Partially Coherent Standard and Elegant Laguerre-Gaussian Beams of All Orders

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Partially coherent standard and elegant Laguerre-Gaussian beams of all orders

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Abstract: Partially coherent standard and elegant Laguerre-Gaussian (LG) beams of all orders are introduced as a natural extension of coherent standard and elegant LG beams to the stochastic domain. By expanding the LG modes into a finite sum of Hermite-Gaussian modes, the analytical formulae are obtained for the cross-spectral densities of partially coherent standard and elegant LG beams in the source plane and after passing through paraxial ABCD optical system, based on the generalized Collins integral formula. A comparative study of the propagation properties of the partially coherent standard and elegant LG beams in free space is carried out via a set of numerical examples. Our results indicate that the intensity and spreading properties of partially coherent standard and elegant LG beams are closely related to their initial coherence states, and are very different from the corresponding results for the coherent standard and elegant LG beams. In particular, an elegant LG beam spreads slower than a standard LG beam, while this advantage disappears when their initial coherences are very small. Our results may find applications in connection with laser beam shaping, singular optics and astrophysical measurements of angular momentum of radiation.

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References and links
1. Introduction

Standard Laguerre-Gaussian (LG) beams are commonly encountered in laser optics, material processing and atomic optics [1]. Such beams were paid enormous attention during the past decades, and various aspects relating to their generation, propagation and applications were profoundly treated in the literature [2–23]. A standard LG beam can be generated by spatial light modulators [2], mode selection [3], cooperative frequency locking in a helium–neon laser [4,5], in fiber-coupled laser diode end-pumped lasers [6,7], through the conversion of Hermite-Gaussian beams by an astigmatic mode converter [8], or through the computer hologram [9]. Standard LG beams have found wide applications in free space optical communications [10,11], atom trapping [12], atom guiding [13], atom interferometers [14],
electron acceleration [15] and nanoparticles trapping [16]. Paraxial and non-paraxial propagation properties of standard LG beams have been studied in details [17–25]. Standard Hermite-Gaussian (HG) and LG modes are eigenmodes of the paraxial wave equation. The Gaussian part of the standard HG or LG modes has a complex argument, but the Hermite or Laguerre part is purely real. Siegman was the first to introduce new HG solutions named elegant HG modes that satisfy the paraxial wave equation but have a more symmetrical form [26]. Later, Takenaka et al. proposed the elegant LG beam as an extension of standard LG beam [27–29]. Relationship between elegant LG and Bessel-Gauss beams was studied in [30]. Elegant LG beams were also proposed as a tool for describing axisymmetric flattened Gaussian beams [31]. Paraxial and non-paraxial propagation of elegant LG beams have been carried out in [32–37].

Coherence is one important property of a laser beam. Laser beams with low coherence named partially coherent beams have advantages over coherent beam in many applications, such as free-space optical communication [38], inertial confinement fusion [39], harmonic generation [40,41], optical trapping [42], optical projection [43], photography [44], optical imaging [45–47].

In the past decades, most research on partially coherent beams is devoted to Gaussian Schell-model beams whose intensity distribution and degree of coherence are Gaussian functions. Recently, more and more attention is being paid to Schell-model beams with special profiles [42,48–63], such as partially coherent Hermite-Gaussian beams, partially coherent dark hollow beams, partially coherent flat-topped beams, partially coherent vortex beams, partially coherent cosh-Gaussian, cos-Gaussian and cosine-Gaussian beams, partially coherent Laguerre-Gaussian (0,1) beams, due to their important potential applications in various fields, including free-space optical communication, optical trapping and singular optics.

Several attempts were previously made in the literature to extend the coherent LG beams to stochastic domain. Ponamorenko [58] used the LG modes for construction, by means of the coherent mode decomposition [59], of the partially coherent fields carrying separable vortexes. This has led to the class of stochastic fields with the degree of coherence in a form of a modified Bessel function. Later this class of beams was further generalized to dark and anti-dark beams with a degree of coherence being a linear combination of Bessel and modified Bessel beams [60].

