Initial Condition Sensitivity and Dynamical Mechanisms of Perturbation Growth in Tropical Cyclones

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INITIAL CONDITION SENSITIVITY AND DYNAMICAL MECHANISMS OF PERTURBATION GROWTH IN TROPICAL CYCLONES

By

Munehiko Yamaguchi

A THESIS

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INITIAL CONDITION SENSITIVITY AND DYNAMICAL MECHANISMS OF PERTURBATION GROWTH IN TROPICAL CYCLONES

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Studying dynamical mechanisms of perturbation growth in tropical cyclones is important from a perspective of designing ensemble prediction system and adaptive observations for tropical cyclones. In this thesis, the role of perturbations in ensemble forecasting and adaptive observations for tropical cyclones is investigated, especially focusing on tropical cyclone tracks. For this purpose, ensemble initial perturbations from operational numerical weather prediction centers are first diagnosed, followed by a study in which the structure and location of singular vectors computed for tropical cyclone-like vortices in a nondivergent barotropic framework are investigated.  

The three most significant findings of this study are that 1) perturbations grow in the vicinity of tropical cyclones through both baroclinic and barotropic energy conversion as seen in mid-latitude dynamics, that 2) those energy conversions lead to the modification in tracks of tropical cyclones, and that 3) the structure and the location of growing perturbations are sensitive to initial temperature and wind profiles of the tropical cyclone vortex.  

The above results (1) and (2) determine a mechanism how the ensemble spread of tropical cyclone tracks evolves with time in ensemble prediction systems at operational
numerical weather prediction centers. In addition, those results identify the reason why the ensemble spread differs from one ensemble prediction system to another. The result (3) gives an insight into understanding what sensitivity analysis guidance targeted for tropical cyclones represents and helps to optimize observation network for tropical cyclones.
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Chapter 1

Introduction

The skill of tropical cyclone (TC) track prediction has improved significantly over the last few decades, due in large part to improvement in numerical weather prediction (NWP) models, data assimilation schemes, and enhanced observations as obtained through satellites and aircraft. However, significant errors can still exist, and are often the result of initial condition errors. For example, errors in 3-day predictions for individual forecasts can vary between less than 50 km and more than 1000 km. For this reason, ensemble techniques are expected to provide useful information on the uncertainty of track predictions, via the ensemble spread, which may differ from one TC to another, and from one initial time to another. Goerss (2000) and Elsberry and Carr (2000) demonstrated the benefit of the ensemble spread as a measure of confidence in ensemble TC predictions; a small spread of tracks is often an indication of a small error of an ensemble mean track prediction. Puri et al. (2001) illustrated reasonable forecast performance in the spread of TC tracks and intensities, using the European Centre for Medium-Range Weather Forecasts (ECMWF) Ensemble Prediction System (EPS; Molteni et al. 1996; Leutbecher and Palmer 2008). The ECMWF EPS uses targeted adiabatic (dry) singular vectors \(^1\) (SVs; Mureau et al. 1993; Buizza 1994; Buizza and Palmer 1995; Palmer et al. 1998) and also targeted diabatic

\(^1\)SVs give the fastest growing perturbation over a prescribed time interval called optimization time under an assumption that the perturbation grows linearly. Mathematical expressions are given in Section 3.2.
(moist) SVs (Barkmeijer et al. 2001) to create ensemble initial perturbations. Majumdar and Finocchio (2010) demonstrated that the ECMWF ensemble exhibited the ability to predict track probabilities for Atlantic Basin tropical cyclones in 2008. Yamaguchi et al. (2009a) constructed a new EPS, the Typhoon EPS, at the Japan Meteorological Agency (JMA), following the philosophy of ECMWF’s moist SV-based EPS. They showed that the ensemble spread can be an effective predictor of confidence information. When the ensemble spread was predicted to be small, the track error was found to be small. When the predicted ensemble spread was large, there was a possibility that the track error would be large.

While the validity of ensemble TC track predictions has been verified in various EPSs, those predictions sometimes contradict each other. Figure 1.1 shows an example in which one EPS predicts relatively small ensemble spread while the other possesses large ensemble spread and vice versa. It is hypothesized that these differences are attributed to the different methods of creating initial perturbations, resulting in different growth of the perturbations. In addition, initial amplitudes of the perturbations may affect the size of the ensemble spread, especially in the early forecast stage. A systematic intercomparison of global model ensembles for TCs has hitherto been difficult because of the limited access to such operational data. However, the recently established THORPEX Interactive Grand Global Ensemble (TIGGE) database makes it possible to conduct an intercomparison study and verify such hypotheses as proposed above.

In Chapter 2, ensemble initial perturbations in and around Typhoon Sinlaku (2008), one of the typhoons heavily sampled during the THORPEX Pacific Asian Regional Campaign (T-PARC) and Tropical Cyclone Structure 2008 (TCS-08) field campaigns, are compared using ECMWF, NCEP and JMA ensembles which are available on the TIGGE database. Vertical and horizontal distributions of perturbation wind, temperature and spe-

\[2\] Typhoon EPS of JMA is not available at the TIGGE website, so the JMA ensemble represents the EPS for the medium-range forecasts.

\[3\] The Met Office (UKMO) is another major operational NWP center. UKMO was excluded from the comparison because the initial perturbations are gradually added to the non-perturbed field over the first 6-h
specific humidity as well as their amplitudes are compared. Moreover, the dynamical mechanisms of the perturbation growth are investigated by comparing the ECMWF and NCEP ensembles to understand how the perturbations change the steering flow and symmetric and asymmetric circulations of Sinlaku. A statistical verification is then conducted to identify whether there exists a relationship between the ECMWF and NCEP ensemble spread of tracks during the 2007 and 2008 seasons and to establish how the initial perturbations and their growth affect the ensemble spread of tracks in each EPS.

Studying dynamical mechanisms of perturbation growth in TCs is important not only from a perspective of designing ensemble prediction systems but also adaptive observations. Since analysis errors in initial conditions for NWP might contain a portion of the growing perturbations, it is believed that, by observing the areas of the growing perturbations and assimilating the observations, the analysis errors that might evolve into large forecast errors would be reduced. As a result, better forecast performance can be expected compared with forecasts that do not include the additional, targeted observations.

The SV method is one of the methods to identify the growing perturbations, and it is used to give sensitivity analysis guidance for adaptive observations (Palmer et al. 1998; Buizza and Montani 1999). This is because SVs corresponding to large singular values represent fast growing perturbations (in a linear sense) over a prescribed optimization time. In order to test the feasibility of SVs as sensitivity analysis guidance for TC prediction, SVs were actually used for targeting in T-PARC, that is, dropsondes were deployed where the vertically accumulated total energy of SVs was large. Prior to T-PARC, Yamaguchi et al. (2009b) conducted an observing system experiment (OSE) to verify the usefulness of JMA SVs for targeting observations like those made in T-PARC. The OSE was performed from an initial time (Bloom et al. 1996), making the comparison difficult. See Bowler et al. (2008) for the initial perturbations of the UKMO ensemble.

4Since the most of the airborne observations during T-PARC and TCS-08 were conducted centered on 0000 UTC, and a follow-up study will focus on comparisons between the typhoon structure in the models and the airborne observations, and the initial perturbations versus sensitivity analysis guidance used to select targeting observations (e.g. Majumdar et al. 2006; Wu et al. 2009), the JMA ensemble, which is initiated only at 1200 UTC, was excluded from the comparison.

5The feasibility is currently being evaluated at many operational and research institutes.
for Typhoon Conson (2004) using dropwindsonde observations from Dropwindsonde Observation for Typhoon surveillance near the Taiwan Region (DOTSTAR). It is found that DOTSTAR observations had a positive impact on the track forecast of Conson, and that the observations within the sensitive region were sufficient to predict the northeastward movement of Conson.

In a previous study on SVs in the vicinity of TC-like vortices, Nolan and Farrell (1999) illustrated the non-modal growth of SVs through the Orr mechanism (Orr 1907) using a barotropic model. The SVs are initially tilted against the shear of the background angular velocity and obtain eddy kinetic energy through wave-mean flow interaction. The growth continues until the up-shear property vanishes. Peng and Reynolds (2006) showed that initial SVs of the U.S. Navy Operation Global Atmosphere Prediction System (NOGAPS) appear about 500 km from storm centers where the radial potential vorticity gradient changes sign (a necessary condition for the barotropic instability of normal modes). It is found that such a condition can be created by the influence of surrounding synoptic features. Peng et al. (2009) identified exponentially growing perturbations (not SVs) in the outer region of a TC-like vortex, which satisfy the necessary condition of the barotropic instability of normal modes. The up-shear tilt of the asymmetric vorticity causes energy transfer continuously from the symmetric mean flow to the asymmetric disturbances. In addition, as will be illustrated in this study, baroclinic energy conversion in a vortex can be a mechanism for SVs to grow in the vicinity of TCs (Yamaguchi and Majumdar 2010). Streamfunction and temperature perturbations in a cylindrical coordinate system centered on a TC collocate with the temperature perturbation 90 degrees ahead of the streamfunction perturbation. Consequently, the perturbation kinetic energy can grow with time through the conversion of azimuthal mean available potential energy into eddy available potential energy and then the conversion of eddy available potential energy into eddy kinetic energy.

The growth of SVs has been studied in many contexts. However, given the weight of the prior work on SVs for mid-latitude dynamics, the basic properties of SVs targeted
for TCs are still not fully understood. In addition, it is hypothesized that the structures and locations of SVs would be highly dependent on initial vortex and temperature profiles and configurations of SV calculations such as 1) optimization time, 2) norm by which the growth of SVs is optimized, and 3) resolution of numerical model for SV computations. It is also hypothesized that the choice of those configurations is critical to determining what physical processes the computed SVs are associated with (e.g. TC motion, intensity, genesis, etc.). In order to provide a fundamental understanding and to offer new hypotheses on the dynamical mechanisms of SV growth in TCs, it is worthwhile to compute SVs under idealized conditions using simplified barotropic and baroclinic models. As a first step of a series of these studies, in this thesis, SVs are computed on a f-plane in an nondivergent barotropic framework. Using an f-plane nondivergent barotropic model is a good starting point for understanding the basic properties of SVs in the vicinity of TCs.

In Chapter 3, SVs are calculated for TC-like vortices using a two dimensional numerical model: a barotropic version of the SPECTRAL ELEMENT OCEAN MODEL (Iskandarani et al. 1995). By considering various initial conditions, the following issues are investigated; 1) sensitivity of SVs to maximum wind speed and radius of maximum wind (RMW) of initial TC-like vortices, 2) sensitivity of SVs to optimization time, and 3) sensitivity of SVs to viscosity. In addition, SVs are calculated for initial vortices which satisfy the necessary condition of the barotropic instability to see if SVs can capture the instability. Furthermore, SVs are computed for an asymmetric initial vortex to account for a more realistic vortex structure.

