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ESSAYS ON PRICE COMPETITION AND STRATEGIES: MARKET ENTRY WITH CAPACITY ALLOCATION, CHANNEL DESIGN AND INFORMATION PROVISION

By

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ESSAYS ON PRICE COMPETITION AND STRATEGIES: MARKET ENTRY WITH CAPACITY ALLOCATION, CHANNEL DESIGN AND INFORMATION PROVISION

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Pricing strategy has attracted great interest among researchers due to its wide application in real world application and in academics. And very often, pricing strategy used along with other strategies such as branding, sales effort, and product information revelation. In this dissertation, we investigate different pricing strategies combined with other important non-price strategies. More specifically, we study how an incumbent firm can adopt different pricing and branding strategies to compete with a capacitated entrant. We also study the interaction between a single retailer and a single manufacturer in different service channels where they both can provide pre-sales effort to increase demand and to reduce the number of consumers coming back for after-sales support. In addition, we discuss the impact of information provision on two firms’ price strategies in a model of simultaneous price competition.
Dedication

This work is dedicated to my wife Fei and my two daughters Ella and Alyssa whose sacrifices, which were realized by our loss of precious time together, were for me the most painful and humbling of all.
I would like to express my heartfelt gratitude and respect to my dissertation advisor, Dr. Haresh Gurnani, for his guidance and support throughout my pursuit of this research. The completion of this dissertation would not have been possible without his encouragement and help.

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Chapter 1

Introduction

1.1. Overview

The first part of this dissertation analyzes the role of pricing and branding in an incumbent firm’s decision when facing competition from an entrant firm with limited capacity. We do so by studying two price competition models (Stackelberg and Nash), where we consider the incumbent’s entry-deterrence pricing strategy based on a potential entrant’s capacity size. In an extension, we also study a branding model, where the incumbent firm, in addition to pricing, can also invest in influencing consumer preference for its product. With these models, we study conditions under which the incumbent firm may block the entrant (i.e., prevent entry without any market actions), deter the entrant (i.e., stop entry with suitable market actions) or accommodate the entrant (i.e., allow entry and compete), and how the entrant will allocate its limited capacity across its own and the new market, if entry occurs. We also study the timing difference between the two different dynamics of the price competition models and find that the incumbent’s first-mover advantage benefits both the incumbent and the entrant. Interestingly, the entrant firm’s profits are not monotone increasing in its capacity even when it is costless to build capacity. In the branding
model, we show that in some cases, the incumbent may even increase its price and successfully deter entry by investing in consumer’s preference for its product.

The second part of this dissertation discusses the role of pricing and pre-sales effort in studying the problem of optimal channel design. An integral part of retailing is design of service channels as it influences the purchasing decision of consumers. Very often, customers buying certain products, for example, complex electronics products, software, etc. may lack full functional knowledge and may need additional service to use the product. This entails an inconvenience cost to consumers as well as additional costs to the provider to better educate the customer. On one hand, the retailer (or manufacturer) may invest in effort at the time of sale. Any customer still seeking assistance after the purchase may be served by the retailer who invests in after-sales service; alternately, the consumers may be referred back to the manufacturer for additional help. In this part, we study the service channel design problem and model three different channel structures depending on the provider of pre-sale effort and after-sales support. We compare and contrast the different channel structures and interestingly show that the retailer would even be worse off if the manufacturer chooses to share the cost of service effort with it.

The third part of this dissertation studies a theoretical game theory model. We study the role of pricing and information provision when two firms compete for a single buyer. We incorporate strategic information provision into a model of simultaneous price competition. We study two ex-ante undifferentiated sellers who want to differentiate themselves make simultaneous price offers to a buyer, interested in purchasing the sellers’ products. Unlike traditional Bertrand’s model, our model assumes that the buyer is uncertain about her valuation for each product and that each seller can provide information of the product to the buyer. We characterize the sub-game perfect equilibria of this game. We find that when the probability of a match
between her tastes and the products is low, firms play a ‘go for broke’ strategy, always providing information and charging high prices. In addition, we show that while firms may be undifferentiated ex ante, at the purchase stage a non-zero probability of differentiation always exists.