An alternative way of obtaining a class of partially coherent beams on the basis of the LG modes is to assume that the cross-spectral density function has the Gaussian Schell-model form [59], in which the intensity part is based on the LG modes but the degree of coherence can be chosen simply Gaussian. To our knowledge, only few papers were dedicated to beams belonging to this class, e.g [61–63]. These studies, however, were aimed at investigating issues like dynamics of optical singularities and of radial polarization and discussed only particular representatives of this rich beam class. No studies were performed so far on partially coherent elegant LG beams.

To our knowledge, up to now, partially coherent standard or elegant LG beam of all orders has not been formulated. Thus it is of practical importance to formulate partially standard or elegant LG beam of all orders and study its propagation properties. Partially coherent vortex beams [61,62] and partially coherent LG(0,1) beams [63] can be regarded as special cases of partially coherent LG beams. Recent research has shown that partially coherent beams with special profiles have advantages over corresponding coherent beams for reducing turbulence-induced intensity fading in laser communication systems [50,52,55–57,62], and for optical trapping [42]. We expect to apply partially coherent standard or elegant LG beam of all orders to free-space optical communication and optical trapping.

The goal of this paper is to establish the unified theoretical model for both standard and elegant stochastic LG beams of all orders (with LG intensity distribution and Gaussian degree of coherence). The paper is organized as follows. We will first derive the formulas for the cross-spectral density function of the new class of beams in the source plane and in a transverse plane after passing through a linear optical ABCD system (Section 2). We will then
carefully study, via numerous examples, the free-space propagation of standard and elegant partially coherent LG beams, which is just a particular case of the general passage of a beam through the ABCD system (Section 3). Finally, the summary of our results will be given (Section 4).

2. Theory

Let us begin by recalling that the electric field of a standard or elegant LG beam in the plane of the source, $z = 0$, is expressed as follows [1–35]

$$E_{pl}(r, \varphi; 0) = \left( \frac{q^p}{\omega_b^p} \right) L_p^l \left( \frac{q^p r^2}{\omega_b^2} \right) \exp \left( -\frac{r^2}{\omega_b^2} \right) \exp (i l \varphi),$$

(1)

where $r$ and $\varphi$ are the radial and azimuthal (angle) coordinates, $L_p^l$ denotes the Laguerre polynomial with mode orders $p$ and $l$, $\omega_b$ is the beam width of the fundamental Gaussian mode. For $q = \sqrt{2}$, Eq. (1) reduces to the electric field of a standard LG beam; for $q = 1$, it gives the electric field of an elegant LG beam; also for $p = 0$ and $l = 0$, Eq. (1) degenerates to the electric field of a fundamental Gaussian beam.

By use of the following relation between an LG mode and an HG mode [64]

$$e^{i p \theta} L_p^l (\rho^2) = \frac{(-1)^p}{2^{p+l}} \sum_{m=0}^{p} \sum_{n=0}^{l} \binom{p}{m} \binom{l}{n} H_{2m+l, -n} (x) H_{2p-2m+l, -n} (y),$$

(2)

with $H(x)$ being the Hermite polynomial, $\binom{p}{m}$ and $\binom{l}{n}$ being binomial coefficients, Eq. (1) can be expressed in following alternative form in Cartesian coordinates

$$E_{pl}(x, y; 0) = \frac{(-1)^p}{2^{p+l}} \sum_{m=0}^{p} \sum_{n=0}^{l} i^n \binom{p}{m} \binom{l}{n} \frac{q^p}{\omega_b^p} \frac{q^l}{\omega_b^l} \exp \left( -\frac{x^2 + y^2}{\omega_b^2} \right) H_{2m+l, -n} \left( \frac{q^p x}{\omega_b^p} \right) \frac{H_{2p-2m+l, -n} \left( \frac{q^l y}{\omega_b^l} \right)}{H_{2p-2m+l, -n} \left( y \right)},$$

