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Beam Criterion for Atmospheric Propagation

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A criterion is introduced for testing whether a beam retains its beam-like form after it propagates any particular distance through the turbulent atmosphere. The criterion applies to monochromatic as well as to partially coherent beams and is illustrated by examples. © 2007 Optical Society of America

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In spite of the increasing use of optical beams for communications and for other purposes, no satisfactory criteria appear to have been established that make it possible to estimate distances of propagation in a random medium over which the radiation remains confined to a narrow solid angle, i.e., is beam-like. Investigations relating to this problem have been made by several authors (see, for example [1–5]), but none of them provide criteria for estimating such distances.

An unsatisfactory aspect of most if not all the previous treatments is the implicit assumption that the paraxial approximation applies, i.e., that the radiation retains its beam-like form at an arbitrary distance from the source. This assumption may not necessarily be justified, because the radiation is "dispersed" as the beam propagates over larger and larger distances. An estimate of the distance of propagation for which this assumption is valid is a basic problem which has not, to our knowledge, been previously adequately addressed.

A new criterion introduced in this paper is based on the concept variably known as "encircled energy," "total illumination," or "radiated power," well-known in the theory of optical image formation (see, for example, [6–8]). Clearly if an appreciable fraction of the radiated power, say 90%, traverses a transverse cross-sectional area at distance, from the source plane, whose linear dimensions subtend an angle of say 1° at the source, one may regard the radiated field to have retained its beam-like form up to that distance. The somewhat arbitrary choice of 90% and 1° may, of course, be replaced by other values, depending on the purpose for which the beam is to be used. We present some analytical and computational results based on this criterion that make it possible to estimate the effective distance from the source throughout which radiation of any state of coherence propagates in the atmosphere as a beam, and we compare them with corresponding results for free-space propagation.

Consider a beam generated by a source located in the plane $z=0$ and propagating into the half-space $z>0$. The fractional power p traversing a circular cross-sectional area σ of radius $\bar{\rho}$ at distance z from the source is, in suitable units, given by the expression

$$p(\bar{\rho},z,\omega)=\frac{L(\bar{\rho},z,\omega)}{L(\infty,z,\omega)}, \tag{1}$$

where

$$L(\bar{\rho},z,\omega)=\int\int_{\sigma} S(\boldsymbol{\rho},z,\omega)d^2\rho. \tag{2}$$

Here $S(\boldsymbol{\rho},z,\omega)$ is the spectral density (intensity at frequency ω) of the beam at a point $(\boldsymbol{\rho},z)$, $\boldsymbol{\rho}=(x,y)$ (see Fig. 1).

Let us first consider the fractional power generated by a monochromatic secondary source in the plane $z=0$, with Gaussian field distribution

$$U^{(0)}(\boldsymbol{\rho})=A\exp\left[-\frac{\rho^2}{w_0^2}\right], \tag{3}$$

generating a beam in free space. Here $\rho=\sqrt{x^2+y^2}$, A is the amplitude, and w_0 is the effective width of the beam in the source plane. After propagating a distance z from the source in free space, the spectral density $S(\boldsymbol{\rho},z)$ of the beam is given by the expression (cf. [9], Eq. (5.6-35))

$$S(\boldsymbol{\rho},z)\equiv|U(\boldsymbol{\rho},z)|^2=A^2\frac{w_0^2}{w^2(z)}\exp\left[-\frac{2\rho^2}{w^2(z)}\right], \tag{4}$$

where

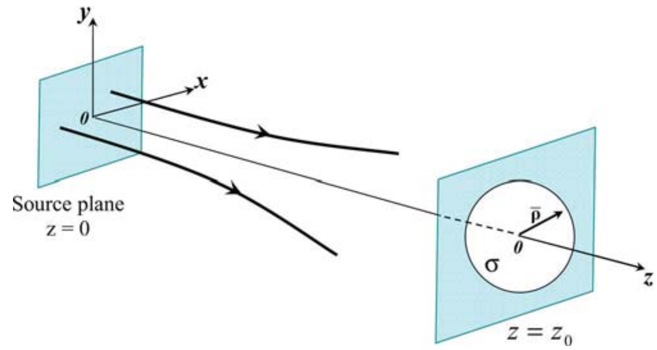

Fig. 1. (Color online) Illustrating the notation and the definition of fractional power $p(\bar{\rho},z,\omega)$ [Eqs. (1) and (2)].

$$w(z) = w_0\sqrt{1 + (2z/kw_0^2)^2}. \qquad (5)$$

Here $k = 2\pi/\lambda = \omega/c$, λ being the wavelength, ω the angular frequency, and c the speed of light in vacuum.

