Advanced Bathymetry Retrieval from Swell Patterns in High-Resolution SAR Images

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ADVANCED BATHYMETRY RETRIEVAL FROM SWELL PATTERNS IN HIGH-RESOLUTION SAR IMAGES

By

Fernando José M. Monteiro

A THESIS

Submitted to the Faculty of the University of Miami in partial fulfillment of the requirements for the degree of Master of Science

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ADVANCED BATHYMETRY RETRIEVAL FROM SWELL PATTERNS IN HIGH-RESOLUTION SAR IMAGES

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We present an enhanced technique to retrieve underwater topography in coastal areas from swell refraction patterns in high-resolution spaceborne synthetic aperture radar (SAR) images. From a SAR scene with swell patterns propagating shoreward, we compute the peak wavelength and wave direction of each sub-scene; we then obtain depth estimates via the linear dispersion relationship, as long as the wave period is known. Compared to previously presented depth retrieval methods based on the same principles, the innovation of our technique consists of implementing a modulation transfer function (MTF) to account for the dependence of the strength of radar signatures of ocean waves on their wavenumber and propagation direction relative to the radar look direction. We compare the SAR-derived depth maps obtained with and without the MTF-based approach and our results indicate that there is an improved accuracy in the former, statistically quantified by a higher correlation coefficient ($r^2=0.85$ with the MTF-based approach, as opposed to $r^2=0.77$ without it) and a lower RMSE (6.37 m with the MTF-based approach, as opposed to 8.74 m without it) with respect to the reference depths.
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CHAPTER 1

INTRODUCTION

Reliable knowledge of coastal bathymetry is relevant for a number of applications. Maritime trade, commercial fishing, inshore exploitation of natural resources, and recreational boating are some of the activities which depend, at different levels of accuracy, on underwater topography information. Militarily speaking, coastal bathymetry can be critical at tactical and operational levels: upon entrance into a harbor or a bay, during submarine operations, in an amphibious landing, among other maneuvers.

Coastal hydrographic survey is a costly project. It requires specialized and well-trained personnel to manage dedicated ships and equipments, such as mono- or multibeam echo sounders, side scan sonars, Differential Global Positioning System (DGPS) receivers, and motion sensors. In very shallow waters, survey boats equipped with portable devices must be employed. Also, weather and man-made events, such as storms and dredging operations, respectively, can make regular chart update hydrographic operations necessary: The more dynamic the events are, the more frequent the updates have to be scheduled.

Hydrographic work is also weather-sensitive: If carried out under rough sea states, the quality of data acquisition may be affected such that it will not meet the accuracy standards established by the International Hydrographic Organization (IHO).

On the other hand, bathymetry retrieval on the basis of remote sensing techniques brings the advantage of a wide coverage of the ocean almost instantaneously, at a relatively low cost. As the title of this thesis implies, we will focus only on active satellite
remote-sensing based methods, which, besides providing an operational / tactical benefit, due to the potential availability of satellite images from coastal regions worldwide, are fairly weather-insensitive: Clouds and rain are effectively transparent to radar waves and imaging does not depend on sunlight.

Two main kinds of bathymetry retrieval techniques derived from Synthetic Aperture Radar (SAR) imagery have been developed based on the imaging mechanisms proposed for the radar signatures related to changes in water depth.

The first kind is based on the interaction between strong tidal currents and the local bottom topography, leading to surface roughness variations. Such methods yield relative variations in water depth and their use is limited to tidal channels with simple bathymetry; moreover, satellite overpasses must happen during favorable tidal phase and wind conditions so that the current field is strong enough to be modulated by the seafloor and the resulting wave-current interaction leads to visible radar signatures.

The second kind of bathymetry retrieval techniques relies on refraction of long surface gravity waves as they propagate shoreward. Since a direct relation between swell patterns and water depth can be established, absolute water depth retrievals are more straightforward with this method than with the above-mentioned one. Another advantage of depth retrievals from wave refraction patterns is that radar image acquisitions can occur at any time, as long as swell patterns are present. Our work will aim at this latter kind.

Pleskachevsky and Lehner (2011) presented a bathymetry estimation technique based on radar observations of refraction patterns; however, it oversimplified the wave imaging mechanism. The purpose of this work is to assess the potential of improving the
technique of bathymetry retrievals from wave refraction patterns in SAR images by accounting for the physics of the wave imaging mechanism more adequately. We compare our results with and without this novel approach to quantitatively assess the improvement in accuracy of the depth maps.

Due to potentially strong nonlinearities of the wave imaging mechanism under certain circumstances, we also discuss limitations to the use of this technique. Another constraint on our method is that it is inherently unable to provide a density of measurements high enough to replace a conventional hydrographic survey, as will be discussed in detail thereinafter. Even so, our method can be employed as a powerful tool to evaluate the need for hydrographic update campaigns. If we take the example of Brazil, with a coastline of about 7,500 km and limited resources allocated to survey activities, developing a method capable of setting priorities to which regions need update on their nautical charts leads to a more efficient use of the means at disposal. From a military perspective, this technique can provide preliminary bathymetric knowledge necessary to plan naval operations in strategic, sensitive areas where cartographic information may be denied.
CHAPTER 2

BACKGROUND

2.1 RADAR SIGNATURES RELATED TO CHANGES IN WATER DEPTH

Launched by the National Aeronautics and Space Administration (NASA) in 1978, Seasat represented a major breakthrough for satellite oceanography because it provided the first high-resolution synthetic aperture radar (SAR) images of the Earth’s surface from earth orbit. One of the capabilities in SAR imagery demonstrated with Seasat was the detection of seafloor expressions at the ocean surface (e.g., Shuchman and Kasischke 1979; Fu and Holt 1982; Lodge 1983a, b; Kenyon 1983). Although the same characteristic had already been shown in real-aperture side-looking airborne radar (SLAR) missions prior to Seasat (De Loor and Brunsveld van Hulten 1978; De Loor 1981), what was remarkable was that it was the first time a wealth of images of the oceans had been made available on a synoptic scale (Born et al. 1979).

Radar features associated with underwater topography may be related to tidal currents in shallow waters, wave refraction, or internal waves propagating over sharp gradients in deep waters (e.g., Fu and Holt 1982; Lodge 1983a, b). Since this work is about bathymetry retrieval in coastal waters, only the first two phenomena will be of interest; they relate changes in radar backscattered power to water depth variations in distinct ways.

At incidence angles between 20° and 70°, covering thus typical values of spaceborne SAR, Bragg scattering is the dominant process for returned power to the radar from the sea surface. This phenomenon can be explained as a constructive interference of
the electromagnetic waves when a given geometry involving the sinusoidal sea surface, the radar waves and the incidence angle is satisfied.

![Figure 1. Diagram of the Bragg resonance geometry (Figure from Ulaby et al. 1982).](image)

In the figure above, the emitted electromagnetic radar wave at an incidence angle of $\theta$ impinges on a sinusoidal sea surface. When the excess distance from the radar to each successive sea surface wave crest ($\Delta R$) equals half the radar wavelength, the contributions from all backscattered signals arrive at the antenna in phase, leading to constructive interference. Only waves with one particular wavelength $L$, propagating in a direction parallel to the radar look direction, satisfy this condition. In terms of incidence angle, sea surface wavelength and radar wavelength, the Bragg condition can be expressed as:

$$L = \frac{\lambda}{2\sin\theta} \quad (1)$$

where $\lambda$ is the radar wavelength.

