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Development and Applications of Second-Order Turbulence Closures for Mixing in Overflows

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UNIVERSITY OF MIAMI

DEVELOPMENT AND APPLICATIONS OF SECOND-ORDER TURBULENCE CLOSURES FOR MIXING IN OVERFLOWS

By

Mehmet Ilicak

A DISSERTATION

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Mixing between overflows and ambient water masses is a crucial problem of deep-water formation in the down-welling branch of the meridional overturning circulation of the ocean. In this dissertation work, performance of second-order turbulence closures in reproducing mixing of overflows is investigated within both hydrostatic and non-hydrostatic models.

First, a 2D non-hydrostatic model is developed to simulate the Red Sea overflow in the northern channel. The model results are compared to the Red Sea Outflow Experiment. It is found that the experiments without sub-grid scale models cannot reproduce the basic structure of the overflow. The k-ε model yields unrealistically thick bottom layer (BL) and interfacial layer (IL). A new technique so-called very large eddy simulation (VLES) which allows the use of k-ε model in non-hydrostatic models is also employed. It is found that VLES results the most realistic reproduction of the observations. Furthermore, the non-hydrostatic model is improved by introducing laterally average terms, so the model can simulate the constrictions not only in the z-direction but also in the y-direction. Observational data from the Bosphorus Strait is employed to test the spatially average 2D non-hydrostatic model (SAM) in a realistic application. The simulations from SAM with a simple Smagorinsky type closure appear
to be excessively diffusive and noisy. We show that SAM can benefit significantly from VLES turbulence closures.

Second, the performance of different second-order turbulence closures is extensively tested in a hydrostatic model. Four different two-equation turbulence closures (k-ε, k-ω, Mellor-Yamada 2.5 (MY2.5) and a modified version of k-ε) and K-Profile Parameterization (KPP) are selected for the comparison of 3D numerical simulations of the Red Sea overflow. All two-equation turbulence models are able to capture the vertical structure of the Red Sea overflow consisting of the BL and IL. MY2.5 with Galperin stability functions produce the largest salinity deviations from the observations along two sections across the overflow and the modified k-ε exhibits the smallest deviations. The rest of the closures fall in between, showing deviations similar to one another.

Four different closures (k-ε, k-ω, MY2.5KC and KPP) are also employed to simulate the Mediterranean outflow. The numerical results are compared with observational data obtained in the 1988 Gulf of Cadiz Expedition. The simulations with two-equation closures reproduce the observed properties of the overflow quite well, especially the evolution of temperature and salinity profiles. The vertically integrated turbulent salt flux displays that the overflow goes under significant mixing outside the west edge of the Strait of Gibraltar. The volume transport and water properties of the outflow are modified significantly in the first 50 km after the overflow exits the strait. The k-ε and k-ω cases show the best agreement with the observations.

Finally, the interaction between the Red Sea overflow and Gulf of Aden (GOA) eddies has been investigated. It is found that the overflow is mainly transported by the undercurrent at the west side of the gulf. The transport of the overflow is episodic
depending strength and location of GOA eddies. The most crucial finding is that the Red Sea overflow leaves the Gulf of Aden in patches rather than one steady current. Multiple GOA eddies induce lateral stirring, thus diapycnal mixing of the Red Sea outflow.
To my family, their sacrifice is more than mine.
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Chapter 1

Introduction

The meridional overturning circulations (MOC) consist of four main branches: deep water formation regions, deep currents, upwelling processes that transport water from the deep to near the ocean surface, and surface currents that transport relatively light water to higher latitudes where they sink again. It is important to understand the physics behind the MOC since it relates to the role of ocean in climate dynamics. Most of the deep and intermediate water masses are transported to depth and modified in overflows from polar seas by cooling (Dickson et al., 1990) and from marginal seas by evaporation Baringer and Price (1997); Bower et al. (2005). The transport and properties (i.e. temperature and salinity) of the deep and intermediate water masses are important for the meridional overturning circulation (MOC). The overflows change the properties of the source water, bottom water in polar seas and marginal seas, by entrainment and mixing into the properties of the product water which is released into the general circulation (Price and Baringer, 1994). Since overflows are important, they have been studied extensively, e.g., Girton et al. (2001) for the Denmark Strait overflow, Gordon et al. (2004) for Antarctic shelves overflows, Baringer and Price (1997) for the Mediterranean Sea overflow, and Peters et al. (2005a) for the Red Sea overflow. One of the primary findings common to all these studies is that not only the properties of the source water but also the small-scale mixing in the overflows and their
ambient stratification determine the product water properties.

The source water modification happens over much smaller spatial and temporal scales than those associated with the MOC and the computational grids of OGCMs. Therefore, the representation of overflows in general ocean circulation models is a challenge. Shear instabilities associated with stably-stratified flows (e.g., Kelvin-Helmholtz vortices) are one of the key turbulent coherent structures responsible for mixing in overflows. These vertical eddies occur on small-scales in time and space ($O(\text{minutes})$ in time and $O(\text{meters})$ in spatial). The small scale nature of the gravity currents prevents their explicit resolution in ocean general circulation models (OGCMs) used in climate models (Wu et al., 2007). The scales separation between overflow mixing and scales those of the MOC is such that not all scales of motion can be numerically integrated simultaneously due to computational expense. Therefore, parameterizations of overflow processes need to be developed, and it seems critical to represent as realistically as possible the net effect of small-scale mixing in these parameterizations.

There are different approaches to develop parameterizations of overflow mixing. The first approach is to conduct laboratory experiments (Ellison and Turner, 1959; Hallworth et al., 1996; Baines, 2001; Cenedese et al., 2004). The main advantage of the lab experiments is that large ensembles of experiments can be conducted with known parameters. The drawbacks include the fact that the effective Reynolds number is orders of magnitude smaller and the topographic slopes are typically much larger than those in the ocean (or simply that oceanic overflows cannot be fit in a tank of a few meters size), and that it is not trivial to measure all prognostic flow variables (in particular the velocity distribution) at high resolutions. Nevertheless, laboratory experiments are extremely useful to understand some of the principal physics of bottom gravity currents (Simpson, 1987). They have been also used to develop simple mixing parameterizations in early models of overflows (Killworth, 1977).

The second approach to develop parameterizations of overflow mixing is using high
resolution numerical models, and this became possible by the improvement of computer power. By recognizing that the resolution of stratified overturning eddies would be a critical step in representing the overflow mixing, a number of studies with numerical models integrating Boussinesq equations (or so-called non-hydrostatic models) have been conducted. Idealized bottom gravity currents are simulated in 2D and 3D settings over smooth topography (Özgökmen and Chassignet, 2002; Özgökmen et al., 2004a), over complex topography (Özgökmen et al., 2004b) and in the presence of ambient stratification (Özgökmen et al., 2006). These studies belong to the category of large eddy simulation (LES), in which the energy containing flow structures are resolved in time and space, and the effect of smaller eddies on the resolved fields is represented by sub-grid scale (SGS) models. The underlying assumption of LES is that large eddies carry most of the Reynolds stress, thus must be resolved. On the other hand, small scale eddies contribute much less to Reynolds stress, and thus can be parameterized. When compared to direct numerical simulation (DNS), in which all scales of motion are resolved and closure assumptions are not needed, LES employs simple SGS closure models and offers significant computational gains (or larger domain size). While most of the SGS model development has been traditionally focused on homogeneous fluids (Sagaut, 2005), it was shown recently that simple forms work reasonably well in stratified flows, provided that the largest overturning (Ozmidov) scale is resolved (Özgökmen et al., 2007).

In contrast to LES, the parameterization of subgrid-scale mixing is much more crucial in ocean general circulation models (OGCMs). This is because OGCMs employ the hydrostatic approximation with coarse mesh sizes. As such, vertical overturning eddies (thus mixing) due to shear instabilities, such as Kelvin-Helmholtz vortices, are not captured. Thus, OGCMs must rely entirely on the accuracy of vertical (or diapycnal) parameterizations to compute mixing.

These parameterizations can be broadly classified into diagnostic and prognostic. The diagnostic models consist of simple algebraic models that are used to evaluate the eddy
diffusivity and eddy viscosity (Large et al., 1994; Large and Gent, 1999; Hallberg, 2000; Chang et al., 2005; Xu et al., 2006). Mixing models of this type are computationally inexpensive. The frequently used K-Profile Parameterization (KPP, Large et al. (1994), Large and Gent (1999)) is an example of a diagnostic model. In KPP, the vertical viscosity is specified as a dimensional constant times a simple function of the gradient Richardson number, $Ri = \frac{N^2}{[(\partial v/\partial z)^2 + (\partial u/\partial z)^2]}$. Another diagnostic model is the Turner parameterization (TP) developed by Hallberg (2000) to prescribe the entrainment velocity as a simple function of the bulk Richardson number. The dynamics behind TP is based on the laboratory study by Ellison and Turner (1959) and Turner (1986). It was further modified by Chang et al. (2005) who used an isopycnic model in an idealized gravity current problem and compared its performance with that of a 3D non-hydrostatic model. Xu et al. (2006) further improved this approach to simulate the Mediterranean overflow.

The prognostic models are essentially based on one or more differential equations. The majority of today’s closure models rely on the Reynolds decomposition of the state variables into mean and fluctuating (or turbulent) components and averaging of the Navier-Stokes equations (RANS). The eddy viscosity and diffusivity are expressed as a function of two turbulent prognostic fields, such as the turbulent kinetic energy $k$ and a turbulent length scale $l$, or dissipation rate $\varepsilon$, or frequency $\omega$. There has been a steady improvement in turbulent closure models used in oceanographic applications (Rodi, 1980; Mellor and Yamada, 1982; Baumert and Radach, 1992; Kantha and Clayson, 1994b; Burchard and Baumert, 1995; Canuto et al., 2001; Baumert and Peters, 2004; Umlauf and Burchard, 2005; Peters et al., 2005a; Baumert et al., 2005). The so-called Mellor-Yamada model (Mellor and Yamada (1982)) has been widely used in overflow studies by Jungclaus and Mellor (2000); Ezer and Mellor (2004); Ezer (2005), Ezer (2005) compared results from a 2.5km resolution hydrostatic model using Mellor-Yamada turbulence closure to those from a 0.5km resolution non-hydrostatic model for an idealized overflow problem. They found that results from non-hydrostatic and hydrostatic models are similar and they concluded
that given enough resolution hydrostatic models with two-equation turbulence closures can simulate the subgrid scale mixing quite well. However, the rest of the two-equation turbulence closures (i.e. $k - \varepsilon$, $k - \omega$) are not tested in overflow simulations. Their performance remains relatively unknown.

Recently, some hybrid models between LES and RANS are developed due to increase in the computational power. Very Large Eddy Simulation (VLES) is one of them. In VLES, only the very large turbulent structures are resolved and the remaining turbulence is parameterized. Since a wider range of scales is modeled in VLES than in LES, the SGS models need to be more comprehensive, and thus turbulence closures developed in the context of RANS provide an attractive solution. The main concepts behind LES, RANS and VLES are summarized in Fig. 1.1. The VLES approach has been used by Magagnato and Gabi (2002) and Ruprecht et al. (2003) for engineering applications and shown to provide better results than RANS models. This is because the mean flow is either independent of time or slowly varying in RANS, and conventional turbulence models using RANS are inadequate for unsteady flows when the turbulent time scale has the same order of buoyancy time scale (i.e. $T \sim N^{-1}$). VLES is designed to capture and model the turbulence in transient flows.

The aim of this dissertation is to investigate the performance of different two-equation turbulence closures in hydrostatic and non-hydrostatic numerical simulations of different overflows. Three different overflows have been investigated. These are the Red Sea, the Mediterranean and the Bosphorus outflows. A brief summary of these outflows is given in Chapter 2. A 2D non-hydrostatic model is developed to simulate the Red Sea overflow in the northern channel. Numerical experiments with VLES and RANS closures have been conducted and we compare them with observations. The comparisons are displayed in Chapter 3. The 2D non-hydrostatic model is improved to simulate not only vertical but also lateral constrictions in Chapter 4. Some idealized cases of exchange flows in complex topographies are compared between 3D LES and 2D VLES models. Modified VLES is
Figure 1.1  Schematic description of the concepts behind RANS, LES and VLES. The energy containing vertical scale is the Ozmidov scale in a stratified flow, and the smallest dynamical scale is the Kolmogorov scale.

employed to reproduce the Bosphorus strait overflow in the same chapter. The simulations of the Bosphorus outflow are compared to the data obtained from Gregg and Özsoy (1999). Then we move to investigate the performance of two-equation turbulence closures in a 3D hydrostatic model. Four different closures are employed in 3D numerical simulations of the Red Sea overflow in Chapter 5. The accuracy of the model results is evaluated using hydrographic and current observations collected in the Red Sea Outflow Experiments (Peters et al., 2005b; Peters and Johns, 2005; Bower et al., 2005). The Mediterranean overflow is also studied using two-equation turbulence closures in 3D hydrostatic model. We compare the simulated Mediterranean outflow to the observations obtained from the 1988 Gulf of Cádiz Expedition (Price and Baringer, 1994; Baringer and Price, 1997) in Chapter 6.
Finally, we investigate the pathways of the Red Sea outflow and its interaction with Gulf of Aden eddies in Chapter 7. The principal conclusions of this work are summarized and discussed in Chapter 8.
Chapter 2

Outflow Characteristics and Dynamics

In this chapter, we briefly describe the essential characteristics of several overflows and give some information about the observational expeditions. Throughout the all thesis we use these observations to compare and evaluate the model results.

2.1 The Red Sea outflow

Although the volume transport of the Red Sea overflow water is the smallest one compared to these major overflows (around annual mean of just 0.37 Sv from Murray and Johns (1997)), observations taken throughout the Indian Ocean during the World Ocean Circulation Experiment shows that Red Sea overflow water has a distinctive and far reaching signal. The influence of Red Sea overflow water has been measured in the Agulhas Current, and also in the Agulhas retroflection region to the south of South Africa (Beal et al. (2000)). The main characteristics of the Red Sea Overflow Water (RSOW) are summarized in the following. The Red Sea region is hot and dry with high evaporation of about 2 m/yr (Bower et al., 2002). The dense Red Sea water is formed at its northern end in winter (Sofianos and Johns, 2001). This dense, warm and salty water leaves the Red Sea through the 150 km long and comparatively narrow strait of Bab el Mandeb (BAM) which has a sill depth of 150 m (Murray and Johns, 1997). South of Bab el Mandeb the outflow divides into
two channels, a Northern Channel (NC) typically 5 km wide and 130 km long paralleling the Yemeni coast, and a wider and shallower Southern Channel (SC) toward the Djibouti coast (Peters et al., 2005b). These channels end at about 800 m depth at the drop-off into 1600 m deep Tadjura Rift as shown in Fig. 2.1.

Figure 2.1  Sea floor topography in the Red Sea outflow area in the western Gulf of Aden. Depth is contoured every 100 m. The Northern and Southern channels are highlighted. Numbers indicate CTD/LADCP stations from the REDSOX 2001.

The Red Sea Outflow Experiment (REDSOX) of 2001 is the first comprehensive study of the RSOW, south of Bab el Mandeb Strait throughout Gulf of Aden, where RSOW equilibrates and spreads into the ambient water. REDSOX was a joint program between Rosenstiel School of Marine and Atmospheric Science and the Woods Hole Oceanographic Institution. It was designed to understand the structure, dynamics and mixing of the Red Sea overflow. Two cruises were performed in 2001, one in the winter (REDSOX-1) when the
outflow is maximum and one in the summer (REDSOX-2) when the outflow is minimum. The observations have three components. (i) 108 conductivity-temperature-depth (CTD) and lowered Acoustic Doppler Current Profilers (LADCP) describe the three dimensional water property distributions and circulation characteristics between BAM and the Tadjura Rift. (ii) Direct measurements of turbulent mixing were made to study the bottom stress in descending plumes. (iii) Acoustically tracked drifters were launched to study the spreading of the RSOW in the entire Gulf of Aden. The reader is referred to Bower et al. (2002); Peters and Johns (2005); Bower et al. (2005); Peters and Johns (2006); Peters et al. (2005b); Matt and Johns (2007) for the complete analysis of the observational results.

One of the major developments during this program concerns the vertical structure of the Red Sea overflow. Peters et al. (2005b) defined two distinct layers to describe the vertical structure of the overflow, the bottom layer (BL), and the interfacial layer (IL). The former reaches from the bottom to a height where the velocity is maximum, and the latter extends from this maximum upward with strong stratification and large shear. The BL is well mixed and maintains high salinities along the Northern Channel. The ratio of stratification to shear leads to low Richardson numbers in both the IL and BL. While the water in the BL remains relatively undiluted along the length of the Northern Channel, the IL shows strong mixing and dilution of RSOW. Entrainment of ambient waters thus mainly affects the IL. It is noteworthy that the IL carries more than half of the vertically integrated volume transport near the lower end of the Northern Channel. The distinct vertical structure and properties of BL and IL determine the vertical distribution of density, temperature and salinity of the product waters. Therefore, if a model is to faithfully reproduce the overflow characteristics, it has to capture its vertical structure consisting of BL and IL.
2.2 The Mediterranean outflow

The Mediterranean overflow (henceforth the Med) exits the Strait of Gibraltar and spreads into the North Atlantic Ocean (Fig. 2.2). It mixes with ambient water and creates a warm saline tongue of water at an intermediate depth of around 1100 m (Levitus and Boyer, 1994). The Med is a bottom trapped gravity current when it flows out the Strait of Gibraltar. The Med follows the Iberian continental shelf due to earth’s rotation. The overflow shows interesting spatial variations and multi-core features at about 7°W, as repeatedly reported (Zenk, 1970; Ambar, 1979; Ambar et al., 2002). Further downstream at Cape St. Vincent, the topography abruptly changes direction, and the flow separation from the bottom slope promotes the generation of MOW anticyclonic vortices, so called meddies (Bower et al., 1997; Serra et al., 2005). Beyond Cape St. Vincent, the Med generally separates into two main branches: one flows northward and the other flows westward. However, the strength and fate of these flows remain largely unknown.

![Image of sea floor topography in the Mediterranean outflow area. Depth is contoured every 200 m. Numbers indicate CTD stations from the Gulf of Cádiz Experiment.](image)

**Figure 2.2** Sea floor topography in the Mediterranean outflow area. Depth is contoured every 200 m. Numbers indicate CTD stations from the Gulf of Cádiz Experiment.

We use the data from the 1988 Gulf of Cadiz Expedition (Price et al., 1993) for our
model-observation comparison. For more details about the observations, the reader is referred to Price and Baringer (1994), Johnson et al. (1994b), Johnson et al. (1994a), Baringer and Price (1997), Baringer and Price (1997). The data consists of 120 CTD profiles, 79 in situ horizontal current profiles gathered with the XCP (expendable current profiler). Multiple drops are obtained at some stations. All repeated profiles show similar velocity and density structures within the plume, implying a negligible tidal influence. The plume appears steady during the survey.

2.3 The Bosphorus outflow

The Turkish Strait System (TSS) is a complex domain formed by the small internal Marmara Sea coupled with the Bosphorus and Dardanelles Straits. Its elongated and tortuous waterway between the European and Asian continents controls the exchange and mixing between the Mediterranean (Aegean) and Black Sea waters, and determines their contrasting physical, chemical and biological properties (Fig. 2.3a). The TSS has been extensively studied to date.

A two-layer stratified flow through the Bosphorus Strait is basically driven by the density difference between the Black and the Aegean Seas. In addition to density differences between basins, remote atmospheric forcing, water budgets and steric volume changes of the adjacent basins as well as the local interactions within the TSS exert dynamical forces on the system. The location of the Turkish Straits System, of which the Bosphorus is the most restrictive member, is shown in Fig. 2.3a,b.

The Bosphorus Strait operates in the full range of weak to strong forcing. Hydraulic controls associated with geometric features of the strait constitute a unique example of the maximal exchange regime that requires a balance with at least two critical sections, a sill and contraction with ideal configuration in the case of the Bosphorus, as foreseen by the cardinal work of Farmer and Armi (1986). Blocking of the flows in either layer
Figure 2.3  (a) Location map for the Turkish Straits System, connecting the Black and the Mediterranean Seas via Dardanelles Strait, Marmara Sea and Bosphorus Strait. (b) The bottom topography of the Bosphorus Strait.
Chapter 3

Performance of two-equation turbulence closures in a non-hydrostatic model

The aim of this chapter is to compare the performance of two RANS type closures representing the mixing process in overflows. Standard RANS and VLES closures are chosen for the investigation. A non-hydrostatic model is required since VLES captures some of the turbulence implicitly. Therefore, we develop a 2D Boussinesq (non-hydrostatic) model to simulate the northern channel of the Red Sea overflow. The 2D approximation is justified because the channel is narrow enough to naturally restrict motions in the lateral direction. Furthermore, the width of the channel is much smaller than the Rossby radius of deformation and the channel length. Thus, the effects of baroclinic instability and rotation on the overflow dynamics are assumed to be small, and neglected.

3.1 Model configuration

3.1.1 The 2D Reynolds solver

The 2D model is based on the vorticity-stream function formulation, and the governing equations are non-dimensionalized as follows:

\[
\begin{align*}
\psi &= U_0 h \psi^*, \\
(x, z) &= h(x^*, z^*), \\
\zeta &= \left(\frac{U_0}{h}\right) \zeta^*, \\
S &= \Delta S S^*, \\
T &= \Delta T T^*, \\
t &= \left(\frac{h}{U_0}\right) t^*.
\end{align*}
\]
Here $\psi$ is the stream function, $\zeta$ is the vorticity, $h$ is the thickness of the overflow at the inlet, $\Delta S$ and $\Delta T$ are the ranges of salinity and temperature in the system obtained from REDSOX data, and $U_0$ is the characteristic velocity scale of the Red Sea overflow, 1 m s$^{-1}$. Dimensional mean velocities ($\bar{U}$ (m/s) and $\bar{W}$ (m/s)) are defined and non-dimensionalized as:

$$\bar{U} = -\frac{\partial \psi}{\partial z} = U_0 \bar{U}^*$$ and $$\bar{W} = \frac{\partial \psi}{\partial x} = U_0 \bar{W}^* .$$

Asterisks and bars are dropped for simplicity, and the final forms of the stream function, vorticity, salinity and temperature equations are (henceforth RAVS will be used for Reynolds Averaged Vorticity Stream-function equations)

$$\frac{D\zeta}{Dt} = \frac{1}{Fr^2} \left( R_\rho \frac{\partial T}{\partial x} - \frac{\partial S}{\partial x} \right) + \frac{1}{Re} \nabla^2 \zeta + \frac{\partial}{\partial x} \left( \frac{1}{Re} \frac{\partial \zeta}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{1}{Re} \frac{\partial \zeta}{\partial z} \right)$$  \hspace{1cm} (3.1)

$$\frac{DT}{Dt} = \frac{1}{RePr} \nabla^2 T + \frac{\partial}{\partial x} \left( \frac{1}{Re} \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{1}{Re} \frac{\partial T}{\partial z} \right)$$ \hspace{1cm} (3.2)

$$\frac{DS}{Dt} = \frac{1}{RePr} \nabla^2 S + \frac{\partial}{\partial x} \left( \frac{1}{Re} \frac{\partial S}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{1}{Re} \frac{\partial S}{\partial z} \right)$$ \hspace{1cm} (3.3)

$$\nabla^2 \psi = \zeta,$$ \hspace{1cm} (3.4)

where $Re = U_0 h / \nu$ is the Reynolds number, the ratio of inertial to viscous forces, and $Fr = U_0 / \sqrt{g \beta \Delta S h}$ is the Froude number, the ratio of the inertial to gravitational forces in the flow; $g = 9.81 \text{m s}^{-2}$ is the gravitational acceleration, $R_\rho \equiv \alpha \Delta T / \beta \Delta S$ is the density ratio, quantifying influence of temperature and salinity on density respectively, where $\alpha$ is the temperature expansion coefficient and $\beta$ is the salinity contraction coefficient for seawater in the linear equation of state $\rho = \rho_0 (1 - \alpha \Delta T + \beta \Delta S)$. $Pr = \nu / K$ is the molecular Prandtl number, the ratio of (molecular values of) viscosity to diffusivity, $\nabla^2$ is the Laplace
The vorticity equation is

\[ \frac{D \Phi}{Dt} = \frac{\partial \Phi}{\partial t} + J(\psi, \Phi). \]

The Jacobian, \( J(\psi, \Phi) \),

\[ J(a, b) = \frac{\partial a \partial b}{\partial x \partial z} - \frac{\partial b \partial a}{\partial x \partial z}, \]

is discretized using the Arakawa (1966) scheme to conserve both energy and enstrophy. \( Re_t \) is the turbulent Reynolds number which is computed from the turbulence closure models, and \( Pr_t \) is the turbulent Prandtl number, discussed below. The last two terms on the right hand side of the Equations (3.1), (3.2), (3.3) contain the turbulent fluxes.