When such a beam propagates in the turbulent atmosphere, filling the half-space $z > 0$, its spectral density in a transverse plane at distance z from the source is given by the formula ([10], Sec. 7.3.3)

$$S(\boldsymbol{\rho},z) = A^2\frac{w_0^2}{w_e^2(z)}\exp\left(-\frac{2\rho^2}{w_e^2(z)}\right), \qquad (6)$$

where

$$w_e(z) = w(z)\sqrt{1 + 4.37C_n^2 l_0^{-1/3} z^3 w^{-2}(z)} \qquad (7)$$

represents the effective radius of the beam in the atmosphere, C_n^2 is the refractive index structure parameter (Ref. [10], p. 65, Eq. (14)), l_0 is the inner scale (Ref. [10], p. 64, Eq. (13)), and $w(z)$ is given by Eq. (5). Formulas (6) and (7) were derived by means of the extended Huygens–Fresnel principle with Tatarskii's power spectrum of atmospheric fluctuations, and they are valid in both weak and strong turbulence.

The fractional power $p(\bar{\rho},z)$ of a monochromatic Gaussian beam propagating in free space can be calculated by substituting from Eq. (4) into Eq. (1). The corresponding expression for propagation of the beam in the atmosphere is obtained by substituting from Eq. (6) into Eq. (1). Figure 2 provides examples of the fractional power of such beams propagating in free space and in the turbulent atmosphere. In calculating the fractional power we have used the expressions given above for the spectral density (spectral intensity), which have been derived by the use of the paraxial approximation. We assume that these expressions may be used under conditions when our power criterion holds. This assumption is hard to justify, but its validity seems intuitively rather obvious.

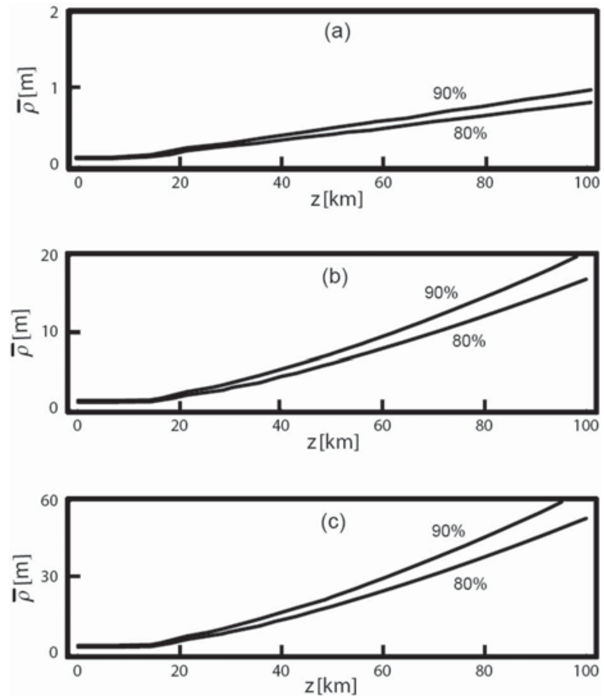

Fig. 2. Contours of the fractional power in transverse cross-sections of a monochromatic Gaussian beam, with $\lambda = 0.633\ \mu$m, $A = 1$, $w_0 = 2$ cm propagating in (a) free space, (b) atmosphere with $C_n^2 = 10^{-14}\ \text{m}^{-2/3}$, $l_0 = 1$ mm, (c) atmosphere with $C_n^2 = 10^{-13}\ \text{m}^{-2/3}$, $l_0 = 1$ mm.