(3)

We will now extend standard and elegant LG beams to the partially coherent case. The second-order statistical properties of a partially coherent beam are generally characterized by the cross-spectral density $W(x_1, y_1; x_2, y_2; z) = \langle E^* (x_1, y_1; z) E (x_2, y_2; z) \rangle$, where $\langle \cdot \rangle$ denotes the ensemble average and $\langle \cdot \rangle^*$ is the complex conjugate [59]. The intensity distribution, of a partially coherent beam at any position $(x, y)$ in plane $z$, $z \geq 0$, can be determined from the relation $I(x, y; z) = W(x, y, x, y; z)$. For a partially coherent beam generated by a Schell-model source (at $z = 0$), the cross-spectral density can be expressed in the following well-known form [59]

$$W(x_1, y_1, x_2, y_2; 0) = \sqrt{I(x_1, y_1; 0) I(x_2, y_2; 0)} g(x_1 - x_2; y_1 - y_2; 0),$$

(4)

where $g(x_1 - x_2; y_1 - y_2; 0)$ is the spectral degree of coherence which we will assume to have Gaussian profile, i.e.

$$g(x_1 - x_2; y_1 - y_2; 0) = \exp \left[ -\frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{2\sigma_g^2} \right],$$

(5)
where $\sigma_g$ is the transverse coherence width. We now assume that the intensity distribution of the Gaussian Schell-model source can be represented by a standard or elegant LG beam. Then $I(x, y; 0)$ can be written as

$$I(x, y; 0) = \left| \frac{(-1)^p}{2^{p+1}p!} \sum_{m=0}^{p} \sum_{n=0}^{p} \sum_{h=0}^{l} \sum_{v=0}^{l} \binom{p}{m} \binom{l}{n} \binom{l}{h} H_{2m+1-s} \frac{q x}{\omega_0} H_{2p-2m+s} \frac{q y}{\omega_0} \exp \left( -\frac{x^2 + y^2}{\omega_0^2} \right) \right|^2. \quad (6)$$

Substituting Eq. (6) into Eq. (4), we can express the cross-spectral density of a partially coherent standard or elegant LG beam as follows

$$W(x_1, y_1, x_2, y_2; 0) = \frac{1}{2^{p+2l}(p!)^2} \sum_{m=0}^{p} \sum_{n=0}^{p} \sum_{h=0}^{l} \sum_{v=0}^{l} \binom{p}{m} \binom{l}{n} \binom{l}{h} \binom{l}{v} \frac{q x_1}{\omega_0} H_{2m+1-s} \frac{q x_2}{\omega_0} H_{2p-2m+n} \frac{q y_1}{\omega_0} H_{2p-2m+s} \frac{q y_2}{\omega_0} \exp \left( -\frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{2\sigma_g^2} \right). \quad (7)$$

Under the condition of $\sigma_g \rightarrow \infty$, Eq. (7) reduces to the expression for a coherent standard or elegant LG beam. Under the condition of $p = 0$ and $l = 0$, Eq. (7) reduces to the expression for a partially coherent Gaussian Schell-model beam [65–70]. By transforming Eq. (1) into Eq. (3) with the help of Eq. (2) and expressing the cross-spectral density of a partially coherent standard or elegant LG beam of all orders in the form of Eq. (7) in the Cartesian coordinates, we can obtain analytical propagation formula for the cross-spectral density and analytical expression for the effective beam size of a partially coherent standard or elegant LG beam in an easy way as shown later. In the cylindrical coordinates, it is very difficult for us to obtain analytical propagation formula of a partially coherent standard or elegant LG beam.

