10-1-2008

Evolution of the Degree of Polarization of an Electromagnetic Gaussian Schell-Model Beam in a Gaussian Cavity

Min Yao
Yangjian Cai
Halil T. Eyyuboglu
Yahya Baykal
Olga Korotkova

*University of Miami, o.korotkova@miami.edu*

---

**Recommended Citation**


http://scholarlyrepository.miami.edu/physics_articles/20

---

This Article is brought to you for free and open access by the Physics at Scholarly Repository. It has been accepted for inclusion in Physics Articles and Papers by an authorized administrator of Scholarly Repository. For more information, please contact repository.library@miami.edu.
Evolution of the degree of polarization of an electromagnetic Gaussian Schell-model beam in a Gaussian cavity

Min Yao,1,2 Yangjian Cai,3,6 Halil T. Eyyuboğlu,4 Yahya Baykal,4 and Olga Korotkova5

1Centre for Optical and Electromagnetic Research, Zhejiang University, Hangzhou, 310058, China
2College of Science and Technology, Zhejiang Education Institute, Hangzhou, 310012, China
3Max-Planck-Research-Group, Institute of Optics, Information and Photonics, University of Erlangen, Staudtstrasse 7/B2 D-91058, Erlangen, Germany
4Department of Electronic and Communication Engineering, Çankaya University, Öğretmenler Cad. 14, Yüzüncüyıl 06530 Balgat Ankara, Turkey
5Department of Physics, University of Miami, Coral Gables, Florida 33146, USA
6Corresponding author: yangjian_cai@yahoo.com.cn

Received May 6, 2008; revised July 11, 2008; accepted August 25, 2008; posted September 10, 2008 (Doc. ID 95767); published September 30, 2008

The interaction of an electromagnetic Gaussian Schell-model (EGSM) beam with a Gaussian cavity is analyzed. In particular, the evolution of the degree of polarization of the EGSM beam is investigated. The results show that the behavior of the degree of polarization depends on both the statistical properties of the source that generates the EGSM beam and the parameters of the cavity. © 2008 Optical Society of America

Over the past several years the class of electromagnetic Gaussian Schell-model (EGSM) beams has been investigated widely owing to its importance in theories of coherence and polarization of light and in some applications, e.g., free-space optical communications [1–17]. The EGSM beams (also sometimes called vectorial GSM beams) were first introduced theoretically as the natural extension of the scalar GSM beams by Gori et al. [1,2] (see also [3–5]), and several methods have been proposed for their synthesis [6,7]. The realizability conditions, i.e., conditions that the parameters of the source should satisfy to produce a physically realizable field, have also been established for this class of beams [8] (see [9] for conditions of more general class of electromagnetic Shell-model beams). Propagation of the EGSM beams through free space [10], turbulent atmosphere [11–13], human tissues [14,15], fractional Fourier transform optical systems [16], and paraxial aligned astigmatic (i.e., nonsymmetrical) and misaligned stigmatic optical systems [17] have also been extensively studied.

The conventional theory of laser resonators was originally confined to completely spatially coherent light fields [18] until Wolf, Agarwal, and Gori generalized it for light fields with any state of coherence [19–21]. Palma and coworkers then studied the behavior of the degree of coherence and of the spectral properties of partially coherent beams in a Gaussian cavity [22,23]. However, almost all previous works on the subject were restricted to the scalar theory, and the behavior of the polarization properties of an electromagnetic partially coherent beam in resonators has not been investigated so far, with a few exceptions (c.f. [24,25]). In this Letter, we analyze the evolution of the degree of polarization (DOP) of an electromagnetic partially coherent beam in a Gaussian cavity. The effects of the physical properties of the cavity and of the statistical properties of the light source are explored in detail. Although our theoretical results are general, the numerical calculations are restricted to the EGSM beams.

For the sake of convenience we will confine our analysis to a bare Gaussian cavity of length $L$. The Gaussian cavity consists of two spherical mirrors, each with radius of curvature $R$ and a Gaussian reflectivity profile with radius $e$, and is equivalent to a sequence of identical thin spherical lenses with focal length $f=R/2$, followed by the amplitude filters with a Gaussian transmission function for the equivalent (unfolded) optical system (see Fig. 1 of [22]). The distance between each lens–filter pair is equal to $L$.