1.2. Organization

In the following chapters, we will discuss each part of the dissertation mentioned in section 1.1 in each specific research topic. Chapter 2 presents the first part of this dissertation in the topic of “Entry Deterrence with Capacitated Competition Using Price and Non-Price Strategies”. Chapter 3 studies the second part of this dissertation in the topic of “Optimal Design of Channel Effort”. Chapter 4 discusses the third part of this dissertation in the topic of “Product Differentiation through Information Provision”. Chapter 5 concludes and outlines some interesting future research directions. All the proofs are in the Appendices.
Chapter 2

Entry Deterrence with Capacitated Competition Using Price and Non-Price Strategies

2.1. Introduction and Literature Review

2.1.1 Introduction

Capacity, a key asset of a firm, plays an important role when it comes to making a decision to enter into a new market and also has been considered as a vital competitive tool. Consider the case of Arcelik, a subsidiary of Koç Holding, which was the dominant supplier of appliances in Turkey. Arcelik was the sixth largest European manufacturer of household appliances with domestic market share in Turkey exceeding 60-70 percent in major white goods categories. There was considerable debate within the company regarding how much emphasis to place on international sales (Root and Quelch 1997). Due to its tight capacity, Koç management was considering its decisions for future international expansion and, more importantly, how to balance its domestic market share against further foreign expansion to optimize its total profits. Another example is of Vina San Pedro (VSP) which was the third-largest vineyard in Chile (Rangan et al. 2000). VSP had recently expanded its capacity and although its sales had previously been almost exclusively domestic, exports accounted for much of the growth in recent years. VSP’s managers were considering how fast
to push into both foreign and domestic markets and how to balance its capacity allocation between both markets.

There are other examples of companies that balance their capacity allocation to certain markets in order to remain competitive and profitable across all markets. For example, Maersk Line, the world’s largest shipping line, announced that it would reduce its capacity on the Asia-Europe trade lanes by 9 percent and expand elsewhere in an effort to restore profitability (2012). In the words of Maersk Line CEO, Søren Skou: “...with this adjustment we are able to reduce our Asia-Europe capacity and improve vessel utilization without giving up any market share we have gained over the past two years.” (Maerskline.com 2012).

The examples above suggest that firms are constantly seeking to explore new markets in search of higher profits. This poses a challenge for incumbent firms facing potential competition as they have to design suitable strategies to deter entry. One of the strategies is the use of pricing to deter away or compete with the entrant. Goolsbee and Syverson (2008) study the relationship between incumbents’ prices and the presence of Southwest on a route and they show that incumbents cut their prices significantly once Southwest threatens a route as a potential entrant. Yamawacki (2002) finds that German car manufacturers cut prices in response to the entry of a Japanese rival in the U.S. market.

While competitive pricing may be one such option, in some cases, the incumbent firm may be better by emphasizing the difference of its brand from the entrant with non-price strategies. Empirical evidences and real world examples abound. Bunch and Smiley (1992) find that incumbent firms often use advertising as a way to build consumer loyalty in order to deter entry. Kadiyali (1996) studies the Kodak’s pricing and advertising strategies for entry, deterrence, and accommodation and shows that Kodak has engaged in limit pricing and ”limit advertising” before Fuji’s entry in the film market and gained good profit margins. With the increase in Fuji’s market share, Kodak was compelled to take an accommodating stance toward Fuji. Ocean Park was the only amusement park in Hong Kong until 2005, when Hong Kong Disney entered its market. The park moved to highlight the differences with Disney rather than
compete on price only (Young et al. 2006). As the CEO Zeman stated, “Disneyland is about fantasy. ... It’s make-believe, ..., Ocean Park is educational. It’s about the environment, sea mammals, conservation.” (Paul Wiseman 2007), We have no intention of trying to out Disney.” By following this strategy, the park was able to be listed as Hong Kong’s Top 5 Tourist Attractions in 2007.

In this chapter, we consider a competition problem between two firms that operate in their own independent markets and one firm - the entrant - which decides whether to enter into the other firm’s (incumbent) market. In our model, the incumbent firm has sufficient capacity to operate as monopoly in its market; the entrant, however, has limited capacity and upon entry, has to allocate it across the two markets to maximize total profit.

