2014-04-07

Tropical Cyclogenesis: Observed Processes and Predictability

William Anthony Komaromi
University of Miami, wkomaromi@rsmas.miami.edu

Follow this and additional works at: https://scholarlyrepository.miami.edu/oa_dissertations

Recommended Citation
https://scholarlyrepository.miami.edu/oa_dissertations/1160

This Open access is brought to you for free and open access by the Electronic Theses and Dissertations at Scholarly Repository. It has been accepted for inclusion in Open Access Dissertations by an authorized administrator of Scholarly Repository. For more information, please contact repository.library@miami.edu.
TROPICAL CYCLOGENESIS: OBSERVED PROCESSES AND PREDICTABILITY

By

William A. Komaromi

A DISSERTATION

Submitted to the Faculty
of the University of Miami
in partial fulfillment of the requirements for
the degree of Doctor of Philosophy

Coral Gables, Florida

May 2014
UNIVERSITY OF MIAMI

A dissertation submitted in partial fulfillment of
the requirements for the degree of
Doctor of Philosophy

TROPICAL CYCLOGENESIS: OBSERVED PROCESSES AND PREDICTABILITY

William A. Komaromi

Approved:

Sharanya J. Majumdar, Ph.D.
Associate Professor of Meteorology
and Physical Oceanography

David S. Nolan, Ph.D.
Professor of Meteorology
and Physical Oceanography

Brian E. Mapes, Ph.D.
Professor of Meteorology
and Physical Oceanography

M. Brian Blake, Ph.D.
Dean of the Graduate School

Ryan D. Torn, Ph.D.
Assistant Professor of Meteorology
University at Albany
This research is a study of the behavior of the observed processes leading up to genesis, and how ensembles can be used to assess their predictability. In the first part of this study, dropwindsonde observations of developing and non-developing tropical waves are examined from the 2010 Pre-Depression Investigation of Cloud Systems in the Tropics (PREDICT) field campaign. Significant results include the development of positive temperature anomalies from 500-200 hPa two days prior to genesis in developing waves, which is not observed in the non-genesis mean. Progressive mesoscale moistening of the column is observed within 150 km of the center of circulation prior to genesis. The genesis composite is found to be significantly more moist than the non-genesis composite at the middle levels, while comparatively drier at low levels, suggesting that dry air is more detrimental to genesis when located at the middle levels. Time-varying tangential wind profiles reveal an initial delay in intensification, followed by an increase in organization 24 hours pre-genesis. The vertical evolution of relative vorticity, in addition to a warm-over-cold thermal structure, is more consistent with a top-down than a bottom-up genesis mechanism.

Thereafter, several metrics are employed to evaluate predictive skill and attempt to quantify predictability using the ECMWF Ensemble Prediction System during the 2010 Atlantic hurricane season, with an emphasis on large-scale variables relevant to tropical
cyclogenesis. These metrics include: (1) Growth and saturation of error; (2) Errors versus climatology; (3) Predicted forecast error standard deviation; and (4) Predictive Power.

Overall, variables that are more directly related to large-scale, slowly-varying phenomena are found to be much more predictable than variables that are inherently related to small-scale convective processes, regardless of the metric. For example, 850-200 hPa wind shear and 200 hPa velocity potential are found to be predictable beyond one week, while 200 hPa divergence and 850 hPa relative vorticity are only predictable to about one day. Similarly, area-averaged quantities such as circulation are much more predictable than non-averaged quantities such as vorticity.

Significant day-to-day and month-to-month variability of predictability for a given metric also exists, likely due to the flow regime. For wind shear, more amplified flow regimes are associated with lower Predictive Power (and thereby lower predictability) than less amplified regimes. Relative humidity is found to be less predictable in the early and late season when there exists greater uncertainty of the timing and location of dry air.

Similarly, significant case-to-case variability is observed in the wave-relative analysis. For some genesis events, predictability of genesis appears to be directly related to the capability of the ensemble to predict an environment favorable for genesis. In other cases, predictability appears to be more directly associated with the strength and location of the initial disturbance in the model. By examining forecast joint distributions of variables, predicted relative humidity values at 700 hPa of less than 60% in the wave core (≤300 km of center of circulation) are found to be a strong limiting factor for genesis in the ensemble, and also tend to be correlated with weak 200 hPa divergence. Genesis is also found to occur in the presence of significant wind shear (~15 ms⁻¹), but generally only
when the core and environment of the wave are both very moist (~85% and 75% 700 hPa RH, respectively).

Lastly, the ensemble demonstrates the potential to predict error standard deviation of variables averaged in 10° x 10° boxes, as well as within 300 km and 1000 km radii about individual tropical waves. Forecasts with greater ensemble standard deviation are on average associated with greater mean error, especially for forecasts with less than 168 h lead time. However, the ensemble also tends to be under-dispersive.
Acknowledgements

I would like to thank my advisor, Dr. Sharanya J. Majumdar, for his advice, guidance and support throughout my pursuit of my Ph.D. Not only is Sharan a very knowledgeable advisor who always provides great input and advise, but he was also friendly, approachable, and easy to get along with, which made working with him that much more enjoyable. I would also like to thank my committee, Dr. David Nolan, Dr. Brian Mapes, and Dr. Ryan Torn for advice and constructive criticism. Their comments have certainly helped to make this dissertation better than it could have been without them. I would also like to thank Dr. Chris Davis and Dr. Rich Rotunno for comments and suggestions, as well as hosting me at NCAR as part of the Graduate ASP Program during summer 2012, as well as David Ahijevych at NCAR for providing GOES IR satellite composites. I would like to thank the TIGGE database for ECMWF ensemble data. Lastly, I would like to gratefully acknowledge funding from the National Science Foundation and the Rosenstiel School of Marine and Atmospheric Science.

I would also like to acknowledge the enormous support, care and friendship from my family, friends, and classmates. I would especially like to thank my Mom, Dad, and my brother Anthony, as well as the great friendships I have made while studying here in Miami, including Falko Judt, Andrew Kough, David Weinstein, Adam and Jenny Greer, Andrew Margolin, Daniel Stern, and many others. Lastly, I would like to acknowledge my wonderful soon-to-be wife, Marcela Ulate, who has stood by me even during the difficult times, has always been fully supportive of everything I do, and has always shown full confidence in me that I would successfully complete my Ph.D.
# TABLE OF CONTENTS

**LIST OF FIGURES** .......................................................................................................................... vi

**LIST OF TABLES** ............................................................................................................................ xiii

**Chapter**

1 **Introduction** ................................................................................................................................. 1  
1.1 Tropical cyclogenesis - the current state of the science and observations .......... 1  
1.2 Predictability and ensemble-based forecasting of tropical cyclogenesis .......... 10

2 **Methodology** ................................................................................................................................. 21  
2.1 Investigating genesis via in-situ data: PREDICT dropwindsondes ............... 21  
2.2 Investigating genesis via ensemble forecasts ....................................................... 26  
2.2.1 ECMWF ensemble and climatology ............................................................... 26  
2.2.2 Wave-relative framework ............................................................................. 30  
2.3 Metrics to quantify error growth and predictability ........................................ 33  
2.3.1 RMS error and error variance ................................................................. 33  
2.3.2 Predictive Power ...................................................................................... 37  
2.3.3 Wave-relative Predictive Power ................................................................. 40

3 **Tropical Cyclogenesis: Observed Physical Processes** ............................................. 46  
3.1 Genesis vs. non-genesis ...................................................................................... 46  
3.2 Time progression leading up to genesis ............................................................ 51  
3.3 Summary of results ............................................................................................ 58

4 **Predictability Results Part I: Basin-Wide Perspective** ........................................ 68  
4.1 Error growth ........................................................................................................... 69  
4.1.1 Monthly evolution of variables .................................................................... 69  
4.1.2 Evolution of errors ..................................................................................... 71  
4.1.3 Forecast error variance ................................................................................ 76  
4.2 Predictive Power ................................................................................................. 77  
4.3 Summary of results ............................................................................................. 86

5 **Predictability Results Part II: Wave-Relative Framework** ................................... 106  
5.1 Individual cases of study ..................................................................................... 107  
5.2 Evolution of variables, errors and variance forecasts for all cases ............... 123  
5.3 Joint distributions, error distributions, and lag-distributions ......................... 126  
5.4 Wave-relative variance prediction .................................................................... 133  
5.5 Wave-relative Predictive Power ........................................................................ 137

6 **Conclusions** ................................................................................................................................. 173  
6.1 Analysis of PREDICT dropwindsonde data ..................................................... 174  
6.2 Basin-wide predictability ..................................................................................... 177
LIST OF FIGURES

Figure 2.1: Map of all dropwindsonde deployment locations during PREDICT and corresponding genesis categories, from August 15 through September 30, 2010................................................................. 43

Figure 2.2: Plots of sounding locations relative to the center of circulation in polar (km, deg) coordinates for each genesis category: (A) genesis, (B) non-genesis and (C) TC stage.................................................. 44

Figure 2.3: Geographic region of genesis domain for Chapter 4 (blue), with region excluded from computations by landmask (left of red line)................................. 45

Figure 2.4: Depiction of the search algorithm, with (A) circulation contours and (B) thickness anomaly contours. Centers of maximum circulation and local maxima in thickness anomaly are found within 1000 km radius (red circle) of the maximum ensemble mean circulation (black star). The maximum ensemble mean circulation is allowed to vary up to 1000 km from the verifying point of genesis (black diamond).............................................................. 45

Figure 3.1: Composite vertical profiles of anomalies relative to the PREDICT mean of (A) temperature, (B) mixing ratio, (C) relative humidity, and (D) tangential component of wind for genesis, non-genesis and TC stage categories.................... 62

Figure 3.2: GOES satellite data in 30-min resolution composited over 6-hour time windows centered temporally on the mean time of each dropwindsonde mission to the nearest half hour, composited over multiple missions for each genesis category. The percentage of total time in each 10x10 km grid point remains below -50°C is depicted for (A) genesis, (B) non-genesis, and (C) TC stage categories (courtesy Dave Ahijevych)............................................................... 63

Figure 3.3: Radial profiles of temperature anomalies (°C) with respect to the PREDICT mean. Data are azimuthally averaged in annuli of 100 km radius for (A) 72+ hours pre-genesis, (B) 48-72 hours pre-genesis, (C) 24-48 hours pre-genesis, and (D) 0-24 hours pre-genesis......................................................... 64

Figure 3.4: Composite vertical profiles of anomalies relative to the PREDICT mean of (A) mixing ratio within 150 km of the center of circulation, (B) relative humidity within 150 km of the center of circulation, (C) radial component of wind at all radii, and (D) tangential component of wind at all radii for 72+ hours pre-genesis, 48-72 hours pre-genesis, 24-48 hours pre-genesis, and 0-24 hours pre-genesis categories................................................................. 65

Figure 3.5: Relative vorticity (s⁻¹) computed in radial coordinates for (A) 72+ hours pre-genesis, (B) 48-72 hours pre-genesis, (C) 24-48 hours pre-genesis, and (D) 0-24 hours pre-genesis......................................................... 66
Figure 3.6: GOES satellite data, as in Figure 3.2 but for (A) 72+ hours pre-genesis, (B) 48-72 hours pre-genesis, (C) 24-48 hours pre-genesis, and (D) 0-24 hours pre-genesis (courtesy Dave Ahijevych). ................................................................. 67

Figure 4.1: Monthly-mean 850-200 hPa wind shear (m s$^{-1}$) for 2010, for the months of (A) June, (B) July, (C) August, (D) September, (E) October, and (F) November. Data to the left of the white land mask line are excluded from calculations. ........................................................................................................... 87

Figure 4.2: Monthly-mean 700 hPa relative humidity (%) for 2010, for the months of (A) June, (B) July, (C) August, (D) September, (E) October, and (F) November ........................................................................................................................................ 88

Figure 4.3: Monthly-mean 850-700 hPa circulation (s$^{-1}$) for 2010, for the months of (A) June, (B) July, (C) August, (D) September, (E) October, and (F) November ........................................................................................................................................ 89

Figure 4.4: Spatial distribution of (A, D) 48 h, (B, E) 96 h, and (C, F) 192 h forecast errors for (A-C) circulation (s$^{-1}$) and (D-F) relative humidity (%), averaged from June-November 2010. Forecast errors are for the control run of the ECMWF ensemble. ........................................................................................................................................ 90

Figure 4.5: Error growth for (A) global forecasts of 500 hPa heights re-plotted from Lorenz (1982), and (B-D) from 2010 using the ECMWF ensemble, for (B) 850-700 hPa circulation, (C) 700 hPa relative humidity, and (D) 850-200 hPa wind shear. Forecast errors are for the control run of the ECMWF ensemble. A linear best-fit line (dashed) is extrapolated from the day 4-6 error growth in each curve. ........................................................................................................................................ 91

Figure 4.6: Comparison of error growth rates for (A) 850-700 hPa circulation, (B) 850 hPa vorticity, (C) 200 hPa divergence, (D) 200 hPa velocity potential, (E) 850-200 hPa wind shear, and (F) 700 hPa relative humidity. Forecasts are made using the ECMWF control forecast (blue), the ECMWF ensemble mean (red), a climatological forecast (green), and a persistence forecast (magenta). ................. 92

Figure 4.7: Evaluation of ensemble variance prediction for 850-200 hPa shear forecasts. (A) Standard deviation of ensemble forecasts versus the absolute error of the ensemble mean forecast as a function of forecast lead-time, in days (colored); Ensemble standard deviation versus error of the ensemble mean forecast (magenta dots), ensemble-mean forecast error standard deviations in 10 equal-sized bins (black circles), and the 1-to-1 line (dashed) for (B) 0-72 h, (C) 84-156 h and (D) 168-240 h. ........................................................................................................................................ 93

Figure 4.8: As in Figure 4.7, but for 850-700 hPa circulation ........................................................................................................................................ 94

Figure 4.9: Probability density functions for climatological differences from the mean or "climatological errors" (A, B) from 1979-2011, and 120 h forecast error distributions (C, D) from Jun-Nov 2010, with Gaussian approximation based upon variance of sample (solid line). Probability density functions are for 850-700 hPa circulation (A, C) and for 700 hPa relative humidity (B, D). .................. 95
Figure 4.10: Diagnosing the predictability of 850-700 hPa circulation and 850 hPa vorticity using Predictive Power. (A) Predictive Power of circulation for each 0000 and 1200 UTC forecast from Aug (blue), Sep (magenta), and Oct (red) 2010. (B) Same as (A), but with the 95% confidence interval and 5% significance level included and individual forecasts removed. (C) The eigenvalue spectrum of forecast error covariance for 10-day through 0-day circulation forecasts of circulation (colored) valid 0000 UTC 09-15-2010, compared to the eigenvalue spectrum of the climatology (black). (D) Same as (B) but for vorticity. In (A), (B) and (D), the mean for all forecasts is indicated by the solid black line. 

Figure 4.11: Ensemble mean 850-700 hPa circulation (magenta contours, $2.5 \times 10^{-5}$ s$^{-1}$ increments) and circulation forecast error variance (shaded) for a low-uncertainty 156 h forecast (A) valid 0000 UTC 09-05-2010, and a high uncertainty 72 h forecast (B) valid 0000 UTC 09-14-2010. Error variance for circulation from (C) ERA-I 1979-2011 is compared against the mean (D) 0 h, (E) 120 h and (F) 240 h forecast error variance from the ECMWF ensemble for Aug-Oct 2010. 

Figure 4.12: (A) Predictive Power for 850-200 hPa wind shear for consecutive 10-day time intervals from Sep 2010. 500 hPa geopotential height anomalies (m) for (B) Sep 1-10 and (C) Sep 21-30, 2010. 

Figure 4.13: Phase diagram of the RMM index, from July (red), August (green) and September (blue) 2010 with dates labeled. Points within the inner circle denote weak or no MJO. Available online at http://www.bom.gov.au/climate/mjo. 

Figure 4.14: (A) 1$^{st}$ and (B) 50$^{th}$ eigenvectors of 120 h forecast error for 850-200 hPa wind shear. 

Figure 4.15: (A) Predictive Power for 700 hPa relative humidity from Aug-Oct 2010, with the 95% confidence interval and the 5% significance level. The 180 h forecast error variance during those 3 months (B) as well as the 1979-2011 climatological error variance (C) for relative humidity are also indicated. 

Figure 4.16: (A) Predictive Power for 200 hPa divergence from Aug-Oct 2010, with the 95% confidence interval and the 5% significance level included. The 24 h (B) and 240 h (C) mean forecast error variance for divergence during those 3 months are indicated. 

Figure 4.17: Ensemble mean forecast 200 hPa divergence (grey contours in $1 \times 10^{-5}$ s$^{-1}$ increments; solid = positive, dashed = negative) and forecast error variance for (A) 2010-08-24 0000 UTC forecast hour 24 (valid 2010-08-25 0000 UTC) and for (C) 2010-09-07 0000 UTC forecast hour 36 (valid 2010-09-08 1200 UTC); GOES-12 and EUMETSAT-8 composite infrared satellite images from (B) 2010-08-25 at 0000 UTC and (D) 2010-09-08 at 1200 UTC (http://catalog.eol.ucar.edu/predict/). 

Figure 4.18: as in Figure 4.16 but for 200 hPa velocity potential. 

Figure 4.19: as in Figure 4.16 but for 850-200 hPa thickness anomaly.
Figure 5.1: Ensemble forecast distributions as a function of lead time (h, x-axis) valid at time of genesis (1200 UTC 25 Aug 2010) for Hurricane Earl, with (A) circulation (s^{-1}), (B) core RH (%) at 700 hPa, (C) environmental 850-200 hPa wind shear (ms^{-1}), and (D) environmental RH (%) at 700 hPa. Each circle represents a different ensemble member with forecast circulation values of <3x10^{-5} s^{-1} (black), ≥3x10^{-5} s^{-1} (blue), ≥6x10^{-5} s^{-1} (green), and ≥9x10^{-5} s^{-1} (magenta). The ensemble mean forecast is also shown (black line).

Figure 5.2: Ensemble mean RH at 700 hPa (contoured), forecast variance for RH (shaded), and center of the tropical wave in each ensemble member (red stars). Included are (A) a 24 h forecast valid at the time of genesis of Earl from 1200 UTC 24 Aug 2010, (B) a 24 h forecast valid at the time of genesis of Fiona from 1200 UTC 29 Aug 2010, (C) a 168 h forecast for Earl from 1200 UTC 18 Aug 2010, and (D) a 108 h forecast for Earl from 0000 UTC 21 Aug 2010.

Figure 5.3: As in Figure 5.1, but for the genesis of Tropical Storm Hermine. Forecasts valid 0000 UTC 06 Sep 2010.

Figure 5.4: Ensemble forecast distributions as a function of lead time (h, x-axis) valid at time of genesis (1200 UTC 30 Aug 2010) for Tropical Storm Fiona, with (A) circulation (s^{-1}), (B) core RH (%) at 700 hPa, (C) environmental 850-200 hPa wind shear (ms^{-1}), and (D) GOES-12 and EUMETSAT-8 composite infrared satellite image of the easterly wave that spawns Fiona exiting the west coast of Africa 108 h prior to genesis (0000 UTC 26 Aug 2010; http://catalog.eol.ucar.edu/predict/). In (A-C), each circle represents a different ensemble member with forecast circulation values of <3x10^{-5} s^{-1} (black), ≥3x10^{-5} s^{-1} (blue), ≥6x10^{-5} s^{-1} (green), and ≥9x10^{-5} s^{-1} (magenta). The ensemble mean forecast is also shown (black line).

Figure 5.5: As in Figure 5.1, but for the genesis of Hurricane Igor. Forecasts valid 1200 UTC 08 Sep 2010.

Figure 5.6: Composite circulation forecasts for all ensemble members valid 48 h prior to the genesis of Igor (A, C) and at the time of genesis of Igor (B, D), contoured in 3x10^{-5} s^{-1} (blue), 6x10^{-5} s^{-1} (green), and 9x10^{-5} s^{-1} (magenta) increments. Included are (A) a 120 h forecast and (B) a 168 h forecast initialized 1200 UTC 01 Sep 2010, and (C) a 0 h forecast and (D) a 48 h forecast initialized 1200 UTC 06 Sep 2010.

Figure 5.7: The predicted location of Igor at time of genesis relative to gradients of moisture (A) and gradients of shear (B) in 36 h forecasts initialized 0000 UTC 07 Sep 2010. Included are (A) ensemble mean RH at 700 hPa (contoured), forecast variance for RH (shaded), and (B) ensemble mean 850-200 hPa shear (black contours), ensemble mean streamlines of 850-200 hPa shear (grey streamlines), and variance of shear (shaded). The center of the tropical wave in each ensemble member is indicated (red stars).

Figure 5.8: Ensemble forecast distributions as a function of lead time (h, x-axis) valid at time of genesis (1200 UTC 22 Jul 2010) for Tropical Storm Bonnie, with
(A) circulation (s\(^{-1}\)), (B) core RH (%) at 700 hPa, (C) environmental 850-200 hPa wind shear (ms\(^{-1}\)), and (D) core 200 hPa divergence (s\(^{-1}\)). Each circle represents a different ensemble member with forecast circulation values of <3x10\(^{-5}\) s\(^{-1}\) (black), ≥3x10\(^{-5}\) s\(^{-1}\) (blue), ≥6x10\(^{-5}\) s\(^{-1}\) (green), and ≥9x10\(^{-5}\) s\(^{-1}\) (magenta). The ensemble mean forecast is also shown (black line).

Figure 5.9: Composite circulation forecasts for all ensemble members valid 48 h prior to the genesis of Bonnie (A, C) and at the time of genesis of Bonnie (B, D), contoured in 3x10\(^{-5}\) s\(^{-1}\) (blue), 6x10\(^{-5}\) s\(^{-1}\) (green), and 9x10\(^{-5}\) s\(^{-1}\) (magenta) increments. Included are (A) a 36 h forecast and (B) an 84 h forecast initialized 0000 UTC 19 Jul 2010, and (C) a 24 h forecast and (D) a 72 h forecast initialized 1200 UTC 19 Jul 2010.

Figure 5.10: As in Figure 5.1, but for the genesis of Tropical Storm Gaston. Forecasts valid 1200 UTC 01 Sep 2010.

Figure 5.11: Composite circulation forecasts for all ensemble members valid 48 h prior to the genesis of Gaston (A, B, C) and at the time of genesis of Gaston (D), contoured in 3x10\(^{-5}\) s\(^{-1}\) (blue), 6x10\(^{-5}\) s\(^{-1}\) (green), and 9x10\(^{-5}\) s\(^{-1}\) (magenta) increments. Included are (A) a 24 h forecast from 1200 UTC 29 Aug 2010, (B) a 12 h forecast from 0000 UTC 30 Aug 2010, (C) a 0 h forecast from 1200 UTC 30 Aug 2010, and (D) a 60 h forecast from 0000 UTC 30 Aug 2010.

Figure 5.12: As in Figure 5.1, but for the genesis of Hurricane Lisa. Forecasts valid 0000 UTC 21 Sep 2010.

Figure 5.13: Ensemble forecast distributions as a function of lead time (h, x-axis) valid at time of genesis (1200 UTC 28 Sep 2010) for Tropical Storm Nicole, with (A) circulation (s\(^{-1}\)), (B) core RH (%) at 700 hPa, and (C) environmental 850-200 hPa wind shear (ms\(^{-1}\)), where each circle represents a different ensemble member with forecast circulation values of <3x10\(^{-5}\) s\(^{-1}\) (black), ≥3x10\(^{-5}\) s\(^{-1}\) (blue), ≥6x10\(^{-5}\) s\(^{-1}\) (green), and ≥9x10\(^{-5}\) s\(^{-1}\) (magenta) and the ensemble mean forecast is indicated (black line). Also included is (D) the ensemble mean 850-200 hPa shear (black contours), ensemble mean streamlines of 850-200 hPa shear (grey streamlines), variance of shear (shaded), and the center of the tropical wave in each ensemble member is indicated (red stars) in an 84 h forecast from 0000 UTC 25 Sep 2010.

Figure 5.14: Shaded 228 h ensemble forecast probabilities valid 1200 UTC 28 Sep 2010 of (A) 700 hPa RH > 70%, with 700 hPa RH contours from control forecast, and (B) probability of 850-200 hPa wind shear > 10 ms\(^{-1}\) with 850-200 hPa mean streamlines.

Figure 5.15: (A,B) 48 h ensemble-mean forecasts valid 1200 UTC 28 Sep 2010 and (C,D) 84 h ensemble mean forecasts valid 1200 UTC 30 Sep 2010 of shaded 850 hPa relative vorticity (x10\(^{4}\) s\(^{-1}\)), contours of 850 hPa heights, and 850 hPa wind vectors for the 10 members with the strongest (A,C) and weakest (B,D) circulations at time of genesis of Tropical Storm Nicole.

Figure 5.16: Ensemble-mean (A) circulation (s\(^{-1}\)), (B) circulation errors (s\(^{-1}\)), and (C) forecast variance for circulation (s\(^{-1}\)) as a function of lead time (x-axis) for each
of 21 genesis events from 2010 (dotted lines), as well as the mean for all events (solid line) .................................................................................................................. 154

Figure 5.17: As in Figure 5.16, but for 200 hPa core divergence ($s^{-1}$). ......................... 155

Figure 5.18: Ensemble-mean (A) environmental 700 hPa RH (%), (B) core 700 hPa RH (%), (C) environmental 700 hPa RH error (%), and (D) core 700 hPa RH error (%) as a function of lead time (x-axis) for each of 21 genesis events from 2010 (dotted lines), as well as the mean for all events (solid line)..................... 156

Figure 5.19: Ensemble-mean (A) 850-200 hPa shear ($ms^{-1}$), (B) 850-500 hPa shear ($ms^{-1}$), (C) 850-200 hPa shear error ($ms^{-1}$), and (D) forecast variance for 850-200 hPa shear ($ms^{-1}$) as a function of lead time (x-axis) for each of 21 genesis events from 2010 (dotted lines), as well as the mean for all events (solid line). All quantities are environmental ($300 \leq r \leq 1000 \text{ km}$) averages about the center of circulation in each ensemble member. ........................................................................ 157

Figure 5.20: (A) Joint distribution, (B) normalized joint distribution, and (C) error distribution of circulation versus 700 hPa core RH; (D) normalized joint distribution of circulation versus 700 hPa environmental RH. In (B) and (D), the total number of elements in each column are labeled (magenta). ................. 158

Figure 5.21: Error joint distributions of circulation versus 700 hPa core RH for (A) 0-72 h, (B) 84-156, and (C) 168-240 h forecast lead times. .............................................. 159

Figure 5.22: Composite 850-200 hPa environmental steering flow (black) and the mean vector wind (red) for all 120 h forecasts valid at time of genesis from 2010, within 1000x1000 km boxes, centered on (A) the 5 ensemble members with the strongest predicted circulation, and (B) the 5 ensemble members with the weakest predicted circulation.......................................................................................................................... 160

Figure 5.23: (A) Normalized joint distribution of circulation versus latitude, (B) normalized joint distribution of 700 hPa environmental RH versus latitude, (C) joint distribution of 700 hPa core RH versus 700 hPa environmental RH, and (D) 3-dimensional joint distribution of 700 hPa core RH, 700 hPa environmental RH, and circulation. In (A) and (B), the total number of elements in each column are labeled (magenta). .............................................. 161

Figure 5.24: Normalized joint distributions of circulation versus (A) 200 hPa core divergence and (B) 850 hPa core convergence; normalized joint distribution of 200 hPa core divergence and 700 hPa core RH; (D) error distribution of 200 hPa core divergence versus 700 hPa error. In (A-C), the total number of elements in each column are labeled (magenta). .............................................. 162

Figure 5.25: (A) Normalized joint distribution and (B) error distribution of thickness anomaly versus circulation. ................................................................................ 163

Figure 5.26: (A) Normalized joint distributions of: (A) 850-200 hPa environmental wind shear versus latitude; (B) circulation versus 850-200 hPa shear; (C) circulation versus 850-500 hPa shear; (D) circulation versus 850-200 hPa zonal shear; (E) 700 hPa environmental RH versus 850-200 hPa zonal wind shear;
(F) 850-200 hPa shear versus 850-200 hPa zonal shear. The total number of elements in each column are labeled (magenta). ................................................ 164

Figure 5.27: Normalized lagged distributions: (A) circulation at time of genesis vs circulation 48 h prior to genesis; (B) latitude at time of genesis vs latitude 48 h prior to genesis; (C) circulation at time of genesis vs 700 hPa core RH 48 h prior to genesis; (D) circulation 48 h prior to genesis vs 700 hPa core RH at time of genesis; (E) 200 hPa divergence at time of genesis vs 700 hPa core RH 48 h prior to genesis; (F) 200 hPa divergence 48 h prior to genesis vs 700 hPa core RH at time of genesis. The total number of elements in each column are labeled (magenta). ............................................................................................... 166

Figure 5.28: Evaluation of ensemble variance prediction for wave-relative circulation forecasts. (A) Standard deviation of ensemble forecasts versus the absolute error of the ensemble mean forecast as a function of forecast lead-time, in days (colored); Ensemble standard deviation versus error of the ensemble mean forecast (magenta dots), 1st confidence interval of the ideal distribution of the sample (whiteish-magenta cone). ensemble-mean forecast error standard deviations in 10 equal-sized bins (black circles), and the 1-to-1 line (dashed) for (B) 0-72 h, (C) 84-156 h and (D) 168-240 h. ................................................ 168

Figure 5.29: As in Figure 5.26, but for 700 hPa environmental RH......................... 169

Figure 5.30: As in Figure 5.26, but for 200 hPa core divergence................................. 170

Figure 5.31: As in Figure 5.26, but for 850-200 hPa environmental wind shear. ....... 171

Figure 5.32: Ensemble-mean wave-relative Predictive Power calculation for 8 genesis events from September 2010 within a 1000x1000 km box centered on the tropical wave for (A) circulation, (B) 700 hPa RH, (C) 850-200 hPa wind shear, and (D) 200 hPa divergence. ................................................................. 172
LIST OF TABLES

Table 2.1: Cases of study during PREDICT comprising the genesis, non-genesis, and TC-stage groups with corresponding dates of G-IV deployments. Timing of mission prior to genesis is included for cases in which genesis occurred. Latitude and longitude of the target center location of each drop pattern, meridional phase speed \( (U_p) \) of the wave, and the number of dropwindsones released are shown. ................................................................. 42

Table 3.1: PREDICT, genesis, non-genesis and TC-stage mean mixing ratio \( (g \text{ kg}^{-1}) \) values, standard deviations \( (\sigma) \), and anomalies versus the PREDICT mean for select levels from 1000 to 200 hPa. ................................................................. 59

Table 3.2: Instability data for different categories. Included are the LFC, EL, CAPE, CIN, standard deviations \( (\sigma) \) of CAPE, and CAPE anomalies versus the PREDICT mean. ................................................................. 60

Table 3.3: 850-500 hPa and 850-200 hPa vertical wind shear values \( (\text{m s}^{-1}) \) and standard deviations \( (\sigma) \) for genesis, non-genesis and TC stage categories. Wind shear data for 72+ hours pre-genesis, 48-72 hours pre-genesis, 24-48 hours pre-genesis, and 0-24 hours pre-genesis subsets also included. .................. 61
Chapter 1

Introduction

1.1 Tropical cyclogenesis - the current state of the science and observations

Tropical cyclogenesis, or the formation of a tropical cyclone (TC), has been a challenge to scientific understanding and prediction for decades (Emanuel 2005). The exact sequence of events culminating in tropical cyclogenesis remains heavily debated, despite well-known necessary conditions associated with an environment that supports genesis. Dynamically favorable conditions include: background cyclonic vorticity, 850-200 hPa tropospheric wind shear of less than 15 m s$^{-1}$ and preferably below 10 m s$^{-1}$, and a sufficiently high Coriolis parameter (Gray 1968). Thermodynamically favorable conditions have also been identified; for a review, please see Raymond et al. (2011) or Nolan and McGauley (2012). While an environment that is favorable for genesis can be predicted many days in advance, the genesis event itself tends to be much less predictable.