This thesis is organized as follows. Chapter 2 describes the comparison of initial ensemble perturbations from operational NWP centers for Typhoon Sinlaku (2008), and demonstrates the dynamical mechanisms for the perturbation growth. Chapter 3 describes the characteristics of SVs which are calculated for idealized initial vortices using a barotropic model. Chapter 4 presents a summary and discusses future work.
Figure 1.1  Ensemble track forecasts (gray lines) by the ECMWF (left) and NCEP (right) ensembles for Typhoon Sinlaku initialized at 1200 UTC of 10th September 2008 (top) and Typhoon Dolphin initialized at 0000 UTC of 13th December 2008 (bottom). The black line is the best track. The black triangles are the forecast positions at 120-h.
Chapter 2

Using TIGGE data to diagnose initial perturbations and their growth for tropical cyclone ensemble forecasts

2.1 Synopsis of Typhoon Sinlaku (2008)

According to the Regional Specialized Meteorological Center (RSMC) Tokyo - Typhoon Center (RSMC Tokyo - Typhoon Center 2008), Sinlaku formed as a tropical depression (TD) over the northwestern Pacific Ocean east of Luzon Island, Philippines at 0000 UTC of 8th September 2008. Moving towards the north-northwest, it was upgraded to tropical storm (TS) intensity over the same waters at 1800 UTC that day. Keeping its north-northwestward track, Sinlaku was upgraded to typhoon (TY) intensity and reached its peak intensity with maximum sustained winds of 100 kt and a central pressure of 935 hPa over the sea northeast of Luzon Island at 1200 UTC of 10th September. Weakening its intensity and turning to the northwest, Sinlaku was downgraded to severe tropical storm (STS) intensity around northern Taiwan at 0600 UTC of 14th September. After recurvature, it was upgraded again to TY intensity off the southern coast of Shikoku Island, Japan at 0000 UTC of 19th September. Keeping its east-northeastward track, Sinlaku transformed into an extratropical cyclone east of Japan at 0000 UTC of 21 September. The track, minimum sea level pressure and sustained maximum wind through Sinlaku’s life cycle analyzed by RSMC Tokyo - Typhoon Center are shown in Fig. 2.1.
The synoptic environments around Sinlaku at 0000 UTC of 10th, 15th and 18th September 2008, which are based on the analysis field of the non-perturbed member of the ECMWF ensemble, are presented in Fig. 2.2. On 10th September (hereafter referred to as the before-recurrence stage of Sinlaku), Sinlaku was located west of the Pacific High. As will be seen in Fig. 2.7, Sinlaku slowly moved northward by the steering flow associated with the Pacific High. On 15th September (hereafter referred to as the during-recurrence stage of Sinlaku), Sinlaku was located in a confluent area induced by the westerly jet and the southerly flow at the west edge of the Pacific High. On 18th September (hereafter referred to as the after-recurrence stage of Sinlaku), Sinlaku was situated between these features; it was located north of the Pacific High and south of the westerly jet, being advected by the confluent westerlies.

2.2 Ensemble Prediction Systems at ECMWF, NCEP and JMA

This section gives a brief overview of the techniques used to create initial perturbations at ECMWF, NCEP and JMA, mainly referring to Leutbecher and Palmer (2008) for ECMWF EPS, Wei et al. (2008) for NCEP EPS, and WMO (2008) for JMA EPS. The techniques are philosophically similar in a sense that the initial perturbations are generated to represent initial uncertainty, but different in terms of the framework and the design of the EPS such as the perturbed areas and a norm that constrains the structures of the perturbations. Table 2.1 summarizes those differences as well as other specifications including the model resolution and the ensemble size.

2.2.1 ECMWF singular vector method

ECMWF creates initial perturbations based on the SV method. SVs with large singular values represent fast-growing perturbations over a prescribed time interval (optimization time) under the assumption that the perturbations grow linearly (Lorenz 1965).
The fast-growing perturbations are considered to be responsible for large forecast uncertainty at the optimization time, thus they lead to sufficient dispersion in the most uncertain directions. The SVs are computed with an optimization of a total energy norm:

\[
\frac{1}{2} \int_{p_0}^{p_1} \int_A (u'^2 + v'^2 + \frac{c_p T'}{T_r} T'^2 + w_q \frac{L_c^2}{c_p T_r} q'^2) dP dA + \frac{1}{2} \int_A \frac{R_d T_r}{P_r} (\ln P')^2 dA
\]  

(2.1)

where \(u', v', T', q'\) and \(P'\) are perturbations of zonal velocity, meridional velocity, temperature, specific humidity and surface pressure, respectively. \(c_p\) is the specific heat of dry air at constant pressure, \(L_c\) is the latent heat of condensation and \(R_d\) is the gas constant for dry air. \(T_r = 300\text{K}\) is a reference temperature, \(P_r = 800\text{ hPa}\) is a reference pressure and \(\int_A\) and \(\int_{p_0}^{p_1}\) denote the horizontal and vertical integrations in a pressure coordinate system. \(w_q\) is a parameter which determines the relative weight of the specific humidity perturbation. ECMWF adopts \(w_q = 0\), so the initial perturbations do not include a specific humidity component.

Dry SVs are targeted for each hemisphere with an optimization time interval of 48 h. The vertical integration of the total energy is limited up to about 100 hPa at a final-time norm. Note that 48 h linearly evolved dry SVs (Barkmeijer et al. 1999; Puri et al. 2001) are also used to construct initial perturbations in order to represent slowly growing perturbations related to large scale flows (there is no constraint on the vertical levels for the evolved SVs). In addition, moist SVs are computed targeted for TCs with an optimization time interval of 48 h \(^6\). The vertical integration of the total energy is limited up to about 500 hPa for the final-time norm. Unlike SVs in the extratropics, the evolved moist SVs are not considered in creating initial perturbations. Note that ECMWF adopts a stochastic physics technique (Buizza et al. 1999). As studied by Puri et al. (2001), however, the stochastic perturbations have little influence on ensemble track predictions.

\(^6\)In order to avoid a duplication, the singular vectors targeted on a tropical cyclone are computed in the subspace orthogonal to the space spanned by the leading 50 extra-tropical singular vectors (Leutbecher 2007).
In order to generate initial perturbations from the initial dry and moist SVs and evolved dry SVs, a Gaussian sampling technique is used, and a total of 50 perturbations are created (a linear combination of 25 orthogonal perturbations are added to and subtracted from the analysis field). The amplitude of the perturbations is determined in an empirical way so that the ensemble predictions have an appropriate ensemble spread for the medium-range forecasts.

2.2.2 NCEP ensemble transform technique

The ensemble transform (ET) technique is the successor of the breeding method at NCEP (Toth and Kalnay 1993; 1997), and was first proposed by Bishop and Toth (1999) for targeting observation studies. The ET method produces initial perturbations along directions that are constrained by the global distribution of analysis error variance. The initial perturbations are computed in a matrix $Z^a$, which is obtained through

$$Z^a = Z^f T$$

(2.2)

where $T$ is an ensemble transformation matrix, and $Z^f$ is a matrix whose columns comprise the differences of state vectors between an ensemble mean and a perturbed forecast. The transformation matrix $T$ is designed so that the initial perturbations reflect the analysis error variances of the data assimilation scheme at NCEP. Unlike SVs where the linear growth of the perturbations over an optimization time interval of 48-h is assumed, the ET method does not explicitly solve for perturbation growth.

The NCEP ensemble does not adopt the plus-minus symmetric addition of initial perturbations used at ECMWF. For the amplitude of the perturbations, the NCEP ensemble considers a regionally varying rescaling to ensure that the amplitude varies in accordance with the uncertainties in the analysis. Finally, the amplitude is empirically adjusted so that the ensemble achieves sufficient spread in the predictions.
2.2.3 JMA singular vector method

JMA also uses the SV method. The main differences from that of ECMWF are the targeted areas and the norm. Rather than for each TC, moist SVs are targeted for the whole tropics (20°S - 30°N), while dry SVs are targeted for the extratropical northern hemisphere only (30°N - 90°N). The total energy norm includes the specific humidity term, and $w_q = 0.04$ is used for both dry and moist SVs. In the initial norm, the vertical integration of the total energy is limited up to about 5 hPa for wind and temperature, and 700 hPa for specific humidity perturbations.

Initial perturbations are created by linearly combining initial dry and moist SVs, 48-h linearly evolved dry SVs and 24-h linearly evolved moist SVs so that the spatial distribution of perturbations becomes large (like ECMWF, there is no constraint on the vertical levels for the evolved SVs). Like the ECMWF and NCEP ensembles, the amplitudes of the perturbations are determined in a statistical way in order to obtain sufficient ensemble spread in the predictions.

2.3 Comparison of initial perturbations

2.3.1 Vertical and horizontal distributions of initial perturbations

The vertical and horizontal distributions of the initial perturbation kinetic energy, the first and second terms in (2.1), are displayed in Fig. 2.3 for the ECMWF, NCEP and JMA ensembles before, during and after recurvature. The horizontal resolution and vertical levels of the TIGGE data are $0.5° \times 0.5°$ and 1000, 925, 850, 700, 500, 300, 250 and 200 hPa, respectively. Prior to computing the perturbation kinetic energy averaged over all the ensemble members, each ensemble member is shifted to a storm-relative Cartesian

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7Note that the verification for the JMA ensemble is based on initial time 1200 UTC because the JMA ensemble is initiated only at 1200 UTC, so the corresponding before-, during- and after-recurrence stages represent 1200 UTC of 10th, 15th and 18th September 2008, respectively.

8The user may specify the horizontal resolution on the TIGGE website.
coordinate system, with the central position determined by the location of the minimum streamfunction of the non-perturbed member at 850 hPa. For the vertical distribution, the perturbation kinetic energy at each level is averaged over the all ensemble members over a 2000 km × 2000 km domain centered on Sinlaku. The horizontal distributions are computed by averaging the energy over the all ensemble members and over the all vertical levels. The corresponding vertical and horizontal distributions of initial perturbation available potential energy, the third term in (2.1), are shown in Fig. 2.4. Similarly, the vertical and horizontal distributions of perturbation specific humidity energy, the fourth term in (2.1), are presented in Fig. 2.5 ($w_q = 1$ in this study). This part of the comparison is only for NCEP and JMA because ECMWF does not include initial perturbations of specific humidity.

ECMWF perturbs only wind and temperature. In the before-recurvature stage, the ECMWF wind perturbation has a peak at 700 hPa on average and is largest in the near environment of Sinlaku (see Fig. 2.3). Looking at each ensemble member (not shown), the maximum amplitude is found to be 4.4 m s$^{-1}$, appearing about 700 km away from the typhoon center while the amplitude within 100 km from the typhoon center is only 1.6 m s$^{-1}$ at most. As the typhoon moves northward, the amplitude of the wind perturbation above 500 hPa becomes larger, corresponding to the change in the area of highest amplitude from the typhoon surroundings to the synoptic features north of the typhoon (see Fig. 2.2). This would imply the upward energy transfer and the conversion of the available potential energy into the kinetic energy by the evolved dry SVs (see Fig. 2.3; Hoskins et al. 2000; Montani and Thorpe 2002). As mentioned before, the ECMWF ensemble considers the evolved SV when creating the initial perturbations. The amplitude of the kinetic energy in the upper troposphere becomes large as Sinlaku moves north due to the energy transfer from the initial-SV available potential energy to the evolved-SV kinetic energy. As with the wind perturbation, the temperature perturbation also has a peak in the mid-troposphere (e.g., the maximum amplitude in the before-recurvature stage is 2.6 K at 500 hPa and about 500 km
away from the typhoon center, implying that the perturbation has little change in the warm core structure in the inner region). The area of the highest temperature perturbation shifts from the typhoon surroundings to the synoptic features as the typhoon moves northward. However, in contrast to the wind perturbation, the altitude with the maximum amplitude of the temperature perturbation is confined to around 500 hPa throughout the life of the typhoon. The vertical profiles of the wind and temperature perturbations are quite similar to those of perturbations seen in the Typhoon EPS at JMA, which also uses moist SVs targeted for TCs (Yamaguchi et al. 2009a).

NCEP perturbs all components: wind, temperature and specific humidity. The amplitude of the wind perturbation is larger than ECMWF, especially in the upper troposphere. For example, the amplitude (not energy) is 9.2 times as large as ECMWF at 200 hPa in the before-recurrence stage; the amplitude averaged over the 2000 km × 2000 km domain about the typhoon center is 3.4 m s\(^{-1}\) (0.33 m s\(^{-1}\) for the ECMWF ensemble and 0.41 m s\(^{-1}\) for the JMA ensemble). This trend is common in the other stages. In the before-recurrence stage, there are large amplitudes in the temperature and specific humidity perturbations within about 300 km from the typhoon center. Looking at each ensemble member (not shown), the maximum amplitude of temperature perturbation is found to be 2.1 K at 250 hPa while that of specific humidity perturbation is 1.8 kg\(^{-1}\) at 700 hPa. Considering that the temperature anomaly due to the warm core structure in the non-perturbed field is about 4.0 K at 250 hPa, the temperature perturbation enhances the warm core structure by about 50 % with respect to the non-perturbed member. The specific humidity perturbation increases the moisture by 16 % at 700 hPa with respect to the non-perturbed field.