Now we study the propagation of beams generated by a partially coherent standard or elegant LG source (7) through a paraxial ABCD optical system. Within the validity of the paraxial approximation, the propagation of the cross-spectral density of a partially coherent beam through an aligned ABCD optical system in free space can be studied with the help of the following generalized Collins formula [49,71]

$$W(u_1, v_1, u_2, v_2, z) = \left( \frac{1}{\lambda |B|} \right)^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W(x_1, y_1, x_2, y_2; 0) \times \exp \left[ -\frac{ik}{2B} (A x_1^2 - 2x_1 u_1 + D u_1^2) - \frac{ik}{2B} (A y_1^2 - 2y_1 v_1 + D v_1^2) \right] \times \exp \left[ \frac{ik}{2B} (Ax_2^2 - 2x_2 u_2 + D u_2^2) + \frac{ik}{2B} (Ay_2^2 - 2y_2 v_2 + D v_2^2) \right] \, dx_1 \, dy_1 \, dx_2 \, dy_2, \quad (8)$$

where $x_1, y_1,$ and $u_1, v_1$ are the position coordinates in the input and output planes, $A, B, C$ and $D$ are the transfer matrix elements of optical system, $k = 2\pi / \lambda$ is the wave number with $\lambda$ being the wavelength. Substituting from Eq. (7) into Eq. (8), we obtain the formula
\[
W(u_1, v_1, u_2, v_2, z) = \left( \frac{1}{|\mathbf{B}|} \right)^2 \frac{1}{2^{2p+2l}(p!^2)} \sum_{m=0}^{p \ast} \sum_{n=0}^{p \ast} \sum_{l=0}^{p \ast} (i^p) \sum_{l=0}^{p \ast} \sum_{l=0}^{p \ast} \left( i^p \right) \exp \left( -\frac{x_i^2 + x_j^2}{\alpha_i^2} \right) \exp \left( -\frac{(x_i - x_j)^2}{2\sigma_i^2} \right) \\
\times \exp \left[-\frac{ikB}{2B} (A' x_i^2 - 2x_i u_i + D' u_i^2) + \frac{ikB}{2B} (A' x_j^2 - 2x_j u_j + D' u_j^2) \right] \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H_{2m+l+n} \left( \frac{q_1}{\alpha_0} \right) H_{2p-2h+s} \left( \frac{q_2}{\alpha_0} \right) \exp \left( -\frac{x_i^2 + x_j^2}{\alpha_i^2} \right) \exp \left( -\frac{(y_i - y_j)^2}{2\sigma_i^2} \right) \\
\times \exp \left[-\frac{ikB}{2B} (A' y_i^2 - 2y_i v_i + D' v_i^2) + \frac{ikB}{2B} (A' y_j^2 - 2y_j v_j + D' v_j^2) \right] dy_i dy_j.
\]

(9)

