10-1-2006

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Received June 29, 2006; revised July 27, 2006; accepted July 27, 2006;
posted August 9, 2006 (Doc. ID 72467); published September 25, 2006

It is shown that two stochastic electromagnetic beams that propagate from the source plane $z=0$ into the half-space $z>0$ may have different degrees of polarization throughout the half-space, even though they have the same sets of Stokes parameters in the source plane. This fact is due to a possible difference in the coherence properties of the field in that plane, but other reasons are also possible. The result is illustrated by an example. © 2006 Optical Society of America

OCIS codes: 260.5430, 030.1690, 350.5500, 260.0260.

Some years ago, James[1] and, later, others[2–4] showed that the degree of polarization of a stochastic (i.e., random) electromagnetic field may change on propagation, even in free space. More recently it was shown both theoretically[5–7] and experimentally[8] that the degree of coherence of an electromagnetic beam at the pinholes, in a Young's interferometer, affects the degree of polarization of the field in the interference pattern. These results are rather surprising, because until the publication of these relatively recent papers, none of the numerous investigations about polarization of light, carried out since the publication of Stokes's classic papers on this subject more than 150 years ago, contained even a hint that the degree of polarization of a light beam might change on propagation.

To obtain a better insight into these "coherence-induced changes of the degree of polarization," we will analyze a somewhat more general problem than those treated previously. Specifically, we consider the following situation: A stochastic electromagnetic beam propagates from the plane $z=0$ (which we will call the source plane) into the half space $z>0$. As is well known, the state of polarization of the field at any point $\mathbf{r}$ in that half-space, at any frequency ω, may be characterized by four Stokes parameters (Ref. 9, Section 10.9.3), which we will denote by $s_0(\mathbf{r},\omega)$, $s_1(\mathbf{r},\omega)$, $s_2(\mathbf{r},\omega)$, and $s_3(\mathbf{r},\omega)$. The degree of polarization is given by the well-known expression [Ref. 9, p. 631, Eq. (68)],

$$P(\mathbf{r},\omega) = \sqrt{s_1^2(\mathbf{r},\omega) + s_2^2(\mathbf{r},\omega) + s_3^2(\mathbf{r},\omega)} \Big/ s_0(\mathbf{r},\omega). \quad (1)$$

The Stokes parameters do not obey any propagation laws. However, as was recently shown,[10] it is possible to determine how they change as the beam propagates by considering certain generalized Stokes parameters $S_\alpha(\mathbf{r}_1,\mathbf{r}_2,\omega)$ $(\alpha=0,1,2,3)$, defined by the formulas:

$$S_0(\mathbf{r}_1,\mathbf{r}_2,\omega) = \langle E_x^*(\mathbf{r}_1,\omega)E_x(\mathbf{r}_2,\omega)\rangle + \langle E_y^*(\mathbf{r}_1,\omega)E_y(\mathbf{r}_2,\omega)\rangle, \quad (2a)$$

$$S_1(\mathbf{r}_1,\mathbf{r}_2,\omega) = \langle E_x^*(\mathbf{r}_1,\omega)E_x(\mathbf{r}_2,\omega)\rangle - \langle E_y^*(\mathbf{r}_1,\omega)E_y(\mathbf{r}_2,\omega)\rangle, \quad (2b)$$

$$S_2(\mathbf{r}_1,\mathbf{r}_2,\omega) = \langle E_x^*(\mathbf{r}_1,\omega)E_y(\mathbf{r}_2,\omega)\rangle + \langle E_y^*(\mathbf{r}_1,\omega)E_x(\mathbf{r}_2,\omega)\rangle, \quad (2c)$$

$$S_3(\mathbf{r}_1,\mathbf{r}_2,\omega) = i[\langle E_y^*(\mathbf{r}_1,\omega)E_x(\mathbf{r}_2,\omega)\rangle - \langle E_x^*(\mathbf{r}_1,\omega)E_y(\mathbf{r}_2,\omega)\rangle]. \quad (2d)$$

Here, $E_x(\mathbf{r},\omega)$ and $E_y(\mathbf{r},\omega)$ are the Cartesian components of the complex electric field component of the electric vector at frequency ω, in two mutually orthogonal directions, perpendicular to the direction of the propagation of the beam, and the angular brackets denote an ensemble average in the sense of coherence theory in the space-frequency domain.[11]