We consider two different models of price competition - Stackelberg and Nash. In the Stackelberg game, we assume that the incumbent, as the prevailing firm in its own market, is the price leader and sets its price first and then the entrant, as follower, sets price in its own market as well as in the incumbent’s market. In the Nash game, both firms set prices simultaneously. The demand of each firm’s product when entry occurs in the incumbent’s market is determined by the firm’s own price and the substitution effect of the other firm’s price. In both price competition models, we study the effects of the capacity size on entry deterrence. Specifically, we study (a) how the entrant’s capacity size would influence the incumbent’s strategy to block the entrant (i.e. prevent entry without any market action), deter the entrant (i.e., stop entry by lowering its price) or accommodate the entrant (i.e., allow entry and compete); (b) how the entrant would allocate its limited capacity into the two markets when entry occurs; (c) the implications of the alternative timings for market outcomes and firm profits.

The study of these models present two interesting observations. While it is intuitive that the incumbent firm would be better off if it were to act as leader and preempt entry (or better compete) by suitably choosing its price, our analysis indicates that the entrant firm, as follower, is also better off by responding to the incumbent’s price as compared to setting prices simultaneously. We show that this is due to the higher
prices set by the two firms in the Stackelberg game as compared to the Nash game. Another counter intuitive result is that in contrast to expectations, the profit for the entrant firm is not monotone in its capacity. Although the competitive threat to the incumbent firm increases in the size of entrant firm, the entrant firm does not necessarily benefit from high capacity even when it is costless to build capacity. Essentially, with large capacity, the entrant cannot credibly commit to selling lower quantity in the incumbent’s market, and in response, the incumbent engages in more intense price competition leading to lower profits for both firms.

In an extension to the pricing models, we also consider the incumbent firm’s branding strategy. Unlike the pricing model where we assume the incumbent’s market size is fixed, in the branding model, the incumbent can increase the size of its customer base through costly investment. As such, the incumbent has another mechanism, besides price, with which to deter entry. Interestingly, we note that in contrast to the price only models, when branding is available, the incumbent may even increase its price and successfully deter entry by suitably investing in branding.

We also incorporate into the branding model the factor of demand uncertainty in the incumbent’s market. We assume that the demand in the incumbent’s market is uncertain but will be realized after the entrant makes its decision in pre-committed quantity. We find that the incumbent can benefit from the demand uncertainty in its market and is more able to keep its monopoly status and to deter the entrant. The incumbent can get higher profit with demand uncertainty, compared to the case of no demand uncertainty. Our results also show that the entrant sells less quantities and gets less profit in the incumbent’s market when there exists market uncertainty.

2.1.2 Literature Review

Our research studies the competition and strategic behavior of a capacitated entrant and an uncapacitated incumbent. The entrant is a monopolist in its own market and considers entering into another market with an incumbent market. As such the entrant faces the problem of optimally allocating its capacity between the two
markets. In response, the incumbent adopts strategies to compete with the entrant. There are three streams of research directly related to our research.

The first stream investigates capacity allocation in different markets. In fact, capacity allocation is a very broad research area and appears in mechanism design (e.g. Maskin and Riley 1989, Cachon and Lariviere 1999) and in the operations research literature (e.g. Wein 1989). Our research interests, however, focus on an entrant’s capacity allocation to different markets. In this research area, for example, Ding et al. (2007) and Ahmed et al. (2012) study a multinational firm that allocates capacity in either one of the two markets or in both markets under uncertain exchange rates. However, they assume that there is no incumbent competitor in the market. In our model, we allow the incumbent’s market to be a duopoly. Specifically, we study the actions both of an entrant and of an incumbent in a duopoly price competition setting.