JMA also perturbs all components: wind, temperature and specific humidity. JMA’s perturbations are characterized by the large amplitude of the specific humidity perturbation. For example, the amplitude averaged over the 2000 km × 2000 km domain about the typhoon center at 925 hPa is 1.25 g kg\(^{-1}\); 3.7 times as large as NCEP at 925 hPa in the before-recurrence stage. However the perturbation area is not in the typhoon surroundings.
but mainly to the south of the typhoon. This is because JMA’s moist SVs are not targeted for each TC, but for the entire tropics. As a result, the amplitude of specific humidity perturbation south of the typhoon becomes smaller as the typhoon moves north. On the other hand, the amplitude of the wind perturbation is small. For example, the amplitude averaged over 2000 km × 2000 km domain about the typhoon center is 0.24 m s⁻¹; a quarter the size of the ECMWF perturbations at 700 hPa in the before-recurvature stage. This trend is common in the other stages.

As seen in Fig. 2.3 to 2.5, the horizontal and vertical structures of the initial perturbations as well as their amplitudes are found to be quite different among the NWP centers. It would then be inferred that those differences cause the different modification of the TC motion at the initial times and subsequent forecast times. In the following section, the evolution of the initial perturbations associated with Sinlaku’s motion and how these perturbations modify the flow field of Sinlaku and its environment are examined.

2.3.2 Modification of the steering flow and symmetric and asymmetric circulations by perturbations

In principle, the total wind in the region of a TC can be decomposed into an environmental steering flow, a symmetric vortex and an asymmetric circulation (Carr and Elsberry 1992). The TC motion can then be governed by 1) the steering flow associated with TC-background synoptic flow and 2) the asymmetric circulation which includes an azimuthal wavenumber one circulation like the beta gyre (George and Gray 1976; Chan and Gray 1982; Fiorino and Elsberry 1989). This section describes how the initial wind perturbation modifies the symmetric vortex of Sinlaku, the steering vector and the advection vector associated with the asymmetric circulation (hereafter referred to as asymmetric propagation vector), using the ECMWF and NCEP ensembles. In addition, the time evolution of the steering and asymmetric propagation vector is compared between the two ensembles.

There is no unique method to define the steering flow because of the difficulty in dividing the background flow from a TC circulation itself. Here, the Lanczos filtering
developed and studied by Kim et al. (2009) is used to separate the total wind into the background flow and the residual from which the symmetric and asymmetric circulations are calculated. The Lanczos filtering filters out components with less than about 12 degrees latitude-longitude in wavelength from the total wind field. In contrast to the method used by Kurihara et al. (1993), no sharp cutoff wavelength exists in the resulting background and residual flow in the Lanczos filtering. For the calculation of the steering and asymmetric propagation vector, the 500 hPa wind field of each ensemble member is used. First, the total wind is separated into the background and residual flow by the Lanczos filtering. Second, a TC center is defined as a minimum streamfunction position using the residual flow. Third, the symmetric circulation centered on the TC center is computed from the residual flow. Fourth, the asymmetric circulation is obtained by extracting the symmetric circulation from the residual flow. Finally, the steering (asymmetric propagation) vector is calculated by averaging the background (asymmetric) flow over 300 km from the TC center. It is confirmed that in each ensemble member, the combination of the steering and asymmetric propagation vector accurately represents Sinlaku’s motion, which is estimated by using 6-hourly forecast position data.

Figure 2.6 shows the radial profile of the symmetric tangential wind at 850 hPa in the before-recurvature stage of Sinlaku by ECMWF (left) and NCEP (right). The following four features can be seen in Fig. 2.6;

1. The size of the typhoon (radial profile of the symmetric tangential wind) is similar among the ensemble members in each EPS;

2. The range of the maximum tangential wind is less than 1 m s\(^{-1}\);

3. The radius of the maximum tangential wind does not change significantly;

4. The differences of the above three quantities between ECMWF and NCEP are much larger than the differences caused by the initial perturbations in each ensemble.
Similar features are seen in the during- and after-recurvature stages and in 500 hPa wind field. Figure 2.7 shows the modification and time evolution of the steering and asymmetric propagation vector by the ECMWF (left) and NCEP (right) perturbations for the before-recurvature stage of Sinlaku. The following four features can be seen in Fig. 2.7:

1. The ECMWF ensemble shows the growth of the steering and asymmetric propagation vector while the NCEP ensemble does not;

2. The spread of the asymmetric propagation vector in ECMWF is larger than in NCEP;

3. Though the spread of the steering vector in ECMWF is smaller than that in NCEP at 0-h, they become almost the same at 48-h;

4. The growth of the steering and asymmetric propagation vector in ECMWF is larger in the early forecast period. The average perturbation wind magnitude of the steering (asymmetric propagation) vector is 0.37 (0.28), 1.04 (0.79), 1.06 (0.69), 0.94 (0.75) and 0.98 (0.86) ms\(^{-1}\) at 0, 12, 24, 36 and 48 hours, respectively.

Similar features are seen in the during- and after-recurvature stages. Additionally, to examine the robustness of the Lanczos filtering, sensitivity tests were conducted by changing the cutoff wavelength in the Lanczos filtering and the radius to calculate the steering and asymmetric propagation vector. Cutoff wavelengths of 10 and 14 degrees in the Lanczos filtering and a radius of 500 km to calculate the steering and asymmetric propagation vector were used. The qualitative features described above are not sensitive to the choice of the cutoff wavelength and the radius.

The differences between the initial modification and growth of the steering and asymmetric propagation vector led to the differences in the spread of ensemble TC track predictions between the two ensembles. However, it remains to be seen what dynamical mechanisms cause the growth of the steering and asymmetric propagation vector in the ECMWF EPS. In the next section, the dynamical mechanisms that lead to the growth of perturbation kinetic energy in the ECMWF EPS are investigated.
2.4 Dynamical mechanisms of the growth of the steering and asymmetric propagation vector

In this Section, the growth of the steering and asymmetric propagation vector in ECMWF EPS are investigated from a perspective of 1) the baroclinic energy conversion in a vortex, 2) the baroclinic energy conversion associated with the mid-latitude waves and 3) the barotropic energy conversion in a vortex.

2.4.1 Baroclinic energy conversion in a vortex

The dynamics of baroclinic energy conversion in the mid-latitude waves can be applied to a TC-like vortex in a cylindrical coordinate system. The upper left in Fig. 2.8 shows the schematic to illustrate the baroclinic energy conversion in a vortex. In the case of the mid-latitude dynamics, the × mark and the circle centered at the mark represent the north pole and a certain latitude, respectively, while in the case of a vortex, they represent a TC center and a circulation at a certain radius, respectively. The only difference between the mid-latitude and a TC vortex is the background temperature gradient; the north is colder than the south in the mid-latitude while the TC center is warmer than the outer region. (the temperature gradient at 500 hPa for the non-perturbed member of ECMWF before recurvature of Sinlaku is shown in Fig. 2.9a). Thus, in the vortex case, a temperature perturbation (wave perturbation) needs to be 90 degrees ahead of a streamfunction perturbation so that the perturbation can obtain available potential energy from mean available potential energy.

The baroclinic energy conversion between azimuthal mean available potential energy ($\overline{A}$) and eddy available potential energy ($\overline{A}'$) in a cylindrical coordinate system is written as (e.g. Kwon and Frank 2005; 2008):

$$\frac{\partial \overline{A}'}{\partial t} = -\left(\frac{h}{s}\right)^2 d'\theta_A \frac{\partial \theta_A}{\partial r} - \left(\frac{h}{s}\right)^2 \omega' \theta_A \frac{\partial \theta_A}{\partial p}$$  \hspace{1cm} (2.3)
where $u'$ is the eddy radial velocity (negative for inflow), $\theta'_A$ and $\bar{\theta}_A$ are the eddy and azimuthal mean potential temperature and $\omega'$ is the eddy vertical velocity (negative for upward flow). $r$ and $p$ denote radius and pressure, and the overbar refers to an azimuthal average. 

$h \equiv (R/p)(P/p_R)^\kappa$, where $R$ is the gas constant, $p_R$ is a reference pressure, and $\kappa$ is $R/c_p$. $s^2 \equiv -h(\partial \theta_0 / \partial p)$, which represents the vertical stability of the atmosphere. In addition, the eddy available potential energy needs to be converted into the eddy kinetic energy ($K'$) in order for the perturbation to obtain kinetic energy through the baroclinic process. This conversion is given by

$$\frac{\partial K'}{\partial t} = -h(\omega'\bar{\theta}_A).$$

(2.4)

Let us focus on the first term in (2.3), the radial eddy heat flux, to demonstrate how ECMWF perturbations cause the growth of the steering and asymmetric propagation vector through the baroclinic energy conversion in a vortex. Suppose that streamfunction and temperature perturbations as described in the upper and middle left in Fig. 2.8 exist in a cylindrical coordinate system. The corresponding radial heat flux is then given in the lower left in Fig. 2.8 (instead of potential temperature, temperature is used for simplicity). Considering that the radial gradient of background temperature is negative ($\frac{\partial \theta_0}{\partial r} < 0$), it turns out that the time change of the eddy available potential energy is positive ($\frac{\partial A'}{\partial t} > 0$).

The figures on the right are equivalent to the figures on the left in Fig. 2.8; streamfunction (shade) and temperature (contour) perturbations at 500 hPa by ECMWF ensemble member 21 for the before-recurvature stage (upper), corresponding azimuthal structures at 500 km from the center of Sinlaku (middle), and the radial heat flux (lower). It is found that the azimuthal mean available potential energy will be converted into the eddy available potential energy from southwest to northeast of Sinlaku, with its peak at around south and east of Sinlaku.

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Note that potential temperature is divided into a basic state, which only depends on height, azimuthal mean and derivative from the mean; $\theta(r, \lambda, p, t) = \theta_0(p) + \bar{\theta}_A(r, p, t) + \theta'_A(r, \lambda, p, t)$.
Figure 2.10 shows the six-hourly time evolution of the wind perturbation (left), the azimuthal structures of streamfunction and temperature perturbations at 500 km from the center of Sinlaku (middle) and the radial heat flux (right). Note that the wind field of the non-perturbed member and ensemble member 21 is shifted to a storm-relative coordinate at each verification time. It is found that where the wind perturbation grows corresponds well to the region where the radial heat flux is positive, especially south and east of Sinlaku. As seen in the flow over the center of the storm, the growth of the wind perturbation leads to the change in the advection flow of Sinlaku. As a result of the significant modification of both the steering and asymmetric propagation vectors, the track of ensemble member 21 becomes one of the most different tracks from that of the non-perturbed member.\footnote{Ensemble member 22 also provides a largely different track from the non-perturbed run because ECMWF EPS adopts plus-minus perturbations (see Section 2b for more details).}

Unfortunately, the second term in (2.3) can not be examined here because vertical velocity fields are not available from TIGGE. Nevertheless, it has been shown that the growth of the wind perturbation occurs at the location where the radial heat flux is positive. Figure 2.11 shows the six-hourly time evolution of the vertical structure of streamfunction and temperature perturbations. Note that the increasing azimuthal angle is in a cyclonic direction, with 0, $\pi/2$, $\pi$ and $3\pi/2$ corresponding to east, north, west and south, respectively. It is found that the streamfunction perturbation is tilted against the shear with height. Because of the vertical tilt of the streamfunction perturbation, it can be inferred that the perturbations in the upper and lower levels interact to increase the amplitude. Given that warm (cold) temperature advection occurs ahead of (behind) the positive streamfunction perturbation, the lower level warm advection amplifies the positive streamfunction perturbation in the upper level. The intensified streamfunction perturbation in the upper level then leads to stronger temperature advection. To respond the enhanced temperature advection in the upper level, the temperature advection in the lower level needs to be further increased (positive feedbacks). In contrast, the temperature perturbation is not tilted like the streamfunction perturbation. At the initial time, warm (cold) temperature perturba-
tions exist at around 300 hPa above cold (warm) temperature perturbations at around 500 hPa. After the initial time, temperature perturbations below 500 hPa begin to appear. It is not clear whether the temperature perturbation results from the vertical structure of the streamfunction perturbation, causing warm (cold) temperature advection ahead of (behind) the positive streamfunction perturbation, or results from other mechanisms such as convection (e.g. difference of thermodynamical process between the non-perturbed member and the perturbed member creates the temperature perturbation). This will be examined in the future using a simplified baroclinic model.

