scalar and electromagnetic twisted GSM beams on a Rayleigh particle

In this section we determine the magnitudes of radiation forces induced by focused scalar and electromagnetic twisted GSM beams on a Rayleigh dielectric sphere with radius $a$, $(a << \lambda)$ and refractive index $n_p$. The reader can refer to Fig. 1 where the schematic diagram is given for trapping a Rayleigh dielectric sphere placed near the focus of the beam.

The radiation force is a combination of the scattering force and the gradient force. The former force component, which is caused by scattering of light by the sphere, is proportional to the beam intensity and acts along its direction of propagation. It can be expressed as [54]

$$ F_{\text{Scat}}(\mathbf{r};z) = \hat{e}_z n_m \alpha I(\mathbf{r};z) / c, $$

where $I(\mathbf{r};z)$ is the intensity of the focused beam at the output plane, $\hat{e}_z$ is a unity vector along the direction of propagation, $\alpha = (8/3) \pi (ka)^4 a^2 [(\chi^2 - 1)/(\chi^2 + 2)]^2$. $\chi = n_p / n_m$ with $n_m$ being the refractive index of the ambient. In the following text, we choose $a = 50 \, \text{nm}$, $n_p = 1.59$ (glass) and $n_m = 1.33$ (water). The gradient force, produced by a non-uniform field, acts along the gradient of light intensity, and can be expressed as [54]

$$ F_{\text{Grad}}(\mathbf{r};z) = 2 \pi n_m \beta N I(\mathbf{r};z) / c, $$

where $\beta = a^4 (\chi^2 - 1)/(\chi^2 + 2)$.

Applying Eqs. (4)-(6) and (16)-(18), we calculate in Fig. 6(a) the scattering force (cross-section $y = 0$) at the real focal plane, in Fig. 6(b) the transverse gradient force (cross-section $y = 0$) at the real focal plane, and in Fig. 6(c) the longitudinal gradient force at $r = 0$ of a scalar twisted GSM beam for different absolute values of the initial twist factor $\mu_0$, with $\sigma_0 = 10 \, \text{mm}$, $\delta_0 = 1 \, \text{mm}$ and $f = 5 \, \text{nm}$. The sign of radiation forces determines the direction of the force: for positive $F_{\text{Scat}}$ the direction of the scattering force is along $+z$ direction; for positive $F_{\text{Grad}_z}$ and $F_{\text{Grad}_x}$ the direction of the gradient force is along $+x$ or $+z$ direction. One finds from Fig. 6 that the scattering force and the gradient force induced by a scalar partially coherent beam are closely related to the initial twist phase of the input beam. We note that the longitudinal gradient force is always much larger than the forward scattering force [compare Fig. 6(a) and 6(c)], which means the scattering force can be neglected.

Fig. 6. (a) Scattering force (cross-section $y=0$) at the real focal plane, (b) transverse gradient force (cross-section $y=0$) at the real focal plane, and (c) longitudinal gradient force at $r = 0$ of a scalar twisted GSM beam for different absolute values of the initial twist factor $\mu_0$ normalized with $\gamma = [\sigma^2 \delta^2]^{1/2} = 0.1 \, \text{m}^{-1}$. The insets in Fig. 6(a) and 6(b) show scaled down radiation forces for the case $1 \mu_0 / \gamma = 1$. The inset in Fig. 6(c) shows the zoomed region where the crossing of the radiation forces with zero occurs.
Figure 6 can also be of use for analyzing trapping ranges and trapping stability. Trapping range along a certain direction is the distance from the equilibrium position to the position at which trapping starts to occur. By trapping stability along a certain direction we mean the difference between the magnitudes of the force at the real focal plane and at the boundary of the trapping region. Since the absolute values of the scattering force and of the gradient force generally decrease as the absolute value of the twist factor increases, the trapping stability of the beam is deteriorated. From Fig. 6(b) and 6(c), we see that one stable equilibrium point exists at the real focus which is not located at \( z = f \) (i.e., geometrical focus), which implies that we can use focused scalar twisted GSM beam to trap a Rayleigh particle whose refractive index is larger than the ambient at the real focus. Furthermore, with increase of the absolute value of the twist factor, although the trapping stability becomes worse, as shown by Fig. 6(b) and 6(c), both transverse trapping range and longitudinal trapping range becomes larger (i.e., the positions of peak values deviate away from the focus). The scattering and gradient forces decrease and the trapping range increases as the twist parameter increases follow directly from the property that increased twist parameter implies reduced effective spatial coherence of the beam and therefore a more broadly distributed focal spot. Tables 1 and 2 show the trapping ranges corresponding to Fig. 6(b) and 6(c) for different absolute values of \( \mu_0 \), from which we find that both transverse and longitudinal trapping ranges increase remarkably as the absolute values of \( \mu_0 \) increases. We should point out that the Rayleigh particle will be diffused instead of being trapped when the absolute value of \( \mu_0 \) is very large because the radiation forces becomes comparable, in this regime, to the Brownian force, as will be shown later. Thus for particle trapping the absolute values of \( \mu_0 \) must be limited from above.