Suppose next that the source is partially coherent. We may characterize it by the cross-spectral density function ([9], Secs. 2.4.4 and 4.7)

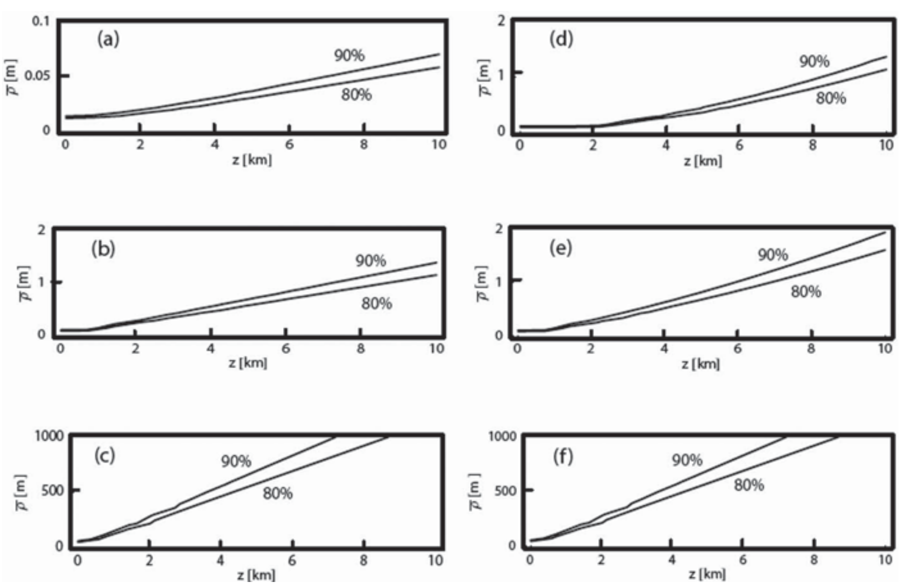

Fig. 3. Contours of the fractional power in cross-sections of a Gaussian Schell-model beam, with $\lambda = 0.633\ \mu$m, $\sigma_S = 1$ cm propagating in free space and in the atmosphere, with $C_n^2 = 10^{-13}\ m^{-2/3}$. (a)–(c) in free space, (d)–(f) in the turbulent atmosphere (Kolmogorov spectrum). (a) and (d): $\sigma_\mu/\sigma_S = \infty$ (completely spatially coherent source), (b) and (e): $\sigma_\mu/\sigma_S = 0.1$ (partially coherent source), (c) and (f): $\sigma_\mu/\sigma_S \to 0$ (incoherent source).

Table 1. Distances of Propagation Calculated from Eqs. (15) and (16) at Which the Directionality of the Gaussian Schell-Model Beam Starts to Exceed Angle θ=1 mrad≈0°3′26″ for Selected Degrees of Source Coherence and Strengths of Turbulence[a]

Degree of Source Coherence	Turbulence with $C_n^2=10^{-13}\ m^{-2/3}$ $p(\bar{\rho},z,\omega)=90\%$	Turbulence with $C_n^2=10^{-13}\ m^{-2/3}$ $p(\bar{\rho},z,\omega)=80\%$	Turbulence with $C_n^2=10^{-14}\ m^{-2/3}$ $p(\bar{\rho},z,\omega)=90\%$	Turbulence with $C_n^2=10^{-14}\ m^{-2/3}$ $p(\bar{\rho},z,\omega)=80\%$
$\sigma_\mu/\sigma_S\to\infty$	280 km	330 km	3000 km	3500 km
$\sigma_\mu/\sigma_S=0.1$	270 km	320 km	2200 km	2500 km
$\sigma_\mu/\sigma_S=0.02$	150 km	250 km	1800 km	2200 km

[a]Estimated error of calculation: ≤5%.