The usual Stokes parameters are evidently special cases of these generalized Stokes parameters, viz.,

$$s_\alpha(\mathbf{r},\omega) = S_\alpha(\mathbf{r},\mathbf{r},\omega) \quad (\alpha = 0,1,2,3). \quad (3)$$

However, unlike the ordinary Stokes parameters, the generalized Stokes parameters obey precise propagation laws. For propagation in free space, one finds that in the paraxial approximation [Ref. 10, Eq. (7)],

$$S_\alpha(\mathbf{r}_1,\mathbf{r}_2,\omega) = \int\int_{z=0} S_\alpha^{(0)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2',\omega) G^*(\mathbf{r}_1 - \boldsymbol{\rho}_1', z;\omega) \times G(\mathbf{r}_2 - \boldsymbol{\rho}_2', z;\omega) \mathrm{d}^2\boldsymbol{\rho}_1' \mathrm{d}^2\boldsymbol{\rho}_2', \quad (4)$$

Here, $\boldsymbol{\rho}_1'$, $\boldsymbol{\rho}_2'$ are position vectors of two points in the source plane and G is the free-space paraxial propagator of the Helmholtz operator $G(\boldsymbol{\rho}-\boldsymbol{\rho}',z;\omega) = -(ik/2\pi z)\exp[ik(\boldsymbol{\rho}-\boldsymbol{\rho}')/2z]$ for propagation from the points $\boldsymbol{\rho}_1'$, $\boldsymbol{\rho}_2'$ in the source plane to two field points $\mathbf{r}_1=(\boldsymbol{\rho}_1,z)$ and $\mathbf{r}_2=(\boldsymbol{\rho}_2,z)$ in the plane at distance z from the source, $k=\omega/c$, c is the speed of light in vacuum.

Before proceeding further we recall that, according to Eq. (13) of Ref. 10, the spectral degree of coherence of the field at any two points $\mathbf{r}_1$, $\mathbf{r}_2$ in the half-space $z>0$ is given by the expression

$$\eta(\mathbf{r}_1,\mathbf{r}_2,\omega) = \frac{S_0(\mathbf{r}_1,\mathbf{r}_2,\omega)}{\sqrt{S_0(\mathbf{r}_1,\mathbf{r}_1,\omega)}\sqrt{S_0(\mathbf{r}_2,\mathbf{r}_2,\omega)}}. \quad (5)$$

We see that the degree of coherence is entirely determined by a single generalized Stokes parameter, namely, $S_0(\mathbf{r}_1,\mathbf{r}_2,\omega)$.

Consider now two sources both located in the plane $z=0$. We distinguish the parameters relating to them by superscripts (1) and (2). Suppose that the two sources have the same sets of Stokes parameters, i.e., that

$$s_\alpha^{(2)}(\boldsymbol{\rho}',\omega) = s_\alpha^{(1)}(\boldsymbol{\rho}',\omega) \quad (\alpha=0,1,2,3), \quad (6)$$

but that

$$S_\alpha^{(2)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2',\omega) \neq S_\alpha^{(1)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2',\omega) \quad (\alpha=0,1,2,3). \quad (7)$$

Equation (6) implies that the two sources have the same sets of the four Stokes parameters, and hence, according to Eqs. (1) and (6), they have the same degree of polarization, i.e.,

$$P^{(2)}(\boldsymbol{\rho}',\omega) = P^{(1)}(\boldsymbol{\rho}',\omega). \quad (8)$$

However, Eqs. (5) and (7) imply that the two sources will have different coherence properties, i.e., that $\eta^{(2)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2',\omega) \neq \eta^{(1)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2',\omega)$.

Let us now consider the field in the half-space $z>0$. According to Eqs. (3), (4), and (7), we then have, at any point $\mathbf{r}$ in that half-space,

$$s_\alpha^{(2)}(\mathbf{r},\omega) \neq s_\alpha^{(1)}(\mathbf{r},\omega) \quad (\alpha=0,1,2,3), \quad (9)$$

except possibly at some particular points. Consequently, it follows from Eqs. (1) and (8) that, in general, $P^{(2)}(\mathbf{r},\omega) \neq P^{(1)}(\mathbf{r},\omega)$ $(z>0)$, i.e., the degrees of polarization of the two beams will be different throughout the half-space into which they propagate.