A conformal mapping is used to convert complex topography in physical coordinates \((x, z)\) into a rectangular domain in computational coordinates \((\xi, \eta)\) (Figs. 3,4). The ensuing equations are discretized with finite differences. The Poisson Equation (3.4) is solved using a direct Fast Fourier transform, which is not only a more accurate method than iterative solution techniques, but also faster for a large number of discretization points. The vorticity equation (3.1) is advanced in time using Adams-Bashforth-3 method. Temperature and salinity transport equations (3.2) and (3.3) rely on the Flux Corrected Transport Zalesak method to eliminate Gibbs oscillations (Zalesak (1978)).

### 3.1.2 The \(k-\varepsilon\) closure model

Two different closure models are applied. The first one is the \(k-\varepsilon\) model based on Baumert and Peters (2000), Peters et al. (2005a) and Warner et al. (2005) and the second one is VLES.

The turbulent kinetic energy, \( k \), and its dissipation rate, \( \varepsilon \), are non-dimensionalized by \( k = U_0^2 k^* \), and \( \varepsilon = (U_0^3/h)\varepsilon^* \), respectively. The asterisks are omitted for simplicity. The transport equations for nondimensional turbulent kinetic energy and the dissipation rate are
then given by

\[
\begin{align*}
\frac{Dk}{Dt} &= P + B - \varepsilon + \frac{1}{Re} \nabla^2 k + \frac{\partial}{\partial x} \left( \frac{1}{Re, \sigma_k} \frac{\partial k}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{1}{Re, \sigma_k} \frac{\partial k}{\partial z} \right), \\
\frac{De}{Dt} &= \frac{\varepsilon}{k} [c_{\varepsilon 1} (P + c_{\varepsilon 3} B) - c_{\varepsilon 2} \varepsilon] + \frac{\nabla^2 \varepsilon}{Re} + \frac{\partial}{\partial x} \left( \frac{1}{Re, \sigma_\varepsilon} \frac{\partial \varepsilon}{\partial x} \right) \\
&\quad + \frac{\partial}{\partial z} \left( \frac{1}{Re, \sigma_\varepsilon} \frac{\partial \varepsilon}{\partial z} \right).
\end{align*}
\] (3.5) (3.6)

In equations 3.5 and 3.6, \( P \) and \( B \) are the production term due to shear and buoyancy forces, respectively:

\[
\begin{align*}
P &= \frac{1}{Re} \left[ \left( \frac{\partial U}{\partial z} + \frac{\partial W}{\partial x} \right)^2 + 2 \left( \frac{\partial U}{\partial x} \right)^2 + 2 \left( \frac{\partial W}{\partial z} \right)^2 \right], \\
B &= \frac{1}{Fr^2 Re_t Pr_t} \left( \frac{\partial S}{\partial z} - R \frac{\partial T}{\partial z} \right).
\end{align*}
\] (3.7) (3.8)

The coefficients used in this \( k - \varepsilon \) model are listed in Table 3.1.

<table>
<thead>
<tr>
<th>( \sigma_k )</th>
<th>( \sigma_\varepsilon )</th>
<th>( c_{\varepsilon 1} )</th>
<th>( c_{\varepsilon 2} )</th>
<th>( c_{\varepsilon 3} )</th>
<th>( c_\mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>1.3</td>
<td>1.44</td>
<td>1.92</td>
<td>-1.0</td>
<td>0.09</td>
</tr>
</tbody>
</table>

The turbulent Reynolds number, \( Re_t \), and turbulent Prandtl number, \( Pr_t \), are defined as

\[
\begin{align*}
Re_t &= \left( c_\mu \frac{k^2}{\varepsilon} \right)^{-1}, \\
Pr_t &= \frac{S_M}{S_H},
\end{align*}
\] (3.9) (3.10)

where \( S_H \) and \( S_M \) are the stability functions taken from Kantha and Clayson (1994b), and
\[ S_H = \frac{A_2(1 - 6A_1/B_1)}{1 - 3A_2G_h(6A_1 + B_2(1 - C_3))}, \quad (3.11) \]

\[ S_M = \frac{B_1^{-1/3} + (18A_1A_1 + 9A_1A_2(1 - C_2))S_HG_h}{1 - 9A_1A_2G_h}, \quad (3.12) \]

\[ G_h = \frac{G_{h,unlimit} - (G_{h,unlimit} - G_{h,crit})^2}{G_{h,unlimit} + G_{h0} - 2G_{h,crit}}, \quad (3.13) \]

\[ G_{h,unlimit} = -\frac{N^2L^2}{2k}, \quad (3.15) \]

where \( N \) is the buoyancy frequency, and \( L \) is the length scale of the energy containing eddies defined as

\[ L = c_1 \frac{k^{3/2}}{\varepsilon}, \quad (3.16) \]

where \( c_1 \) may have a wide range, from 0.161 (Baumert and Peters (2000)) to 1 (Buntić O et al. (2006)). In this study \( c_1 \) is taken as 1. Parameters used in stability functions are given in Table 3.2. Further information about the stability functions (such as plots of \( S_H, S_M \) versus \( G_h \), and variation of \( S_H, S_M \) with flux Richardson number) can be found in Kantha and Clayson (1994b).

<table>
<thead>
<tr>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( B_1 )</th>
<th>( B_2 )</th>
<th>( C_2 )</th>
<th>( C_3 )</th>
<th>( G_{h,min} )</th>
<th>( G_{h,0} )</th>
<th>( G_{h,crit} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.92</td>
<td>0.74</td>
<td>16.6</td>
<td>10.1</td>
<td>0.7</td>
<td>0.2</td>
<td>-0.28</td>
<td>0.0233</td>
<td>0.02</td>
</tr>
</tbody>
</table>

### 3.1.3 VLES closure model

The second turbulence closure model is VLES. In this model, part of the turbulence spectrum can be resolved, and the influence of the unresolved part on the resolved part has to be modeled. In order to distinguish the resolved and the modeled parts, a spatial filtering technique is used (Buntić O et al., 2006). In this method a length scale filter, \( \delta \), is applied...
to the standard $k - \varepsilon$ closure. This length scale depends on either the computational time step and local velocity or local grid size. For the turbulent kinetic energy, large scales are already resolved and the small scales are filtered and obtained from

$$\hat{k} = \begin{cases} k & \text{if } \delta \geq L, \\ (\delta\varepsilon)^{2/3} & \text{if } \delta < L, \end{cases}$$  \hspace{1cm} (3.17)

where the hat symbol, “^”, represents spatial filtering and

$$\hat{\delta} = \alpha \cdot \max \left\{ \frac{|u| \cdot \Delta t}{\sqrt{\Delta V}} \right\},$$  \hspace{1cm} (3.18)

where $\alpha = 2$ is a model constant, $u$ is the local velocity, $\Delta t$ is the computational time step, and $\Delta V$ is the local volume (i.e. area in 2D). According to Kolmogorov (1942) theory, the dissipation rate is constant for all scales in the whole spectrum, which leads to

$$\varepsilon = \hat{\varepsilon}. \hspace{1cm} (3.19)$$

The filtered turbulent kinetic energy, $\hat{k}$, and its dissipation rate, $\hat{\varepsilon}$, were re-substituted into the equations 3.5 and 3.6, and turbulent Reynolds number computed as

$$Re_t \equiv \left( \frac{\hat{k}^2}{c_{\mu} \hat{\varepsilon}} \right)^{-1}.$$

(3.20)

The same stability functions are also used in VLES. Thus, the filtering procedure (3.17) is the only difference between the $k - \varepsilon$ and VLES turbulence closure models.
3.2 Model setup and parameters

The topography in the model is derived from multi-beam echosounder data of the northern channel of the Red Sea (Fig. 2.1) between Station 35 and Station 39. The station positions are shown in Fig. 3.1. The domain is 102 km long and has a maximum depth of 1.28 km. The flow is forced by observed temperature and salinity profiles at the inlet, the left boundary coinciding with the location of Station 35. There is also a net transport specified at the top boundary for the stream function. REDSOX-1 observations clearly show a surface mixed layer, which is approximately 100 m thick, salty and warm because of heating and evaporation. The surface mixed layer is important since it occupies a significant portion of the water depth. As this layer has nearly constant properties along the channel, it is used as part of the initial condition and maintained by relaxation to the observed data (Fig. 3.5). A buffer zone is implemented downstream of Station 9 to absorb the incoming flow. This buffer zone is for salinity and temperature, while an Orlanski-type boundary condition (Orlanski (1976)) is used for vorticity.

The realistic Pr based on molecular values for heat is $Pr \approx 7$ and $Pr \approx 700$ for salt. The effect of such high Pr is to extend active tracer cascade to smaller spatial scales beyond the dissipation scale of momentum. As such, active tracers tend to exhibit thinner boundary layers, which can lead to various interesting phenomena at small scales, such as Hölmboe shear instability (Hölmboe, 1962) or double-diffusive instability (Stern, 1960). The resolution of such small-scale instabilities as well as development of appropriate SGS models is beyond the scope of the present study. Their consideration would particularly violate our overall VLES philosophy which implicitly rests on very high local Reynolds numbers which thus implies $Pr = 1$ for all scalars. The Froude number $Fr = U_0/Nh_0 = U_0/\sqrt{g \Delta \rho h_0/\rho_0}$ is set as follows. The velocity scale is assumed as that of the initial gravity current speed, $U_0 = \sqrt{g \Delta \rho' h/\rho_0}$. Also, initially $h_0 = 2h$ since $h_0$ is the initial depth at the model inlet. Therefore, the Froude number is set to $Fr = 1/\sqrt{2}$, and this value is in the range of observations from Peters et al. (2005b). Using a thermal expansion
Figure 3.1  Model domain size, topography, locations and numbers of the observational stations.

Figure 3.2  Sample model domain discretization using $101 \times 16$ (on average $\Delta x \approx 1000$ m, and $\Delta z \approx 66$ m) points with conformal mapping.

coefficient of $\alpha = 1.7 \times 10^{-4}$ °C$^{-1}$ and saline contraction coefficient $\beta = 7.5 \times 10^{-4}$ psu$^{-1}$ estimated in the temperature range of $14.5 - 25.5$ °C and the salinity range of $35.6 - 39.9$ psu, the density ratio is set to $0.579$.

Insulated (zero flux) boundary conditions are applied for salinity and temperature at the bottom and top surface (i.e. $\partial S/\partial n = \partial T/\partial n = 0$, where $n$ is normal to the surface). A net transport is also specified as $\psi_{net} = 105$ m$^2$s$^{-1}$ at the top surface; detailed information can be found in Özgökmen et al. (2003). A free-slip boundary condition is applied for the top surface. For the bottom boundary, wall-layer condition is applied to vorticity, in that

$$\zeta = -\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \quad \text{at } z = z_{bottom}$$

(3.21)

$$\tau_{bottom} = \nu \frac{\partial u}{\partial z} = C_d U |U|,$$

(3.22)
where \( C_d \) is the drag coefficient which is governed by the vertical mesh size and the length of the bottom-roughness elements (Baumert and Radach (1992)). \( \tau_{\text{bottom}} \) is the shear at the bottom. From (3.21) and (3.22), we get the nondimensional form of the vorticity boundary condition at the bottom,

\[
\zeta_{\text{bottom}} = -Re_t \cdot C_d U |U|,
\]

(3.23)

where \( U \) is taken at the first grid point (\( \Delta z/2 \)).

The boundary conditions for \( k \) and \( \varepsilon \) at the bottom are obtained by assuming a local balance between production and dissipation from Eq. (3.5), namely \( P = \varepsilon \) (see Baumert and Radach (1992); Burchard and Baumert (1995) for details):

\[
k = |U_f|^2 \mu^{-1/2},
\]

(3.24)

\[
\varepsilon = \frac{U_f^3}{\kappa z},
\]

(3.25)

where \( U_f = u_f/U_0 \) is the nondimensional bottom friction velocity found from \( U_f = \sqrt{C_d U^2} \) and the von Kármán constant \( \kappa = 1/\sqrt{2\pi} \approx 0.399 \).

Estimates of the bottom drag coefficient from overflow observations indicate a broad range of \( 1 \times 10^{-3} \leq C_d \leq 10 \times 10^{-3} \) (Girton and Sanford, 2003; Peters and Johns, 2006). Modeling of the drag coefficient for flows over irregular topography is a complex problem in its own right. Here, a single value of \( C_d \) is used for simplicity. In order to decide on this value, several experiments with different Reynolds numbers and resolutions are conducted. First, low resolution and low Reynolds number (\( \Delta x \approx 200 \text{ m}, \Delta z \approx 10 \text{ m}, Re = 5000 \)) experiments have been conducted with three different drag coefficient values: \( C_d = 1 \times 10^{-3} \), \( C_d = 5 \times 10^{-3} \), and \( C_d = 8 \times 10^{-3} \). The results are compared with data from REDSOX stations, which show that \( C_d = 5 \times 10^{-3} \) and \( C_d = 8 \times 10^{-3} \) achieve better results than \( C_d = 1 \times 10^{-3} \) (Fig. 3.3). Then, both the Reynolds number and horizontal resolution are increased (\( \Delta x \approx 100 \text{ m}, \Delta z \approx 10 \text{ m}, Re = 10^4 \)) to carry out further comparisons between these two under different circumstances. As shown in Fig. 3.4, \( C_d = 5 \times 10^{-3} \) gives some-
what better results than $C_d = 8 \times 10^{-3}$, and therefore $C_d = 5 \times 10^{-3}$ is used in all the following experiments.

**Figure 3.3** Comparison of salinity profiles from REDSOX-1 stations 58, 37, 82 and 83 to those obtained with VLES using $C_d$ values of $1 \times 10^{-3}$ (dotted lines), $5 \times 10^{-3}$ (lines with +), and $8 \times 10^{-3}$ (lines with *) for $Re = 5000$, $\Delta x \approx 200$ m, $\Delta z \approx 10$ m.

The model parameters are summarized in Table 3.3.

**Table 3.3** Parameters of the numerical simulation.

<table>
<thead>
<tr>
<th>$L_x$</th>
<th>$H$</th>
<th>$Fr$</th>
<th>$R_p$</th>
<th>$Pr$</th>
<th>$C_d$</th>
<th>$\psi_{net}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>102 km</td>
<td>1.28 km</td>
<td>0.707</td>
<td>0.579</td>
<td>1</td>
<td>$5 \times 10^{-3}$</td>
<td>105 m$^2$/s</td>
</tr>
</tbody>
</table>

The main experimental matrix consist of 30 cases, with three different models, at three Reynolds numbers, and three different horizontal resolutions, and two different vertical resolutions (Table 3.4). The models are standard $k-\varepsilon$, VLES and one without SGS stresses (or under-resolved 2D DNS). The latter is important to understand whether turbulence models are necessarily needed in the simulations. The main task is to explore which set of model
Figure 3.4 Comparison of salinity profiles from REDSOX-1 stations 58, 37, 82 and 83 to those obtained with VLES using $C_d$ values of $5 \times 10^{-3}$ (lines with +) and $8 \times 10^{-3}$ (lines with *) for $Re = 10000$, $\Delta x \approx 100$ m, $\Delta z \approx 10$ m.

Equations leads to the best agreement with REDSOX-1 observations. It is also important to investigate the dependence of the results on $Re$. This is because oceanic $Re$ is extremely high. If results from $k - \varepsilon$ or VLES change with $Re$, this will obviously pose some questions regarding the utility of these approaches for oceanic overflow simulations. Finally, resolution dependence is explored. The $k - \varepsilon$ model presumably parameterizes the effect of all turbulence. As such, there is no information about resolved scales in such models. On the other hand, VLES incorporates information about the model resolution via (3.17). The performance of these models in a non-hydrostatic setting must be explored.

Two different approaches can be followed in evaluating the performance of turbulence closure models (Sagaut, 2005). The first may be called a priori approach, in which the turbulent kinetic energy $k$ and its dissipation rate $\varepsilon$ are directly compared to measurements.
An example of this approach can be found in the recent study by Peters and Baumert (2007). This approach requires the availability of proper turbulence observations which are quite rare and which are often incomplete as either only the dissipation rate is measured or only the Thorpe scale or the Reynolds stress are available. Generally, this approach rests on measurement of second moments of the hydro-thermodynamic fields.

The second approach may be called *a posteriori* method and aims at an indirect evaluation using only first moments of the state variables for comparison with observations, namely simply the water temperature or current or salinity or even quantities of still higher integrative character like the mass or volume transport through certain cross sections.

We conduct *a posteriori* testing, by comparing modeled salinity, temperature and velocity profiles and overflow mass transport to those from REDSOX-1 observations.

### 3.3 Results

Snapshots of the salinity distribution from the high resolution-high $Re$ number ($\Delta x = 50 \text{ m}$ and $Re=20000$, henceforth HighRe) experiments are plotted in Fig. 3.5 to describe the
time evolution of the outflow. The formation of a characteristic head is observed in the case without SGS parameterization. This is a transient feature associated with the initial propagation and does not carry much significance for overflows in statistical equilibrium. The overflow travels over the channel’s topography and reaches the eastern boundary after 31.7 hours (≈ 1.3 days). Further integration shows the emergence of a hydraulic-jump like transition approximately 30 km into the domain (Fig. 3.5d). This transition coincides with the increase in the slope angle, and appears to result in a significant dilution of the overflow, removing the distinction between the IL and BL seen in the observations. Such high mixing as in Fig. 3.5b appears to be inconsistent with the REDSOX-1 observations. As such, we conclude that one of the key requirements for the turbulence closures is to impact the solution such that the distinction between IL and BL is preserved.

Salinity snapshots from experiments with the same resolutions and Reynolds numbers (HighRe case) but with turbulence closures $k - \varepsilon$ and VLES are plotted in Fig. 3.5 as well. Both $k - \varepsilon$ and VLES satisfy the above requirement, namely they result in distinct IL and BL in the overflow. While the characteristics of results from $k - \varepsilon$ and VLES are similar qualitatively, they differ quantitatively. For instance, the overflow in the equilibrium state in VLES (Fig. 3.5f) is characterized by a layer at the bottom approximately 100m thick, which is well mixed and transports the dense water signal along the channel with little dilution. Above this bottom layer, there is another layer approximately 200m thick where temperature and salinity values gradually decreases away from the bottom due to mixing with ambient water masses. The realism of these solutions can be assessed using the salinity distribution from REDSOX-1 observations, illustrated in Fig. 3.5g. The REDSOX-1 salinity field is plotted by interpolating the data between the stations. Observed fields are not shown after Station 9 since that region coincides with the model’s radiation boundary condition zone, therefore model fields do not reflect the underlying physics in that regime. The results from VLES appear to show a better agreement with the observations than that from $k - \varepsilon$. In particular, the $k - \varepsilon$ model seems to result in unrealistic thicknesses for
both BL and IL. The dramatic change in the flow pattern from the case without SGS, and those with turbulence closures appears to be related to the turbulent Prandtl number ($Pr_t$). The stability functions play important roles to calculate the $Pr_t$ (see the equation 3.10). When $Pr_t$ is set equal to 1 explicitly, $k-\varepsilon$ models produce very similar results to the model without SGS (not shown in here). More quantitative and extensive model-data comparisons at individual stations follow next.

### 3.3.1 Comparison of modeled and observed scalar fields

The temperature and salinity profiles are compared with the REDSOX-1 observations along the northern channel. Four stations along the northern channel (Fig. 3.1) are chosen to compare the vertical structure of the plume. The salinity and temperature profiles are compared for the LowRe case in Figs. 3.6 and 3.7, respectively, and also for the HighRe case in Figs. 3.8 and 3.9. In these figures, data collected in the REDSOX-1 cruise are compared with time mean of the computed profiles. Time averaged salinity and temperature fields are calculated from $\bar{X} = (\tau_2 - \tau_1)^{-1} \int_{\tau_1}^{\tau_2} X dt$, where the time interval is chosen as $\tau_1=1.3$ days and $\tau_2=4.3$ days, which corresponds to the period after the modeled outflows reach a quasi-steady state. In order to show the variability of the profiles during this time period, 95% confidence intervals are also shown around the mean profiles. The behavior of the temperature with different models (Figs. 3.7, 3.9) is similar to that of the salinity (Figs. 3.6, 3.8). Thus, the following comparisons and discussions are based on salinity profiles.

When the model is run without any SGS parameterization, results are not satisfactory because of excessive mixing and variability, and it is really hard to achieve even a quasi-steady solution. For LowRe, mixing is not enough and there is formation of an IL (green curve in figure 3.6), while for HighRe there is adequate mixing to form an IL, however the BL is dramatically decreased (green curve in figure 3.8), and outflow signal becomes weak. In the standard $k-\varepsilon$, the IL is thicker than observed, especially in LowRe case. One of the limitations of $k-\varepsilon$ models is that they are designed for flows with steady mean
Figure 3.5  Salinity distribution of the HighRe model with w/out SGS ((a) and (d)), standard $k − \varepsilon$ ((b) and (e)) schemes, VLES ((c) and (f)), respectively as a function of time at $t = 7.82$ h ((a)-(c)) and $t = 79.63$ h ((d)-(e)). (g) The salinity distribution based on REDSOX-1 observations is plotted in (g).
flows. As such, this transient problem could be tough to handle. This may help explain the resulting high diffusivities, and a thicker IL (red curves in Figs. 3.6, 3.8). The BL is also thicker than the observational data. As such, the $k - \varepsilon$ model appears to cause overly thick BL and IL. Finally, the variability of the profiles around the mean is also quite high with the $k - \varepsilon$ model. Salinity profiles obtained using VLES show the most faithful agreement with observations (blue curves in Figs. 3.6, 3.8). Even in low Reynolds number - low resolution configuration, results are quite promising. High salinity and temperature signals at the bottom layer advected throughout the channel with little dilution, both the BL and IL are consistent with the REDSOX-1 observations.

### 3.3.2 Sensitivity to grid spacing

Different horizontal resolutions are used to examine the grid sensitivity of the three different models. For the lowest resolution $\Delta x = 200$ m and $\Delta z = 10$ m, and for mid-resolution $\Delta x = 100$ m and $\Delta z = 10$ m are used. The finest resolution is $\Delta x = 50$ m and $\Delta z = 10$ m. For most of the experiments, the vertical resolution is 10 m, since according to Peters and Johns (2006), the average bottom layer is 100 m thick, thus an adequate number of grid points has been used to capture this layer. Salinity comparisons at different resolutions for $Re = 20000$ are plotted in Fig. 3.10. The first impression from this figure is that increasing the resolution gives better results for VLES and $k - \varepsilon$.

In order to quantify the difference between the results of the different resolutions of the different models, an error function is defined as

$$
Error_S \equiv \frac{1}{n} \sum_{i=1}^{n} \frac{1}{\Delta S_r} \sqrt{\sum_{j=1}^{N_i} \left[ \bar{S}_{model}^i(j) - S_{obs}^i(j) \right]^2 / N_i},
$$

(3.26)

where $\bar{S}_{model}^i$ is time-averaged model output, and $S_{obs}^i$ is REDSOX-1 data at station $i$, $N_i$ is the number vertical sampling points at each data station, $n$ is the number of stations, $Error_S$ is the root mean square (rms) error normalized by salinity range $\Delta S = 3.5$ psu and averaged
Figure 3.6 Comparison of the salinity profiles for LowRe case. Dashed lines represent variability of the means with 95% confidence.
Figure 3.7  Comparison of the temperature profiles for LowRe case. Dashed lines represent variability of the means with 95% confidence.
Figure 3.8  Comparison of the salinity profiles for HighRe case. Dashed lines represent variability of the means with 95% confidence.
Figure 3.9  Comparison of the temperature profiles for HighRe case. Dashed lines represent variability of the means with 95% confidence.
over selected stations. Only observations within the overflow plume, defined by a salinity interface larger than 36.5 psu, are used to compute the error. Using four stations, namely 58, 37, 82 and 83, the average Errors is computed for different closures, and plotted as a function of the horizontal resolution in Fig. 3.11 for the cases with \( Re = 20000 \). It can be seen that the most consistent model is VLES, since error (\%) is lowest and nearly resolution-independent (decreases slightly with increasing resolution). The average error of VLES is around 10\%, and it has a very similar profile (structure of IL and BL) compared to the real data. However, average errors in both \( k - \varepsilon \), and without SGS are quite high (around 20\%); the former overshoots the real data, and the latter underestimates it. The error is quite low at low resolution in the model without SGS parameterization, when none of the flow turbulence is resolved and the model fields are basically laminar. The net effect of this is to transport to some extent the basic inflow structure consisting of a BL and an IL. When the resolution is increased, the model starts to capture finer turbulent structures, yet these structures appear to destroy the overflow structure. Presumably, in the case of DNS, all the turbulent effects will be captured in a way to restore the BL and IL, but we appear to be very far away from such a simulation without a closure model. Errors for temperature (not shown in here) are very similar to the errors for salinity, and this appears to confirm the model consistency. In order to quantify how errors change with \( Re \), they are also plotted for \( Re = 10000 \) in Fig. 3.11. It should be mentioned here that two-equation turbulence closures are theoretically designed for very high Reynolds number. As such, when the Reynolds number is increased from 10000 to 20000, errors from standard \( k - \varepsilon \) and VLES change only by a few percent, on average. This also shows the consistency of the models.