Perturbation structures as seen in Fig. 2.8 are not unique to ensemble member 21, but are common features for many ensemble members. However, they become less distinctive at the during- and after-recurvature stages of Sinlaku when the TC is more influenced by the mid-latitude waves.

As studied by Peng and Reynolds (2006), Reynolds et al. (2007), Yamaguchi et al. (2009a) and Wu et al. (2009), SVs targeted for TCs tend to appear away from the center of TCs; about 500 to 700 km away from the center. In addition, the SVs are asymmetric about the storm center; SVs are more likely to appear toward the north to the east of TCs rather than the west. These might be related to the fact that the amplitude of the azimuthal mean radial temperature gradient is relatively large around 500 to 700 km away from the center of the simulated TCs, and that the amplitude is locally magnified by the synoptic features around TCs. The Lanczos filtering is applied to the temperature field of the non-perturbed member of the ECMWF ensemble for the before-recurvature stage of Sinlaku (see Fig. 2.9a). The total temperature field is divided into the high- and low-frequency component to find the azimuthal mean temperature gradient (Fig. 2.9b) and temperature gradient created by the surrounding synoptic features (Fig. 2.9c). It is found that the amplitude of the azimuthal mean temperature gradient takes a relatively large value about 500 km away from the storm center. It is locally amplified by the low-frequency component of the total temperature field, which probably represents the surrounding synoptic features.
like the Pacific High and mid-latitude waves. The local amplification would more likely occur toward the north to the east of TCs rather than the west because, in general in the western north Pacific, the Pacific High is located to the east of TCs, and mid-latitude waves located to the north of TCs.

2.4.2 Baroclinic energy conversion associated with the mid-latitude waves

As previously studied by Buizza (1994), Palmer et al. (1998) and Buizza and Montani (1999), SVs capture baroclinic energy conversion in mid-latitude waves. Figure 2.12 shows an example where a streamfunction perturbation at 500 hPa for ECMWF ensemble member 50 during the after-recurvature stage is associated with the mid-latitude waves at the initial time, and then leads to the change in the advection flow of Sinlaku, mainly modifying the steering flow. Figure 2.13 shows another example where both features, the baroclinic energy conversion in a vortex and the mid-latitude jet, appear simultaneously in one ensemble member (ECMWF ensemble member 35 at the during-recurvature stage). There are positive and negative streamfunction perturbations south and east of Sinlaku, respectively, and a positive temperature perturbation in between. This is characteristic of the baroclinic energy conversion in a vortex as seen in Section 2.4.1. Meanwhile, there also exist perturbations north of Sinlaku, which capture the baroclinic energy conversion of the mid-latitude waves. In fact, the perturbations along 100°E to 140°E at 33°N are tilted towards the west, which is a feature of baroclinic energy conversion in mid-latitude waves. Consequently, both the steering and asymmetric propagation vectors are modified during the period of the perturbation growth.
2.4.3 Barotropic energy conversion in a vortex

The total eddy kinetic energy can also grow through the radial eddy momentum flux. This is described as;

$$\frac{\partial \tilde{K'}}{\partial t} = -u'^2 \frac{\partial \overrightarrow{v}}{\partial r} - \frac{v'^2}{r} \overrightarrow{v} - u'v' \left( \frac{\partial \overrightarrow{v}}{\partial r} - \frac{\overrightarrow{v}}{r} \right)$$  (2.5)

where $\overrightarrow{v}$ and $\overrightarrow{V}$ are the eddy and azimuthal mean tangential velocity, and $\overrightarrow{u}$ and $\overrightarrow{U}$ are the eddy and azimuthal mean radial velocity. Figure 2.14 shows the unaveraged terms of (2.5).

The difference of the time change of local perturbation kinetic energy ($\frac{\partial \tilde{K'}}{\partial t}$) between a non-perturbed member and an ensemble member is shown. Ensemble members 21 and 11 in the before-recurvature stage are selected for the ECMWF and NCEP cases, respectively. It is found that the barotropic energy conversion occurs in the ECMWF member in addition to the baroclinic energy conversion as seen in Fig. 2.8. This fact indicates that SVs are constructed to find the growth mechanism, or a combination of growth mechanisms, that achieve the largest growth in a given time. Note that the streamfunction perturbation in Fig. 2.8 has an up-shear property, implying positive momentum fluxes. On the other hand, the rate of change of the perturbation kinetic energy of the NCEP ensemble member is smaller than that of the ECMWF ensemble member in the region of the TC center, say 300 km from the center. These features are distinctive in the ensemble members as shown in Fig. 2.8, but common in other ensemble members. As a result, the growth of the asymmetric propagation vector is different between the ECMWF and NCEP ensembles.

Figure 2.15 shows the comparison of energy conversion between barotropic and baroclinic processes at 500 hPa by ECMWF ensemble member 21 before the recurvature of Typhoon Sinlaku. A sum of the whole three terms of the right hand side in (2.5) and the first term of the right hand side in (2.3) are compared at the initial time as a function of radius from the typhoon center. The energy conversion by baroclinic process is found to be
smaller than that by barotropic process on average, but in the same order as that by barotropic process.

The baroclinic features as described in Section 2.4.1-2.4.2 are less distinctive in the NCEP EPS compared with ECMWF EPS. In addition, the barotropic energy conversion is less likely to occur in NCEP EPS than ECMWF EPS as illustrated in Fig. 2.14. As Fig. 2.3 shows, the amplitude of the wind perturbation is larger in NCEP in and around Sinlaku. However, the perturbations are not efficient in modifying the growth of the eddy kinetic energy. Rather, as Fig. 2.7 shows, perturbations associated with the motion of Sinlaku seem to be saturated at the initial time in the NCEP ensemble.

### 2.5 Evolution of ensemble spread of tracks in 2007 and 2008 seasons

In order to compare quantitatively how the initial perturbations and their growth affect the ensemble spread of tracks in ECMWF and NCEP EPS, a statistical relationship between their respective spreads of ensemble track predictions is examined for the 2007 and 2008 seasons. Figure 2.16 shows the verification results for 1-day (top) and 3-day (bottom) forecasts from the 2007 (left) and 2008 (right) seasons. The verification only includes TCs whose intensity is tropical storm or stronger. First, the correlation of the spread between the ECMWF and NCEP ensembles is found to be weak. The correlation coefficient of 1-day forecasts is 0.26 and 0.27 for 2007 and 2008 while that of 3-day forecasts is 0.56 and 0.21, respectively. The low correlation may arise due to the different methods of creating initial perturbations in the respective ensembles, as well as their respective differences in initial amplitudes and the different growth of ensemble perturbations as presented above. Second, NCEP’s spread for 1-day forecasts is larger than ECMWF on average. This can be attributed to the fact that the spread in the initial steering wind is larger in the NCEP ensemble. Third, the spread of ECMWF usually becomes larger than NCEP for 3-day forecasts. This likely arises due to the differences of the energy growth between ECMWF
and NCEP. While ECMWF starts from relatively small amplitudes of initial perturbations, the growth of the perturbations help to amplify the ensemble spread of tracks. On the other hand, it is only the relatively large amplitudes of initial perturbations that seem to play a role in producing the ensemble spread of tracks in NCEP.

### 2.6 Implications of Chapter 2 and links to Chapter 3

The significant findings in this chapter are that 1) perturbations grow in the vicinity of TCs through both baroclinic and barotropic energy conversion as seen in mid-latitude dynamics, that 2) those energy conversion processes lead to the modification in tracks, and that 3) the difference of perturbation growth causes the difference of the ensemble spread of tracks. The above results determine a mechanism how the ensemble spread of TC tracks evolves with time in the EPSs at the operational NWP centers. In addition, those results identify the reason why the ensemble spread differs from one EPS to another.

The growth of SV-based perturbations was investigated in detail for a single typhoon case. The structures and locations of SVs change depending on initial vortex and temperature profiles as well as the specifications of SV calculations such as norm and optimization time. It is of great importance to investigate the structures and locations of SVs, not only for understanding the dynamical mechanisms of perturbation growth in TCs but also for the purpose of adaptive observations. In the next chapter, SVs will be investigated in a non-divergent barotropic framework on an f-plane. Using an f-plane nondivergent barotropic model is a good starting point for understanding the basic properties of SVs which grow in the vicinity of TCs.
Table 2.1 Specifications of ECMWF, NCEP and JMA EPS. Perturbed variables, $u, v, T, q$ and $P_s$ represent horizontal wind, meridional wind, temperature, specific humidity and surface pressure, respectively. Note that TL255 ($0.70^\circ \times 0.70^\circ$) is used for 10 to 15 day forecasts in ECMWF.

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<td>Extratropics ($&lt; 30^\circ S, &gt; 30^\circ N$) and up to 6 tropical areas</td>
<td>Global</td>
<td>Northern hemisphere ($&gt; 30^\circ N$) and tropics ($20^\circ S - 30^\circ N$)</td>
</tr>
<tr>
<td>Type of SVs</td>
<td>Dry (moist) SVs in extratropics (tropical areas)</td>
<td>-</td>
<td>Dry (moist) SVs in northern hemisphere (tropics)</td>
</tr>
<tr>
<td>Resolution of SVs</td>
<td>T42 ($2.86^\circ \times 2.86^\circ$) with 62 layers</td>
<td>-</td>
<td>T63 ($1.88^\circ \times 1.88^\circ$) with 40 layers</td>
</tr>
<tr>
<td>Optimization time</td>
<td>48 hours for Extratropics and tropical areas</td>
<td>6 hour cycle</td>
<td>48 (24) hours for Northern hemisphere (tropics)</td>
</tr>
<tr>
<td>Perturbed variables</td>
<td>$u, v, T, P_s$</td>
<td>$u, v, T, q, P_s$</td>
<td>$u, v, T, q, P_s$</td>
</tr>
<tr>
<td>Model uncertainty perturbations</td>
<td>Stochastic perturbations</td>
<td>Not used</td>
<td>Not used</td>
</tr>
</tbody>
</table>
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Chapter 3

Singular vectors for tropical cyclone-like vortices in a nondivergent barotropic framework

3.1 Introductory outline

In the previous chapter, the dynamical mechanisms of SV-based perturbation growth were investigated in detail, but only for a single typhoon case. As mentioned in Chapter 1, the structures and locations of SVs should be highly sensitive to initial vortex and temperature profiles as well as the parameters of the SV calculations, such as the optimization time, norm and model resolution. It is also hypothesized that the choice of those parameters is essential to control the impact of SVs on TC dynamics (e.g. TC motion, intensity, genesis, etc.). Therefore, it is of great importance and interest to investigate SV dependency on initial conditions and the parameters of the SV calculations.

