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It is shown how the phenomenon of correlation-induced spectral changes generated on scattering of a polychromatic plane wave on a spatially homogeneous random medium may be used to determine the correlation function of the scattering potential of the medium.

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Since the discovery in 1986 that the spectrum of light generated by partially coherent light sources may change on propagation even in free space [1] and subsequent demonstration that a similar effect may also arise in scattering [2], numerous papers on these subjects have been published [3]. However, no applications of these phenomena have been made up to now, although it was shown that when light is scattered by a so-called quasi-homogeneous medium, information about properties of the medium may be obtained from measurements of spectra and of the degree of coherence of the scattered field in the far zone of the scatterer [4,5]. It has also been suggested [6,7] that from measurements of correlation-induced spectral changes generated by scattering of light on a system of particles one might obtain structural information about the system; and explicit calculation for scattering from a many particle system with a high degree of symmetry has been reported in Ref. [8].

In the present Letter we consider what is probably the simplest inverse problem of this kind, namely that of determining the correlation function of a *homogeneous* random medium illuminated by a polychromatic plane wave with Gaussian spectral distribution, from the knowledge of the spectrum of the far field, generated by scattering on the medium.

Consider scattering of a polychromatic plane wave that propagates in a direction specified by a real unit vector $\mathbf{s}_0$ and is incident to a scattering medium occupying a finite domain D (see Fig. 1). The cross-spectral density function $W^{(i)}(\mathbf{r}_1,\mathbf{r}_2,\omega)$ of the incident field at points specified by position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ may be expressed in the form

$$W^{(i)}(\mathbf{r}_1,\mathbf{r}_2,\omega)=\langle U^{(i)^*}(\mathbf{r}_1,\omega)U^{(i)}(\mathbf{r}_2,\omega)\rangle, \tag{1}$$

where $U^{(i)}(\mathbf{r},\omega)$ is a member of a statistical ensemble of random functions, all of the form

$$U^{(i)}(\mathbf{r},\omega)=a(\omega)e^{ik\mathbf{s}_0\cdot\mathbf{r}}, \tag{2}$$

where $k=\omega/c$, ω denoting the angular frequency and c is the speed of light in vacuum. In Eq. (2) $a(\omega)$ is a (generally complex) random variable, and the angular brackets in Eq. (1) denote expectation value, taken over the ensemble of the incident field, the ensemble being understood in the sense of coherence theory in the space-frequency domain ([9], Sec. 4.1 and [10], Sec. 4.7).

Let $F(\mathbf{r},\omega)$ represent the scattering potential of the random medium. We assume that the scatterer is weak, so that the scattering may be analyzed within the accuracy of the first-order Born approximation ([11], Sec. 13.1.2). Let $C_F(\mathbf{r}_1,\mathbf{r}_2,\omega)$ be the correlation function of the scattering potential, viz.,

$$C_F(\mathbf{r}_1,\mathbf{r}_2,\omega)=\langle F^*(\mathbf{r}_1,\omega)F(\mathbf{r}_2,\omega)\rangle_m, \tag{3}$$

where the angle brackets with subscript m denote the average, taken over the ensemble of the scattering medium. If, as we now assume, the medium is homogeneous, the correlation function will have the form

$$C_F(\mathbf{r}_1,\mathbf{r}_2,\omega)\equiv C_F(\mathbf{r}_2-\mathbf{r}_1,\omega). \tag{4}$$

Let $S^{(i)}(\omega)=\langle a^*(\omega)a(\omega)\rangle$ represent the spectrum of the incident field and $S^{(\infty)}(r\mathbf{s},\omega)$ the spectrum of the scattered field at a point P in the far zone, at distance

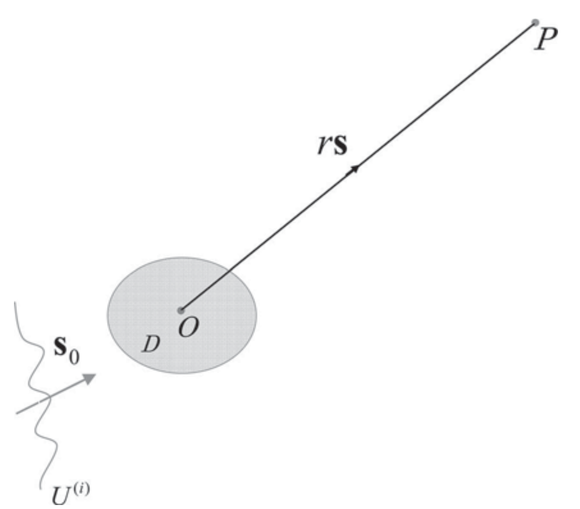

Fig. 1. Illustrating the notation.

r from an origin in the region of the scatterer, in the direction specified by the unit vector $\mathbf{s}$ (see Fig. 1). It follows at once from Eqs. (3.7) and (2.7) of Ref. [2] that

$$S^{(\infty)}(r\mathbf{s},\omega)=\frac{V}{r^2}\tilde{C}_F[k(\mathbf{s}-\mathbf{s}_0),\omega]S^{(i)}(\omega), \tag{5}$$

where V denotes the volume of the scatterer and

$$\tilde{C}_F(\mathbf{K},\omega)=\int C_F(\mathbf{r}',\omega)e^{-i\mathbf{K}\cdot\mathbf{r}'}d^3r' \tag{6}$$

is the three-dimensional Fourier transform of the correlation function $C_F(\mathbf{r}',\omega)$ of the scattering medium.