The dependence of the model performances on the vertical grid spacing is also investigated. In Fig. 3.12, results from different vertical resolutions are plotted with \( \Delta x = 200 \) m and \( Re = 20000 \). The boundary layer becomes very weak and strongly diluted in the case without a SGS model, when the vertical resolution is decreased (dashed green lines
in Fig. 3.12. The performance of the $k - \varepsilon$ model does not seem to change much with different vertical resolutions (see the red lines of Stations 58, 82, and 83). In VLES, high resolution case gives better results only in Stations 37, and 82. However, differences among stations are not significant for different vertical resolutions. Therefore, we conclude that $k - \varepsilon$ and VLES models are fairly insensitive to the vertical resolution when the overflow is adequately resolved, that is for $\Delta z = 10$ m and $\Delta z = 20$ m corresponding to 15 to 30 points in the overflow layer. We did not try a coarser mesh, such as $\Delta z = 40$ m, since the oceanic BL is approximately 100-150 m thick, and trying to represent it with only three grid points is not likely to be fruitful.

### 3.3.3 Comparison of modeled and observed velocity

A comparison of the modeled and observed velocity distributions at selected stations along the northern channel are depicted in Fig. 3.13. As in temperature and velocity profiles, VLES results in the best fit with data. In particular, the maximum velocity coincides with the starting point of the IL, and its depth and magnitude are comparable with REDSOX-1 data. In $k - \varepsilon$ model, the location of the velocity maximum is too far up in the water column and the ambient fluid speed is too high, whereas the case without an SGS model generally leads to a thinner and faster overflow than observed. One of the striking features is that there is a significant reduction in the overflow speed from REDSOX-1 data at station 37. This raises the issue of sampling and synopticity in the velocity field compared to tracer field in the observations. A good illustration of the concept that the velocity field may be more variable than the tracer fields is given in Fig. 9 of Peters et al. (2005b).

### 3.4 Summary and discussion

The main objective of this chapter is to extensively test the performance of two-equation turbulence closures, or specifically $k - \varepsilon$ models, for overflow simulations in a nonhydro-
Figure 3.10  Comparison of the salinity profiles for Re=20000. Solid lines, dashed lines, and dotted-dashed lines indicate resolutions of $\Delta x = 200$ m and $\Delta z = 10$ m, $\Delta x = 100$ m and $\Delta z = 10$ m, and $\Delta x = 50$ m and $\Delta z = 10$ m, respectively.
static model. Since the non-hydrostatic models can capture some of the small scale turbulence such as vertical overturns, it is crucial to adopt $k-\varepsilon$ turbulence closures to be used in these models. This can be done by applying a spatial filter to the $k-\varepsilon$ equations, which allows the effect of this parameterization to vary depending on the range of resolved turbulence with the main fluid model. This is the main concept behind VLES, and to our knowledge, this is the first time that VLES is tested for an oceanographic application.

A large set of experiments are conducted using not only $k-\varepsilon$ and VLES but also without using any SGS model, which is important to assess the importance of the turbulence closure terms. Sensitivity of the results to $Re$ and model resolution were also explored.

The comparison of modeled and observed salinity and temperature distributions yields the following conclusions. It is found that the experiments without SGS models cannot reproduce the basic structure of the northern channel overflow, because of excessive mixing throughout the overflow. As such, turbulence closures appear to be a necessity. The $k-\varepsilon$
Figure 3.12  Comparison of the salinity profiles for Re=20000. Solid lines, dashed lines, indicate resolutions of $\Delta x = 200\text{m}$ and $\Delta z = 10\text{m}$, $\Delta x = 200\text{m}$ and $\Delta z = 20\text{m}$, respectively.
Figure 3.13  Comparison of streamwise velocity profiles between three models with $Re=20000$ and REDSOX-1 observed data.
model yields unrealistically thick BL and IL, while VLES gets things just about right. The primary reason appears to be that RANS-type closure models parameterize all the turbulence, while non-hydrostatic models can resolve some part of the turbulence given adequate resolution. Thus, the combination of $k – \varepsilon$ with non-hydrostatic dynamics appears to provide excessive mixing, namely without the balancing of the contributions presented via VLES. VLES results are found to be fairly consistent within the ranges of $10000 \leq Re \leq 20000$, $50 \text{ m} \leq \Delta x \leq 200 \text{ m}$ and $10 \text{ m} \leq \Delta z \leq 20 \text{ m}$. We conclude that VLES is more suitable than standard RANS in simulating the mixing of overflow using a non-hydrostatic model.
Chapter 4

Simulation of exchange flows in complex topographies

Flows between ocean and marginal sea basins are often connected by narrow channels and shallow sills. The Mediterranean overflow exits to the Atlantic Ocean over a shallow sill, Camarinal Sill, in the Strait of Gibraltar. The Red Sea overflow opens to Gulf of Aden from Bab el Mandeb strait which has a 150 m depth sill. The exchange flow in the Bosphorus strait is constrained by a sill and a contraction. The 2D non-hydrostatic model described in the previous chapter cannot model the exchange flows in a narrow strait with lateral contractions. Thus, we laterally average the 3D equations in the spanwise directions and modify our 2D model introducing a new term. Our aim in this chapter is to capture the effect of lateral geometric variations on the flow field using 2D non-hydrostatic model. We approach this problem by first conducting 3D large eddy simulations, LES, of the lock-exchange problem in the presence of lateral and vertical contractions. We compare the results of 3D LES with the results of the 2D model in terms of shape of the density interface and the time evolution of the background potential energy that quantifies the cumulative effects of the stratified mixing in the system. Then, the non-hydrostatic 2D model is used to model the exchange flow through the Bosphorus Strait.
4.1 Numerical Models

4.1.1 3D LES Model

3D computations are carried out using the non-hydrostatic spectral element model Nek5000 (Fischer, 1997; Fischer et al., 2000) which combines the geometrical flexibility of the finite-element method with the numerical accuracy of spectral expansions. For the LES part, the dynamic Smagorinsky sub-grid scale (SGS) model (Germano et al., 1991; Porté-Agel et al., 2000; Meneveau et al., 1996) is used, in which the test filtering exploits the high-order discretization of the spectral element model.

In LES, only the turbulent coherent structures are resolved and subgrid-scale closures are used to model the effect of unresolved scales on the resolved scales of motion. The resolved scales are obtained by applying a spatial filtering to the original Boussinesq equations. In the present set-up, the filtered non-dimensional equations become (Özgökmen et al., 2007):

\begin{align}
\bar{\mathbf{u}}_t + \mathbf{u} \cdot \nabla \mathbf{u} + \nabla \bar{p} - Re^{-1} \nabla^2 \bar{\mathbf{u}} + Fr^{-2} \bar{\rho} \mathbf{k} &= -\nabla \cdot \tau \\
\nabla \cdot \bar{\mathbf{u}} &= 0 \\
\rho_t' + \mathbf{u} \cdot \nabla \rho' - (Re Pr)^{-1} \nabla^2 \rho' &= -\nabla \cdot \sigma,
\end{align}

where \( \tau = \bar{\mathbf{u}} \mathbf{u} - \bar{\mathbf{u}} \bar{\mathbf{u}} \) and \( \sigma = \bar{\mathbf{u}} \bar{\rho} - \bar{\mathbf{u}} \bar{\rho} \) are the subgrid-scale (SGS) stresses exerted by the unresolved scale on the resolved scales of motion. In practice, a filter is not actually applied to the computational field, but it is assumed that the restriction of the computed fields to the numerical grid constitutes the filtering procedure. The Prandtl number, Pr, and Reynolds number are as defined in previous chapter.

Here, the dynamic Smagorinsky SGS model is used:

\[ \tau = -2(c_{ds} \delta)^2 |\nabla^5 \bar{\mathbf{u}}| \nabla^5 \bar{\mathbf{u}}, \]
where $\nabla^s \mathbf{u} := (\nabla \mathbf{u} + \nabla \mathbf{u}^T)/2$ is the deformation tensor of $\mathbf{u}$. The Smagorinsky constant $c_{ds}$ is computed dynamically from (Germano et al., 1991; Porté-Agel et al., 2000; Meneveau et al., 1996):

$$c_{ds}^2 = \frac{1}{2} \frac{\langle M_{ij} L_{ij} \rangle}{\langle M_{kl} M_{kl} \rangle}.$$  
(4.5)

In (4.5), $\langle \cdot \rangle$ denotes spanwise-averaging,

$$M_{ij} := \delta^2 |\nabla^s \tilde{\mathbf{u}}| \nabla^s \tilde{\mathbf{u}}_{ij} - \delta^2 |\nabla^s \mathbf{u}| \nabla^s \mathbf{u}_{ij} = \delta^2 \left( \alpha^2 |\nabla^s \tilde{\mathbf{u}}| \nabla^s \tilde{\mathbf{u}}_{ij} - |\nabla^s \mathbf{u}| \nabla^s \mathbf{u}_{ij} \right)$$  
(4.6)

$$L_{ij} = \tilde{u}_i \tilde{u}_j - \tilde{u}_i \tilde{u}_j,$$  
(4.7)

where a tilde denotes filtering with a test filter of radius $\tilde{\delta}$ and $\alpha := \tilde{\delta}/\delta$ is the filter ratio. Defining $\hat{M}_{ij} := M_{ij}/\delta^2$, $c_{ds}$ is computed from

$$c_{ds}^2 = \frac{1}{2} \frac{\langle \hat{M}_{ij} L_{ij} \rangle}{\langle M_{kl} M_{kl} \rangle},$$  
(4.8)

which depends only on the ratio of filter widths $\alpha$ and has no explicit dependence on $\delta$. If one constructs a test filter that projects onto half of the number of modes, then $\alpha = 2$. In the context of the spectral element method used here, we project from the $N$th-order local basis functions onto a basis of order $\tilde{N}$, with corresponding $\alpha = N/\tilde{N}$. This allows the test filter regime of $\alpha < 2$. In the LES computations here, $1.18 \leq \alpha \leq 1.5$. The main advantage of the dynamic estimation procedure with respect to a constant-coefficient implementation (typically $0.05 \leq c_s \leq 0.17$) is its ability to correctly yield $c_{ds} = 0$ in laminar regions of the flow, thereby avoiding excessive dissipation.

### 4.1.2 Spanwise-Averaged Boussinesq Model

The laterally-averaged 2D non-dimensionalized model equations are
\[ \frac{\partial \zeta}{\partial t} + \frac{1}{B} J(\psi, \zeta) + \zeta \frac{J(B, \psi)}{B^2} = -\frac{1}{Fr^2} \left( \frac{\partial \rho'}{\partial x} \right) + \frac{1}{Re} \nabla^2 \zeta \]

\[ + \frac{1}{B} \frac{\partial}{\partial x} \left( \frac{B}{Re} \frac{\partial \zeta}{\partial x} \right) + \frac{1}{B} \frac{\partial}{\partial z} \left( \frac{B}{Re} \frac{\partial \zeta}{\partial z} \right) \]

\[ \quad (4.9) \]

\[ \frac{\partial \rho'}{\partial t} + \frac{1}{B} J(\psi, \rho') = \frac{1}{RePr} \nabla^2 \rho' + \frac{1}{B} \frac{\partial}{\partial x} \left( \frac{B}{RePr} \frac{\partial \rho'}{\partial x} \right) \]

\[ + \frac{1}{B} \frac{\partial}{\partial z} \left( \frac{B}{RePr} \frac{\partial \rho'}{\partial z} \right), \quad (4.10) \]

\[ \nabla \cdot \left( \frac{1}{B} \nabla \psi \right) = \zeta, \quad (4.11) \]

where \( B \) is the width of the domain. The rest of the terms in Eq. 4.9-4.11 are the same terms used in the previous chapter. The Poisson Equation (4.11) is solved using a multigrid solver called MUDPACK developed by Adams (1990). More details about the numerical solution of this 2D model can also be found in Ilıcak et al. (2008a). Here, we refer to this 2D model given in (4.9)-(4.11) as the spanwise-averaged model or SAM.

Two types of mixing closures are incorporated and tested in SAM. In the first one, the turbulent Reynolds number, \( Re_t \), is computed from a Richardson number dependent Smagorinsky model,

\[ Re_t = \left[ (C_S \delta_{2D})^2 |\bar{S}| f(Ri) \right]^{-1}, \quad (4.12) \]

where \( C_S = 0.17 \) is the Smagorinsky constant and \( \delta_{2D} = \sqrt{\Delta x \Delta z} \) is the filter scale as a function of model resolution. \( \bar{S} = \frac{\partial U}{\partial z} + \frac{\partial W}{\partial x} \) is the nondimensional shear, where \( U = -\frac{\partial W}{\partial z} \) and \( W = \frac{\partial U}{\partial x} \). The Richardson number \( Ri = Fr^{-2}N^2/\left( \frac{\partial U}{\partial z} \right)^2 \) is the ratio of the square of the nondimensional buoyancy frequency \( N^2 = -\frac{\partial \rho'}{\partial z} \) and vertical shear. \( f(Ri) \) is a function...
used in 2D lock-exchange studies by Özgökmen et al. (2007),

\[
f(Ri) = \begin{cases} 
1 & \text{for } Ri < 0, \\
\sqrt{1 - \frac{Ri}{Ri_c}} & \text{for } 0 \leq Ri \leq Ri_c, \\
0 & \text{for } Ri > Ri_c,
\end{cases}
\] (4.13)

where \(Ri_c = 0.25\) is the critical Richardson number. Turbulent Prandtl number, \(Pr_t\), is taken as 1 which corresponds to the case where eddy viscosity is equal to eddy diffusivity. The closure (4.12) is essentially a variant of the LES approach, albeit in 2D.

We have also tested the VLES closure introduced in the previous chapter. The transport equations for non-dimensional turbulent kinetic energy, \(k\), and its dissipation rate, \(\varepsilon\), are also laterally averaged and given by

\[
\frac{\partial k}{\partial t} + \frac{1}{B} J(\psi, k) = P + B - \varepsilon + \frac{1}{Re} \nabla^2 k + \frac{1}{B} \frac{\partial}{\partial x} \left( \frac{B}{Re_t, \sigma_k} \frac{\partial k}{\partial x} \right) + \frac{1}{B} \frac{\partial}{\partial z} \left( \frac{B}{Re_t, \sigma_k} \frac{\partial k}{\partial z} \right),
\] (4.14)

\[
\frac{\partial \varepsilon}{\partial t} + \frac{1}{B} J(\psi, \varepsilon) = \frac{\varepsilon}{k} \left[ c_{e1} (P + c_{e3} B) - c_{e2} \varepsilon \right] + \frac{1}{Re} \nabla^2 \varepsilon + \frac{1}{B} \frac{\partial}{\partial x} \left( \frac{B}{Re_t, \sigma_\varepsilon} \frac{\partial \varepsilon}{\partial x} \right) + \frac{1}{B} \frac{\partial}{\partial z} \left( \frac{B}{Re_t, \sigma_\varepsilon} \frac{\partial \varepsilon}{\partial z} \right). \] (4.15)

In equations (4.14) and (4.15), \(P\) is the production term due to shear forces, and \(B\) is due to buoyancy forces defined in Eq. 3.7 and Eq. 3.8, respectively. The only difference in VLES is the spatial filter used for turbulent kinetic energy. The length scale filter is now computed from

\[
\delta = \alpha \cdot \max \left\{ \frac{|u| \cdot \Delta t}{\sqrt{\Delta x \Delta z B(x, z)}} \right\}, \] (4.16)

where \(\alpha = 2\) is a model constant.
4.1.3 Model setup and parameters

The domain for the 3D computations is $-\frac{L}{2} \leq x \leq \frac{L}{2}$, $0 \leq y \leq W$ and $0 \leq z \leq H$, where $L/H = 16$ and $W/H = 2$, so that in the non-dimensionalized setting, $-8 \leq x \leq 8$, $0 \leq y \leq 2$ and $0 \leq z \leq 1$. At the bottom boundary, no-slip and no-flow conditions are used, while free-slip and no-flow conditions are set at all other boundaries for the velocity components. No-flux (insulation) conditions are used for the density perturbation $\rho'$. 

Geometrical deformations of the domain are as follows. In order to introduce a vertical contraction for $x_1 \leq x \leq x_2$ (e.g., sill), the following mapping is used in the mesh generation:

$$
z' = 1 - (1-z) \left( 1 - A \sin \left( \frac{\pi x - x_1}{x_2 - x_1} \right) \right) \quad \text{for} \quad x_1 \leq x \leq x_2,
$$

where $z' \to 1$ as $z \to 1$, so that only the bottom boundary is deformed and the top boundary remains unperturbed.

The lateral contractions for $x_3 \leq x \leq x_4$ are introduced using:

$$
y' = y - (y + 1) A \sin \left( \frac{\pi x - x_3}{x_4 - x_3} \right) \quad \text{for} \quad x_3 \leq x \leq x_4.
$$

Two additional shapes are used that are characteristic of many oceanic channels. First is the vertical expansion of the geometry. A $\sqrt{\cdot}$-shaped channel is obtained from:

$$
y' = y - (y + 1) A (1 - z).
$$

Second is the $S$-shaped curvature along the streamwise direction, which is generated using:

$$
y' = y + A \cos \left( l \pi \frac{x}{x_5} \right).
$$

The exchange problem is initialized with dense fluid on the left that is separated from the light fluid on the right by a sharp transition layer:
\[
\frac{\rho'(x, y, z, 0)}{\Delta \rho'} = \begin{cases} 
1 & \text{for } -8 \leq x < -(0.005 + \eta)16, \\
100(0.005 - x - \eta) & \text{for } -(0.005 + \eta)16 \leq x < +(0.005 - \eta)16, \\
0 & \text{for } +(0.005 - \eta)16 \leq x \leq +8.
\end{cases}
\]

The perturbation superimposed on the density interface in order to facilitate transition to 3D flows (Fig. 4.1a) is \( \eta = -0.02 \sin (2\pi y) \).

The 3D experiments are conducted in seven geometrically-different settings (Table 4.1). First, \( A = 0 \) (geom1) which corresponds to the flat reference case (Fig. 4.1a). Then, a sill and a constriction are superimposed at \( x = 0 \) with \( A = 0.15 \) (geom2, Fig. 4.1b) and then with an increased geometrical deformation amplitude of \( A = 0.5 \) (geom3, Fig. 4.1c). Two constrictions at different locations are used (\( A = 0.5 \), geom4, Fig. 4.1d). One sill and one constriction are placed at different locations (\( A = 0.5 \), geom5, Fig. 4.1e). Finally, \( \sqrt{} \)-shaped (geom6, Fig. 4.1f) and \( S \)-shaped (geom7, Fig. 4.1g) channels are configured.

<table>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>flat reference case</td>
</tr>
<tr>
<td>geom2</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>0.15</td>
<td>overlapping sill and constriction</td>
</tr>
<tr>
<td>geom3</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>0.50</td>
<td></td>
</tr>
<tr>
<td>geom4</td>
<td>-</td>
<td>-</td>
<td>-4 and 2</td>
<td>-2 and 4</td>
<td>-</td>
<td>-</td>
<td>0.50</td>
<td>two constrictions</td>
</tr>
<tr>
<td>geom5</td>
<td>-4</td>
<td>-2</td>
<td>2</td>
<td>4</td>
<td>-</td>
<td>-</td>
<td>0.50</td>
<td>one sill, one constriction</td>
</tr>
<tr>
<td>geom6</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>16</td>
<td>0.80</td>
<td>( \sqrt{} )-shaped channel</td>
</tr>
<tr>
<td>geom7</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>5</td>
<td>1.00</td>
<td>( S )-shaped channel</td>
</tr>
</tbody>
</table>

Table 4.1 Different geometries used in the 3D simulations, where \( x_1/x_2 \) and \( x_3/x_4 \) refer of the beginning/ending locations of a sill and a constriction, respectively. \( x_5 \) and \( l \) are the parameters of \( S \)-shaped channel, and \( A \) is the amplitude of the topographic perturbation.

SAM is configured in a similar way, except that no boundary conditions are needed in the span-wise direction. Vertical deformations are generated by the conformal mapping of the physical coordinates \((x, z)\) to computational coordinates, whereas lateral deformations are introduced via the width term, \( B \). Nondimensional \( B \) is defined as its counterpart in the
Figure 4.1  Types of geometries and initializations in the 3D model.

3D model:

\[ B(x) = \begin{cases} 
1 & \text{for } -8 \leq x < x_3, \\
1 - A \sin \left( \frac{\pi (x - x_3)}{x_4 - x_3} \right) & \text{for } x_3 \leq x \leq x_4, \\
1 & \text{for } x_4 < x \leq 8, 
\end{cases} \quad (4.21) \]

where \( B = 1 \) is the width of the uncontracting channel, \( A \) is the amplitude. Different cases conducted with three different geometries are shown in Fig. 4.2. (i) \( A = 0 \): no contraction; (ii) \( A = 0.15 \): minimum width of the channel is 0.85; and (iii) \( A = 0.5 \): minimum width of the channel is 0.5.

Model parameters are set as follows. The Froude number is taken as \( Fr = 1 \) in or-
Figure 4.2  Types of geometries and initializations in the 2D model.

der facilitate critical flow and subsequent hydraulic control and mixing. It is known that
lock-exchange flows naturally tend towards critical Froude numbers (Benjamin, 1968). The
Prandtl number and Reynolds number are taken as $Pr = 1$ and $Re = 10,000$, respectively.
We employ three different models: 3D LES, 3D DNS* and 2D SAM. In here, DNS* corre-

cponds to the model without any SGS used in the previous chapter. Convergence tests are
carried out by changing the resolutions of DNS*, LES, and SAM in the reference setting
geom1. This allows us to settle at a resolution for all the other experiments. The conver-
gence is evaluated on the basis of a metric that quantifies mixing in the system, which is
one of our primary interests.

Mixing in an enclosed system is best quantified by changes in the background (or refer-
ence) potential energy (RPE) (Winters et al., 1995). RPE is the minimum potential energy
that can be obtained through an adiabatic redistribution of the water masses, and it is com-
puted using the probability density function approach introduced by Tseng and Ferziger
(2001). In this method, the fluid is scanned every time step and the fluid parcels with a
density perturbation $\rho'$ within the range of $[0, \Delta \rho']$ are assigned into bins between $\rho$ and
$\rho + d\rho$. The normalized number of control volumes in each bin gives the probability den-
sity function $P(\rho) = V^{-1} \int \delta(\rho - \rho')dV$, where $\delta(\cdot)$ is the Dirac delta function. $P(\rho)$ is
the probability of a fluid parcel density to be in the bin of density $\rho$. Next, we define $z_r(\rho')$
to be the height of fluid of density $\rho'$ in the minimum potential energy state, which can be
computed as $z_r(p') = H \int_0^{\Delta p'} P(p) dp$. The background potential energy is calculated from

$$RPE = gLW \int_0^H p'(z_r) z_r dz_r,$$

by splitting the $p'$ distribution into 51 bins at each time step. It is convenient to use the non-dimensional background potential energy here:

$$RPE^*(t) \equiv \frac{RPE(t) - RPE(0)}{RPE(0)}.$$  \hfill (4.23)

Fig. 4.3a shows $RPE^*(t)$ in geom1 from DNS$^*$ and LES at several resolutions, namely coarse-res LES, low-res LES, coarse-res DNS$^*$, low-res DNS$^*$ and high-res DNS$^*$ (Table 4.2). The curve from high-res DNS$^*$ is assumed to be the truth. The differences between the curves are quite small, but coarse-res DNS$^*$ and low-res DNS$^*$ lead to slightly more mixing than the truth, while LES results show a nearly identical time evolution. We find from results plotted in Fig. 4.3 that LES results do not show any sensitivity to spatial resolution and those from coarse-res LES are in good agreement with low-res LES.