As a first step of a series of these studies, in this chapter, SVs are computed on in a nondivergent barotropic framework. Using the barotropic model, the following issues will be addressed from a perspective of the SV dependency on initial conditions:

1. Sensitivity of SVs to maximum wind speed and radius of maximum wind;

2. SVs for initial vortices that meet and do not meet a necessary condition of barotropic instability;
3. SVs for an asymmetric initial vortex (symmetric initial vortices will be used in the above two experiments);

and from a perspective of the SV dependency on the parameters of SV the computations:

1. Sensitivity of SVs to the optimization time;

2. Sensitivity of SVs to the viscosity in the model.

Two dynamical mechanisms for SVs to grow can be expected in a nondivergent barotropic framework; one is the Orr mechanism (Orr 1907) and the other is the barotropic instability of normal modes (Rayleigh 1880). In the Orr mechanism, the growth of a perturbation is determined solely by how far back against the shear the perturbation is tilted. The growth continues until the up-shear property vanishes, meaning that the perturbation growing through the Orr mechanism exhibits transient growth. In TCs, there is radial shear in angular velocity and I hypothesize that SVs will be able to identify perturbations that grow through the Orr mechanism. In the barotropic instability of normal modes, the perturbation grows through momentum fluxes like the Orr mechanism, but grows exponentially. In order for barotropic instability to occur, the necessary condition has to be satisfied, that the vorticity gradient changes sign. As will be seen in Section 3.7, SVs will be calculated for initial TC-like vorticesthat meet and do not meet the necessary condition of the barotropic instability. In addition, SVs are calculated for an asymmetric initial vortex to account for a more realistic vortex structure.

For the optimization time, it is not clear what optimization time is suitable for SVs targeted for TCs. It would likely be dependent on the initial conditions and/or forecast interests (e.g., track, intensity, genesis, etc). For the viscosity, it is also not clear how the viscosity affects the structure of SVs, especially azimuthal wavenumber of the fastest growing SVs. Thus, it is worthwhile to investigate the sensitivity of SVs to the optimization time and the viscosity.
3.2 Methodology

3.2.1 General overview

Let us consider a following $n$th order dynamical system;

$$\frac{du}{dt} = f(u), \quad u \in \mathbb{R}^n. \quad (3.1)$$

The evolution of a small perturbation to $u$ can be written as a linear dynamical system;

$$\frac{dy}{dt} = Ty, \quad (3.2)$$

where $y$ is a state vector of a perturbation, and $T$ is the time evolution operator. The linearized dynamical matrix, $T$, is the Jacobian matrix and written as;

$$T = \frac{\partial f}{\partial u} = \begin{pmatrix}
\frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \cdots & \frac{\partial f_1}{\partial u_n} \\
\frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} & \cdots & \frac{\partial f_2}{\partial u_n} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial f_n}{\partial u_1} & \frac{\partial f_n}{\partial u_2} & \cdots & \frac{\partial f_n}{\partial u_n}
\end{pmatrix}. \quad (3.3)$$

By introducing a positive-definite Hermitian operator $M$, the energy of the system can be written as

$$E = y^* M y. \quad (3.4)$$

Then I perform the following useful change into generalized velocity coordinates,

$$x = M^{1/2} y, \quad (3.5)$$

$$A = M^{1/2} T M^{-1/2}, \quad (3.6)$$
so that in generalized velocity coordinates,

\[
\frac{dx}{dt} = Ax, \quad (3.7)
\]

\[
E = x^*x. \quad (3.8)
\]

One can obtain the fastest growing perturbations over finite time in the tangent linear system in (3.8) by singular vector decomposition (SVD). Consider the normalized growth of a perturbation \( x \);

\[
GR = \sqrt{\frac{(x(t),x(t))}{(x(0),x(0))}}. \quad (3.9)
\]

By introducing the tangent linear propagator \( L \)

\[
L = e^{At}, \quad (3.10)
\]

the growth can be written as

\[
GR = \sqrt{\frac{(Lx(0),Lx(0))}{(x(0),x(0))}}, \quad (3.11)
\]

\[
= \sqrt{\frac{(x(0),L^*Lx(0))}{(x(0),x(0))}}, \quad (3.12)
\]

\[
= \sigma, \quad (3.13)
\]

where

\[
L^*Lx(0) = \sigma^2 x(0). \quad (3.14)
\]

Thus, the SVs of \( L \), or the eigenvectors of \( L^*L \), give a set of initial perturbations with amplification factors (singular values) of \( \sigma \) in the system, and the SV with the largest value of \( \sigma \) corresponds to the fastest growing perturbation (leading SV).
3.2.2 Singular vector computations in a nondivergent barotropic framework

The governing equation in this study is a nondivergent barotropic vorticity equation with a viscosity term;

\[ \frac{\partial \zeta}{\partial t} + \mathbf{v} \cdot \nabla \zeta = \nu \nabla^2 \zeta \]  

(3.15)

where \( \zeta \) is the relative vorticity, \( \mathbf{v} \) is a horizontal velocity vector, and \( \nu \) is the kinematic viscosity.

In order to construct the Jacobian matrix in (3.3) for SV computations, (3.15) is solved for an initial TC-like vortex by adding a point vortex perturbation at each discretized grid point of a numerical model. For this purpose, a two-dimensional version of the SPECTRAL ELEMENT OCEAN MODEL (SEOM; Iskandarani et al. 1995) is used. The SEOM has an advantage in the flexibility of the design of the model grid spacing. As it allows non-uniform grid spacing, so that more grid points can be allocated in the vicinity of an initial vortex with fewer grid points near the boundaries.

The model domain is 4000 km \( \times \) 4000 km in the zonal and meridional directions. The total number of grid points is 6561 \((81 \times 81)\) and the averaged horizontal resolution within 500 km from the domain center is 19 km. The boundary condition is periodic in the zonal direction and impenetrable \((\psi = 0)\) at the northern and southern boundaries. The kinematic viscosity is 75000 m\(^2\)s\(^{-1}\). The amplitude of each point vortex perturbation is \(10^{-4} \) s\(^{-2}\).

A kinetic energy is used to evaluate the growth of the SVs. Using streamfunction and vorticity, the kinetic energy over the model domain is written as;

\[ KE = -\frac{1}{2} \int_A (\psi \cdot \zeta) dA \]  

(3.16)
where \( \int_A dA \) represents an integration over the model domain. To obtain the kinetic energy, an operator, \( G \), is constructed so that \( G \) can be used as a norm operator \( M \) as defined in (3.4);

\[
\psi = G\zeta
\]

where \( \psi \) and \( \zeta \) represent a state vector of streamfunction and vorticity, respectively.

The procedure to construct the Jacobian matrix, \( T \), is:

1. Set an initial TC-like vortex;
2. Run the SEOM with no perturbation for only one time step;
3. Calculate the tendency, \( \frac{\partial \zeta}{\partial t} \), at each grid point of the model;
4. Run the SEOM for only one time step by adding a point vortex perturbation at a certain grid point;
5. Calculate the tendency, \( \frac{\partial \zeta}{\partial t} \), at each grid point of the model;
6. Take the difference between the results of 3 and 5. The difference makes one column of the Jacobian matrix;
7. Repeat 4 to 6 for all grid points.

And the following is the subsequent procedure to calculate SVs:

1. Compute the tangent linear operator, \( L \). The optimization time interval is defined by \( t \) in (3.10);
2. Compute the SVs of \( L \) using a MATLAB function named SVDS, which gives some leading SVs and takes much less time than a function named SVD, which gives a whole set of SVs.
3.3 Reproduction of Fiorino and Elsberry (1989)

To test the validity of the SEOM as a nondiagonal barotropic model, I repeated some of the calculations of Fiorino and Elsberry (1989) (hereafter referred to as FE89).

In FE89, the sensitivity of TC structure to vortex motion was examined in a beta-plane nondiagonal barotropic framework. It was found through the analysis of the model streamfunction tendency that the linear beta term is responsible for the initial formation of beta gyres, and that the vortex moves northwestward because the gyres are twisted by the nonlinear advection of the asymmetric circulation by the symmetric vortex and because this effect is compensated by the linear beta forcing, creating a quasi-steady state of the beta gyres. One of the main goals of FE89 was to illustrate that tropical cyclone motion is more sensitive to the outer structure than the inner structure. For this purpose, numerical experiments were conducted using a nondiagonal barotropic model. The governing equation of the model is written as

$$\frac{\partial \zeta}{\partial t} + v \cdot \nabla \zeta + \beta v = 0$$ (3.18)

where $\beta$ is the meridional gradient of the Coriolis parameter and $v$ is meridional velocity. Following FE89, $\beta = 2.2103 \times 10^{-11}$ (a value at 15 $^\circ$N), and viscosity is zero. The model is solved on a rectangular Cartesian grid with uniform grid spacing. A typical configuration is $101 \times 101$ grid points in the north-south and east-west directions with a horizontal grid increment of 40 km. The lateral boundary conditions are no slip (zero gradient in $\psi$) along the northern and southern boundaries, and are cyclic in the east-west direction. The initial conditions in FE89 have symmetric tangential wind profiles with no mean radial flow. The tangential wind profile is given by

$$v(r) = V_{max} \left( \frac{r}{RMW} \right) \exp \left\{ \frac{1}{b} \left[ 1 - \left( \frac{r}{RMW} \right)^b \right] \right\}$$ (3.19)
where $r$ is a radial position, $v(r)$ is tangential wind, $V_{\text{max}}$ is maximum wind speed, $RMW$ is the radius of maximum wind and $b$ is a parameter which determines the shape of the profile. For larger $b$, the tangential wind goes to zero outside $RMW$ more quickly. Table 3.1 shows some of the initial conditions used in FE89. The initial conditions 1, 2, and 3 were used to test the sensitivity of the vortex motion to the intensity while the initial conditions 1, 4 and 5 were used to test the sensitivity to the outer structure of tangential wind. As Fig. 3.1 shows, the vortex motion was found to be more sensitive to the outer structure than the intensity.

In order to reproduce these track results, I added the beta term to the governing equation, (3.15), turned off viscosity, and ran the SEOM from the same initial conditions as FE89. As Fig. 3.2 shows, the SEOM produced nearly identical track results as FE89, even though the configurations of SEOM are different from those of the model used in FE89 regarding the model grid spacing, spectral or grid model, and the boundary conditions at the north and south.

### 3.4 Reproduction of Nolan and Farrell (1999)

To test the validity of the SV calculation procedures in this study, I conducted a similar experiment to one performed by Nolan and Farrell (1999) (hereafter referred to as NF99).

In NF99, the dynamics of asymmetric perturbations in two-dimensional vortices (one- and two-celled vortices) that are maintained by radial inflow were studied. In the one-celled vortex case, they showed that the perturbations grow through the Orr mechanism, and that the transient decaying perturbations ultimately increased the kinetic energy of the background mean flow. In the two-celled vortex that satisfied the necessary condition of barotropic instability, NF99 captured this exponentially growing instability. Unlike previous studies, NF99 studied the influence of the radial inflow on the perturbation growth.
It was found that vorticity amplification through stretching caused by the radial inflow is comparable to the actual growth. Finally, NF99 gave a physical insight into the importance of a wavenumber 1 perturbation. Physically, the azimuthal wavenumber 1 perturbation corresponds to a linear displacement of some or all of the vorticity of the total flow. Therefore, the wavenumber 1 perturbation corresponds to deviations in the path of the vortex, which has particularly important applications in the forecasting of TC tracks.

The upper figures of Fig. 3.3 shows one result of NF99: a wavenumber 1 perturbation (SV) for one-celled vortex including the radial inflow. The left (right) figure shows the streamfunction (vorticity) field. The perturbations are located immediately outside the RMW (not shown) and are tilted against the shear of the background flow (recall the radial gradient of the angular velocity is negative). There are two major differences in the SV calculation framework between NF99 and this study. One is the radial inflow: NF99 had while this study does not. The other is the governing equations in numerical models: NF99 used linear dynamics while the SEOM is a fully nonlinear model. Though there are such differences, I conducted a SV calculation to examine whether the SEOM and the SV calculation procedure mentioned in Section 3.2 can capture the similar perturbation structure as NF99. Using initial condition 1 of the above FE89 study as an initial vortex, SVs were calculated. The lower figures of Fig. 3.3 shows the result: a wavenumber 1 SV (a leading SV in this case). It was confirmed that the SEOM successfully reproduced the up-shear property of SVs as seen in the upper figures of Fig. 3.3.

3.5 Sensitivity of singular vectors to maximum wind speed and radius of maximum wind of initial tropical cyclone-like vortices.