Therefore, waves traveling in the cross-track (range) direction towards the radar or away from it that satisfy the Bragg geometry are the ones that will effectively
contribute to radar backscattering; they are selected by the Bragg mechanism to represent
the measure of the local surface roughness. These waves are called the Bragg waves. To
first order, the backscattered power is proportional to the intensity (squared amplitude) of
the Bragg waves (Wright 1968; Valenzuela 1978).

Equation (1) implies that the Bragg waves are comparable in size with the radar
wavelength: In other words, the sea surface waves relevant for radar backscattering are
small gravity waves and capillary-gravity waves.

Prior to the Seasat mission, De Loor (1981) had already observed radar signatures
associated with the underwater bottom topography in shallow water regions when
moderate winds and strong tidal currents were present. Nevertheless, the imaging
mechanism was not understood and thus the description of such effects was qualitative.

Alpers and Hennings (1984) proposed a first-order theory of such imaging
mechanism as a three-step process:

i) The bottom topography induces spatial variations in the surface tidal current as
a consequence of the depth-averaged continuity equation.

ii) Changes in the surface tidal current (i.e., a horizontal current gradient) interact
with the short ripple waves, causing a spatial modulation of their energy spectra
(hydrodynamic wave-current interaction).

iii) Due to Bragg scattering, variations in the short-scale waves energy cause
corresponding spatial variations in the backscattered signal and thus in the radar image
intensity.

The Figure below depicts the three steps previously described:
Another physical process observed in radar images in coastal areas and connected to changes in local bathymetry is wave refraction. It occurs due to the interaction between the surface gravity waves and the shoaling topography over which it propagates. Under these circumstances, waves go through changes in their wavelength and wave direction (if they are not propagating perpendicularly to the isobaths), and the connection between the former and the local depth can be established via the linear dispersion relationship.

Microwave imaging radars have been extensively used to measure long ocean surface waves over the few decades, and they have the potential of gauging their key properties, such as the peak wavelength and wave direction (Alpers et al. 1981). Therefore, their capability of capturing them and their alterations as they propagate
shoreward can be used as an indirect way of estimating local underwater topography.

The linear dispersion relationship – derived from the linear wave theory – reads as follows:

\[ \omega^2 = g k \tanh (k h) \]  

(2)

where \( \omega \) is the intrinsic angular frequency (\( \omega = 2\pi / T \), where \( T \) is the wave period), \( g \) is the acceleration of gravity, \( k \) is the wavenumber (\( k = 2\pi / \lambda \), \( \lambda \) being the wavelength) and \( h \) is the local depth.

This theory can be considered suitable for analyzing swell patterns, since the latter are decoupled from the local wind field, which implies their behavior to be close to that of free waves. Knowing the wave period from an external source (e.g., a forecast model) along with the wavelength information from the radar image, we are able to estimate the water depth.

In the absence of currents, the intrinsic angular wave frequency as given by (2) is conserved. Theoretically, at the region where the bottom depth reaches half the peak wavelength, refraction effects begin to manifest: The peak wavelength begins to decrease, as we can see in (2), and the peak wave direction tends to bend towards the depth gradient.

In the case of a non-negligible surface current, a correction term, known as the Doppler shift, must be inserted to account for changes in the wave frequency due to a nonuniform background flow:

\[ \omega' = \omega + k \cdot u = \text{constant} \]  

(3)
where \( \omega' \) is the apparent angular frequency, \( \omega \) is the intrinsic angular frequency, defined in (2), and \( k \cdot u \) is the dot product between the local wavenumber and current vectors (Doppler shift term).

A clear distinction between both kinds of depth retrieval techniques must be made: the method based on observation of swell pattern uses resolved patterns in the radar images, while the other, applied in tidal channels, uses the large-scale spatial variations of the sub-resolution-scale (i.e., unresolved) surface roughness.

In sections 2.2 and 2.3, we present the main bathymetry retrieval methods arisen from the analysis of the aforementioned radar features: the Bathymetry Assessment System (BAS) and the one developed by Pleskachevsky and Lehner (2011), respectively. In chapter 3, we present our own technique, based on the same principles as Pleskachevsky and Lehner’s, but with additional physical considerations.

2.2 BATHYMETRY RETRIEVAL FROM SAR IMAGE INTENSITY VARIATIONS

Based on the imaging mechanism of underwater bottom topography in tidal channels proposed by Alpers and Hennings (1984), the BAS was developed by Calkoen et al (2001).

Such method comprises two main parts: the imaging (or forward) model and the data assimilation scheme (or inverse model). The system requires one or more SAR images as well as a limited number of reference depths (usually obtained from conventional echo soundings) as input and was shown to yield reliable depth maps using
tracklines with a spacing of five to ten times the spacing that would be necessary in a conventional hydrographic survey.

There are two reasons why conventional depth measurements must be used in the BAS: Since SAR images show the intensity modulations of the Bragg waves induced by the current gradients, only relative depth variations are detected; therefore, absolute reference depths must come from a different source. Moreover, depth information is also necessary to generate a first-guess depth map to start the data assimilation scheme.

The BAS’ forward model is composed of three modules, each one devised to account for each step of the imaging mechanism previously described: the flow, the wave and the radar backscatter ones. Besides the SAR images and the depth measurements, it also needs information about the tidal regime (water level and tidal flow), the wind velocity and direction to be used in the wave model as well as the radar wavelength, incidence angle, look direction, and polarization as input parameters in the radar backscatter model.

The one-dimensional flow model aims at describing the tidal surface flow in shallow coastal areas, under the assumption that the water column is well mixed. Two types of depth variations are considered under this simplified flow model: Either they are mainly perpendicular or parallel to the flow. Thus, simplified shallow water equations are used to describe the surface current.

The wave model depicts the interactions between the slowly spatially varying surface current and space and time scales of the short Bragg waves by means of the action balance equation (Keller and Wright 1975) in the stationary case. The relaxation-time approximation is employed to describe the source term that makes the Bragg wave
spectrum return to its equilibrium shape in reaction to the distortion induced by the current gradients.

The radar backscatter model makes use of the first-order Bragg scattering to represent the backscatter variations brought about by the hydrodynamic modulation.

After inserting the environmental and radar parameters into the forward model and running its three modules, it eventually produces a simulated SAR image. This image is then used in the inverse model to be compared with the actual observed image through data assimilation techniques. The inversion part of the system analyzes both images using a penalty function that compares the modeled and the observed image, the estimated depths and those from the soundings as well as imposes a constraint on steep slopes to smooth the effects of speckle noise. In the areas where topography is known, model parameters, among which the relaxation rate (the inverse of the relaxation time) is the most relevant, can be optimized to produce the best depth assessment; the model is then run iteratively so as to minimize the deviations of estimated depths with respect to the sea truth. The goal is to make the penalty function come to a minimum, which means producing the most consistent depth map with the available data and imposed constraints.

An important limitation of the BAS is the fact that it uses a one-dimensional flow model. Although this feature enables a tractable description of the underwater topography and the data assimilation scheme, it restricts its application to regions with a quite simple sea bottom. Even if the area under analysis can be divided in sub-areas in which either of the two types of the flow model will be more suitable, there will still be some regions which will fit in with neither of the idealized bathymetric patterns. Moreover, non bottom-related backscatter modulations, such as the ones associated with surface films,
ships and ship wakes, and oceanic and atmospheric fronts, must be identified and removed to avoid misinterpretation as signatures of depth variations by the system. Both aspects raised make human interventions in the model a challenging and quite frequent task, which might increase the likelihood of inaccurate results.