<table>
<thead>
<tr>
<th>Exp</th>
<th># of grid points</th>
<th>CPU hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>DNS$^*$ coarse-res</td>
<td>924,385</td>
<td>79</td>
</tr>
<tr>
<td>DNS$^*$ low-res</td>
<td>2,167,425</td>
<td>252</td>
</tr>
<tr>
<td>DNS$^*$ high-res</td>
<td>14,070,001</td>
<td>3938</td>
</tr>
<tr>
<td>LES coarse-res</td>
<td>924,385</td>
<td>128</td>
</tr>
<tr>
<td>LES low-res</td>
<td>2,167,425</td>
<td>384</td>
</tr>
<tr>
<td>LES mid-res</td>
<td>9,183,825</td>
<td>3319</td>
</tr>
</tbody>
</table>

| Table 4.2   | Table of number of grid points and computational cost for DNS$^*$ and LES with nek5000. |

The convergence of SAM is investigated using the resolutions listed in Table 4.3. Attention needs to be paid to several issues regarding a direct comparison of SAM and DNS$^*$. First, the 3D model has lateral boundaries, which may reduce somewhat the development of shear instabilities in the flow fields. Second, the break-down of Kelvin-Helmholtz rollers due to secondary instabilities is a dynamical mechanism that is not represented in a 2D
Third, one would expect the convergence to be slower in a second-order finite difference model than in higher order spectral element model. Nevertheless, the comparison is necessary in order to assess the practical utility of SAM for oceanic applications, such as exchange flows in narrow straits. The $RPE^*(t)$ curves from SAM show a significant decrease with increasing resolution, with results from high-res SAM being very close to those from high-res DNS*. In general, SAM shows a much wider range than DNS* and LES results, making the selection of spatial resolution non-trivial, given particularly when taking computational time into consideration. Based on Fig. 4.4, we decided to run the following experiments at two spatial resolutions of SAM (Table 4); with low-res because this is the 2D equivalent of the LES, and at mid-res3 because these results are quite close to those from high-res but have significantly less computation time and storage requirements.
<table>
<thead>
<tr>
<th>Exp</th>
<th># of grid points</th>
<th>CPU hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAM low-res</td>
<td>32,768</td>
<td>2</td>
</tr>
<tr>
<td>SAM mid-res1</td>
<td>131,072</td>
<td>8</td>
</tr>
<tr>
<td>SAM mid-res2</td>
<td>524,288</td>
<td>30</td>
</tr>
<tr>
<td>SAM mid-res3</td>
<td>2,097,152</td>
<td>136</td>
</tr>
<tr>
<td>SAM high-res</td>
<td>8,388,608</td>
<td>560</td>
</tr>
</tbody>
</table>

Table 4.3 Table of number of grid points and computational cost for experiments with SAM.

4.2 Results

4.2.1 Comparison of SAM and Inviscid Two-Layer Theory

SAM is compared with inviscid two-layer theory by Farmer and Armi (1986). To this end, SAM is integrated in the domain setting geom3 with open boundary conditions until an approximately steady state is established. The mid-isopycnal, $\Delta \rho/2$, is chosen to separate the flow into two layers and to form layer Froude numbers. The lower layer thickness, $h_2(x)$, is taken from the bottom to the height where $\rho' = \Delta \rho/2$, and the upper layer thickness $h_1(x)$ is then $H - h_2(x)$. Layer velocities $u_1(x)$ and $u_2(x)$ are computed by vertically averaging local velocities within each layer. Layer transports $q_1(x) = u_1(x)h_1(x)B_1(x)$ and $q_2(x) = u_2(x)h_2(x)B_2(x)$ are then computed, and the average value of $q_r \equiv q_1/|q_2|$ is taken to characterize the flow for comparison with analytical predictions. After $q_r$ and Fr numbers are known in the domain, non-dimensional interface height $h_i$ can be computed from

$$h_i = \Delta H' = \frac{Fr_2^{-2/3} (1 + \frac{1}{2} Fr_2^2) - \frac{1}{2} q_r^{-2/3} Fr_1^{-2/3} Fr_2^2}{Fr_1^{-2/3} + Fr_2^{-2/3}}.$$ (4.24)

For further information about the analytical formulations, the reader is referred to Winters and Seim (2000).

In Fig. 4.5, we compare the prediction of $h_i$ on the basis of the inviscid theory to $\Delta \rho/2$ interface from SAM. The model result coincides with the inviscid theory well at the contraction and sill area. It is clearly seen that there is a hydraulic jump at $x \approx 1.5$. The
Figure 4.4  Time evolution of background potential energy $RPE^*(t)$ from SAM with different resolutions and high-res DNS* for geom1.

offset between the two lines is probably mainly due to mixing in the domain. Although results between model and theory are quite similar, there are two main issues that can’t be addressed in the context of this theory. The first is that the theory does not provide any information about mixing between the layers, which is usually of great interest. The second is that the flow has to be in steady-state, while oceanic flows are transient in nature. As such, we proceed to a comparison of results from SAM to those from LES.

4.2.2  Comparison of SAM and LES

Snapshots of the salinity field from simulations with low-res LES for all geom1 to geom7 are depicted in Fig. 4.6. The reference case geom1 has been well studied in the literature (e.g. Härl et al., 2000; Monaghan, 2007; Cantero et al., 2007). Let’s look at the main features. After the interface is removed, two gravity currents are generated and they prop-
agate in opposite directions. The heads of the gravity currents propagating along the upper and lower boundaries differ because of the boundary conditions. No-slip boundary condition is applied for the bottom of the domain, thus the head of the current at the bottom is composed of a complex pattern of so-called lobes and clefts. In contrast, the upper gravity current traveling along a free-slip boundary does not experience such instabilities and arrives at the side boundary somewhat faster than the bottom gravity current (Fig. 4.6a). The shear along the interface between the two gravity currents leads to instabilities. The initial instability is in the form of 2D Kelvin-Helmholtz (KH) rolls, which constitute the usual first step toward the onset of more complex evolution of the density field. The initial 2D KH rolls are susceptible to secondary convective instability (Klaassen and Peltier, 1989, 1991), in which the stream-wise vortices stretch and tilt the span-wise vorticity concentrated in KH rolls. Consequently, KH rolls cannot sustain their lateral coherence and break down (e.g., Fritts et al. (1998); Andreassen et al. (1998)). The spanwise instability of KH rolls leads to increasingly complex turbulent interactions and smaller overturning scales; a manifestation of forward energy cascade in 3D flows. On the other hand, KH rolls tend to merge by pairing in 2D dynamics (Corcos and Sherman, 1984), which is an indication of the inverse energy cascade from small to larger scales. This difference is likely to lead to a difference of amount of mixing between 2D and 3D simulations.

In order to compare qualitatively the results from low-res LES to those from SAM,
Figure 4.6  Density perturbation fields at the end of experiments with low-res LES.

spanwise-averaged density perturbation fields are plotted in Figs. 4.7,4.8 with respect to those from low-res and mid-res3 SAM at the same times. Comparison of Fig. 4.7a-c shows that the density interface in SAM appears to be thicker than that in LES and is characterized by pairing KH rolls, as anticipated. The frontal positions of the gravity currents are in good agreement with one another. The lower resolution SAM experiments appear to be more diffusive than the higher resolution case, which may affect mixing.

When the domain configuration is changed to geom2, the impact is clearly visible in the flow field in the form of a hydraulic control region centered at x = 0 (Figs. 4.6b, 4.7d). This
Figure 4.7  Comparison of spanwise-averaged density perturbation fields from low-res LES to those low-res and mid-res3 SAM from $RPE^+(t)$ from SAM for geom1 to geom3.
results in a reduction of the thickness of the top and bottom gravity currents, and therefore a slower propagation speed so that the upper front arrives at the left boundary at $t = 16.6$ as opposed to $t = 16.2$ in the reference case. Hydraulic control region is less defined in SAM due to the presence of large overturning eddies, nevertheless the effect of the geometric deformation clearly changes the gravity currents propagating along both directions (Figs. 4.7e,f). The effect is quite significant for $\text{geom3}$, which leads to much thinner and slower gravity currents in both LES and SAM (Figs. 4.6c, 4.7g-i).

The hydraulic control regions created by the two constrictions ($\text{geom4}$) are clearly visible in LES (Figs. 4.6d, 4.8a). There is also enhanced mixing in between the two constrictions, which appears to be induced by breaking internal waves associated with flows that do not pass through the middle part of the domain, but get reflected from the constrictions. The enhanced mixing in the region between the two constrictions appears to be greater in SAM as to obscure somewhat the hydraulic control points (Figs. 4.8b,c). But they are essentially in agreement with LES, which is a confirmation that spanwise-averaged formulation (Eq. 4.9-4.11) has an effect on the flow field. Similar results are obtained for $\text{geom5}$ (Figs. 4.6e, 4.8d-f).

The narrowing cross section in the $\sqrt{\ }$-shaped channel ($\text{geom6}$) acts to cause a faster propagating gravity current in the bottom than near the surface (Fig. 4.6f). This effect is fairly accurately reproduced in SAM and the positions of the leading edges of the gravity currents are in good agreement with that from LES (Figs. 4.8g-i). As in all the previous cases, the vertical scale of the interfacial layer in SAM appears to be greater than that in LES due to the inability of KH rolls to break down into small-scale turbulence via lateral vortex stretching and secondary instabilities.

$S$-shaped channel ($\text{geom7}$) with LES is an example the dynamics of which cannot be represented in a 2D model, and therefore it is of interest to know whether there is a significant difference in mixing with respect to the reference case $\text{geom1}$. The time evolutions of the background potential energy $RPE^*(t)$ from low-res LES are shown in Fig. 4.9a, which
Figure 4.8  Comparison of spanwise-averaged density perturbation fields from low-res LES to those low-res and mid-res3 SAM from $RPE^*(t)$ from SAM for geom4 to geom6.
demonstrates significant differences in mixing in between different domain geometries. In particular, S-shaped channel geom7 leads to the highest and geom3 to the lowest levels of mixing. Overall, the overlapping sill and constriction configurations (geom2 and geom3), and V-shaped channel (geom6) act to reduce the mixing with respect to the reference case geom1. In the former, this takes place by a reduction of volume transport across the obstacle. In cases of offset obstacles (geom4 and geom5), and S-shaped channel (geom7), there is a significantly higher mixing by the time one of the gravity current reaches the end of the domain.

**Figure 4.9** Time evolution of the background potential energy $RPE^*(t)$ for all geometrically-different cases from (a) low-res LES and (b) mid-res3 SAM.
The relative proportions of mixing in different geometries is reproduced in mid-res3 SAM in reasonable agreement with low-res LES (Fig. 4.9b). In other words, geom4 and geom5 lead to more mixing than geom1, while geom3 and geom6 result in less mixing. Only in the case of geom2, the result is opposite of that in LES, which could be related to large overturns seen in SAM (Fig. 4.7f). Overall, mid-res3 SAM tends to overestimate mixing with respect to low-res LES by a factor of 1.27 to 1.97 (Table 4.4).

<table>
<thead>
<tr>
<th>Geometry</th>
<th>RPE* × 100 low-res LES</th>
<th>RPE* × 100 mid-res3 SAM</th>
<th>RPE*(SAM)/RPE*(LES)</th>
</tr>
</thead>
<tbody>
<tr>
<td>geom1</td>
<td>3.19</td>
<td>4.05</td>
<td>1.27</td>
</tr>
<tr>
<td>geom2</td>
<td>3.00</td>
<td>4.67</td>
<td>1.56</td>
</tr>
<tr>
<td>geom3</td>
<td>1.86</td>
<td>3.68</td>
<td>1.97</td>
</tr>
<tr>
<td>geom4</td>
<td>4.23</td>
<td>6.24</td>
<td>1.48</td>
</tr>
<tr>
<td>geom5</td>
<td>3.65</td>
<td>5.21</td>
<td>1.43</td>
</tr>
<tr>
<td>geom6</td>
<td>1.83</td>
<td>2.53</td>
<td>1.38</td>
</tr>
<tr>
<td>geom7</td>
<td>4.34</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

**Table 4.4** Final values of the background potential energy $RPE^*$ at the end of the experiments for all domain geometries from low-res LES and mid-res3 SAM.

In conclusion, we find that SAM appears to be accurate in identifying the hydraulic control regions due to both vertical and horizontal constrictions, and also provides useful indications of the mixing in domains with complex geometries, provided high spatial resolutions are employed. We also conduct some of the SAM simulation by VLES closure. Since the domain size is relatively small and the resolution is high enough, there was no difference between VLES and SGS in SAM (not shown here).

### 4.2.3 Simulation of Bosphorus Strait using SAM

We also conduct a realistic oceanographic problem using SAM. An important component of the the Turkish Strait System (TSS) is selected. TSS is a complex domain formed by the small internal domain of the Marmara Sea coupled with the Straits of Bosphorus and Dardanelles, an elongated and tortuous waterway between the European and Asian continents controlling the exchange and mixing between the Mediterranean (Aegean) and Black Sea.
waters, destined to determine their contrasting physical, chemical and biological properties (Fig. 2.3a).

The bathymetric data used in here (Fig. 4.10) has been made available by the Turkish Navy Office of Navigation, Hydrography and Oceanography, based on the work of Gökçaşan et al. (2005). Because the Bosphorus geometry is extremely complex, a simplified method was used to estimate $B(x,z)$. The depth and width data were computed with reference to the deepest point on the channel (i.e. thalweg). However, in order to avoid excessive channel curvature displayed by the thalweg, width and depth data were collected on channel cross sections perpendicular to a stretched length-wise coordinate shown above. A maximum width value of 50 m was added to the width at the deepest point of each section, decreasing linearly to zero at the surface, to avoid having zero width at the bottom. This also helped reduce some noise in the determination of $B(x,z)$. The function $B(x,z)$ is shown in Fig. 4.10.

![Bosphorus width B(x,z) (km)](image-url)

**Figure 4.10** The width term of the Bosphorus Strait.

The data from the 1994 cruise of the R/V BİLİM is plotted in Fig. 4.11a. The potential density, $\rho$ is non-dimensionalized using the minimum ($\rho_{min} = 1010.39 \text{ kgm}^{-3}$) and the
maximum ($\rho_{\text{max}} = 1029.24 \text{ kgm}^{-3}$) values as $\bar{\rho} = (\rho - \rho_{\text{min}}) / (\rho_{\text{max}} - \rho_{\text{min}})$ to conduct a model-data comparison. From these observations, one of the most pronounced features is the hydraulic jump just after the sill around $x = 12 \text{ km}$, where at first isopycnals drop down then rise up again and some mixing is evident. This location also coincides with the maximum contraction point in the strait. Therefore, both geometric effects due to both the sill and contraction are likely to be important in the density structure. Another possible control point is located at $x = 40 \text{ km}$. Finally, the exchange flow from these observations is characterized by nearly a two-layer structure separated by a thin mixing layer.

The model is initialized as a lock-exchange problem, centered at $x = 22.5 \text{ km}$, where dense fluid is on the left representing the saline water originating from the Marmara Sea and light fluid is on the right representing the fresh water entering from the Black Sea (Fig. 4.11b). The domain is 45.3 km long and has a maximum depth of 83.4 m. There are 8192 grid points used in $x$-direction and 64 points in $z$-direction ($\Delta x \approx 5.5 \text{ m}$ and $\Delta z \approx 1.3 \text{ m}$). Free-slip boundary condition is applied for the top boundary. For the bottom boundary, a wall layer approximation is employed with a linear drag coefficient of $C_d = 2 \times 10^{-3}$. The Froude number $Fr = U / \sqrt{g' h}$ is estimated using $U \approx 1 \text{ ms}^{-1}$, $g' = g \Delta \rho / \rho_0 \approx 0.18 \text{ m}^2\text{s}^{-1}$, $h \approx 25 \text{ m}$, and the Froude number is taken as $Fr = 0.5$. $Pr$ number is set to 1 as in all the experiments above. Three sets of computations are conducted. First with $Re = 10^4$, based on model to model comparison studies from previous section. But, it is noted that the effective $Re$ for the Bosphorus Strait is about $Re = U h / \nu \approx 25 \times 10^6$, which is of course beyond our present computational capabilities. Nevertheless, it is important to know what types of changes occur in the simulated fields when $Re$ is increased beyond $10^4$. Therefore, another simulation is conducted by setting $Re = 10^5$. We also conduct an experiment using VLES closure.

Model results are shown in Fig. 4.11c,e. The density field reaches nearly a steady state after non-dimensional time of 421, when the light water reaches the Marmara Sea ($x \leq 8 \text{ km}$), $B$ becomes quite large and propagation of the density front slows down significantly.
Figure 4.11  (a) Distribution of potential density from observations by Özsoy et al. (2001). (b) Density perturbation at the initial state of SAM. Density perturbation at the final state of SAM with (c) $Re = 10^4$, (d) $Re = 10^5$ and (e) VLES closure.
The same behavior occurs when the dense water reaches the Black Sea ($x \geq 37$ km). Overall, the model shows a pronounced change in thickness across the first sill/constriction region at $12 \leq x \leq 18$ km, and another at $x = 38$ km. In the intermediate region of $20 \leq x \leq 35$ km, where the bottom topography is relatively flat, the density structure appears to be correlated with the structure of $B$ (Fig. 4.10). The higher Re case (Fig. 4.11d) appears to result in a somewhat better preservation of the layer density structure than the lower Re case (Fig. 4.11c). Nevertheless, the model still shows far more structure than the observations and results in a significantly more mixing, namely the water masses originating from the Black Sea do not reach the Marmara Sea and vice versa.

The discrepancy of the SAM results from the observations is likely to be related to four reasons. The first is that the observations along the strait do not reflect span-wise averaged quantities but point-wise measurements so that it is not a comparison of identical quantities. The second is the reduction of really complex 3D geometry to 2D via the $B$ function, for which there is not a unique solution. Third is the excessive mixing taking place in the 2D model because of lacking instabilities related to span-wise vortex stretching. Fourth is the generic problem of being confined to a significantly lower Re regime than the oceanic flows, which results in excessive diffusive mixing throughout the simulation period.

The VLES closure is tested in order to explore whether it can make a difference regarding the performance of SAM, and it is seen to lead to a significant improvement (Fig. 4.11e). In particular, the dense bottom water mass is much better preserved through the Bosphorus and the results appear quite satisfactory.

### 4.3 Summary and discussion

The aim of this chapter is to investigate the exchange flows in complex topography. To this end, 2D VLES is improved by introducing laterally averaged terms. The model can then simulate the constrictions not only in the $z$–direction but also in the $y$–direction.
The study was conducted in two stages. First, we focussed on a set of idealized cases in a domain with non-dimensional lengths of \(16 \times 1 \times 2\) in the horizontal, vertical and lateral directions, respectively. A set of seven cases are considered, in which geometric deformations are introduced that are generically observed in straits, namely lateral and vertical constrictions, \(\bigtriangledown\)- and \(S\)-shaped channels. For these idealized cases, LES using a high-order spectral element model Nek5000 is taken as the ground truth. Non-dimensional parameters are set to \(Re = 10^4\), \(Pr = 1\) and \(Fr = 1\). The dynamic Smagorinsky model is used as the SGS model. Mixing is calculated using the background potential energy method developed by Tseng and Ferziger (2001). We also tested briefly SAM in comparison to results from two-layer inviscid theory, which predicts the shape of the density interface in steady state flows, but not the mixing or the transient behavior. A good agreement is found between SAM and two-layer theory regarding the steady-state location of the density interface.

On the basis of the idealized experiments using LES, significant differences in the amount of mixing are found among the different domain geometries. In particular, \(S\)-shaped channel \(geom7\) is shown to lead to the highest (1.36-fold more than the reference case) level of mixing, possibly because of the additional shears caused by side to side sloshing due to the centrifugal force. Vertical and horizontal constrictions \(geom4-geom5\) that are off set with respect to the location of the initial density interface also act to enhance the mixing by the trapping and breaking of the internal waves in between the obstacles. On the other hand, vertical and horizontal constrictions overlapping with the initial density interface \(geom2-geom3\) restrain the rate of the exchange flow, reduce vertical shears and therefore mixing. The lowest mixing (some 1.74-fold less than the reference case) is encountered \(\bigtriangledown\)-shaped channel \(geom6\).

Counterparts of these experiments are conducted with SAM in a domain of \(16 \times 1\) in the horizontal and vertical directions, respectively, and using a specification of the domain width via the \(B\)-function. SAM experiments are run at two spatial resolutions; using 32,768
points (low-res) corresponding to a single streamwise slice of low-res LES, and also at a much higher resolution of 2,097,152 points (mid-res3). By comparing the spanwise-averaged density perturbation fields from LES to those from SAM, it is found that SAM can successfully represent the net effect of lateral constrictions on the general shape of the interface reasonably well. But SAM tends to overestimate mixing when compared to LES, by some 1.38 to 1.97 times for mid-res3, and significantly more for low-res. This difference appears to be mainly due to the inability of the 2D dynamics to facilitate the break down of KH rollers into smaller scales because of the lack of vortex dynamics in the span-wise direction. The difference in the size of the stratified overturns between SAM and LES is clearly visible in the snapshots of the density perturbation fields.

Finally, observational data from the Bosphorus Strait is employed to test SAM in a realistic application. One of the main challenges at this stage is to express the complex width data in the form of a $B$-function. The simulations from SAM with a simple closure appear to be excessively diffusive and noisy. The excess diffusion/mixing is likely to be related to the dynamical simplifications due to the 2D nature of the model, as discussed above, while the noise appears to correlate well with the $B$ function. As such, further post-processing of the $B$ function can possibly reduce the noise, but this avenue is not pursued here. However, it is of interest to find out whether the VLES closure can improve the solution. We show that SAM can benefit significantly from such comprehensive turbulence closures. Overall, we conclude that exchange flows in narrow straits pose significant computational challenges due to the details of domain geometry and their impact on mixing and hydraulic effects. SAM with the VLES turbulence closure might be an alternative to simulate the exchange flows.
Chapter 5

Performance of two-equation turbulence closures in a hydrostatic model

The main goal of this chapter is to investigate performance of various two-equation turbulence closures representing the overflow mixing in a hydrostatic model. Four turbulence closures are selected for the comparison of 3D numerical simulations of the Red Sea overflow. Three of them, $k - \varepsilon$, $k - \omega$, and $MY2.5$, are the turbulence closures with complex stability functions representing the effects of buoyancy and shear. The fourth scheme is the modified $k - \varepsilon$ turbulence closure of Peters and Baumert (2007). In their model, a simple gradient Richardson number-dependent turbulent Prandtl number is used instead of complex stability functions. Peters and Baumert (2007) validate this modified closure by comparing it with microstructure observations from a stratified and sheared tidal estuary and with laboratory experiments. We also employ KPP since it is a widely used diagnostic parameterization in climate models. We further performed a control experiment in which both vertical eddy viscosity and eddy diffusivity were set to zero in an effort to understand the performance of the turbulence schemes. The accuracy of the model results is evaluated using hydrographic and current observations collected in the Red Sea Outflow Experiments (Peters et al., 2005b; Peters and Johns, 2005; Bower et al., 2005).
5.1 Model configuration

The Regional Ocean Modeling System (ROMS) was chosen as the hydrostatic model mainly because ROMS has the generic length scale scheme of Umlauf and Burchard (2003) which allows us to easily test different two-equation turbulence closures. ROMS is a free-surface, hydrostatic, primitive-equations ocean model that uses orthogonal curvilinear horizontal coordinates on an Arakawa C grid. The primitive equations are discretized over topography in the vertical using stretched terrain-following, or “sigma”, coordinates (Song and Haidvogel, 1994; Shchepetkin and McWilliams, 2005).

The computational domain covers the area between the longitudes 43°E and 46°E, and latitudes 11.7°N and 13°N (Fig. 5.1). The sea floor topography is crucial for the model configuration because of the narrow channels which constrain the flow of the gravity currents. A detailed discussion of the topography can be found in Chang et al. (2008), who used the same topography as herein. The topography is based on multi-beam echosoundings taken during REDSOX and a cruise of the French R/V L’Atalante (Hébert et al., 2001). These measurements have a resolution of approximately 30 m. Far field areas and gaps were filled from the 2′ grid sea floor data of Smith and Sandwell (1997). The east and the south sides of the domain are extended to use open boundary conditions effectively. The north and the west boundaries are closed. A nonuniform grid is used in the model with 250 m resolution around the channels and the bifurcation point where the two channels separate at approximately 43.5°E and 12.5°N (Fig. 5.1). The resolution gradually coarsens to 3 km toward the end of the domain in all experiments. In the vertical, 30 sigma layers are used. With the help of a logarithmic distribution, most of the layers are placed near the bottom. All experiments are performed with $320 \times 180 \times 30$ grid points in $x$, $y$, and $z$ directions, respectively. (Fig. 5.1). The baroclinic time step is set to 40 seconds, and 20 barotropic time steps are used between each baroclinic time step. Pressure-gradient error is a well-known problem in terrain-following models. To minimize this error, bathymetry is smoothed so that the Beckmann-Haidvogel (Beckmann and Haidvogel, 1993) criterion is satisfied. We
also use parabolic splines density Jacobian method (Shchepetkin and McWilliams, 2003) to calculate pressure-gradient forces. The vertical advection of tracers is discretized by a 4th order center difference scheme. A test case was run with a stable stratification and closed boundaries which should not produce any velocities. Erroneous speeds were less than 0.01 m s\(^{-1}\), and they were generated only in the Tadjura Rift outside areas of interest. Since baroclinic velocities of 0.4-0.6 m s\(^{-1}\) are to be expected around that area, these errors are acceptable.