We will illustrate this result by an example. Consider two electromagnetic Gaussian Schell-model sources (see, for example, Ref. 3) with diagonal elements of the cross-spectral density matrices given by the formulas

$$W_{ii}^{(m)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2',\omega) = A_i^2 \exp\left[-\frac{\boldsymbol{\rho}_1'^2+\boldsymbol{\rho}_2'^2}{4\sigma^2}\right] \times \exp\left\{-\frac{(\boldsymbol{\rho}_2'-\boldsymbol{\rho}_1')^2}{2[\delta_i^{(m)}]^2}\right\} \quad (m=1,2,\ i=x,y). \quad (10)$$

A_i, σ, and $\delta_i^{(m)}$ are constants, while the off-diagonal elements of the matrices are zeros. Let us assume that $A_i=1$ and that

$$\delta_x^{(1)} = \delta_x^{(2)} \equiv \delta_x \neq \delta_y^{(1)} \neq \delta_y^{(2)}. \quad (11)$$

The Stokes parameters of the two sources are

$$s_0^{(m)}(\boldsymbol{\rho}',\omega) = 2\exp[-\boldsymbol{\rho}'^2/2\sigma^2], \quad s_{1,2,3}^{(m)}(\boldsymbol{\rho}',\omega) = 0, \quad (12)$$

i.e., they are the same for any point $\boldsymbol{\rho}'$ of the source plane. Consequently, the degrees of polarization of the two sources are also the same and are equal to zero, i.e., $P^{(1)}(\boldsymbol{\rho}',\omega)=P^{(2)}(\boldsymbol{\rho}',\omega)=0$ indicating that the sources are unpolarized.

According to Eqs. (2) and (10), the generalized Stokes parameters of the two sources are

$$S_{0,1}^{(m)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2',\omega) = \exp\left[-\frac{\boldsymbol{\rho}_1'^2+\boldsymbol{\rho}_2'^2}{4\sigma^2}\right] \times \exp\left[-\frac{(\boldsymbol{\rho}_2'-\boldsymbol{\rho}_1')^2}{2\delta_x^2}\right] \pm \exp\left[-\frac{\boldsymbol{\rho}_1'^2+\boldsymbol{\rho}_2'^2}{4\sigma^2}\right]\exp\left\{-\frac{(\boldsymbol{\rho}_2'-\boldsymbol{\rho}_1')^2}{2[\delta_y^{(m)}]^2}\right\},$$

$$S_2^{(m)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2',\omega) = S_3^{(m)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2',\omega) = 0, \quad (m=1,2). \quad (13)$$

From Eq. (5), for the spectral degree of coherence $\eta(\mathbf{r}_1,\mathbf{r}_2,\omega)$, one readily finds that for the two sources,

$$\eta^{(m)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2',\omega) = \exp[-(\boldsymbol{\rho}_2'-\boldsymbol{\rho}_1')^2/2\delta_x^2] + \exp[-(\boldsymbol{\rho}_2'-\boldsymbol{\rho}_1')^2/2(\delta_y^{(m)})^2], \quad (m=1,2). \quad (14)$$

Using Eqs. (13) and (14) and the conditions (11), one can see that the generalized Stokes parameters and also the spectral degrees of coherence of the two sources differ from each other.

Let us determine the Stokes parameters of the two beams at a point $(\boldsymbol{\rho}_1=0,\ \boldsymbol{\rho}_2=0,\ z>0)$, i.e., at an arbitrary point along the axis of the beam in the half-space $z>0$. On substituting from Eq. (13) into the propagation law (4), for the generalized Stokes parameters in the source plane, performing the integration, and evaluating the resulting expression for the point $\boldsymbol{\rho}_1=\boldsymbol{\rho}_2=0$, we find that

$$s_{0,1}^{(m)}(0,z,\omega) = \frac{1}{\Delta_x^2(z)} \pm \frac{1}{[\Delta_y^{(m)}(z)]^2}, \quad (15a)$$