Likewise, the radar backscattering model also uses a simplified parameterization to represent the backscattered power from the sea surface: It accounts for pure Bragg scattering theory, neglecting higher-order contributions due to the presence of longer waves; the composite surface model (Brown 1978; Lyzenga and Bennett 1988; Romeiser and Alpers 1997; Romeiser et al. 1997) is not considered. To compensate for such a simplification, the relaxation rate is tuned, albeit in an artificial way, in order to make the intensity variations of the simulated image approach those of the observed image as if they result from pure Bragg scattering. This unrealistic representation of actual physical processes leads to the need of different tuning for different images.

Other restrictions of the method come from the own physical aspects it takes into account: It can only be applied in coastal tidal channels under the influence of a specific dynamical framework (e.g., strong tidal currents and wind speeds up to 12 m/s; optimally, between 3 and 5 m/s); even in these limited areas, not all satellite overpasses are useful for such technique: only the ones under favorable flow conditions. Besides, the BAS needs a number of conventional depth data points (i.e., an external data source) as input. All these constraints make BAS a challenging operational system.
2.3 BATHYMETRY RETRIEVAL FROM SWELL PATTERNS IN SAR IMAGES

Another approach to underwater topography retrieval can be implemented by observing swell patterns at sea as they travel shoreward. This is made possible due to the fact that SAR images are able to image long waves in the ocean and their wavelength can be connected to the local depth by means of Equation (2).

Due to the physics involved in this approach, some limitations on water depth retrieval can be pointed out, amongst which two will be briefly described: the “upper” and “lower” depth ones.

The “upper” depth limitation can be explained in the case of a wave propagating over deep-water regions, (i.e., for large \( k h \)), in which the value of \( \tanh(kh) \) approaches 1; under such conditions, the wavelength loses its dependency on water depth and no connection between the latter and the former can be made through the linear dispersion relationship: The wave must “feel” the bottom so that such relation holds. In a typical swell, the wave period ranges from 10 to 15 s; making use of the dispersion relationship in its deep-water approximation, wavelengths at those areas span from 150 to 350 m. Due to the fact that waves start feeling the sea bottom at depths on the order of half their wavelengths, we can assume that this method will be effective at water depths under 75 m. Since most continental shelves – the region of interest to such a technique – lie within that depth limit, this should not be considered a strong limitation.

However, this limitation can have a positive effect, in the case of a single wave system dominating the region: In the deep waters covered by the radar image, the wavelength computed can be used to determine the intrinsic frequency and, consequently, the period of the dominant waves, making the approach independent of external wave
period information. If we use the deep-water approximation for the linear dispersion, the intrinsic frequency and period are expressed as, respectively:

\[ \omega = \sqrt{gk} \quad (4) \]

\[ T = \frac{2\pi\lambda}{g} \quad (5) \]

The “lower” depth limitation applies to waves close to the surf zone; in such areas, linear theory no longer holds, since the asymmetric shape of the waves becomes more pronounced, their height increases, and they eventually break. Pleskachevsky and Lehner (2011) claim that, depending on sea state and image resolution, a linear analysis of wave patterns could be made at water depths as low as 10 m.

2.3.1 THE MODULATION TRANSFER FUNCTION

Ocean surface wave imaging by SAR is possible because the long waves modulate the normalized radar backscattering cross section (NRCS) such that wavelike patterns are created. The main modulation mechanisms, which can be described by a linear modulation transfer function (MTF), are the tilt modulation and the hydrodynamic modulation (both known as the cross-section or amplitude modulation), plus the effect of velocity bunching (also known as the phase modulation), as described by Alpers et al. (1981). These modulation mechanisms together are regarded as an adequate representation of the imaging process in low to moderate seas. All three components of the MTF are explained qualitatively in this chapter. In chapter 3, when the MTF array is presented, they will be depicted in a quantitative sense.
The tilt modulation is a geometric effect due to the fact that the Bragg scattering waves are seen at different incidence angles subject to their position with respect to the long surface wave, as we can see in Figure 3:

![Figure 3. Illustration of the tilt modulation (Figure from Robinson 1985).](image)

The backscattered power resulting from Bragg scattering varies with the local incidence angle. Thus, the long-wave induced periodic change of slope produces banded, wavelike patterns on the image. The tilt modulation is stronger for HH polarization than for VV polarization (Wright 1968; Valenzuela 1978), and for long waves propagating in the radar look (range) direction, reaching zero when they are propagating in the azimuth (flight) direction. Maximum positive tilt modulation occurs at the face of the ocean wave that is tilted towards the radar.

The hydrodynamic modulation can be explained as the effect of the nonuniform distribution of the short-scale waves along long waves due to hydrodynamic interactions with the long wave orbital currents. In a similar way to the tidal current approach (section 2.1), the weak hydrodynamic interaction between long and short waves can be described by the modulation of the short waves’ energy spectra in the relaxation-time approximation (Alpers and Hasselmann 1978). The large-scale-waves’ spatially varying
orbital motion creates regions of convergent flow – which pile up the short-scale waves – and divergent flow – able to stretch them, smoothing the surface, also producing a double sign, wavelike intensity pattern on the image. Figure 4 displays the alternating regions of convergent and divergent flow present in the hydrodynamic modulation mechanism:

![Diagram of swell wave direction, surface divergence, and surface convergence.]

Figure 4. Illustration of the hydrodynamic modulation (Figure from Robinson 1985).

It can be noted from the formulation of the hydrodynamic modulation that it is proportional to $\sin^2 \Phi$ ($\Phi$ being the angle between the flight direction and the propagation direction of the long waves – the azimuth angle). Therefore, for long waves propagating in the azimuth direction, the Bragg scattering waves, in the range direction, will not be modulated at all. Furthermore, the maximum of the hydrodynamic modulation is always found on the forward face of a wave, where the surface becomes rougher due to the convergent flow. While the phase of this modulation in the radar image is reversed for waves approaching the radar and propagating away from it, the tilt modulation is the same for both kinds of waves.

Velocity bunching effects are apparent when facets of the scene present varying radial velocity, and is the only imaging mechanism, to a first-order description, that enables the visibility of ocean waves propagating in azimuth direction. As the fine azimuthal resolution in SAR is reached by recording the phase (Doppler) history of the backscattered signal over a finite time (integration time), scattering elements at a varying
radial velocity lead to a nonuniform displacement of them in the image plane. Consequently, a spatially varying density of the backscattered power from such elements will be present in the image. Contingent on the long-wave field parameters as well as the duration of the integration time, velocity-bunching effects can be constructive, generating wavelike patterns, or destructive, leading to a smearing in the image. Figure 5 illustrates a sea surface wave in the azimuth direction yielding a constructive imaging effect:

![Image](image.png)

Figure 5. Illustration of the velocity bunching mechanism (Figure from Robinson 1985).

On low to moderate sea states, the modulations described above can be approximated by linear processes, and, as a result, the concept of a linear MTF, meant to establish the connection between the ocean waves and wave patterns in the SAR image, can be applicable as the sum of the individual modulation components.

In addition to Alpers et al. (1981), there have been several other contributions to the field of ocean wave imaging by SAR: Hasselmann et al. (1985) analyzed the range of models applied to the SAR ocean imaging problem so as to present a more complete view of the SAR imaging phenomenon. In the case of a broad-bandwidth ocean wave field spectrum, the assumption of a linear MTF no longer holds: In order to retrieve the ocean
wave field taking the nonlinearities in the imaging mechanism into account, SAR inversion techniques have been proposed (e.g., Brüning et al. 1990; Hasselmann and Hasselmann 1991; Krogstad et al. 1994; Schulz-Stellenfleth et al. 2005).