$$W^{(0)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2',\omega)=\langle U^{(0)^*}(\boldsymbol{\rho}_1',\omega)U^{(0)}(\boldsymbol{\rho}_2',\omega)\rangle, \tag{8}$$

where the angular brackets represent average over an ensemble of space-frequency realizations $U(\mathbf{r},\omega)$ of the field. We will only consider the so-called Gaussian Schell-model beams, i.e., beams whose cross-spectral density in the source plane is given by expression of the form

$$W^{(0)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2',\omega)=S^{(0)}\left(\frac{\boldsymbol{\rho}_1'+\boldsymbol{\rho}_2'}{2},\omega\right)\mu^{(0)}(\boldsymbol{\rho}_2'-\boldsymbol{\rho}_1',\omega), \tag{9}$$

where

$$S^{(0)}(\boldsymbol{\rho}',\omega)=A^2\exp(-\boldsymbol{\rho}'^2/2\sigma_S^2), \tag{10}$$

$$\mu^{(0)}(\boldsymbol{\rho}_2'-\boldsymbol{\rho}_1',\omega)=\exp\left[-\frac{(\boldsymbol{\rho}_2'-\boldsymbol{\rho}_1')^2}{2\sigma_\mu^2}\right]. \tag{11}$$

$S^{(0)}(\boldsymbol{\rho}',\omega)$ represents the spectral density of the field in the source plane, $\mu^{(0)}(\boldsymbol{\rho}_1'-\boldsymbol{\rho}_2',\omega)$ represents its spectral degree of coherence, A, σ_S, and σ_μ being constants. For a beam generated by such a source the spectral density at a point $(\boldsymbol{\rho},z)$ in free space is given by the formula ([9], Eq. (5.6-98))

$$S(\boldsymbol{\rho},z,\omega)=\frac{A^2}{\Delta^2(z,\omega)}\exp\left[-\frac{\rho^2}{2\sigma_S^2\Delta^2(z,\omega)}\right], \tag{12}$$

where

$$\Delta^2(z,\omega)=1+\alpha z^2 \tag{13}$$

is the so-called expansion coefficient of the beam, with

$$\alpha=\frac{1}{(k\sigma)^2}\left(\frac{1}{4\sigma_S^2}+\frac{1}{\sigma_\mu^2}\right). \tag{14}$$

If such a beam propagates in a turbulent atmosphere with a Kolmogorov power spectrum its spectral density is given by the expression [4]

$$S(\boldsymbol{\rho},z,\omega)=\frac{A^2}{\Delta_e^2(z,\omega)}\exp\left[-\frac{\rho^2}{2\sigma_S^2\Delta_e^2(z,\omega)}\right], \tag{15}$$

where the effective expansion coefficient is now given by the expression

$$\Delta_e^2=1+\alpha z^2+0.98(C_n^2)^{6/5}k^{2/5}\sigma_S^{-2}z^{16/5}. \tag{16}$$

Formula (15) was derived in Ref. [4] by use of the Extended Huygens–Fresnel principle for atmospheric turbulence described by Kolmogorov's model and is valid in both weak and strong fluctuation regimes.

The fractional power $p(\bar{\rho},z,\omega)$ of a Gaussian Schell-model beam in free space can be obtained by substituting from Eqs. (12) into Eq. (1). The corresponding expression for propagation in the atmosphere with Kolmogorov spectrum is obtained by substituting from Eq. (15) into Eq. (1).

Figure 3 shows the fractional power of Gaussian Schell-model beam propagating in free space and also in the atmosphere calculated from these formulas for several values of the ratio σ_μ/σ_S. When $\sigma_\mu\ll\sigma_S$ the source is fairly incoherent, when $\sigma_\mu\gg\sigma_S$ it is spatially highly coherent.

Some of our results for propagation in the turbulent atmosphere are also shown in Table 1. The data clearly reveal how coherence properties of the source and the strength of turbulence affect the critical distance at which the atmosphere begins to destroy the beam-like form of the radiated field.

We conclude by saying that we have formulated a criterion based on the concept of fractional transmitted power that makes it possible to test whether a beam of any state of coherence retains its beam-like form after propagating any particular distance through the turbulent atmosphere, and we have illustrated the results by several examples.

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