Numerical experiments are conducted to investigate the sensitivity of the amplification factor (singular value) and the location of initial SVs to maximum wind speed and RMW of initial TC-like vortices. For this purpose, nine experiments are performed using
Rankine vortices as initial conditions. The tangential wind profile of Rankine vortex is written as;

\[ v(r) = \begin{cases} 
\frac{V_{\text{max}}}{R_{\text{MW}}} r, & r < R_{\text{MW}} \\
\frac{V_{\text{max}} R_{\text{MW}}}{r}, & r \geq R_{\text{MW}}.
\end{cases} \]  

(3.20)

The values of \( V_{\text{max}} \) and \( R_{\text{MW}} \) tested here are 20, 35 and 50 ms\(^{-1}\) and 100, 200 and 300 km, respectively.

Table 3.2 shows the singular value of the 1st SV for each experiment. The optimization time used in the SV calculations is 6 hours. It turns out that the singular value increases as \( V_{\text{max}} \) increases, and decreases as \( R_{\text{MW}} \) increases. This result can be explained by an equation for the rate of change of perturbation kinetic energy (\( \overline{K'} \)):

\[ \frac{d\overline{K'}}{dt} = -\overline{u'v'} \left( \frac{\partial \overline{\Omega}}{\partial r} + \frac{\overline{v'}}{r} \right), \]

(3.21)

\[ = -\overline{u'v'} \left( \frac{\partial \overline{v}}{\partial r} - \frac{\overline{v'}}{r} \right) \]

(3.22)

where \( u' \) and \( v' \) are perturbation radial and tangential velocity, \( \overline{\Omega} \) is azimuthal mean angular velocity, \( \overline{v} \) is azimuthal mean tangential velocity, and the overbar refers to an azimuthal average. This equation represents the growth or decay of perturbation kinetic energy through momentum fluxes.

Using (3.20), (3.22) is written as;

\[ \frac{d\overline{K'}}{dt} = \begin{cases} 
0, & r < R_{\text{MW}} \\
\frac{u'v' V_{\text{max}} R_{\text{MW}}}{r^2}, & r \geq R_{\text{MW}}.
\end{cases} \]

(3.23)

As the circulation inside \( R_{\text{MW}} \) is a solid rotation, the rate of change of perturbation kinetic energy is zero. As (3.23) shows, the rate of change of perturbation kinetic energy increases with \( V_{\text{max}} \), indicating that the singular value increases with \( V_{\text{max}} \). It is also deduced from
(3.23) that the location just beyond $RMW$ is favorable for SVs to grow because the rate of change of perturbation kinetic energy is proportional to $1/r^2$. In other words, the most favorable location for SVs to grow is where the radial gradient of the background angular velocity is large. In fact, as Figs. 3.4 to 3.6 show, SVs are located just beyond the $RMW$. By substituting $r = RMW$ into (3.23), it is found that the rate of change of perturbation kinetic energy decreases with $RMW$.

As mentioned above, SVs appear right beyond $RMW$, implying that the location of SVs are sensitive to $RMW$. Meanwhile, the location is not sensitive to $V_{max}$. The structure of SVs also changes with $RMW$. For example, the azimuthal wavenumber of SVs for $RMW = 100$ km is 1 while that for $RMW = 200$ and 300 km is 2. The sensitivity of the wavenumber to $RMW$ will be investigated in Section 3.7.

### 3.6 Sensitivity of singular vectors to optimization time

Another set of numerical experiments is conducted to understand the sensitivity of SVs to optimization time. Using a Rankine vortex with $V_{max} = 35$ ms$^{-1}$ and $RMW = 100$ km as an initial TC-like vortex, optimization times of 6, 12, 24, 48, 72 and 96 hours are tested.

Figure 3.7 shows the initial kinetic energy fields of the 1st SV at each experiment. It is found that the location of maximum kinetic energy moves outward with the increasing optimization time. For example, in the case of the 6-hour optimization time, it is located at around $r = 150$ km, which is right beyond the $RMW$. Meanwhile it is at around $r = 500$ km in the case of a 96-hour optimization time. As discussed in Section 3.4, the location immediately beyond $RMW$ is a favorable location for SVs. Nevertheless, the SVs appear far away from the $RMW$ when optimization time is long. Why is this?

With zero viscosity, a perturbation whose phase lines are tilted all the way back becoming nearly parallel with the flow can exist just beyond the $RMW$ (Nolan and Farrell
However, viscosity does not allow this because viscosity quickly diffuses such a tight perturbation (which is created by high radial wavenumbers). In other words, the extent of the up-shear tilt is limited by viscosity. A perturbation right beyond RMW relatively quickly becomes radially aligned because the tangential wind is relatively fast. Thus, in the case of a longer optimization time interval, a perturbation seeks out the part of the flow which allows the perturbation to grow for a longer period.

3.7 What determines the azimuthal wavenumber of singular vectors?

As Figs. 3.4 to 3.6 show, the leading SV (with optimization time of 6 hours) sometimes has wavenumber 1 or wavenumber 2 structures. In order to understand what determines the azimuthal wavenumber of SVs, a sensitivity test is conducted.

With zero viscosity, a perturbation with higher wavenumbers can have a larger singular value. This can be explained by substituting a perturbation streamfunction \( \psi' \):

\[
\psi' = C \exp \{i(n\lambda + kr - \omega t)\}
\]

into the equation for the rate of change of perturbation kinetic energy (3.22), to yield

\[
\frac{dK'}{dt} = nk\psi'^2 \left( \frac{\partial v}{\partial r} - \frac{v}{r} \right).
\]

In (3.28), \( C \) is the amplitude of the perturbation, \( n \) is azimuthal wavenumber, \( k \) is radial wavenumber and \( \omega \) is angular velocity of perturbation. Note that when the perturbation is tilted against (along) the radial shear of the background angular velocity, \( nk \) takes a negative (positive) value, leading to the growth (decay) of the perturbation. As (3.25) shows, the rate of change of perturbation kinetic energy increases with azimuthal wavenumber.

In reality, however, the growth of SVs with higher wavenumbers is not allowed because the magnitude of the viscous term increases with the second power of wavenumber.
Thus, such a low wavenumber as one or two is dominant in the leading SVs as seen in Figs. 3.4 to 3.6.

From the above discussion, it can be inferred that the azimuthal wavenumber of leading SV would increase (decrease) as viscosity decreases (increases). To test this, two Rankine vortices with $V_{\text{max}} = 35$ ms$^{-1}$ and $RMW = 100$ km (hereafter referred to as Initial 1) and $V_{\text{max}} = 35$ ms$^{-1}$ and $RMW = 300$ km (hereafter referred to as Initial 2) are selected as initial conditions. Note that Initial 1 (2) has an azimuthal wavenumber 1 (2) structure when viscosity is 75000 m$^2$s$^{-1}$ (see Fig. 3.5). In the case of Initial 1, viscosity of 20000 m$^2$s$^{-1}$ is tested while viscosity of 300000 and 20000 m$^2$s$^{-1}$ are tested for Initial 2. Figure 3.8 shows the streamfunction fields of the 1st SV at each experiment. As expected, the azimuthal wavenumber becomes 2 in the case of Initial 1 while, in the case of Initial 2, it becomes 1 (3) when viscosity has increased (decreased).

When viscosity is constant, the azimuthal wavenumber of SVs is also sensitive to $RMW$. As Fig. 3.5 shows, the azimuthal wavenumber is 1 when $RMW = 100$ km while it is 2 when $RMW = 200$ and 300 km. This is due to the fact that the magnitude of the viscous term weakens with the second power of radial location;

$$\nabla^2 \zeta' = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \zeta'}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \zeta'}{\partial \lambda^2} \quad (3.26)$$

$$= -(k^2 + \frac{n^2}{r^2}) \zeta' + ik \frac{\zeta'}{r^2}, \quad (3.27)$$

where $\zeta'$ is;

$$\zeta' = C \exp \left\{ i(n \lambda + kr - \omega t) \right\}. \quad (3.28)$$

For fixed viscosity, therefore, the azimuthal wavenumber of the leading SV increases with $RMW$. 

3.8 Singular vectors capturing the barotropic instability

The initial vortices used in Section 3.3 to 3.6 do not satisfy the necessary condition for the barotropic instability of normal modes, that the vorticity gradient changes sign. Thus the only mechanism for SVs to grow is the Orr mechanism, in other words, a perturbation is tilted against the background shear and gains eddy kinetic energy through the perturbation momentum fluxes. As studied by Peng and Reynolds (2006), the initial SVs of the U.S. Navy Operation Global Atmosphere Prediction System (NOGAPS) tend to occur at around 500 km from storm centers where the radial potential vorticity gradient changes sign. They found that such a condition can be created by the influence of surrounding synoptic features. In this section, SVs are calculated for initial vortices that meet the necessary condition for the barotropic instability to examine whether SVs capture the barotropic instability.

For this purpose, three initial vortices are created using a cubic Hermitian polynomial:

$$\zeta(r) = \begin{cases} 
\zeta_1 S \left( \frac{r}{d_1} \right) + \zeta_2 S \left( \frac{d_1 - r}{d_1} \right), & 0 \leq r \leq d_1 \\
\zeta_2 S \left( \frac{r - d_1}{d_2 - d_1} \right) + \zeta_3 S \left( \frac{d_2 - r}{d_2 - d_1} \right), & d_1 \leq r \leq d_2 \\
\zeta_3, & d_2 \leq r 
\end{cases}$$  

(3.29)

where $S(x) = 1 - 3x^2 + 2x^3$ is the cubic Hermitian polynomial that satisfies $S(0) = 1$, $S(1) = 0$, and $S'(0) = S'(1) = 0$. Values of $\zeta_1$ to $\zeta_3$ and $d_1$ and $d_2$ for the three vortex profiles are shown in Table 3.3. As Fig. 3.9 shows, one of the vortices does not satisfy the necessary condition of the barotropic instability (hereafter referred to as Initial A) while two of them do satisfy the condition. The initial condition which weakly (strongly) meets the necessary condition is hereafter referred to as Initial B (C).

Figures 3.10 to 3.12 show the kinetic energy fields of the leading SV for Initial A, B, and C, respectively. As in Section 3.6 where SV dependency on the optimization time is investigated, optimization times of 6, 12, 24, 48, 72, and 96 hours are tested for each
initial vortex. Table 3.4 shows the singular value of the leading SV at each experiment. In the case where the barotropic instability is not allowed, the location of the SV shifts outward as the optimization time increases (see Fig. 3.10). As already seen in Fig. 3.7, this is a characteristic of SVs for a vortex that does not satisfy the necessary condition for barotropic instability. Meanwhile, in the case where the necessary condition for barotropic instability is strongly met, the SVs occur at around $r = 400$ km where the radial vorticity gradient changes sign (see Fig. 3.9 and Fig. 3.12) for all optimization times, implying that the SVs capture barotropic instability. In fact, as Table 3.4 shows, the singular values for Initial C are much larger than those for Initial A, especially for the longer optimization time. It can be inferred that, for a relatively short optimization time, the transient growth by the Orr mechanism is dominant in the growth of SVs while the exponential growth by the barotropic instability plays a primary role for a longer optimization time. Note that, even in the case of barotropic instability, the initial SVs have an up-shear property, implying the existence of the Orr mechanism during the early stage of the growth. For Initial B, which satisfies the necessary condition weakly, the results are similar to Initial A, whose initial condition does not allow the barotropic instability. This indicates that vortex profiles that satisfy the necessary condition do not always lead to the instability because the condition is necessary, but not sufficient. In fact, the structure of the evolved SVs for Initial B does not look like that for Initial C, where two vortex waves interact with each other across the location where the radial gradient of vorticity changes sign (see Fig. 3.13).

### 3.9 Evolution of singular vectors in a tangent linear model

In this section, the evolution of SVs in a tangent linear model is explored, and how this evolution changes depending on initial vortex structures are investigated.