| Table 1. The trapping ranges from Fig. 6(b) for different absolute values of \( \mu_0 \) |
|-----------------|---------|
| \( |\mu_0/\gamma| = 1 \) | 202.4 \( a \) |
| \( |\mu_0/\gamma| = 0.2 \) | 45.2 \( a \) |
| \( |\mu_0/\gamma| = 0.1 \) | 28.4 \( a \) |
| \( |\mu_0/\gamma| = 0.05 \) | 22.4 \( a \) |
| \( |\mu_0/\gamma| = 0 \) | 20 \( a \) |
Table 2. The trapping ranges from Fig. 6(c) for different absolute values of $\mu_0$.

| $|\mu_0/\gamma|$ | Trapping Range D |
|-----------------|------------------|
| 1               | 58.4 $a$         |
| 0.2             | 13.2 $a$         |
| 0.1             | 8.4 $a$          |
| 0.05            | 6.4 $a$          |
| 0               | 6 $a$            |

Fig. 7. (a) the scattering force (cross-section $y=0$) at the real focal plane, in Fig. 7(b) the transverse gradient force (cross-section $y=0$) at the real focal plane, and in Fig. 7(c) the longitudinal gradient force at $r = 0$ of a scalar twisted GSM beam for different values of $\chi = n_p / n_m$ with $n_m = 1.33$ , $|\mu_0/\gamma| = 0.05$ and $\gamma = \left(k^2\sigma^4\right)^{-1/2} = 0.1 \text{m}^{-1}$ . The inset in Fig. 7(c) shows the zoomed region where the crossing of the radiation forces with zero occurs.

To learn about the dependence of the radiation forces on the refractive index of the particle, we calculate in Fig. 7(a) the scattering force (cross-section $y=0$) at the real focal plane, in Fig. 7(b) the transverse gradient force (cross-section $y=0$) at the real focal plane, and in Fig. 7(c) the longitudinal gradient force at $r = 0$ of a scalar twisted GSM beam for different values of $\chi = n_p / n_m$ with $n_m = 1.33$ , $|\mu_0/\gamma| = 0.05$ and $\gamma = \left(k^2\sigma^4\right)^{-1/2} = 0.1 \text{m}^{-1}$ .

One finds from Fig. 7 that the radiation forces are closely related to the refractive index of the particle. When $\chi > 1$, one stable equilibrium point always exists at the real focus, so we can use the highly focused scalar twisted GSM beam to trap the particle with the refractive index larger than the ambient stably. In the case of $\chi < 1$, it is clear from Fig. 7(b) and 7(c), there is no stable equilibrium points at the real focus, thus we can’t use the highly focused scalar twisted GSM beam to trap the particle with the refractive index smaller than the ambient stably. In order to trap a particle with the refractive index smaller than the ambient stably, we can use doughnut beams (such as Laguerre-Gaussian modes, Bessel Gaussian beam and dark hollow beams) to trap the particle [65].
Fig. 8. (a) Scattering force (cross-section y = 0) at the real focal plane, (b) transverse gradient force (cross-section y=0) at the real focal plane, and (c) longitudinal gradient force at r = 0 of an electromagnetic twisted GSM beam for different absolute values of the initial twist factors $\mu_x$ and $\mu_y$ normalized with $\gamma_x = \left( k \delta_x^2 \right)^{1/2} = 0.1 \text{m}^{-1}$ and $\gamma_y = \left( k \delta_y^2 \right)^{1/2} = 0.025 \text{m}^{-1}$, respectively. The inset in Fig. 8(c) shows the zoomed region where the crossing of the radiation forces with zero occurs.

Fig. 9. (a) Scattering force (cross-section y=0) at the real focal plane, (b) transverse gradient force (cross-section y=0) at the real focal plane, and (c) longitudinal gradient force at r = 0 of an electromagnetic twisted GSM beam for different values of the initial degree of polarization. In Fig. 9(c) the inset shows the zoomed region where the crossing of the radiation forces with zero occurs.

Fig. 10. (a) Scattering force (cross-section y=0) at the real focal plane, (b) transverse gradient force (cross-section y=0) at the real focal plane, and (c) longitudinal gradient force at r = 0 of an electromagnetic twisted GSM beam for different values of the initial correlation coefficients $\rho_{xx}$ and $\rho_{yy}$. In Fig. 10(c), the inset shows the zoomed region where the crossing of the radiation forces with zero occurs.