The first obstacle we face in improving our understanding of tropical cyclogenesis is an inability to differentiate the often subtle physical differences between developing and non-developing tropical cyclones (Dunkerton et al. 2009). This is primarily a consequence of a lack of observations of the genesis process. Before 2010, field campaigns with observations of tropical cyclogenesis were limited to: the Tropical Experiment in Mexico (TEXMEX; Bister and Emanuel 1997, Raymond et al. 1998), the
Tropical Cloud Systems and Processes (TCSP) experiment in 2005 (Halverson et al. 2007), the NASA component of the African Monsoon Multidisciplinary Analyses (AMMA) project in 2006 (Zipser et al. 2009), the Tropical Cyclone Structure experiment in 2008 (TCS-08, Elsberry and Harr 2008), and a few observations from the RAINEX experiment of 2005 (Houze et al. 2006). However, in these studies, observations of tropical cyclogenesis were often secondary to rapid intensification, structure of mature TCs, or sampling the Saharan Air Layer (SAL).

Nonetheless, data gathered from TC genesis events during these field campaigns has advanced the science of tropical cyclogenesis. Zipser et al. (2009) emphasized the difficulty of achieving genesis in excessively dry, dusty airmasses. Ritchie and Holland (1997), Davis et al. (2008), Houze et al. (2009), and Braun et al. (2010), showed that the progressive strengthening of a mid-level vortex, a gradual moistening of the column in a region of deep convection, and the development of a warm core were all evident in observations of various tropical cyclones during and shortly following genesis. While these studies alluded to the development of a warm core, the altitude of the warm core maxima and the timing of the development of the warm core were generally neglected in these studies. Earlier observational studies such as La Seur and Hawkins (1963) and Hawkins and Rubsam (1968) found maximum warm anomalies at around 250 hPa in mature TCs, while Hawkins and Imbembo (1976) and Stern and Nolan (2012) suggested that the primary warm core is located from 500 hPa to as low as 650 hPa. The level of maximum warm anomalies for pre-genesis disturbances remains to be determined.

The most recent of the field campaigns involving genesis, known as the PRE-Depression Investigation of Cloud-systems in the Tropics (PREDICT) experiment, was an
expansive investigation of tropical cyclogenesis in the Atlantic during the 2010 season (Montgomery et al. 2012). One of the overarching goals of PREDICT was to gather in-situ observations necessary to examine the marsupial theory of genesis (Dunkerton et al. 2009). The marsupial paradigm proposes that tropical depression formation from a pre-depression wave trough in the lower troposphere is greatly favored within the critical-layer “pouch”, a region of closed material contours wherein the parent easterly wave’s phase speed equals the mean flow. A young disturbance within the pouch is repeatedly moistened by deep moist convection within the critical layer while remaining somewhat protected from lateral intrusion of dry air and vertical shear due to its location near the critical latitude. This proto-vortex is co-located with the critical latitude, which is the latitude within the critical layer in that the pouch-relative (Lagrangian) horizontal wind is equal to zero. The disturbance is thereby able to keep pace with the parent wave until it has strengthened into a self-maintaining entity.

Ultimately, the PREDICT experiment has provided the most expansive in-situ dataset comprising both developing and non-developing tropical waves prior to genesis, complemented by simultaneous observations from the Intensity Forecasting Experiment (IFEX) and Genesis and Rapid Intensification Processes (GRIP) campaigns. During PREDICT, the NSF/NCAR G-V aircraft provided once- and occasionally twice-daily sampling of tropical waves for up to five consecutive days. Unlike in previous field campaigns, except for perhaps TCS-08, it was possible to target a potential genesis region much smaller than an entire tropical wave by targeting the critical layer. In total, 25 flight missions were completed between August 15 and September 30, 2010 (plus one calibration flight). The strength of this dataset comes not only from the unprecedented quantity of
developing and non-developing tropical waves sampled, but also from the temporally-evolving nature of data associated with distinct pouches.

Two differing views of tropical cyclone formation are the top-down and the bottom-up hypotheses. Ritchie and Holland (1997) and Simpson et al. (1997) described a top-down mechanism for genesis by which successive mergers of mesoscale convective systems (MCSs) increase the size and/or strength of the mid-level vortex, which induces a surface circulation through vertical penetration and vortex stretching. Similarly, Bister and Emanuel (1997) proposed that a stratiform rain region associated with an existing MCS acts to moisten and cool the mid to lower levels. The level of peak cooling descends within the stratiform rain region, thereby lowering the level of maximum potential vorticity (PV) production, while moistening acts to limit the occurrence of dry downdrafts. This mid-level vortex creates a cold core at low levels, which alters deep convection as to facilitate spin-up. Along with the necessity of a strengthening mid-level circulation, Nolan (2007) also found humidification of the inner core due to moist detrainment and precipitation from deep convective towers preceding genesis. However, Nolan (2007) did not necessitate a top-down genesis process.

While the Ritchie and Holland (1997) and Bister and Emanuel (1997) top-down framework is appealing, there have been no observations to support the idea of cyclonic relative vorticity descending in a tropical cyclone. In fact, theoretical analysis shows that this is impossible (Tory and Montgomery 2006). However, this does not preclude the general idea of the existence of a mid-level vortex preceding the development of a low-level vortex. Raymond et al. (2011) suggested that the formation of a strong midlevel
circulation, with its associated cold core at low levels and warm core aloft, greatly aids the spin-up of a tropical cyclone by changing the profile of vertical mass flux,

\[ M = \rho w \]  

from top heavy to bottom heavy as \( w \) increases at low levels. This in turn maximizes \( \frac{\partial M}{\partial z} \) at the low levels, and by continuity, results in horizontal convergence \( \nabla_h \cdot \mathbf{U} \). Vorticity is thereby enhanced at the center of circulation as this increase in horizontal convergence occurs in an environment of pre-existing cyclonic vorticity. This process also acts to suppress the lateral export of moist entropy by deep convective inflows and outflows from the core of the developing system.

A slightly differing sequence, known as bottom-up genesis, was proposed by Hendricks et al. (2004) and Montgomery et al. (2006), in which individual deep moist convective updrafts or vortical hot towers (VHTs) develop within the tropical wave, amplify pre-existing cyclonic vorticity, and gradually consolidate to form a low-level center of circulation. The physical mechanism by which VHTs induce genesis can be understood in terms of the prognostic potential vorticity (PV) equation in material form (Holton 1992):

\[
\frac{D(PV)}{Dt} = \frac{\zeta_a}{\rho} \cdot \nabla \hat{\theta} + \frac{1}{\rho} (\nabla \times F) \cdot \nabla \theta
\]  

(1.2)

where \( \zeta_a \) is the absolute vorticity, \( \hat{\theta} \) is the change in potential temperature with time or heating term, and \( F \) is friction. Hendricks et al. (2004) perform a scale analysis for convective hot towers and find that the dominant terms are

\[
\frac{D(PV)}{Dt} = \frac{f + \zeta}{\rho} \frac{\partial \hat{\theta}}{\partial z}
\]  

(1.3)
Hence, PV increases with time below the level of maximum latent heating (i.e. $\frac{\partial h}{\partial z} > 0$) and PV decreases above. If these hot towers are also vortical, a complementary process will occur in which cyclonic low-level vorticity is vertically stretched by rising surface parcels, thereby increasing positive vorticity at the middle levels. Finally, latent heat released within these VHTs also aids in the development of the mid-level warm core.

While the top-down and bottom-up hypotheses have generally been defined and studied as independent processes in the literature, in reality it likely that some compromise between the two is the most common scenario. Observational evidence supporting a top-down mechanism for genesis is presented by Ritchie and Holland (1997), Mapes and Houze (1995) and Raymond et al. (2011), while Houze et al. (2009) find evidence that supports the VHT argument. Some commonality between the two theories could occur as follows: a seemingly disorganized pattern of convective towers gradually consolidates into an organized convective system within a pre-existing region of cyclonic circulation maximized at the mid-levels. Simultaneously, changes in the vertical mass flux profile of deep convection enhances low-level convergence of mass and vorticity, while suppressing lateral export of moist entropy by deep convective inflows and outflows from the core of the developing system. Once there is sufficient saturation for updrafts to exceed downdrafts, latent heat released by convective hot towers will generate positive PV below and negative PV above the layer of maximum heating, strengthening the low-level cyclone and generating an upper-level anticyclone. The mid-level vortex continues to strengthen as vorticity is stretched vertically in regions of convection through (and beyond) the time of genesis.
Regardless of the exact genesis mechanism, a favorable environment is critical in order for genesis to occur. A significant environmental factor constraining the favorability of the environment for genesis is wind shear. As previously stated, Gray (1968) asserted the claim that low tropospheric wind shear values tend to be more favorable to TC genesis than greater wind shear. Bracken and Bosart (2000) and Nolan and McGauley (2012, hereafter NM12) examine the climate record and find that genesis historically occurs most frequently with shear between 5 and 10 ms$^{-1}$, and significantly less frequently with very low shear near 0 ms$^{-1}$. However, once NM12 account for the fact that very low shear occurs extremely infrequently, they determine that 2.5-3.75 ms$^{-1}$ shear is this most favorable condition. While 0 ms$^{-1}$ shear is not the most favorable condition, it is nonetheless quite favorable and comparable to a 7.5 ms$^{-1}$ shear environment. This is because a small amount of shear helps to focus the convection to one side of the pre-genesis vortex, allowing for a more concentrated region of latent heating and release of moisture than would otherwise occur in the presence of zero shear. NM12 and Cheung (2004) also find from the climatological record that easterly shear is significantly more favorable to genesis than westerly shear, with peak favorableness from 5-7.5 ms$^{-1}$. However, the fact that easterly shear is found to be more favorable to genesis may simply be due to the covariance of favorable environmental parameters, since easterly shear occurs more frequently in environments with high SSTs and high RH (Tory and Frank 2010). NM12 confirm that easterly shear regions are typically associated with higher maximum potential intensity (MPI, Emanuel 1995) than westerly shear regions.

NM12 also explore the physical process of TC genesis in wind shear using idealized, high-resolution numerical simulations. Genesis occurs most rapidly in the
presence of 5 ms\(^{-1}\) westerly shear, where the shear acts to organize convection into a mesoscale convective vortex displaced to one side of the circulation center. However, continued intensification beyond genesis is most pronounced in the case of 2.5 ms\(^{-1}\) westerly shear, where the distribution of convection more evenly about the entire circulation ultimately results in a more organized tropical cyclone. Westerly shear is more favorable than easterly shear, under the realistic assumption that surface winds are from the east, because convection develops on the north side of the circulation superimposed with the region of greatest surface fluxes. Finally, the effects of varying moisture in the presence of varying amounts of shear are also presented in NM12. The primary result is that dry air perturbations in conjunction with even modest 5 ms\(^{-1}\) westerly shear are hostile and even prohibitive to TC genesis. However, in the absence of shear, genesis can still occur in idealized simulations with a \(-40\%\) RH perturbation at 600 hPa to the initial sounding (Dunion 2010 moist-tropical sounding). Overall, these experiments reveal that, all other environmental parameters equal, westerly shear is more favorable for genesis than easterly shear or zero shear and that dry air in conjunction with shear is particularly hostile for genesis.

In addition to dynamical constraints such as wind shear, there are also thermodynamic conditions of varying degrees of favorability for genesis. By National Hurricane Center definitions, in order for genesis to occur, the cyclone must exhibit sustained deep convection. Therefore, it is natural to hypothesize that there exists some minimum instability requirement for genesis. Using in-situ data, Molinari and Vollaro (2010) find that highly sheared, generally weaker tropical cyclones tend to be associated with higher convective available potential energy (CAPE) than their non-sheared,
generally stronger counterparts. Similarly, Braun (2010) found higher CAPE in environments for weakening TCs compared to strengthening TCs in the days following genesis. In idealized numerical simulations, Nolan et al. (2007) found that greater Maximum Potential Intensity (MPI) resulted in greater likelihood of genesis, while greater CAPE did not. Nonetheless, the question of whether genesis becomes increasingly favored with increasing instability, or whether there is some threshold beyond which decreasing stability is detrimental to genesis, has not been conclusively answered via observational evidence.

Two recent papers, Smith and Montgomery (2011) and Davis and Ahijevych (2012), examined PREDICT dropwindsonde data for Tropical Storm (TS) Matthew, Hurricane Karl, and ex-TS Gaston. Smith and Montgomery (2011) found lower values of equivalent potential temperature between the surface and 3 km of non-developing ex-Gaston than in developing pre-Karl and pre-Matthew. The authors found evidence that dry air for the non-developing case was not necessarily associated with stronger downdrafts but rather that the drier mid-level air weakened the convective updrafts. A weakening of convective updrafts results in less vertical stretching of low-level vorticity and less latent heat release, thereby preventing sufficient amplification of system relative vorticity necessary for development. Lastly, greater CAPE and convective inhibition (CIN) were associated with ex-Gaston than either genesis event. Davis and Ahijevych (2012) found that a misalignment of the mid- and low-level circulation centers, due to vertical shear, made TS Gaston more susceptible to intrusion of dry air. They found that Karl and Matthew developed in a more moist environment, with mid- to upper-level moisture
increasing with time. Lastly, an initial vertical misalignment of the vortex delayed genesis of Karl until the vortex could subsequently realign.

In-situ observations have proven critical for verifying model output, supporting or refuting existing theories with truth, and in data assimilation as a means to improve upon the model "first guess". Numerical simulation, on the other hand, makes it possible to predict tropical cyclogenesis prior to its occurrence, and also allows analysis of the full 3-dimensional structure of an event that is only sampled in limited locations and at finite time intervals. While numerous atmospheric phenomena have been simulated and predicted to varying degrees, the difficulty in doing so is not always equal. The most challenging processes to correctly predict are typically those of shorter time scales and/or smaller spatial scales. As such, modeling and the prediction of tropical cyclogenesis remains a difficult task due in part to the stochastic nature of convection and the uncertainty in the feedback of mesoscale processes onto the larger scales. This leads us to the topic of predictability, the concepts from which have been established in the literature for some time but have only recently been applied to the study of tropical cyclones.

1.2 Predictability and ensemble-based forecasting of tropical cyclogenesis

Much of the foundation of our current understanding of predictability begins with the Lorenz (1963) solutions to deterministic ordinary nonlinear differential equations. In this study, it was shown that if a system, such as the atmosphere, contains any non-periodic component, and if the present state of the system is not known to complete accuracy, then the ability to forecast the instantaneous state of the system will eventually deteriorate to
the point at which only the periodic component can be predicted. Lorenz (1965) expanded
upon this idea by introducing a 28-variable atmospheric model and comparing the growth
rate of small initial errors. It was found that the rate of error growth was strongly dependent
upon the circulation pattern. Therefore, certain initial states of the atmosphere are
inherently more predictable than others.

Mesoscale processes play a crucial role in the development of a tropical cyclone,
and the degree to which errors on the mesoscale or even convective scale prevent the model
from developing a TC or failing to do so (“false alarms”) remains largely unexplored.
Lorenz (1969) suggested that even the smallest of errors will result in a loss of
predictability at the convective scales after a few minutes to a few hours, even with an
arbitrarily near-perfect model and assimilation scheme. On the other hand, larger scale
synoptic patterns are theoretically predictable out to a week or more. This was further
demonstrated in a later study in which Lorenz (1982) computed globally averaged errors
of 500 hPa geopotential height forecasts from the ECMWF deterministic model. Whether
or not it is necessary to accurately forecast the mesoscale in order to capture tropical
cyclogenesis remains an open question. Examination of the spatial and temporal scales
associated with error growth may provide some insight. Prior to cloud-resolving modeling,
the fastest growing errors were found to be associated with baroclinic or barotropic
instabilities (Farrell 1985, 1989; Buizza and Palmer 1995). The most rapidly growing
disturbances can also often be attributed to wave phase errors (Snyder 1999), emphasizing
the importance of correctly initializing the position of the waves in a model, whether it be
a baroclinic wave or a tropical wave. Zhang et al. (2007) find that errors in a cloud-
resolving model grow from small-scale convective instability and saturate at the convective
scale on the order of a single hour for a moist baroclinic wave. They find that some of these errors are radiated away from the location of convection in the form of gravity waves, and only after tens of hours to a few days does the character of the errors change from the small scales to large-scale unbalanced motions. However, Rotunno and Snyder (2008) have shown that strong downscale error-energy spreading exists when there is high base-state energy in the small scales. Therefore, both upscale and downscale cascades of errors should be considered. In terms of tropical cyclogenesis, this means that inadequate representation of the parent circulation will lead to errors in the mesoscale details (structure of the core of the system, rainbands, etc.) within the circulation, while errors associated with mesoscale features will also degrade the forecast of the large-scale circulation.

A multi-scale modeling framework is necessary to handle the multi-scale nature of the TC genesis problem. Differences in initial conditions comparable to analysis errors can result in missed genesis forecasts or false alarm forecasts, even at short lead-times (Sippel and Zhang 2008). While a deterministic model is capable of capturing genesis, the use of an ensemble system provides the ability to represent uncertainty in the mesoscale processes, larger-scale uncertainty associated with the tropical wave itself, and uncertainty in the scale interactions between the two. While the convective scales are not resolved by the ECMWF ensemble utilized in this study, uncertainty at the convective scales is accounted for via a stochastic kinetic-energy backscatter scheme (SKEB). These convective scale errors grow rapidly and eventually contribute to mesoscale and synoptic scale errors.

Ensemble forecasts have proven to be a useful forecasting tool for predicting tropical cyclone tracks while conveying a sense of uncertainty through the spread in the
ensemble, as demonstrated by Goerss (2000) and Elsberry and Carr (2000). The 50-member ECMWF ensemble was found to produce reasonable TC intensity and track forecasts (Puri et al. 2001). While the number of studies examining the uncertainty in TC track via ensemble techniques has increased considerably over the last decade, only very recently have studies on the prediction of TC genesis via ensemble methods begun to emerge, due to advancements in numerical modeling and data assimilation (Halperin et al. 2013). In 2013, the National Hurricane Center (NHC) introduced a new operational product issuing the probability of tropical cyclogenesis within 5 days, underlying the increased confidence in genesis prediction\(^1\). Additionally, several recent studies have assessed the ability of ensemble forecasts to accurately predict genesis. Snyder et al. (2010) found greater than 50% overall accuracy of genesis forecasts made by the NCEP global ensemble initialized during the pregenesis phase. In a similar study using the NOGAPS ensemble, Snyder et al. (2011) found that the addition of stochastic convection increases the ability to correctly detect genesis events, although at the cost of a slightly higher false alarm ratio. Examining longer lead times, Tsai et al. (2013) find that 32-day ECMWF ensemble forecasts completely miss very few tropical cyclones, and the ones that are missed are often not significant. That said, the forecast distribution increasingly resembles climatology during weeks 3 and 4, especially during peak season for TC activity. Despite a recent increase in studies that examine the predictability of whether or not genesis will occur, few if any examine the predictability of the pregenesis environment.

\(^1\) Genesis probabilities out to 5 days are provided every 6 h by the NHC on http://www.nhc.noaa.gov/gtwo_atl.shtml
Since the predictability and modeling component of this study will utilize the ECMWF ensemble, it is worth a brief discussion on how the ensemble is generated. A summary of the current operational version of the ECMWF ensemble appears in Lang et al. (2012). Initial perturbations are produced via a combination of Singular Vectors and an ensemble of data assimilations. The Singular Vector technique identifies the fastest-growing linear perturbations within a particular optimization time, which ECMWF sets to be 48 h. The ensemble of data assimilations consists of ten 4D-Var analyses in which observations are perturbed by randomly-generated numbers from a Gaussian distribution corresponding to the observation error variance. While the model is running, a stochastic kinetic-energy backscatter scheme (SKEB) perturbs streamfunction forcing in an attempt to account for uncertainty associated with sub-gridscale convective processes. Lastly, a stochastic perturbation of parameterized tendencies (SPPT) perturbs model tendencies produced by physics parameterizations, as biases or flaws in parameterization schemes may contribute to ensemble forecasts that converge to a particular erroneous solution (Palmer et al. 2009). All of the above techniques have been tuned to produce realistic forecast variance in the extratropics. While the ECMWF ensemble is widely regarded as one of the best in the industry, tuning for the extratropics may contribute to the ensemble having an incorrect (too large or too small) forecast variance in the tropics. Other issues, such as the fact that the total energy norm used in the SV computation does not account for moisture (Leutbecher and Palmer 2008), or the fact that SVs are computed at T42 or ~320 km resolution (Lang et al. 2012), may also lead to unrealistic variance in the tropics, where moist convective-scale processes often dominate.
The majority of predictability studies prior to the 1990s neglect moisture in their models when computing predictability limits. However, doing so may be neglecting a significant source of errors. Ehrendorfer et al. (1999) find faster singular-vector perturbation growth when moisture is included, and Zhang et al. (2002, 2003) have found significant error growth to be associated with moist, small-scale processes in regional simulations with parameterized convection. These studies find that convective-scale errors contaminate the mesoscale within lead times of interest to numerical weather prediction (NWP), effectively eliminating the predictability of the mesoscale. Zhang et al. (2007) later extended the examination to error growth in a convection-permitting (3.3 km resolution) experiment. In a series of numerical integrations of moist baroclinic waves, they find that errors initially grow from small-scale convective instability then quickly saturate on the order of one hour. Thereafter, errors begin to saturate after only a single day for intermediate wavelengths of 200-1000 km. Lastly, the large-scale, balanced components of the errors grow with the background baroclinic instability, albeit not in the tropics. While the degree to which these conclusions apply to tropical cyclogenesis remains to be seen, the fact that moist convection in a convectively unstable environment is a precursor to genesis suggests that parallels can be drawn. The operational ECMWF model used in this study accounts for moisture, and relative humidity will be included as a metric in diagnosing predictability from the model.

An accurate model analysis is necessary to produce a realistic genesis forecast. Correspondingly, an ensemble that produces a meaningful spread of solutions is necessary to convey the uncertainty in the forecast. Several operational forecast centers, including ECMWF, use an advanced data assimilation technique known as four-dimensional
variational data assimilation (4DVAR; Rabier et al. 2000) to initialize their model, and a singular vector technique to initialize ensemble members with perturbations that maximize the fastest-growing linear error growth (Buizza et al. 2010). Output from the operational 51-member ECMWF ensemble (50 perturbations plus 1 control) will be utilized in determining the predictability limit of tropical cyclogenesis in this study. Palmer (2000) demonstrated that the overall performance of an ensemble could be evaluated via a measure of spread versus skill. The philosophy is that predictions are least predictable when the mean state is most unstable (and vice versa). By this logic, forecasts subject to the greatest errors should be associated with the greatest uncertainty. Whether or not forecasts subject to the greatest errors also correspond to forecasts with the most unstable flows (the criteria for uncertainty in aforementioned studies) remains to be seen. Ideally, greater uncertainty will be conveyed through a greater spread of ensemble solutions. Therefore, if the ensemble is conveying meaningful information to the user, then the forecasts with the greatest verifying error will be associated with the greatest predicted forecast error variance.

Numerous predictability studies, including Shukla (1981, 1985), Hayashi (1986), Murphy (1988), and Griffies and Bryan (1997), have all demonstrated that the ratio of root-mean-squared (RMS) errors of a particular model prediction to the RMS error of a climatological prediction provides a measure for the degree of uncertainty in a forecast. Griffies and Bryan (1997) find predictability of a coupled oceanic-atmospheric climate system on the order of a decade or longer. However, for a purely atmospheric system, eddy turnover times or wave domain-crossing times are much shorter, and one would expect a predictability limit to be orders of magnitude less. A generally-observed theme,
demonstrated more rigorously by Palmer (1996) and Palmer et al. (1998), is that the outcome of a particular predictability study is dependent upon choice of metric, which is often chosen arbitrarily. Metrics or indices associated with more slowly-varying oscillations tend to be predictable out to longer timescales than metrics associated with high-frequency variability. It is also important to note that all of these studies assume Gaussian error distributions. In the event that error distributions are non-Gaussian, Anderson and Stern (1996) propose that comparisons can still be made between the error distributions / shape of the probability density function (PDF) of the forecast as compared to climatology.

It is also necessary to assess whether or not the variance of the ensemble forecast distribution is accurately conveying the uncertainty in the forecast. Forecasts associated with greater uncertainty should, on average, be associated with greater ensemble-mean forecast error, and this enhanced uncertainty should be conveyed to the user by enhanced spread in the ensemble forecast. Theoretically, the ensemble forecast standard deviation to forecast error of the ensemble mean ratio should behave following a one-to-one monotonic increasing relationship when averaged over a sufficiently-large number of cases (e.g. Wang and Bishop 2003, Grimit and Mass 2007). In this study, the ability of the ECMWF ensemble to predict variances for a number of different metrics will be analyzed.

Schneider and Griffies (1997) propose a measure called predictive power (PP) that allows one to quantify how similar or dissimilar the forecast PDF is from the climatological PDF. More precisely, PP is a measure of the ratio of the determinants of the ensemble and climatological error covariance matrices, where the determinant of the ensemble error covariance is small at analysis time and eventually grows to reach or even exceed the
determinant of the climatological error covariance matrix. This philosophy is that the PDF of the ensemble forecast, assumed to be close to Gaussian, is initially tightly clustered about a single solution for whatever particular metric is being analyzed. With increasing lead-time, the forecast PDF gradually broadens until the forecast variance resembles or even exceeds a climatological PDF (DelSole 2004).

Complementary to the idea of PP, relative entropy is a metric to quantify predictability that is similar and slightly simpler to calculate (Kleeman 2002). Both PP and relative entropy are calculated from the same climatological and ensemble forecast PDFs. However, while PP is based upon the determinant of the inner product of the forecast and climatological distributions with their own means, relative entropy is based upon the log of the ratio of the forecast distribution to the climatological distribution. There is no right answer as to whether PP or relative entropy is a "better" metric for quantifying predictability, but it would be enlightening to at least investigate whether or not results are consistent between the two metrics. Kleeman (2008) uses relative entropy to diagnose predictability of the mid-latitude atmosphere, finding a quasi-linear decline out 25 days and skill over climatology out to 45 days. However, Kleeman (2008) neglects convective-scale processes, makes a perfect-model assumption, and generates the climatology from a very long run of the model itself. These factors would likely inflate the predictability limit beyond what one would expect when comparing to a real climatology with an imperfect model.

Several studies have utilized PP and similar methods. Stephenson and Doblas-Reyes (2000) found that the Relative Entropy of probabilistic forecasts could be a useful predictor of forecast skill. Schneider and Held (2001) followed a variation of the
predictable component analysis of Schneider and Griffies (1999) to quantify warming in the climate record. Waliser et al. (2003) examined the predictability of the Asian summer monsoon by comparing the ratio of the signal to the mean square error for velocity potential and precipitation. DelSole (2004) presented idealized examples utilizing “predictive information” from the climatological and forecast error covariances. Abramov et al. (2005) utilized Relative Entropy in a low-resolution barotropic model to quantify the effect of a lack of information and information flow associated with leading Empirical Orthogonal Functions (EOFs), which corresponded to the North Atlantic Oscillation, the Pacific-North American pattern, and the Arctic Oscillation. Again with a low-resolution model, Kleeman (2007) used ensemble forecasts to track the flow of Mutual Information to determine synoptically ‘sensitive’ regions. Finally, Giannakis and Majda (2012) quantified skill in long-range ocean forecasting using the PP framework. In all the cited studies, computational constraints restricted the resolution at which PP and related metrics could be calculated. Also, the vast majority of these studies focused on more slowly varying systems than those considered in this study.

Overall, there is an apparent lack of studies on the topic of the predictability of tropical cyclogenesis. One relevant study is Sippel and Zhang (2008), in which the predictability of a non-developing tropical wave in the Gulf of Mexico is assessed using an MM5 ensemble. While this wave failed to develop, several ensemble members did predict genesis. They found that whether or not genesis occurred in individual ensemble members was most sensitive to moisture and CAPE. A more recent study by Torn and Cook (2012) examines the initial-condition sensitivity of two genesis events from the 2010 season, Danielle and Karl, to the environment and to the initial vortex. The genesis of Karl
is found to be most sensitive to the initial vortex, while the genesis of Danielle is found to be more sensitive to the upper-level divergence and wind shear. Here, while we do not seek to investigate any single genesis event to the breadth of Torn and Cook (2012), we will explore a much larger set of genesis events from the 2010 season.

This study will be comprised of three different research chapters all related to tropical cyclone formation and/or predictability. In Chapter 3, PREDICT dropwindsonde data will be analyzed with an emphasis on the physical differences between developing and non-developing tropical waves, as well as the temporal evolution of the genesis cases. In Chapter 4, the predictability of variables relevant to tropical cyclogenesis will be quantified on a large scale, basin-wide domain using the ECMWF ensemble via predictability measures such as error growth, variance prediction, and Predictive Power. Lastly, in Chapter 5, the predictability of tropical cyclogenesis will be examined through a wave-relative framework, again using the ECMWF ensemble. Here, correlations between dynamically-related variables will be computed, along with wave-relative variance prediction and a closer inspection of a variety of individual genesis events. Conclusions will be discussed in Chapter 6.
Chapter 2

Methodology

2.1 Investigating genesis via in-situ data: PREDICT dropwindsondes

During PREDICT, 547 dropwindsondes were deployed over the course of 25 aircraft missions investigating tropical waves in the Caribbean and western Atlantic (Figure 2.1). Five cases of genesis, 3 cases of non-genesis, and four TCs named during or prior to investigation (TC stage) comprise the PREDICT data set (Table 2.1). The TC stage category overlaps with three of the five genesis cases: TSs Fiona, Matthew and Nicole, in addition to an investigation of TS Gaston prior to weakening to a remnant low. Cases are sorted by genesis or non-genesis based upon whether or not the tropical wave under investigation eventually yields a tropical storm / depression as declared by the National Hurricane Center (NHC). The genesis category is further separated temporally into missions that occur 0-24 h pre-genesis, 24-48 h pre-genesis, 48-72 h pre-genesis and 72+ h pre-genesis. Missions that begin during one such time period and end in another are assigned to the period during which the majority of dropwindsondes are deployed. As such, the 0-24 h pre-genesis category includes data from the 8/30 flight into pre-TS Fiona, the 9/14 flight into pre-Hurricane Karl, and the 9/27 flight into pre-TS Nicole. The 72+ h pre-genesis category includes data from the 9/10 and 9/11 flights into pre-TS Karl, the 9/20 flight into pre-TS Matthew, and the 9/30 flight into pre-Hurricane Otto. Tropical Storm
Gaston is a special case. The first mission into Gaston occurred at a time when the storm was already a named system, and therefore data from this mission is assigned to the TC category. However, subsequent PREDICT flights occurred after Gaston was downgraded by the NHC to a remnant cyclone. Since Gaston was no longer a named system during these missions, and was given a 70% chance to redevelop by NHC but failed to do so, missions into the remnants of Gaston are added to the non-genesis category.