Figure 3.13 shows the evolution of an azimuthal wavenumber 2 SV for Initial A (left) and C (right) at 0 (top), 24 (middle) and 48 (bottom) hours, respectively. The opti-
mization time of the SVs is 24 hours, and the wavenumber 2 SV for Initial A (C) is the 3rd (1st) SV. It is found that, in the case that barotropic instability is not allowed, the up-shear tilt at the initial time almost vanishes at the optimization time, and that the perturbation takes on a down-shear property after the optimization time. Meanwhile, in the case that the barotropic instability is allowed, the SV is initially tilted like Initial A, but the evolved SV at the optimization time has two wave perturbations across the location where the vorticity gradient changes sign, and the perturbation outside is located downstream with respect to the perturbation inside. Furthermore, this phase relationship between the inside and outside perturbations does not change even after the optimization time. This result indicates that a SV capturing the barotropic instability is initially tilted against the background shear and grows through transient growth during the first 6 hours while it keeps the same structure after that time. At a certain time during the initial growth, the SV “locks in” to a normal mode structure and stays there so that it can continuously grow with time. The result of the normal mode-like evolution is identical to Peng et al. (2009) which illustrate the existence of a normal mode perturbation with an azimuthal wavenumber 2 structure when the barotropic instability is allowed.

As can be seen in Fig. 3.13, the SVs propagate radially outward. The location of a vorticity maximum which is tracked every 6 hours for both SVs (see Fig. 3.13) is shown in Fig. 3.14. In the case of Initial C (red), the SV moves inward during the first 6 hour evolution and then remains at the same radius. As stated above, this is due to the phase-lock property of a normal mode. Meanwhile, in the case of Initial A (blue), the SV moves inward during the first 6 hour evolution while it moves outward after the optimization time. This inward propagation can be explained by the group velocity of vortex Rossby-waves (Montgomery and Kallenbach 1997);

\[ C_{gr} = \frac{-2kn\frac{\partial \zeta}{\partial r}}{R(k^2 + \frac{n^2}{Re})^2} \]  

(3.30)
where $C_{gr}$ is the radial group velocity of vortex Rossby-waves, $k$ and $n$ are radial and azimuthal wavenumber, respectively, $\frac{\partial \zeta}{\partial r}$ is background vorticity gradient, and $R$ is radius. Given that the background vorticity gradient, $\frac{\partial \zeta}{\partial r}$, is negative, perturbations with an up-shear tilt propagate inward because $kn$ is negative, making $C_{gr}$ negative. On the other hand, if perturbations have a down-shear tilt, they move outward because $kn$ is positive. As seen in Fig. 3.9 and Fig. 3.13, both initial SVs in Fig. 3.13 are located where the background vorticity gradient is negative, and have the up-shear property. For this reason, the SVs move inward. In NF99, the inward propagation was explained by the radial inflow. However, it has been shown here that SVs propagate inward even without radial inflow. Meanwhile, in the case when the barotropic instability is not allowed, the SV has the down-shear tilt after the optimization time due to the radial shear of background angular velocity. However, the regions where the SV moves outward extend to the location where the background vorticity gradient is zero (see Fig. 3.9). The outward propagation must then be an apparent propagation due to the fact that the viscous term works strongly near the vortex center. After the optimization time, the SV starts to decay not only by the downshear tilt, but also by viscosity. Since the viscous term is proportional to $1/r^2$, the SV appears to propagate outward.

It is confirmed that the radial propagation can be seen for SVs with an azimuthal wavenumber 3 (not shown). How about for a wavenumber 1 SV? Fig. 3.15 shows the evolution of SV (vorticity fields) with an azimuthal wavenumber 1 structure at 0, 1, 2, 3, 4, 5, 6, 12, 18, 24, 36 and 48 hours. The SV is the leading SV calculated for Initial A with optimization time of 24 hours. As can be seen from figures for the evolution time 3 or 4 hours, a radially aligned wavenumber 1 structure emerges near the center of the initial vortex. This would be a pseudomode, which is an azimuthal wavenumber 1 mode resulting from the displacement of a vortex (Nolan and Montgomery 2001; Nolan et al. 2001). It can be inferred that the wavenumber 1 SV would create flow over the vortex center, leading to the displacement of the vortex. Since the perturbation structure gets trapped in
a pseudomode at and after the optimization time, it does not decay by being sheared over like a wavenumber 2 SV.

### 3.10 Singular vectors for an asymmetric initial vortex

In order to understand the basic properties of SVs in a two dimensional barotropic framework, all the initial vortices used so far had a symmetric structure. In reality, however, tropical cyclones have an asymmetric structure, which is induced by convection, the beta effect, surface inhomogeneity, etc. Thus, in this section, SVs are calculated for an asymmetric initial vortex to account for SVs for more realistic TC-like vortex.

For this purpose, the SEOM is integrated for 72 hours on a beta plane from an initially symmetric vortex. Because of the beta effect, the initially symmetric vortex becomes asymmetric and moves toward the northwest. SVs are then calculated for this prediction field at 72-h. Note that the SV calculation is conducted on an f-plane. Initial condition 1 shown in Table 3.1 is chosen as the initial symmetric vortex. Following FE89, $\beta = 2.2103 \times 10^{-11}$ is used, which is the value at 15°N. Figure 3.16 shows the initial condition (kinetic energy fields) for the SV calculation. Due to the beta effect, the initially symmetric vortex becomes asymmetric and is located to the northwest.

Figure 3.17 shows the vorticity fields of the 1st SV for optimization times of 6, 12, 24, 48, 72 and 96 hours. As seen in Fig. 3.7, the location of SVs moves outward with the optimization time. The distance between the vortex center (see the × mark in Fig. 3.17) and the location of the maximum amplitude of the vorticity of the SV is 358 km and 472 km for the optimization time of 6 and 96 hours, respectively. Unlike the SVs calculated for symmetric TC-like vortices, the SVs have an asymmetric structure. The SVs appear toward the north to the east of the vortex rather than the west. This result is consistent with previous studies by Peng and Reynolds (2006), Reynolds et al. (2007), Yamaguchi et al. (2009a) and Wu et al. (2009) where SVs used in NWP centers are investigated.
Figure 3.18 shows the displacement of the initial (asymmetric) vortex by the SV with the optimization time of 24 hours. In order to examine the effect of the SV on the initial vortex, the initial SV, 24-hour linearly evolved SV, and 96-hour linearly evolved SV are added to the initial vortex. As Fig. 3.18 shows, the SV makes a little shift of the initial vortex at the initial time. At the optimization time (24 hours), the displacement of the initial vortex becomes larger, and even after the optimization time, the initial vortex is still shifted by the SV. The same result is seen in the SVs with other optimization times. This would be a reason why SVs work for ensemble prediction systems and adaptive observations for TC track prediction. Since SVs can explain the evolution of the displacement of vortex, SVs can be used as initial perturbations in ensemble prediction systems and as sensitivity analysis guidance in adaptive observations (though it will be needed to investigate to what extent SVs explain analysis errors in NWP models, and to explore more effective configurations of SV calculations by changing parameters such as the optimization time, norm and model resolution).

3.11 Implications of Chapter 3

The significant outcome in Chapter 3 is that both the structures and locations of SVs are highly sensitive to initial vortex structures, viscosity and optimization time. It is also confirmed that SVs propagate radially inward and outward due to their tilting property.

For a given TC-like vortex, the favorable location for SVs to grow is just beyond the RMW. If the vortex satisfies the necessary condition of barotropic instability, SVs may be located at the radius at which the vorticity gradient changes sign. Furthermore, SVs are found to appear toward the north to the east of TC-like vortex rather than the west for an asymmetric vortex. This is in agreement with previous studies. From the perspective of adaptive observations for TCs, these results would give an insight into understanding what SV-based sensitivity analysis guidance represents.
It is of great interest to examine how these results would change if SVs are calculated on a beta plane with and without the background steering flows. As the vortex evolves (moves) with time on a beta plane due to the beta gyres and/or the steering flow, SVs might be related to the beta gyres and/or the background flows that modify the track of the vortex. Moreover, the structures and locations of SVs would change for baroclinic vortices. In the baroclinic case, the radial temperature gradient would play an important role as discussed in Section 2.4.1. These will be studied in the future by using a baroclinic model.
Table 3.1  Parameters for initial tangential wind profiles used in Figs. 2a and 3a of Fiorino and Elsberry (1989).

<table>
<thead>
<tr>
<th></th>
<th>$V_{\text{max}} (r &lt; 300\text{km})$</th>
<th>$b(r &lt; 300\text{km})$</th>
<th>$V_{\text{max}} (r \geq 300\text{km})$</th>
<th>$b(r \geq 300\text{km})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35 ms$^{-1}$</td>
<td>0.9575</td>
<td>35 ms$^{-1}$</td>
<td>0.9575</td>
</tr>
<tr>
<td>2</td>
<td>50 ms$^{-1}$</td>
<td>1.2141</td>
<td>35 ms$^{-1}$</td>
<td>0.9575</td>
</tr>
<tr>
<td>3</td>
<td>20 ms$^{-1}$</td>
<td>0.4082</td>
<td>35 ms$^{-1}$</td>
<td>0.9575</td>
</tr>
<tr>
<td>4</td>
<td>35 ms$^{-1}$</td>
<td>0.9575</td>
<td>50 ms$^{-1}$</td>
<td>1.2141</td>
</tr>
<tr>
<td>5</td>
<td>35 ms$^{-1}$</td>
<td>0.9575</td>
<td>20 ms$^{-1}$</td>
<td>0.4082</td>
</tr>
</tbody>
</table>
Table 3.2  Sensitivity of singular values to maximum wind speed and radius of maximum wind (RMW) of initial Rankine vortices.

<table>
<thead>
<tr>
<th>Max. Wind/ RMW</th>
<th>20 ms⁻¹</th>
<th>35 ms⁻¹</th>
<th>50 ms⁻¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 km</td>
<td>1.7</td>
<td>2.6</td>
<td>3.8</td>
</tr>
<tr>
<td>200 km</td>
<td>1.5</td>
<td>2.2</td>
<td>3.2</td>
</tr>
<tr>
<td>300 km</td>
<td>1.4</td>
<td>2.0</td>
<td>2.7</td>
</tr>
</tbody>
</table>
Table 3.3  Parameters for initial vortex profiles for Initial A to C. Initial A does not satisfy the necessary condition of the barotropic instability. Initial B (C) satisfies the condition weakly (strongly).

<table>
<thead>
<tr>
<th></th>
<th>$\zeta_1$</th>
<th>$\zeta_2$</th>
<th>$\zeta_3$</th>
<th>$d_1$</th>
<th>$d_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial A</td>
<td>$3.5 \times 10^{-4}$ s$^{-2}$</td>
<td>0 s$^{-2}$</td>
<td>0 s$^{-2}$</td>
<td>250 km</td>
<td>-</td>
</tr>
<tr>
<td>Initial B</td>
<td>$3.5 \times 10^{-4}$ s$^{-2}$</td>
<td>$-0.35 \times 10^{-2}$ s$^{-2}$</td>
<td>0 s$^{-2}$</td>
<td>400 km</td>
<td>600 km</td>
</tr>
<tr>
<td>Initial C</td>
<td>$3.5 \times 10^{-4}$ s$^{-2}$</td>
<td>$-1.05 \times 10^{-2}$ s$^{-2}$</td>
<td>0 s$^{-2}$</td>
<td>400 km</td>
<td>600 km</td>
</tr>
</tbody>
</table>

Initial B $(-0.1 \times \zeta_1)$

Initial C $(-0.3 \times \zeta_1)$
Table 3.4  Singular values for Initial A, B and C with different optimization times of 6, 12, 24, 48, 72 and 96 hours.