Pleskachevsky and Lehner (2011) presented a SAR-based bathymetry retrieval method based on long wave refraction in a coastal area. In order to assess changes in wave patterns, the image was divided into sub-scenes, in which a 2-D Fast Fourier Transform (FFT) was computed. Multiplying the Fourier coefficients found with their complex conjugates yields a power spectrum, from which the peak wavelength and wave direction can be obtained for each FFT box. After comparing sonar depth measurements with SAR-based depth estimates in the study area – Rottnest Island, West Australia – Pleskachevsky and Lehner (2011) claimed that accuracy of the order of 15% was achieved for depths from 20 to 60 m, depending on image resolution, sea state, wave pattern, and bathymetry complexity.

Nevertheless, the developed method assumed a direct relation between ocean wave spectrum and image spectrum: No modulation correction factor was applied to the wave spectra with respect to either their peak direction relative to the satellite flight direction or to their peak wavelength. The aim of the present study is to analyze initially the same SAR image Pleskachevsky and Lehner (2011) worked with and evaluate if a correction by means of an MTF for the computed long-wave parameters can reduce the deviations in terms of depth estimation, when compared with sea-truth data. Such weighting of the different image spectrum components is expected to enhance the robustness and accuracy of the method.

The MTF-based approach – as opposed to a complete SAR inversion technique –
can be justified by the fact that our aim is not to recover a complete 2-D wave spectrum: it is to track the peak of the wave spectrum (i.e., the most prominent wave pattern in the image) for depth retrievals. Therefore, a relatively simple MTF – such as that proposed by Alpers et al. (1981) – to track the peak wavenumber and direction more accurately without making the spectral interpretation a complicated task seems the best procedure to adopt.
CHAPTER 3

DATA AND METHODS

3.1 STUDY AREA

Rottnest Island is located 18 kilometers off the Western Australian coast, near Fremantle. The island is 11.0 kilometers long and 4.5 kilometers wide at its widest point. Its total land area is 19 square kilometers, and it lies within longitudes from 115° 26’44”E to 115° 33’34”E, and latitudes from 32° 01’45”S to 31° 59’12”S.

Geologically speaking, the island in question is in the Southern Australia continental margin, a notably swell-dominated region, due to the extra-tropical storms generated in the Southern Ocean. Such a continental margin is exposed to some of the largest waves of the global ocean (Sterl and Caires 2005), which renders the study area an adequate location for applying the proposed depth retrieval technique.

The image we work with in this thesis comes from TerraSAR-X (TSX), a German Earth-observation satellite, launched in June 2007, whose primary payload consists of an X-band radar sensor that can operate under a variety of modes, depending on the desired swath width and resolution. It is in a sun-synchronous near-polar orbit, at an altitude of 514 kilometers, and the radar emits 9.65-GHz pulses.

Figure 6 shows the TSX image for which we perform the water depth estimation. The image was acquired on October 20, 2009, in Spotlight mode (area size = 11,778 m × 10,440 m, pixel size = 0.75 m × 0.75 m). Due to such high-quality satellite data, we can see a clear swell pattern whose wave trains undergo refraction as they approach the shore.
Figure 6. TerraSAR-X image (Spotlight mode) of Rottnest Island and the adjacent Western Australian continental shelf, acquired on October 20, 2009 (area size = 11,778 m × 10,440 m, pixel size = 0.75 m × 0.75 m). © 2013 German Aerospace Center (DLR).

The bathymetric reference data used in this thesis come from three different sources:

- a LiDAR survey dataset, distributed by the Western Australian Department of Planning and Department of Transport, collected at 5 m × 5 m resolution and acquired between April and May 2009;

- a joint project by two Australian federal agencies, Geoscience Australia and the National Oceans Office, to produce a bathymetric grid at a resolution of 0.0025°
(approximately 235 m in longitude × 280 m in latitude for the area of study), carried out in 2009. This grid is an improved version of its 2005 counterpart and incorporates data from several systems at different levels of density and accuracy (ship-track, swath, and satellite altimetry surveys) such that 90% of the grid cells are guaranteed to yield depths within one cell of their measured position; and

- a compilation of swath-bathymetry surveys courteously carried out by Dr. Sascha Frydman and her team, with the CSIRO (Commonwealth Scientific and Industrial Research Organisation, Australia’s national science agency), upon our request. The processed file is provided on a 0.001° grid (approximately 95 m in longitude × 110 m in latitude for the area of study).

Figure 7 represents a selection from the datasets above described of sea-truth data pertaining to the study area. They were interpolated so as to match the spatial resolution of the SAR-derived depth map (approximately 770 m × 770 m) and displayed in a 200 m × 200 m grid – the same used for the computed depths – to enable a comparison between predicted and expected bathymetry. A discussion about the spatial resolution and grid spacing of our method will be held in sections 3.2.1 and 4.1, respectively.
Figure 7. Interpolated reference bathymetric map of the study area. The grid spacing is 200 m × 200 m and each point represents the mean depth averaged over an area of approximately 770 m × 770 m.

For the sake of consistency, depth reference points with an interpolation area that encompasses the shore zone (i.e., closer than 770 m to the island) are not represented.

3.2 IMAGE PROCESSING STEPS

3.2.1 SUB-WINDOWS LAYOUT

The first step in the image processing is to define an adequate size for the sub-windows within which the image spectrum will be computed. The larger the sub-scene, the finer the spectral resolution to determine the peak wavenumber (and, consequently,
the wavelength) and the wave direction, and the better the vertical resolution: computation relies on more Fourier coefficients and the derived depth values are averaged over a larger number of waves, respectively. Nevertheless, if the sub-scene becomes excessively large, the estimated depth fails to represent variations in bathymetry in the area in question; in addition to that, there will be fewer sub-scenes covering the image, and, as a result, fewer independent depth estimates, degrading the horizontal resolution. Hence, we can identify another limitation on this water depth retrieval technique besides the ones already described: the inevitable trade-off between spectral / vertical resolution and horizontal resolution.

Considering the wavelength extrema for a typical swell – between 150 m and 350 m, depending on the wave period – the worst-case scenario to estimate depth map (due to the smallest number of waves within the sub-boxes), our method is expected to be most effective if applied in regions of smooth depth gradients on the length scale of 1 km.

Another factor to keep in mind when deciding about FFT window sizes is that the best computational efficiency is obtained with arrays with power-of-2 line and column numbers.

In light of the arguments presented, some tests with different sizes and configurations of the sub-windows were made and the following layout was adopted:

– 5 overlapping 1024 × 1024-wavenumber-bin (768 m × 768 m) sub-windows, of which:

– 4 sub-windows were arranged in a 2 × 2 layout with a half window width / height overlap; and
– 1 central sub-window overlapping with all other four sub-windows to give the center the most weighting.

An image spectrum computation of a given region uses the average of the power spectra of all 5 overlapping sub-windows, which make up a 1536 × 1536-pixel (1152 m × 1152 m) area. Making use of overlapping sub-windows aims at reducing noise.

Another publication on dynamically similar regions, in which surface wave refraction was observed in TSX imagery, used a similar sub-window size (Li et al 2010).