The second stream of relevant literature, accordingly, are the papers that study problems related to price duopoly with capacity constraints. Indeed, it is a well-studied problem in economics literature with a seminal paper by Levitan and Shubik (1972). Singh and Vives (1984) analyze a Bertrand price competition in a differentiated duopoly model with no capacity constraint. Osborne and Pitchik (1986) study the equilibrium in a model of price-setting duopoly in which each firm has limited capacity. Van Mieghem and Dada (1999) study a competitive price postponement model under capacity-constrained condition. However, these papers only consider competition in a single market. We contribute to this line of literature by considering the interaction of firms across the two markets, one of which is a duopoly and the other a monopoly. We study two different dynamics of price competition in the duopoly market and show that the dynamics of the price game have a significant impact on the two players’ behavior in both markets.

Gal-Or (1985) investigates the conditions under which a player will prefer to be the leader or a follower in a game of sequential moves (with complementary or substitutable products) under price competition and under quantity competition. In our model, we also analyze the strategies and payoffs of a player in a first-mover or
simultaneous game. However, we differ in our analysis by considering two different players: one player (the entrant) with constrained capacity serving two markets and the other (the incumbent) operating in its home market with unconstrained capacity. By investigating the impact of two different dynamics of pricing games on both incumbent’s and entrant’s pricing strategies and profits, this research contributes to the stream of research on first-mover advantage.

The third stream of literature that is relevant to our research is the one which studies the problems of strategic entry-deterrence in economics. A large stream of the economics literature has been concerned with strategic entry-deterrence since Spence (1977) and Dixit’s (1980) seminal papers that study the use of limit pricing. Entry-deterrence models in this line emphasize the use of limit pricing in price competition or limit quantity in quantity competition as the incumbent’s strategic tool for deterring entry.

We have some common research goals in this regard but our research differs from most previous analyses of entry-deterrence in some important aspects relating to the role of both the incumbent and the entrant firm. Research from the entrant’s point of view in the entry-deterrence model has been relatively neglected. Previous literature assumes that the entrant has no resource constraint. However, as the examples discussed earlier in the section suggest, entry decision also involves allocation of capacity across different markets. Specifically, we focus on the implications for adopting different pricing (or branding) strategies when the incumbent firm faces competition from an entrant with constrained-capacity. In addition, unlike previous entry-deterrence models that pay little attention to the status in the entrant’s own market, our model considers a scenario where the entrant has to make capacity allocation decisions. This allows us to understand the interplay between the incumbent’s entry-deterrence policies and the pricing and capacity allocation strategies employed by the entrant in its own market. As in the Koç Holding example discussed in the previous section, this setting is more realistic since it is rare for an entrant to enter into the incumbent’s market with no consideration for its own market operations. Our results show that
the entrant’s profit in its own monopoly market is indeed affected by the dynamics of competition in the incumbent’s market.

Previous literature considers one mechanism such as price or output level (quantity) in the post-entry game. Dixit (1980), Spulber (1981), and Bulow et al. (1985) examine the entry problem when the post-entry game is Cournot competition. Basu and Singh (1990) study entry when the post-entry game is Stackelberg quantity competition. Allen et al. (2000) consider a model of Bertrand price competition in the post-entry game. However, in our research, by building on a case where post-entry game is Stackelberg price competition, we allow the incumbent to have two levers: price and branding, both of which affect demand and thus affect the entrant’s entry and pricing decisions. Chayet and Hopp (2005) study firms’ post-entry behavior in a simultaneous price game with delivery time determined endogenously. In their model, the cost of expected leadtime is part of the full price that affects the determination of the demand allocation. However, our research differs from their paper in three important ways. First, in their model, the cost of expected leadtime is decided by a queuing system, where the only decision variable is the service rate. As a result, the expected leadtime in itself is not a decision variable. In our model, the branding is a decision made by the incumbent to compete with the entrant. Second, in addition to the simultaneous price competition, we study Stackelberg price competition in post-entry game. Third, they assume that the incumbent’s market is homogeneous, but we assume that it is heterogeneous.

The rest of this chapter is organized as follows. In section 2.2, we present the model assumptions and define the notation. In section 2.3, we introduce the Stackelberg competition model, and in section 2.4, the Nash competition model is discussed and we compare and contrast the differences between the two pricing models. In section 2.5, we extend the analysis to consider the branding model. Finally, in section 2.6, we conclude and discuss future research. All proofs are presented in the Appendix A.
2.2. Model Formulation

The base model comprises of two firms operating in their own markets: the entrant’s market and the incumbent’s market. Both markets are independent, but one firm - a potential entrant - plans to enter into the other firm’s market. The entrant firm has limited capacity, and upon entry, would need to allocate its capacity across the two markets. Moreover, the firms would engage in price competition if entry does occur.