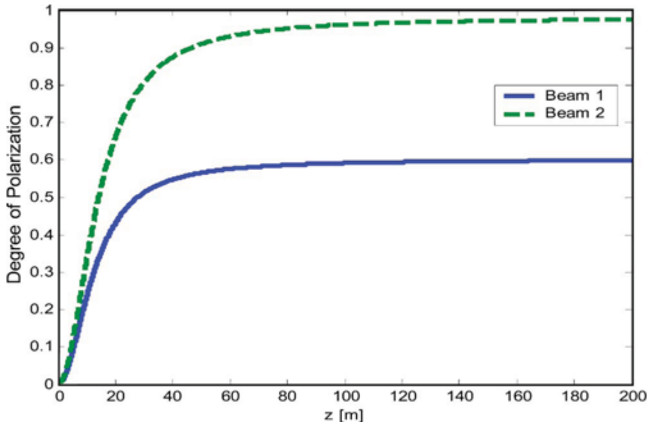

Fig. 1. Degree of polarization, calculated from Eq. (18), of beams generated by two unpolarized sources along the axis as a function of the propagation distance z from the source plane. The parameters of the two sources have been chosen as: Beam 1, $\sigma=1$ cm, $\lambda=0.633$ μm, $\delta_x=0.1$ mm, $\delta_y^{(1)}=0.2$ mm. Beam 2, $\sigma=1$ cm, $\lambda=0.633$ μm, $\delta_x=0.1$ mm, $\delta_y^{(2)}=1$ mm.

$$s_2^{(m)}(0,z,\omega) = s_3^{(m)}(0,z,\omega) = 0, \qquad (15b)$$

where

$$\Delta_x^2(z) = 1 + \frac{z^2}{k^2\sigma^2}\left(\frac{1}{4\sigma^2} + \frac{1}{\delta_x^2}\right),$$

$$[\Delta_y^{(m)}(z)]^2 = 1 + \frac{z^2}{k^2\sigma^2}\left(\frac{1}{4\sigma^2} + \frac{1}{[\delta_y^{(m)}]^2}\right) \quad (m=1,2). \qquad (16)$$

From conditions (11) and Eq. (16), it follows that $\Delta_y^{(1)}(z) \neq \Delta_y^{(2)}(z)$ and, consequently, it follows from Eqs. (15) that

$$s_0^{(2)}(0,z,\omega) \neq s_0^{(1)}(0,z,\omega), \quad s_1^{(2)}(0,z,\omega) \neq s_1^{(1)}(0,z,\omega), \qquad (17)$$

i.e., the Stokes parameters s_0 of the two beams are different, and the same is true about the Stokes parameters s_1 while according to the last two formulas in Eq. (15), the last two parameters are the same. Thus we have shown that two of the Stokes parameters of the beams generated by the two sources are different, at least along the axis. The degrees of polarization of the two beams along the axis are given by the formulas:

$$P^{(m)}(0,z,\omega) = \left|\frac{1}{\Delta_x^2(z)} - \frac{1}{[\Delta_y^{(m)}(z)]^2}\right| \Bigg/ \left(\frac{1}{\Delta_x^2(z)} + \frac{1}{[\Delta_y^{(m)}(z)]^2}\right) \quad (m=1,2). \qquad (18)$$

Figure 1 shows how the degrees of polarization, calculated from Eq. (18), of the beams generated by the two unpolarized sources, vary along the axis.

In conclusion, we may say that we have shown that two stochastic electromagnetic beams propagating from the plane $z=0$ into the half-space $z>0$ and which have the same Stokes parameters in that plane will, in general, have different degrees of polarization in the half-space. The difference is due to the coherence properties of the source, but other reasons for such a difference (not discussed in this paper) are also possible.

The research was supported by the U.S. Air Force Office of Scientific Research under grant FA9550-06-1-0032, by the Engineering Research Program of the Office of Basic Energy Sciences at the U.S. Department of Energy under grant DE-FG02-2ER45992, and by the Air Force Research Laboratory (AFRC) under contract FA 95550-06-0032. O. Korotkova's e-mail address is korotkov@pas.rochester.edu.

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