Dropwindsonde data are composited into a single vertical profile for each group and genesis time category. Compositing of temperature ($T$), mixing ratio ($q$) and relative humidity ($RH$) involves a simple separation of dropwindsondes by genesis or non-genesis categories, interpolating onto a common pressure grid in 5 hPa increments, and averaging. For stability calculations, the virtual temperature adjustment $T_v = T(1 + \varepsilon q)$ will be applied, where $\varepsilon = 0.608$ when $q$ is expressed in kg kg$^{-1}$. Convective available potential energy will be calculated via

$$CAPE = g \int_{z_{LFC}}^{z_{EL}} \left( \frac{T_{v, parcel} - T_v}{T_v} \right) dz$$

where $T_{v, parcel}$ is the virtual temperature of a surface parcel lifted dry adiabatically below the level of free convection (LFC) and moist adiabatically above. The EL is the equilibrium level or level of neutral buoyancy for a virtual surface-based parcel. Calculation of CIN will be equivalent to (3) except that integration will be from the lowest level of negative buoyancy ($T_{v, parcel} < T_v$) to the LFC (Doswell and Rasmussen 1994).

Vortex-relative tangential ($V_{tan}$) and radial ($V_{rad}$) components of wind are also calculated. Since the tropical waves are moving within some background flow, and winds from the dropwindsonde data are Earth-relative, it is necessary to remove the parent wave’s zonal phase speed, $U_p$ in vortex-relative wind calculations. Zonal phase speed is calculated
from Hovmöller diagrams using a consensus of NCEP Global Forecast System (GFS) and European Centre for Medium-Range Weather Forecasts (ECMWF) analyses and forecasts of $v$ and RH (Table 2.1). The values used in this study were determined operationally during PREDICT (available online at http://met.nps.edu/~mtmontgo/storms2010.html). Since meridional phase speed is generally much weaker than $U_p$ for westward-moving tropical waves and was not calculated operationally during PREDICT, it is not included here. As such, the mean $V_{tan}$ profile is then computed as the sum of the component of wind orthogonal to the center of circulation with $U_p$ removed, normalized by the total number of dropwindsondes. For $V_{rad}$, the component of wind from each dropwindsonde blowing away from (towards) the center of circulation contributes positively (negatively).

Computation of $V_{tan}$ and $V_{rad}$ requires selection of a center of circulation, which is chosen to be the point at which mean 850-700 hPa $V_{tan}$ is maximized for each flight. The methodology follows Marks et al. (1992) with one exception: since the radius of maximum winds (RMW) is poorly defined for many cases in this study, mean $V_{tan}$ will be computed with respect to all dropwindsonde locations, rather than only those within an annulus around the RMW. Computation of $V_{tan}$ is performed in one tenth-of-a-degree iterations over a 10° by 10° latitude/longitude box centered on the flight pattern. Dropwindsondes are distributed relatively even in space within 300 km about the center of circulation (Figure 2.2). Further out, there is a tendency for greater data coverage to the east and southeast, with less coverage to the west. A sensitivity test was performed to examine the sensitivity of computed $V_{tan}$ and $V_{rad}$ profiles to choice of center location. All center locations are perturbed by ± 1° and 5° latitude and longitude, and $V_{tan}$ and $V_{rad}$ are calculated with respect to each new possible choice of center. The magnitude of the wind
anomalies, or the difference between each perturbed state and the control, are averaged over all cases. For $V_{\text{tan}}$, center perturbations of $1^\circ$ and $5^\circ$ result in $0.4 \text{ m s}^{-1}$ and $2.2 \text{ m s}^{-1}$ mean anomalies, respectively. Similarly, perturbations result in $0.4$ and $1.5 \text{ m s}^{-1}$ anomalies for $V_{\text{rad}}$. These results indicate that the wind metrics used in this study are not particularly sensitive to possible errors in the chosen center location if these errors are approximately $1^\circ$ latitude or longitude. Errors on the order of $5^\circ$ may be more problematic, but are unlikely to occur.

Vertical wind shear is calculated as the vector wind difference between the 850-500 hPa and 850-200 hPa pressure levels. Prior to computing the total wind shear vector, an average within each quadrant relative to the circulation center is performed in order to eliminate any false signal associated with asymmetries in the drop pattern. Quadrant averages are then averaged together. However, vertical asymmetries of the vortex itself are not removed from the shear calculation.

In addition to the previously mentioned vertical mean profiles, azimuthal averages of temperature, mixing ratio, relative humidity, tangential winds and relative vorticity are calculated in order to depict changes in radial structure of the tropical wave leading up to genesis. Data are binned in annuli of 100 km radius centered on the nearest 100 km grid point, with the exception of winds at the center of circulation where $V_{\text{tan}}$ is set to zero at $r = 0 \text{ km}$. Relative vorticity is computed in radial coordinates as

$$\zeta = \frac{1}{r} \frac{\partial (r V_{\text{tan}})}{\partial r}$$

(2.2)

using a centered difference approximation, where $dr = 100 \text{ km}$, beginning at $r = 50 \text{ km}$ since $\frac{1}{r}$ is not defined at $r = 0$. 

Vertical profiles will be depicted as an anomaly with respect to the PREDICT mean profile. Using this framework, it will be possible to investigate and quantify the differences between the vertical profiles of developing and non-developing tropical systems. While there are certainly large-scale synoptic, mesoscale, and convective-scale differences between the two scenarios that cannot be captured in a mean dropwindsonde profile, a number of significant results can nonetheless be drawn.

Mid-level moisture can often vary immensely over relatively small distances over the spatial extent of a tropical wave, due to a number of factors including: advection of moist or dry air, drying associated with subsidence, convective moistening from detrainment and precipitation processes, or drying associated with dry downdrafts. Much of both the top-down and bottom-up literature note a general trend of increasing convection near the center of the cyclone. However, averaging over the full areal extent of any one case might lead to a net cancellation of numerous moistening and drying processes, masking mesoscale variability. Therefore, in addition to domain-wide averaging, profiles of $q$ and RH from dropwindsondes located within a 150 km radius of the approximate center of circulation will be composited in order to investigate localized moisture anomalies. This will only be performed for genesis cases, and of these, only for cases in which the center is well defined. This limited dataset includes 8-12 dropwindsondes for each genesis time block (0-24 h pre-genesis, 24-48 h, etc.) consisting of data from the pre-Fiona, pre-Karl, pre-Matthew, pre-Nicole, and pre-Otto missions.

Lastly, Geostationary Operational Environmental Satellite (GOES) infrared data are investigated in order to relate any dynamic and thermodynamic phenomena observed in the dropwindsonde data to the convective structure of the tropical waves. GOES cloud-
top imagery in full 30-min resolution is composited over 6-hour time windows centered temporally on the mean time of each dropwindsonde mission to the nearest half hour, and then further composited over all cases in each genesis category. While accurate estimates of static center locations were determined from dropwindsonde data alone, model data was preferred when attempting to locate the time-evolving center location. As such, satellite composites are centered geographically on the center of the pouch, determined by the intersection of the disturbance critical line with the axis of the wave trough, from a consensus of GFS and ECMWF analyses interpolated linearly between analysis times. Similar to Davis and Ahijevych (2011), convective activity is depicted as a fraction of the total time comprising each category within each 6-h period, using half-hourly data in the co-moving frame of reference, that a grid box 10 km on a side exhibits an IR temperature less than -50°C.

2.2 Investigating genesis via ensemble forecasts

2.2.1 ECMWF ensemble and climatology

Analyses and 10-day forecasts from the operational 50-member ECMWF Ensemble Prediction System at T639 L62 resolution, interpolated onto a horizontal grid with 0.25° spacing are used in this study. The period considered is from 1 June – 30 November 2010. The ensemble data, produced twice daily, were provided via the THORPEX Interactive Grand Global Ensemble (TIGGE) database (Bougeault et al. 2010, http://tigge.ecmwf.int/). The ensemble is initialized through a combination of (i) initial-
time dry global singular vectors (SVs) maximized over the Northern Hemisphere, (ii) an “ensemble of data assimilations” to create initial spread in the ensemble, and (iii) initial-time tropical SVs with a dry total energy norm and additional linearized physics relevant to tropical processes, optimized over the Caribbean area (0-25°N, 100-60°W). The SVs are not optimized on regions centered on tropical waves and disturbances prior to their existence as TCs. Model error perturbations are included via the Stochastic Kinetic-Energy Backscatter scheme (SKEB, Berner et al. 2009) and the Stochastic Perturbed Parameterization Tendencies scheme (SPPT, Palmer et al. 2009).

The choice of variables was restricted by availability within TIGGE, as well as available disk space. Fields of geopotential height (denoted by Z) at the 200, 500 and 850 hPa pressure levels, together with horizontal wind components at 200, 700 and 850 hPa, and 700 hPa relative humidity were archived. The following quantities are used in this study:

(i) Relative vorticity: 850 hPa;
(ii) Vertical shear of horizontal wind: 850-200 hPa and 850-500 hPa;
(iii) Divergence: 850 hPa and 200 hPa;
(iv) Relative humidity: 700 hPa;
(v) Velocity potential: 200 hPa;
(vi) Circulation: average 850-700 hPa relative vorticity within 200 km of each grid point; and
(vii) Thickness anomaly, defined by

\[ \Delta Z = Z_{r=100\,\text{km}} - Z_{r=1000\,\text{km}} \]  \hspace{1cm} (2.3)
The first four variables are conventionally used in various diagnostics relevant to tropical meteorology. Positive relative vorticity, low vertical wind shear, positive upper-level divergence / negative low-level divergence, and high ambient relative humidity have all been found to be associated with TC formation (Gray 1979). Specific threshold ranges have been mapped to genesis parameters which incorporate these variables either directly or indirectly (DeMaria et al. 2001), such as through instability calculations, and are included in operational products produced at the Colorado State Regional and Mesoscale Meteorology Branch (RAMMB) to predict genesis probabilities (Schumacher et al. 2009).

The fifth, velocity potential, describes the irrotational flow, with the field being smoother and more slowly varying than divergence in both space and time. For these reasons, it is hypothesized that velocity potential will prove to be more predictable than divergence. Velocity potential $\phi$ is computed via

$$\delta = \nabla^2 \phi$$  \hspace{1cm} (2.4)

where $\delta$ is the horizontal divergence and the Laplacian is inverted using a successive overrelaxation technique (Bijlsma et al. 1986). The advantage of using velocity potential is that it preferentially emphasizes the planetary-scale aspects of the divergent circulation while smoothing the convective-driven component (Ventrice et al. 2013).

The circulation and thickness anomaly were chosen since they were found to be relatively unambiguous in objectively identifying a tropical disturbance or cyclone in numerical model fields (Majumdar and Torn 2014). First, the circulation was found to be easier to unambiguously track temporally than relative vorticity, given its distinct local maxima in easterly waves and gyres compared with multiple filaments with adjacent local maxima of vorticity. Second, the 850-200 hPa thickness anomaly $\Delta Z$ yields a distinct and
unambiguous signal of a local warm anomaly, which may correspond to a warm core building in a developing tropical cyclone. The exceedance of circulation and thickness anomaly by given threshold values in ECMWF analyses (investigated in Majumdar and Torn 2014) was found to be consistent with whether the tropical disturbance had attained tropical cyclone status (as declared by NHC).

The domain used throughout this study comprises the Atlantic Ocean from 0-40°N and 10-100°W. Attention in this study is emphasized on tropical weather, incorporating tropical waves, dry air masses originating from Africa, and gyres and inverted troughs. Less emphasis is given to conditions that may lead to cyclogenesis of disturbances of baroclinic origin or of a subtropical nature. As will be discussed later, the latitudinal extent is restricted further to prevent baroclinic processes from contributing substantially to the variables under investigation.

It should be noted that all calculations include a land mask to ensure that results are relevant to only the tropical Atlantic without contamination from South America or the east Pacific. An example of the geographic area excluded by the land mask is depicted in Figure 2.3.

Finally, the climatology is generated using 33 years of ERA-Interim data from 1979-2011. ERA-Interim is a re-analysis dataset that uses the ECMWF data assimilation system and forecast model from 2006, but with T255 spectral resolution and 60 vertical levels (Dee et al. 2011). The climatological data, originally at the equivalent of 0.703° x 0.702° resolution on a Gaussian grid, are interpolated to a 0.25° fixed grid using spherical harmonics (via NCL software).
2.2.2 Wave-relative framework

In addition to estimating predictability on the large scale, a number of variables and predictability metrics will be calculated over a much smaller domain centered on individual tropical waves prior to genesis. These experiments, hereafter referred to as the wave-relative predictability experiments, are cases of study which include all genesis events from the 2010 Atlantic hurricane season. Verification will be defined as the ECMWF control analysis at the time of genesis as declared by the National Hurricane Center. Genesis events that occur between analyses are assigned to the next available analysis, as the ECMWF is only initialized at 00Z and 12Z and genesis is often declared between the two. Since tropical cyclogenesis is inherently a multiscale problem, each variable is evaluated with respect to a wave-relative 300 km radius “core” about the center location of the wave in each member, as well as a 300-1000 km radius “environment” ring. The only variables that are not included in the environmental domain are circulation and thickness anomaly, since (a) values are generally at least 2 orders of magnitude smaller than they are in the core, and (b) a physical interpretation of the results is not always clear.

It is first necessary to define how wave centers are determined in the model output. First, circulation is calculated as described in section 2.2.1 in each ensemble member at all lead-times. A 200 km radius is used for circulation, rather than 300 km as used for the remainder of storm-relative metrics, as several cases exhibited anticyclonic vorticity within a larger 300 km radius disk which resulted in erroneously low circulation estimates. Since ensemble forecasts may have a significant position bias, particularly at large lead times, the location of the tropical wave in the model is chosen to be the location of the greatest
ensemble-mean circulation (Figure 2.4, black star) within a 1000-km radius (Figure 2.4, red circle) of the verifying genesis location (Figure 2.4, black diamond). Centers of circulation are then determined from each individual ensemble member forecast as the location of maximum circulation within 1000 km of the maximum ensemble mean circulation (Figure 2.4A). Complementary to the circulation criterion, center locations are also determined by finding the warm core. All regions of maximum thickness anomaly within 1000 km of the center of maximum ensemble mean circulation are also located (Figure 2.4B). Finally, the working center location is taken to be an average of the positions determined via the circulation and thickness criteria. The two methods are in good agreement, as they are found to rarely (~1% of the time) differ by more than 100 km.

Once the center of circulation is determined, computation of relative humidity, upper-level divergence and low-level convergence within the inner disk and outer ring is fairly straightforward. Computation of shear is slightly more complicated. In order to obtain a clean representation of environmental shear without spurious influences from the presence of the vortex, the wind field associated with the vortex itself is removed following the methodology of Davis et al. (2008). Environmental shear is derived from regional subsets of ECMWF analyses and forecasts by solving a Poisson equation for the streamfunction (2.5) and velocity potential (2.6)

\[
\nabla^2 (\psi_2 - \psi_1) = \begin{cases} 
\zeta_2 - \zeta_1 & \text{for } r \leq r_0 \\
0 & \text{for } r > r_0
\end{cases}; \\
\psi_2 - \psi_1 = 0 \text{ on lateral boundaries,}
\]

\[
\nabla^2 (\chi_2 - \chi_1) = \begin{cases} 
\delta_2 - \delta_1 & \text{for } r \leq r_0 \\
0 & \text{for } r > r_0
\end{cases}; \\
\chi_2 - \chi_1 = 0 \text{ on lateral boundaries,}
\]
associated with vorticity $\zeta$ and divergence $\delta$ within $5^\circ$ of the circulation center. Subscripts "1" and "2" refer to the bottom and top levels of the shear layer, respectively. In this study, subscript "1" is $850 \text{ hPa}$, while subscript "2" is either $500$ or $200 \text{ hPa}$. The streamfunction and velocity potential are recovered via inversion of the Laplacian operator through the use of a successive overrelaxation technique (Tannehill et al. 1997). Once known, $(\psi_2 - \psi_1)$ and $(\chi_2 - \chi_1)$ are used to compute nondivergent and irrotational wind shear vectors representing the disturbance:

$$\Delta \mathbf{v}_\psi = \mathbf{k} \times \nabla(\psi_2 - \psi_1) \quad \text{and} \quad \Delta \mathbf{v}_\chi = \nabla(\chi_2 - \chi_1)$$

Lastly, the resulting nondivergent and irrotational winds are subtracted from the full winds, and the resultant winds are used to compute the shear without the disturbance:

$$\Delta \mathbf{v}_{\text{env}} = \mathbf{v}_2 - \mathbf{v}_1 - \Delta \mathbf{v}_\psi - \Delta \mathbf{v}_\chi$$

Note that the vortex removal was not applied when calculating shear from the dropwindsonde data, as doing so requires having continuous 3-dimensional fields of $\psi$ and $\chi$, neither of which exist without having to make assumptions about interpolation between drop points or incorporating model fields. While dropwindsonde coverage was sufficient to produce azimuthal mean radius-height diagrams, it was very likely insufficient to reproduce the full streamfunction surrounding the TC.

Every ensemble member forecast for a wave and its surrounding environment has a particular area-averaged circulation, thickness anomaly, divergence, shear, and moisture value associated with it. However, it is difficult if not impossible to visualize all of these variables simultaneously. For this reason, lower-order (2-dimensional, and in a few cases 3-dimensional) joint distributions are examined to isolate the relationship between one variable and another without reference to the other variables. As will be elaborated upon
in Chapter 5, these joint distributions will be normalized by the number of entries in each column to account for the fact that we have an unequal distribution of samples within a larger array of possible environments. Due to this normalization, the integral of each column is equal to 1. Thus they are similar to, but not equivalent to, marginal distributions, in which the entire joint distribution is normalized by the total number of points in all rows and columns, and therefore the integral over the entire distribution is equal to 1. In this study, joint distributions, joint distributions of error, and lagged joint distributions are all calculated. Joint distributions of error are a simple way of examining the relationship between the forecast errors of one variable with the forecast errors of another, and are expected to depict the same physical relationships as are seen in the variables themselves. As an example, one would expect the forecast circulation to be weaker than verification if the forecast is also drier than verification. Lastly, lagged joint distributions relate the values of one variable 24 and 48 h before genesis with the value of another variable at the time of genesis. By computing lagged-A vs B and comparing it to A vs lagged-B, it is often possible to separate the effect of A on B versus B on A.

2.3 Metrics to quantify error growth and predictability

2.3.1 RMS error and error variance

Previous studies have traditionally summarized the time-evolution of domain-integrated ‘difference total energy’ (DTE) in order to quantify error growth, with (Hodyss and Majumdar 2007) or without (Zhang et al. 2002, 2003, 2007, Hodyss and Majumdar
2007) a moisture term. However, doing so does not allow one to separate errors associated with different metrics, as high kinetic and low thermal energy error is indistinguishable from high thermal and low kinetic energy error when only examining DTE. To avoid this issue, we will compute error, error growth, and forecast error variance with respect to each metric independently.

The RMS error $\varepsilon(t)$ of the forecast of a meteorological variable $x^f(t)$ at time $t$ is defined as

$$
\varepsilon(t) = \sqrt{[x^f(t) - x^a(t)]^2}
$$

(2.9)

where $x^a(t)$ is the corresponding ECMWF analysis (e.g. Piccolo 2011). In this study, the RMS error of the control forecast (an ensemble member integrated from the unperturbed analysis) is computed, i.e. $x^f \rightarrow x^c$. The ensemble mean is also compared against the average RMS error of a corresponding ‘forecast’ that simply uses climatology, i.e. $x^f \rightarrow \bar{x}^f$.

The ensemble offers additional information via the PDF. The most basic higher moment is the ensemble variance, which serves as a prediction of the forecast error variance $\sigma^2_{M-1}$:

$$
\sigma^2_{M-1}(t) = \frac{1}{M-1} \sum_{i=1}^{M} [x^f_i(t) - \bar{x}^f(t)]^2
$$

(2.10)

where $M$ is the total number of ensemble members (50 members plus 1 control run here), and $\bar{x}^f(t)$ is the ensemble mean forecast of $x$ at time $t$. Note that Equation 4 is normalized by $M-1$ to produce an unbiased estimate. Equations (2.9) and (2.10) are calculated over sixteen 10° X 10° regions within 5-25°N and 100-10°W, excluding the area from 100-80°W south of 15°N to ensure that the East Pacific does not factor into these calculations.
There has been considerable debate about how to evaluate the reliability of ensemble predictions via their statistical consistency versus actual errors. In other words, the ensemble needs to be able to discriminate between different error distributions, with forecasts of higher error corresponding to larger error spread in the ensemble predictions. One possible technique would be to relate the predicted forecast error variance to the actual forecast error variance, given by:

$$E^2(t) = \frac{1}{M-1} \sum_{i=1}^{M} \left[ x_i^f(t) - x_i^a(t) \right]^2$$

or

$$E^2(t) = \frac{1}{M-1} \sum_{t=1}^{M} \varepsilon_i(t)^2$$

(2.11)

If the ensemble is well constructed, then the variance of solutions in individual ensemble members should reflect the actual uncertainty in the forecast. In other words, a PDF of predicted forecast errors should closely resemble the PDF actual forecast errors, and the predicted forecast error variance will be similar to the actual forecast error variance. If the predicted forecast error variance is much lower than the actual forecast error variance, then the ensemble forecast is underdispersive and depicts greater confidence in the forecast than it should. However, stemming from the fact that $$\left( x_i^a(t) - \overline{x^f(t)} \right)^2$$ is always $$\geq 0$$, the actual forecast error variance will always be equal or greater than the predicted forecast error variance. Thus, when plotted, all data points will fall above the 1-to-1 line (if predicted forecast error variance is on the x-axis) and the linear regression will yield an inflated correlation and higher $$R^2$$ value than one would expect.

A superior approach that has been adopted over the past decade is to determine whether a relationship exists between the predicted error variance (equation (2.10), or the standard deviation which is its square root) and the corresponding variance (or standard deviation) of actual errors in the ensemble mean (e.g. Wang and Bishop 2003), given by:
for the $b^{th}$ forecast in variance bin of size $B$. This is accomplished by first plotting the
distribution of forecast errors of the ensemble mean versus their corresponding predictions
of the variance (or standard deviation) of the ensemble mean, for each forecast case. By
averaging the data points in equally sized bins of increasing predicted forecast error
variance, the variance (or standard deviation) of the actual errors is computed for each bin.
Ideally, a linear, increasing relationship of slope 1 between the predicted and actual error
variance (or standard deviation) is found. In this study, we elect to plot the error of the
ensemble mean, retaining the sign of the errors, versus the ensemble standard deviation,
following the lower panel of Fig. 5 of Grimit and Mass (2007). In this way, the distribution
of forecast errors, including any biases and departures from Gaussianity, are illustrated,
while retaining the ability to formally evaluate the relationship between the predicted and
actual error range.

While the traditional approach is to examine error growth from a fixed initialization
time out to some time at which error is maximized, this study will instead fix verification
time and allow initializations and subsequent forecasts verifying at this time to vary.
Therefore, time of initialization will flow chronologically from left to right, while forecast
time and subsequent error will increase from right to left. Mean values of actual squared
forecast error will be calculated and plotted for the lower, middle, and upper terciles of
variance, and a best fit line with 99% chi-squared confidence interval, following Majumdar
et al. (2001), but for forecast error and forecast error variance. The bias of the ensemble
mean forecast of each metric, relative to the verifying analysis, is also calculated as:

$$bias(t) = \overline{x^f(t)} - x^a(t)$$

(2.13)
It is necessary to calculate bias, as comparisons of error to variance assume an unbiased estimate.

2.3.2 Predictive Power

Predictive Power (PP) takes advantage of information contained in ensemble forecasts to quantify the difference between the ensemble distribution and a reference climatological distribution (Schneider and Griffies 1999). It is defined as

\[
PP = 1 - e^{-R_v},
\]

where the predictive information \( R_v \) is calculated as

\[
R_v = -\frac{1}{2m} \log \left( \frac{|C_v|}{|\Sigma|} \right)
\]

and \( m \) is the dimension of the state space and \( C_v \) and \( \Sigma \) are the sample covariance matrix of the ensemble perturbations and climatological covariance matrix, respectively. The sample covariance matrix of the ensemble perturbations is given by

\[
C_v = \frac{1}{M-1} E_v (E_v)^T
\]

where \( M \) is the number of ensemble members (51 in this study), and \( E_v = (e^1_v \ e^2_v \ \cdots \ e^M_v) = (x^1_v - \bar{x}_v \ x^2_v - \bar{x}_v \ \cdots \ x^M_v - \bar{x}_v) \) is a matrix of differences between state vector \( x^m \) for each ensemble member \( m \) and ensemble mean \( \bar{x} \), all calculated at lead time \( v \). Note that Schneider and Griffies (1999) compute \( E_v (E_v)^T \) as \( \sum_{i=1}^{M} e^i_v (e^i_v)^T \), but they are mathematically identical (for proof that the two formulations are equal, please see Appendix A.1.) Next, the climatological covariance matrix is computed as

\[
\Sigma = \frac{1}{N-1} E_v (E_v)^T
\]
where $E_v = (e^1, e^2, \ldots, e^N) = (x^1 - \bar{x}, x^2 - \bar{x}, \ldots, x^N - \bar{x})$ is a matrix of differences between state vector $x^n$ for each day $n$ and the monthly mean $\bar{x}$. We use the monthly mean instead of the climatological mean so that the difference $x^N - \bar{x}$ does not contain inter-seasonal variability (here we are only interested in short-term through intra-seasonal errors).

As forecast errors grow, the numerical values contained in $C_v$ also grow. Schneider and Griffies (1999) suggest that once the determinant of this error covariance matrix exceeds the determinant of $\Sigma$, predictability has been lost.

In calculating PP, we restrict the latitudinal extent of our domain to 5-25° N while retaining the 10-100° W dimension. This is done to focus on the tropical Atlantic, and to eliminate most spurious regions of enhanced circulation or thickness anomalies associated with non-tropical systems. To compute PP with so many data points (due to a combination of high resolution and large domain size), it is necessary to truncate the computations to find the determinants of $C_v$ and $\Sigma$. Since $E_v(E_v)^T$ is a $[28800 \times 28800]$ matrix, computation of the determinant is expensive and prone to large rounding errors. Instead, we calculate $(E_v)^T E_v$ to produce a $[51 \times 51]$ solution (50 perturbed members plus 1 control), and compute the determinant by taking the product of the eigenvalues $\lambda$ given by 

$$\det(-\lambda I + (E_v)^T E_v) = 0.$$ 

This can be done because the non-zero eigenvalues of $E_v(E_v)^T$ and $(E_v)^T E_v$ are equal, and there are only 50 modes of variability in $C_v$ (see Appendix A.2 for proof). Therefore, $m = 50$ in Equation 6. Here we denote eigenvalues and eigenvectors of $E_v(E_v)^T$ as 

$$E_v(E_v)^T = \xi \lambda \xi^T \quad \text{(2.18)}$$

and eigenvalues and eigenvectors of $(E_v)^T E_v$ as
where \( \lambda \) is again a diagonal matrix of eigenvalues \( \geq 0 \) and \( \mathbf{e} \) are orthonormal matrices of eigenvectors. We can solve for \( \mathbf{e} \) via a singular value decomposition, where \( \mathbf{e} = \mathbf{E}_v \Gamma \lambda^{-\frac{1}{2}} \) and verify the result as follows:

\[
\epsilon \lambda \epsilon^T = \mathbf{E}_v \Gamma \lambda^{-\frac{1}{2}} \epsilon \lambda^{-\frac{1}{2}} \Gamma^T (\mathbf{E}_v)^T
\]

\[
\epsilon \lambda \epsilon^T = \mathbf{E}_v \Gamma \mathbf{I}^T (\mathbf{E}_v)^T
\]

\[
\epsilon \lambda \epsilon^T = \mathbf{E}_v (\mathbf{E}_v)^T
\]

In summary, we can compute \((\mathbf{E}_v)^T \mathbf{E}_v\) instead of \(\mathbf{E}_v (\mathbf{E}_v)^T\) to yield a \([51 \times 51]\) matrix instead of a \([28800 \times 28800]\) matrix and still obtain the same eigenvalues. Likewise, Schneider and Griffies (1999) do not use the full determinant, but instead truncate their state space and only retain leading principal components of \(\mathbf{C}_v \Sigma^{-1}\). Since the determinant of a square matrix equals the product of the eigenvalues, or \(\det(m \times m) = \prod_{i=1}^{m} \lambda_i\), we can compute \(\det(\mathbf{C}_v)\) from \(\lambda\) in order to compute the PP. Lastly, we can recover eigenvectors of \(\mathbf{E}_v (\mathbf{E}_v)^T\) from the relation \(\mathbf{e} = \mathbf{E}_v \Gamma \lambda^{-\frac{1}{2}}\) and examine their structure. The structure of the eigenvectors will reveal geographically where the greatest sources of error associated with each metric are originating.

The Relative Entropy measure was also considered for use in this study. The primary difference between PP and Relative Entropy is that Relative Entropy has an additional term for the difference in the means of the two distributions (DelSole 2004). If the forecast distribution becomes broader than the climatological distribution, but if its mean differs sufficiently from the climatological mean, then it may still retain predictability according to the Relative Entropy but not according to PP (DelSole and Tippett 2007).
Our attempt to compute Relative Entropy following DelSole and Tippett (2007) turned out to be computationally intractable. One must compute the term $\text{Tr}[\Sigma^{-1}(C_v - \Sigma)]$, which cannot be factored into much smaller terms as was possible for the determinant in PP. In addition to the difficulty of computing the trace of a $[28800 \times 28800]$ matrix, computation of the inverse of $\Sigma$ was found to be prone to large machine error for covariance matrices of any meaningful size at 0.25° resolution. We expect that the results between PP and Relative Entropy would be similar for most variables, including shear and moisture, as the mean of the forecast distribution is often similar to the mean of the climatological distribution. The predictability for circulation would perhaps be extended somewhat according to Relative Entropy during particularly active or inactive periods, in terms of TC activity, as during those periods the mean of the circulation forecast is occasionally much above or much below the climatological mean.