After integration over \( x_i, x_j, y_i, y_j \), Eq. (9) becomes

\[
W(u_1, u_2, v_1, v_2, z) = \left( \frac{1}{|\mathbf{B}|} \right)^2 \frac{1}{2^{4p+2l}(p!^2)} \pi^2 \frac{\alpha_i^2}{2M_2 \alpha_0^2} \left( \frac{2q}{\alpha_0} \right)^{2p+l} \\
\times \exp \left[-\frac{ikD^*}{2B} u_i^2 + \frac{ikD^*}{2B} u_j^2 \right] \exp \left(-\frac{k^2 u_i^2}{4M_1 B^2} \right) \exp \left(-\frac{k^2 u_j^2}{4M_1 B^2} \right) \\
\times \exp \left[-\frac{ikD^*}{2B} v_i^2 + \frac{ikD^*}{2B} v_j^2 \right] \exp \left(-\frac{k^2 v_i^2}{4M_1 B^2} \right) \exp \left(-\frac{k^2 v_j^2}{4M_1 B^2} \right) \\
\times \sum_{m=0}^{p} \sum_{n=0}^{p} \sum_{l=0}^{p} \sum_{l=0}^{p} \sum_{l=0}^{p} \sum_{l=0}^{p} (i^p) \sum_{l=0}^{p} \sum_{l=0}^{p} \sum_{l=0}^{p} \sum_{l=0}^{p} \left( i^p \right) \exp \left( -\frac{x_i^2 + x_j^2}{\alpha_i^2} \right) \exp \left( -\frac{(x_i - x_j)^2}{2\sigma_i^2} \right) \\
\times \frac{\left( \frac{p}{h} \right)^{2h+l-s} \left( \frac{2p-2h+s}{d} \right)}{(2m+p+n)!} \left( \frac{(2m+l-n)!}{c_1 !(2m+l-n-2c_1)!} \right) \\
\times \frac{d!}{c_2 !(d-2c_2)!} e_1 !(2p-2m+n-2c_1)! e_2 !(d_1 -2e_2)! \\
\times \frac{d_2!(d_1 -2e_2)!}{c_2 !(d-2e_2)!} e_1 !(2p-2m+n-2c_1)! e_2 !(d_1 -2e_2)! \\
\times \left( \frac{1}{\sqrt{M_2}} \right)^{d} \left( \frac{2q}{\sqrt{2\sigma_i^2 M_1 \alpha_0^2 - q^2 M_1}} \right)^{2q} \\
H_{2h+l-s-d} \left( \frac{i q u_2}{\sqrt{2B M_1 \alpha_0^2 - q^2 M_1}} \right) H_{2h+l-m+n-d-2c_2} \left( \frac{k u_2}{4M_1 \sqrt{M_1 \sigma_i^2 B - 2\sqrt{M_2 B^2}}} \right) \\
H_{2p-2m+n-d-2c_2} \left( \frac{-i q v_2}{\sqrt{2B M_1 \alpha_0^2 - q^2 M_1}} \right) \\
H_{2p-2m+n-d-2c_2} \left( \frac{k v_2}{4M_1 \sqrt{M_1 \sigma_i^2 B - 2\sqrt{M_2 B^2}}} \right).
\]

(10)

with \( M_1 = 1/\alpha_0^2 + 1/(2\sigma_i^2) - i k A / (2B), M_2 = 1/\alpha_0^2 + 1/(2\sigma_i^2) + i k A' / (2B') - 1/(4M_1 \sigma_i^2) \). In above derivations, we have used the following integral and expansion formulae [72,73].
\[
\int_{-\infty}^{\infty} \exp\left[-(x-y)^2\right] H_n(ax) dx = \sqrt{\pi} (1-a^2)^{n/2} H_n \left( \frac{ay}{(1-a^2)^{1/2}} \right), \tag{11}
\]

\[
\int_{-\infty}^{\infty} x^n \exp\left[-(x-\beta)^2\right] dx = (2i)^n \sqrt{\pi} H_n(i\beta), \tag{12}
\]

\[
H_n(x+y) = \frac{1}{2^{n/2}} \sum_{k=0}^{n} \binom{n}{k} H_k(\sqrt{2}x) H_{n-k}(\sqrt{2}y), \tag{13}
\]

\[
H_n(x_i) = \sum_{m=0}^{[n/2]} (-1)^m \frac{n!}{m!(n-2m)!} (2x_i)^{n-2m}. \tag{14}
\]

Thus, Eq. (10) is the analytical formulae for the cross-spectral density of a partially coherent standard or elegant LG beam passing a paraxial ABCD optical system. The intensity of the partially coherent standard or elegant LG beam at the output plane can be obtained by setting, in Eq. (10), \(u_1 = u\), and \(v_1 = v\).

The effective beam size is a useful parameter for characterizing the spreading properties of a beam. According to [74], by use of twice the variance of \(x\) or \(y\), the effective beam size of a partially coherent standard or elegant LG beam at plane \(z\) is defined as

\[
W_{\text{sc}}(z) = \sqrt{\frac{2}{\pi}} \frac{s^2 \langle I(x, y, z) \rangle dx dy}{\int \langle I(x, y, z) \rangle dx dy} \quad (s=x, y). \tag{15}
\]