It is seen from Eq. (5) that with the spectrum $S^{(i)}(\omega)$ being known, measurements of the scattered field in direction $\mathbf{s}$ will provide the value of the Fourier component $\tilde{C}_F[\mathbf{K},\omega]$ of the scattering potential for

$$\mathbf{K}=k(\mathbf{s}-\mathbf{s}_0). \tag{7}$$

This formula is the analog in classical theory of the momentum transfer equation of potential scattering ([12], pp. 156, 210). From observations made in different directions of scattering and in different directions of incidence, $\mathbf{s}_0$, one can determine all the Fourier components of the correlation function of the scattering potential for which

$$|\mathbf{K}|\leqslant 2k, \tag{8}$$

i.e., all its "low" spatial-frequency components. This procedure is strictly analogous to the classic "Ewald-sphere construction" ([11], Sec. 13.1.2), which is used to determine the structure of crystals from measurements on the diffracted field [13].

We will illustrate the preceding analysis by an example. Let us first consider a direct scattering problem. Suppose that the correlation function of the scattering potential is the Gaussian distribution

$$C_F(\mathbf{r}',\omega)=\frac{A}{(2\pi\sigma^2)^{3/2}}\exp(-\mathbf{r}'^2/2\sigma^2). \tag{9}$$

Suppose further that the incident field is a polychromatic plane wave that propagates in the direction of a unit vector $\mathbf{s}_0$, whose spectral density is

$$S^{(i)}(\omega)=B\exp\left[-\frac{(\omega-\omega_0)^2}{2\Gamma_0^2}\right]. \tag{10}$$

It was shown in Ref. [2] that in this case the spectrum of the scattered field in the far zone is given by the expression

$$S^{(\infty)}(r\mathbf{s},\omega)=\frac{AV}{r^2}k^4 f(\theta,\omega)S^{(i)}(\omega), \tag{11}$$

where $k=\omega/c$ is the wavenumber associated with frequency ω. Apart from a constant, $f(\theta,\omega)$ is the scattering amplitude, given by the expression

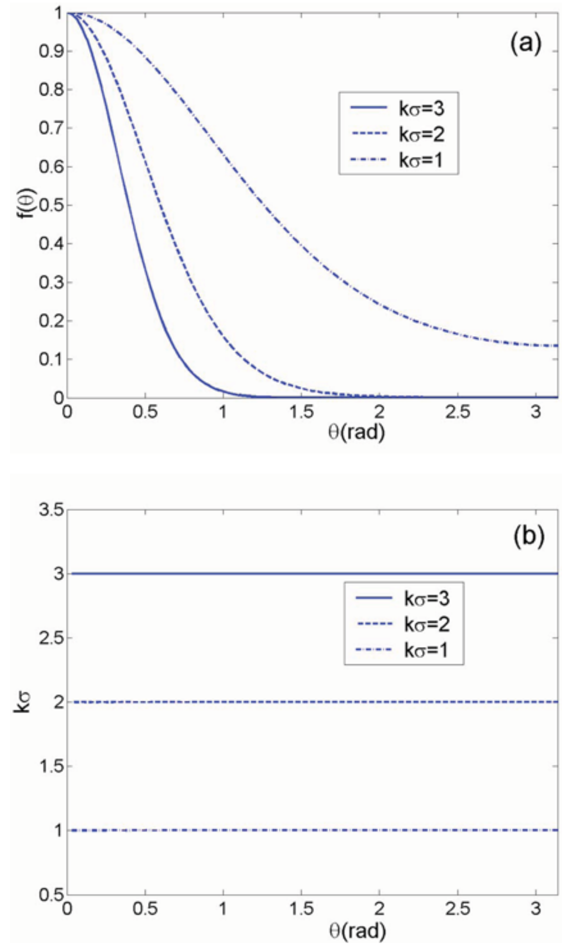

Fig. 2. (Color online) Scattering of a polychromatic plane wave with spectral density given by Eq. (10) on a medium whose correlation function is given by Eq. (9). (a) The direct problem: the scattering amplitude $f(\theta,\omega)$ [Eq. (12)]. (b) The inverse problem: the value of the effective normalized size parameter $k\sigma$, determined from the scattering amplitude $f(\theta,\omega)$ by the use of the inversion formula (13).

$$f(\theta,\omega)=\exp\left[-2k^2\sigma^2\sin^2\left(\frac{\theta}{2}\right)\right], \tag{12}$$

θ being the angle which the direction of scattering $\mathbf{s}$ makes with the direction of incidence $\mathbf{s}_0$, i.e., $\mathbf{s}\cdot\mathbf{s}_0=\cos\theta$.

Let us now consider the inverse problem. It follows from Eq. (12) that

$$k\sigma=\frac{1}{\sqrt{2}\sin\left(\frac{\theta}{2}\right)}\sqrt{-\ln f(\theta,\omega)}. \tag{13}$$

This formula provides solution to the inverse problem of determining the scaled r.m.s. width $k\sigma$ of the correlation function (9) of the scattering potential from measurements of the scattering amplitude. Because of the very simple nature of the correlation function of the scattering potential, assumed in this case [Eq. (9)], the parameter σ may be obtained from measurements of the scattering amplitude to *any* direction of scattering and for *any* frequency ω. Results of such calculations are illustrated by examples in Fig. 2.

Figure 2(a) presents plots of the scattering amplitude, given by Eq. (12), for scattering on a medium for which the correlation function of the scattering potential is given by Eq. (9), for the selected values of the effective, normalized size parameter $k\sigma$. Figure 2(b) shows the values $k\sigma$ obtained from the inversion formula (13). We see that indeed the correct values of $k\sigma$ were obtained from values of the scattering amplitude in a range of angles of scattering.

This simple example clearly demonstrates that correlation-induced spectral changes generated by scattering in random media may be used to provide information about the correlation function of the scatterer.

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