**Figure 5.1** a) Model domain and bottom topography. Crosses represent the stations selected for the model comparisons. Two sections across the outflow, Sec1 and Sec2, are also used to compare models results to those from the REDSOX-1 cruise. b) Model grid plotted at every fifth point.

Temperature (\(T\)) and salinity (\(S\)) profiles from a REDSOX station denoted \(T\)1 in Fig. 5.1, located near the deepest part of the Tadjura Rift and outside of the overflows pathways, are used as an initial condition for the stratification in the model domain setup. Station \(T\)1 is
selected since it is as deep or deeper than the entire domain, and it is the least affected from
the overflows in the Tadjura Rift. The profile from this location is cropped to the local water
depth at any other location. Dense overflow water is introduced into the domain as follows.
At the north-western side of the domain, dense water which represents the overflow coming
from Bab el Mandeb Strait is released. T-S profiles in this area are relaxed towards profiles
taken from the REDSOX observations in the Bab el Mandeb Strait. We further ensure
that downstream of the relaxation zone simulated T-S and velocity profiles show a good
agreement with the observations at REDSOX-1 station 30 (T0 in the Fig. 5.1). This station
serves as a checkpoint to confirm the realism of the overflow source water T-S properties
and the velocity distributions.

A nonlinear equation of state (Jackett and McDougall, 1995) is used to compute den-
sity. Open boundary conditions (Marchesiello et al., 2001) are employed at the south and
east boundaries. Once the dense water is released from the top of the domain, the model
is integrated for 34 days. This integration time is chosen for the overflow to reach a quasi-
steady state along the channels since we know from the observations that most of the mix-
ing occurs in the northern and southern channel. Wind forcing as well as evaporation,
precipitation and radiative heat fluxes are set to zero everywhere. We have thus simplified
the regional atmospheric forcing such as to be represented exclusively through the over-
flow source properties. Bottom drag coefficients from overflow observations indicate a
wide range of $1 \times 10^{-3} \leq C_d \leq 10 \times 10^{-3}$ (Girton and Sanford, 2003; Peters and Johns,
2006). Here, a quadratic bottom drag formulation with an intermediate drag coefficient of
$C_d = 5 \times 10^{-3}$ is used to incorporate the bottom shear stress since this value gives better
results in our 2D simulations presented in the Chapter 3.

5.1.1 Turbulence models

Warner et al. (2005) incorporated the “generic length scale algorithm” in ROMS, which
allows the use of different turbulence closures by changing the coefficients in the equations
The first equation in the generic length scale (GLS) scheme is for the turbulent kinetic energy, $k$, and the second one is for a generic parameter, $\psi$, which is used to calculate the turbulent length scale, $l$. The GLS equations are

$$\frac{\partial k}{\partial t} + U_i \frac{\partial k}{\partial x_i} = \frac{\partial}{\partial z} \left( \frac{K_M}{\sigma_k} \frac{\partial k}{\partial z} \right) + P + B - \varepsilon \quad (5.1)$$

$$\frac{\partial \psi}{\partial t} + U_i \frac{\partial \psi}{\partial x_i} = \frac{\partial}{\partial z} \left( \frac{K_M}{\sigma_\psi} \frac{\partial \psi}{\partial z} \right) + \frac{\psi}{k} (c_1 P + c_3 B - c_2 \varepsilon F_{\text{wall}}), \quad (5.2)$$

where $\sigma_k$ and $\sigma_\psi$ are the turbulence Schmidt number for $k$ and $\psi$, respectively. $P$ and $B$ represent production due to shear and buoyancy as

$$P = -\langle u'w' \rangle \frac{\partial U}{\partial z} - \langle v'w' \rangle \frac{\partial V}{\partial z} = K_M M^2, \quad M^2 = \left( \frac{\partial U}{\partial z} \right)^2 + \left( \frac{\partial V}{\partial z} \right)^2, \quad (5.3)$$

$$B = -\frac{g}{\rho_0} \langle \rho'w' \rangle = -K_H N^2, \quad N^2 = -\frac{g}{\rho_0} \frac{\partial \rho}{\partial z}, \quad (5.4)$$

where $N$ is the buoyancy frequency, $M$ is the vertical shear, $g$ is the gravitational acceleration, $\rho_0$ is the reference density, $\langle \cdot \rangle$ is the ensemble averaging operator, $U$ and $V$ are the mean horizontal velocity components, $u'$, $v'$, $w'$ and $\rho'$ are the turbulent velocity and density components, respectively. $F_{\text{wall}}$ is the wall proximity function suggested by Mellor and Yamada (1982) and it is used only in Mellor-Yamada model and defined as

$$F_{\text{wall}} = \left( 1 + E_2 \left( \frac{1}{\kappa} \frac{d_b + d_s}{d_b d_s} \right)^2 \right), \quad (5.5)$$

where $\kappa = 0.41$ is the von Kármán’s constant and $E_2 = 1.33$. The parameters $d_b$ and $d_s$ are the distances from the bottom and surface, respectively. $F_{\text{wall}}$ is taken as 1 for the rest of the turbulence models.

In the framework of the ‘generic algorithm’ turbulent dissipation rate, $\varepsilon$, is defined
according to
\[
\varepsilon = (c_\mu^0)^3 + p/nk^{3/2} + m/n \psi^{-1/n},
\] (5.6)
where \(c_\mu^0\) is a coefficient based on experimental data for unstratified channel flow with a log-layer solution (Warner et al., 2005). The value of \(c_\mu^0\) changes according to which stability functions have been used. For instance, it is 0.5540 and 0.5270 for Canuto-A and Canuto-B, respectively. The definitions of generic parameter, \(\psi\), turbulent length scale, \(l\), and turbulent frequency, \(\omega\), are
\[
\psi = (c_\mu^0)^p k^m l^n \quad (5.7)
\]
\[
l = (c_\mu^0)^3 k^{3/2} \varepsilon^{-1} \quad (5.8)
\]
\[
\omega = (c_\mu^0)^{-1} k^{1/2} l^{-1}. \quad (5.9)
\]

In here, three different two-equation turbulence closures are used. These are \(k - \varepsilon\) (Burchard and Baumert, 1995), \(k - \omega\) (Umlauf et al., 2003), and the Mellor-Yamada level 2.5 scheme (Mellor and Yamada, 1982). Different closures can be obtained from equations (5.1) and (5.2) by changing the coefficients. However, we use the original Mellor-Yamada model in a stand alone subroutine. The coefficients used in this study are taken from Warner et al. (2005) and listed in the Table 5.1. The parameter \(c_3\) has two values; one for unstable stratification positive, \(c_3^+\) and one for stable stratification negative, \(c_3^-\).

<table>
<thead>
<tr>
<th></th>
<th>(k - \varepsilon)</th>
<th>(k - \omega)</th>
<th>MY2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p)</td>
<td>3.0</td>
<td>-1.0</td>
<td>0.0</td>
</tr>
<tr>
<td>(m)</td>
<td>1.5</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>(n)</td>
<td>-1.0</td>
<td>-1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>(\sigma_k)</td>
<td>1.0</td>
<td>2.0</td>
<td>2.44</td>
</tr>
<tr>
<td>(\sigma_\psi)</td>
<td>1.3</td>
<td>2.0</td>
<td>2.44</td>
</tr>
<tr>
<td>(c_1)</td>
<td>1.44</td>
<td>0.555</td>
<td>0.9</td>
</tr>
<tr>
<td>(c_2)</td>
<td>1.92</td>
<td>0.833</td>
<td>0.5</td>
</tr>
<tr>
<td>(c_3^+)</td>
<td>1.0</td>
<td>1.0</td>
<td>0.9</td>
</tr>
<tr>
<td>(c_3^-)</td>
<td>-0.4</td>
<td>-0.6</td>
<td>0.9</td>
</tr>
</tbody>
</table>

**Table 5.1** Coefficients used in \(k - \varepsilon\), \(k - \omega\) and MY2.5 closures.
In the hydrostatic primitive equations resulting from Reynolds averaging of the Navier-Stokes equations, there are Reynolds stress terms (i.e. \( \langle u'_i u'_j \rangle \)). If the transport equations for the Reynolds stresses are derived, third-order moments and pressure strain correlations are encountered. This is the famous turbulence closure problem since the number of unknowns are more than the number of equations. One way to close the system is to parameterize the third-order moment and pressure strain correlation terms. After the parameterization of these terms, so-called stability functions can be derived algebraically from the Reynolds stresses equations for steady state, homogeneous conditions. The stability functions describe the effect of shear and stratification. ROMS provides three different stability functions: KC (Kantha and Clayson, 1994a), Canuto-A, CA, and Canuto-B, CB, (Canuto et al., 2001), which are used to calculate the eddy viscosity, \( K_M \), and the eddy diffusivity, \( K_H \) as

\[
K_M = c\sqrt{2kl}S_M + \nu, \quad K_H = c\sqrt{2kl}S_H + \nu_\theta,
\]

(5.10)

where \( c \) is a coefficient with a value of \( c = 1.0 \) for Galperin et al. (1988) stability functions and \( c = \sqrt{2}(c'_\mu)^3 \) for CA and CB. \( S_M \) and \( S_H \) are the stability functions, and \( \nu \) and \( \nu_\theta \) are the molecular viscosity and diffusivity, respectively. Stability functions depend on \( k, l, N, \) and \( M \). For detailed information about the stability functions, the reader is referred to Warner et al. (2005) and Burchard and Bolding (2001).

In addition to these three closures, the K-profile parameterization (KPP) and a modified \( k-\varepsilon \) turbulence closure validated by Peters and Baumert (2007) (hereafter PB07) have been used. KPP is employed since most of the general circulation ocean models have this parameterization and it is widely used in the community. The PB07 closure does not contain separate stability functions. It rather uses, as an independent function of the gradient Richardson number, only their ratio, turbulent Prandtl number:

\[
Pr_t = \frac{K_M}{K_H} = \frac{S_M}{S_H}.
\]

(5.11)
In PB07, the turbulent Prandtl number depends on the Richardson number (Schumann and Gerz, 1995) as follows

\[ Pr_t = Pr_0 \exp\left( -\frac{R_i_g}{Pr_0 Pr_f^\infty} \right) + \frac{R_i_g}{Pr_f^\infty}, \]  

(5.12)

where \( R_i_g \) is the gradient Richardson number, \( Pr_0 = 0.63 \), and \( Pr_f^\infty = 0.2 \). In this approach, the eddy viscosity and diffusivity are calculated from

\[ K_M = \pi^{-2} \frac{k^2}{\epsilon}, \quad K_H = \frac{K_M}{Pr_t}. \]  

(5.13)

\[ \text{Figure 5.2} \quad \text{Inverse Prandtl number, } Pr_t^{-1}, \text{ versus gradient Richardson number, } R_i_g, \text{ for different turbulence models. Quasi-equilibrium versions of CA=Canuto-A stability function, CB=Canuto-B stability function, G88=Galperin stability function, PB07= modified } k-\epsilon \text{ model, KC=Kantha-Clayson stability function.} \]

In order to provide a perspective of differences between the turbulence models, the inverse of turbulent Prandtl number, \( Pr_t^{-1} \), is plotted versus the gradient Richardson number, \( R_i_g \), in Fig. 5.2. The most striking feature in this plot is that CA, CB, and PB07 almost coincide around \( R_i_g = 0.25 \). The Galperin et al. (1988), G88 and Kantha and Clayson
(1994a), KC stability functions used in MY2.5 cause turbulence to collapse for \( \text{Ri}_g \geq 0.19 \) and \( \text{Ri}_g \geq 0.23 \), respectively. CA and CB also have cut-off Richardson numbers, \( \text{Ri}_c = 0.847 \) for CA and \( \text{Ri}_c = 1.02 \) for CB (Burchard and Bolding, 2001). At \( \text{Ri} > \text{Ri}_c \) diffusivity and viscosity assume background values (i.e. \( K_H \sim 10^{-5} \text{ m}^2\text{s}^{-1} \) and \( K_M \sim 10^{-4} \text{ m}^2\text{s}^{-1} \)). Thus, turbulent Prandtl number for CA, CB, G88, KC and KPP become \( \text{Pr}_t = 10 \) for large Richardson numbers.

Stationary Richardson number \( \text{Ri}_{st} \) (Umlauf and Burchard, 2003) is another important parameter in two-equation models, which controls the entrainment in stably-stratified shear flows. \( \text{Ri}_{st} \) can be evaluated using

\[
\text{Ri}_{st} = \frac{S_M c_2 - c_1}{S_H c_2 - c_3}.
\]

Stationary Richardson numbers for different closures and for different stability functions are shown in Table 5.2. The MY2.5 model exhibits no homogeneous and stationary solution, and thus no stationary Richardson number, if the original parameters are used (Baumert and Peters, 2000). However, Mellor-Yamada using KC stability function and \( c_3 = 2.55 \) has a stationary Richardson number which is very close to \( \text{Ri}_c = 0.23 \).

<table>
<thead>
<tr>
<th>Turbulence Closure</th>
<th>Stability Function</th>
<th>( \text{Ri}_{st} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k - \varepsilon )</td>
<td>CA</td>
<td>0.303</td>
</tr>
<tr>
<td>( k - \omega )</td>
<td>CA</td>
<td>0.265</td>
</tr>
<tr>
<td>( k - \omega )</td>
<td>CB</td>
<td>0.251</td>
</tr>
<tr>
<td>( k - \varepsilon )</td>
<td>CB</td>
<td>0.278</td>
</tr>
<tr>
<td>( k - \varepsilon )</td>
<td>PB07</td>
<td>0.220</td>
</tr>
<tr>
<td>( \text{MY2.5} )</td>
<td>G88</td>
<td>none</td>
</tr>
<tr>
<td>( \text{MY2.5} )</td>
<td>KC</td>
<td>0.210</td>
</tr>
</tbody>
</table>

Table 5.2 Stationary Richardson numbers for different turbulence closures and stability functions.

A total of nine experiments are conducted with different turbulence closure models and stability functions (Table 5.3). These models are standard \( k - \varepsilon \) with CA and CB stability functions, standard \( k - \omega \) with CA and CB stability functions, K-profile parameterization
(KPP), the Mellor-Yamada level 2.5 scheme (MY2.5) with G88 and KC stability functions, and \( k - \varepsilon \) with PB07 turbulent Prandtl number. Additionally, a control experiment is conducted with \( K_M \) and \( K_H \) set to zero. The objective of the control experiment is to investigate the flow dynamics when the vertical transport is limited. We will try to address the question how \( K_M = K_H = 0 \) will affect the overflow in the sense of mixing and flow patterns. The main task of all experiments is to explore which turbulence model leads to the best agreement with REDSOX-1 observations. In all the simulations, we use zero explicit horizontal numerical viscosity but just the implicit viscosity built into the third-order, upstream-biased advection operator (Shchepetkin and McWilliams, 1998). The implicit viscosity is such that the effective Reynolds number at that grid resolution takes the largest value possible while still ensuring stability.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Turbulence Closure</th>
<th>Stability Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp1</td>
<td>( k - \varepsilon )</td>
<td>CA</td>
</tr>
<tr>
<td>Exp2</td>
<td>( k - \omega )</td>
<td>CA</td>
</tr>
<tr>
<td>Exp3</td>
<td>( k - \omega )</td>
<td>CB</td>
</tr>
<tr>
<td>Exp4</td>
<td>( k - \varepsilon )</td>
<td>CB</td>
</tr>
<tr>
<td>Exp5</td>
<td>KPP</td>
<td>none</td>
</tr>
<tr>
<td>Exp6</td>
<td>MY2.5</td>
<td>G88</td>
</tr>
<tr>
<td>Exp7</td>
<td>( k - \varepsilon )</td>
<td>PB07</td>
</tr>
<tr>
<td>Exp8</td>
<td>( K_M = K_H = 0 )</td>
<td>none</td>
</tr>
<tr>
<td>Exp9</td>
<td>MY2.5</td>
<td>KC</td>
</tr>
</tbody>
</table>

Table 5.3 List of experiments, consisting of different turbulence closures, stability functions; CA=Canuto-A, CB=Canuto-B, G88=Galperin, KC=Kantha-Clayson stability functions.

We compare one of the primary variables of prognostic turbulence closure equations to those from measurements. In this case, it is the eddy viscosity \( K_M \). We also compare simulated water temperature, salinity, and velocity to observations.
5.2 Results

A three-dimensional view of the propagation of the outflow is shown in Fig. 5.3. This view is complemented by Fig. 5.4, which depicts plan view of salinity distributions of the bottom layer of the Exp1 model run at four different times, from day 0 to 15. After the dense water is released at $t = 0$, it takes approximately one day to reach the bifurcation point of the northern and southern channels. At this point, the overflow flows into the northern channel first (Figs. 5.3(a) and 5.4(b)) and into the southern channel only later. This feature has also been observed in the REDSOX experiments. Matt and Johns (2007) mentioned that the flow into the southern channel is delayed because it has to overcome a sill 33 m above the floor of the northern channel. The resolution of this phenomena shows that our model resolution is sufficiently high. After reaching the bifurcation point, it takes the outflow five days to reach the Tadjura Rift (Fig. 5.4(c)). In the Tadjura Rift, the two branches equilibrate at different depths; 800 m for the northern channel flow, and around 550 m southern channel flow (Fig. 5.5).

The plan view of the bottom layer in the control experiment is plotted in Fig. 5.6. The difference with respect to Fig. 5.4(d) is drastic. In figure 5.6 there is obvious lateral dispersion around SC. In the narrow NC the lateral dispersion is limited and thus there is somewhat better agreement with the observations than in the SC. Longitudinal sections of the control experiment (Figs. 5.16i and 5.17i) clearly show the horizontal dispersion. There is high saline water between the NC and SC around the latitude 12.3° in the Fig. 5.6. Since vertical eddy diffusivity is set to zero and there is only background vertical diffusivity, the overflow cannot thicken enough to overcome the sill in front of the SC. Therefore, it goes directly to the NC. Bathmetry at the entrance of the NC allows the overflow to leak from the side walls. Lateral dispersion after leaking from the NC is due to topography. We conclude that the main overflow pathways are not realistically reproduced in the control experiment.

Next, modeled eddy diffusivity profiles from the northern and southern channels are compared with REDSOX-I data. Peters and Johns (2005) estimate turbulence variables
from turbulent overturning scales extracted from regular CTD profiles. Estimates of $\varepsilon$, $K_M$ and $K_H$ are provided only as averages over the IL. We follow the definition of the IL given in Peters and Johns (2005) and compute average IL eddy diffusivities as illustrated in an example in Fig. 5.7. In the model output, the eddy diffusivity has a minimum at the velocity maximum. REDSOX-1 observations also imply this phenomena (see Peters and Johns (2005)).

In the NC, REDSOX-1 stations 35, 78, 62, 69, 82, 83, and 81 (Fig. 2.1) are used to compute the eddy diffusivity in the IL (Fig. 5.8). Even though the observations as well as
modeled data show large variations along the channel, it is clear that $K_H$ increases near the end of the northern channel. All models except KPP capture this increase. Mean values of the eddy diffusivity of all the stations are around $2 \times 10^{-2} \text{ m}^2\text{s}^{-1}$ in the observation and in the models except for KPP. Such high values of $K_H$ are common in swift, stratified shear flows (Wesson and Gregg, 1994; Peters, 1999). These average values are three orders of magnitude larger than the background mixing encountered in the ocean away from rough topography (Polzin et al., 1997).

In the SC, REDSOX-1 stations 29, 56, 28, 44, 27, and 19 (Fig. 2.1) are used to compute

Figure 5.4 Salinity distribution of the bottom layer as function of longitude and latitude of the Exp1 model at times $t = 0, 1, 5,$ and 15 days.
the eddy diffusivity in the IL (Fig. 5.9). In the first 20 km of the SC the values are comparable to the NC values, but at greater distances, mixing is reduced and eddy diffusivity values are lower than in the NC in a thinner and slower flow. The main reason for the reduction in the flow speed and mixing appears to be that the SC widens near the exit. Mean values of the eddy diffusivity of the models and observation are around $5 \times 10^{-3}$ m$^2$s$^{-1}$. Although these average values are one order of magnitude smaller than the NC values, they are still large compared to ocean background mixing.

The differing levels of $K_H$ in different turbulence models have implications for $T$-$S$ profiles which we re-examine further below. From Figs. 5.8 and 5.9, it is clear that KPP has relatively low $K_H$ values except in the SC at 30 km. This indicates low mixing in the IL. In contrast, MY2.5 with G88 shows relatively high $K_H$ values indicating strong mixing.
in the IL. The rest of the models have similar $K_H$ values. This suggests that similar results are to be expected for simulated velocity, salinity and temperature fields as well.

### 5.2.1 Comparison of modeled and observed scalar fields

Simulated salinity ($S$) and temperature ($T$) profiles are compared with the REDSOX-1 observations at four stations each, in the northern and the southern channel. The stations chosen are 35, 58, 37, and 81 in the northern channel and 29, 28, 57, and 19 in the southern channel (for the locations of the stations see Figs. 2.1 and 5.1). Salinity and temperature profiles from our 8 experiments are compared with the REDSOX data in Figs. 5.10–5.13. We compare the observations with time averages of the model profiles. Time averages are computed as $\bar{X} = (\tau_2 - \tau_1)^{-1} \int_{\tau_1}^{\tau_2} X dt$ with $\tau_1=20$ days and $\tau_2=30$ days. Here, $X$ stands for $T$ and $S$. The structure of the temperature profiles (Figs. 5.12 and 5.13) from different
Figure 5.7  Profiles of salinity, velocity (left panel), and eddy diffusivity (right panel) at Station 35 from Exp7. The shaded area is the layer referred to as the IL. The mean eddy diffusivity, $K_H$, of this area is compared with ocean observations.

experiments is similar to that of salinity (Figs. 5.10 and 5.11). Therefore, the following comparisons and discussions are based on salinity profiles alone. In the observations, there is a distinct high salinity layer at 150 m depth at the entrances of the channels (station 35 and station 29). It is thought that the formation of this saline water mass is related to tidal stirring within the BAM Strait (Matt and Johns, 2007). Herein, the flow within the Strait of BAM is not modeled. Discrepancy at station 81 is due to the initial salinity profile (see the dashed line in Fig. 5.10).

The first noticeable feature is that all the models capture the vertical structure of the overflow consisting of BL and IL. There are slight differences between the models, for instance, KPP has a thinner IL compared to the others in both channels due to weaker overall mixing. This feature can be noticed especially in temperature profiles (cyan curves
Figure 5.8  Modeled and observed average eddy diffusivity in the IL of the northern channel overflow. Distances along the channel are measured from REDSOX-1 station 35.

in Figs. 5.12 and 5.13). In MY2.5 with the Galperin et al. (1988) stability functions, the salinity in the BL is small (green curves in Figs. 5.10, 5.11, 5.12 and 5.13). Near the exit of the NC, MY2.5 with G88 has a thick IL as a result of excessive mixing of the BL with the overlying water (ST81 in Fig. 5.10). Five models ($k-\varepsilon$ CA, $k-\omega$ CA, $k-\omega$ CB, $k-\varepsilon$ CB, MY2.5KC) show similar salinity profiles. These findings are consistent with implications from $K_H$ levels discussed in previous section. The model PB07 exhibits reasonable agreement compared to the observations just like the other five models.

5.2.2 Comparison of modeled and observed velocity

Comparisons of the modeled and observed velocity profiles at selected stations along the northern and the southern channels are shown in Figs. 5.14 and 5.15. The horizontal velocity components, $u$ and $v$, are projected into the flow direction so that only one streamwise velocity component is shown. The modeled velocities are also time-averaged between
Figure 5.9 Modeled and observed average eddy diffusivity in the IL of the southern channel overflow. Distances along the channel are measured from REDSOX-1 station 29.

Model days 20 and 30. The flow direction from the other experiments is also used for the control experiment. However, as the flow spreads laterally in the control case, maximum velocities of the control experiment are higher than in the observations; since there is no vertical momentum transfer the flow undergoes excessive acceleration.

Modeled velocity profiles are similar to the observations in the northern channel; the maximum velocities in all models are approximately 0.9 m/s. Among the output from different closures the comparatively small velocities from MY2.5 with G88 toward the end of the NC stand out (green curve at Station 81 in Figure 5.14).

In the southern channel, modeled maximum velocities are slightly larger than those of observation. There is a good agreement with REDSOX-1 data at the end of the southern channel (Station 19 in Figure 5.15). Maximum velocities in all models are approximately 0.6 m/s, and maximum velocity in the observation is around 0.4 m/s.

The model velocity profiles are highly sensitive to the boundary conditions at Bab el
Figure 5.10  Comparison of salinity profiles from different models and REDSOX-1 observations in the northern channel. The dashed lines are initial profiles.
Figure 5.11  Comparison of salinity profiles from different models and REDSOX-1 observations in the southern channel. The dashed lines are initial profiles.
Figure 5.12  Comparison of temperature profiles from different models and REDSOX-1 observations in the northern channel. The dashed lines are initial profiles.
Figure 5.13 Comparison of temperature profiles from different models and REDSOX-1 observations in the southern channel. The dashed lines are initial profiles.
Figure 5.14  Comparison of the streamwise velocity profiles from different models and REDSOX-1 observations in the northern channel.