Applying Eqs. (14)-(18), we calculate in Fig. 8(a) the scattering force (cross-section y=0) at real focal plane, in Fig. 8(b) the transverse gradient force (cross-section y=0) at real focal plane, and in Fig. 8(c) the longitudinal gradient force at r = 0 of a twisted EGSM beam for different absolute values of the initial twist factors $\mu_x$ and $\mu_y$ normalized with $\gamma_x = \left( k \delta_x^2 \right)^{1/2} = 0.1 \text{m}^{-1}$ and $\gamma_y = \left( k \delta_y^2 \right)^{1/2} = 0.025 \text{m}^{-1}$, respectively. The inset in Fig. 8(c) shows the zoomed region where the crossing of the radiation forces with zero occurs.
different absolute values of the initial twist factors $\mu_x$ and $\mu_y$ with $\sigma_{xx} = \sigma_{yy} = 10\text{mm}$, $\delta_{xx} = 1\text{mm}$, $\delta_{yy} = 2\text{mm}$ and $P_0 = 0.333$ and $f = 5\text{mm}$. By comparing Figs. 6 and 8, we come to the conclusion that the effect of the twist phase on the radiation force induced by a twisted EGSM beam is similar to that induced by a scalar twisted GSM beam, i.e., the radiation force decreases with the increase of the twist factors but both transverse and longitudinal trapping increases. We calculate in Fig. 9(a) the scattering force (cross-section $y=0$) at real focal plane, in Fig. 9(b) the transverse gradient force (cross-section $y=0$) at real focal plane, and in Fig. 9(c) the longitudinal gradient force at $r = 0$ of a twisted EGSM beam for different values of the initial degree of polarization with $\sigma_{xx} = \sigma_{yy} = 5\text{mm}$, $\delta_{xx} = 1\text{mm}$ and $\delta_{yy} = 2\text{mm}$. Further, we calculate in Fig. 10(a) the scattering force (cross-section $y=0$) at real focal plane, in Fig. 10(b) the transverse gradient force (cross-section $y=0$) at real focal plane, and in Fig. 10(c) the longitudinal gradient force at $r = 0$ of an electromagnetic twisted GSM beam for different values of the initial correlation coefficients $\delta_{xx}$ and $\delta_{yy}$ with $\sigma_{xx} = \sigma_{yy} = 5\text{mm}$. In calculation of Figs. 9 and 10, we have chosen the twist factors to be $\mu_x / \gamma_x = 0.04$, $\mu_y / \gamma_y = 0.1$ with $\gamma_x = \left[ k^2 \delta_{xx}^4 \right]^{1/2} = 0.1 \text{m}^{-1}$ and $\gamma_y = \left[ k^2 \delta_{yy}^4 \right]^{1/2} = 0.025 \text{m}^{-1}$. From Figs. 9 and 10 one finds that the radiation force induced by a twisted EGSM beam is also closely determined by its initial degree of coherence and correlation coefficients. As the values of the initial degree of polarization or correlation coefficients decrease, the radiation force decreases (i.e., the trapping stability becomes worse), while the positions of peak values of the gradient forces deviate away from the focus (i.e., trapping ranges become larger). From above discussions, we come to the conclusion that we can control the trapping stability and the trapping ranges by choosing suitable values of the twist phase, degree of polarization and correlation coefficients of the partially coherent beam at the input plane.

4. Analysis of the trapping stability

Fig. 11. Dependence of the radiation forces $F_{\text{Max}}$, $F_{\text{Max}}$, and $F_{\text{Max}}$ induced by a scalar twisted GSM beam on the absolute value of the normalized initial twist factor $| \mu_x / \gamma_x |$ at the real focal plane. $F_{\text{B}}$ is the Brownian force.