On substituting from Eq. (10) into Eq. (15), we obtain the following expression for the effective beam size of a partially coherent standard or elegant LG beam after propagation

\[
W_{\text{sc}} = W_{\text{sc}} = \sqrt{\frac{A_1(z)}{A_2(z)}}. \tag{16}
\]

where
\[ A_1(z) = \left( \frac{1}{\lambda |B|} \right)^2 \frac{1}{2^{k+s+2}} \frac{1}{(p!)^2} \pi^3 \left( \frac{1}{M_1 M_2 M_3} \left( \frac{1}{2M_2} - \frac{q^2}{2M_2 a_0^2} \right) \right)^{2(p+s)/2} \left( \frac{2q}{a_0} \right)^{2(p+s)/2} \]

\[ \times \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \sum_{s=0}^{d-1} \sum_{t=0}^{\min(2n, 2m, 2s, k-2)} \frac{1}{2^{(k+s)/2}} \frac{1}{2^{(k-s)/2}} \frac{1}{2^{(m-t)/2}} \frac{1}{2^{(n-r)/2}} \frac{1}{2^{(l-s)/2}} \left( \frac{p}{m} \right)^{l} \left( \frac{l}{n} \right)^{h} \]

\[ \left( \frac{2h + l - s}{d} \right)^{d + h - 2e_1 - 2e_2} \]

\[ \left( \frac{2h + l - s - d}{2f_1} \right)^{2h + l - s - d - 2f_1} \]

\[ \frac{2q}{\sqrt{2\sigma^2 \frac{1}{M_1} a_0^2 - q^2 M_1}} \left( \frac{2iqk}{\sqrt{2B \frac{1}{M_1} a_0^2 - q^2 M_1}} \right)^{2p + 2h + s - d - 2g_1} \]

\[ \frac{k}{2M_1 \sqrt{\sigma^2 B}} - \frac{k}{2M_1 \sqrt{\sigma^2 B}} \]

\[ \frac{2^{m+n+d-2e_1-2e_2-2f_1}}{2^{m+n+d-2e_1-2e_2-2f_1}} \]

\[ \times \frac{2^{m+n+d-2e_1-2e_2}}{2^{m+n+d-2e_1-2e_2}} \]

\[ \times H_{2h+2m+2s+n-2e_1-2e_2-2f_1-2g_1+1} (0) \]

and

\[ (17) \]
\[ A_e(z) = \left( \frac{1}{\lambda |B|} \right)^2 \frac{1}{2^{p+1}} \left( \frac{p!}{2} \right)^2 \frac{\pi^{3/2}}{M_1 M_2 M_3} \left( \frac{1}{2M_2} - \frac{q^2}{2M_2 o_0^2} \right)^{(2p+1)/2} \left( \frac{2q}{o_0} \right)^{2p} \times \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{c_1=0}^{\infty} \sum_{c_2=0}^{\infty} \sum_{e_1=0}^{\infty} \sum_{e_2=0}^{\infty} \sum_{f_1=0}^{\infty} \sum_{f_2=0}^{\infty} \left[ \begin{array}{c} 2h+l-s \\ d \end{array} \right] \left[ \begin{array}{c} 2p-2h+s \\ d_1 \end{array} \right] (-1)^{e_1+e_2+e_1+e_2+f_1+f_2+e_1+e_2} \frac{(2m+l-n)!}{c_1!(2m+l-n-2c_1)!} \frac{d!}{c_1!(d-2c_1)!} \\
\times \frac{(2p-2m+n)!}{e_1!(2p-2m-n-2e_1)!} \frac{d_1!}{f_1!(2m+l-n+d-2e_1-2e_2)!} \frac{(2p-2h+s-d)!}{g_1!(2p-2m+n+d_1-2e_1-2e_2-2g_1)!} \frac{(2p-2h+s-d)!}{g_1!(2p-2h+s-d-2g_1)!} \\
\times (2p)^{e_1+4e_2+4e_1+4e_2+2e_1+2e_2+2f_1+2f_2-6p} \left( \frac{1}{\sqrt{M_2}} \right)^{d+d_1-2e_1-2e_2} \left( \frac{2q}{o_0} \right)^{2e_1-2e_2} H_{2h+l+s+d-2f_1} \left( 0 \right) H_{2p-2h-2m+n+d-2e_1-2e_2} \left( 0 \right), \]