2.3.3 Wave-relative Predictive Power

Computation of a wave-relative Predictive Power is also attempted. Whereas the Predictive Power computation for our Atlantic genesis region encompasses a 90° x 20° domain for $C_v$ and $\Sigma$, a 1000 km x 1000 km box centered on the tropical wave at time of genesis is employed for the wave-relative computations instead. Here, forecast error covariance $C_v$ only includes ECMWF ensemble forecasts verifying at times of genesis during 2010. Climatological error covariance $\Sigma$ includes all genesis events in ERA-I from 1979-2011, with genesis times and locations provided by HURDAT (McAdie et al. 2009). Note that $C_v$ and $\Sigma$ are prescribed to vary from month to month, since the covariance of the
distribution of genesis events during June would likely differ from that of October genesis events.
<table>
<thead>
<tr>
<th>Group</th>
<th>Case</th>
<th>Date</th>
<th>Lat (°N)</th>
<th>Lon (°W)</th>
<th>$U_p$ (m s$^{-1}$)</th>
<th>No. drops</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-Genesis</td>
<td>PGI-27L</td>
<td>08/17</td>
<td>14.5</td>
<td>69.3</td>
<td>-7.9</td>
<td>23</td>
</tr>
<tr>
<td>Non-Genesis</td>
<td>PGI-27L</td>
<td>08/18</td>
<td>16.2</td>
<td>77.2</td>
<td>-7.7</td>
<td>24</td>
</tr>
<tr>
<td>Non-Genesis</td>
<td>PGI-30L</td>
<td>08/21</td>
<td>20.3</td>
<td>52.9</td>
<td>-8.9</td>
<td>16</td>
</tr>
<tr>
<td>Non-Genesis</td>
<td>PGI-30L</td>
<td>08/23</td>
<td>20.8</td>
<td>69.4</td>
<td>-7.9</td>
<td>12</td>
</tr>
<tr>
<td>Genesis (0-24 h)</td>
<td>pre-Fiona</td>
<td>08/30</td>
<td>14.9</td>
<td>46.8</td>
<td>-9.0</td>
<td>30</td>
</tr>
<tr>
<td>TC Stage</td>
<td>TS Fiona</td>
<td>08/31</td>
<td>16.0</td>
<td>56.8</td>
<td>-7.0</td>
<td>30</td>
</tr>
<tr>
<td>TC Stage</td>
<td>TS Fiona</td>
<td>09/01</td>
<td>19.1</td>
<td>62.6</td>
<td>-5.1</td>
<td>21</td>
</tr>
<tr>
<td>TC Stage</td>
<td>TS Gaston</td>
<td>09/02</td>
<td>13.9</td>
<td>39.7</td>
<td>-3.5</td>
<td>20</td>
</tr>
<tr>
<td>Non-Genesis</td>
<td>PGI-38L</td>
<td>09/03</td>
<td>15.2</td>
<td>42.6</td>
<td>-3.6</td>
<td>22</td>
</tr>
<tr>
<td>Non-Genesis</td>
<td>PGI-38L</td>
<td>09/05</td>
<td>17.4</td>
<td>51.3</td>
<td>-7.4</td>
<td>21</td>
</tr>
<tr>
<td>Non-Genesis</td>
<td>PGI-38L</td>
<td>09/06</td>
<td>15.8</td>
<td>56.9</td>
<td>-6.4</td>
<td>23</td>
</tr>
<tr>
<td>Non-Genesis</td>
<td>PGI-38L</td>
<td>09/07</td>
<td>16.1</td>
<td>64.3</td>
<td>-6.1</td>
<td>22</td>
</tr>
<tr>
<td>Genesis (72+ h)</td>
<td>pre-Karl</td>
<td>09/10 (1)</td>
<td>13.5</td>
<td>61.1</td>
<td>-4.8</td>
<td>21</td>
</tr>
<tr>
<td>Genesis (72+ h)</td>
<td>pre-Karl</td>
<td>09/10 (2)</td>
<td>14.0</td>
<td>61.3</td>
<td>-5.2</td>
<td>21</td>
</tr>
<tr>
<td>Genesis (72+ h)</td>
<td>pre-Karl</td>
<td>09/11</td>
<td>15.5</td>
<td>67.8</td>
<td>-6.4</td>
<td>22</td>
</tr>
<tr>
<td>Genesis (48-72 h)</td>
<td>pre-Karl</td>
<td>09/12</td>
<td>15.0</td>
<td>72.0</td>
<td>-6.0</td>
<td>22</td>
</tr>
<tr>
<td>Genesis (24-48 h)</td>
<td>pre-Karl</td>
<td>09/13</td>
<td>16.8</td>
<td>76.9</td>
<td>-6.7</td>
<td>20</td>
</tr>
<tr>
<td>Genesis (0-24 h)</td>
<td>pre-Karl</td>
<td>09/14</td>
<td>18.4</td>
<td>83.6</td>
<td>-6.4</td>
<td>21</td>
</tr>
<tr>
<td>Genesis (72+ h)</td>
<td>pre-Matthew</td>
<td>09/20</td>
<td>11.2</td>
<td>58.6</td>
<td>-6.4</td>
<td>21</td>
</tr>
<tr>
<td>Genesis (48-72 h)</td>
<td>pre-Matthew</td>
<td>09/21</td>
<td>12.2</td>
<td>62.8</td>
<td>-6.9</td>
<td>22</td>
</tr>
<tr>
<td>Genesis (24-48 h)</td>
<td>pre-Matthew</td>
<td>09/22</td>
<td>13.3</td>
<td>70.7</td>
<td>-7.1</td>
<td>18</td>
</tr>
<tr>
<td>TC Stage</td>
<td>TS Matthew</td>
<td>09/24</td>
<td>14.9</td>
<td>82.6</td>
<td>-8.5</td>
<td>22</td>
</tr>
<tr>
<td>Genesis (0-24 h)</td>
<td>pre-Nicole</td>
<td>09/27</td>
<td>19.6</td>
<td>86.9</td>
<td>2.5</td>
<td>23</td>
</tr>
<tr>
<td>TC Stage</td>
<td>TS Nicole</td>
<td>09/28</td>
<td>19.5</td>
<td>84.6</td>
<td>2.6</td>
<td>24</td>
</tr>
<tr>
<td>Genesis (72+ h)</td>
<td>pre-Otto</td>
<td>09/30</td>
<td>15.8</td>
<td>58.1</td>
<td>-11.3</td>
<td>26</td>
</tr>
</tbody>
</table>

Table 2.1: Cases of study during PREDICT comprising the genesis, non-genesis, and TC-stage groups with corresponding dates of G-IV deployments. Timing of mission prior to genesis is included for cases in which genesis occurred. Latitude and longitude of the target center location of each drop pattern, meridional phase speed ($U_p$) of the wave, and the number of dropwindsondes released are shown.
Figure 2.1: Map of all dropwindsonde deployment locations during PREDICT and corresponding genesis categories, from August 15 through September 30, 2010.
Figure 2.2: Plots of sounding locations relative to the center of circulation in polar (km, deg) coordinates for each genesis category: (A) genesis, (B) non-genesis and (C) TC stage.
Figure 2.3: Geographic region of genesis domain for Chapter 4 (blue), with region excluded from computations by landmask (left of red line).

Figure 2.4: Depiction of the search algorithm, with (A) circulation contours and (B) thickness anomaly contours. Centers of maximum circulation and local maxima in thickness anomaly are found within 1000 km radius (red circle) of the maximum ensemble mean circulation (black star). The maximum ensemble mean circulation is allowed to vary up to 1000 km from the verifying point of genesis (black diamond).
Chapter 3

Tropical Cyclogenesis: Observed Physical Processes

In this chapter, the physical differences between developing and non-developing tropical cyclones are explored using dropwindsonde observations from PREDICT. The temporal evolution of dynamic and thermodynamic changes that take place within the core of a developing tropical wave is also sought. A few key questions that will be addressed:

• What are the distinguishing physical characteristics between developing and non-developing tropical waves in terms of temperature, moisture, instability, shear, radial wind, tangential wind and relative vorticity?

• How do each of the aforementioned fields evolve in developing waves?

• Is there any clear evidence for either a top-down or bottom-up genesis mechanism?

• Are differences in moisture or instability reflected in a greater coverage of cold cloud tops using infrared satellite imagery?

• How long before genesis does the warm core begin to develop, and at what altitude?

3.1 Genesis vs. non-genesis

We begin our investigation by comparing $T$ profiles of genesis and non-genesis cases with the PREDICT mean. Non-genesis cases are associated with slight warm anomalies of 0.1-0.2°C below 600 hPa, while the genesis mean is associated with cold
anomalies of -0.2 to -0.3°C (Figure 3.1A). Above 600 hPa, $T$ anomalies steadily decrease with height in the non-genesis profile, while the genesis profile is instead associated with warm anomalies. While of a much lower magnitude, 0.2°C versus 0.8°C, the greatest positive $T$ anomalies for genesis appear within the same layer as for the TC stage profile between 400-200 hPa. It is possible that at least part of the warming associated with the genesis profile is the beginnings of the development of the warm core. Radial composites reveal the greatest warm anomalies occurring within 200 km radius of the center of circulation for genesis and TC samples, supporting this hypothesis (the development of which is shown in section 3.2). Alternatively, the non-genesis mean profile is associated with cold anomalies of -0.1 to -0.9°C from 600-200 hPa. It should be noted that standard deviations of these data are large, and only cold anomalies for non-genesis above 500 hPa are greater than one standard deviation from the PREDICT mean. We hypothesize at this time, and later results will support the idea, that cold anomalies in non-genesis cases are due in part to a lack of sustained latent heating from deep convection and the lack of a developing warm core. This is apparent since anomalies are taken against the PREDICT mean, which includes many genesis cases featuring both sustained deep convection and developing warm cores.

Maximum anomalies of $q$, both positive and negative, exist between 700-500 hPa for genesis, non-genesis and TC stage cases (Figure 3.1B). TC and pre-genesis profiles are associated with moist $q$ anomalies of +0.1 to +0.5 g kg$^{-1}$ while non-developing systems are associated with dry $q$ anomalies of -0.1 to -1.0 g kg$^{-1}$ in this layer. While these values appear to be small, the non-genesis profile is 25% drier than the PREDICT mean $q$ of 3.24 g kg$^{-1}$ with a -0.81 g kg$^{-1}$ anomaly at 500 hPa. Radial composites reveal dry anomalies as
large as -1.8 g kg$^{-1}$ at greater than 400 km radius for non-genesis, with non-genesis drier than genesis through the center of circulation, suggesting an influence of the dry air on the core of the tropical wave. Interestingly, profiles of $q$ indicate that the non-genesis mean below 850 hPa is greater than 0.5 g kg$^{-1}$ more moist than the genesis mean, suggesting that perhaps dry air at the mid levels was more detrimental to genesis than drier air at the surface during PREDICT. The PREDICT mean is greater than one standard deviation more moist than the non-genesis mean from 700-500 hPa, suggestive of the relative significance of this dry air (Table 3.1).

While comparing $q$ profiles allows for direct comparisons of the mass of water vapor in a column of atmosphere, examination of RH (Figure 3.1C) is necessary to identify near-saturation of the lower and mid levels, a significant criterion for genesis in Bister and Emanuel (1997) and Nolan (2007). The presence of low ambient RH in the non-genesis cases from PREDICT suggests that the developing tropical wave is vulnerable to the potential detrimental effects of entrainment on parcel buoyancy. Consistent with $q$ results, RH is notably lower by 10 to 20% from 700-300 hPa for non-genesis than genesis, indicating greater potential for entrainment of drier air.

Values of CAPE reveal an interesting result, in that not only are the pre-genesis and TC profiles no more unstable than the non-genesis sounding, but the non-genesis profile is in fact much more unstable than the genesis profile (Table 3.2). Mean CAPE anomalies are +336 J kg$^{-1}$ for non-genesis, -171 J kg$^{-1}$ for genesis, and -42 J kg$^{-1}$ for TC cases. All departures are relative to the PREDICT mean CAPE of 2096 J kg$^{-1}$ when integrated to the height of the EL or the highest available pressure level. Large CAPE values of 1900-2500 J kg$^{-1}$, in conjunction with low LFCs of 940-920 hPa and very low CIN of -2 to -10 J kg$^{-1}$
are not unexpected given that calculations are with respect to rather moist surface-based parcels. In fact, many genesis and non-genesis dropwindsondes exhibited no CIN at all. While it is true that the height at which the dropwindsonde is deployed may be below the EL in some cases, $T_{v,\text{parcel}} - T_v$ in (1) is close to zero for most cases at this altitude, and therefore the amount of CAPE “missed” should be small. Overall, these results suggest that the availability of additional instability in an already otherwise unstable tropical environment does not increase the likelihood of tropical cyclogenesis, consistent with Nolan et al. (2007). Smith and Montgomery (2011) also find greater CAPE associated with ex-Gaston than either genesis case they studied. These results are also not inconsistent with Molinari and Vollaro (2010), that highly sheared, generally weaker tropical cyclones tend to be associated with higher CAPE than their non-sheared, generally stronger counterparts, as well as the findings of Braun (2010), that CAPE generally tends to be higher in environments of weakening TCs compared to strengthening TCs. Satellite imagery suggests that lower CAPE values for genesis cases may be due, at least in part, to greater consumption of CAPE by more widespread deep convection (Figure 3.2). Cold upper-levels are required to produce strong CAPE, and continued latent heat release is eroding this cold air.

Average $V_{tan}$ values are 2-3.5 m s$^{-1}$ for non-genesis and 3-5 m s$^{-1}$ for genesis profiles from the surface through 500 hPa (Figure 3.1D). While significant variability exists, with standard deviations greater than 2 m s$^{-1}$, the developing tropical waves are generally associated with stronger circulations. As expected, these circulations are still significantly weaker than those of TC stage systems. For genesis, non-genesis, and TC cases, $V_{tan}$ is cyclonic from the surface through 300 hPa and anticyclonic above.
Radial wind $V_{rad}$ is also calculated (not shown). Maximum negative values of up to -1 m s$^{-1}$ for TC stage and -0.3 m s$^{-1}$ for genesis and non-genesis occur between the surface and 900 hPa, suggesting a shallow layer of inflow and boundary layer convergence. While these values appear to be small, they are averages over the entire sampled region of the tropical wave, and do not correspond to maxima at a particular radius. All three profiles are associated with positive $V_{rad}$ above 300 hPa, indicative of divergence. Time-averaged genesis and non-genesis $V_{rad}$ profiles are generally indistinguishable, and $V_{rad}$ does not appear to be of much value as a discriminating characteristic for genesis. However, as will be demonstrated in section b, the genesis $V_{rad}$ profile evolves considerably with time.

Mid-level, 850-500 hPa vertical wind shear is slightly greater for genesis than non-genesis with 2.70 m s$^{-1}$ shear ± a standard deviation of 0.22 m s$^{-1}$ as compared to 2.22 ± 0.64 m s$^{-1}$ for non-genesis (Table 3.3). Alternatively, 850-200 hPa deep layer shear is slightly more hostile for the non-genesis than the genesis cases, with 7.39 ± 0.71 m s$^{-1}$ compared to 6.97 ± 0.40 m s$^{-1}$. However, wind shear did not appear to be the primary factor in differentiating genesis from non-genesis cases in 2010 as values of wind shear are statistically identical between the two cases. Unfortunately there were no cases sampled with less than 4 m s$^{-1}$ or shear greater than 10 m s$^{-1}$. However, perhaps this should have been expected, as Nolan and McGauley (2012) find that very low shear occurs very infrequently in nature, and shear greater than 10 m s$^{-1}$ is less favorable for genesis. Given the high percentage of dropwindsondes deployed within 400 km of the center of circulation and comparatively few outside of this region, it is also possible that these data are not fully representative of the true environmental wind shear.
GOES satellite composites clearly demonstrate persistently colder cloud tops over a much larger area for genesis than non-genesis (Figure 3.2). In fact, the area of cloud tops colder than -50°C more than 60% of the time is larger for genesis than our sample of TCs. Genesis is clearly associated with more consistent deep convection over a larger spatial area than non-genesis, which, in turn, would promote the development of a sustained mid-level vortex. Greater spatial coverage of deep convection is also consistent with the building of upper-level warm anomalies as a result of additional latent heating. The presence of sustained deep convection evident in satellite imagery, in the presence of significant positive mid-level moisture anomalies as demonstrated from the dropwindsonde data, suggests that humidification of the inner core is occurring due to moist detrainment and precipitation from deep convective towers preceding genesis.

3.2 Time progression leading up to genesis

A major benefit of the pouch tracking framework developed for the PREDICT field campaign was an ability to routinely identify and sample regions of potential genesis daily beginning a few days prior to the development of a tropical depression. This tracking and sampling technique yielded a useful temporal genesis dataset, in which it is possible to sub-categorize genesis profiles by time leading up to genesis. In this section, we will continue to examine differences between various mean profiles and the PREDICT mean, but from the perspective of a temporal progression.

Examination of radial profiles of $T$ reveals widespread cold anomalies at large lead times for all radii gradually transitioning to warm anomalies at small lead times through
much of the troposphere (Figure 3.3A-D). The 72+ h pre-genesis profile is associated with negative $T$ anomalies ranging from -0.1 to -1.0°C, with local minima between 800-600 and 400-200 hPa. Conversely, the entire 0-24 h pre-genesis profile is associated with deep warm anomalies ranging from +0.1°C at 1000 hPa to as large as +2.0°C at 300 hPa within 100 km of the center of circulation. The most obvious warming trend occurs from 400-200 hPa, evident during every successive 24 h time increment. However, cold anomalies persist from 900-700 hPa through 24-48 h pre-genesis, until finally warming rapidly 0-24 h pre-genesis. Warm anomalies are observed from 500-200 hPa above cold anomalies below 500 hPa from 24-48 h and 48-72 h pre-genesis, characteristic of the development of a low-level cold core prior to genesis as described by Bister and Emanuel (1997) and Nolan (2007). The observed upper-level warming can likely be attributed to a combination of latent heat released by convection and warm core development via subsidence induced by this latent heating. The predominant warm anomalies 0-24 h pre-genesis are maximized near 300 hPa, at a similar altitude to what was found in La Seur and Hawkins (1963) and Hawkins and Rubsam (1968). These results suggest that warm core development commences prior to genesis and at the same altitude one would expect to find the warm core in a mature TC. There even exists some hint of a secondary $T$ maximum developing between 600-500 hPa within 200 km of the center of circulation, as suggested by Hawkins and Imbembo (1976) and Stern and Nolan (2012), although much weaker than the primary warm core.

In contrast with temperature, neither $q$ nor RH increase on average with time. Mixing ratio differences between 72+ h pre-genesis and 0-24 h pre-genesis cases tend to be small, demonstrating low variability with no evident large-scale humidification.
However, the full wave mean for cyclogenesis events during PREDICT is simply more moist than for non-genesis events to begin with, even 4 or more days in advance. In a separate calculation, $q$ and RH are composited for only a small subset of the total sample, only including 8-12 dropwindsondes for each 24 h period within 150 km radius of the center of circulation. Local averaging reveals a moistening trend (Figure 3.4A), with $q$ increasing from +0.5 to +0.7 g kg$^{-1}$ at 72+ h pre-genesis to +1.2 to +1.6 g kg$^{-1}$ 0-24 h pre-genesis between 800-500 hPa, with the greatest increase in moisture occurring 24-48 h pre-genesis. Radial cross sections of $q$ azimuthally averaged in annuli reveal similar trends (not shown). This observation is consistent with Nolan (2007) in that moist detrainment and precipitation from deep convective towers are acting locally to humidify the center of the wave, although the greater low-level moistening observed here suggests a greater fraction of moist detrainment associated with shallow convection. The fact that this trend does not appear in the full composite may simply be reflective of the larger area whose profile is difficult to modify given the small area occupied by convection. This result is also consistent with pre-genesis moistening local to the inner 60 km above 2 km observed in the numerical experiments of Bister and Emanuel (1997). A threshold of 60 km radius is not used here, as decreasing the radius by 40 km reduces the number of dropwindsondes encompassed by about 75%, which leads to a very small sample size. While Bister and Emanuel (1997) and Nolan (2007) suggest that a similar local moistening trend should be evident in the RH field, this was not observed in the limited dataset of this study (Figure 3.4B). Instead, a very small increase RH from 800-600 hPa is observed 24-48 h pre-genesis, followed by a decrease in RH 0-24 h pre-genesis. Overall, the added moisture
content nearly exactly cancels with the increased saturation deficit associated with warming temperatures such that the profile of RH remains relatively steady.

A time-progression of CAPE reveals maximum instability 72+ h pre-genesis (Table 3.2), at a time when upper-level temperatures are the coldest. A sudden decrease in CAPE by approximately 500 J kg\(^{-1}\) occurs 48-72 h pre-genesis as upper levels warm. The mean sounding becomes slightly more unstable again 24-48 h pre-genesis as the boundary layer warms and moistens rapidly enough to offset continued warming at the upper levels, followed by a small decrease in CAPE again 0-24 h pre-genesis.

Mid-level, 850-500 hPa wind shear gradually subsides from 2.82 m s\(^{-1}\) 72+ h pre-genesis to 2.45 m s\(^{-1}\) 0-24 h pre-genesis (Table 3.3), although both values are associated with very low shear and are highly favorable for tropical cyclogenesis. Upper-level, 850-200 hPa wind shear does not depict a coherent signal, fluctuating between 6.8 and 7.2 m s\(^{-1}\) between pre-genesis time bins. However, as was the case for the comparison between genesis and non-genesis shear values, these values are statistically indistinguishable.

Profiles of \(V_{\text{rad}}\) indicate that mean 300-200 hPa upper-level outflow fluctuates significantly with time, but with no obvious trend (Figure 3.4C). It would appear that level of organization of divergent flow aloft is not critical to any particular stage of genesis. Radial winds of +1.5 m s\(^{-1}\) at 200 hPa are observed both 72+ h pre-genesis and at the time of genesis, indicating that upper-level conditions were favorable for genesis several days before it occurred, but the tropical waves required additional time to organize. At lower levels, positive values of \(V_{\text{rad}}\) of +0.5 to +1 m s\(^{-1}\) are observed below 850 hPa for both 72+ h pre-genesis and 48-72 h pre-genesis composites, indicating surface divergence. This suggests that, while upper-level outflow was favorable for genesis, lack of surface
convergence and/or an overabundance of downdrafts may have hindered the process. While not evident in these composites, Davis and Ahijevych (2012) suggested that an initial vertical misalignment of vortex centers may have also delayed genesis of Karl. By 24-48 h pre-genesis, radial outflow reverses to -1 m s\(^{-1}\) inflow, increasing to -2 m s\(^{-1}\) 0-24 h pre-genesis. While it is likely that some of the large-scale surface divergence 48-72 h pre-genesis is associated with convective or mesoscale downdrafts, there does not appear to be sufficient dry air near the core of the system at any time for dry air induced downdrafts to be a prohibitive factor for genesis (Figure 3.4B). Thereafter, a secondary circulation begins to develop within 48 h of genesis, and surface convergence increases rapidly with time.

Tangential winds \(V_{tan}\) (Figure 3.4D) progress from weaker to stronger and more cyclonic with time above 600 hPa. This result is consistent with the progressive building of a mid- to low-level vortex described by Nolan (2007) and Raymond et al. (2011). However, the region of maximum \(V_{tan}\) exists between 850-550 hPa, depending upon the timeframe examined, and there is no well-defined peak. The altitude of the tangential wind maximum is of lower altitude than the warm core, consistent with thermal wind balance. While it is difficult to determine an exact threshold, genesis appears imminent when system-wide deep-layer positive tangential wind anomalies of 6-7 m s\(^{-1}\) develop between 850-700 hPa, with much weaker values observed 24 or more hours earlier. Below 700 hPa, the strength of the mean tangential wind actually fluctuates from 72-24 h pre-genesis, followed thereafter by an abrupt increase 24 h pre-genesis. This is consistent with Nolan (2007) in that the transition to the intensification stage can be sharp, having less to do with a continuous strengthening of the mean wind and perhaps more to do with moistening of
the core. These results show a 24 hour lag between the greatest increase in moisture 24-48 h pre-genesis and a strengthening of the vortex 0-24 h pre-genesis. It is particularly striking that this sudden increase in organization is evident in the mean profile of five sampled systems, but is well within convective heating timescales.

While average tangential wind profiles depict a temporal time progression of the strengthening vortex, they do not clearly demonstrate a top-down or bottom-up genesis evolution. To further investigate this issue, the time progression of relative vorticity is computed in radial coordinates relative to the center of circulation (Figure 3.5A-D) from tangential wind data binned in annuli of 100 km radius. As expected, the greatest values of $\zeta$ are observed within $r < 100$ km near the center of circulation at all times. The initial altitude of the $\zeta$ maximum is not well defined 72+ h pre-genesis, with relatively weak local maxima between the surface and 700 hPa (Figure 3.5A). By 48-72 h pre-genesis, vorticity has amplified considerably through a deep layer from 925-500 hPa, with the greatest amplification between 900-600 hPa (Figure 3.5B). Vorticity remains only marginally cyclonic at the surface, likely due to a combination of previously-demonstrated low-level divergence occurring in a region of cold anomalies and surface friction. Relative vorticity continues to strengthen at all levels below 400 hPa within 24-48 and 0-24 h pre-genesis, although what was previously a broad region of maximum $\zeta$ from 900-600 hPa has evolved to become a distinct maximum slightly near 800 hPa (Figure 3.5C,D). While we find no evidence of vorticity descending, a clear intensification of mid-level vorticity prior to the development of a robust surface circulation is more consistent with a Bister and Emanuel (1997), Nolan (2007) and Raymond et al. (2011) genesis framework.
The satellite presentation of the temporal progression leading up to genesis is not as intuitive as the satellite comparison between genesis and non-genesis. From 72+ h pre-genesis to 48-72 h pre-genesis, there is a significant expansion of the persistent cloud tops colder than -50°C, indicating greater coverage of deep convection and presumably a more mature system approaching genesis (Figure 3.6A,B). However, during subsequent 24-48 h and 0-24 h pre-genesis time increments, the coverage of deep convection diminishes considerably, both spatially and temporally (Figure 3.6C,D). Despite the fact that significant changes are ongoing within the pre-genesis vortex, including the development of a warm core, moistening of the core, and amplification of system relative vorticity, the organization and persistence of temporally-averaged -50°C cloud tops is no more indicative of genesis 0-24 h pre-genesis than it is 72+ h pre-genesis. This result demonstrates the obvious advantage of having available in-situ data when assessing how close a tropical disturbance is to developing into a tropical cyclone. While not obvious in the satellite composites, examination of satellite images at individual times suggests that non-genesis cases were associated with bursts of deep convection that collapsed before a robust circulation could develop. Therefore, even if the mean cloud-top temperatures are similar between genesis and non-genesis, the non-genesis cases featured more intense but shorter-lived convection than genesis cases.

While the PREDICT dropwindsonde observations provided an excellent dataset for examining the internal dynamics and thermodynamics of developing tropical waves, the sample size is nonetheless limited. In order to explore the predictability of genesis, we will move from observations to modeling, as well as expand the dataset to include all developing systems from 2010. An examination of individual systems from 2010 will be
performed in Chapter 5. However, in order to set the stage for Chapter 5, the overall evolution of the 2010 hurricane season and predictability associated with variables relevant to genesis on the large scale will first be analyzed in Chapter 4.

3.3 Summary of results

While a number of interesting findings were uncovered in this chapter, three in particular stand out amongst the rest. First, it was found that genesis events were significantly more moist than non-genesis events at the middle levels, but drier at the low levels, suggesting that dry air is more detrimental to genesis when located at middle levels. Second, time-varying tangential wind profiles reveal an initial delay in intensification, followed by an increase in organization 24 h prior to genesis. Lastly, the vertical evolution of vorticity and the warm-over-cold thermal structure are more consistent with a top-down than a bottom-up genesis mechanism. In the next chapter, Chapter 4, the predictability of variables relevant to genesis on the large scale will be assessed. Finally, in Chapter 5, we will combine the concepts of Chapter 3 with the findings of Chapter 4 and assess the predictability of the physical processes of genesis via ensemble forecasts. Amongst improved modeling and data assimilation, improved observations, particularly in the data-starved regions of the eastern Atlantic and western Africa, is paramount to decreasing uncertainty in genesis forecasts.
<table>
<thead>
<tr>
<th>Pressure (hPa)</th>
<th>Sounding</th>
<th>No. Cases</th>
<th>Mixing Ratio (g kg(^{-1}))</th>
<th>(\sigma) (g kg(^{-1}))</th>
<th>Anom. vs PREDICT mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>PREDICT mean</td>
<td>12</td>
<td>0.02</td>
<td>0.01</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>Genesis</td>
<td>5</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>Non-genesis</td>
<td>3</td>
<td>0.01</td>
<td>0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>TC stage</td>
<td>4</td>
<td>0.02</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>500</td>
<td>PREDICT mean</td>
<td>12</td>
<td>3.24</td>
<td>0.75</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>Genesis</td>
<td>5</td>
<td>3.41</td>
<td>0.38</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>Non-genesis</td>
<td>3</td>
<td>2.43</td>
<td>0.48</td>
<td>-0.81</td>
</tr>
<tr>
<td></td>
<td>TC stage</td>
<td>4</td>
<td>3.63</td>
<td>0.89</td>
<td>0.39</td>
</tr>
<tr>
<td>700</td>
<td>PREDICT mean</td>
<td>12</td>
<td>7.86</td>
<td>0.81</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>Genesis</td>
<td>5</td>
<td>8.01</td>
<td>0.56</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>Non-genesis</td>
<td>3</td>
<td>7.16</td>
<td>0.57</td>
<td>-0.70</td>
</tr>
<tr>
<td></td>
<td>TC stage</td>
<td>4</td>
<td>8.20</td>
<td>1.03</td>
<td>0.34</td>
</tr>
<tr>
<td>850</td>
<td>PREDICT mean</td>
<td>12</td>
<td>12.78</td>
<td>0.65</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>Genesis</td>
<td>5</td>
<td>12.56</td>
<td>0.78</td>
<td>-0.22</td>
</tr>
<tr>
<td></td>
<td>Non-genesis</td>
<td>3</td>
<td>12.78</td>
<td>0.20</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>TC stage</td>
<td>4</td>
<td>13.05</td>
<td>0.73</td>
<td>0.27</td>
</tr>
<tr>
<td>1000</td>
<td>PREDICT mean</td>
<td>12</td>
<td>18.68</td>
<td>0.68</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>Genesis</td>
<td>5</td>
<td>18.35</td>
<td>0.52</td>
<td>-0.33</td>
</tr>
<tr>
<td></td>
<td>Non-genesis</td>
<td>3</td>
<td>19.18</td>
<td>0.40</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>TC stage</td>
<td>4</td>
<td>18.72</td>
<td>0.88</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Table 3.1: PREDICT, genesis, non-genesis and TC-stage mean mixing ratio (g kg\(^{-1}\)) values, standard deviations (\(\sigma\)), and anomalies versus the PREDICT mean for select levels from 1000 to 200 hPa.
Sounding  | No. Cases | LFC (hPa) | EL (hPa) | CIN (J kg\(^{-1}\)) | CAPE (J kg\(^{-1}\)) | SD of CAPE (J kg\(^{-1}\)) | Anom. vs PREDICT mean |
--- | --- | --- | --- | --- | --- | --- | --- |
PREDICT mean | 12 | 928 | 199 | -8 | 2096 | 539 | --- |
Genesis | 5 | 920 | 200 | -10 | 1925 | 298 | -171 |
Non-genesis | 3 | 940 | 196 | -7 | 2433 | 314 | 336 |
TC stage | 4 | 932 | 202 | -7 | 2054 | 571 | -42 |
72+ h pre-gen | 5 | 924 | 197 | -10 | 2076 | 117 | 151 |
48-72 h pre-gen | 2 | 900 | 210 | -16 | 1550 | 453 | -882 |
24-48 h pre-gen | 2 | 935 | 193 | -8 | 1980 | 576 | -75 |
0-24 h pre-gen | 3 | 919 | 204 | -7 | 1868 | 296 | -208 |

Table 3.2: Instability data for different categories. Included are the LFC, EL, CAPE, CIN, standard deviations (\(\sigma\)) of CAPE, and CAPE anomalies versus the PREDICT mean.
<table>
<thead>
<tr>
<th>Group</th>
<th>850-500 hPa shear (m s⁻¹)</th>
<th>850-500 hPa σ (m s⁻²)</th>
<th>850-200 hPa shear (m s⁻¹)</th>
<th>850-200 hPa σ (m s⁻²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Genesis</td>
<td>2.70</td>
<td>0.22</td>
<td>6.97</td>
<td>0.40</td>
</tr>
<tr>
<td>Non-genesis</td>
<td>2.22</td>
<td>0.64</td>
<td>7.39</td>
<td>0.71</td>
</tr>
<tr>
<td>TC Stage</td>
<td>2.35</td>
<td>0.35</td>
<td>6.92</td>
<td>0.52</td>
</tr>
<tr>
<td>72+ h pre-gen</td>
<td>2.82</td>
<td>0.14</td>
<td>6.93</td>
<td>0.31</td>
</tr>
<tr>
<td>48-72 h pre-gen</td>
<td>2.77</td>
<td>0.10</td>
<td>7.12</td>
<td>0.22</td>
</tr>
<tr>
<td>24-48 h pre-gen</td>
<td>2.74</td>
<td>0.04</td>
<td>7.11</td>
<td>0.21</td>
</tr>
<tr>
<td>0-24 h pre-gen</td>
<td>2.45</td>
<td>0.31</td>
<td>6.85</td>
<td>0.76</td>
</tr>
</tbody>
</table>

Table 3.3: 850-500 hPa and 850-200 hPa vertical wind shear values (m s⁻¹) and standard deviations (σ) for genesis, non-genesis and TC stage categories. Wind shear data for 72+ hours pre-genesis, 48-72 hours pre-genesis, 24-48 hours pre-genesis, and 0-24 hours pre-genesis subsets also included.
Figure 3.1: Composite vertical profiles of anomalies relative to the PREDICT mean of (A) temperature, (B) mixing ratio, (C) relative humidity, and (D) tangential component of wind for genesis, non-genesis and TC stage categories.
Figure 3.2: GOES satellite data in 30-min resolution composited over 6-hour time windows centered temporally on the mean time of each dropwindsonde mission to the nearest half hour, composited over multiple missions for each genesis category. The percentage of total time in each 10x10 km grid point remains below -50°C is depicted for (A) genesis, (B) non-genesis, and (C) TC stage categories (courtesy Dave Ahijevych).
Figure 3.3: Radial profiles of temperature anomalies (°C) with respect to the PREDICT mean. Data are azimuthally averaged in annuli of 100 km radius for (A) 72+ hours pre-genesis, (B) 48-72 hours pre-genesis, (C) 24-48 hours pre-genesis, and (D) 0-24 hours pre-genesis.
Figure 3.4: Composite vertical profiles of anomalies relative to the PREDICT mean of (A) mixing ratio within 150 km of the center of circulation, (B) relative humidity within 150 km of the center of circulation, (C) radial component of wind at all radii, and (D) tangential component of wind at all radii for 72+ hours pre-genesis, 48-72 hours pre-genesis, 24-48 hours pre-genesis, and 0-24 hours pre-genesis categories.
Figure 3.5: Relative vorticity (s⁻¹) computed in radial coordinates for (A) 72+ hours pre-genesis, (B) 48-72 hours pre-genesis, (C) 24-48 hours pre-genesis, and (D) 0-24 hours pre-genesis.
Figure 3.6: GOES satellite data, as in Figure 3.2 but for (A) 72+ hours pre-genesis, (B) 48-72 hours pre-genesis, (C) 24-48 hours pre-genesis, and (D) 0-24 hours pre-genesis (courtesy Dave Ahijevych).
Chapter 4

Predictability Results Part I: Basin-Wide Perspective

We now deviate from the focus of Chapter 3, which was primarily on the internal dynamics and thermodynamics of developing tropical waves, and shift our focus to the large scale. In this chapter, we also transition from observational data to modeling, and seek to explore the predictability associated with a number of environmental variables deemed relevant to tropical cyclogenesis. Several metrics are employed to evaluate predictive skill and attempt to quantify predictability using the ECMWF Ensemble Prediction System during the 2010 Atlantic hurricane season, with an emphasis on large-scale variables relevant to tropical cyclogenesis. These metrics include: (1) Growth and saturation of error; (2) Errors versus climatology; (3) Predicted forecast error standard deviation; and (4) Predictive Power. Several questions will be addressed, including:

- Are some variables more predictable than others? If so, which ones?
- Are the results consistent across multiple metrics employed to assess predictability?
- What are the geographical distributions of climatological and forecast errors across the tropical north Atlantic? What is the spatial distribution of the variances of these errors? Are these errors Gaussian in distribution?
- How much does predictability of a given variable change with time or with varying flow regime?
4.1 Error growth

4.1.1 Monthly evolution of variables

To give perspective to the results, we first document the evolution of the relevant variables through the 2010 Atlantic hurricane season, based on ECMWF analysis fields. The evolution of the environment during 2010 is overall consistent with the climatological progression, although conditions were slightly more favorable during peak-season than the climatology, likely contributing to anomalously-high TC activity that year.