Figure 8 depicts the overlapping sub-window layout previously described. The shaded red rectangles highlight the overlap between the upper side-by-side sub-scenes and the shaded yellow rectangles indicate the overlap between the upper and the lower ones on the left. For the sake of clarity, overlapping regions related to the lower side-by-side sub-scenes, the upper and lower ones on the right, and the central sub-window are not shown.
In order to account for computational artifacts that a limited sub-window size can introduce in the spectral analysis (i.e., high-order harmonics due to a non-integer number of wavelengths within a sub-window), a bell-shaped tapering function was applied to each sub-window.
The first step to compute the image power spectrum of the region comprised by the five sub-scenes is to demean all arrays and then taper them; afterward, they are again demeaned. Figure 9 displays the same sub-windows as in Figure 8 after a tapering function has been applied to them:

![Figure 9](image)

Figure 9. Same sub-windows as those shown in Figure 8, plus the central one, now tapered.
The next step is to compute a 2-D FFT and multiply the complex Fourier components found with their respective conjugates for each sub-window. Finally, the image power spectrum for that 1536 × 1536-pixel area is the average of the five power spectra calculated. Since the FFT computation is done over the space domain, the image spectrum is expressed in terms of the range and azimuth components of the wavenumber, which will be discussed in section 3.2.3.

Figures 10 and 11 display image power spectra of the area depicted in Figures 8 / 9: the power spectrum of Figures 10 and 11a is calculated from the sub-boxes of Figure 8, while that of Figure 11b is computed from the tapered ones, shown in Figure 9.

Figure 10. Power spectrum obtained from the data displayed in Figure 8.
Since the image columns and rows are oriented in the azimuth (flight) and range directions, respectively, the computed image spectra follow the same orientation. In Figure 10, the whole wavenumber space is represented: the graph shows $1024 \times 1024$ wavenumber-bin arrays. From the power spectrum pattern shown in that figure, we can clearly note that the image energy is mostly associated with long waves, within the first hundredths of the wavenumber spectrum; the level of energy of the remainder part of the spectrum is negligible. Therefore, the display of the power spectrum array in its entirety makes the main characteristics within the region of interest as well as the differences of calculation with and without tapering the sub-scenes impossible to identify.

Figure 11 is a $31 \times 31$-spectral-bin blowup of the power spectrum array, centered at the DC component, with its corresponding pixel highlighted by a white circle. Under such magnification, the image spectrum is then depicted in a useful way:

Figure 11. Blowup of the power spectra obtained from the data displayed in Figure 8 (a) and Figure 9 (b).

The wavenumber range represented in Figure 11 goes from 0 (the DC component) to 0.1227 rad/m, either in the flight or in the range direction, corresponding to
wavelengths from infinity to 51.2 m. Since our study aims at observing spatial changes in surface gravity waves, such wavenumber range would suffice to represent their power spectra. Taking the shallow-water approximation of the linear dispersion relationship to evaluate the shortest wavelengths expected in the shallowest regions of our study area, about 5 m deep, a 10- to 15-s swell has a wavelength between 70 and 105 m, having thus a smaller wavenumber than the maximum value shown in Figure 11.

The color table used in the figures ranges from dark blue (lowest values) to red (highest values). It is noticeable that the spectrum of Figure 11b, due to the tapering effect, shows smoother peaks than the one in Figure 11a.

### 3.2.2 THE MTF ARRAY

To compute the wave spectrum from the image spectrum, the wave imaging mechanisms should be taken into account. On the assumption of a linear process, it can be expressed by an MTF made up of a sum of three kinds of modulation: tilt, hydrodynamic and velocity bunching:

$$MTF_{SAR} = MTF_{\text{tilt}} + MTF_{\text{hydr}} + MTF_{\text{bunching}}$$  \hspace{1cm} (6)

Performing a dimensional analysis of the modulations discussed in Alpers et al. (1981), we find that $\frac{MTF}{k}$ is dimensionless, where $k$ is the long ocean wavenumber vector. Arguing with a monochromatic wave, the long-wave-driven gray level (i.e., intensity) modulation within a SAR image is proportional to the local surface slope $a \cdot k$, where $a$ is the wave amplitude. Hence, when we compute an image power spectrum, our result is in fact proportional to $(a \cdot k)^2$, not $a^2$ – the waveheight power spectrum. The
relation between image spectrum and waveheight spectrum then becomes (Alpers et al. 1981):

\[ P_I(k) = \left| MTF_{SAR}(k) \right|^2 \cdot \Psi(k) \]  

(7)

where \( P_I(k) \) is the intensity power spectrum of the image, \( MTF_{SAR}(k) \) is the modulation transfer function in its dimensional form, and \( \Psi(k) \), the waveheight power spectrum.

Our plan is to introduce the MTF array in our depth retrieval technique. For that purpose, the image spectrum will be divided by the dimensionless squared MTF, yielding the wave slope spectrum; this simple correction takes the rotation and shift of the spectral peak driven by the imaging mechanism into account. We will then compare the bathymetric maps obtained with and without this initial correction.

Figures 12, 13, and 14 depict, respectively, the dimensionless components of tilt, hydrodynamic, and velocity bunching squared modulation. Figure 15 shows the combined effect of all modulations. All figures show the center \( 31 \times 31 \) spectral bins of the full \( 1024 \times 1024 \)-bin array. The horizontal axis represents the wavenumber component in range direction, while the vertical axis represents the one in azimuth direction.
Figure 12. Squared magnitude of the dimensionless tilt MTF.
Figures 12 and 13 show that waves traveling in the range direction have stronger radar signatures than waves with a component in azimuth direction, due to the amplitude modulation, comprised by the tilt and hydrodynamic components. We can also state that both contributions are of the same order of magnitude.
On the other hand, we observe in Figure 14 the opposite situation: the velocity bunching effect favors the detection of ocean waves traveling in the azimuth direction.

Another aspect to be highlighted is the magnitude of this kind of modulation: the maximum value reached is two orders of magnitude greater than the other arrays within the same wavenumber range. Indeed, this modulation is a function of the $R/V$ ratio, where $R$ is the slant range and $V$ is the satellite velocity. In the case of TSX, typical values of this ratio lie between 80 and 100 s, depending on the incidence angle. Such a strong velocity-bunching-related modulation suggests a narrow wave direction range where its effects can be regarded as linear.
By inspection of the patterns shown when all modulations are considered, we note the striking predominance of velocity bunching effects over the other modulations for wave components outside the range direction.

### 3.2.3 WAVE PARAMETERS COMPUTATION

After computing the image spectrum array (or the wave slope spectrum, if divided by the squared dimensionless MTF array), it is smoothed twice with a $3 \times 3$-element smoothing window, to reduce noise and to obtain more stable peak wave directions and wavelengths. The latter properties are then derived from the spectrum.
An initial peak wavenumber \( k_{p0} \) can be readily obtained by finding the location of the maximum of the power spectral density in wavenumber space, at \((k_{p0,x}, k_{p0,y})\). The magnitude of \( k_{p0} \) is then given as

\[
k_{p0} = \sqrt{k_{p0,x}^2 + k_{p0,y}^2}
\]  

(8)

In a second step, we compute a more robust power-weighted peak wavenumber \( k_p \) by using a mask around \((k_{p0,x}, k_{p0,y})\), allowing only spectral components in its vicinity to be taken into account. For that purpose, an isotropic mask including wavenumbers with a difference of up to three times the spectral resolution relative to \( k_{p0} \) was found to work best.

The power-weighted peak wavelength \( L_p \) and direction \( \theta_p \) are found as follows:

\[
L_p = \frac{2\pi}{\sqrt{k_{p,x}^2 + k_{p,y}^2}}
\]  

(9)

\[
\theta_p = \arctan \left( \frac{k_{p,y}}{k_{p,x}} \right)
\]  

(10)

where \( k_{p,x} \) is the range component of the power-weighted peak wavenumber, while \( k_{p,y} \) is its azimuth component.