We assume that originally the beam in the resonator was produced by an EGSM source. The second-order statistical properties of the initial beam can then be characterized by the $2 \times 2$ cross-spectral density matrix $\hat{W}(\mathbf{r}_1,\mathbf{r}_2)$ specified at any two points with transverse position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ in the input plane with elements [1–5,10,16,17,26]

$$W_{\alpha\beta}(\mathbf{r}) = A_{\alpha} A_{\beta} B_{\alpha\beta} \exp \left[ -\frac{i k}{2} \mathbf{r}^T \mathbf{M}_{\alpha\beta}^{-1} \mathbf{r} \right],$$

(1)

where $A_\alpha$, $B_{\alpha\beta}$, $B_{\alpha\beta}(\phi_{\alpha\beta}) = B_{\alpha\beta}^* \sigma_{\alpha}$ and $\delta_{\alpha\beta}$ are independent of position but, in general, depend on the frequency. In Eq. (1) and everywhere else in this Letter we omitted the dependence on the oscillation frequency for conciseness. The nine real parameters $A_x$, $A_y$, $\sigma_x$, $\sigma_y$, $|B_{xy}|$, $\phi_{xy}$, $\delta_{xx}$, $\delta_{xy}$ and $\delta_{yy}$ entering the general model are shown to satisfy several intrinsic constraints and obey some simplifying assumptions (e.g.,

\[0146-9592/08/192266-3/$15.00 © 2008 Optical Society of America\]
the phase difference between the x and y components of the field is removable, i.e., \( \varphi_{00} = 0 \) \(^7\text{–}^9\). \( k = 2\pi/\lambda \) is the wavenumber, \( \lambda \) is the wavelength, \( \tilde{r}^r = (\tilde{r}^r_x, \tilde{r}^r_y) \), and \( M^−1_{\alpha\beta} \) has the form

\[
M^−1_{\alpha\beta} = \begin{pmatrix}
\frac{1}{ik} & \frac{1}{\delta^2_{\alpha\beta}} & \frac{1}{ik} & \frac{1}{\delta^2_{\alpha\beta}} \\
\frac{i}{k\delta^2_{\alpha\beta}} & \frac{1}{\delta^2_{\alpha\beta}} \\
\frac{1}{ik} & \frac{1}{\delta^2_{\alpha\beta}} & \frac{1}{ik} & \frac{1}{\delta^2_{\alpha\beta}} \\
\frac{i}{k\delta^2_{\alpha\beta}} & \frac{1}{\delta^2_{\alpha\beta}} & \frac{i}{k\delta^2_{\alpha\beta}} & \frac{1}{\delta^2_{\alpha\beta}}
\end{pmatrix},
\]

\( I \) being a \( 2 \times 2 \) unit matrix. After propagating through a general astigmatic ABCD optical system, the elements of the cross-spectral density matrix can be expressed in the following tensor form \(^17,26\):

\[
W_{\alpha\beta}(\rho) = A_{\alpha} A_{\beta} B_{\alpha\beta} [\text{det} (\tilde{\mathbf{A}} + \tilde{\mathbf{B}} M^−1_{\alpha\beta})]^{−1/2} \times \exp \left[ −\frac{ik}{2} \tilde{\rho}^r (\tilde{C} + \tilde{D} M^−1_{\alpha\beta}) (\tilde{A} + \tilde{B} M^−1_{\alpha\beta})^{-1} \tilde{\rho} \right],
\]

where \( \text{det} \) stands for the determinant of a matrix, \( \tilde{\rho}^r = (\rho^r_x, \rho^r_y) \), \( \rho_1 \) and \( \rho_2 \) being the transverse position vectors in the output plane. \( \tilde{\mathbf{A}}, \tilde{\mathbf{B}}, \tilde{\mathbf{C}}, \) and \( \tilde{\mathbf{D}} \) are \( 4 \times 4 \) matrices of the form

\[
\tilde{\mathbf{A}} = \begin{pmatrix}
A & 0I \\
0I & A^r
\end{pmatrix}, \quad \tilde{\mathbf{B}} = \begin{pmatrix}
B & 0I \\
0I & -B^r
\end{pmatrix},
\]

\[
\tilde{\mathbf{C}} = \begin{pmatrix}
C & 0I \\
0I & -C^r
\end{pmatrix}, \quad \tilde{\mathbf{D}} = \begin{pmatrix}
D & 0I \\
0I & D^r
\end{pmatrix},
\]

where \( \mathbf{A}, \mathbf{B}, \mathbf{C}, \) and \( \mathbf{D} \) are the \( 2 \times 2 \) submatrices of the general astigmatic ABCD optical system and * denotes the complex conjugate being required for a general optical system with loss or gain, although it does not appear in Eq. (13) of \(^{26}\).