with \( M_3 = \frac{k^2}{4M_2 B^2} + \frac{k^2}{4M_2} - \frac{1}{2M_2 \sigma^2 B^2} \left( \frac{1}{B'} - \frac{1}{2M_2 \sigma^2 B} \right)^2 + \frac{ikD}{2B} - \frac{ikD'}{2B'} \). Equations (10) and (16)-(18) are the main analytical results of the paper. Although Eqs. (10) and (16)-(18) involve up to 14 nested sums of binomial coefficients and Hermite function, it takes only several minutes to calculate each line of Figs. 1-4. If we want to calculate each line of Figs. 1-4 by Eqs. (7) and (8) through direct numerical integration, it is very time consuming due to four integrals, usually its take several hours or more to do this. More comparisons between calculation by analytical formula and by direct numerical calculation can be found in [75].

3. Propagation properties of partially coherent standard and elegant LG beams in free space

In this section we will carry out a comparative study of the properties of partially coherent standard and elegant LG beams propagating in free space by using the formulae derived in Section 2.

The ray transfer matrix relating to free-space propagation between the source plane \((z = 0)\) and output plane \((z \geq 0)\) takes the form

\[ \text{Equations (10) and (16)-(18) are the main analytical results of the paper. Although Eqs. (10) and (16)-(18) involve up to 14 nested sums of binomial coefficients and Hermite function, it takes only several minutes to calculate each line of Figs. 1-4. If we want to calculate each line of Figs. 1-4 by Eqs. (7) and (8) through direct numerical integration, it is very time consuming due to four integrals, usually its take several hours or more to do this. More comparisons between calculation by analytical formula and by direct numerical calculation can be found in [75].} \]
Substituting from Eq. (19) into Eq. (10), we calculate in Fig. 1 the normalized intensity distribution (cross line v = 0) of a partially coherent standard LG beam for different values of the initial coherence width $\sigma_s$ at several propagation distances in free space with $p = 1, l = 1, \alpha_0 = 2\text{mm}$ and $\lambda = 632.8\text{nm}$. One finds from Fig. 1 that the propagation properties of a partially coherent LG beam are very different from those of a coherent standard LG beam. For a coherent standard LG beam, its initial source beam profile remains invariant on propagation although its beam spot increases, the result being in good agreement with previous results of [1–25]. For a partially coherent LG beam, its propagation properties are closely related to its initial degree of coherence. The initial source beam profile of a partially coherent standard LG beam does not remain invariant on propagation, but gradually disappears on propagation and eventually takes a Gaussian shape. As the initial degree of coherence decreases, the transition from a standard LG beam into a Gaussian beam occurs sooner and the beam spreads more rapidly.

For a convenient comparison, we calculate in Fig. 2 the normalized intensity distribution (cross line v = 0) of a partially coherent elegant LG beam for different values of the initial coherence width $\sigma_s$ at several propagation distances in free space with $p = 1, l = 1, \alpha_0 = 2\text{mm}$ and $\lambda = 632.8\text{nm}$. We find from Fig. 2 that the propagation properties of an elegant LG beam are also largely determined by its initial degree of coherence. The beam profile of a coherent elegant LG beam transforms into a dark hollow beam profile in the far field, as expected [27–37]. Similar to a partially coherent standard LG beam, the beam profile of a partially coherent elegant LG beam also gradually disappears on propagation and eventually takes a Gaussian shape. As the initial coherence decreases, the conversion from an elegant LG beam into a Gaussian beam occurs more quickly and the beam spreads more rapidly. From Figs. 1 and 2, one comes to the conclusion that by degrading the coherence of a standard or elegant LG beam it possible to perform beam shaping in the far field.
Fig. 2. Normalized intensity distribution (cross line $v = 0$) of a partially coherent elegant LG beam for different values of the initial coherence width $\sigma_x$ with $p = 1$, $l = 1$ at several propagation distances in free space. (a) $z = 0$, (b) $z = 3m$, (c) $z = 10m$, (d) $z = 30m$