Since there is no time information available in the image, each wave spectrum is point symmetric with a corresponding 180° ambiguity of the spectral peak, as shown in Figure 11. However, this ambiguity can be eliminated on the assumption that the waves
are propagating shoreward, which is very plausible for the refraction pattern exhibited in the satellite image (Figure 6).

The last step is to link the computed wavelength to the water depth via the linear dispersion relationship, provided the wave period is known.

3.2.4 MODES OF OPERATION

According to the dynamical characteristics of the study region, wavelength and depth maps can be computed according to two basic modes of operation: ray tracing or fixed grid.

In the ray-tracing mode, once the main parameters of the wave – the power-weighted peak wavelength and direction – are computed, the sub-windows are displaced by a fraction of the computed peak wavelength in the peak direction and the process is repeated over and over, until the sub-boxes reach a set distance to the shore. The displacement from one set of sub-windows to the other within the wave ray can also be set to a fixed value. Pleskachevsky and Lehner (2011) used this approach in their work.

Besides making refraction patterns clearly visible throughout the image, as seen in the next figure, this mode may offer an advantage in the case of different wave systems coming from different directions, with different periods. As every ray or group of rays, under such conditions, represents a given wave system, depth maps can be produced taking that aspect into account, by assigning to the rays the corresponding frequency of the wave system to which they belong; this is justified by the fact that the intrinsic wave frequency remains constant along a wave ray.
Figure 16 shows a wavelength map in which every set of sub-windows, along a given wave ray, is apart from the next one by half the calculated peak wavelength in the former region. Each colored dot indicates the wavelength value found for the respective sub-region defined by the sub-windows layout and is located in the center of that area.

![Image of wavelength map](image)

Figure 16. Wavelength map computed in the ray-tracing mode.

Another contribution that the ray-tracing mode may bring to generate more robust depth maps is an inherent quality control system: under certain wave conditions, wave rays from different systems may cross each other, yielding, at the crossing point, two independent depth computations. A comparison between the depth values found at the intersection of the rays can be used as an indicator of the method accuracy.
In a more uniform wave field (e.g., under the influence of a single wave system), the fixed grid mode can be used. In this mode, wave parameters computation is done throughout an equally spaced grid, both in range and azimuth direction.

The advantages this mode brings are the use of a simpler algorithm to deal with the calculations and the fact that it provides a uniformly dense mapping of the region, which does not happen in the previous mode, due to wave refraction.

As the radar image of our study area depicts a single wave system, the depth map presented and discussed in the next chapter is computed using the fixed grid approach.
CHAPTER 4

RESULTS AND DISCUSSION

Based on the assumption of the presence of a single wave system in the radar image of our study area, we present the depth map computed in a fixed grid, initially without an MTF-based correction, and compare it with the interpolated sea truth data. Scatter plots of both datasets as well as a statistical analysis are the tools we use to evaluate the quality of our method.

Next, we present our results considering an MTF-based correction, with its formulation based on the theoretical description by Alpers et al (1981), and compare them with the ones obtained without any correction to assess the improvement achieved by using a wave slope spectrum – as opposed to an image spectrum – for retrieving depth information.

4.1 SAR- DERIVED DEPTH MAP FROM IMAGE SPECTRA

After calculating the wave parameters of the image of our study area, we retrieve the depth map by means of the linear dispersion relationship.

However, the wave period has to be known for that conversion. In the absence of such information from an external source, this parameter should be derived from the radar image. Due to the fact that there is no region within our study area where the deep-water approximation of the linear dispersion relationship can be applied to derive the wave period (Equation (5)), that parameter is tuned based on a statistical analysis of the SAR-derived and reference depth values.
Denoting $y$ as the SAR-derived depth and $x$ as the sea truth data, we can assume a linear relationship between these variables of the form

$$y = cx$$

where $c$ is the regression coefficient – the slope of the best-fitting line approximating both variables.

Let us consider a set of pairs of variables $(x_i, y_i), i = 1, 2, \ldots, N$. If both $x$ and $y$ are independent variables, we can compute the unbiased, symmetric regression coefficients, defined by

$$c^2 = \frac{1}{N} \sum_{i=1}^{N} \frac{y_i^2}{x_i^2}$$

The first step of the wave period tuning is to select a first-guess wave period and apply the linear dispersion relationship to the SAR-derived wave parameters in order to compute the predicted depth map; then, we assess both predicted and expected bathymetry by computing the symmetric regression coefficient. If it is not 1, the whole cycle of computations is carried out with a slightly incremented (or decremented) wave period and the regression coefficient is computed again; this routine is repeated until $c$ converges to 1.

Therefore, the wave period tuned for the computation of the derived depth map is imposed to yield the best possible agreement between both predicted and expected
bathymetry as measured by $c$. Such criterion implies that both mentioned datasets are given the same weight in our analysis (i.e., both can be considered independent variables); hence, the symmetric regression coefficient seems to be the best statistical parameter to be evaluated.

A threshold, first-guess wave period can be computed for the study area as follows: Since wavelength computations show variations throughout the image, it can be assumed that the wave field interacts with the underwater topography, not being, thus, on a deep-water domain. Therefore, in order to compute the shortest theoretical wave period to exist under such conditions, we evaluate Equation (5) with the longest smoothed computed wavelength found in the image, of 225.3 m; the shortest wave period is then set to 12.01 s.

Other statistics are also calculated: the correlation coefficient $r$ and the root-mean-square error (RMSE), evaluated by the following respective formulas:

$$r^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i y_i)^2 - \frac{1}{N} \sum_{i=1}^{N} x_i^2 \frac{1}{N} \sum_{i=1}^{N} y_i^2$$

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - y_i)^2}$$

The former parameter is a measure of the interrelationship between the variables and their relative strength, while the latter evaluates the dispersion between them.

The following table presents all previously mentioned statistical parameters computed as a function of the wave period:
<table>
<thead>
<tr>
<th>Wave period (s)</th>
<th>( r^2 )</th>
<th>( c^2 )</th>
<th>RMSE (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.90</td>
<td>0.769866</td>
<td>1.0286136</td>
<td>8.87059</td>
</tr>
<tr>
<td>11.92</td>
<td>0.768092</td>
<td>1.0035072</td>
<td>8.73880</td>
</tr>
<tr>
<td>11.95</td>
<td>0.768524</td>
<td>0.96727427</td>
<td>8.53972</td>
</tr>
<tr>
<td>12.00</td>
<td>0.774081</td>
<td>0.91030849</td>
<td>8.27238</td>
</tr>
<tr>
<td>12.10</td>
<td>0.791518</td>
<td>0.81486287</td>
<td>8.27664</td>
</tr>
<tr>
<td>12.20</td>
<td>0.801379</td>
<td>0.74132012</td>
<td>8.91764</td>
</tr>
</tbody>
</table>

Table 1. Computation of the correlation coefficient, symmetric regression coefficient, and root-mean-square error between the derived and observed depth variables, as a function of the wave period.

The table highlights the chosen adjustable wave period in boldface type. As mentioned before, the criterion for the selection of the wave period was based on the value of the symmetric regression coefficient.

As we adopt the same procedure for determining the wave period when considering an MTF-based correction in our depth retrieval, the comparison between the correlation coefficient \( r \) and the RMSE obtained in both approaches can be used as a measure of the improvement (if any) in the accuracy of our depth retrieval technique by applying the proposed correction.