Now we analyze the trapping stability in greater detail by taking into consideration the Brownian motion of the trapped particle. The magnitude of the Brownian force is expressed...
as \( |F_b| = (12 \pi \kappa \alpha a T)^{1/2} \) according to the fluctuation-dissipation theorem of Einstein [66]. Here \( \kappa \) is the viscosity of the ambient (in our case, for water [67], \( \kappa = 7.977 \times 10^{-4} \) Pas at \( T = 300K \)), \( \alpha \) is the radius of the particle and \( k_B \) is the Boltzmann constant. Thus, the magnitude of the Brownian force becomes \( F_b = 2.5 \times 10^{-3} \) pN. Comparing this value with the values of the scattering and the gradient components of the radiation force in Fig. 6 we can find that both components of the radiation force are much larger than the Brownian force, provided the absolute value of the twist factor is small. We calculate in Fig. 11 the dependence of the radiation forces \( F_{\text{Max,Scat}} \), \( F_{\text{Max,Grad-x}} \) and \( F_{\text{Max,Grad-z}} \) induced by a scalar twisted GSM beam on the absolute value of the twist factor \( \mu_0 \) at the real focal plane. For comparison, Brownian force \( F_b \) is also shown in Fig. 11. From Fig. 11, one finds that both scattering force and gradient force decrease as \( |\mu_0| \) increases, which is consistent with Fig. 6. When \( |\mu_0| \) is very large, then the scalar twisted GSM beam may no longer be used for trapping a Rayleigh particle because the radiation forces becomes smaller than the Brownian force. The line Q in Fig. 11 can be regarded as the critical line. Our numerical results show that the dependence of the radiation forces of a twisted EGSM beam on the twist factors is similar to that of a scalar twisted GSM beam (not present here to save space). We illustrate in Fig. 12(a) the dependence of the radiation forces \( F_{\text{Max,Scat}} \), \( F_{\text{Max,Grad-x}} \) and \( F_{\text{Max,Grad-z}} \) on the initial degree of polarization at the real focal plane with \( \sigma_{xx} = \sigma_{yy} = 5 \text{mm}, \delta_{xx} = 1 \text{mm}, \delta_{yy} = 2.5 \text{mm} \), and in Fig. 12(b) the dependence of the radiation forces \( F_{\text{Max,Scat}} \), \( F_{\text{Max,Grad-x}} \) and \( F_{\text{Max,Grad-z}} \) on the initial correlation coefficients with \( \eta = \delta_{xx} / \sigma_{xx} \), \( \delta_{yy} = 2.5 \delta_{xx}, \sigma_{xx} = \sigma_{yy} = 5 \text{mm} \). In calculation of Figs. 12(a) and 12(b), we have chosen the twist factors to be \( \mu_{xx} / \gamma_{xx} = 0.04 \), \( \mu_{yy} / \gamma_{yy} = 0.1 \) with \( \gamma_{xx} = \left[ k^2 \delta_{xx}^4 \right]^{-1/2} = 0.1 \text{m}^{-1} \) and \( \gamma_{yy} = \left[ k^2 \delta_{yy}^4 \right]^{-1/2} = 0.025 \text{m}^{-1} \). One finds from Fig. 12(a) that although the radiation force decreases as the degree of polarization increases, it remains larger than the Brownian force. Hence, by tuning the degree of polarization of the input partially coherent beam it is possible to control particle trapping. From Fig. 12(b), we see that the radiation force decreases as the values of the correlation coefficients decrease. If the correlation coefficients are smaller than certain values, the radiation force becomes smaller than the Brownian force, and the particle cannot be trapped. The line Q in Fig. 12(b) also represents critical line. From above discussion, we conclude that it is necessary to choose suitable values of the twist phase, degree of polarization and correlation coefficients of a partially coherent beam for particle trapping.
Fig. 12. (a) Dependence of the radiation forces $F_{\text{Max}}^{\text{Max}}$, $F_{\text{Max}}^{\text{Grad-x}}$, and $F_{\text{Max}}^{\text{Grad-z}}$ induced by an electromagnetic twisted GSM beam on the initial degree of polarization at $z_1 = 0$, (b) dependencies of the radiation forces $F_{\text{Max}}^{\text{Max}}$, $F_{\text{Max}}^{\text{Grad-x}}$, and $F_{\text{Max}}^{\text{Grad-z}}$ on the initial correlation coefficients with $\xi = \delta_\omega / \sigma_\omega$ and $\delta_\omega = 2.5\delta_\omega$. In Fig. 12 (b) the inset shows the zoomed region where the crossing of the three radiation forces with the Brownian force occurs.

5. Conclusion

In conclusion, we have studied the focusing properties of scalar and electromagnetic twisted GSM beams, and the radiation force induced by such beams on a Rayleigh dielectric sphere. Our results have shown that the induced radiation force is closely related to the initial twist phase, degree of polarization and correlation coefficients of the beam. In particular we found that the transverse and longitudinal trapping ranges can be increased at the real focus by increasing the values of the twist factor, degree of polarization or by decreasing the values of correlation coefficients. We have also found that the trapping stability decreases as the trapping ranges increases. Thus it is necessary to choose suitable twist phase, degree of polarization and correlation coefficients of the initial partially coherent beam in order to trap a particle stably.

Acknowledgements

Yangjian Cai acknowledges the support by the National Natural Science Foundation of China under Grant No. 10904102, the Foundation for the Author of National Excellent Doctoral Dissertation of PR China under Grant No. 200928 and the Natural Science of Jiangsu Province under Grant No. BK2009114. O. Korotkova’s research is funded by the AFOSR (grant FA 95500810102).