In June, much of the central Atlantic is dominated by relatively high values of 850-200 hPa vertical wind shear (exceeding 15 ms\(^{-1}\)) (Figure 4.1A). The shear decreases rapidly through July and remains favorable, in the 7.5-12.5 ms\(^{-1}\) range, for the majority of August and September (Figure 4.1B-D). By October, shear again increases rapidly, especially north of 15°N (Figure 4.1E), and high shear values dominate much of the tropical Atlantic in November (Figure 4.1F). The 850-500 hPa vertical wind shear exhibits a similar spatial distribution and evolution to 850-200 hPa vertical wind shear (not shown). Interestingly, a local maximum in mid- and deep-layer shear evident in the central Caribbean in the 1979-2011 climatology is not evident in the 2010 composites. This region traditionally corresponds to a local minimum in genesis frequency, also known as the Caribbean "dead zone" (McGauley and Nolan 2011). An acceleration of the low-level easterlies in this region, combined with slightly stronger mid-level winds than neighboring regions, results in a maximum in mid-level wind shear (Shieh and Colucci 2010). Despite
the fact that wind shear was anomalously low in the central Caribbean in 2010, only one
genesis event, Matthew, occurred in this region.

An examination of 700 hPa relative humidity reveals that the northeast quadrant of
the tropical Atlantic is generally driest throughout the season (Figure 4.2). This minimum
in moisture is particularly pronounced during June and July (Figure 4.2A-B) when the
Saharan Air Layer (SAL) is most prominent (Dunion 2011). The basin, and in particular
the Main Development Region (MDR) and the Caribbean, possesses the greatest average
700 hPa RH from August-October (Figure 4.2C-E), with noticeable drying in the Gulf of
Mexico and western Atlantic during October-November as mid-latitude intrusions of dry
air become increasingly frequent (Figure 4.2E-F).

Lastly, the evolution of 850-700 hPa circulation is examined. Negative values of
circulation, corresponding to anticyclonic vorticity, dominate the northern half of the
domain from June-August (Figure 4.3A-C). Positive values of circulation progressively
build with time during the first half of hurricane season, particularly within 10-20° N, 15-
50° W (Figure 4.3C-D), before diminishing rapidly in October-November (Figure 4.3E-F).
The African easterly wave train and tropical cyclones contribute substantially to the mean
circulation field. Monthly-mean circulation values at or above 1.5x10^-5 s^-1 in August and
September 2010 exceed climatological values due to easterly waves that tended to be larger
and stronger than average (Philip Klotzbach, personal communication). A maximum in
cyclonic circulation in the western Caribbean is also evident throughout the 2010 season.
The genesis of tropical cyclones Alex, Karl, Matthew, Nicole, Paula and Richard all
occurred in this region.
4.1.2 Evolution of errors

The next stage is to document errors in ensemble-mean forecasts between 0-10 days. Between days 2 and 4, errors in circulation tend to grow most quickly in the eastern MDR along the preferred path of easterly waves (Figure 4.4A, B). Circulation errors throughout the rest of the basin begin to "catch up" by day 8 (Figure 4.4C). Note that regions of maximum circulation error generally correspond to regions of maximum wave and TC activity. Meanwhile, errors in both 700 hPa relative humidity (Figure 4.4D-F) and deep-layer shear (not shown) are generally maximized north of 15°N. Differences in the phase, strength and evolution of mid-latitude dry air intrusions, upper-level troughs, and SAL outbreaks are likely culprits. Such events introduce errors in the forecast that rarely occur equatorward of 15°N, where the shear is more persistently low and moisture is more persistently high.

While there are geographically preferred regions for slightly faster error growth, the errors are growing everywhere with increasing lead-time. By computing the mean of the forecast errors from June-November at every grid point in our Atlantic domain, we compare error growth characteristics against the hallmark study of error growth in the atmosphere (Lorenz 1982). Our results in Figure 4.5 appear qualitatively similar to those of Figure 1 of Lorenz (1982) who examined globally averaged 500 hPa geopotential height (here we re-plot his data in Figure 4.5A), in contrast to our regional averages of circulation, relative humidity, and vertical wind shear (Figure 4.5B-D). In each case, the most rapid error growth occurs during the first 24 h of model integration due to the rapid growth of errors on the small scales, then declines as these small-scale errors saturate and/or energy...
is transferred to larger scales. In both the case of Lorenz (1982) and our study, the error growth rate becomes quasi-linear from around Day 2. The error doubling time from this point is about 4 days in Lorenz (1982), whereas using our metrics the errors may saturate beyond 10 days without having doubled. In other words, errors are growing considerably more slowly in the tropics as expected. Beyond 8 days, as the errors at all spatial scales approach a maximum, the error curve slowly begins to saturate and asymptote towards constant error. Note that there is significant variability in the error growth rate from one forecast to the next, indicated by the large 95% confidence interval accounting for the error growth curve for all forecasts from June-November 2010. If forecast errors were calculated beyond 10 days, eventually a maximum error state would be reached. Beyond this timeframe, a particular deterministic forecast becomes no more skillful than a forecast made with arbitrary initial conditions, and all predictability has been lost (Lorenz 1965).

A linear best-fit trend line is extrapolated from the day 4-6 error growth in each curve. By 240 h, the error growth rate has slowed below the linear rate for all our metrics as large-scale errors saturate. It has dropped the most substantially for circulation (Figure 4.5B), while it remains close to linear for shear (Figure 4.5D). One may interpret this result as suggesting that basin-wide shear on average possesses a more extended range of predictability than circulation.

The error growth rates also vary from month to month (Figure 4.5). For circulation, the greatest errors occur during August and September, where higher uncertainty associated with an abundance of mature TCs contributes to larger errors. It is hypothesized that circulation errors for 2010 are overall greater than average for most seasons due to heightened wave activity. For shear and RH, the greatest errors occur during October and
November, where uncertainty that is likely associated with fronts and mid-latitude systems affecting the tropics results in greater forecast uncertainty. Also note that since the much larger 95% confidence interval is for the full spread of all days within the 6-month period and not just the monthly averages, the variability from one forecast to the next is even greater than what the variability of the monthly means suggests. Therefore, it appears that total error and error growth rates are highly regime-dependent.

A test was also performed to discern the sensitivity of the results to the choice of domain. When the land mask is not used, all of the error growth rate curves are shifted very slightly towards lower error (not shown). However, the change is small and consistent across all variables such that the main result is the same. The greater mean error when including the land mask can be attributed to the fact that errors are especially small in the area removed by the land mask (Figure 4.4). This is likely due to the fact that moisture is persistently high, shear is persistently low, and there are few easterly waves to track in this part of the domain, all factors that contribute to less uncertainty in the forecast.

The philosophy introduced by Lorenz (1982) abides by one particular definition of predictability. However, it may not be the most practical definition. If errors are still growing for a particular forecast, but said forecast has lost skill versus climatology, then is the forecast still useful? To quantify this, we compare ECMWF control and ensemble-mean forecasts for August-October 2010 against ‘forecasts’ during that same time frame using the 1979-2011 climatology as the forecast (Figure 4.6). It is found that the point at which the ECMWF control run forecast errors exceed errors of the climatological forecast varies greatly depending upon choice of variable. By this criterion, forecast skill as a proxy for predictability is lost in the medium range, or 120 h, for circulation (Figure 4.6A). For
some variables, predictability is lost much more rapidly, with forecast error of the control run exceeding that of the climatological forecast within 48 h for 850 hPa vorticity (Figure 4.6B) or only 24 h for 200 hPa divergence (Figure 4.6C). Meanwhile, other variables are associated with much greater estimated predictability, with forecast error not exceeding climatological error until >240 h for 850-200 hPa thickness anomaly (not shown), 240 h for 200 hPa velocity potential (Figure 4.6D), 192 h for 850-200 hPa wind shear (Figure 4.6E), 180 h for 700 hPa relative humidity (Figure 4.6F), and 108 h for 850-500 hPa wind shear (not shown). The much lower errors for circulation versus vorticity forecasts in comparison to climatology are likely a result of the fact that it is much easier to forecast an area-average quantity than it is to correctly predict a value at every possible grid point. The lower predictability associated with 850-500 hPa shear than 850-200 hPa shear is likely due to the greater uncertainty in the overall steering pattern at 500 hPa than at 200 hPa (along with, to some degree, the fact that forecasting climatology for mid-level shear in 2010 actually made a good forecast: only a 2.6 ms$^{-1}$ error on average from Aug-Oct). As expected, forecasts of velocity potential are associated with much lower error than divergence forecasts. The order of magnitude difference between the predictability ‘limit’ (by this definition) of divergence and velocity potential is a direct consequence of the variability of their respective spatial and temporal scales.

The ensemble mean forecast performs notably better on average than the control forecast. For example, using the ensemble mean forecast rather than the control forecast greatly extends the predictive skill for circulation from 108 h to 240 h (Figure 4.6A), or from 180 h to >240 h for 700 hPa relative humidity (Figure 4.6F). The lead time at which a skillful forecast can be made versus climatology only increases by a small amount for
850-200 hPa wind shear, although the control forecast is already very skillful to begin with (Figure 4.6E). These results suggest that the ensemble mean forecast could be useful in predicting environments favorable for tropical cyclones to develop with greater than one week lead time.

Errors associated with a persistence forecast (repeatedly forecasting using the ECMWF analysis) are also included for comparison. For most variables, persistence forecasts tends to only be associated with lower errors than a climatological forecast for <36 h, beyond which the state of the Atlantic has changed significantly from the initial analysis and the forecast is no longer useful. It should be emphasized that errors are defined here by a mean square error as opposed to a mean absolute error, consistent with the majority of the literature on predictability and uncertainty quantification. This is worthy of consideration as it may mean that errors in the vorticity and divergence fields are strongly influenced by extreme local maxima, such as those that would occur associated with tropical cyclones, tropical waves and convective areas.

While comparing errors in the ensemble mean forecast to climatological errors addresses the mean error and error growth rate, it does not address the fact that there is significant variability from the climatological mean state. The ensemble forecast, with each member constructed to represent an equal-likelihood event, is also associated with an increasing variance with increasing lead-time. Therefore, it is perhaps more appropriate to be comparing the probability distribution, or more specifically, the probability density function (PDF) of the ensemble forecast to the PDF of all possible states comprising the climatology. We address these issues by evaluating forecast error variance and Predictive Power.
4.1.3 Forecast error variance

Compiling all forecasts of 850-200 hPa wind shear from August - October 2010 in 10° x 10° boxes and plotting the absolute error of the ensemble mean versus the square root of the predicted forecast error variance (the standard deviation), a modest but unconvincing positive relationship is observed (Figure 4.7A). Low values of predicted error standard deviation should yield a low error of the ensemble mean. High values of predicted error standard deviation can yield low and high errors of the ensemble mean, though the average error of the ensemble mean over a large sample should be larger than when the predicted error standard deviation is low. Not surprisingly, the distribution changes as a function of forecast time, with generally lower (higher) predicted error standard deviations and errors of the ensemble mean at shorter (longer) forecast times. This relationship (black dots) is mostly monotonic increasing and possesses a slope near one for 0-72 h forecasts (Figure 4.7B), 84-156 h (Figure 4.7C) and 168-240 h forecasts (Figure 4.7D). Additionally, all of the predicted forecast error standard deviations lie above the line of unit slope, indicating that the variance of actual outcomes is greater than the variance of the forecasts. In other words, the ensemble is under-dispersive given typical forecast errors. A similar near-monotonic increasing relationship is found for the 850-700 hPa circulation (Figure 4.8), though the slopes for the different times vary. The slope is steeper than one for 0-72 h forecasts (excluding the final bin containing the largest forecast standard deviations) and flattens to a slope less than one as the forecast time increases.
In general, the ECMWF ensemble has demonstrated the utility to discriminate between forecast cases of high mean error and cases of low mean error out to 10 days, for our chosen variables averaged within 10° x 10° boxes. The results are quite robust, even for divergence and relative vorticity. Averaging within 10° x 10° boxes tends to filter out the small scales in the variance predictions as well as mean error, such that the results for vorticity and circulation are nearly identical. Despite overall positive results, the ensemble is found to be under-dispersive for all variables.

4.2 Predictive Power

Predictive Power (PP) is developed such that it is invariant under linear coordinate transformations and applies to multivariate predictions irrespective of whether or not error distributions are Gaussian (Schneider and Griffies 1999). However, for the form used in this study, along with confidence intervals and significance levels, it is assumed that error PDFs are approximately Gaussian. Climatological error distributions, $x^n - \bar{x}$ from each day from June-November 1979-2011 (Figure 4.9A, B) are compared with 120 h forecast error distributions, $x^1_{120} - \hat{x}_{120}$ from each ensemble member forecast from June-November 2010 (Figure 4.9C, D) in a 5°x5° box in the MDR from 55-60° W and 15-20° N. Overall, error distributions for both 850-700 hPa circulation (Figure 4.9A, C) and 700 hPa relative humidity (Figure 4.9B, D) are approximately Gaussian, albeit with some degree of skewness, especially for RH. Note that the best-fit Gaussian curves are forced to be centered on zero, while the peaks in the actual distributions may be 'biased' slightly above or below zero.
From Figure 4.9, it is apparent that the overall variance or width of the climatological distribution is greater than that of the 120 h forecast error distribution. However, if error PDFs at multiple lead times were plotted, they would gradually widen with time from \( t = 0 \) h to \( t = 240 \) h, eventually reaching or exceeding the width of the climatological PDF. There are also several cases in which the circulation error significantly exceeds the climatological mean, corresponding to a peak in the distribution along the tails of the PDFs in Figure 4.9A and Figure 4.9C. The right-most peak along the positive tail in Figure 4.9A corresponds to times when tropical cyclones pass through the domain, as the circulation becomes much greater than climatology. The positive (negative) peak at the end of the PDF tail in Figure 4.9C corresponds to when a TC forms (fails to form) inside the domain when the ensemble does not predict it (predicts it), or when a TC inside the specified domain is stronger (weaker) than predicted. Since the width of the plotted Gaussian fit curve is based upon the variance of the distribution, and the variance is somewhat skewed by high and low outliers, much of the PDF in Figure 4.9C lies “inside” the bell curve.

In order to estimate the predictability of 850-700 hPa circulation versus climatology using the ECMWF ensemble, PP is computed from \( t = 0 \) h to \( t = 240 \) h in 12 h increments twice per day from August-October 2010 (Figure 4.10A). Just as Schneider and Griffies (1999) run a Monte Carlo simulation to produce a confidence interval for the Predictive Power given a small number of actual forecasts, each of these independent forecasts from 2010 is treated as a sample of the true PP. The resulting mean PP crosses the 5% significance level, or the point at which the forecast error distribution and the climatological error distribution are no longer statistically-distinguishable datasets, at 180
h (Figure 4.10B). This implies a predictability limit for circulation at the previous forecast time of 168 h. This is shorter than the predictability for circulation found when using the saturation of error growth to determine predictability, but longer than the limit of predictability found when comparing the RMS forecast error to the RMS climatological error. However, there is significant variability within the 95% confidence interval for PP, and the true predictability for circulation could range between 84 h to >240 h. This broad range can potentially be reduced in future work if additional years are included.

The PP calculation can be understood further by examining the eigenvalue spectrum (Figure 4.10C). PP becomes negative once the product of the first 50 eigenvalues (since we only have 51 ensemble members) of the forecast error exceeds the product of the eigenvalues of climatological error. This corresponds to a broadening of the forecast PDF beyond the width of the climatological PDF. All eigenvalues grow as errors grow with time from day 0 (blue) through day 10 (red), while the climatological eigenvalue distribution remains fixed.

The PP for relative vorticity is also included for comparison (Figure 4.10D). Consistent with our results from Section 3, PP suggests a lack of predictability for relative vorticity in <24 h, suggesting again that area-averaged quantities are substantially more predictable than non-averaged quantities.

Predictive information is calculated in order to quantify the broadening of the forecast PDF with increasing forecast time and compare it to the relative width of the climatological PDF (DelSole 2004). As such, some insight should be provided by examining the structure of the forecast error variance over the tropical Atlantic. Recall that while the mean PP for circulation became negative at 180 h, there were individual forecasts
where the PP became negative at <48 h while other forecasts had positive PP through 240 h (Figure 4.10A). The 156 h predicted forecast error variance for the highest PP valid at 0000 UTC 5 September 2010 forecast (Figure 4.11A, denoted ‘A’) is compared with the 48 h predicted forecast error variance for the lowest PP forecast valid at 0000 UTC 14 September 2010 forecast (Figure 4.11B, denoted ‘B’). It is apparent that, despite the fact that 'A' is from a much longer-range forecast than 'B', there is overall much lower forecast error variance in 'A' than in 'B'. On 5 September, there is only a small amount of error variance associated with the location and strength of a tropical wave over the Leeward Islands and Puerto Rico, which happens to be the remnants of Tropical Storm Gaston, along with another wave that has yet to emerge off of Africa. On the other hand, on September 14, there is a high amount of error variance associated with three TCs across the basin, including Igor, Julia and Karl. This comparison illustrates a common example of when forecasts associated with greater (lesser) error variance are associated with a greater (lesser) determinant of the forecast error covariance matrix and thereby lower (greater) PP. Note that a high ensemble mean circulation does not guarantee a high forecast error variance, as Karl in the Gulf of Mexico on the September 14 at 00Z (Figure 4.11B) was associated with lower forecast error variance than ex-Gaston did on the 5th at 00Z (Figure 4.11A), despite the fact that Karl had a much stronger forecast circulation in its ensemble members than ex-Gaston. Forecast error variance is instead a more direct measure of the spread, and thereby the uncertainty, in the ensemble forecasts.

The above example demonstrates the utility of PP in individual test cases. However, in order to abide by the Gaussian assumption and for the 5% confidence interval to have meaning, it is necessary to examine the season as a whole and compare against the
33-year climatology. The 33-year August-October climatological error variance for circulation shows a local maximum right off the coast of Africa near the Cape Verde islands, as well as along the dominant recurving TC track within the northern MDR (Figure 4.11C). The mean August-October 2010 0-h forecast error variance (i.e. analysis error variance) for circulation indicates overall low uncertainty across the basin, but a maximum near the Cape Verde islands (Figure 4.11D). The mean forecast error variance increases rapidly through 120 h (Figure 4.11E), continuing to increase but at a slightly lower growth rate through 240 h (Figure 4.11F). The overall structures of the climatological and forecast error variance plots are similar, but with greater error variance closer to 25°N in the forecast distribution and greater error variance closer to 10°N in the climatological error variance (along and to the north of the mean September circulation exceeding 5 x 10^{-6} s^{-1} from Figure 4.3D). This is likely a result of there being only a few strong low-latitude tropical waves, and numerous stronger waves and tropical cyclones further north in 2010. The basin mean forecast error variance does not exceed the climatological error variance at 0 h, is of comparable magnitude at 120 h, and definitely does so by 240 h. Therefore, it is not surprising that the determinant of the forecast error covariance matrix exceeds the determinant of the climatological error covariance matrix in the PP calculation at 180 h.

Next, PP for 850-200 hPa wind shear is examined. Overall, the estimated predictability limit for shear is very similar to that of circulation, approximately 168 h (Figure 4.12A). However, the upper and lower bounds of the PP estimate for shear are significantly narrower than for circulation. This is likely due to the fact that, while uncertainties associated with tropical waves and cyclones introduce extreme local maxima in error variance for circulation (and pronounced minima in PP) there is no analog for
shear. However, there is still some flow-dependent variability associated with PP for shear. For example, during September 2010, most ensemble forecasts associated with greatest PP are from September 1-10 2010, and those ensemble forecasts associated with the lowest PP are from September 21-30 2010. Correspondingly, the average 500 hPa geopotential height pattern during the first 10 days of September (Figure 4.12B) was much less amplified than the average pattern during the final 10 days (Figure 4.12C). Similar results were found for various 10-day periods during August and October 2010. These results demonstrate that PP is highly dependent on the flow regime, with a general trend that PP for shear and thereby predictability of shear decrease during more amplified flow regimes. The greater forecast error variance for shear is found in the northern half of the domain (not shown), where greater day-to-day variability will lead to greater uncertainty in the forecast. This is also the region associated with the greatest month-to-month variability for shear (Figure 4.1).

It is possible that sub-seasonal potential predictability values for vertical wind shear can be related to the Madden-Julian oscillation (MJO, Madden and Julian 1971, 1972) or other equatorial waves, perhaps via amplification of geopotential height patterns (Gloeckler and Roundy 2013). If a model or set of models does an adequate job reproducing the MJO or equatorial waves, then predictability will likely be enhanced. If the models do a poor job reproducing the MJO, then there will likely be more difficulty during active periods of the MJO. During September 2010, for example, there does not appear to be a substantial increase or decrease in MJO activity from the beginning to the end of the month (blue line, Figure 4.13). More generally, the literature suggests that the passage of the westerly phase of the MJO reduces vertical wind shear by weakening surface
easterlies and weakening upper-level westerlies (e.g. Mo 2000, Bessafi and Wheeler 2006). If the passage of the westerly phase of the MJO can be accurately predicted, then predictability for shear would certainly be increased. We expect similar results for equatorial waves that can temporarily strengthen (weaken) the low-level easterlies and thereby predictably strengthen (weaken) the shear, again assuming that they are adequately represented in the models. However, further exploring this question in more detail using our dataset is currently beyond the scope of this study.

Eigenvalues of forecast error covariance for shear are also examined. Recall that the 50 eigenvalues $\lambda$ of the truncated forecast error covariance matrix $(E_v)^T E_v$ are equal to the non-zero eigenvalues $\lambda$ of the full error covariance matrix $E_v (E_v)^T$. However, eigenvectors $\varepsilon$ of $E_v (E_v)^T$ are not equal to eigenvectors $\Gamma$ of $(E_v)^T E_v$. As shown in Chapter 2, $\varepsilon$ can be computed from $E_v$, $\Gamma$ and $\lambda$ as $\varepsilon = E_v \Gamma \lambda^{-\frac{1}{2}}$. Doing so for 850-200 hPa wind shear, it is immediately obvious that the first few eigenvalues are associated with larger spatial scale structures with lower peak amplitude, while the last few eigenvalues are associated with smaller spatial scales but with larger amplitudes (Figure 4.14). However, the lack of organization to these structures makes it difficult to draw any robust conclusions. Eigenvectors were also computed for each of the other variables without yielding any particularly insightful results, and we will not discuss them further.

Predictive Power suggests a predictability limit of <144 h for 700 hPa relative humidity (Figure 4.15A), lower than that for either circulation or shear. It is possible that smaller-scale variability associated with regions of convection lead to faster growth of forecast error variance for RH than for either circulation or shear, for which the error variance is dominated by mesoscale or synoptic scale variability. However, in practice, it
can be very difficult to separate the component of error variance for RH associated with individual grid points that either may or may not be convectively active from the synoptic-scale error variance associated with the movement of moist and dry air masses. The overall structure of the Aug-Oct 2010 forecast error variance (Figure 4.15B) closely resembles the August-October 1979-2011 climatological error variance across the basin (Figure 4.15C). The greatest error variance for RH is generally confined to the northern half of the domain, where there are alternating moist and dry periods. Further south, particularly along the ITCZ into the Caribbean, error variance for RH is very low. This is due to the fact that these regions are almost always very moist (Figure 4.2), leading to both a relatively uniform climatology as well as low uncertainty in the forecast.

Predictive Power for divergence decreases rapidly during the first 24 h, then remains steadily negative (Figure 4.16A). This is in agreement with the fact that forecast error variance for divergence grows very large by $t = 24$ h (Figure 4.16B), particularly along the ITCZ and within the southern MDR, but barely amplifies at all and even decreases in some regions by 240 h (Figure 4.16C). Of all of our variables, forecast error variance of 200 hPa divergence is the most closely co-located with regions of convection (Figure 4.17). The ECMWF ensemble forecast error variance for divergence from the 2010-08-24 at 00Z 24 h forecast (Figure 4.17A) identifies regions of greatest error variance in regions associated with the greatest convective activity at the valid time, 2010-08-25 00Z (Figure 4.17B). Particularly high values of forecast error variance are associated with hurricane Danielle, easterly wave activity near Africa, and active convection in the Gulf of Mexico and Caribbean. A similar correlation is observed between the 36 h forecast from 2010-09-07 at 00Z (Figure 4.17C) and regions of enhanced convective activity (Figure
These results are consistent throughout the Aug-Oct period. However, results for longer-range forecasts begin to degrade, as the ensemble has more difficulty correctly identifying regions of greater convective activity beyond a couple of days. Since convective-scale predictability is closely tied to the small scales (Zhang et al. 2007) and errors at the small scales grow and saturate the most rapidly (Lorenz 1982), it is not surprising that divergence becomes unpredictable in less than 24 h. Similar results are also found for 850 hPa convergence.

Consistent with the aforementioned results examining error growth (Figure 4.6), the 200 hPa velocity potential is found to have much greater Predictive Power than divergence (Figure 4.18A). Velocity potential is found to be predictable out to 180 h, on average. Regions of maximum forecast error variance for velocity potential (Figure 4.18B, C) correspond with regions that had the greatest forecast error variance for divergence (Figure 4.16B, C), which also happen to be regions that are the most persistently convectively active. However, unlike 200 hPa divergence, forecast error variance for velocity potential is extremely small at 24 h in comparison with what it grows to be by 240 h.

Lastly, 850-200 hPa thickness anomaly is associated with the greatest Predictive Power of all the chosen variables, to beyond 240 h (Figure 4.19A). Thickness anomaly appears to be highly predictable due to its association with slow-moving large-scale synoptic patterns. There is a remarkable increase in forecast error variance from $t = 24$ h (Figure 4.19B) through $t = 240$ h (Figure 4.19C), with errors continuing to grow out to 10 days (and likely beyond). Similar to both RH and shear, the greatest forecast error variance for thickness anomalies is confined to the northern half of the domain, where greater
uncertainty associated with synoptic features decreases predictability. Further south, where there is significantly-less variability in thickness anomaly, the forecast error variance is correspondingly low. Unlike for circulation, in which tropical cyclones have a significant impact on the total error variance, the majority of the error variance for thickness anomalies is not associated with warm cores of tropical cyclones. Instead, the amplitude and location of anomalously warm and cold air masses, including the subtropical ridge and cold-core lows, appear to be much more significant contributors.

4.3 Summary of results

In this chapter, it was found that ECMWF ensemble forecast errors in the tropical Atlantic are approximately Gaussian. Variables more directly related to large-scale, slowly-varying phenomena (e.g. shear and velocity potential) are found to be more predictable than variables related to small-scale / convective processes (e.g. divergence and vorticity). Additionally, area-averaged quantities, such as circulation, are much more predictable than non-averaged quantities, such as vorticity. Lastly, significant day-to-day and month-to-month variability of predictability exists, likely due to changing flow regimes. For example, more amplified height patterns are associated with lower predictability for shear than less amplified patterns.