![Diagram of Competition Model]

Figure 2.1: The Competition Model

We study two models of price competition in this research. First, we model competition as a Stackelberg game, where the entrant firm sets its price after observing the incumbent’s prevailing price in its own market. In the second case, we model simultaneous price competition as a Nash game where the two firms set prices simultaneously. In the literature on price (or quantity) competition between firms with differentiated goods, it is common to find linear demand models (see Singh and Vives 1984, Vives 1984, Häckner 2000, Symeonidis 2003, Choi and Coughlan 2006, etc.)
that we use in our model. We refer to these papers for discussion of the linear demand model which is derived from a more structured approach of a utility maximizing consumer. A detailed list of notation is given in Table 2.1.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i$</td>
<td>Firm index $i = 1$ for entrant in its own market, $i = 2$ for incumbent</td>
</tr>
<tr>
<td>$j$</td>
<td>Index $j = n$ for Nash competition, $j = s$ for Stackelberg competition</td>
</tr>
<tr>
<td>$p_i^j$</td>
<td>Entrant’s price in its own market in Nash, Stackelberg</td>
</tr>
<tr>
<td>$q_i^j$</td>
<td>Entrant’s quantity sold in its own market in Nash, Stackelberg</td>
</tr>
<tr>
<td>$\pi_i^j$</td>
<td>Entrant’s profit in the incumbent’s market in Nash, Stackelberg</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>Market i’s size</td>
</tr>
<tr>
<td>$c_i$</td>
<td>Firm i’s unit production cost, $c_i \geq 0$</td>
</tr>
<tr>
<td>$k_x^s$</td>
<td>$x \in {1, 2, 3}$. Threshold value of capacity in Nash and Stackelberg competition</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Market preference, $0 \leq \mu \leq 1$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Product’s substitution effect on demand $0 &lt; \theta &lt; 1$</td>
</tr>
<tr>
<td>$t$</td>
<td>Entrant’s transportation cost, $t &gt; 0$</td>
</tr>
</tbody>
</table>

If entry does occur, we assume that the entrant’s (firm 1) demand functions in the two markets are given by:

$$q_1^j = \alpha_1 - p_1^j, \quad (2.1)$$

$$q_t^j = (1 - \mu)\alpha_2 - p_t^j + \theta p_2^j, \quad (2.2)$$

where $q_1^j + q_t^j \leq k$, $j = s$ or $n$ for Stackelberg and Nash competition, respectively. We also assume that demand function for the incumbent (firm 2) is given by:

$$q_2^j = \mu\alpha_2 - p_2^j + \theta p_1^j. \quad (2.3)$$

The total size of the incumbent’s market is equal to $\alpha_2$, and $\mu$ represents the consumer’s preference for its product. Similarly, $\theta$ represents the substitution effect on demand. Each firm’s demand in market 2 is decreasing in its own price level and increasing in the competitor’s price. The entrant firm also incurs transportation cost $t$ per unit if it enters into the incumbent’s market.

If entry does not occur, the incumbent firm acts as a monopoly in its market.
and its demand function is suitably modified. Essentially, the transformation in the demand function is based on setting $q_t = 0$, and substituting the expression for $p_j^t$ back into $q_j^t$ defined above (see Ingene and Perry 2004):

$$q_j^m = [\mu + \theta(1 - \mu)]\alpha_2 - (1 - \theta^2)p_2^m.$$  \hfill (2.4)

With this demand function, the monopoly profit function for the incumbent firm is given by:

$$\pi_2^m = (p_2^m - c_2)[(\mu + \theta(1 - \mu))\alpha_2 - (1 - \theta^2)p_2^m],$$  \hfill (2.5)

and the optimal monopoly price is:

$$p_2^m = \frac{[\mu + \theta(1 - \mu)]\alpha_2 + c_2(1 - \theta^2)}{2(1 - \theta^2)}.$$  \hfill (2.6)