By applying the ABCD-matrix approach for a Gaussian aperture \(^{27}\), we find that after the EGS system travels between the two mirrors for \( N \) times, \( \mathbf{A}, \mathbf{B}, \mathbf{C}, \) and \( \mathbf{D} \) for the equivalent optical system become

\[
\begin{pmatrix}
\mathbf{A} & \mathbf{B} \\
\mathbf{C} & \mathbf{D}
\end{pmatrix} = \left( \begin{pmatrix}
\mathbf{I} \\
L^{-1} \mathbf{I}
\end{pmatrix} \right)^N.
\]

Here \( \varepsilon \) can be regarded as the “soft” mirror spot size. On substituting from Eq. (5) into Eqs. (3) and (4), we can determine how various statistical properties of an EGS system evolve in a Gaussian cavity with the increasing number of passages between the mirrors.

We will now study numerically the behavior of the DOP of a typical EGS beam in a Gaussian cavity. The resonator is characterized by the classical cavity parameter \( g = 1 - L/R \), the resonators with \( 0 \leq g < 1 \) being stable and the ones with \( g \geq 1 \) being unstable \(^{28}\). The DOP of the beam is defined by the expression \(^5\)

\[
P(\mathbf{r}) = \sqrt{1 - \frac{4 \text{det} \bar{\mathbf{W}}(\mathbf{r}, \mathbf{r})}{[\text{Tr} \bar{\mathbf{W}}(\mathbf{r}, \mathbf{r})]^2}},
\]

where \( \text{Tr} \) denotes the trace of the matrix. In the following numerical examples, the initial beam parameters and cavity length are chosen to have the following values: \( \lambda = 590 \text{ nm}, A_0 = \lambda = 0.707, B_{xy} = B_{yx} = 0.2, \sigma_x = \sigma_y = 1 \text{ mm}, \) and \( L = 30 \text{ cm}. \) In this case the polarization properties are uniform across the source plane with \( P(\mathbf{r}) = 0.2 \).

In Fig. 1 we calculate the DOP (on-axis) versus \( N \) for different values of cavity parameter \( g \) and the initial correlation coefficients with \( \varepsilon = 0.8 \text{ mm}. \) One finds from Figs. 1(a) and 1(b) that the DOP increases as \( N \) increases, and its value approaches different constant values for different resonators when \( N \) is large enough \((N > 30)\). The DOP exhibits growth with oscillations in stable resonators \((0 < g < 1)\) but asymptotically saturates when \( N \) is large enough \((N > 30)\), while growth is monotonic for unstable resonators \((g > 1)\). Also, for unstable resonators the DOP decreases for higher values of \( g \). This behavior is similar to that of a spectral shift of a scalar GSM beam in a Gaussian cavity \(^{22}\). One finds from Figs. 2(c) and 2(d) that the DOP decreases as the correlation coefficients in the input plane take larger values, both in stable and unstable resonators. Physically it can be explained as follows: when the cavity is stable then periodical focusing and free-space diffraction (including the edge diffraction caused by the Gaussian aperture) act on the beam as two competing mechanisms changing the value of the DOP in an oscillatory manner, depending on which mechanism prevails for certain point between the mirrors. Meanwhile, for unstable resonators the effect of focusing is negligible and the behavior of the DOP is monotonic, just like in the case of free-space propagation (see \(^{2}\)).
To better understand how the mirror spot size affects the DOP, we calculate in Fig. 2 the DOP (on-axis) versus \( N \) for different values of \( \varepsilon \) in a Gaussian plane-parallel cavity \((g=1)\) with \( \delta_{xx}=\delta_{yy}=0.1 \text{ mm} \) and \( \delta_{xy}=\delta_{yx}=0.2 \text{ mm} \).

To better understand how the mirror spot size \( \varepsilon \) affects the DOP, we calculate in Fig. 2 the DOP (on-axis) versus \( N \) for different values of \( \varepsilon \) in a Gaussian plane-parallel cavity \((g=1)\). We can observe that the growth of the DOP is more pronounced in response to higher values of \( \varepsilon \) when \( N \) is sufficiently large. We can explain this by the fact that as the value of the mirror size decreases, the edge diffraction caused by the Gaussian aperture increases (i.e., the loss caused by the aperture increases), thus leading to the dependence of the DOP on the mirror size as shown in Fig. 2.

In conclusion, we have formulated the propagation laws of the cross-spectral density matrix for an electromagnetic partially coherent beam, in particular, an EGSM beam in a Gaussian cavity, and have studied the evolution of its DOP. Our results show that we can control the polarization properties of an EGSM beam within a Gaussian cavity by controlling the cavity parameters and the correlation properties of the source. Our formula can also be used for the analysis of the spectral properties and of the state of coherence of an EGSM in a Gaussian cavity.

Y. Cai gratefully acknowledges support from the Alexander von Humboldt Foundation. O. Korotkova’s research is funded by the Air Force Office of Scientific Research (AFOSR) (grant FA 95500810102).

References