The following scatter plot displays the SAR-derived against the reference depths for the tuned wave period of 11.92 s:
Figure 17. Scatter plot of SAR-derived vs. reference depth data, for a wave period of 11.92 s, and some statistical parameters.

The scatter plot suggests that the depth retrieval procedure without any correction to the image spectrum works best at shallow- to mid-water areas (up to 50 m), while the SAR-derived depth in the deepest part of the image is scattered and less reliable.

In the next figure, the SAR-derived depths used in the previous scatter plot are now displayed in the fixed grid mode: The resulting depth array is displayed over a 200 m × 200 m grid and the SAR-derived depth information of each grid cell is located at the
center of each region defined by the sub-windows. Using a finer grid spacing than the spatial resolution (770 m × 770 m) aims at gradually depicting variations in bathymetry. Such array was computed using a tuned wave period of 11.92 s, as previously explained.

![SAR-derived depth map](image)

Figure 18. SAR-derived depth map, computed in the fixed grid mode, without any correction to the image spectra. The grid spacing is 200 m × 200 m and the tunable wave period is set to 11.92 s.

If we compare the depth map in Figure 18 with the reference map, in Figure 6, in a qualitative way, we note that the former is able to represent the general bathymetric features present in the latter: In the vicinity of the island, the isobaths tend to follow its shape and the deepest regions are concentrated on the lower left corner of the image.
Nevertheless, the SAR-derived depth map shows a sudden depth discontinuity on the center left of the image that does not seem to be present in the actual bathymetry.

Such a sharp submarine valley occurs due to the fact that the chosen tuned wave period (11.92 s) is shorter than the theoretical minimum value (12.01 s); hence, the areas where the longest waves are found will be considered infinitely deep.

The occurrence of anomalously long waves in that region could be explained by an interference pattern of two wave systems, with slightly different peak wavelengths and wave directions that can be observed in the image.

In order to identify some kind of pattern in the deviations of the derived depths with respect to the expected values, a map indicating the ratio between derived depth and sea truth data is shown in Figure 19. Perfect agreement between both arrays is reached when that ratio is 1; the blue points indicate where our technique overestimates the local depth, while the red points show where the latter is underestimated.
Figure 19. Map of the ratio between SAR-derived and reference water depths. No correction to the image spectra has been applied. The grid spacing is 200 m × 200 m and the tunable wave period is set to 11.92 s.

The figure also shows that most computed depth points have an accuracy of 20% with respect to the expected value, comparable to Pleskachevsky and Lehner’s results. Furthermore, it suggests that there is not a connection between depth ratio and local water depth: our method yields indistinctively deeper and shallower depths than the reference data either in deep or shallow water regions.

In an attempt to detect depth deviations from the expected value as a function of wave parameters that are relevant for the imaging mechanism and, thus, could be
minimized with an MTF-based correction, the depth ratios of Figure 19 are shown as a function of wavenumber and wave direction in Figure 20a:

![Diagram of the ratio between SAR-derived and reference depths as a function of wavenumber and wave direction. No correction to the image spectra has been applied.](image)

The black points in the diagram – out of the colorbar scale – indicate the wavenumbers associated to infinitely deep regions for the tuned wave period chosen for the wave field in question; their occurrence as well as a possible explanation for them have been previously discussed.

Blue points in Figure 20a indicate that the SAR-derived water depth is greater than the reference data, while red points show the opposite.
Figure 20a suggests a systematic dependence of the depth retrieval error on wavenumber and wave direction that does not seem to be a random effect. This justifies our expectations of better results by including an MTF in the interpretation of the image spectra.

In addition to depth ratios, our results can be also expressed in terms of wavenumber ratios, as shown in Figure 20b: Reference depths can be converted into reference wavenumbers via Equation (2), evaluated at the tuned wave period of 11.92 s.

![Figure 20b](image_url)  
Figure 20b. Same as Figure 20a, but showing the ratio between SAR-derived and reference wavenumbers.

The color code adopted in Figure 20b follows the same pattern as in the previous figure: Blue points indicate that the SAR-derived wavenumber is lower than the reference
wavenumber – hence, the retrieved wavelength is longer than the reference value, which, for a given wave period, implies a greater retrieved water depth. Likewise, red points indicate higher SAR-derived wavenumbers – hence, shorter retrieved wavelengths, which, for a given wave period, implies shallower retrieved depths.

4.2 SAR-DERIVED DEPTH MAP FROM WAVE SLOPE SPECTRA

In a first attempt to improve our technique, we compute the wave parameters from their wave slope spectra by dividing the image spectra by the theoretical squared dimensionless MTF. By taking the rotation and shift of the spectral peak induced by the imaging mechanism into account, we expect a better agreement between SAR-derived and reference depths.

The threshold, first-guess wave period for the area is calculated taking the longest smoothed wavelength found with the new computation – 237.5 m – and is set to 12.33 s.

The following table presents the values of the statistical parameters evaluated to define the tunable wave period:
Wave period (s) | $r^2$ | $c^2$ | RMSE (m)  
---|---|---|---
12.00 | 0.826399 | 2.2602634 | 24.5921 
12.50 | 0.854036 | 1.1111003 | 7.17418 
12.60 | **0.853411** | **1.0012739** | **6.37030** 
12.62 | 0.853255 | 0.98233265 | 6.31833 
12.65 | 0.852931 | 0.95537720 | 6.29808 
12.70 | 0.852461 | 0.91404964 | 6.38563 

Table 2. Computation of the correlation coefficient, symmetric regression coefficient, and root-mean-square error between the derived and observed depth variables, as a function of the wave period. The derived depth dataset is computed with an MTF-based correction.

The chosen adjustable wave period is also highlighted in boldface type. Unlike the previous set of computations, discussed in section 4.1, the tuned wave period is greater than the threshold value. Such a desirable situation prevents sharp depth discontinuities – as seen in Figure 18 – from happening in our newly derived depth array.

The following scatter plot displays the MTF-based SAR-derived depth against the reference depths for the tuned wave period of 12.60 s:
Figure 21. Scatter plot of SAR-derived vs. reference depth data, for a wave period of 12.60 s, and some statistical parameters. In contrast to Figure 17, the SAR-derived depths shown have been obtained with the MTF-based approach.

In Figure 21, we can still observe the same behavior found in the previous scatter plot (Figure 17): better depth estimates at shallow to mid waters and greater deviations at deep waters. However, there is an overall improvement, albeit more pronounced in the deepest areas of the image. Such enhancement can be quantified by the new computed values of $r^2$ (0.85, as opposed to 0.77) and RMSE (6.37 m, as opposed to 8.74 m). Hence, there is evidence that the MTF-based SAR-derived depth data are more accurate.
The resulting corrected depth array is displayed in the same way as Figure 18, for a tuned wave period of 12.60s:

Figure 22. Same as Figure 18, but for SAR-derived depths obtained from the MTF-based approach. The wave period is set to 12.60 s.

As anticipated from the analysis of Figure 21, the general bathymetric features of the area are better represented in the depth map obtained from the MTF-based approach. Moreover, the effects of the constructive interference of two slightly different wave systems producing longer wavelengths – speculated explanation for the sharp depth discontinuity observed in Figure 17 and previously discussed – seem to be reduced by the correction of the image spectrum into a wave slope spectrum.
Figure 23 shows the map indicating the ratio between SAR-derived depths obtained from the MTF-based approach and reference data:

![Map showing ratio of SAR-derived depths](image)

Figure 23. Same as Figure 19, but for SAR-derived depths obtained from the MTF-based approach. The wave period is set to 12.60 s.