While the large scale predictability of a number of variables relevant to tropical cyclogenesis was assessed, whether or not the predictability associated with individual genesis events is equal, greater or lesser remains to be seen. An assessment of wave-relative predictability for the 2010 season follows in Chapter 5.
Figure 4.1: Monthly-mean 850-200 hPa wind shear (m s\(^{-1}\)) for 2010, for the months of (A) June, (B) July, (C) August, (D) September, (E) October, and (F) November. Data to the left of the white land mask line are excluded from calculations.
Figure 4.2: Monthly-mean 700 hPa relative humidity (%) for 2010, for the months of (A) June, (B) July, (C) August, (D) September, (E) October, and (F) November.
Figure 4.3: Monthly-mean 850-700 hPa circulation (s⁻¹) for 2010, for the months of (A) June, (B) July, (C) August, (D) September, (E) October, and (F) November.
Figure 4.4: Spatial distribution of (A, D) 48 h, (B, E) 96 h, and (C, F) 192 h forecast errors for (A-C) circulation (s$^{-1}$) and (D-F) relative humidity (%), averaged from June-November 2010. Forecast errors are for the control run of the ECMWF ensemble.
Figure 4.5: Error growth for (A) global forecasts of 500 hPa heights re-plotted from Lorenz (1982), and (B-D) from 2010 using the ECMWF ensemble, for (B) 850-700 hPa circulation, (C) 700 hPa relative humidity, and (D) 850-200 hPa wind shear. Forecast errors are for the control run of the ECMWF ensemble. A linear best-fit line (dashed) is extrapolated from the day 4-6 error growth in each curve.
Figure 4.6: Comparison of error growth rates for (A) 850-700 hPa circulation, (B) 850 hPa vorticity, (C) 200 hPa divergence, (D) 200 hPa velocity potential, (E) 850-200 hPa wind shear, and (F) 700 hPa relative humidity. Forecasts are made using the ECMWF control forecast (blue), the ECMWF ensemble mean (red), a climatological forecast (green), and a persistence forecast (magenta).
Figure 4.7: Evaluation of ensemble variance prediction for 850-200 hPa shear forecasts. (A) Standard deviation of ensemble forecasts versus the absolute error of the ensemble mean forecast as a function of forecast lead-time, in days (colored); Ensemble standard deviation versus error of the ensemble mean forecast (magenta dots), ensemble-mean forecast error standard deviations in 10 equal-sized bins (black circles), and the 1-to-1 line (dashed) for (B) 0-72 h, (C) 84-156 h and (D) 168-240 h.
Figure 4.8: As in Figure 4.7, but for 850-700 hPa circulation.
Figure 4.9: Probability density functions for climatological differences from the mean or "climatological errors" (A, B) from 1979-2011, and 120 h forecast error distributions (C, D) from Jun-Nov 2010, with Gaussian approximation based upon variance of sample (solid line). Probability density functions are for 850-700 hPa circulation (A, C) and for 700 hPa relative humidity (B, D).
Figure 4.10: Diagnosing the predictability of 850-700 hPa circulation and 850 hPa vorticity using Predictive Power. (A) Predictive Power of circulation for each 0000 and 1200 UTC forecast from Aug (blue), Sep (magenta), and Oct (red) 2010. (B) Same as (A), but with the 95% confidence interval and 5% significance level included and individual forecasts removed. (C) The eigenvalue spectrum of forecast error covariance for 10-day through 0-day circulation forecasts of circulation (colored) valid 0000 UTC 09-15-2010, compared to the eigenvalue spectrum of the climatology (black). (D) Same as (B) but for vorticity. In (A), (B) and (D), the mean for all forecasts is indicated by the solid black line.
Figure 4.11: Ensemble mean 850-700 hPa circulation (magenta contours, $2.5 \times 10^{-5}$ s$^{-1}$ increments) and circulation forecast error variance (shaded) for a low-uncertainty 156 h forecast (A) valid 0000 UTC 09-05-2010, and a high uncertainty 72 h forecast (B) valid 0000 UTC 09-14-2010. Error variance for circulation from (C) ERA-I 1979-2011 is compared against the mean (D) 0 h, (E) 120 h and (F) 240 h forecast error variance from the ECMWF ensemble for Aug-Oct 2010.
Figure 4.12: (A) Predictive Power for 850-200 hPa wind shear for consecutive 10-day time intervals from Sep 2010. 500 hPa geopotential height anomalies (m) for (B) Sep 1-10 and (C) Sep 21-30, 2010.
Figure 4.13: Phase diagram of the RMM index, from July (red), August (green) and September (blue) 2010 with dates labeled. Points within the inner circle denote weak or no MJO. Available online at http://www.bom.gov.au/climate/mjo
Figure 4.14: (A) 1st and (B) 50th eigenvectors of 120 h forecast error for 850-200 hPa wind shear.
Figure 4.15: (A) Predictive Power for 700 hPa relative humidity from Aug-Oct 2010, with the 95% confidence interval and the 5% significance level. The 180 h forecast error variance during those 3 months (B) as well as the 1979-2011 climatological error variance (C) for relative humidity are also indicated.
Figure 4.16: (A) Predictive Power for 200 hPa divergence from Aug-Oct 2010, with the 95% confidence interval and the 5% significance level included. The 24 h (B) and 240 h (C) mean forecast error variance for divergence during those 3 months are indicated.
Figure 4.17: Ensemble mean forecast 200 hPa divergence (grey contours in $1 \times 10^{-5}$ s$^{-1}$ increments; solid = positive, dashed = negative) and forecast error variance for (A) 2010-08-24 0000 UTC forecast hour 24 (valid 2010-08-25 0000 UTC) and for (C) 2010-09-07 0000 UTC forecast hour 36 (valid 2010-09-08 1200 UTC); GOES-12 and EUMETSAT-8 composite infrared satellite images from (B) 2010-08-25 at 0000 UTC and (D) 2010-09-08 at 1200 UTC (http://catalog.eol.ucar.edu/predict/).
Figure 4.18: as in Figure 4.16 but for 200 hPa velocity potential.
Figure 4.19: as in Figure 4.16 but for 850-200 hPa thickness anomaly.
Chapter 5

Predictability Results Part II: Wave-Relative Framework

In this chapter, predictability is assessed from a wave-relative framework to focus more specifically on the genesis events from 2010. First, results from several individual cases are presented to explore which aspects of predictability may be consistent amongst many cases, as well as how they differ from case-to-case. Next, using the statistics from the entire sample of tropical waves, the evolution of forecast variables leading up to genesis, as well as the errors and forecast variance of these variables, is explored. Thereafter, joint distributions of variables, joint error distributions and lagged-joint distributions between two (and occasionally three) variables are examined. Lastly, variance prediction is computed from a wave-relative framework and compared against the results of section 4.1.3. A few key questions in this chapter include:

- How consistent are results from one genesis event to the next?
- What environmental factors may limit predictability?
- Does the predicted forecast variance accurately convey the amount of uncertainty in the forecast? Similarly, to what extent does uncertainty in the environment of the tropical wave limit the predictability of the genesis event?
5.1 Individual cases of study

While an examination of the predictability of large-scale genesis “ingredients” has revealed some insight into the predictability of tropical cyclogenesis, but only on a basin-averaged sense, it is necessary to examine the wave core and environment for individual cases to gain a deeper understanding of the predictability of the dynamically-related variables and processes that culminate in genesis. In this section, all forecast data are valid at time of genesis (as determined by NHC) for the particular case being discussed, unless otherwise noted. Plots are arranged with forecast lead time prior to genesis along the x-axis, with decreasing lead time from left to right. All environmental data are calculated with respect to locations of the maximum circulation and thickness anomaly in each ensemble member. Several genesis events will first be presented where there is a clear relationship between the ensemble forecast for the environment and the predicted circulation. Thereafter, a few outliers and more difficult cases to understand will be presented.

i. Earl

The first genesis event examined is that of Earl (2010), as it is a case in which the relationship between the favorability of the forecast environment and the forecast strength of circulation at time of genesis is fairly straightforward. For Earl, there is a reasonably strong signal for a circulation of at least moderate \((6 \times 10^{-5} \text{ s}^{-1})\) strength within the verifying genesis region even 192-240 h before genesis (Figure 5.1A). With the exception of a few
odd drop-offs in strength of circulation that occur at, e.g. 120 h, the strength of the ensemble-mean forecast circulation (solid black line) increases fairly steadily from long-range forecasts to short-range forecasts, right on through genesis at $t = 0$ h. However, at this particular time (120 h), the verification falls outside the entire ensemble, suggesting a complete failure of the ensemble and a lack of predictability at and beyond this lead time.

As expected, the number of ensemble members indicating very weak (black), weak (blue) or moderate (green) circulations decreases while the number of members indicating a stronger (magenta) circulation increases with decreasing lead time prior to genesis. Although an increase with time in the number of ensemble members indicating a moderate to strong circulation is expected, the clustering toward circulation values as strong as $\geq 1.25 \times 10^{-4} \text{s}^{-1}$ in the final 84 h prior to genesis is stronger than average for 2010. With verification of $1.24 \times 10^{-4} \text{s}^{-1}$ in the control run analysis, Earl was associated with a particularly strong circulation value already at time of genesis, likely due at least in part to the large size of the tropical wave from which it formed.

While there is good agreement very early on amongst ensemble members that 850-200 hPa environmental wind shear will be low and 700 hPa core relative humidity will be high, a few outliers predicting a high shear environment or dry core do exist more than 96 h before genesis (Figure 5.1B,C). Also note that at any particular forecast lead time, the ensemble members associated with the lowest (highest) shear or the moistest (driest) core are often associated with the strongest (weakest) circulation\textsuperscript{2}; good examples of this relationship for either variable are evident in the 144 h and 84 h forecasts. Despite a very

\textsuperscript{2} Note that while the y-axis now indicates either shear or RH in B, C, and D, the color coding still indicates strength of circulation.
clear positive relationship between core moisture and strength of circulation, a near-opposite relationship between environmental moisture and strength of circulation is observed (Figure 5.1D). This apparent paradox is actually an artifact of the fact that ensemble members that strengthened the circulation more quickly produced a solution in which the wave began to gain latitude more rapidly. This is consistent with the fact that the magnitude of the “Beta drift”, or the northwestward drift of a vortex due to the development of Beta gyres which form via the advection of planetary vorticity by the storm-scale cyclonic circulation, is proportional to the strength of the total circulation associated with the vortex (Fiorino and Elsberry 1989). Season-wide relationships between latitude, strength of circulation, and environmental moisture will be elaborated upon in the marginal distributions that will appear in section 5.3.

In addition to the aforementioned contrast between core RH (Figure 5.1B) and environmental RH (Figure 5.1D), there is also a stark difference in magnitudes. While core RH is between 80-90%, environmental RH is between 50-60%. This phenomenon occurred for Earl as well as several other easterly waves during 2010. Occasionally, the easterly wave becomes cut off from a larger moisture source (often to the south) as the wave penetrates deep into a large airmass of dry air (Figure 5.2A). While the environment can be extremely dry, an insignificant amount of dry air actually penetrates the core and the system still develops. Limited penetration of dry environmental air appears to be especially common for larger circulations associated with larger regions of core moisture. This result is consistent with the marsupial paradigm, in which a recirculating region of moisture co-located with the critical layer of the pre-depression wave trough avoids interaction with the drier environmental air located outside of the critical layer (Dunkerton
et al. 2009). Observations (Braun et al. 2013) and idealized simulations (Braun et al. 2012) also suggest that dry air >300 km from storm center does not enter recirculating region, consistent with these results for pre-Earl. However, these ideas have only been developed for cases in which shear is low (Earl happens to be such a case). The applicability of the marsupial paradigm and whether or not the system can avoid interaction with a dry environment in a more sheared regime has not been explored. Fiona, which will be discussed in greater detail below, was another such case in which a pocket of higher RH values became detached from a broader environment of high RH values further southeast (Figure 5.2B).

Another important concept illustrated by Earl is the relationship between uncertainty in the environment and how this translates to uncertainty in the strength of circulation at time of genesis. More specifically, sharp gradients in environmental shear or moisture are often associated with increased forecast variance, due to greater uncertainty in where that gradient will be located. When a tropical wave such as Earl is projected to be located right along this gradient with members on both the dry side and the moist side (Figure 5.2C), the result is a very large variance in the projected environmental moisture, such as Earl in the 168 h forecast (Figure 5.1B). This translates to increased variance and thereby uncertainty in the strength of the predicted circulation (Figure 5.1A). On the other hand, once the ensemble reaches an agreement as to which side of the moisture (or shear) gradient the wave will be located (Figure 5.2D), the variance in the moisture forecast drops considerably. This ultimately contributes to a reduction in the variance in the circulation forecast (Figure 5.1A). Therefore, there is a clear dynamical link between the uncertainty
in the strength of circulation and the uncertainty in the location of the wave and/or location of gradients of environmental variables.

\textit{ii. Hermine}

Forecasts for the genesis of Tropical Storm Hermine also exhibited a clear relationship between the number of ensemble members depicting a favorable environment for genesis and the number of members producing a strong circulation at time of genesis (Figure 5.3). While it is clear that an increasing number of ensemble members depicting genesis corresponded to a subtle decrease in the number of forecasts with a drier core (Figure 5.3B), RH remains favorable throughout the forecast. On the other hand, a substantial drop in forecast deep layer (Figure 5.3C) and mid-level (not shown) shear occurs between 120-96 h lead time. Also note that, again, many of the strongest circulation forecasts at a particular lead time correspond to the individual ensemble members with the lowest forecast shear. Lastly, there is once again a distinct inverse relationship between environmental moisture and strength of circulation (Figure 5.3D). However, as with Earl, the strongest members were also the furthest north, likely due to increased Beta drift, and are thereby located closer to dry mid-latitude air at verifying time of genesis. For both Earl and Hermine, there appears to be no correlation between predicted 200 hPa divergence and strength of circulation (not shown).
iii. Fiona

In addition to being a case with a very moist core embedded in an otherwise very dry environment, Fiona was also an interesting case in that there was virtually no genesis signal in the ensemble between 144-240 h prior to genesis, followed by a rapid transition towards a very strong signal for genesis from 96-120 h lead-time (Figure 5.4A). In fact, the ensemble over-adjusted as the predicted strength of circulation was too strong in most members from 48-96 h lead time. It is likely that a more accurate forecast for these environmental parameters contributed to a superior genesis forecast at shorter lead times. Fewer ensemble members predict a dry core (Figure 5.4B), a dry environment (not shown), or a high shear environment (Figure 5.4C) as an increasing number of members simultaneously produce a stronger circulation.

While changes in predicted environmental conditions likely contributed somewhat to changes in predicted strength of circulation, they were likely too subtle to be the only factor. Interestingly, 108 h prior to the genesis of Fiona also corresponds to the timing in which the pre-Fiona wave first emerged into the Atlantic from the coast of Africa (Figure 5.4D). This corresponds with the time at which the pre-Fiona vortex passed over the Dakar radiosonde location (station ID #61641). As upper-air data is particularly sparse over Africa, the vortex was likely poorly initialized in many successive model runs up until additional data provided by the Dakar sounding improved the initial representation of the easterly wave. In addition to direct observations, it is also likely that the initialization of the ECMWF ensemble improved once the wave emerged over water as data retrieved via microwave profiles from polar orbiters, as total precipitable water (TPW), are not available
over land. For these reasons, it appears that the predictability of genesis for Fiona is directly related to whether the parent wave is over land or sea at the time of model initialization, particularly if that landmass is the data-sparse continent of Africa. While Fiona was perhaps the most obvious case of such a phenomenon occurring during 2010, several other genesis events, including Lisa, and to a lesser degree, Tomas, appear to have been associated with decreased predictability while their associated pre-genesis vortices were still located over Africa.

iv. Igor

Igor was an interesting case in that genesis occurred in an environment of unusually high shear, with 850-200 hPa shear exceeding 14 ms\(^{-1}\) (Figure 5.5C). This was the greatest value of shear to be associated with any genesis event in 2010. Meanwhile, Igor was also associated with one of the strongest circulation values at the time of genesis (Figure 5.5A), and was a rare case in that the strength of predicted shear actually increased with decreasing lead time before genesis. While forecasts for higher values of environmental shear often prevent particular ensemble members from predicting genesis for most other events in 2010, Igor appears to be a case where a large and strong initial circulation was able to overcome higher values of shear. The strength of the predicted circulation at time of genesis from Igor appears to have been strongly related to the strength of the circulation 48 h prior to genesis in 120 h and 168 h forecasts. A moderate-strength vortex (in most ensemble members) exiting the African coast resulted in a moderate to strong vortex at time of genesis (Figure 5.6A, B). At shorter lead times, better agreement for a strong wave
exiting the coast of western Africa resulted in a very strong genesis signal (Figure 5.6C, D). Igor was also associated with not only a very moist core (Figure 5.5B), but also a very moist surrounding environment (Figure 5.5D). In fact, Igor developed in the most moist environment at 700 hPa of all the systems in 2010. Igor demonstrates that a very moist environment can help a system to overcome other challenges during development, such as high shear.

Another noteworthy aspect of Igor was the fact that the circulation forecasts for the genesis of Igor were associated with unusually high variance even at very short lead times (Figure 5.5A). While there was low uncertainty in the forecast for moisture (Figure 5.5B) due to a combination of low ensemble variance and a lack of any sharp moisture gradients near the center of circulation in any ensemble member (Figure 5.7A), there was considerable uncertainty even at short lead times in the strength of the shear (Figure 5.5C). This was due to the fact that the wave associated with Igor was traversing through a very sharp gradient in shear and a local maximum in forecast variance of shear, evident in forecasts from, e.g. 36 h lead time (Figure 5.7B). So while Igor managed to develop in a seemingly unfavorable environment despite higher values of shear, it nonetheless appears that uncertainty in the strength of the shear still translated to uncertainty in the strength of circulation.

v. Bonnie

The predictability of the genesis of Bonnie was particularly difficult to relate to changes in its environment. While in many other cases the ensemble distribution shifts
from weak to strong circulation at a time when the environmental forecast becomes more favorable, a small but sudden jump in the circulation forecasts occur between 72-96 h pre-genesis for Bonnie (Figure 5.8A) with seemingly no major changes in predicted moisture or shear (Figure 5.8B,C). Not only were the changes with time in predicted moisture and shear fairly insignificant, but the actual verifying values were not particularly noteworthy either. The core and environment were slightly drier than average for 2010 at 82% and 58%, respectively, but well within the variance of the sample. Shear was also neither unusually high nor low at 6 ms$^{-1}$. However, Bonnie was one of only two systems in 2010 (the other being Shary) in which predictability appears to have been closely tied to the divergence forecast (Figure 5.8D). Early on, the majority of forecasts have only weak divergence or even net upper-level convergence. At lead times greater than 72 h, there is little relationship between divergence and circulation, as the initial vortex is simply too weak to generate a TC in the ensemble. However, with less than 72 h lead time, the ensemble initializes with a sufficiently-strong vortex such that a clear relationship between divergence and circulation develops; the members associated with the strongest circulation are often, but not always, those with the strongest upper-level divergence.

In addition to the relationship between uncertainty in divergence and strength of circulation, what other factors might have contributed to making Bonnie such an unusual case? Another attribute about Bonnie was the fact that it originated from an anomalously small wave, especially for 2010. It is likely that its small size made Bonnie difficult for the ensemble to accurately predict more than 72 h before genesis, due to sampling issues, resolution issues, or a combination of the two. It is apparent by $t = 36$ h that the forecast initialized 07/19/2010 at 00Z (Figure 5.9A) will only result in a modest genesis signal by $t$
= 84 h (Figure 5.9B), at verifying time of genesis. However, the forecast initialized just 12 h later on 07/19/2010 at 12Z already has a much stronger wave at $t = 24$ h (Figure 5.9C), which ultimately yields a much stronger genesis signal by $t = 72$ h (Figure 5.9D). So while many genesis events in 2010 showed considerable sensitivity to the environment, the dominant sensitivity for Bonnie appears to be to the strength of the initial circulation. Perhaps it is more difficult for the ensemble to initialize weak waves accurately.

**vi. Gaston**

Forecasts for the genesis of Tropical Storm Gaston were even worse than those for Bonnie, failing to produce more than a rogue ensemble member with a strong circulation more than 24 h prior to genesis (Figure 5.10A). Immediately apparent is the fact that the forecast environment (as well as the verifying environment) 300-1000 km from the center of circulation was quite dry. However, unlike for larger tropical waves such as those associated with Earl and Igor, forecasts for a dry environment (Figure 5.10D) also translated to a very dry core (Figure 5.10B). Ultimately, these forecasts were much drier than verification, particularly those produced 5-7 days prior to genesis which had ensemble-mean core RH values of < 50%. It is likely that the entire ensemble was analyzing too much dry air within the wave pouch, with verification much more moist than the entire forecast distribution. Interestingly, there was little change in forecast shear with time, and the shear forecast in the 5-7 day range was actually too low (Figure 5.10C). Therefore, these forecasts for a dry core associated with a weak vortex cannot be attributed to a high shear bias. In addition to the dry bias in the core, initial strength of circulation
also appears to have been a major forecast issue. This problem was apparent earlier in the wave’s track across the Atlantic as well. Even 24 h and 12 h forecasts initialized 3.0 and 2.5 days, respectively, prior to genesis were verifying poorly due to a low-bias in the initial strength of circulation, with predicted circulations too weak and with maxima in the wrong locations (Figure 5.11A,B) relative to verification (Figure 5.11C). Not surprisingly, a poor 12 h forecast with a large low-intensity bias (Figure 5.11B) resulted in a very weak genesis signal at 60 h, the verifying time of genesis (Figure 5.11D). In this sense, Gaston was a similar case to Bonnie in that predictability was highly limited by poor initializations and short-range forecasts for circulation.

\[ \text{vii. Lisa} \]

Upon first inspection, Lisa appears to be a fairly straightforward case. The signal for development is initially very weak from 180-240 h prior to genesis (Figure 5.12A). Thereafter, there is a significant increase in the ensemble mean forecast circulation, albeit with enormous spread, from 120-168 h pre-genesis. Similar to pre-Fiona, the pre-Lisa vortex emerged from the African coast on September 15, approximately 144 h prior to genesis, during the time at which there was the greatest increase in ensemble-mean circulation. As was the case for pre-Fiona, it is likely that a combination of in-situ data from the Dakar sounding and microwave data that are only available over the ocean acted to improve the ensemble analysis and decreased uncertainty in the forecast. An increase in the number of ensemble members producing a strong circulation corresponds to a decrease in shear (Figure 5.12C), a modest increase in core moisture (Figure 5.12B), and a
lack of any members depicting a very dry environment of less than 50% RH (Figure 5.12D). However, at 84 h prior to genesis, the entire ensemble appears to lose the genesis signal, suggesting that perhaps genesis will either be delayed or is less likely to occur at all. The ensemble soon corrects itself by the next model cycle, albeit still weaker than verification. This sudden drop in predicted circulation does not correspond to any noticeable changes in environment. What caused this? This was also a time where there was a noticeable drop in strength of circulation at initialization time, which resulted in a much weaker vortex 48 h prior to time of genesis, and ultimately at time of verification (not shown). It is hypothesized that the vortex was poorly initialized at this timeframe, perhaps due to sampling or data assimilation issues. However, as pre-Lisa was already well off the coast of Africa by this time and not located near any radiosonde sites, it is difficult to relate any specific observations (or lack thereof) to predictability of genesis as was done for pre-Fiona. It is possible that there was a problem with either satellite retrievals or the assimilation of satellite data at this time, but confirmation of this hypothesis is currently beyond the scope of this study.

viii. Nicole

While there was a strong genesis signal for Nicole even from very early lead times, there was also an unusually high spread in the strength of circulation (Figure 5.13A). However, unlike for many earlier cases, this large variance for circulation cannot be directly associated with variance for shear or RH, as there was very good agreement for a low-shear (<5 ms\(^{-1}\)), high-moisture (core RH > 75%) environment even 7-10 days prior to
genesis (Figure 5.13B,C). Also note that, due to the monsoon-trough like environment in which Nicole developed, the wave was very far from any sharp gradients in shear (not shown) or RH (Figure 5.13D) that could have possibly contributed to uncertainty in circulation. Motivated by this oddity, a slightly more in-depth investigation of the genesis of Tropical Storm Nicole was performed.

The genesis of Nicole was quite complicated, with some of the incipient low-level vorticity originating from a vorticity separation from TS Matthew over Central America, all occurring within a somewhat unusual monsoon-like gyre in the western Caribbean. Due to the complexity of this particular genesis event, both errors and ensemble variance of circulation maxima were unusually high. However, this was also a case where the presence of an environment that would be favorable to genesis, i.e. the center of the monsoon trough, was well-modeled many days in advance. The monsoon trough or gyre was characterized by relatively high shear of 10-20 ms\(^{-1}\) on its periphery, but very low shear of 5 ms\(^{-1}\) or less within 200-300 km of its center. The monsoon gyre was also associated with a very moist environment, with RH>70% at 700 hPa and >85% at 850 hPa within 400 km of the center of the trough, particularly on the southeast side. The ECMWF ensemble performed very well in predicting this environment, forecasting >70% probabilities of having RH>70% at 700 hPa (Figure 5.14A) and >85% at 850 hPa, co-located with only a 30% chance of 850-200 hPa shear >10 ms\(^{-1}\) (Figure 5.14B), or a 70% chance of shear <10 ms\(^{-1}\) as many as 8-10 days pre-genesis over a large region in the western Caribbean. Despite how well this favorable environment for genesis was predicted, the actual genesis event was not well predicted. Forecasts from days 5-10 depict TS Matthew either making landfall in Central America and dissipating, or taking a more northerly route into the Gulf of Mexico. In either
case, there is no vorticity separation from Matthew and no subsequent genesis of Nicole in any ensemble member. Even the ensemble initialized only 60 h prior to genesis exemplifies a lack of predictability. This run forecasts a regeneration of the remnant vorticity associated with Matthew, but the vast majority of the ensemble members have this occurring in the East Pacific. Only five out of 51 ensemble members have a definitive genesis signal in the Caribbean at this time. The overall trend is that the circulation forecasts are too strong and too far east at earlier lead times, and too weak and too far west at later lead times.

Nicole was also somewhat unusual in that the ensemble members initialized with the strongest vortex 48 h prior to genesis were not the same members associated with the strongest vortex at the time of genesis. In fact, the circulation strengthens the most rapidly in many ensemble members initialized with a weak vortex. Both of these occurrences appear to be due at least in part to land interaction and distance from the coast of Belize and the Yucatan Peninsula, as the initial circulation is over land in the forecast initialized 48 h prior to genesis. When comparing the 10 ensemble members with the strongest circulation at time of genesis with the 10 members with weakest circulation at time of genesis, the members that eventually produce the strongest circulation do not necessarily correspond with the 10 ensemble members with the strongest initial circulation. Instead, the members with the strongest genesis signal simply develop a new circulation over the open waters of the Caribbean more quickly than the other ensemble members, consistent with a breakdown or splitting of the gyre. The ensemble mean of the 10 strongest members in the 48 h forecast verifying at the time of genesis indicates a contour of relative vorticity $> 3 \times 10^{-4} \text{s}^{-1}$ with radius $> 250 \text{ km}$ and a closed 144 dm contour at 850 hPa (Figure 5.15A),
while much weaker relative vorticity and a much shallower height minimum exist for the 10 weakest members (Figure 5.15B). The resulting center of circulation for the stronger genesis members is several hundred km further northeast than it is in the weakest members. Note that the verifying point of genesis was even further northeast than the mean of the 10 strongest members. Considering the fact that pre-Nicole was moving in a northeasterly direction, the 10 members with the strongest genesis signal had too slow a forward speed, and the 10 members with the weakest genesis signal had much too slow a forward speed. Wind and height fields at 850 and 500 hPa indicate greater interaction between the monsoon trough enveloping Nicole with the mid-latitude trough over the southeastern United States in the ensemble members with a faster forward motion, with little to no interaction in the slowest members. Verifying analyses indicate even greater interaction between the two troughs than modeled 48 h prior. Forty-eight hours later in the forecast, while still slower and further from verification than the 10 strongest members (Figure 5.15C), all of the members that do not have any robust genesis signal in the vorticity, thickness or height fields at the verifying time of genesis do eventually produce an apparent genesis signal (Figure 5.15D). In other words, genesis is delayed but not denied, but only in model world. Therefore, the environment within the monsoon trough remains favorable for genesis for a multi-day period. It is likely a matter of timing the co-location of the predecessor vorticity with the moist, low-shear center of the monsoon trough and, more importantly, having the circulation located over open water and higher moist enthalpy surface fluxes.

In summary, the genesis of Tropical Storm Nicole (2010) was a particularly complicated meteorological event due to uncertainty associated with a predecessor TC (TS
Matthew), the presence of a monsoon-like gyre, and land interaction. The presence of high moisture and relatively low shear associated with the monsoon trough appear to be very predictable out to a week or longer lead times. The fact that a tropical cyclone will exist somewhere within the monsoon trough is also quite predictable out 7+ days. However, the location and strength (in terms of circulation) of this tropical cyclone, as well as if and where genesis will even occur, remain uncertain even two and a half days prior to genesis. Land interaction exacerbated implications of uncertainty in the track forecast, as a small difference in the location of the center of circulation of the precursor disturbance meant the difference between genesis and non-genesis in the western Caribbean.

ix. Overarching results

The results from this section suggest that, while there are some similarities amongst multiple genesis cases, there is also enormous case-to-case variability. For some waves, predictability is clearly limited by uncertainty in the environment. For other cases, predictability is more obviously limited by the strength and location of the initial vortex. In general, waves traversing tighter gradients of RH or shear (perpendicular to the gradient) have greater forecast variance, and thereby uncertainty, in circulation forecasts. Lastly, the genesis of several events that occur in a low-shear environment accompanied by a very moist core but a dry environment is consistent with the marsupial paradigm (Dunkerton et al. 2009). The fact that dry air >300 km from the center of circulation does not enter the recirculating region is consistent with previous studies (Braun et al. 2012, 2013).
5.2 Evolution of variables, errors and variance forecasts for all cases

There is much to be learned about the predictability of tropical cyclogenesis from the individual cases. However, the case-to-case variability is so large that it can become difficult to separate the general trends and consistencies (or inconsistencies) throughout multiple cases. In this section, the evolution of variables, errors, and variance forecasts for all cases from 2010 is examined.

We begin by examining the evolution of the ensemble mean forecast circulation from the ECMWF ensemble for each developing system in 2010, along with the mean for all systems (Figure 5.16A). Overall, the ensemble mean forecasts for virtually all cases are too weak at earlier lead times. This reflects an initial weak-bias in the genesis signal. As lead time decreases, the ensemble mean circulation strengthens as an increasing number of ensemble members begin to predict genesis to occur. However, it is clear that there is an enormous spread in the verifying circulation values, from 7.5x10^{-5} s^{-1} to 1.6x10^{-4} s^{-1}, making it impossible to define an exact circulation threshold for genesis that corroborates with genesis as determined by NHC.

Error growth for circulation is also examined (Figure 5.16B). Unlike in traditional error growth plots in which lead time increases from left to right (e.g. Lorenz 1982), lead time decreases in these plots to be consistent with the rest of the wave-relative results presented. Perhaps a more appropriate terminology would be “error decay” diagrams. Nevertheless, results are consistent with those of Chapter 4 in that errors grow more rapidly

\footnote{While verification is defined as the verifying analysis of the control run, the control and ensemble mean are nearly identical at \( t = 0 \).}
at shorter lead times than longer lead times. However, error growth here is fairly linear out to 96 h, as opposed to the rapid error growth during the first 48 h followed by a slowing of error growth observed in the basin wide computation. Perhaps the centering of the domain on the center of circulation in each ensemble member somehow acts to smooth the transition from short term to medium range errors. Lastly, change in variance of the ensemble forecast versus time is examined (Figure 5.16C), which indicates, as expected, a decrease in variance with decreasing lead time. However, it is unclear upon visual inspection alone what the relationship is between the variance and errors for any particular case, and if the variance is meaningfully conveying the uncertainty in the forecast. These points will be analyzed further in section 5.4.

The evolution of core divergence depicts an unexpected result in that ensemble mean divergence increases very rapidly in approximately half of the cases in the final 12-24 h prior to genesis (Figure 5.17A). The fact that the ensemble mean forecast for divergence is so far from verification for so many waves just 24 h prior to genesis suggests that there is either very little predictability for divergence, or that there is something fundamentally wrong with the ensemble and how it represents convection and/or outflow (and perhaps both). Not surprisingly, divergence errors grow much more rapidly than circulation errors during the initial 0-12 h (Figure 5.17B). However, they do not appear to saturate any more quickly than circulation errors, only beginning to asymptote after 192 h. This apparent increase in predictability in comparison with the results of Chapter 4 is likely due to the spatial averaging of divergence within a 300 km radius. The plot of variance of divergence versus time depicts a minimum in divergence at 24 h and an unexpected maximum at 0 h, but is otherwise mostly flat (Figure 5.17C). This implies that the
uncertainty for divergence at short lead times is just as great as it is at long lead times, and is thereby an unpredictable metric.