### 2.3. Stackelberg Competition

In this section, we consider the case where the incumbent (firm 2) is the price leader and the entrant is the follower. The sequence of events are as follows: (1) Facing the threat of potential entry, the incumbent firm sets the price in its own market; (2) Observing the incumbent’s price, the entrant decides whether to enter the market or not. If it decides to enter, it sets price $p_1^s$ in the incumbent’s market and $p_2^s$ in its own market to maximize total profits. As is standard, we solve the problem by backward induction. All proofs are in the Appendix.

If entry occurs, each player’s optimization problem is given by:

$$\max_{p_1^s, p_2^s} \pi_1^s = (p_1^s - c_1)(\alpha_1 - p_1^s) + (p_t^s - t - c_1)[(1 - \mu)\alpha_2 - p_t^s + \theta p_2^s]$$  \hfill (2.7)

s.t. \hspace{1cm} [\alpha_1 - p_1^s] + [(1 - \mu)\alpha_2 - p_t^s + \theta p_2^s] \leq k, \hspace{1cm} (2.8)

and

$$\max_{p_2^s} \pi_2^s = (p_2^s - c_2)(\mu \alpha_2 - p_2^s + \theta p_1^s).$$  \hfill (2.9)

Starting with the entrant’s (firm 1) problem, when the capacity constraint is
activated, we get:

$$p^*_1(p^*_2) = \frac{-2k - t + 3\alpha_1 + \alpha_2 + p^*_2\theta - \alpha_2\mu}{4} \quad (2.10)$$

and

$$p^*_2(p^*_2) = \frac{-2k + t + \alpha_1 + 3\alpha_2 + 3p^*_2\theta - 3\alpha_2\mu}{4}. \quad (2.11)$$

On substituting $p^*_1(p^*_2)$ into $q^*_2 = (1 - \mu)\alpha_2 - p^*_i + \theta p^*_2$ we get:

$$q^*_i(p^*_2) = \frac{2k - t - \alpha_1 + \alpha_2 + p^*_2\theta - \alpha_2\mu}{4}. \quad (2.12)$$

Note that in equation (2.12), $q^*_i$ is a function of $p^*_2$. There may exist some values of $k$ such that the incumbent can choose price $p^*_2$ to make $q^*_i \leq 0$ (as long as it is more profitable for the incumbent to do so compared to other strategies), which means that the entrant would not enter the incumbent’s market. As shown next, the capacity size $k$ plays a critical role in shaping the incumbent’s selection of price and accordingly its competition strategy.

**Proposition 2.1.** In equilibrium, there exist three thresholds ($k^*_1 < k^*_2 < k^*_3$) in capacity size with four regions such that:

(a) **Blockaded Entry region** ($k \leq k^*_1$): The entrant does not enter the incumbent’s market and the incumbent sets the monopoly price.

(b) **Effectively Impeded Entry region** ($k^*_1 < k \leq k^*_2$): The incumbent is able to lower price and the entrant does not enter the incumbent market.

(c) **Ineffectively Impeded Entry region** ($k^*_2 < k \leq k^*_3$): Both players compete in the incumbent’s market.

(d) **Unlimited Capacity region** ($k^*_3 < k$): The capacitated problem switches to the uncapacitated case.

The four regions in capacity size for the entrant firm are as follows:

**Blockaded Entry Region** ($k \leq k^*_1$): In this region, the potential entrant’s capacity is small and therefore if the incumbent uses its monopoly price ($p^*_m$), the expression for $q^*_i(p^*_m)$ becomes negative. Consequently the entrant would not enter
into the incumbent’s market. The incumbent can make its monopoly profit by setting price at 
\[ p^m_2 = \frac{c_2 - c_2\theta^2 + \alpha_2(\theta + \mu - \theta\mu)}{2 - 2\theta^2} \] and get profits 
\[ \pi^m_2 = \frac{|\alpha_2(\theta + \mu - \theta\mu) - c_2(1 - \theta)^2|^2}{4(1 - \theta^2)}. \] The entrant’s capacity is so low that devoting any output to the incumbent market (where he faces competition) is not worthwhile.