By comparing Figure 19 with Figure 23, we can observe that the blue points in the former – indicating a deeper region than expected – become less blue in the latter; also, the same effect happens to the red points. Therefore, we can conclude that introducing a simple, theoretical MTF-based correction to our depth retrieval method leads to a reduction of absolute deviations of the SAR-derived depths from the reference depths.

Such an improvement is also depicted as a function of wavenumber and wave direction in Figure 24a:
Figure 24a. Same as Figure 20a, but for SAR-derived depths obtained from the MTF-based approach.

Similarly to what has been done in the previous section, the results obtained from the MTF-based approach can also be expressed in terms of wavenumber ratios, as shown in Figure 24b: Reference depths can be converted into wavenumbers via Equation (2), evaluated at the tuned wave period of 12.60 s.
Figure 24b. Same as Figure 20b, but for SAR-derived depths obtained from the MTF-based approach.

4.3 SENSITIVITY ANALYSIS

In our technique, there are two distinct processes of which a sensitivity analysis should be carried out: the estimates of the tunable wave period and the water depth. The linear dispersion relationship is used in both cases to evaluate how uncertainties in the input parameters data affect the estimates.

In the first process, there are two input variables: the SAR-derived wavelength and the reference water depth, used to estimate the wave period.

Since our method aims at estimating the bathymetry in coastal regions, the latter is not implied to be known: Hence, determining the wave period by adjusting all derived
depths to the reference data – available throughout the radar image – in the least-square sense is not an operational procedure; it is adopted for the sole purpose of assessing the potential of improving our technique by computing the wave parameters from their wave slope spectra – as opposed to their image spectra – such that all the improvement comes from the use of the MTF-based approach, not due to a mistuning of the wave period that accidentally yields better results in one situation than in the other.

In a more realistic scenario, knowledge of the local bathymetry is available only in limited areas, more likely at the deeper regions, further from the shore: therefore, our analysis will be based on that assumption.

In our study, we assume errors of ±5% both in wavelength and water depth estimates; a wavelength of 200 m will be evaluated at a depth of 50 m and variations around these central values will be allowed to evaluate the degree of sensitivity of each input parameter to wave period retrieval, as shown in Figure 25:
Figure 25. Curves showing variations of the tuned wave period as a function of wavelength derived at a given water depth.

If we consider a reference depth of 50 m free of errors and allow fluctuations only in the SAR-derived wavelength, Figure 25 shows that variations of ±10 m in the wavelength lead to variations in the wave period of approximately ±0.35 s. Shallower regions are more sensitive to wavelength fluctuations: the same initial conditions cause the wave period to vary about ±0.4 s; on the other hand, deeper waters exhibit the opposite behavior: wave period changes are about ±0.3 s. Moreover, the degree of
sensitivity seems to be constant throughout the depth and wavelength ranges depicted in the figure.

Likewise, if we consider our wavelength retrieval perfectly accurate and allow fluctuations only in the depth estimates, Figure 25 shows that, at a wavelength of 200 m, depth changes of ±2.5 m causes the wave period to vary about ±0.15 s. In the presence of longer waves, higher wave period fluctuations are induced: At 260 m long waves, the above-mentioned depth variations lead to wave period changes of about ±0.25 s; in turn, shorter waves yield smaller wave period variations: For 160 m long waves, wave period changes induced by depth changes of ±2.5 m are less than ±0.1 s.

Thus, we can conclude that the wave period retrieval process yields better results at the deepest regions, with the shortest wavelengths – in other words, its accuracy is proportional to $k h$, as defined by Equation (2).

Next, we evaluate the sensitivity of the depth retrieval of our method by propagating the uncertainties due to the SAR-derived wavelength and tuned wave period. Figure 26 shows how variations in the derived wavelength affect depth retrieval, for a given wave period. The increments in wave period shown in the curves are compatible with the expected variations in their estimates, as previously discussed:
Figure 26. Curves showing variations of water depth estimates as a function of wavelength and wave period.

Arguing with a SAR-derived wavelength of 200 m at a tuned wave period of 12 s (red curve), we observe that assumed wavelength errors of ±10 m lead to depth absolute variations of about 20 m; furthermore, if we allow the same wavelength error range but with longer wavelengths, at the same wave period, the retrieved depth absolute fluctuations become greater, eventually reaching the deep-water section of the curve, where infinitesimal increments in wavelength produces infinitely great water depth
variations. As a result, the curves in Figure 26 clearly show that our depth retrieval method is more sensitive to wavelength errors in deep-water regions.

Back to the initial hypothetical condition of a SAR-derived wavelength of 200 m with assumed errors of ±10 m, at a tuned wave period of 12 s, let us analyze how depth absolute variations vary with different wave periods: In the latter situation, they are of about 20 m; at a wave period of 11.65 s, they grow up to about 46 m, while at 12.35 s they are as low as 13 m. Such results also support our findings that the bathymetry retrieval technique in study is more accurate in shallower waters.
CHAPTER 5

CONCLUSIONS AND OUTLOOK

We have tested a technique for water depth retrievals in coastal regions from swell refraction patterns in high-resolution SAR images, based on the work by Pleskachevsky and Lehner (2011). While these authors derived peak wavelengths and wave directions straight from the SAR image spectra, without accounting for the SAR imaging mechanism of ocean waves, we have investigated possibilities of obtaining more accurate results by using a linear modulation transfer function (MTF) to convert the SAR image spectra into ocean surface slope spectra and deriving peak wavelengths and wave directions from the slope spectra. The retrieved wave parameters were then converted into water depths by means of the linear dispersion relationship evaluated at a wave period derived such that the best fit between SAR-derived and reference depths was reached, in the least-square sense.

We used a TerraSAR-X image of Rottnest Island (Australia), acquired in Spotlight mode (area size = 11,778 m × 10,440 m, pixel size = 0.75 m × 0.75 m) – the same image in Pleskachevsky and Lehner’s (2011) work – and a reference depth map based on LiDAR measurements, echosoundings, and satellite altimeter data analysis, provided by Australian agencies. We compared SAR-derived depth maps of the test area obtained with and without the MTF-based approach. Our results indicate that there is an improved accuracy in the former, statistically quantified by a higher correlation coefficient ($r^2=0.85$ with the MTF-based approach, as opposed to $r^2=0.77$ without it) and a lower RMSE (6.37 m with the MTF-based approach, as opposed to 8.74 m without it) with respect to the reference depths.
The use of a linear MTF to account for the physics involved in the wave imaging by SAR was speculative due to the fact that the linearization is known to be accurate for a very limited wavenumber and wave direction range only. We justified the MTF-based approach by the fact that our main objective is an improved estimation of peak wavenumbers and directions, not necessarily a correct quantitative retrieval of complete two-dimensional ocean wave spectra. Therefore, the most important finding of this work is that the use of a linear MTF in deriving the wave parameters necessary for depth retrievals can indeed improve the results.

The technique should be developed further in future work. As a first step, it is desirable to apply it to a variety of SAR images of the same test area and other areas to assess the consistency of our method and to identify remaining weaknesses. In this context, it should be tested if better results can be obtained if ocean waveheight or action spectra are used for the peak retrievals instead of surface slope spectra. It is well known that observed SAR image spectra are not always consistent with the parameterization of the MTF according to Alpers et al. (1981). An example of a work in which the use of modified MTFs led to significant improvement was presented by Brüning et al. (1994), who compared observed airborne SAR image spectra with simulated image spectra based on ocean wave spectra measured by a pitch-and-roll buoy.
REFERENCES