To learn about the spreading properties of the partially coherent standard and elegant LG beams on propagation in free space, we calculate in Fig. 3 the effective beam sizes of partially coherent standard and elegant LG beams versus the propagation distance $z$ in free space for different values of the initial coherence width with $p = 1$, $l = 1$ and $\lambda = 632.8nm$. For the convenience of comparison, we have set $\omega_0 = 10mm$ for a partially coherent elegant LG beam and $\omega_0 = 5.773mm$ for a partially coherent standard LG beam, so that they have the same
effective beam sizes in the source plane (z = 0). One finds from Fig. 3 that the spreading properties of standard and elegant LG beams are also closely related to their initial coherence widths. A coherent standard LG beam spreads more rapidly than a partially coherent elegant LG beam (see Fig. 3(a)), as expected [27–37]. When the coherence width is large, a partially coherent elegant LG beam still spreads slower than a partially coherent standard LG beam (see Fig. 3(b) and (c)). When the coherence width is small, there is no distinct difference between the spreading properties of a partially coherent standard LG beam and that of a partially coherent elegant LG beam (see Fig. 3(d)).

In Fig. 4, we calculate the effective beam sizes of partially coherent standard LG beams and elegant LG beams versus the propagation distance z in free space for different values of mode orders $p$ and $l$. For the convenience of comparison, in Fig. 4(a)-(c), we have chosen $\omega_0 = 10.0\text{mm}$ for $p = 1, l = 1$, $\omega_0 = 10.54\text{mm}$ for $p = 2, l = 1$, $\omega_0 = 10.80\text{mm}$ for $p = 3, l = 1$, $\omega_0 = 8.165\text{mm}$ for $p = 1, l = 2$, $\omega_0 = 6.90\text{mm}$ for $p = 1, l = 3$, respectively, so that all partially coherent elegant LG beams have the same effective beam sizes on the source plane. In Fig. 4 (d)-(f), we have chosen $\omega_0 = 10.0\text{mm}$ for $p = 1, l = 1$, $\omega_0 = 8.165\text{mm}$ for $p = 2, l = 1$, $\omega_0 = 7.07\text{mm}$ for $p = 3, l = 1$, $\omega_0 = 8.944\text{mm}$ for $p = 1, l = 2$, $\omega_0 = 8.166\text{mm}$ for $p = 1, l = 3$ respectively. It is evident from Fig. 4 that the mode orders $p$ and $l$ of partially coherent standard and elegant LG beams affect their spreading properties strongly when the
initial degree of coherence is high. Both partially coherent standard and elegant LG beams spread more rapidly as their mode orders $p$ and $l$ increase when the initial degree of coherence is high (see Fig. 4(a), (b), (d) and (e))). When the initial coherence is small, the partially coherent LG beams, both standard and elegant, with different mode orders exhibit almost the same spreading features (see Fig. 4 (c) and (f)).

4. Summary

We have proposed theoretical model to describe partially coherent standard and elegant LG beams, and have derived the analytical formulae for the cross-spectral densities of such beams propagating through paraxial ABCD optical systems. By numerical examples, we have studied the intensity and spreading properties of partially coherent standard and elegant LG beams in free space, comparatively. We have found that the properties of standard and elegant LG beams on free-space propagation are much different from those pertaining to coherent standard and elegant LG beams. As a general rule, a partially coherent elegant LG beam spreads more slowly in free space than a partially coherent standard LG beam. The advantage of a partially coherent elegant LG beam over a partially coherent standard LG beam disappears when the initial coherence in the source plane is very low. Thus, coherent or partially coherent elegant LG beams have some advantages over the corresponding standard LG beams, the result which can be employed in applications, such as free-space optical communications and remote sensing.

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