**Effectively Impeded Entry Region** \((k^s_1 < k \leq k^s_2)\): In this region, if the incumbent uses its monopoly price \(p^m_2\), the expression for \(q^s_t(p^m_2)\) is positive, which means the entrant would be interested in entering the incumbent’s market. On substituting \(p^m_2\) into \(p^d_t\), we get \(p^d_t = \frac{-2k + t + \alpha_1 + 3\alpha_2 + 3p_2\theta - 3\alpha_2\mu}{4}\), which is the entrant’s price in the incumbent market when the incumbent allows entry by using its monopoly price. (Here we abuse the notation by creating \(p^d_t\) and \(\pi^d_2\)). Then, substituting \(p^m_2\) and \(p^d_t\) into \(\pi^d_2 = (p^m_2 - c_2)(\mu\alpha_2 - p^m_2 + \theta p^d_t)\), we get the incumbent’s profit. However, if the incumbent lowers its price to stop entry, then substituting \(p^*_t\) into the equation \(q^*_t = (1 - \mu)\alpha_2 - p^*_t + \theta p^*_2\) and solving for \(p^*_2\) that makes \(q^*_t = 0\), we get:

\[ p^m'_2(k) = \frac{-2k + t + \alpha_1 - \alpha_2 + \alpha_2\mu}{\theta}. \] (2.13)

Using the above price in the incumbent’s monopoly profit function \(\pi^m_2\), we get a modified monopoly profit function \(\pi^m'_2(k)\). Further, if \(k^s_1 < k \leq k^s_2\), we have \(\pi^d_2 < \pi^m'_2(k)\), that is, the incumbent has the incentive to lower its price to stop entry.

**Ineffectively Impeded Entry Region** \((k^s_2 < k \leq k^s_3)\): If the incumbent continues to lower price to stop entry in this region, its profit would be lower than the profit from allowing entry by setting the optimal price with competition. In this case, we need to calculate the incumbent’s profit with competition. Substituting \(p^*_t\) into \(\pi^*_2 = (p^*_2 - c_2)(\mu\alpha_2 - p^*_2 + \theta p^*_t)\), we get:

\[ \max_{p^*_2} \pi^*_2 = (c_2 + p^*_2)[\alpha_2\mu - p^*_2 + \frac{1}{4}\theta(-2k + t + \alpha_1 + 3\alpha_2 + 3p^*_2\theta - 3\alpha_2\mu)] \] (2.14)

Solving for \(p^*_2\), we get:

\[ p^*_2 = \frac{4c_2 - 2k\theta + t\theta + \alpha_1\theta + 3\alpha_2\theta - 3c_2\theta^2 + 4\alpha_2\mu - 3\alpha_2\theta\mu}{2(4 - 3\theta^2)}. \] (2.15)
Substituting $p^*_2$ back into equations (2.7), (2.9), (2.10) and (2.11), we get $\pi_1^I(k)$, $\pi_2^I(k)$, $p_1^I(k)$, $p_2^I(k)$ as given in Table 2.2. On comparing the profits $\pi_2^I(k)$ and $\pi_2^{mI}(k)$, we find the threshold value $k^*_2$ such that if $k > k^*_2$ then $\pi_2^{mI} < \pi_2^I(k)$. Therefore, instead of lowering its price any further to stop entry, the incumbent would allow entry and set the optimal price to compete with the entrant.

**Unlimited Capacity Region** ($k^*_3 < k$): To facilitate our capacity-constrained analysis, we study the case where the entrant’s capacity is unconstrained (i.e., relax the capacity constraint condition by eliminating equation (2.8)). We employ this assumption to find the maximum capacity in capacitated scenario that the entrant could utilize to satisfy demand in both markets. Solving equations (2.7) and (2.9) by backward induction, we obtain the incumbent’s profit in the uncapacitated case $\pi_2^{us}$. (Here we use $\pi_2^{us}$ to denote the incumbent’s profit in the uncapacitated Stackelberg competition case). If the incumbent continues to lower price, its profit would be lower than $\pi_2^{us}$. As such, the incumbent will set price equal to $p^*_2^{us}$ and the game switches to the uncapacitated case. On setting $\pi_2^{us} = \pi_2^I(k)$, we get $k^*_3$, the threshold capacity from which the game switches to competition as if the entrant has unlimited capacity.