Mandeb. We have modified these conditions such as to accurately reproduce the flow at station T0, which is downstream from the boundary conditions at BAM but upstream of the channel sections where we probe the model performance. If, for instance, velocities at T0 are even modestly too large, the flow in the SC can become as swift as 1.1 m/s.

It cannot be expected that the model simulations match the observations perfectly. The CTD/LADC profiles represent an incoherent, possibly aliased survey taken in a time-variable flow field (see Fig. 9 in Peters et al., 2005b). Considering this, reasonable agreement with REDSOX-1 observations is found in both channels for all models.
Figure 5.15  Comparison of the streamwise velocity profiles from different models and REDSOX-1 observations in the southern channel.
5.2.3 Error analysis of different turbulence closures

In this section, we examine salinity deviations between the numerical experiments and the observations. First, salinity sections are compared visually and qualitatively, and then the deviations are quantified.

Salinity distributions along Sections 1 and 2 introduced in Fig. 5.1 are plotted for the different experiments and the REDSOX observations in Figures 5.16 and 5.17, respectively. It is clearly seen in the observations (Fig. 5.16a) that the overflow water in the northern channel is denser than that in the southern channel. This pattern is adequately captured by all closures except MY2.5 with G88. There is not enough dense water in the southern channel in MY2.5 with G88. Experiments 1 to 4 show very similar salinity sections to each other (Figs. 5.16b, 5.16c, 5.16g, and 5.16h). In Section 2, mixing is high for Exp6 and bottom, dense flow signal becomes weak (Fig. 5.17i). On the contrary, KPP does not mix enough in the SC (Fig. 5.17f). This can be also seen in individual salinity profiles in the SC (Fig. 5.11). Section 2 is more important than Section 1, since it is far away from the bifurcation point. In Section 2, MY2.5 with G88 and KPP deviate much more from the observed $S$ than the other closures. Dense water is absent in the channels in the control run (Fig. 5.17j), since the control experiment spreads horizontally. Further, there is high saline water between the channels (red stripes between the channels in Figs. 5.16j and 5.17j) not seen in the observations. There is a distinct difference between MY2.5KC and MY2.5 with G88 (5.17d and 5.17i), the former shows reasonable agreement with the observations and the other models.

We will now quantify the model deviation from the observed $S$ and try to answer the question if some closures perform significantly better or worse than the others. This is a difficult question because of the uncertainty in the sparse observations. For instance, the observed maximum salinity in Section 2 is 39.55 psu while the maximum is only 38.6 psu in the MY2.5 with G88 simulations. Does this single out MY2.5 with G88 for poor performance?
Figure 5.16  Salinity distribution of all model experiments and REDSOX-1 data along Section 1 (see Fig. 5.1 for the position of the section).
Figure 5.17  Same as Fig. 5.16 but for Section 2.
First, we compare the BL salinity near the ends of channels. Three stations (38, 81, and 101 in Fig. 2.1) were taken near the lower end of the NC at different times. We compute a mean salinity profile, $\bar{S}$, from these stations along with its standard deviation, $\sigma$, (Fig. 5.18). Since the BL is well-mixed, it is sufficient to analyze the statistics of the BL mean, $\bar{S}$, in observations and simulations. Results appear in Table 5.4. We take two standard deviations as confidence bounds of the observations. Only KPP and the $k - \varepsilon$ PB07 fall into the confidence bounds of the REDSOX data (gray shaded areas in REDSOX row in Table 5.4). Maximum salinities from $k - \varepsilon$ CA, $k - \omega$ CA, $k - \omega$ CB and $k - \varepsilon$ CB are 0.1 psu smaller than the bottom limit of REDSOX-1 values. The maximum value of MY2.5 with G88 is still smaller by 0.4 psu than the bottom limit of the REDSOX-1 values. We conclude that the MY2.5 with G88 scheme falls outside of the variance range of the REDSOX data at the lower end of the NC. However, MY2.5KC is significantly better than MY with G88 stability functions in this analysis. MY2.5KC is only 0.16 psu smaller than the bottom limit of REDSOX-1 values. This value is very similar to the other model’s performance. This clearly shows that stability functions have a clear impact on the overall performance of the MY2.5 turbulence model.

Given the stratification in the IL, comparing simulated and observed mean salinities from the IL makes little sense. The salinity variance in the IL is much larger in the observations than in any of the model runs. The large observed variability of $S$ in the IL is probably caused by time variations in flow and mixing. These cannot be captured in the model since its boundary forcing is constant and there is no wind forcing.

In order to quantify the difference between the observations and the results of the different models, an error function is defined as

$$\text{Error} = \frac{1}{n} \sum_{i=1}^{i=n} \frac{1}{\Delta S} \sqrt{\sum_{j=1}^{j=M^i} \left[ \bar{S}_{\text{model}}^i(j) - S_{\text{obs}}^i(j) \right]^2 / M^i}, \quad (5.15)$$

where $\bar{S}_{\text{model}}^i$ is the time-averaged model output, $S_{\text{obs}}^i$ is the observed salinity at station $i$, $M^i$
Figure 5.18  Observed salinity profiles of Station 38, 81, and 101 together with the mean profile from these 3 stations and the 2-σ spread.

is the number of vertical sampling points at each data station, \( n \) is the number of stations. Section 1 is made by interpolating eleven REDSOX-1 stations, these stations are 67, 58, 61, 60, 59, 70, 71, 72, 73, 74, 75, 76. Therefore, we used these stations to calculate the root mean square error (\( Error \)) for Section 1. Section 2 is made by interpolating fifteen REDSOX-1 stations, these stations are 07, 08, 09, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24. We also used these stations to calculate the \( Error \) for Section 2. Thus, the number of stations, \( n \), is 12 for Section 1 and 18 for Section 2. Salinity deviations are normalized by a salinity range of \( \Delta S = 3.5 \) psu, since only the overflow plume, defined by a salinity interface larger than 36.5 psu, is used to compute the deviation. Results are shown in Table 5.5, salinity deviations at Section 1 are larger than at Section 2, since there is the intrusion layer in the observations (Fig. 5.16 a). The models cannot capture this
Table 5.4 Mean salinity value, $\bar{S}$ and its variance, $\sigma$, in the BL for different experiments and REDSOX-1 data.

layer as already mentioned above. Salinities from KPP, MY2.5 with G88 and the control experiment have the higher deviations from the observations at both sections than the other experiments which have smaller, similar errors.

Table 5.5 Average of normalized salinity errors (%) of different experiments at the Section 1 and the Section 2.

As a further and final test of the model performance, we calculate the mass transport of different salinity classes. The mass transport indicates the overall entrainment occur in the plume. We divide the overflow into ten equal salinity classes from 36.5 psu to 39.5 psu and we calculate the mass transport, $\int udz$, of each class. Fig. 5.19 depicts the mass transport of the different models at stations 81 and 19 near the lower ends of the NC and SC, respectively. In the NC, only $k - \varepsilon$ models have comparable transports with the observed
transport (Fig. 5.19a) at the maximum salinity class (i.e. 39.2 < S ≤ 39.5 psu). MY2.5 with G88 and control cases have zero transport for that class since maximum salinities in these models are less than 39.2 psu. KPP and two $k - \omega$ models behave similarly to each other by transporting the same salinity classes. In the SC, only $k - \varepsilon$ PB07 and REDSOX-1 transport the second most densest class (i.e. 38.9 < S ≤ 39.2 psu). MY2.5 with G88 and control cases again have zero transport at denser classes, and these models transport intermediate salt classes compared to the other models (Fig. 5.19b). For instance, the control experiment has the highest transport at the class where salinity falls between 38.0 and 38.3 psu. Also, MY2.5 with G88 has two peaks at the classes where salinity falls between 37.7 and 38.0 psu, and between 38.3 and 38.6 psu.

5.3 Summary and discussion

In this chapter, we analyze the performances of a set of second-order turbulence models in a study of the Red Sea overflow using a hydrostatic circulation model. The turbulence models examined are conventional $k - \varepsilon$, $k - \omega$, MY2.5, and a modified $k - \varepsilon$. The model simulations are compared to temperature, salinity, velocity and eddy diffusivity observations from the 2001 REDSOX-1 cruise. As the overflow is contained in two narrow channels, we compare CTD/LADCP stations from the channels with corresponding model data. Model runs are integrated for 34 days during which the flow becomes quasi-stationary. Time averages over model days 20 – 30 of primary variables ($T, S, u$) and secondary variables ($K_H$) are compared to the observations.

A set of nine experiments is conducted with different turbulence closures using different stability functions. These are conventional $k - \varepsilon$ using CA and CB stability functions, $k - \omega$ using CA and CB stability functions, MY2.5 using G88 and KC stability functions, and a control experiment where $K_M = K_H$ is set to zero implicitly. Since it is widely used in the community, the KPP scheme is also tested even though it is a diagnostic model. In addition,
Figure 5.19  Transport of the different salinity classes for different models and REDSOX - 1 observations in the northern channel (left panel) and in the southern channel (right panel).
a modified $k - \varepsilon$ model without stability functions but with Richardson number-dependent turbulent Prandtl number is also employed.

The comparison of modeled and observed salinity, temperature, and eddy diffusivity distribution can be summarized as follows. All two-equation turbulence models are able to capture the vertical structure of the overflow consisting of the BL, which transports salty, dense water along the channels, and the IL, where most of the mixing and entrainment takes place. Mean eddy diffusivities in most closures are $O(10^{-2})$ in the NC and $O(10^{-3})$ in the SC. These values compare well with observational data. Eddy diffusivities are too small in KPP and too large in MY2.5 with G88. In consequence of the large $K_H$, the BL salinity becomes diluted in MY2.5 with G88. The control experiment exhibits poor results mainly because of horizontal dispersion of the flow beyond the confining channels.

Salinity deviations between model and observations are quantified along two sections across the overflow. KPP and MY2.5 with G88 produce the largest deviations from the observations and the modified $k - \varepsilon$ exhibits the smallest deviations. The other five closures fall in between, showing deviations similar to one another. MY2.5KC displays better performance than the MY2.5 with G88. The main reason behind this is that the version with KC stability functions has a $Ri_{st} = 0.21$, whereas the one with G88 does not have a stationary Richardson number. The importance of $Ri_{st}$ on entrainment in stably-stratified shear flows was emphasized by Umlauf and Burchard (2003), and this seems to play a critical role in improving the performance of the MY2.5 model.

With respect to the ventilation of the deep ocean, it is the properties of gravity currents at their point of equilibration that are the most important. That is, our model should adequately reproduce the transport of all salinity classes in the overflow. However, with respect to the lateral spreading along ocean basins of a saline water mass, the higher $S$ classes, and especially the maximum $S$, matter most. On this background, success and failure of some closures of transporting the most saline and dense water are important. Only $k - \varepsilon$ models transport the densest salinity class in the NC and only the modified $k - \varepsilon$ transport the dense-
est salinity class in the SC. KPP and $k - \omega$ models have correct salinity classes but weak transports compared to the observations. However, the control experiment and MY2.5 with G88 transport intermediate salinity classes in the NC and SC, therefore we can say that only $k - \varepsilon$ models have successfully reproduced transport of the high salinity classes.

This study indicates that two-equation closures can perform well in the low-$Ri_g$, large shear setting of the overflows. Thus, it extends the range of applications for such closures to overflows. It is important to note that the quantitative performance relative to the observations of our model depends on much more than just the turbulence parameterization employed. Model results depend critically on such factors as horizontal and vertical resolution, ambient stratification, and proper forcing at the source of the overflow. Within such model sensitivities we find only modest differences in the performances of a range of tested two-equation closures, and overall reasonable agreement with the observations. In conclusion, we find that most turbulence closures lead to a satisfactory reproduction of the Red Sea overflow, within the temporal and spatial sampling uncertainties of the REDSOX data, provided that fairly high-resolution regional models are used.
Chapter 6

Simulation of the Mediterranean outflow

In the previous chapter, performance of different two-equation turbulence closures has been investigated. It is found that there are slight differences between closures. Besides, other model factors such as initialization, forcing, resolution etc. are as important as the turbulence parameterization. The aim of this chapter is to investigate the same closures in another overflow. For this purpose, the Mediterranean outflow is chosen since it is one of the best-observed (Ambar, 1979; Johnson et al., 1994b; Bower et al., 1997; Baringer and Price, 1997; Ambar et al., 2002; Serra et al., 2005). The Mediterranean outflow is so dense ($\sigma_\theta = 28.95 \text{ kgm}^{-3}$) that it would sink directly to the bottom of the North Atlantic Ocean in the absence of entrainment. However, the product water equilibrates at an intermediate depth of 1100 m (Price and Baringer, 1994). Thus, the modification of the outflow properties is sensitive to the strength of mixing and vertical turbulence parameterization.

Four turbulence closures are selected for the comparison of 3D numerical simulations of the Mediterranean overflow. These are $k - \varepsilon$ CB, $k - \omega$ CB, MY2.5 KC and KPP. The first three closures relatively give better agreement in the Red Sea simulations and KPP is a control experiment. The accuracy of the model results is evaluated using hydrographic and current observations collected in the Gulf of Cádiz Experiment (Johnson et al., 1994b; Price and Baringer, 1994; Bower et al., 1997). Fig. 6.1 displays the locations of the CTD
stations during the Gulf of Cádiz Expedition.

![Figure 6.1](image)

**Figure 6.1** The locations of CTD stations during the 1988 Gulf of Cádiz Expedition.

### 6.1 Model configuration

ROMS is employed as the numerical model. The domain covers the area between longitude 5°W and 14°W, latitudes 33°N and 40°N (Fig. 6.2a). A nonuniform grid resolution is used in the model with 950 m until 9°W, after that grid resolution gradually coarsens to 10 km toward the end of the domain (Fig. 6.2b). The west, north and south sides of the domain are extended to use open boundary conditions effectively. The vertical resolution relies on 40 sigma layers, unevenly spaced and clustered near the bottom and intermediate level of the water column since the major interest is in these areas. The bottom topography is extracted from ETOPO2 2 minute world bathymetry. The model is initialized using the temperature and salinity fields from the climatology "World Ocean Atlas 2005". Monthly Comprehensive Ocean-Atmosphere Data Set (COADS) winds are applied as the surface forcing. All experiments are performed with 378 × 286 × 40 grid points in x, y, and z directions, respectively. The model starts from rest and is integrated for 400 days. A total of four experiments are conducted with different turbulence closure models and stability functions.
(Table 6.1). The closures employed in here are the closures which perform reasonably well in the previous chapter.

![Image of bathymetry and model grids](image)

**Figure 6.2**  a) East of Atlantic Ocean and Gulf of Cádiz bathymetry in [m] b) Model grids in every 4 points.

### 6.2 Results

At first, the exchange through the Strait of Gibraltar is computed by volume transport at Lon = 6°W. *Bryden et al.* (1994) estimate volume transports of 0.7 Sv. However, *Candela* (2001) estimates a higher volume transport of 1.0 Sv which is consistent with the higher
Turbulence Closure | Stability Function
---|---
Exp1 | \(k - \epsilon\) | CB
Exp2 | \(k - \omega\) | CB
Exp3 | \(KPP\) | 
Exp4 | \(MY2.5\) | KC

Table 6.1 List of experiments, consisting of different turbulence closures, stability functions

observed outflow velocity. All the numerical simulations conducted in this chapter exhibit mean value of 0.78 Sv for volume transport in agreement with the observations.

The salinity field of the bottom layer of the Exp1 model run is shown in Fig. 6.3 at four different times, from day 2 to 20. After the overflow released at \(t = 0\), it takes approximately 2 days to leave the Strait of Gibraltar. Some of the Atlantic Ocean water is moving north to replace the overflow water (Fig. 6.3a). The overflow is constrained to the narrow channel and the velocities are directed along-channel. After seven days, the overflow turns north due to earth’s rotation and becomes an undercurrent. The overflow follows the continental shelf and flows more nearly parallel to isobaths (Fig. 6.3b). A cyclone forms and stays at \(7.2^\circ\text{W}\) and \(36.25^\circ\text{N}\) due to potential vorticity conservation. This will be described in detail in the next chapter. After the cyclone, the overflow separates into three branches because of the topography. At time = 10 days, the cyclone moves northwest and the high saline water of the overflow stays off-shore (Fig. 6.3c). Three branches of the overflow can be clearly seen 20 days after the initial release 6.3d). There are two sills located around \(7.7^\circ\text{W}, 36.5^\circ\text{N}\) and \(7.45^\circ\text{W}, 36.65^\circ\text{N}\). The relatively low saline water of the overflow comes on-shore and the high saline water remains off-shore. However, the bottom layer temperature displays that the part of the overflow that approaches shore is warmer than the part that stay off-shore (Fig. 6.4). It is found that the overflow separates into two branches; one stays on-shore and the other stays off-shore. The on-shore part consists of low saline and high temperature values and the off-shore part has high saline and cold temperature values.

The two branches separating east of the Gulf of Cádiz rejoin at \(8.5^\circ\text{W}\). Vertical sections
Figure 6.3  Salinity distribution of the bottom layer as function of longitude and latitude of the Exp1 model at times $t = 2, 7, 10, \text{and } 20$ days.

of salinity and temperature are displayed in Fig. 6.5. Two cores structure can been seen in the velocity contours. The bottom core of the velocity field coincides with high saline overflow water and it is around 1000 m (Fig. 6.5a). The upper core of the velocity field is around 700 m. The warm branch of the overflow water is also around the same depth (Fig. 6.5b). The Mediterranean undercurrent double core structure is also reported in the literature (e.g. Ambar, 1979; Chérubin et al., 2000). The latter described that two cores of the overflow come together at $8.5^\circ W$ and induce instabilities.
6.2.1 Comparison of modeled and observed scalar fields

In this section, observed and simulated salinity and temperature profiles are compared. The model results are linearly interpolated to the location of each station and a time average of the last 50 days of the model simulations was calculated. However, it has to be mentioned in here that the spreading of the observed plume is different than the modeled ones at the exit of the Strait. To this end, two Gulf of Cádiz (henceforth GOC) CTD stations are displayed in 6.6. The station 67 and 70 represent the northern and the southern edge of the plume in the observations, respectively. However in all models, the southern edge of the plume is more to the north in the models than in the data.

In light of this information, stations 64, 83, 96 and 104 are selected for the comparison. The structure of the temperature profiles from different experiments is similar to that of salinity (Fig. 6.7). Therefore, the following comparisons and discussions are based on salinity profiles alone. All models except KPP capture the vertical salinity structure of the
overflow consisting of BL and IL (see stations 64, 83 and 104). KPP has a thin IL compared to the others just like the Red Sea overflow simulations. The maximum salinity in the BL is always bigger than the observations in the KPP simulation. Thin IL and bigger maximum salinity values indicate that KPP has less mixing in the overflow. This result is consistent with the result found in the previous chapter. The other three models ($k - \varepsilon$ CB, $k - \omega$ CB, MY2.5KC) show similar salinity profiles.

2D vertical structure of the overflow is computed to investigate the performance of the turbulence closures in more detail. Salinity distributions along two sections at Lon = 8°W and Lon = 8.6°W are introduced in Figs. 6.8 and 6.9, respectively. The sections for the observations are computed by Ocean Data View (ODV). It uses an advanced gridding...
Figure 6.6  Comparison of salinity profiles of CTD stations 67 (left panel) and 70 (right panel) between observation and models.

Figure 6.7  Comparison of salinity and temperature profiles of CTD stations 64, 83, 96 and 104 between observation and models.
software so-called Data-Interpolating Variational Analysis (DIVA) for the integration of the gridded data. For more information, the reader is referred to read http://odv.awi.de/.

In Fig. 6.8, $k - \varepsilon$ and $k - \omega$ closures display similar structure with the GOC data. The overflow stays on the shelf and it reaches around 1200 m depth. However, the maximum depth which the overflow can be traced is almost 1500 m in the KPP run (Fig. 6.8d). There is more saline water in the overflow and the thickness of the outflow is significantly less than the one in GOC data (Fig. 6.8a). In the MY2.5KC case, the overflow reaches to 1100 m, however the thickness of the overflow is comparable with the observations. In Fig. 6.9, salinity distribution is plotted at Lon = 8.6°W. The simulated outflow water mass equilibrates deeper than observed with KPP and shallower with MY2.5KC (Fig. 6.9d, e).

The bottom of the overflow water is around 1500 m, 1650 m and 1300 m in the GOC data, KPP case and MY2.5KC case, respectively. The other experiments perform similar to each other and maximum depth of the overflow is around 1400 m. In all the salinity profiles and sections, it is found that the entrainment is under-estimated in the KPP case and over-estimated in the MY2.5KC case. The overflow water expands till 35.9°N in the data and the depth is around 1100 m. The only model displaying the same behavior is the $k - \varepsilon$ case. In the KPP case, the expansion of the overflow is much deeper. There is a patch of overflow water around Lat = 35°N in the GOC data. This is probably one of the overflow-induced eddies. The patch of the overflow water cannot be seen in the numerical simulations since they are averaged over 50 days.

### 6.2.2 Mixing

In this section, we try to address two important questions; the amount of mixing in the overflow using different turbulence closures and where the mixing occurs in the Gulf of Cádiz. Burchard et al. (2007) described that the vertically integrated turbulent salt flux $\int K_H \partial_z S dz$ is a measure for the potential energy gain due to turbulent mixing. It has to be mentioned that the vertical turbulent salt flux is not a fully consistent estimate of
Figure 6.8  Vertical salinity distribution at Lon = 8°W of (top to bottom) GOC observations, $k -\varepsilon$, $k - \omega$, KPP, MY2.5KC.
Figure 6.9  Vertical salinity distribution at Lon = 8.6°W of (top to bottom) GOC observations, $k - \epsilon$, $k - \omega$, KPP, MY2.5KC.
turbulent mixing, since it may consist of positive or negative contributions, depending on the sign on the vertical salt gradient. However, Burchard et al. (2007) showed that the time averaged $- \int K_H \partial_z S dz$ can be used to estimate mixing in dense bottom currents. In Fig. 6.10, the decadal logarithms of the vertically integrated upward turbulent salt flux for different cases are shown. It can be seen that the highest values occur just at the exit of the Strait of Gibraltar. This indicates the strong amount of mixing occurs in the first tens of kilometers outside the Strait. Baringer and Price (1997) described that there is intense localized mixing with fresher, overlying North Atlantic water in the same region. Furthermore, the complete dense water pathways till Cape St. Vincent are characterized by mixing. It is evident that the entrainment is less in the KPP case compared to the other cases (Fig. 6.10c). The other models ($k-\varepsilon$, $k-\omega$, MY2.5KC) show similar values which indicate that the mixing is similar.

All the comparisons of temperature/salinity profiles, meridional sections and vertically integrated turbulent salt flux indicate that the KPP model mixes less than the two-equation turbulence closures. Finally, to show the difference between KPP and two-equation turbulence closures, the volume transports of different salinity classes are shown in Fig. 6.11. for only $k-\varepsilon$ and KPP cases. For the salinity classes, the transport is divided into 20 bins of 0.175 psu. In both cases, an eastward return flow is present. The observations also show the same return flow in the Gulf of Cádiz. This confirms that the numerical simulations represent the upper ocean circulation accurately. The KPP case has the transport of the maximum salinity class in the sections A,B and E. However, in the $k-\varepsilon$ case, strong entrainment takes place east of section A, thus the maximum salinity class disappeared. As the outflow moved to section B, the maximum transport appeared in a slightly lighter salinity class indicating internal mixing. In the section F, there are still transports of 37.1875 and 37.3625 psu bins in the KPP case (Fig. 6.11). However, the maximum salinity bin which has a transport is 37.0125 psu in the $k-\varepsilon$ case.
Figure 6.10  Simulated decadal logarithm of vertically integrated upward turbulent salt flux $\log_{10}(\int K_H \partial_z S dz)$ of $k - \varepsilon, k - \omega, KPP, MY2.5KC$, respectively.