There is a large spread in ensemble-mean 700 hPa environmental RH at time of genesis, from 37-78% (Figure 5.18A). By comparison, core RH only varies by about half as much, between 69-90% (Figure 5.18B). Perhaps more importantly, the mean for all cases appears to converge to a “threshold” value of 85%. Also, with only one notable exception (Gaston), medium- to long-range forecasts for core moisture also exhibit much lower spread than medium- to long-range forecasts for environmental moisture. However, errors of environmental moisture appear to be continuing to grow out to day 10 while errors for core moisture do not (Figure 5.18C,D). Therefore, while the actual errors are larger within the ring that defines the environment, there may nonetheless be greater predictability associated with this domain due to its larger areal extent. Also apparent is the fact that the ensemble mean for all cases is slightly below their verifying values even 12-24 h prior to genesis, suggesting that not only is there a dry bias, but it begins to appear at very short lead times. Similar to circulation, the difference in forecast variance distributions between cases varies immensely for both core and environmental moisture at longer lead times (not shown).

Shear varies between 2-15 ms\(^{-1}\) in the 850-200 hPa layer and between 1-6 ms\(^{-1}\) in the 500-200 hPa layer for genesis events in 2010 (Figure 5.19A,B). Errors in shear continue to grow between 192-240 h, suggesting a longer range of predictability (Figure 5.19C), consistent with the results of Chapter 4. Lastly, the forecast variance distribution for shear converges towards low values at short lead times, suggesting lower case-to-case variability of the uncertainty than occurs for most of the other variables (Figure 5.19D).
5.3 Joint distributions, error distributions, and lag-distributions

While it is illuminating to examine the evolution of individual variables as was done in section 5.2, the reality is that many of our variables are dynamically related to each other. Therefore, joint distributions that account for the relationship between variables for all genesis events in 2010 are examined in this section. Here, joint distributions are the composites of the relationships between variables for all 21 genesis events in all 51 ensemble members. Also, unless otherwise stated, forecasts over the entire 0-240 h time frame are included.

We begin our exploration of the highly multivariate phase space of variables relevant to genesis by examining the relationship between moisture and strength of circulation. The joint distribution for 700 hPa core RH and circulation reveals the expected results of an increase in the number of cases with strong circulations given higher values of moisture (Figure 5.20A). While higher moisture does not necessarily preclude the existence of weaker circulation, as there are still a few counts in the bins of high RH / low circulation, it appears that core RH < 60% almost certainly prevents circulation from reaching $1 \times 10^{-4}$ s$^{-1}$. The sample also appears to be heavily biased towards cases with core RH between 70-90%. There are very few samples of low RH since only those cases in which genesis actually occurs are included. On the other hand, there are very few cases of $\leq 90\%$ RH simply because it is very difficult to sustain such a high moisture content over a region 600 km in diameter. In this sense, the depicted joint distribution may give the false impression that $\leq 90\%$ RH is actually less favorable than 70-90\% RH at 700 hPa. Therefore, to account for unequal samples within each RH bin, each column is normalized
by the total number of cases contained within it. Doing so gives a very clear depiction that the higher the RH, the greater the potential for a system to have a stronger circulation, due to an increased fraction of that column being associated with stronger circulation values (Figure 5.20B). Similarly, if RH is $< 60\%$, it is most likely that the circulation of the wave will fall between $2.5-5.0 \times 10^{-5} \text{ s}^{-1}$. Note that there are fewer cases of circulation $< 2.5 \times 10^{-5} \text{ s}^{-1}$ because circulations that weak are close to no longer trackable.

The joint distributions of forecast errors for circulation and 700 hPa core RH (Figure 5.20C), where we now subtract verification from each case, depict an overall similar structure to the “non-error” version of that same plot (Figure 5.20A). However, by computing errors, the dry bias becomes particularly pronounced. In general, cases associated with this dry bias are also associated with a weak circulation bias. However, it may not be particularly surprising that there are many more cases with dry errors than moist errors since only cases in which genesis actually occurred are included. With a sample of cases in which verification for RH is consistently in the 80-90\% range, it is physically impossible to get errors much greater than $+20\%$, but it is not difficult to have errors of $-40\%$. That said, the fact that the dry bias has already appeared in short-range, 0-72 h forecasts, suggests a fundamental problem in the model physics or parameterizations (Figure 5.21A). Thereafter, the dry bias becomes progressively worse with increasing lead times (Figure 5.21B,C). There is also an apparent weak bias for circulation (Figure 5.20D) which grows with increasing lead times (Figure 5.21). While this weak bias is likely in part dynamically-related to the dry bias, the fact that the sample only includes developing systems is also a likely contributor. In future work, non-developing waves will be included and should produce a more even distribution about zero error for circulation. Lastly, note
that for errors of $< -20\%$ RH (dry bias), the forecast is so dry that it is nearly physically impossible to produce a forecast with a positive circulation error (weak bias).

As already alluded to in section 5.1, a clear relationship between $300 \leq r \leq 1000$ km “environmental” RH and strength of circulation is much less obvious (Figure 5.20D). In this case, normalizing the data in each column by number of cases does not help matters. RH less than 30% appears to be a hard cut off for non-development, but other than that, 30-40% RH appears to be almost as favorable as 80-90% RH. However, these variable phase spaces are highly multi-dimensional. Ensemble members with stronger waves also yielded further north systems at time of genesis. As already established, this is likely due to the fact that the strength of the northwestward Beta drift is a function of total circulation (Fiorino and Elsberry 1989). A composite of 120 h forecast 850-500 hPa winds valid at time of genesis from the 5 ensemble members with the strongest wave in 1000x1000 km boxes centered on all genesis times and locations agrees with this result, with a domain-mean meridional wind of $v = +1.78 \text{ ms}^{-1}$, as opposed to $v = +0.78 \text{ ms}^{-1}$ for the 5 weakest members (Figure 5.22). If this meridional wind magnitude remained constant throughout that 5 day period, it would translate to having the 5 strongest members 432 km north of the 5 weakest members, on average. Therefore, the tropical wave will gain more latitude in ensemble members with stronger circulations. Not surprisingly, there is an apparent correlation between latitude and strength of circulation, although a distinct cut-off does not exist (Figure 5.23A). There is also an apparent negative correlation between latitude and environmental RH, particularly if one examines “edge” (blue) values (Figure 5.23B). Therefore, it is most likely that a dry environment is not necessarily favorable to genesis,
but instead that strengthening systems tend to gain latitude and often enter a drier environment as they do so.

Joint distributions can occasionally be more complicated than the preceding cases. One such example is the distribution of core RH versus environmental RH (Figure 5.23C). While the distribution exhibits a clear maximum of counts in the 60-80% environmental RH and 70-90% core RH bins, there are actually two local maxima in the core RH distribution for low environmental RH in the 30-50% range. However, recall that these joint distributions are actually just 2-dimensional representations of a multi-dimensional data set. To explain the odd maxima of high core RH at low environmental RH, all other variables were compared one-at-a-time against the two moisture variables. The clearest signal in any of these 3-dimensional joint distributions arises when circulation is plotted against core and environmental moisture (Figure 5.23D). This result indicates that dry environment / dry core cases are associated with weak circulations, while dry environment / moist core cases are associated with strong circulations due to the northward shift. Further examination suggests that most if not all of the dry environment / moist core cases can be attributed to waves such as Fiona and Earl in which a localized region of higher RH becomes detached from a moist environment to the south and/or east but remains co-located with the circulation maximum (Figure 5.2A,B). In the absence of hostile shear or significant dry air penetrating the core, the system remains unaffected by the dry environment and undergoes genesis.

As expected, stronger upper-level divergence and low-level convergence both tend to be associated with stronger circulations, on average, although the relationship is not nearly as clear for low-level convergence (Figure 5.24A,B). Somewhat unexpectedly, a
non-negligible number of cases still exist with net upper-level convergence and/or low-level divergence. However, this result is not inconsistent with the results of the dropwindsonde study: Figure 3.4C indicated that developing tropical cyclones are often associated with net low-level divergence > 48 h prior to genesis. Perhaps these cases in the ECMWF ensemble are examples of forecasts in which the dynamics are properly represented, but genesis is simply too slow to occur. Divergence at 200 hPa also exhibits a clear positive correlation with 700 hPa RH (Figure 5.24C). The more moist the core of the system, the stronger the upper-level divergence can potentially be. Errors in RH and divergence depict a similar relationship (Figure 5.24D); a general dry bias in the forecast is associated with a weak bias in the area-averaged divergence. This relationship is likely tied to the strength of the convection and strength of the secondary circulation, either of which are weakened by downdrafts caused by evaporation triggered by dry air. However, it is unclear if the presence of dry air necessarily leads to weaker upper-level divergence, or if this distribution also includes cases of weak upper-level divergence resulting in weaker updrafts and less convection, which appears as less moisture at 700 hPa.

The normalized joint distribution for circulation versus thickness anomaly depicts the expected result that stronger waves are associated with both stronger circulations and stronger warm cores (Figure 5.25A). The correlation is also quite linear. Somewhat unexpectedly, while all cases in the data set are associated with positive values for circulation, there are a few cases of weak circulations with near-zero or even negative thickness anomalies⁴. This demonstrates that some pre-genesis tropical waves in the

⁴ While the search algorithm requires that all thickness anomalies be local maxima, there is no requirement that the maxima be positive in values.
ECMWF ensemble are associated with a clear cyclonic circulation but do not yet have a warm core, which is consistent with the results of Chapter 3. The joint distribution of error for circulation and thickness anomaly reveals a similar positive linear relationship (Figure 5.25B). While the relationships between various environmental parameters and the strength of the warm thickness anomaly were also examined, results were almost entirely redundant with the results for circulation since both are a proxy for the strength of the tropical wave. Therefore, these results will not be shown.

Next, relationships that involve vertical wind shear are examined. While one might expect there to be a strong relationship between latitude and wind shear, as was evident in Figure 4.1, the normalized joint distribution for genesis cases does not indicate an obvious trend (Figure 5.26A). Stronger wind shear most likely destroys the majority of the waves that gain too much latitude. Since only developing cases are included in this data set, the sample is biased towards cases where anomalously low shear exists at higher latitudes in which genesis can actually occur. The normalized distributions for 850-200 hPa and 850-500 hPa wind shear indicate definite trends of stronger circulations for weaker values of shear (Figure 5.26B,C). However, there is no definitive cutoff for shear beyond which a circulation >1.0x10^{-4} s^{-1} is virtually impossible, as there was for moisture. Also note that 0-2.5 ms^{-1} of 850-200 hPa shear is virtually identical to 7.5-10 ms^{-1} of shear in terms of favorability for genesis. For comparison, Nolan and McGauley (2012) find that 2.5-3.75 ms^{-1} shear is this most favorable condition, while 0 ms^{-1} shear about as favorable as 7.5 ms^{-1} shear. The joint distribution is not shown for 850-500 hPa shear above 10 ms^{-1} because there is an insignificant number of cases above that range.
Lastly, the effects of the zonal component of wind shear are examined. First, it is worth noting that there are more genesis forecasts in the sample that occur with easterly shear than westerly shear. However, while Nolan and McGauley (2012) found westerly shear to be more favorable for genesis, neither easterly nor westerly shear appear to be quantifiably more favorable for genesis than the other in this data set (Figure 5.26D). It should be noted that westerly shear tends to be associated with a drier environment than easterly shear (Figure 5.26E), as well as stronger total shear (Figure 5.26F), which implies a greater meridional component to shear for westerly shear cases. Therefore, if all cases were in otherwise equal environments, genesis events in westerly shear may have perhaps been stronger on average. Alternatively, westerly shear cases also tend to be further north on average than the easterly shear cases (not shown). This implies that at least some of the sample of stronger cases associated with westerly shear were associated with ensemble members that strengthened the wave more quickly, bringing it northward (again, due to increased Beta drift) into the westerly shear as an already stronger system.

Finally, normalized lagged distributions are examined in order to help to assess causality, as well as time-varying relationships. For example, the strength of the vortex at time of genesis is strongly correlated with the strength of the vortex 48 h prior to genesis (Figure 5.27A). In general, the vortex also strengthens somewhat during this time period, such that the greatest frequency of circulation values of $1.00-1.25 \times 10^{-4} \text{ s}^{-1}$ at 48 h prior to genesis fall in the $1.25-1.50 \times 10^{-4} \text{ s}^{-1}$ bin at time of genesis. There are also several outlier cases where the circulation is actually stronger 48 h prior to verifying time of genesis. However, these are predominantly long-range forecasts in which the timing of genesis is too early, and the vortex begins to weaken as it enters a less favorable environment.
Similarly, there is also a strong positive relationship between latitude of the initial vortex 48 h prior to genesis and latitude of the vortex at time of genesis (Figure 5.27B). There are a few cases in which a wave rapidly gains (e.g. Otto, Richard) or loses (e.g. TD5) latitude during the final 48 h prior to genesis, but the vast majority of cases are primarily propagating to the west-northwest and therefore fall close to a (slightly tilted) positive 1-to-1 slope.

Normalized lagged distributions for circulation and core moisture depict a coherent signal for causality. It appears that strength of circulation at time of genesis is fairly-well correlated with magnitude of the 700 hPa core RH 48 h prior to genesis (Figure 5.27C). Conversely, the relationship between core RH at time of genesis and strength of circulation 48 h prior to genesis is not as evident (Figure 5.27D). Therefore, it appears that the presence of a moist core contributes to a stronger genesis signal more so than a strong initial vortex results in a moist core at time of genesis. Similar methodology applied to core RH and core upper-level divergence does not produce as clear a result. It appears that the degree to which a moist core 48 h prior to genesis contributes to stronger upper-level divergence at time of genesis (Figure 5.27E) is roughly equal to the degree to which stronger upper-level divergence 48 h prior to genesis contributes to a moist core at time of genesis (Figure 5.27F).

5.4 Wave-relative variance prediction

Next, variance prediction is examined in a similar fashion to section 4.1.3, except that only forecast errors and variances within 0-300 km and 300-1000 km disks centered
on tropical waves are included here. One goal of this section is to determine whether or not the ensemble has greater difficulty conveying uncertainty via ensemble variance in the core of the tropical wave than it does in the nearby environment, perhaps due to unresolved convection or other poorly-represented processes occurring within the wave. We also seek to investigate the change (deterioration) in variance prediction with increasing lead-times.

First, wave-relative variance prediction for circulation is examined. In general, forecasts with shorter lead times are associated with lower ensemble standard deviation and verify with lower absolute error of the ensemble mean (Figure 5.28A). Note that, while a least-squares linear best fit does not yield statistically-significant results, there is nonetheless a positive relationship between the two. There are also a relatively low number of cases that fall within the category of poor ensemble performance associated with low standard deviation and high error, and the cases that do are all with 6 or more days lead time. Further examination reveals that virtually all of these cases are associated with missed genesis forecasts, in which there is very good agreement within the ensemble that genesis will not occur, but it does. Note that this region of the diagram could also correspond to cases where there is very good agreement that genesis will occur, but it fails to do so, if false alarms were included in the sample.

When comparing ensemble-mean error standard deviation with error of the ensemble mean, there is an overall increasing relationship for all lead times (Figure 5.28B-D). However, the slope tends to be less than the desired 1-to-1 relationship (dashed line), which becomes increasingly obvious at longer (168-240 h) lead times. In other words, the

5 Recall that this is one variable in which an environmental counterpart is not calculated, as values are at least 2 orders of magnitude smaller than core circulation.
ensemble spread is too low for high-error circulation forecasts and is actually slightly too high for low-error circulation forecasts. This “flattening” of the forecast error standard deviation with time, or a transition from a slope of 1 to a slope of 0, is indicative of a model bias, likely the result of a weak bias at longer lead times for circulation. If the ensemble is well constructed and unbiased, 68.27% or 1 confidence interval of the sample (magenta dots) should fall within the “ideal” confidence interval (whitish-magenta cone). Many of the magenta dots fall below the “ideal” distribution at later lead times and with incorrect variance (too much variance for low-error cases, too little variance for high-error cases), indicating that the ensemble forecast bias has become large at 7+ days lead time and that the system is no longer predictable.

The relationship between ensemble standard deviation and absolute error of the ensemble mean is positive for environmental RH at 700 hPa (Figure 5.29A). Unlike for circulation, all environmental RH forecasts associated with high ensemble mean error are associated with high standard deviation at the time the forecast was issued, meaning that the ensemble is increasing spread in less certain (and thereby occasionally high error) scenarios. A comparison between ensemble standard deviation and ensemble-mean error standard deviation also reveals roughly a 1-to-1 increasing relationship at short, medium and longer lead times (Figure 5.29B-D). This is in agreement with the result for environmental RH from the basin-wide part of the study.

Ensemble standard deviation and absolute error for core RH is overall quite similar to that of environmental RH, with an obvious positive relationship (not shown). The relationship between ensemble mean standard deviation and ensemble-mean error standard deviation is also increasing for core RH for both 0-72 h and 84-156 h lead times. However,
the relationship begins to fail at 168-240 h, in which the slope is much more shallow than 1-to-1, implying that forecasts with higher forecast variance are only slightly more likely to verify with higher mean error than forecasts with low variance. This results is at odds with the results for the environmental rings, as well as the results from Chapter 4 for the 10x10° basin-wide boxes for moisture, for both of which the relationship was still monotonic increasing at 168-240 h. This demonstrates that the ensemble has greater difficulty predicting environmental moisture variance for tropical waves than the surrounding environment at longer lead times. As was the case for circulation, the forecast error standard deviation points approaching a slope of zero at longer lead times (as well as a large number of samples well below the 1st confidence interval) is indicative of a bias that becomes worse with greater lead times. For wave core moisture, this is a dry bias.

Both environmental (not shown) and core divergence (Figure 5.30A) do not exhibit any obvious positive relationship between ensemble standard deviation and absolute error of the ensemble mean. Results are overall decent for the 10 equal-sized ensemble-mean error standard deviation bins (Figure 5.30B-D). The fact that forecast errors and variance are roughly equal at 168-240 h to their values at 0-72 h for both core and environmental divergence is discouraging, implying that there is roughly equal uncertainty in the forecast divergence at short lead times as there is at longer lead times. However, higher-variance divergence forecasts tend to be associated, on average, with higher error forecasts, implying that the ensemble is adequately conveying the uncertainty in the forecast. The somewhat surprising fact that there is some predictability in these divergence fields at 168-240 h is likely due to the fact that area-averaging greatly extends predictability. That said,
the fact that results are just as good for core divergence as they are for environmental divergence is somewhat unexpected.

There is overall very little difference between the variance prediction results for shear averaged over 300 km radius and shear over 300-1000 km radius. For this reason, only the results over the larger ring will be discussed. Results for the ensemble standard deviation versus absolute error of the ensemble mean are encouraging, with very few cases of low variance / higher error forecasts and a net positive increasing relationship (Figure 5.31A). The relationship between ensemble standard deviation and ensemble mean error standard deviation also suggests high predictability for 0-72 h and 84-156 h forecasts (Figure 5.31B-C). However, the relationship has failed by 168-240 h, suggesting that the limit of predictability for shear has been reached (Figure 5.31D). However, this result is at odds with the shear results for the 10x10° bins in the basin-wide study, which included a much larger sample size, so perhaps this is simply a sampling issue. Nevertheless, the results suggest a moderate to high degree of predictability for area-averaged shear, with no appreciable differences between shear calculated over the wave core versus shear calculated over the environment.

5.5 Wave-relative Predictive Power

Motivated by Chapter 4 but with the goal of ascertaining results more directly associated with genesis, Predictive Power was calculated in a wave-relative framework. Rather than compute PP over the entire tropical Atlantic genesis domain, PP is calculated within 1000x1000 km boxes centered on the positional-averaged maximum in circulation
and thickness anomaly, for each ensemble member forecast from 240-0 h lead time for $C_v$ and with respect to each genesis event in ERA-Interim from 1979-2011 for $\Sigma$. Genesis events are defined as the first instance of tropical storm or tropical depression designation that falls on either 00Z or 12Z (to coincide with ERA-Interim analyses) within our geographic domain for each system included in the HURDAT database.

Results indicate enormous case-to-case variability for all variables, including circulation, core 700 hPa RH, 850-200 hPa shear, and 200 hPa divergence (Figure 5.32A-D), likely due in part to the fact that PP was formulated for use on much larger geographic scales and for slower-varying systems. The overall variability is especially pronounced for circulation, as changes in wave size and strength likely contribute significantly to error variance in $C_v$. Another apparent problem is the fact that the forecast error distribution and the climatological error distribution are from such different samples. The forecast distribution is taken from ensemble output verifying at the time of genesis, which, for longer lead times, can include anything from a mature TC to a wave that has already nearly dissipated. The climatological distribution, on the other hand, only includes genesis events at the time that genesis occurs, meaning the variance for circulation and thickness anomaly will almost always be lower than the forecast distribution. There will also inevitably be lower variance in the environment surrounding the wave in the forecast distribution than in the climatological distribution, because hostile environments that preclude genesis are inherently excluded from the climatological sample (since only cases where genesis occurred are included), while the forecast sample includes cases where genesis occurred in reality but may have been too hostile for genesis in the model. This problem is especially pronounced for 168-240 h forecasts where the forecast may deviate from reality.
Figure 5.1: Ensemble forecast distributions as a function of lead time (h, x-axis) valid at time of genesis (1200 UTC 25 Aug 2010) for Hurricane Earl, with (A) circulation (s⁻¹), (B) core RH (%) at 700 hPa, (C) environmental 850-200 hPa wind shear (ms⁻¹), and (D) environmental RH (%) at 700 hPa. Each circle represents a different ensemble member with forecast circulation values of <3x10⁻⁵ s⁻¹ (black), ≥3x10⁻⁵ s⁻¹ (blue), ≥6x10⁻⁵ s⁻¹ (green), and ≥9x10⁻⁵ s⁻¹ (magenta). The ensemble mean forecast is also shown (black line).
Figure 5.2: Ensemble mean RH at 700 hPa (contoured), forecast variance for RH (shaded), and center of the tropical wave in each ensemble member (red stars). Included are (A) a 24 h forecast valid at the time of genesis of Earl from 1200 UTC 24 Aug 2010, (B) a 24 h forecast valid at the time of genesis of Fiona from 1200 UTC 29 Aug 2010, (C) a 168 h forecast for Earl from 1200 UTC 18 Aug 2010, and (D) a 108 h forecast for Earl from 0000 UTC 21 Aug 2010.
Figure 5.3: As in Figure 5.1, but for the genesis of Tropical Storm Hermine. Forecasts valid 0000 UTC 06 Sep 2010.
Figure 5.4: Ensemble forecast distributions as a function of lead time (h, x-axis) valid at time of genesis (1200 UTC 30 Aug 2010) for Tropical Storm Fiona, with (A) circulation ($s^{-1}$), (B) core RH (%) at 700 hPa, (C) environmental 850-200 hPa wind shear (ms$^{-1}$), and (D) GOES-12 and EUMETSAT-8 composite infrared satellite image of the easterly wave that spawns Fiona exiting the west coast of Africa 108 h prior to genesis (0000 UTC 26 Aug 2010; http://catalog.eol.ucar.edu/predict/). In (A-C), each circle represents a different ensemble member with forecast circulation values of $<3\times10^{-5}$ s$^{-1}$ (black), $\geq3\times10^{-5}$ s$^{-1}$ (blue), $\geq6\times10^{-5}$ s$^{-1}$ (green), and $\geq9\times10^{-5}$ s$^{-1}$ (magenta). The ensemble mean forecast is also shown (black line).
Figure 5.5: As in Figure 5.1, but for the genesis of Hurricane Igor. Forecasts valid 1200 UTC 08 Sep 2010.
Figure 5.6: Composite circulation forecasts for all ensemble members valid 48 h prior to the genesis of Igor (A, C) and at the time of genesis of Igor (B, D), contoured in $3 \times 10^{-5}$ s$^{-1}$ (blue), $6 \times 10^{-5}$ s$^{-1}$ (green), and $9 \times 10^{-5}$ s$^{-1}$ (magenta) increments. Included are (A) a 120 h forecast and (B) a 168 h forecast initialized 1200 UTC 01 Sep 2010, and (C) a 0 h forecast and (D) a 48 h forecast initialized 1200 UTC 06 Sep 2010.
Figure 5.7: The predicted location of Igor at time of genesis relative to gradients of moisture (A) and gradients of shear (B) in 36 h forecasts initialized 0000 UTC 07 Sep 2010. Included are (A) ensemble mean RH at 700 hPa (contoured), forecast variance for RH (shaded), and (B) ensemble mean 850-200 hPa shear (black contours), ensemble mean streamlines of 850-200 hPa shear (grey streamlines), and variance of shear (shaded). The center of the tropical wave in each ensemble member is indicated (red stars).
Figure 5.8: Ensemble forecast distributions as a function of lead time (h, x-axis) valid at time of genesis (1200 UTC 22 Jul 2010) for Tropical Storm Bonnie, with (A) circulation (s\(^{-1}\)), (B) core RH (%) at 700 hPa, (C) environmental 850-200 hPa wind shear (ms\(^{-1}\)), and (D) core 200 hPa divergence (s\(^{-1}\)). Each circle represents a different ensemble member with forecast circulation values of \(<3\times10^{-5} \text{ s}^{-1}\) (black), \(\geq3\times10^{-5} \text{ s}^{-1}\) (blue), \(\geq6\times10^{-5} \text{ s}^{-1}\) (green), and \(\geq9\times10^{-5} \text{ s}^{-1}\) (magenta). The ensemble mean forecast is also shown (black line).
Figure 5.9: Composite circulation forecasts for all ensemble members valid 48 h prior to the genesis of Bonnie (A, C) and at the time of genesis of Bonnie (B, D), contoured in $3 \times 10^{-5}$ s$^{-1}$ (blue), $6 \times 10^{-5}$ s$^{-1}$ (green), and $9 \times 10^{-5}$ s$^{-1}$ (magenta) increments. Included are (A) a 36 h forecast and (B) an 84 h forecast initialized 0000 UTC 19 Jul 2010, and (C) a 24 h forecast and (D) a 72 h forecast initialized 1200 UTC 19 Jul 2010.
Figure 5.10: As in Figure 5.1, but for the genesis of Tropical Storm Gaston. Forecasts valid 1200 UTC 01 Sep 2010.
Figure 5.11: Composite circulation forecasts for all ensemble members valid 48 h prior to the genesis of Gaston (A, B, C) and at the time of genesis of Gaston (D), contoured in $3 \times 10^{-5}$ s$^{-1}$ (blue), $6 \times 10^{-5}$ s$^{-1}$ (green), and $9 \times 10^{-5}$ s$^{-1}$ (magenta) increments. Included are (A) a 24 h forecast from 1200 UTC 29 Aug 2010, (B) a 12 h forecast from 0000 UTC 30 Aug 2010, (C) a 0 h forecast from 1200 UTC 30 Aug 2010, and (D) a 60 h forecast from 0000 UTC 30 Aug 2010.
Figure 5.12: As in Figure 5.1, but for the genesis of Hurricane Lisa. Forecasts valid 0000 UTC 21 Sep 2010.
Figure 5.13: Ensemble forecast distributions as a function of lead time (h, x-axis) valid at time of genesis (1200 UTC 28 Sep 2010) for Tropical Storm Nicole, with (A) circulation ($s^{-1}$), (B) core RH (%) at 700 hPa, and (C) environmental 850-200 hPa wind shear (ms$^{-1}$), where each circle represents a different ensemble member with forecast circulation values of $<3\times10^{-5}$ s$^{-1}$ (black), $\geq 3\times10^{-5}$ s$^{-1}$ (blue), $\geq 6\times10^{-5}$ s$^{-1}$ (green), and $\geq 9\times10^{-5}$ s$^{-1}$ (magenta) and the ensemble mean forecast is indicated (black line). Also included is (D) the ensemble mean 850-200 hPa shear (black contours), ensemble mean streamlines of 850-200 hPa shear (grey streamlines), variance of shear (shaded), and the center of the tropical wave in each ensemble member is indicated (red stars) in an 84 h forecast from 0000 UTC 25 Sep 2010.
Figure 5.14: Shaded 228 h ensemble forecast probabilities valid 1200 UTC 28 Sep 2010 of (A) 700 hPa RH > 70%, with 700 hPa RH contours from control forecast, and (B) probability of 850-200 hPa wind shear > 10 ms$^{-1}$ with 850-200 hPa mean streamlines.
Figure 5.15: (A,B) 48 h ensemble-mean forecasts valid 1200 UTC 28 Sep 2010 and (C,D) 84 h ensemble mean forecasts valid 1200 UTC 30 Sep 2010 of shaded 850 hPa relative vorticity ($10^{-4}$ s$^{-1}$), contours of 850 hPa heights, and 850 hPa wind vectors for the 10 members with the strongest (A,C) and weakest (B,D) circulations at time of genesis of Tropical Storm Nicole.
Figure 5.16: Ensemble-mean (A) circulation ($s^{-1}$), (B) circulation errors ($s^{-1}$), and (C) forecast variance for circulation ($s^{-1}$) as a function of lead time (x-axis) for each of 21 genesis events from 2010 (dotted lines), as well as the mean for all events (solid line).
Figure 5.17: As in Figure 5.16, but for 200 hPa core divergence (s\(^{-1}\)).
Figure 5.18: Ensemble-mean (A) environmental 700 hPa RH (%), (B) core 700 hPa RH (%), (C) environmental 700 hPa RH error (%), and (D) core 700 hPa RH error (%) as a function of lead time (x-axis) for each of 21 genesis events from 2010 (dotted lines), as well as the mean for all events (solid line).
Figure 5.19: Ensemble-mean (A) 850-200 hPa shear (ms$^{-1}$), (B) 850-500 hPa shear (ms$^{-1}$), (C) 850-200 hPa shear error (ms$^{-1}$), and (D) forecast variance for 850-200 hPa shear (ms$^{-1}$) as a function of lead time (x-axis) for each of 21 genesis events from 2010 (dotted lines), as well as the mean for all events (solid line). All quantities are environmental (300 $\leq$ r $\leq$ 1000 km) averages about the center of circulation in each ensemble member.
Figure 5.20: (A) Joint distribution, (B) normalized joint distribution, and (C) error distribution of circulation versus 700 hPa core RH; (D) normalized joint distribution of circulation versus 700 hPa environmental RH. In (B) and (D), the total number of elements in each column are labeled (magenta).
Figure 5.21: Error joint distributions of circulation versus 700 hPa core RH for (A) 0-72 h, (B) 84-156, and (C) 168-240 h forecast lead times.
Figure 5.22: Composite 850-200 hPa environmental steering flow (black) and the mean vector wind (red) for all 120 h forecasts valid at time of genesis from 2010, within 1000x1000 km boxes, centered on (A) the 5 ensemble members with the strongest predicted circulation, and (B) the 5 ensemble members with the weakest predicted circulation.
Figure 5.23: (A) Normalized joint distribution of circulation versus latitude, (B) normalized joint distribution of 700 hPa environmental RH versus latitude, (C) joint distribution of 700 hPa core RH versus 700 hPa environmental RH, and (D) 3-dimensional joint distribution of 700 hPa core RH, 700 hPa environmental RH, and circulation. In (A) and (B), the total number of elements in each column are labeled (magenta).
Figure 5.24: Normalized joint distributions of circulation versus (A) 200 hPa core divergence and (B) 850 hPa core convergence; normalized joint distribution of 200 hPa core divergence and 700 hPa core RH; (D) error distribution of 200 hPa core divergence versus 700 hPa error. In (A-C), the total number of elements in each column are labeled (magenta).
Figure 5.25: (A) Normalized joint distribution and (B) error distribution of thickness anomaly versus circulation.
Figure 5.26: (A) Normalized joint distributions of: (A) 850-200 hPa environmental wind shear versus latitude; (B) circulation versus 850-200 hPa shear; (C) circulation versus 850-500 hPa shear; (D) circulation versus 850-200 hPa zonal shear; (E) 700 hPa environmental
RH versus 850-200 hPa zonal wind shear; (F) 850-200 hPa shear versus 850-200 hPa zonal shear. The total number of elements in each column are labeled (magenta).
Figure 5.27: Normalized lagged distributions: (A) circulation at time of genesis vs circulation 48 h prior to genesis; (B) latitude at time of genesis vs latitude 48 h prior to genesis; (C) circulation at time of genesis vs 700 hPa core RH 48 h prior to genesis; (D) circulation 48 h prior to genesis vs 700 hPa core RH at time of genesis; (E) 200 hPa divergence at time of genesis vs 700 hPa core RH 48 h prior to genesis; (F) 200 hPa
divergence 48 h prior to genesis vs 700 hPa core RH at time of genesis. The total number of elements in each column are labeled (magenta).
Figure 5.28: Evaluation of ensemble variance prediction for wave-relative circulation forecasts. (A) Standard deviation of ensemble forecasts versus the absolute error of the ensemble mean forecast as a function of forecast lead-time, in days (colored); Ensemble standard deviation versus error of the ensemble mean forecast (magenta dots), 1st confidence interval of the ideal distribution of the sample (whiteish-magenta cone). ensemble-mean forecast error standard deviations in 10 equal-sized bins (black circles), and the 1-to-1 line (dashed) for (B) 0-72 h, (C) 84-156 h and (D) 168-240 h.
Figure 5.29: As in Figure 5.28, but for 700 hPa environmental RH.
Figure 5.30: As in Figure 5.28, but for 200 hPa core divergence.
Figure 5.31: As in Figure 5.28, but for 850-200 hPa environmental wind shear.
Figure 5.32: Ensemble-mean wave-relative Predictive Power calculation for 8 genesis events from September 2010 within a 1000x1000 km box centered on the tropical wave for (A) circulation, (B) 700 hPa RH, (C) 850-200 hPa wind shear, and (D) 200 hPa divergence.
Chapter 6

Conclusions

In this study, we made an attempt to further our understanding of tropical cyclogenesis and the predictability of tropical cyclogenesis via examination of in-situ data in Chapter 3 and ensemble forecasts in Chapters 4 and 5. As tropical cyclogenesis is inherently a multiscale problem that depends upon both a favorable large scale environment as well as local and mesoscale organization and dynamics, both basin-wide scales (Chapter 4) as well as wave-local scales (Chapters 3 and 5) were explored. Having both observational and modeling sections in this research has made it possible to first observe physical processes associated with genesis and several environments in which genesis has occurred, and then apply this knowledge to (i) identify these environments, and (ii) evaluate whether or not these physical processes are adequately represented in ensemble forecasts. Once it was demonstrated that these favorable environments, as well as processes (e.g. development of a warm core, strengthening of circulation, and moistening of core) are identifiable in the ensemble, it was then possible to quantify their predictability. Conclusions specific to each chapter individually appear below. Finally, some over-arching conclusions and synthesized results appear at the end of the conclusions.
6.1 Analysis of PREDICT dropwindsonde data

Observations from the 2010 PREDICT field campaign, when analyzed from a composite mean framework, offer discernible differences between developing and non-developing tropical waves that may be advantageous to the understanding and prediction of tropical cyclogenesis. Temperature, mixing ratio, relative humidity, radial and tangential components of wind, relative vorticity and CAPE are examined.