<table>
<thead>
<tr>
<th></th>
<th>6</th>
<th>12</th>
<th>24</th>
<th>48</th>
<th>72</th>
<th>96</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial A</td>
<td>1.5</td>
<td>1.9</td>
<td>2.8</td>
<td>4.6</td>
<td>6.5</td>
<td>8.5</td>
</tr>
<tr>
<td>Initial B</td>
<td>1.7</td>
<td>2.4</td>
<td>3.6</td>
<td>6.5</td>
<td>10.1</td>
<td>14.6</td>
</tr>
<tr>
<td>Initial C</td>
<td>1.9</td>
<td>2.8</td>
<td>5.7</td>
<td>22.4</td>
<td>88.1</td>
<td>347.3</td>
</tr>
</tbody>
</table>
Figure 3.1 Results of Fiorino and Elsberry (1989). Figure on the upper left shows the tangential wind profiles of initial tropical cyclone-like vortex. Figure on the lower left is the result of 3-day integrations regarding the vortex motion. Figures on the right are the same as left except for the initial conditions.
Figure 3.2  Same as Fig. 3.1 except for the SEOM.
Figure 3.3  Upper: Results of Nolan and Farrell (1999). Lower: Results using the SEOM. The figures show a singular vector for a tropical cyclone-like vortex (left: streamfunction, right: vorticity).
Figure 3.4  Sensitivity of the leading singular vectors (vorticity fields) to maximum wind speed and radius of maximum wind (RMW).
Figure 3.5  Same as Fig. 3.4 except for streamfunction fields.
Figure 3.6  Same as Fig. 3.4 except for kinetic energy fields.
Figure 3.7  Sensitivity of the leading singular vectors (kinetic energy fields) to optimization time.
Figure 3.8  Sensitivity of the azimuthal wavenumber of leading singular vectors (stream-function fields) to viscosity.
Figure 3.9  Radial profiles of initial vortex for Initial A to C. Initial A does not satisfy the necessary condition of the barotropic instability. Initial B (C) satisfies the condition weakly (strongly).
Optimization time = 6 hours

Optimization time = 12 hours

Optimization time = 24 hours

Optimization time = 48 hours

Optimization time = 72 hours

Optimization time = 96 hours

Figure 3.10  Leading singular vectors (kinetic energy fields) for an initial vortex (Initial A) which does not satisfy the necessary condition for the barotropic instability.
Figure 3.11  Same as Fig. 3.10 except for the initial vortex (Initial B) which weakly satisfies the necessary condition for the barotropic instability.
Figure 3.12  Same as Fig. 3.10 except for the initial vortex (Initial C) which strongly satisfies the necessary condition for the barotropic instability.
Figure 3.13  Time evolution of azimuthal wavenumber 2 singular vectors (vorticity fields) for Initial A (left) and C (right). Top: initial time, middle: optimization time (24 hours), bottom: 48 hours.
Figure 3.14  Radial propagation of azimuthal wavenumber 2 singular vectors for Initial A (blue) and C (red).
Figure 3.15  Time evolution of azimuthal wavenumber 1 singular vector (vorticity fields) for Initial A. The evolution times are 0, 1, 2, 3, 4, 5, 6, 12, 18, 24, 36 and 48 hours. See the figures from left to right and from top to bottom. The optimization time is 24 hours.
Initially asymmetric vortex (kinetic energy field)

Figure 3.16  Kinetic energy field of an initial asymmetric vortex.
Figure 3.17  The leading singular vectors (vorticity fields) for an asymmetric vortex with optimization time of 6, 12, 24, 48, 72 and 96 hours. The × mark is plotted at the center of the asymmetric vortex.
Figure 3.18 Displacement of the initial asymmetric vortex by the SV with the optimization time of 24 hours. Streamfunction field of the initial asymmetric vortex is drawn in blue, and streamfunction field after adding the initial SV (left), 24-hour linearly evolved SV (middle), and 96-hour linearly evolved SV (right) to the initial vortex is drawn in red.
Chapter 4

Summary and future work

In this thesis, initial condition sensitivity and dynamical mechanisms of perturbation growth in tropical cyclones were investigated using ensembles and singular vectors.

In Chapter 2, ensemble initial perturbations and their growth were investigated in order to understand the ensemble spread of tracks. Using the recently established TIGGE database, vertical and horizontal distributions of initial perturbations around Typhoon Sinlaku (2008) were first compared among ECMWF, NCEP and JMA, before, during and after recurvature. The initial amplitude of NCEP wind perturbations was found to be generally larger than that of ECMWF, particularly in the upper troposphere. For example, the 200 hPa NCEP perturbations were nearly 10 times as large as ECMWF in the before-recurvature stage of Sinlaku. Accordingly, the modification of the advection vector by the initial perturbations was larger in the NCEP ensemble than the ECMWF ensemble. ECMWF wind perturbations were of peak amplitude in the mid-troposphere in the before-recurvature stage. As Sinlaku moved northward, the amplitude of the upper tropospheric wind perturbations increased due to upward energy transfer and the conversion of the available potential energy into kinetic energy by the evolved SVs (Hoskins et al. 2000; Montani and Thorpe 2002), which are associated with the synoptic features north of the typhoon that played a role in its recurvature. Conversely, the peak amplitude of ECMWF temperature
perturbations was confined to the mid-troposphere. JMA perturbations were characterized by a large amplitude of specific humidity, which was nearly 4 times as large as NCEP at 925 hPa in the before-recurvature stage. In contrast, the JMA wind perturbations were small, being only a quarter the size of the ECMWF perturbations at 700 hPa prior to recurvature.

The subsequent flow perturbations for Sinlaku were found to be generally larger in ECMWF than NCEP. The perturbation growth in the ECMWF ensemble was found to be associated with 1) baroclinic energy conversion in a vortex, 2) baroclinic energy conversion associated with mid-latitude waves, and 3) barotropic energy conversion in a vortex. For baroclinic energy conversion in the vortex, the temperature perturbation was 90 degrees ahead of the streamfunction perturbation so that the perturbation could obtain eddy available potential energy from mean available potential energy, leading to the modification of the steering and asymmetric propagation vector. As previously studied, the baroclinic energy conversion associated with the mid-latitude waves caused the change in the steering flow. In addition, the radial eddy momentum flux near the center of Sinlaku was larger in the ECMWF ensemble than the NCEP ensemble. This barotropic process would result in the difference of the growth of the asymmetric propagation vector between them. A statistical verification demonstrated that NCEP’s spread for 1-day forecasts was larger than ECMWF on average, likely due to the relatively large initial perturbation amplitudes and the accordingly large modification of the steering flow at the initial time. For 3-day forecasts, however, the spread of ECMWF became larger than that of NCEP, due to the larger energy growth in ECMWF. In summary, it appeared that though the ECMWF initial perturbation amplitudes were small, the growth of the perturbations helped to obtain an appropriately large ensemble spread of tracks. Meanwhile, the relatively large amplitudes of initial perturbations seemed to play a role in obtaining the ensemble spread of tracks in NCEP. Those results are comparable to those of Magnusson et al. (2008), who compared the skill of two versions of the ECMWF EPS: one based on SVs and the other based on bred vectors (BV). They found that the initial amplitude of BVs needed to be amplified
significantly to secure a sufficient ensemble forecast spread while the growth of SVs, whose amplitude is on the order of analysis error, played an important role in the SV-based EPS.

Future work includes investigating more typhoon cases to obtain statistical significance, and extending the verification into other basins such as the Atlantic in which the results may differ. For example, as noted in Majumdar and Finochio (2010), the ability of the ECMWF ensemble to produce appropriately dispersive probabilistic forecasts of tropical cyclone track was significantly higher in the Atlantic basin than in the Northwestern Pacific basin for the 2008 season. Further studies are also required to determine optimal perturbation methods to produce a consistently reliable spread of tracks. An extension to the studies of Magnusson et al. (2008) and Buizza et al. (2008) who investigated different initial perturbation techniques in the ECMWF EPS would be recommended for tropical cyclones. In parallel, new methods to parameterize model error, for example via stochastic perturbations (McLay et al. 2007) warrant further investigation.

In Chapter 3, basic properties of SVs for TC-like vortices were investigated in a nondivergent barotropic framework. For initial vortices that did not satisfy the necessary condition of the barotropic instability, the only mechanism for SVs to grow was the Orr mechanism (through momentum fluxes). For a short optimization time, the initial SVs tended to appear immediately beyond the radius of maximum wind \((\text{RMW})\), where the radial shear of background angular velocity was large. Meanwhile, for a longer optimization time, the initial SVs were located far beyond the \(\text{RMW}\), where the background tangential flow was relatively slow, in order for the perturbation to grow over a longer time period. In the case where the barotropic instability was allowed, SVs captured this instability; the initial SVs were located where the vorticity gradient changes sign regardless of the optimization time. The SVs capturing the barotropic instability were initially tilted against the background shear and grew through transient growth for a short time, after which they evolved into a perturbation that looked like a normal mode structure. The SV “locked in” to a normal mode structure at a certain moment during the initial growth, and kept the
phase-lock structure so that it could exponentially (continuously) grow with time. It was of interest that the initial SVs had an up-shear property without relation to the necessary condition of the barotropic instability, then leading to the inward propagation of SVs according to a theory for vortex Rossby-waves (Montgomery and Kallenbach 1997).

For the azimuthal wavenumber of SVs, it was found that viscosity played an important role. With zero viscosity, a perturbation with higher wavenumbers can have a large singular value. In reality, however, as the viscous term works with the second power of wavenumber, the growth with higher wavenumbers is diminished. Thus such a low wavenumber as 1 or 2 was dominant in the leading SVs as seen in Figs. 3.4 to 3.6. When viscosity was constant, the azimuthal wavenumber of SVs was also sensitive to RMW. This was due to the fact that the magnitude of the viscous term decreases with the second power of radial location. For fixed viscosity, therefore, the azimuthal wavenumber of the leading SV increased with RMW.

Future work includes calculating SVs on a beta plane. An azimuthal wavenumber one asymmetry, that is, the beta-gyre is created on a beta plane, leading to the displacement of the vortex. It is interesting to investigate how SVs on a beta plane respond to the beta gyre, and modify the vortex motion. This will be studied with and without the background steering flows. Another interest is to interpret the relationship between the growth of SVs and the modification of track of vortex. As investigated in this study, various dynamical mechanisms can lead to the growth of SVs; the Orr mechanism, barotropic instability and baroclinic energy conversion in a vortex. It is not clear which dynamical mechanism is the most efficient in terms of modifying the track of the vortex. Those will be investigated using a simplified baroclinic model by changing initial conditions and the configurations of SV calculations.

To close, the significance of this thesis is summarized in the context of TC prediction, and important issues remaining for the TC ensemble and targeting community to explore are stated. This study has illustrated that the difference of ensemble spread of
tracks among NWP centers is attributed to the difference of the perturbation growth as well as the initial amplitude of the perturbations. Unlike previous studies, which focused on barotropic mechanisms, this study identified that baroclinically growing perturbations cause the modification of the advection flow of TCs. Furthermore, this study gave an insight into understanding what SV-based sensitivity analysis targeted for TCs represents. One of the challenging topics for our ensemble and targeting community would be to further investigate perturbations for TC ensemble track forecasts to obtain better probabilistic forecasts.

It is of great importance to survey error sources in track forecasting for various TC cases (Carr and Elsberry 2000a; 2000b) and reflect the structure into initial ensemble perturbations. Another challenging issue would be to design perturbations that are effective on other forecast interests like TC intensity and tropical cyclogenesis. Even though a state-of-the-art numerical model has difficulty in predicting TC intensity, it would be expected that if the resolution of SVs are highly increased, SVs would capture some dynamical mechanisms which are associated with axisymmetrization of an inner core structure of TCs. Those perturbations might help the intensity forecasts of TCs. However, unlike track forecasts which are mostly governed by the steering and asymmetric propagation vector, many aspects such as moist process and air-sea interaction can influence the intensity change of TCs. Accordingly, the uncertainty in initial conditions must be considered from various aspects. The most challenging issue (at least to me) is to optimize the observation network for TCs; how, when and where to observe to obtain better forecast performance. For this goal, it is important 1) to study error sources in TC forecasting, 2) to design targeting techniques so that sensitivity analysis guidance targeted for TCs can capture the error sources and 3) to make best use of targeted observations with help from a data assimilation scheme.
References