6.3 Summary and discussion

To investigate performance of different turbulence closures in simulations of another realistic overflows, a regional model of the Mediterranean outflow is conducted. Four different closures ($k - \varepsilon, k - \omega, MY2.5KC$ and $KPP$) are employed in this chapter. The numerical results are compared with observational data obtained in the 1988 Gulf of Cádiz Expedition. The simulations with two-equation closures reproduce the observed properties of the Med quite well, especially the evolution of temperature and salinity profiles. All the simulations capture the spreading of the overflow as it descends along the continental shelf and the transition of the overflow from a bottom-trapped gravity current to a wall-bounded undercurrent. In the $KPP$ simulation, the overflow does not entrain in comparison to that
Figure 6.11 Volume transport of salinity (in bins of 0.175 psu) at sections A, B, E and F. The upper panel is from the $k-\varepsilon$ case and the lower panel is from the $KPP$ case.

in the two-equation cases. The vertically integrated turbulent salt flux displays that the Med undergoes significant mixing outside the west edge of the strait. The volume transport and water properties of the outflow are modified significantly in the first 50 km after the overflow exits the strait. The $k-\varepsilon$ and $k-\omega$ cases show the best agreement with the observations.
Chapter 7

Interaction of the Red Sea overflow with GOA eddies

Until now, we focussed on the performance of different turbulence closures. One of the conclusions is that not only the turbulence closure but also the numerical setup is crucial to reproduce the outflow. The important parameters are spatial resolution, initial conditions and forcing. There is also one more mechanism that is important to modify the product water properties in the case of the Red Sea outflow; the interior dynamics of the Gulf of Aden (GOA). The surface circulation in the GOA is dominated by wind and a series of mesoscale eddies (Fratantoni et al., 2006; Al Saafani et al., 2007). The size of these eddies can be as large as the width of the gulf (∼ 200 – 300 km) and their azimuthal speed can be as high as 0.2 – 0.3 m/s (Fig. 7.1). Bower et al. (2002) records that these cyclonic and anticyclonic eddies are in fact barotropic and they can reach as deep as 1500 m.

In this chapter, we use a very high resolution 3D regional model and look at the interaction between the Red Sea overflow and GOA eddies, systematically. The main objective of this section is to address the following questions:

- What is the pathway of the Red Sea overflow with and without eddies?
- What is the effect of an isolated cyclone/anticyclone in terms of mixing?
- How does the overflow transport evolve in different cases?
- What is the influence of multiple realistic eddies on the Red Sea overflow?
Figure 7.1 Sea level anomaly (6 January 1999) displaying the wide range of mesoscale activity in western tropical Indian Ocean.

- How does the Red Sea overflow find its way out of the GOA when it is full of realistic eddies?

In order to reply to these questions we employ an expanded version of the numerical model used in Chapter 5 (Ilıcak et al., 2008b). Since the interaction between the overflow and multiple eddies can be complex, we start with a simplified problem. To isolate the effect of eddies, at first we only consider the overflow without any eddies in the GOA. After that, an idealized shielded cyclone or anticyclone is initialized with the overflow in two cases. Our focus is on the pathway of the overflow in the presence of an idealized eddy. We then move to more realistic simulation using a global ocean circulation model for the forcing of multiple eddies in the GOA. We also compute the overflow transport and mixing in different simulations for the quantitative analysis.
7.1 Generation of Gulf of Aden Eddies

There are different theories for the generation of GOA eddies. Frantoni et al. (2006) described a passage of series eddies into the gulf mostly during transition between summer and winter monsoons. They showed that the dominant mechanisms for these eddies are the instabilities and retroflection of the Somali Current due to monsoon winds. Al Saafani et al. (2007), however showed that some GOA eddies are come from the Arabian Sea. The dominant mechanism for these eddies consists of westward propagating Rossby waves generated in the Indian Ocean by poleward propagating Kelvin waves during the winter monsoon.

During the northeastern monsoon, the Somali Current changes its direction and accelerates northward through the Socotra Passage (Molinari et al., 1990). Frantoni et al. (2006) showed that a part of the Somali Current retroflects at the Socotra Passage and forms cyclones and anticyclones which move westward into the gulf. Formation of an anticyclone and a cyclone can be seen in Fig. 7.2 which shows sea level anomaly obtained from satellite altimetry data. Sea level anomaly (SLA) and geostrophic velocities are obtained and quality controlled by AVISO (http://www.aviso.oceanobs.com) which contains a composite data of several satellite missions TOPEX/Poseidon, GFO, Jason-I, ERS I/II, and ENVISAT. Satellite altimeters measure the sea surface height and its spatial derivative, thus geostrophic velocity fields can be obtained. Details of the altimetric measurements and procedure of the data processing can be found in Le Traon and Dibarboure (1999). There is a narrow jet of Somali Current that reaches the coast of Yemen on the 28th of October. The Somali Current jet turns sharply to the east and south (positive SLA at 52°E and 14°N). On the 2nd of December, the same retroflecting jet collapses upon itself and generates an anticyclone. During January, the anticyclone moves slowly westward (~ 5 cm/s) into the GOA. There is another anticyclone on the 6th of January coming into the domain from the north (positive SLA at 54°E and 15.5°N). The anticyclone moves to west on the 27th of January and it is more evident in Fig. 7.2d. It seems that this anticyclone is associated with
westward propagating Rossby waves coming from the Arabian Sea. It is probably one of the eddies Al Saafani et al. (2007) described. Rossby waves in the Arabian Sea have been observed in earlier studies. They have been produced by forcing due to Ekman pumping over the Arabian Sea (McCreary et al., 1993; Shankar et al., 2002). For further details about the generation of GOA eddies, the reader is referred to Frantoni et al. (2006) and Al Saafani et al. (2007).

7.2 Numerical Setup

ROMS is employed as the numerical model. The domain covers the area between longitude 43°E and 48°E, latitudes 10.2°N and 13°N (Fig. 7.3a). The numerical domain is an extended version of the domain of Ilıcak et al. (2008b) (henceforth IL08). The same initial conditions and forcing of the overflow from IL08 are employed in this study. We also use $k - \epsilon$ turbulence closure with Canuto-B (Canuto et al., 2001) stability functions. A nonuniform grid resolution is used in the model with 450 m until 46°E, after that grid resolution gradually coarsens to 10 km toward the end of the domain (Fig. 7.3b). The vertical resolution relies on 30 sigma layers, unevenly spaced and clustered near the bottom and intermediate level of the water column since the major interest is in these areas.

According to REDSOX observations water properties (temperature and salinity) of the east and west side of the Gulf of Aden are different. Therefore, the model domain is initialized by temperature (T) and salinity (S) profiles from two different REDSOX stations; one is Station 93 (circle dot in Fig. 7.3a) which is in the Tadjura Rift and was also used in IL08, the other is Station 182 (square dot in Fig. 7.3a) which is in the Gulf of Aden away from the Tadjura Rift. The profiles from these stations are cropped to the local water depth at any other location. The same method described in Chapter 5 is used to introduce the dense overflow. Until now, all the numerical simulations indicate that the Red Sea overflow is a salinity-driven outflow, hence all the following discussions are based on the salinity
Figure 7.2  Time sequence of sea level anomaly (indicated by colors) and geostrophic velocity anomaly (indicated by vectors).
A nonlinear equation of state (Jackett and McDougall, 1995) is used to compute density. Open boundary conditions (Marchesiello et al., 2001) are employed at the eastern boundary. Once the dense water is released from the top of the domain, the model is integrated for 120 days. Wind forcing as well as evaporation, precipitation and radiative heat fluxes are set to zero everywhere. In here, a quadratic bottom drag formulation with a drag coefficient of $C_d = 5 \times 10^{-3}$ is used to incorporate the bottom shear stress. All of the parameters defined above have been tested in Ilıcak et al. (2008b). All experiments are performed with

**Figure 7.3** a) Gulf of Aden bathymetry in [m] and idealized eddy location (red circle). b) Model grids in every 10 points until 46°E and in every 5 points after that.
800 \times 702 \times 30 \text{ grid points in x, y, and z directions, respectively. The baroclinic time step is set to 30 seconds, and 20 barotropic time steps are used between each baroclinic time step.}

### 7.2.1 Idealized Eddies in Open Ocean

Since satellite data show that westward propagation of Indian Ocean eddies into the gulf takes months (Fig. 7.2), it is important for us to model an eddy that does not dissipate in a short amount of time. In this section, we try to find a velocity profile to initialize the modeled eddies, so that the vortices are stable and they do not lose their coherence structures. We conduct five idealized experiments to be assured that cyclones and anticyclone survives in the open ocean away from boundaries. The tests are positioned at the same longitudes and latitudes of the Gulf of Aden domain, however bathymetry is constant (2000 m) and the flow is homogenous ($S = 35 \text{ psu and } T = 12.5^\circ \text{C}$). Two of the experiments have been completed on so-called $f$–plane where the Coriolis parameter is constant (i.e. $f \approx f_0 = 2\Omega \sin \lambda_0 = 3.032 \times 10^{-5} \text{ s}^{-1}$, where $\Omega = 7.2921 \times 10^{-5} \text{ rad/s}$ and $\lambda_0 = 12^\circ$). For the rest of the experiments, we use variable Coriolis frequency.

At first, we employ an experiment using a linear velocity profile. The tangential velocity of the initial eddy is defined as

$$v_\theta = A_0 \frac{r}{R} \quad \text{for} \quad r \leq R, \quad (7.1)$$

where $A_0 = 0.5 \text{ m/s}$ is the magnitude and $R = 50 \text{ km}$. Fig. 7.4a displays the initial condition of the cyclone with the linear profile. The run is completed using variable Coriolis frequency. The initial cyclone propagates northward due to $f$ changes with latitude (i.e. $\beta$–effect). At time = 10 days, the initial eddy has generated wavenumber-three, $k_3$, instabilities (Fig. 7.4b). Three anticyclones formed due to these instabilities. The initial cyclone gets smaller and its azimuthal speed decreases with time since three anticyclones extract kinetic energy from the initial cyclone (Fig. 7.4c). Forty five days later, the initial
cyclone has completely disappeared (not shown here). We also conduct different experiments for cyclone/anticyclone using linear profiles on the $f-$plane or variable Coriolis frequency (not shown here). All of them indicate that a linear velocity profile induces $k^3$ instabilities and the initial eddies do not last more than 45 days.

Next, we employ an isolated (or shielded) cyclone for the initial velocity structure. A necessary and sufficient condition for an isolated vortex is that the area integral of the relative vorticity ($\zeta = \partial_x v - \partial_y u = \nabla^2 \psi$) vanishes at all depths (Morel and McWilliams, 1997). Thus, the vortex has zero total circulation in all horizontal planes. We choose a barotropic eddy (i.e. vortex structure is independent of depth) with a tangential velocity as

$$v_0 = A_0 \frac{r}{R} e^{-\frac{r^2}{R^2}}$$

(7.2)

to satisfy this necessary condition. The magnitude $A_0 = 1.1$ m/s and $R = 50$ km are selected in order to provide that the maximum velocity is 0.5 m/s. Figs. 7.5a and 7.6a exhibit the initial conditions of a cyclone and an anticyclone, respectively. Mixed barotropic/baroclinic instabilities might occur if the radial gradient of potential vorticity (PV) changes its sign somewhere in the domain (Rayleigh, 1880). When we plot the potential vorticity of the vortex structure defined above (i.e. derivative of Eq. 7.2), we see that it changes its sign in the domain (not shown in here). This fits the necessity criteria of barotropic/baroclinic instabilities, thus the vortices initialized using Eq. 7.2 might be unstable. In the $f-$plane vortices, however these instabilities are weak and do not lead to breakdown of the initial vortex into smaller vortices. Instead, vortices become more stable due to rearrangement of PV (Figs. 7.5b and 7.6b). The vortices do not propagate because $\beta = 0$ and stay at their initial location for more than 45 days.

For the variable Coriolis case, since $f$ increases in meridional direction there is a northward gradient in background PV. The $\beta-$effect causes the development of a secondary circulation (beta-gyre) due to advection of the background PV. Beta-gyre deforms the initial vortex structure (Figs. 7.7b and 7.8b) and extracts kinetic energy (McWilliams and Flierl,
Figure 7.4  Linear profile cyclone using the variable Coriolis frequency at time=0, 10 and 35 days.
Figure 7.5  Cyclone on the $f-$plane at time=0 day and time=45 days.
Figure 7.6  Anticyclone on the $f-$plane at time=0 day and time=45 days.
Cyclone propagates to the west-northwest (anticyclone to the west-southwest). Although, vortices last more than 45 days in the variable Coriolis frequency plane, their strength is less in comparison to their counterparts in the $f-$plane case. Detailed dynamics such as weakening and instabilities of the isolated vortices are beyond the scope of this dissertation. We are only interested in the life time of a vortex and its pathway. As a result, idealized numerical simulations indicate that isolated (shielded) vortices survive more than 45 days and pathways of vortices depend on the $\beta-$effect and the sign of the maximum vorticity in the center (McWilliams and Flierl, 1979; Morel and McWilliams, 1997). Therefore, we select the shielded vorticity profile (Eq. 7.2) rather than the linear one since eddies are more stable and they last longer in the open ocean away from boundaries.

We perform two additional simulations of a cyclone and an anticyclone using real topography and initial conditions. The isolated velocity profiles have been used in these idealized cases. The eddy center is located at 45.16°E and 11.3°N (Fig. 7.3) with 100 km diameter and maximum azimuthal speed 0.5 m/s (i.e. $A_0 = 1.1$ m/s and $R = 50$ km). The location, magnitude and dimension of this numerical eddy is similar to one of the eddies observed in 2001 REDSOX cruise (see C1 in Fig. 1 of Bower et al. (2002)). The Red Sea overflow was not released in these runs. We are interested in the evolution of the vortices, their interaction with the real topography, and their pathways in the absence of the Red Sea overflow. In addition to this, it is also crucial to assess how long the eddies stay coherent in the presence of topography. Figs. 7.9a and 7.10a display initial positions of the vortices.

The cyclonic eddy was stable at the beginning, then it started to propagate north-west due to reason described in previous section. Twenty five days later, the cyclone reaches the Tadjura Spurs which are sea mountains at the exit of Tadjura Rift (Fig. 7.9b). The Tadjura Spurs act as a barrier, the cyclone cannot propagate northward anymore and starts to get squeezed, and its diameter decreases. The beta-gyre also forms at the east side of the cyclone. Due to interaction with the topography, the vortex is not shielded anymore, and decays rapidly and almost disappears at time = 40 days (Fig. 7.9c).
Figure 7.7  Cyclone on the variable Coriolis plane at time=0 day and time=45 days.
Figure 7.8  Anticyclone on the variable Coriolis plane at time=0 day and time=45 days.
On the other hand, the anticyclone moves to south-west and meets with continental shelf of Somalia at time = 25 days (Fig. 7.10b). It is clearly evident that the anticyclone is more stable compared to the cyclone 25 days later (Fig. 7.10b vs. Fig. 7.9b). The anticyclone continues to propagate southwest and it gets smaller but not exactly diminishes at 40 days (Fig. 7.10c). Since we know that in the open ocean cases, eddies last more than 45 days, we conclude that decreasing size of eddies is purely due to topography.

### 7.3 Results

In this section, we perform a sequence of experiments to understand the interaction between the Red Sea overflow and idealized GOA eddies. Three different experiments are conducted. These are overflow without any eddies (O), overflow with an idealized cyclone (O+C) and overflow with an idealized anticyclone (O+A) as shown in Table 7.1.

<table>
<thead>
<tr>
<th>Case</th>
<th>Overflow</th>
<th>Cyclone</th>
<th>Anticyclone</th>
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<tbody>
<tr>
<td>O</td>
<td>x</td>
<td></td>
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<tr>
<td>O+C</td>
<td>x</td>
<td>x</td>
<td></td>
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<tr>
<td>O+A</td>
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**Table 7.1** Three different experiments with the overflow using real topography

In the case O, the overflow is released in the absence of any eddies in the GOA domain. Initially, horizontal salinity section at 800 m displays two different water properties (Fig. 7.11a); one is saline water inside the Tadjura Rift (station 93) and the other is fresh water inside the GOA (station 182). At time=40 days, the overflow passes through the Tadjura Spurs and a cyclone is formed (Fig. 7.11b) just after the Spurs. Previous studies (Spall and Price, 1998; Ezer, 2006) also show cyclogenesis in the overflow systems. The overflow induces a cyclone due to Potential Vorticity (PV) conservation. The explanation is as follows. Assume at first we have a steady water column ($h_0$) with zero relative vorticity ($\xi_0$), when the overflow is injected into mid-layer depth, the water column thickness decreases. By PV conservation, this has to be balanced by negative vorticity (i.e. anticyclone). How-
Figure 7.9  Horizontal section for salinity and velocity vectors at $z = 800$ m for cyclone without overflow at time=0, 25 and 40 days, respectively.
Figure 7.10  Horizontal section for salinity and velocity vectors at $z = 800$ m for anticyclone without overflow at time=0, 25 and 40 days, respectively.
ever, after a while the overflow and the upper layer water column are coupled to each other and the overflow carries the upper layer into deeper waters ($h_{final} > h_0$). Therefore, a cyclone (positive vorticity) has to be formed. At time = 80 days, the overflow is more evident and attached on the continental shelf (Fig. 7.11c). The center of the cyclone stays around 45.2°E and 11.1°N. Four months later, the cyclone generated by the overflow stays at the same location in the western part of the GOA. The overflow itself becomes an attached current that stays on the shelf. Bower et al. (2002) expected that the Red Sea overflow might be a boundary undercurrent such as the Mediterranean outflow before they discovered the presence of eddies. The case O clearly shows that the Red Sea overflow follows the shelf in the absence of GOA eddies.

In the case O+C, the overflow is released with an idealized cyclone in the GOA (Fig. 7.12a). The cyclone moves north-northwest as shown in the former section. At time = 40 days the initial cyclone has already disappeared and there are two anticyclones and a cyclone formed because of secondary instabilities of the initial idealized cyclone. One of the anticyclones carries high saline water in its core into the western part of the GOA. There are also some salinity filaments advected north around 12.5°N due to the other anticyclone (Fig. 7.12b). This is an important mechanism for the transportation of tracers. The saline water is encircled around an anticyclone and advected up north. Eighty days after the release of the overflow, the picture is quite different. The initial cyclone disappeared before the overflow passed the Tadjura Spurs, thus there is no direct interaction between the initial cyclone and the overflow. However, there are now two new eddies in the GOA, and the main part of the overflow is transported between these eddies (north-south stripe of high saline water between vortices in Fig. 7.12c). The overflow reaches the Somalia shelf and separates into two branches; one goes east and joins into the anticyclone and the other goes west and becomes a shelf-attached current. At time = 120 days the overflow has spread towards the western part of the GOA. The anticyclone diminishes and there is a cyclone around 44.7°E and 11.0°N. It seems that not only PV conservation but also arch-shaped
Figure 7.11  Horizontal section for salinity field and velocity vectors at $z = 800$ m for case O at time (from top to bottom) 0, 40, 80, 120 days.
topography favors the cyclone generation. The overflow becomes attached on the shelf till 46°E and detaches from the boundary close to the east side of the numerical domain. We believe that the cyclonic movement around 46.7°E, 12.0°N is a problem of boundary conditions. That’s why we will exclude that part from our analysis.

The third case O+A consists of the overflow and an idealized anticyclone in the GOA (Fig. 7.13a). Forty days later, the overflow passes through the Tadjura Rift and starts to surround the anticyclone. At this time southwest propagation of the eddy seems to stop because of the overflow (Fig. 7.13b). At time = 80 days filaments of high saline water have almost encircled the anticyclone. The rest of the overflow stays on the arch-shaped shelf and generated a new cyclone (Fig. 7.13c). There is a little transport of the overflow near the shelf to the east. After four months the anticyclone stays still in the GOA. In the previous section, the anticyclone without the overflow was getting smaller and almost diminished. Therefore, not only the eddy affects the Red Sea overflow but also the overflow affects the eddy dramatically especially in this case. We do not see any saline water east of 46°E. Even though some of the overflow filaments have encircled the anticyclone and been advected through it, most of the overflow water is trapped on the west side of the GOA. The anticyclone stays at 45.6°E and acts like a barrier for the overflow water (Fig. 7.13d). The overflow blocking by the anticyclone is very crucial because the same amount of high saline water is released from the north part of the domain at every time step (i.e. the source is constant). Since the overflow cannot go eastward, its pathway has to change. One possible mechanism is that the overflow needs to mix or advect vertically, so that it can go to upper layers. The cyclone generated by the overflow/arch-shaped topography is also evident for the case O+A. It is clear that the overflow generates its own cyclone in all idealized experiments.

To look at the overflow structure in more detail, vertical sections at 46°E are plotted at time=120 days (Fig. 7.14). Vertical section of the case O shows a distinct shelf-attached under current on the south side of the domain (≈ 11°N). This current has two cores; one
Figure 7.12  Horizontal section for salinity field and velocity vectors at $z = 800$ m for case O+C at time (from top to bottom) 0, 40, 80, 120 days.
Figure 7.13  Horizontal section for salinity field and velocity vectors at $z = 800$ m for case O+A at time (from top to bottom) 0, 40, 80, 120 days.
is around 500-550 m and the other is around 850-900 m (Fig. 7.14a). These cores are probably overflow waters coming from the southern and the northern channels. High salinity values (≈ 37 psu) that reaches up to 12°N is a part of the cyclonic eddy generated by the overflow. Fig. 7.14b displays vertical section for the case O+C. Shelf-attached under current can be also seen in this section, however the current is rather thin compared to one in the previous case. The core of the current is equilibrated approximately around 950 m. Eddies in the west side of the GOA transport some of the middle class saline water (≈ 36.4 psu) up to 12.4°N. This saline water is 0.6 psu fresher than the one advected in the case O. Therefore we can say that there is an additional lateral stirring and mixing in the case O+C. As a matter of fact the inside of the GOA is more homogenized in this case (there is much more saline water in Fig. 7.14a). In the case O+A, there is no current on the south side of the domain (Fig. 7.14c). Most of the overflow water is on the north side since they encircled the anticyclone and transported. The rest of the overflow water is dispersed throughout 600 m depth. We know that the main part of the overflow did not pass 46°E, yet. This means that high saline water on the northern side is just a filament not a current related to the overflow.

In Fig. 7.15, 3D views of vorticity and salinity fields for different cases are shown. Figs. 7.15a, c, and e display 37 psu salinity iso-surface and two horizontal sections of vorticity fields at depths 400 and 800 meters. Figs. 7.15c,d display the case O+C. The initial cyclone can be still seen at depth z=400 m (positive vorticity field in upper layer in the Fig. 7.15c), however it has disappeared at depth z=800 m. As we discussed before, at first both upper and lower layer cyclones move northwest but then lower layer eddy gets smaller and reduces its radius through the Tadjura Spurs. The upper layer eddy survives and stays in the southwest of the domain. We know that the upper layer eddy is a part of the initial barotropic cyclone because high saline water surrounds the cyclone but does not get into the core of it (Fig. 7.15d upper layer). Figs. 7.15e,f show the case O+A. At first, the barotropic anticyclone tilts due to topography and moves southwest. During
the same time, the overflow passes through the Tadjura Spurs and pushes the lower part of the anticyclone to the east side of the domain. The 3D vorticity field clearly indicates that upper and lower layer eddies are separated (not shown here) and there are two different baroclinic anticyclones at different depths. In this experiment, both upper and lower layer eddies survive for four months (negative vorticity fields in Figs. 7.15e) and are surrounded by high saline water.

In summary, the basic features of the three different experiments are as follows. The overflow stays attached to the continental shelf and follows the Somali coast in the case O.
Figure 7.15  From top to bottom; O, O+C and O+A. On the left; horizontal relative vorticity [1/s] sections at 400 and 800 m with 37 psu salinity field (red cloud). On the right; horizontal salinity [psu] sections at 400 and 800 m with velocity arrows.
Injection of the Red Sea water into resting GOA water column induces a cyclone formation due to PV conservation. This cyclone can be seen in all the cases. In the O+C experiment, the initial cyclone does not affect the overflow’s path and the eddy moves northwest and get smaller and looses its coherence after it interacts Tadjura Spurs. There were a cyclone and an anticyclone formed just after the initial idealized cyclone diminished. These eddies stir the overflow laterally (there is more saline water in Fig. 7.11d compared to Fig. 7.12d). The most dramatic event is in the O+A case. The anticyclone does not propagate west because of the overflow and the eddy stays on the east side. It acts like a barrier for the overflow, only some filaments surround the anticyclone and are advected further east. The rest of the overflow probably mixes or advects vertically.

7.3.1 Transport and Mixing

It is important to quantify the effect of GOA eddies on the overflow. In this section, we look at transport and mixing of the overflow due to the idealized cyclone and the anticyclone. We want to quantify how much overflow water is leaving the domain and what are the water properties (for instance maximum salinity, depth of maximum salinity and T/S values) for the different cases.

Transport of the overflow is computed at longitude 46°E because this is end of the high resolution grid size and it is also a convenient location to measure the shelf-attached boundary current. We need to use a salinity value to define the overflow water. We choose 36.5 psu as used in other studies to define the Red Sea overflow plume (Peters et al., 2005b; Ilıcak et al., 2008a). Mass transport, \( \int u dz dy \), is computed at 46°E for 120 days (Fig. 7.16). In the base experiment, case O, for the first 70 days the transport is zero since there is no saline water larger than 36.5 psu that passes through 46°E. This means the overflow has not arrived 46°E, yet. After 70 days, the transport starts to increase and in 10 days it reaches a steady value. The mean transport of the Red Sea overflow is around 1 Sv (10^6 m^3/s). This is a reasonable number in comparison to observations. Peters et al. (2005b)
computes the Red Sea overflow transport as 0.7 Sv in the channels. The total volume in the numerical simulation for 120 days (i.e. the area under the transport line) is $3.68 \times 10^{12}$ m$^3$.