Temperature profiles reveal a progressive building of warm anomalies from 500-200 hPa, relative to the PREDICT mean, of +0.5 to +1.0°C at 24-48 h pre-genesis, increasing to +1.0 to +2.0°C 0-24 h pre-genesis within 200 km of the center of circulation. While the existence of a warm core in mature TCs has been well-established in previous literature, the magnitude and timing of the warm core development with respect to time of genesis has not. The observation of maximum warm anomalies just below tropopause level pre-genesis suggests that warm core development occurs at the same altitude as observed in mature TCs by La Seur and Hawkins (1963) and Hawkins and Rubsam (1968). A local maximum in warm anomalies below 500 hPa also suggests that formation of a secondary warm core is possible pre-genesis, at a similar altitude as the Hawkins and Imbembo (1976) secondary warm core. This is also consistent with the level of the Stern and Nolan (2012) warm core, although it is not the primary warm core as they suggest. It should be noted that any presence of a secondary warm core is much weaker than the primary warm core found at higher altitude. In contrast with the genesis cases, negative $T$ anomalies of -0.5 to -1.0°C exist from 500-200 hPa for non-developing systems.
In terms of moisture, positive $q$ anomalies of +0.1 to +0.5 g kg$^{-1}$ from 800-300 hPa are observed in developing systems, even 72 or more hours pre-genesis. Moisture does not increase significantly with time on the spatial scale of the entire tropical wave. Meanwhile, non-developing systems are associated with significant dry anomalies from 800-300 hPa. When only examining dropwindsondes located within 150 km from the center of circulation, moist convective processes appear to increase moisture as the tropical wave approaches genesis, as suggested by Bister and Emanuel (1997), Nolan (2007), and others. The maximum increase in moisture of 1 g kg$^{-1}$ from 800-600 hPa occurs 24-48 h pre-genesis. This trend is likely washed-out when all dropwindsondes are included due to the large spatial area of averaging in the full composite, possibly coupled with some large-scale entrainment of dry air into the wave circulation. Nonetheless, the full $q$ composite still demonstrates that time-evolving genesis profiles are all significantly more moist than non-developing systems, even more than 72 h prior to genesis. Non-genesis RH profiles are on the order of 10-20% drier than the PREDICT mean from 700-500 hPa, suggesting a greater potential for dry air entrainment into convective towers. Conversely, the non-genesis mean is actually more moist than the genesis mean from the surface through 850 hPa, possibly suggesting that dry air at the mid-levels is more detrimental to genesis than dry air at the low levels.

Examination of the wind field reveals a progressive strengthening of the vortex above 600 hPa, with an initial delay in intensification from 850-700 hPa. Tangential wind at these levels fluctuates between 3-5 m s$^{-1}$ from 72 through 24 hours pre-genesis, before jumping suddenly to 6-7 m s$^{-1}$ less than 24 hours pre-genesis. This sudden intensification of the vortex appears to lag the greatest increase in moisture by 24 hours. Radial wind
profiles suggest that many cases of genesis may have been delayed by low-level outflow. Alternatively, an initial stage of low-level outflow may instead be a consequence of the process of moistening the core by convection, induced by cool sinking air in the stratiform precipitation region as in Bister and Emanuel (1997). During the final 48 hours before genesis, low-level inflow of 1-2 m s$^{-1}$ develops and strengthens with time.

Vorticity fields reveal a broad region of maximum $\zeta$ from 900-600 hPa at 48-72 h and 24-48 h pre-genesis. This feature appears simultaneously with low-level cold anomalies 24-72 h pre-genesis, as well as divergence 48-72 h pre-genesis. Thereafter, a distinct $\zeta$ maximum develops near 800 hPa 0-24 h pre-genesis. This evolution of the vortex is consistent with a process described by Bister and Emanuel (1997), in which the level of maximum PV production descends as the level of peak cooling descends in the stratiform rain region. Ritchie and Holland (1997) and Simpson et al. (1997) propose a similar mechanism by which a mid-level vortex induces a surface circulation through vertical penetration and vortex stretching. While vertically-descending vorticity was not identified, the development of a robust mid-level vortex prior to the intensification of a surface vortex was more consistent with these studies, along with Nolan (2007) and Raymond et al. (2011), than with studies that suggest a bottom-up genesis mechanism. Differences between genesis and non-genesis $V_{tan}$ and $\zeta$ also reveal that developing waves are, on average, associated with a slightly stronger circulation than non-developing waves.

Genesis cases are associated with slightly greater 850-500 hPa mid-level wind shear than non-genesis cases: 2.70 m s$^{-1}$ versus 2.22 m s$^{-1}$. On the other hand, genesis cases are associated with slightly lower 850-200 hPa deep layer wind shear: 6.97 m s$^{-1}$ compared to 7.39 m s$^{-1}$. Mid-level wind shear also gradually decreases with time from 2.82 m s$^{-1}$ to
2.45 m s\(^{-1}\) during the time progression towards genesis. However, in both cases, differences in wind shear are statistically indistinguishable.

Lastly, calculation of virtual CAPE indicates significantly greater instability associated with non-genesis profiles than with either pre-genesis or TC-stage profiles. Results suggest that 2000 J kg\(^{-1}\) of CAPE may be sufficient for tropical cyclogenesis, and additional instability does not aid in the genesis process.

While other recent studies have examined and compared individual cases sampled during PREDICT, we have presented an alternative perspective in comparing genesis to non-genesis cases via creating composite vertical profiles for all sampled tropical waves, as well as examining the day-to-day evolution of multi-case pre-genesis composites. Further in-depth investigation of tropical waves with new aircraft data and corroboration with model analyses could potentially increase the robustness of these results.

These observations help to narrow the focus of the predictability study, emphasizing the need to predict the development of the warm core and intensification of the pre-genesis vortex circulation at least 24-48 h prior to genesis. It is also important to note that observations are not just necessary to verify or refute model results, they are also critical in real-time in order for the model to properly initialize the tropical wave.

### 6.2 Basin-wide predictability

Several metrics to evaluate predictive skill and attempt to quantify predictability have been explored using the ECMWF Ensemble Prediction System during the 2010 Atlantic hurricane season, in the context of large-scale variables in the tropical Atlantic
basin that are relevant to tropical cyclogenesis. These metrics include: (1) Growth and saturation of RMS error; (2) RMS errors versus climatology; (3) Predicted forecast error standard deviation (or variance) and (4) Predictive Power.

When error growth was computed in the same vein as the classical study of Lorenz (1982), basin-wide errors were found to grow linearly beyond Day 2, and began to saturate slowly after Day 8. On average, the errors in the ensemble mean forecast were smaller than those from the corresponding control forecast, thereby extending the range of predictability. For variables including 850-700 hPa circulation, 200 hPa velocity potential, 700 hPa relative humidity and 850-200 hPa vertical wind shear, the ensemble mean still exhibited skill (compared with climatology) beyond 9 days. In contrast, the 200 hPa divergence, 850 hPa convergence and 850 hPa relative vorticity were virtually indistinguishable from climatology beyond 2 days. An evaluation of the ability of the ensemble to predict the standard deviation of the forecast error in 10° x 10° boxes across the domain revealed a mostly linear increasing relationship between the predicted and actual standard deviations for all investigated variables, for forecast times up to and beyond a week.

Using Predictive Power, the 850-200 hPa thickness anomaly was found to possess the highest predictability, extending beyond 10 days. This is likely due to the fact that changes in thickness at any given point are often slow to occur in the tropics, and are associated with the movement of large-scale troughs as well as the strength and orientation of the subtropical ridge, since thickness is really just a measure of vertically-integrated temperature. Similarly, the 850-200 and 850-500 hPa vertical wind shear were found to be predictable in the 7-10 day range. These quantities are also often associated with the
variability of large-scale features such as troughs and ridges. Relative humidity at 700 hPa was also found to be quite predictable, out to about a week, but slightly less so than either shear or thickness anomaly. This is likely due to the fact that prediction of moisture involves many scales. The large-scale slowly-varying nature of mid-latitude dry-air intrusions or moist Caribbean monsoon gyres lends greater predictability to moisture. However, small-scale variations in regions of convection lead to rapid error growth at small scales, decreasing the predictability. Consistent with the earlier metrics, the predictability of rapidly varying fields driven by small-scale processes such as 200 hPa divergence was found to be very short, only 12 h with little case-to-case variability using Predictive Power. Although the magnitude and variance of quantities such as divergence and vorticity is normally maximized in regions of active convection and is therefore governed by small-scale processes, physically related variables with larger-scale characteristics such as velocity potential and circulation possess a longer range of predictability (typically several days) and can be considered for use in identifying the potential for tropical cyclogenesis. The high predictability of thickness anomaly is also encouraging for tropical cyclogenesis forecasts, suggesting that warm core development may be predictable out to longer time ranges. However, care must be taken in future work to isolate the predictability associated with tropical cyclogenesis from that of synoptic-scale warm and cold anomalies that appear to dominate the climatological and forecast error variance on larger scales.

There exists a considerable degree of variability from day-to-day and between successive initial conditions. This is the result of the evolving flow, with some flow regimes being more predictable than others as first put forth by Lorenz (1965). The degree of variability also depends on the atmospheric variables under consideration. While the
Predictive Power of metrics such as wind shear exhibited relative low variability, the Predictive Power for circulation was found to decrease considerably in the presence of multiple tropical waves and tropical cyclones.

Although the conclusions may differ due to the metric used and the flow, the predictability is generally dependent on the scales of motion (following Lorenz 1969). Those metrics typically associated with larger spatial scales (and slower temporal scales of variability) were generally found to be more predictable than those associated with smaller spatial scales (and faster temporal scales). For these reasons, properties of the atmospheric flow that are mostly driven by the large scales (such as tropical cyclone track) are more predictable than those quantities that are mostly driven by the small scales (such as tropical cyclone intensity). Tropical cyclogenesis is an inherently complicated problem due to the numerous scale interactions involved. However, this investigation of predictability at large scales establishes the foundation for understanding what limits predictability in the wave core and local environment. Motivated by these issues, the predictability of tropical cyclogenesis was subsequently explored from a wave-relative framework.

6.3 Wave-relative predictability

Easterly waves during the 2010 season were spaced with sufficient latitudinal separation with low enough positional forecast variance in the ensemble as to make it possible to separate forecasts associated with individual waves. An examination of the wave-relative nature of the predictability of tropical cyclogenesis has yielded results that are overall consistent with the basin-wide study, as well as several new insights. One result
that is immediately apparent is that some genesis events are more predictable than others, and the factors that most limit the predictability for some cases are not the same as for others. In some cases, there is a clear and consistent relationship between the ensemble distribution of circulation at time of genesis and the overall favorability of the predicted moisture and shear at time of genesis. Classic examples included Earl and Hermine. In these cases, the predictability of genesis is directly related to the ability of the ensemble to correctly depict the environment at time of genesis, and the greater the forecast variance in the environment, the greater the forecast variance for circulation at time of genesis. The failure of the ensemble to produce a genesis signal for Gaston more than 36 h prior to genesis appears to have been strongly related to a severe dry bias within the core of the wave. While Gaston was a particularly obvious example, the ensemble exhibited a dry bias to varying degree prior to genesis for all cases in 2010, even at shorter lead times. In general, the strength of the predicted circulation at time of genesis exhibited no significant relationship with the strength of the predicted core divergence. Two exceptions to this rule were Bonnie and Shary (although Shary was not discussed in this dissertation).

In contrast to the more classic examples, in approximately half of the cases, or 11 out of 21, there is no clear relationship between strength of genesis signal and the favorability of the predicted shear and moisture. These cases, some of which were discussed in this manuscript while others were not, include: Tropical Depression 2, Bonnie, Tropical Depression 5, Fiona, Igor, Julia, Karl, Lisa, Nicole, Paula, and Richard. In some, but not all, of these cases, there appears to be a stronger relationship between the strength of the initial vortex (or the vortex 48 h prior to time of genesis) and the strength of the verifying circulation. Examples include Bonnie, Lisa, and Igor. In other cases, such as
Nicole, the forecast strength of circulation at time of genesis was strongly dependent upon the location, not strength, of the initial vortex 48-84 h prior to genesis. The genesis of Nicole, however, was an example of land interaction with Central America limiting predictability. The final subset of genesis cases included those in which there was a clear relationship between the uncertainty in the genesis signal and the longitudinal location of the easterly wave. In the cases of Fiona and Lisa, there was virtually no genesis signal in forecasts initialized while the easterly waves were still located over Africa. Thereafter, all subsequent forecasts resulted in a strong genesis signal within a day of the wave emerging over water from the West African coast, likely due to improved data coverage and especially data from the Dakar radiosonde location. This case-to-case disparity between sensitivity to the initial vortex versus sensitivity to the environment was also found in Torn and Cook (2012).

Despite the significant case-to-case variability, some general characteristics associated with predictability of genesis were also observed. In general, tropical waves co-located with or traversing sharp gradients in shear or moisture were associated with greater forecast variance and thereby uncertainty for those variables, which ultimately resulted in greater variance in the predicted strength of circulation. Similarly, tropical waves far from any significant gradients tended to be associated with a lower uncertainty environment, which translated to lower variance in the predicted circulation. Another common result was that tropical waves that maintained a moist core while traversing an otherwise hostile / dry environment maintained a consistent genesis signal. In two notable examples, Earl and Fiona, an enclosed region of high moisture detached from a larger moisture source to the south and east while maintaining moisture as it moved northwestward into a hostile
environment. It is likely that genesis would not have occurred had shear been greater in either of these cases.

While the case-to-case variability was made apparent via examination of these waves individually, several overarching conclusions were drawn by compositing wave-relative joint distributions for all genesis events from 2010. As expected, there is a positive correlation between 200 hPa divergence / 850 hPa convergence and strength of circulation. However, there were also a non-negligible number of forecasts associated with net low-level core divergence despite having positive circulation. It is likely that some of these cases are instances where the forecast is exhibiting a slow-bias, and the predicted wave is still at the low-level outflow dominant pre-genesis stage as described in Chapter 3. There is also little to no correlation between environmental moisture and strength of circulation, but a strong positive correlation between core moisture and strength of circulation. This relationship is non-linear, as there is a distinct jump in the strength of the genesis signal associated with core 700 hPa RH > 60%. There is also a sharp distinction between cases with weak and strong UL divergence at 60% RH. Lower moisture values are likely associated with enhanced evaporation, with downdrafts weakening convection and thereby diminishing UL divergence.

Core RH is overall positively correlated with environmental RH. However, a secondary peak given low environmental RH of high core RH was found. Three-dimensional joint distributions incorporating latitude and circulation reveal these cases to be those in which the wave begins to rapidly gain latitude as cyclogenesis commences and the wave strengthens, bringing a stronger vortex associated with a moist core northward into a much drier environment. In many of these cases, the waves appears to be “protected”
from dry environmental air within the recirculating pouch region. However, these none of these cases occurred in particularly sheared environments. It is likely that in a more sheared environment, the critical layer would become tilted with height, allowing more dry air to enter the system and perhaps prevent genesis from occurring. However, the author does not know of any studies that investigate the vertical coherence of the pouch in adverse conditions to cite. This problem will need to be investigated in future studies.

Greater values of 850-200 hPa and 850-500 hPa wind shear are found to be associated with weaker predicted circulation at time of genesis. However, a few ensemble members do develop stronger circulations with up to 20-25 ms$^{-1}$ deep-layer shear, demonstrating that greater shear values may suppress but not totally prevent genesis from occurring. Neither westerly nor easterly shear was found to be much more favorable than the other in this data set.

A few interesting results were also demonstrated through lagged joint distributions and error distributions, including some causality in the relationships. One particularly notable result was that circulation at time of genesis is more strongly correlated with core RH 48 h prior to genesis than circulation 48 h prior to genesis is with core RH at time of genesis. In other words, a dry core is more likely to result in a weaker genesis signal than it is to be the result of a weak vortex. Also, a stronger circulation 48 h prior to genesis was generally associated with a stronger circulation at time of genesis. However, this is not always the case, as some systems develop too quickly in the model and begin to weaken by verifying time of genesis, especially in long-range forecasts where time of genesis may be inaccurate. While not shown, joint distributions of error yield overall consistent results with the aforementioned relationships. Just as a stronger (weaker) predicted circulation
was generally associated with a stronger (weaker) warm core, a strong (weak) bias for circulation was almost always associated with a strong (weak) bias for the predicted warm core.

By averaging forecasts over core and environmental area rings, wave-relative variance prediction yielded generally favorable results. For all variables, including circulation, shear, RH and even divergence, the relationship between ensemble standard deviation and ensemble-mean error standard deviation was approximately monotonic increasing and close to the 1-to-1 line. There were several cases of low ensemble standard deviation and high mean error for circulation associated with underdispersive genesis forecasts, but only at longer lead times (7-10 days). The 1-to-1 relationship is also beginning to fail at 168-240 h for RH and shear, suggesting that forecasts at this lead time are approaching the predictability limit for those variables. The 168-240 h variance prediction for 200 hPa divergence was actually quite good, indicative of the advantages of area-averaging.

Lastly, the overall “flattening” of the forecast error standard deviations with time (i.e. a progression of the black circles from a slope of 1 to a slope of 0) is likely due to the growth of model biases at increasing lead times. In an unbiased ensemble, this relationship should remain 1-to-1 out to arbitrarily-long lead times, regardless of how large errors become. This issue could be investigated in future work by verifying all forecasts against a 240 h forecast from a single ensemble member rather than a verifying model analysis, thereby making a perfect model assumption and eliminating any biases.
6.4 Final remarks

While this investigation explored multiple metrics to quantify predictability for a variety of variables deemed relevant to tropical cyclogenesis, the fundamental question of the limit of predictability for genesis is far from solved. It has been demonstrated that predictability, especially in the context of tropical cyclogenesis, needs to be thought of in a multi-scale way. It has also been shown that there is a dependence of the results for predictability upon the performance of the ensemble: it is difficult to estimate the predictability when the ensemble fails outright, i.e. verification falls outside of the entire forecast distribution. This problem was particularly pronounced in regards to a dry bias for core RH and a weak bias for predicted circulation, contributing to low predictability and poor variance predictions at 168-240 h lead times.

Whether the relationship between the large-scale predictability of variables relevant to tropical cyclogenesis and the sub-seasonal potential predictability may be connected to the Madden-Julian oscillation, other equatorial waves, or large-scale oscillations such as the North Atlantic Oscillation requires investigation. Future studies that estimate large-scale predictability in the tropical Atlantic using additional hurricane seasons and with different ensemble systems are encouraged to determine the robustness of the presented results. It will also be necessary in future work to include non-developing tropical waves in the wave-relative predictability investigation. Lastly, additional in-situ observations of genesis are necessary to further verify the results and hypothesis set forth during PREDICT. Additional observations, especially over the data-sparse regions of the eastern
Atlantic and western Africa, would likely improve the representation of tropical waves in the model and contribute to improved genesis forecasts.
Appendix A

A.1 Computing $C_v$

First, let $\mathbf{x}$ be a state vector of length $K$. Also let $\mathbf{x}_v^i$ be the state vector for the $i$th ensemble member with forecast lead-time $v$. There are $M$ total ensemble members and $v$ is in 12 h increments from 0→$N$. In this study, $M = 51$ members and $N = 240$ h. The mean of all ensemble members at time $v$ can be expressed as $\mathbf{\hat{x}}_v$. The differences between each ensemble member and the ensemble mean $e_v^i = \mathbf{x}_v^i - \mathbf{\hat{x}}_v$ are collectively referred to as the perturbations. Schneider and Griffies (1999) tell us that the error covariance of the ensemble is given by

$$
C_v = \frac{1}{M - 1} \sum_{i=1}^{M} (e_v^i (e_v^i)^T)
$$

Since $e_v^i$ is $[K \times 1]$ and $(e_v^i)^T$ is $[1 \times K]$, $C_v$ is $[K \times K]$. Rather than having to deal with a state vector associated with each ensemble member individually, we can instead combine $M$ state vectors associated with each member in the ensemble into a single matrix $\mathbf{X}_v = (\mathbf{x}_v^1 \quad \mathbf{x}_v^2 \quad \cdots \quad \mathbf{x}_v^M)$. Therefore $\mathbf{E}_v = (\mathbf{x}_v^1 - \mathbf{\hat{x}}_v \quad \mathbf{x}_v^2 - \mathbf{\hat{x}}_v \quad \cdots \quad \mathbf{x}_v^M - \mathbf{\hat{x}}_v) = (e_v^1 \quad e_v^2 \quad \cdots \quad e_v^M)$

To make $C_v$ easier to manipulate, we will show that $\frac{1}{M-1} \sum_{i=1}^{M} e_v^i (e_v^i)^T$ is equal to $\frac{1}{M-1} \mathbf{E}_v (\mathbf{E}_v)^T$.

First expand $\frac{1}{M-1} \sum_{i=1}^{M} e_v^i (e_v^i)^T$
\[
\begin{align*}
\frac{1}{M-1} \sum_{l=1}^{M} \begin{pmatrix}
  e(1)_v^l \\
  e(2)_v^l \\
  \vdots \\
  e(K)_v^l 
\end{pmatrix}
\begin{pmatrix}
  e(1)_v \\
  e(2)_v \\
  \vdots \\
  e(K)_v 
\end{pmatrix} =
\begin{pmatrix}
  e(1)_v^l e(1)_v \\
  e(1)_v^l e(2)_v \\
  \vdots \\
  e(1)_v^l e(K)_v \\
  \end{pmatrix} \\
\end{align*}
\]

Perform the matrix multiplication:
\[
\begin{align*}
\frac{1}{M-1} \sum_{l=1}^{M} \begin{pmatrix}
  e(1)_v^l e(1)_v^l & e(1)_v^l e(2)_v^l & \cdots & e(1)_v^l e(K)_v^l \\
  e(2)_v^l e(1)_v^l & e(2)_v^l e(2)_v^l & \cdots & e(2)_v^l e(K)_v^l \\
  \vdots & \vdots & \ddots & \vdots \\
  e(K)_v^l e(1)_v^l & e(K)_v^l e(2)_v^l & \cdots & e(K)_v^l e(K)_v^l 
\end{pmatrix}
\end{align*}
\]

Sum over \( M \) ensemble members:
\[
\begin{align*}
\frac{1}{M-1} \begin{pmatrix}
  e(1)_v^l e(1)_v^1 + \cdots + e(1)_v^l e(1)_v^M & \cdots & e(1)_v^l e(K)_v^1 + \cdots + e(1)_v^l e(K)_v^M \\
  \vdots & \ddots & \vdots \\
  e(K)_v^l e(1)_v^1 + \cdots + e(K)_v^l e(1)_v^M & \cdots & e(K)_v^l e(K)_v^1 + \cdots + e(K)_v^l e(K)_v^M 
\end{pmatrix}
\end{align*}
\]

We will call this "result 1".

This time, instead compute the inner product of \( \mathbf{E}_v \) and \( (\mathbf{E}_v)^T \):
\[
\begin{align*}
\frac{1}{M-1} \mathbf{E}_v (\mathbf{E}_v)^T
\end{align*}
\]

Expand terms in \( \mathbf{E}_v \):
\[
\begin{align*}
\frac{1}{M-1} \begin{pmatrix}
  \mathbf{e}_v^1 & \mathbf{e}_v^2 & \cdots & \mathbf{e}_v^M
\end{pmatrix}
\begin{pmatrix}
  \mathbf{e}_v^1 & \mathbf{e}_v^2 & \cdots & \mathbf{e}_v^M
\end{pmatrix}^T
\end{align*}
\]

Expand each state vector of the perturbations into \( K \) total components:
\[
\begin{align*}
\frac{1}{M-1} \begin{pmatrix}
  e(1)_v^1 & e(1)_v^2 & \cdots & e(1)_v^M \\
  e(2)_v^1 & e(2)_v^2 & \cdots & e(2)_v^M \\
  \vdots & \vdots & \ddots & \vdots \\
  e(K)_v^1 & e(K)_v^2 & \cdots & e(K)_v^M 
\end{pmatrix}
\begin{pmatrix}
  e(1)_v^1 & e(1)_v^2 & \cdots & e(1)_v^M \end{pmatrix}^T
\end{align*}
\]

Take the transpose of the second matrix:
\[
\begin{align*}
\frac{1}{M-1} \begin{pmatrix}
  e(1)_v^1 & e(1)_v^2 & \cdots & e(1)_v^M \\
  e(2)_v^1 & e(2)_v^2 & \cdots & e(2)_v^M \\
  \vdots & \vdots & \ddots & \vdots \\
  e(K)_v^1 & e(K)_v^2 & \cdots & e(K)_v^M 
\end{pmatrix}
\begin{pmatrix}
  e(1)_v^1 & e(1)_v^2 & \cdots & e(1)_v^M \end{pmatrix}^T
\end{align*}
\]
Perform the matrix multiplication:

\[
\frac{1}{M-1} \begin{pmatrix}
\cdots & \cdots \\
\cdots & \cdots \\
\end{pmatrix}
\cdot
\begin{pmatrix}
e(1)^1_v e(1)^1_v + \cdots + e(1)^M_v e(1)^M_v \\
e(K)^1_v e(1)^1_v + \cdots + e(K)^M_v e(1)^M_v \\
\end{pmatrix}
\]

We will call this "result 2".

Since result 1 = result 2,

\[
\frac{1}{M-1} \sum_{i=1}^M e_i^T (e_i^i)^T = \frac{1}{M-1} E_v (E_v)^T
\]

and therefore

\[
C_v = \frac{1}{M-1} E_v (E_v)^T
\]

A.2 Proof that eigenvalues of 
\( E_v (E_v)^T \) are equal to eigenvalues of 
\( (E_v)^T E_v \)

We seek eigenvalues and eigenvectors of \( E_v (E_v)^T \), where \( E_v \) is a \([K \times M]\) matrix and \( K \gg M \), via:

\[
E_v (E_v)^T = \varepsilon \lambda \varepsilon^T
\]

where \( \lambda \) is a diagonal matrix of eigenvalues \( \geq 0 \) and \( \varepsilon \) is an orthonormal matrix of eigenvectors, i.e. \( \varepsilon^T \varepsilon = I \). There are typically \( K \) eigenvalues associated with a \([K \times K]\) matrix. However, in the special case of \( E_v (E_v)^T \) there are only \( M \) eigenvalues. To show this, we begin with the generalized form of the matrix determinant lemma (Theorem 18.1.1 of Harville 1997), which states that for a \([K \times M]\) matrix \( A \), an \([M \times K]\) matrix \( B \), and any invertible \([K \times K]\) matrix \( X \)

\[
\det(X + AB) = \det(X) \det(I_M + BX^{-1}A)
\]

So if \( X = -\lambda I_K \), \( A = E_v \) and \( B = (E_v)^T \)
\[
\det(-\lambda I_K + E_v(E_v)^T) = \det(-\lambda I_K) \det(I_M + (E_v)^T(-\lambda I_K)^{-1}E_v)
\]

The first term

\[
\det(-\lambda I_K) = \begin{vmatrix} -\lambda & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & -\lambda \end{vmatrix} = (-\lambda)^K
\]

\[
(-\lambda)^K \det(I_M + (E_v)^T(-\lambda I_K)^{-1}E_v)
\]

assuming \(\lambda \neq 0\)

\[
\det(I_M + (E_v)^T(-\lambda I_K)^{-1}E_v)
\]

Since \(\lambda\) is a constant:

\[
\det \left( I_M - \frac{1}{\lambda} (E_v)^T I_K E_v \right)
\]

Since \(\det(cA) = c^n \det(A)\) for an \([n \times n]\) matrix

\[
\left( -\frac{1}{\lambda} \right)^M \det(-\lambda I_M + (E_v)^T E_v)
\]

\[
\det(-\lambda I_M + (E_v)^T E_v)
\]

Where the characteristic polynomial of the above determinant is order \(M\). As such, there are only \(M\) unique nonzero eigenvalues of \(\det(-\lambda I_K) \det(I_M + (E_v)^T(-\lambda I_K)^{-1}E_v)\) and therefore only \(M\) unique eigenvalues of \(\det(-\lambda I_K + E_v(E_v)^T)\).
Works Cited


Tory, K. J., and M. T. Montgomery, 2006: Internal influences on tropical cyclone formation, The Sixth WMO International Workshop on Tropical Cyclones (IWTC-VI), San José, Costa Rica.