In the case O+C, there are small and intermittent transport values around 50 days. These transports are due to filaments advected by the cyclone and anticyclone as we discussed before. After 80 days, the transport starts to increase linearly and reaches a maximum 1.72 Sv, before it drops to 1 Sv. Mean transport of the overflow (between 80 days and end of the simulation) is 0.9 Sv and the total volume is $2.66 \times 10^{12}$ m$^3$. The transport in the last case O+A is different in comparison to the other cases. As we discuss in the previous section, the anticyclone acts as a barrier and does not let the overflow move to the east. Therefore the transport at 46°E is small; it is around 0.15 Sv and the total volume during the entire simulation time is only $0.6 \times 10^{12}$ m$^3$. It is noted that the total volume in the case O+A is 6.1 and 4.4 times lower than the total volume in the case O and O+C, respectively. Fig. 7.16 clearly shows the importance of eddies and how they affect the transport dramatically. Two eddies in the case O+C increase the transport up to 1.72 Sv. On the contrary, anticyclone pushed outside by the overflow decreases transport down to 0.15 Sv.

![Figure 7.16](image)

Figure 7.16  Transport of the Red Sea overflow at 46°E for different cases.

In the last part of this section, we look at temperature-salinity ($T/S$) diagrams for the different cases. Fig. 7.17 displays water properties for different cases at 46°E and time = 120 days. In addition to these, there are also one profile from source of the overflow (43.3°E and 12.9°N) and one profile from the entrance of the Tadjura Spurs (44.5°E and 11.9°N). The last two profiles help us to identify water properties at the releasing point and exit of the Tadjura Rift area. Maximum salinity is 40 psu at the starting point of the
overflow (the source in Fig. 7.17). After the overflow is released, entrainment takes place throughout the southern and the northern channels. Water properties at the entrance of the Tadjura Spurs are as follows. Maximum salinity becomes 38.9 psu due to vertical mixing. In the O case, two cores of the overflow can be distinguished. The maximum salinity value is around 37.2 psu in this case. The overflow mixes laterally because of the multiple eddies in the O+C case, thus the maximum salinity value is below 37.0 psu. Maximum salinity value in the O+A is lower in comparison to that in the O case. The overflow is also laterally transported by an eddy in the O+A case just like in the O+C case. Therefore, we conclude that interaction of the overflow with two eddies induces diapycnal mixing.

![Figure 7.17](image_url)  

Figure 7.17  

*T/S diagrams at 46°E for cases O, O+C, O+A, source of the overflow and Tadjura Spurs*
7.3.2 Realistic Eddies and Overflow in Real Topography

Finally, we consider a realistic simulation to look at the interaction between the Red Sea overflow and multiple GOA eddies. An ocean general circulation model is required to use as an outer nest to provide synoptic mesoscale boundary conditions. The 1/12° Hybrid Coordinate Ocean Model (HYCOM) is employed since there is a global HYCOM simulation available between November 2003 to February 2009. Some information about the global HYCOM are as follows. The simulation is being performed using a Mercator grid between 78°S and 47°N. A bipolar patch is used for regions north of 47°N. There are 4500 x 3298 x 32 grid points in x, y and z, respectively. Navy Operational Global Atmospheric Prediction System (NOGAPS) which includes wind stress, wind speed, heat flux, precipitation is used as surface forcing. This global run also uses the Navy Coupled Ocean Data Assimilation (NCODA) system (Cummings, 2005) for data assimilation. Although HYCOM with NCODA data assimilation provides us realistic surface and deep circulations of the Indian Ocean, it does not include Red Sea overflow because of the relatively coarse resolution. Besides, the global HYCOM model does not contain the channels at the north of the Tadjura Rift.

One year data of salinity, temperature and horizontal velocities is downloaded from http://hycom.rsmas.miami.edu/dataserver/. The data is employed to provide initial and boundary conditions for the high resolution regional model (Fig. 7.18). The numerical simulation is integrated for 345 days (from January 1st 2007 to December 15th 2007). Since this is a whole year simulation, the seasonality of the overflow needs to be taken into account. Matt and Johns (2007) reported that the Red Sea overflow shuts down for three months in the summer. Therefore, the relaxation of the overflow water has been stopped between 165 and 260 days (i.e. between June 15 and September 20). We employ 50 km of the east part of the domain as a buffer zone (between 47.5°E and 48°E). Barotropic and baroclinic velocities, temperature and salinity fields of the global model are relaxed to the regional model in this area.
There are two cyclones and an anticyclone at $z = 700$ m at the initial time. It is evident that there is no Red Sea overflow water in the HYCOM simulation (Fig. 7.18). We define three sections to compute the transport of the overflow. There are two coast to coast sections, S1 and S2, and a cross section, S3 between S1 and S2. These sections are chosen based on pathways of the overflow in previous simulations. S1 and S2 are also separated into two parts. First parts of these sections start from Somali coast and end around 80 km offshore (gray lines in S1 and S2) and the second parts are the rest of the S1 and S2 (black lines). Transports of the overflow across the gray lines give us an idea about how much overflow water is carried attached to boundary. Transports across the black lines represent the interior transport of the overflow. The goal of the section S3 (magenta line) is to understand the transport of the overflow water between/through GOA eddies. Snapshots of the salinity field at time = 15, 45, 75, 105, 135, 165, 195, 225, 255, 285, 315 and 345 days are shown in the Fig. 7.19. On January 15, the overflow just passes the Tadjura Rift and

**Figure 7.18** Initial (January 1st 2007) salinity and velocity fields from global HYCOM simulation at depth=700 m. Three sections across the outflow (S1, S2 and S3) are also used to compute the transport.
a basin wide cyclone dominates the interior (Fig. 7.19a). After 45 days, the cyclone continues to propagate westward and an anticyclone forms at the east side of the domain (Fig. 7.19b). The overflow becomes a shelf-attached undercurrent and reaches till 46°E, then it separates from the boundary and flows into the interior. It is clearly seen that the overflow is advected between the cyclone and the anticyclone. On March 15 (time = 75 days), there is a shelf-attached boundary current on the south, and there is also some overflow water on the north boundary (Fig. 7.19c). The overflow is transported around the northern rim of the anticyclone and reaches to the boundary. On the 15th of April, some of the overflow water reaches the east side of the boundary (Fig. 7.19d). A branch of overflow water separates from the boundary current at 45.8°E and moves north. An anticyclone forms on the west side and its center is located at 44.6°E and 11°N. Entrance of a strong cyclone into the domain can be seen at the east side. At time = 135, the boundary current becomes very thin after 45.8°E. A part of the overflow is transported north because of the anticyclone in the previous figure. Even more, a filament of the overflow reaches up to 12.5°N (Fig. 7.19e). There is a small cyclone in the overflow around the arch-shaped boundary. The big cyclone on the east moves to the west-northwest due to the sign of its relative vorticity. When the cyclone reaches the middle of the domain, the overflow surrounds the southern rim of the big cyclone and it is transported by the eddy. A trace of the overflow water can be seen around 47.4°E, 12.9°N(Fig. 7.19f). The boundary undercurrent is evident again at the last snapshot of the first six months. The source of the overflow is shut down at this time.

On July 15, the cyclone moves further west and its radius is reduced (Fig. 7.19g). An anticyclone has been formed at the eastern boundary. In addition, a small cyclone located 46.4°E, 11.5°N carries the overflow to the east. A patch of overflow is present around 46.4°E, 12.9°N. It is noted that merging and diminishing of eddies are frequently observed dynamics in the GOA. At time = 225 days, the cyclone strengthens and transports the overflow water at the southern boundary (Fig. 7.19h). The shelf-attached undercurrent gets really thicker, thus transport gets higher. It is clearly seen that there is a strong lateral
Figure 7.19  700 m horizontal sections for salinity field and velocity vectors at time = 15, 45, 75, 105, 135, 165, 195, 225, 255, 285, 315, 345 days.
mixing in the GOA since the mean salinity value at $z = 700$ m becomes 0.35 psu higher. At the initial time (Fig. 7.19a) the mean salinity is around 35.35 psu. In Fig. 7.19h, however the mean salinity is around 35.7 psu. Since the overflow is shut down, there is no high saline water at the exit of the northern channel. There is a 20-30 days delay between shutting down the overflow and its effect in the GOA. The Tadjura Rift water is less saline in comparison to the previous times on the 15th of September. It is clearly evident that there is almost no overflow water (Fig. 7.19i). The undercurrent has almost disappeared. A new cyclone coming from outside covers the eastern side of the domain.

Since the overflow is released back on the 20th of September, it can be seen in the GOA at time = 285 days (Fig. 7.19j). It follows the continental shelf to become an undercurrent. The cyclone moves west and a new anticyclone forms on the east side of the domain. This new anticyclone gets bigger as it propagates west. On the 15th of November, there are four eddies with various sizes located in the middle of the domain (Fig. 7.19k). Two of them (eddies at north) are the cyclone and the anticyclone from the previous figure. The other two are a relatively small anticyclone and a small cyclone below the two large eddies. At the end of the year (15 of December) these four eddies stir and transport the overflow to the middle of the domain (Fig. 7.19l).

In brief, the overflow is mostly transported along the southern boundary throughout the whole year. Some of the overflow water surround GOA eddies and are advected into the interior of the GOA. However, it is clear that the favorable pathway of the overflow is as a shelf-attached boundary current. Besides, all the idealized cases in the previous section indicate the same thing. Eddies in the gulf are merging and diminishing continuously. This behavior is similar to what happened in the idealized case $O+C$; the initial cyclone dies, then two new eddies form. We can conclude that interaction with multiple vortices with the overflow also induce lateral mixing. In all idealized cases, the overflow is advected around rims of the eddies. It does not penetrate into the core of eddies. However, it is more complex in the real GOA simulation because of multiple eddies. The transport of the
overflow is episodic and depends on strength and position of GOA eddies.

The transport of the Red Sea overflow through different sections is plotted in Fig. 7.20. 
Section I, S1, is the western section and it is separated into two components. S1 south (gray line) represents the transport of the overflow in the shelf-attached boundary current. In the first 28 days, S1 south is zero since the overflow has not arrived, yet. It then starts to increase and the mean transport value is 1.05 Sv. Maximum transport is around 6 Sv when the boundary current is also pushed by a stronger cyclone in the domain (Fig. 7.19h). The overflow is shut down between 165 and 260 days, thus the transport decreases after 240 days and it becomes zero after 270 days. After that day, the south transport increases (blue line in 7.20a) since GOA eddies carries the overflow water (Fig. 7.19l). Total volume of the overflow through the boundary undercurrent is around $28.1 \times 10^{12}$ m$^3$ at S1, while in the northern part, where eddies are dominant, it is around $7.9 \times 10^{12}$ m$^3$. Thus, the total volume at the Section I is $36 \times 10^{12}$ m$^3$. Section II, S2, is the eastern section and it is also separated into two components. Maximum transport at the south part of S2 is around 8.2 Sv which indicates entrainment in the overflow from S1 to S2. However total overflow volume at the shelf-attached current drops to $16.14 \times 10^{12}$ m$^3$ while the total overflow volume through GOA eddies increases to $11.25 \times 10^{12}$ m$^3$. We conclude that at the western part of the GOA the overflow is mostly carried by the shelf-attached current (%78 of the total volume), then it decreases to %58.9 at the eastern part of the GOA. The volume of the overflow through GOA eddies increases from %22 of the total volume at the S1 to %41.1 of the total volume at the S2. Total volume at the Section III, S3, is around $6.23 \times 10^{12}$ m$^3$. This means that there is a net transport of the overflow from boundary current to the interior of the GOA domain.

The transport of the overflow between 44.5°E and 47.5°E is also depicted in the form of Hovmöller diagram in Fig 7.21. There are two strong events that increase the transport: one is around 160 days (i.e. beginning of summer) and the other is around 240 days (i.e. beginning of fall). The transport of the overflow goes up to 3 Sv and 6 Sv, respectively.
Fig. 7.19 continued.
in these events. These events can be also seen in Fig. 7.20. The first event is probably due to the basin size GOA cyclone in Fig 7.19f. The cyclone pushes the overflow, thus the transport increases. Around 190 days, transport of the overflow is negative at the east of 46.5°E due to anticyclone entered the domain (Fig. 7.19g). Since there is a strong cyclone in the GOA (Fig. 7.19g) around 240 days, the overflow entrains and it transport increases up to $6 - 7$ Sv. This is a very large amount of transport for the Red Sea overflow. It is known that the mean overflow transport is around 1 Sv (Fig. 7.16). A volume budget is computed to investigate the situation. The total volume transport for the first 220 days is around $6.2 \times 10^{12}$ m$^3$ at 47.5°E. However, the amount of the overflow sunked is around $14.2 \times 10^{12}$ m$^3$ (1 [Sv] $\times$ 165 [days] $\times$ 86400 [s], note that the volume is computed for only 165 days since the overflow is shut down between 165 days and 220 days) which means that only a small portion of the overflow has left the domain. However, the total volume
between 220 days and 270 days is approximately \(12.1 \times 10^{12} \text{ m}^3\) at 47.5°E. It is evident that the overflow water leaves the GOA in less than two months. After 270 days, the transport goes to zero since the overflow is shut down. The transport of the overflow can be negative or positive at the end of the domain (47.5°E). This is crucial since it shows the importance of GOA eddies and their effect on the overflow. It is also clear that the Red Sea overflow is not a steady current that leaves the GOA at a steady flow rate. On the contrary, it exits the Gulf of Aden like patches of Red Sea water due to presence of eddies and this happens in less than 50 days.

![Figure 7.21](image.png)

**Figure 7.21** Hovmoller diagram of the overflow transport (Sv) in time.
7.4 Summary and discussion

The Red Sea overflow is traced as far south as the Agulhas Current. However, the water properties of the overflow are set by the small scale mixing in the channels north of Tadjura Rift and Gulf of Aden eddies coming from the Indian Ocean.

Analysis of the pathways, the transport and the mixing of the Red Sea overflow yields the following conclusions. In the base experiment, the case O, the overflow is released without any eddies. The overflow follows the southern boundary and becomes a shelf-attached boundary current. The overflow induces a cyclone due to the potential vorticity constraint. The O+C case consists of the overflow with an idealized cyclone. The initial cyclone moves to north and disappears within 40 days. Two eddies forms after the initial cyclone dies, they carry the some of the overflow water to the north. Filaments of the overflow surround the rim of the eddies and do not penetrate into the core of the vortices. The favorable pathway is still as a boundary undercurrent. The last idealized experiment, O+A, initializes the overflow with an idealized anticyclone. The overflow pushes the anticyclone towards the east of GOA. The eddy acts as a blockage and does not let the overflow flow to eastward. In this case, the overflow water is mostly transported by the advection due to the anticyclone rather than shelf-attached boundary current. In all three idealized experiments, the overflow generates a cyclone due to PV conservation.

We also look at T/S properties of the overflow for different cases. Maximum salinity value in the O case is around 37.2 psu. In the O+C case, the presence of the initial cyclone and two other eddies stir the overflow which reduces the maximum salinity to below 37.0 psu. In the O+A case, the anticyclone barrier forces the overflow to mix laterally. Thus, maximum salinity value in the O+A is lower in comparison to that in the O case.

For the last part, we employ HYCOM as a nesting model to reproduce multiple GOA eddies. HYCOM provides us initial and boundary conditions. The model is integrated 345 days and the overflow is shut down in the summer due to seasonality of the Red Sea outflow. It is found that even though there are some similarities between idealized and
realistic simulations, the idealized experiments with single eddies cannot provide detailed information of the realistic simulation with multiple GOA eddies. The overflow is mainly transported by the boundary undercurrent at the western side of the domain. However, GOA eddies play more important roles when the overflow propagates into interior of the GOA. At the eastern side of the domain, %60 of the overflow is transported by the shelf-attached current and the rest is transported by GOA eddies. The transport of the overflow is episodic depending on the strength and location of GOA eddies. The most crucial finding is that the Red Sea overflow leaves the Gulf of Aden in patches rather than one steady current. Multiple GOA eddies induce lateral stirring, thus diapycnal mixing of the Red Sea outflow.
Chapter 8

Conclusions

Deep and intermediate water formations are crucial for the large scale ocean circulation and therefore in Earth’s climate. These dense waters originate in marginal seas and form outflows that flow out from narrow channels/straits and settle in the open ocean at levels determined by entrainment. The representation of overflows in general ocean circulation models is a challenge due to small scale mixing in time and space. The primary objective of the present work is to investigate the development and applications of two-equation turbulence closures for mixing in overflows. Three different overflows have been investigated; the Red Sea, the Mediterranean and the Bosphorus outflows. We look at the performance of second-order turbulence closures in both non-hydrostatic and hydrostatic models. A 2D non-hydrostatic model is developed to simulate the Red Sea overflow in the northern channel. Numerical experiments with Very Large Eddy Simulation (VLES) and Reynolds Average Navier Stokes (RANS) closures have been conducted in Chapter 3. The 2D non-hydrostatic model is improved to simulate not only vertical but also lateral constrictions in Chapter 4. VLES is employed to reproduce the Bosphorus strait overflow in the same chapter. Then we move to investigate the performance of two-equation turbulence closures in a 3D hydrostatic model. Four different closures are employed in 3D numerical simulations of the Red Sea overflow in Chapter 5. The Mediterranean overflow is also studied
using two-equation turbulence closures in 3D hydrostatic model. Finally, we investigate the
The principal results are reviewed in this concluding section, and their significance both to
ocean modeling and to the understanding of outflow in large-scale ocean is discussed.

The first aim in this dissertation is to investigate the two-equation turbulence closures in
a non-hydrostatic model. To this end, a 2D Boussinesq model is developed and employed
to simulate the Red Sea overflow in the northern channel. It is found that the experiments
without SGS models cannot reproduce the basic structure of the northern channel overflow,
because of excessive mixing throughout the overflow. As such, turbulence closures appear
to be a necessity. The \( k - \varepsilon \) model yields unrealistically thick boundary layer (BL), which
transports salty, dense water along the channels, and the interfacial layer (IL), where most
of the mixing and entrainment takes place. It is found that VLES is more suitable than
standard RANS in simulating the mixing of overflow using a non-hydrostatic model. Fur-
thermore, the non-hydrostatic model is improved by introducing laterally average terms,
so the model can simulate the constrictions not only in the \( z \)--direction but also in the
\( y \)--direction. Observational data from the Bosphorus Strait is employed to test the spatially
average 2D non-hydrostatic model (SAM) in a realistic application. The simulations from
SAM with a simple Smagorinsky type closure appear to be excessively diffusive and noisy.
We show that SAM can benefit significantly from VLES turbulence closures. SAM with the
VLES turbulence closure might be an alternative to simulate the exchange flows. However,
it has to be mentioned that simulations of exchange flows in narrow straits remains a signif-
ificant computational challenges due to the details of domain geometry and their impact on
mixing and hydraulic effects. Some of the geometries such as twisting straits cannot even
be simulated using 2D models.

The second goal is to extensively test the performance of different second-order tur-
bulence closures in a hydrostatic model. The Regional Ocean Modeling System (ROMS)
is employed to simulate the 3D dynamics of the Red Sea overflow since ROMS has the
generic length scale scheme which allows us to easily test different two-equation turbulence closures. Four different two-equation turbulence closures are selected for the comparison of 3D numerical simulations of the Red Sea overflow. Three of them, $k - \varepsilon$, $k - \omega$, and $MY2.5$, are the turbulence closures with complex stability functions representing the effects of buoyancy and shear. The fourth scheme is the modified $k - \varepsilon$ turbulence closure of Peters and Baumert (2007).

All two-equation turbulence models are able to capture the vertical structure of the Red Sea overflow consisting of the BL and IL. Mean eddy diffusivities in most closures are $O(10^{-2})$ in the NC and $O(10^{-3})$ in the SC. These values compare well with observational data. Eddy diffusivities are too small in KPP and too large in MY2.5 with G88. As a consequence of the large $K_H$, the BL salinity becomes diluted in MY2.5 with G88. KPP and MY2.5 with G88 produce the largest salinity deviations from the observations along two sections across the overflow and the modified $k - \varepsilon$ exhibits the smallest deviations. The rest of the closures fall in between, showing deviations similar to one another. MY2.5KC displays better performance than the MY2.5 with G88. The main reason behind this is that the version with KC stability functions has a $Ri_{st} = 0.21$, whereas the one with G88 does not have a stationary Richardson number.

The transport of different salinity classes in the overflow is also computed and compared to the observations. Only $k - \varepsilon$ models transport the densest salinity class in the NC and only the modified $k - \varepsilon$ transport the densest salinity class in the SC. KPP and $k - \omega$ models have correct salinity classes but weak transports compared to observations. However, the control experiment and MY2.5 with G88 transport intermediate salinity classes in the NC and SC.

It has to be mentioned in here that other factors, such as high resolution around the channels, ambient stratification and/or correct forcing, are as important as the turbulence closures. Thus, the Red Sea overflow might not be a highly discriminating case study to evaluate the performances of the different closures. Another fundamental problem is
using small, sparse, and non-synoptic observational data sets to compare to the numerical simulations. Thus, we move to investigate the performance of the turbulence closures in another overflow. The Mediterranean outflow is chosen since it is one of the best-observed.

Four different closures ($k - \varepsilon$, $k - \omega$, $MY2.5KC$ and $KPP$) are also employed to simulate the Mediterranean outflow. The numerical results are compared with observational data obtained in the 1988 Gulf of Cádiz Expedition. The simulations with two-equation closures reproduce the observed properties of the overflow quite well, especially the evolution of temperature and salinity profiles. All the simulations capture the spreading of the overflow as it descends along the continental shelf and the transition of the overflow from a bottom-trapped gravity current to a wall-bounded undercurrent. In the $KPP$ simulation, the overflow does not entrain in comparison to that in the two-equation cases. The vertically integrated turbulent salt flux displays that the overflow undergoes significant mixing outside the west edge of the Strait of Gibraltar. The volume transport and water properties of the outflow are modified significantly in the first 50 km after the overflow exits the strait. The $k - \varepsilon$ and $k - \omega$ cases show the best agreement with observations.

Former overflow studies describe that the small scale mixing is crucial to modify the product water properties. However, in the Red Sea overflow case Gulf of Aden Eddies are also important to set the properties of product water that leaves the gulf. We employ a high resolution 3D regional model and look at the interaction between the Red Sea overflow and GOA eddies. To isolate the effect of eddies, at first we only consider the overflow without any eddies in the GOA. After that, an idealized shielded cyclone or anticyclone is initialized with the overflow in two cases. Our focus is on the pathway of the overflow in the presence of an idealized eddy. We then move to more realistic simulation using a global ocean circulation model for the forcing of multiple eddies in the GOA. In the idealized cases, the overflow follows the southern boundary and becomes a shelf-attached boundary current. The overflow induces a cyclone due to the potential vorticity constraint. There are some similarities between idealized and realistic simulations, however the idealized experiments
with single eddies cannot provide realistic information of the realistic simulation with multiple GOA eddies. The overflow is mainly transported by the boundary undercurrent at the western side of the domain in the realistic simulation. However, GOA eddies play more important roles when the overflow propagates into interior of the GOA. In the eastern side of the domain, 60% of the overflow is transported by the shelf-attached current and the rest is transport by GOA eddies. The transport of the overflow is episodic depending on the strength and location of GOA eddies. The most crucial finding is that the Red Sea overflow leaves the Gulf of Aden in patches rather than as a steady current. Multiple GOA eddies induce lateral stirring, thus enhance mixing of the Red Sea outflow.

A general conclusion arising from all these idealized and realistic simulations is that second-order turbulence closures can simulate the mixing in the observed outflow plumes given sufficient resolution, initial condition and forcing. However it is also found that the turbulence closure alone does not play a decisive role for overflow simulations. Resolving the topography plays an important role in simulating overflows since they are bottom-trapped currents that flow across isobaths. The Red Sea outflow simulation is a perfect example because of the channels that separate the overflow into two branches. Initial conditions of the interior ocean is also crucial to capture the right amount of mixing. Xu et al. (2007) showed that variations of the ambient ocean water properties have a greater impact on the outflow product water properties than variations of the overflow source water. In both the Red Sea and the Mediterranean Sea overflows, the mixing and stirring processes that take place after exiting the straits set the properties of the overflow waters. Thus, the interior dynamics in the Gulf of Aden and the Gulf of Cádiz play a crucial role before the overflows spread into the open ocean. It is found that the Red Sea overflow water properties are significantly modified by Gulf of Aden eddies. Despite all these factors, this study is a needed step to pursue the performances of second-order turbulence models for overflow simulations.
Bibliography