In this region, allocating more capacities into the incumbent’s market would hurt the profitability for the entrant, as the example of Maersk Line shows in the introduction.

We define the following and list all the expressions in Table 2.2:

\[
A = t\theta + \alpha_1\theta + 3\alpha_2\theta - c_2(4 - 3\theta^2) + 4\alpha_2\mu - 3\alpha_2\theta\mu ,
B = c_1\theta + t\theta + \alpha_2\theta - c_2(2 - \theta^2) + 2\alpha_2\mu - \alpha_2\theta\mu ,
C = t + \alpha_1 - \alpha_2 + \alpha_2\mu ,
D = c_2(1 - \theta^2) + \alpha_2(\theta + \mu - \theta\mu).
\]

<table>
<thead>
<tr>
<th>Blockaded Entry</th>
<th>Effectively Impeded Entry</th>
<th>Ineff. Impeded</th>
<th>Unlimited</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p^*_1$</td>
<td>$\alpha_1 - k$</td>
<td>see Table A.1*</td>
<td>$\alpha_1^* + \frac{c_1}{2} + 2\alpha_2 - \alpha_2\mu + \frac{\theta}{2}(\frac{B}{2(2-\theta^2)} + c_2)$</td>
</tr>
<tr>
<td>$p^*_2$</td>
<td>$-\frac{D}{2(2-\theta^2)}$</td>
<td>see Table A.1</td>
<td>$\frac{B}{2(2-\theta^2)} + c_2$</td>
</tr>
<tr>
<td>$q^*_1$</td>
<td>$k$</td>
<td>see Table A.1</td>
<td>$\frac{B}{2(2-\theta^2)} + c_2$</td>
</tr>
<tr>
<td>$q^*_2$</td>
<td>$c_2(\theta^2 - 1) + \alpha_2(\theta + \theta\mu) - (1 - \theta^2)(-2k + C)$</td>
<td>see Table A.1</td>
<td>$\frac{B}{2} + \frac{\theta}{2}(\frac{B}{2(2-\theta^2)} + c_2)$</td>
</tr>
<tr>
<td>$\pi^*_1$</td>
<td>$k(\alpha_1 - k - c_1)$</td>
<td>see Equation (A.14)</td>
<td>$\alpha_1^* + \frac{c_1}{2} + 2\alpha_2 - \alpha_2\mu + \frac{\theta}{2}(\frac{B}{2(2-\theta^2)} + c_2)$</td>
</tr>
<tr>
<td>$\pi^*_2$</td>
<td>$\frac{(D)^2}{4(1-\theta^2)}$</td>
<td>see Table A.1</td>
<td>$\frac{B}{2}$</td>
</tr>
</tbody>
</table>

* in Appendix A ** Ineffectively Impeded Entry
Figure 2.2: Stackelberg Competition

Figure 2.2 is based on a numerical analysis with parameters $\alpha_1 = 100, \alpha_2 = 100, \theta = 0.5, c_1 = 1, c_2 = 1.2, \mu = 0.5$ and $t = 0.5$. As shown in the figure, when the entrant’s capacity level goes beyond the Blockaded Entry region, it would be profitable if the entrant enters the incumbent’s market. However, the incumbent can lower its price to deter entry for $k < k_2$. As the entrant’s capacity increases, the incumbent would be worse off if it lowers the price to stop entry. Consequently, the firms would compete on price, and the entrant would optimally allocate its limited capacity into the two markets. It is interesting to note that the entrant firm’s profit is not monotone in its capacity size, as shown in the figure above. This is a consequence of the fact that
as capacity increases, the entrant aims to sell more quantity in the incumbent’s market by lower the selling price, and therefore, the profit is not monotonely increasing. This leads to the next lemma.

**Lemma 2.1.** The profit for the entrant firm is not monotone in its capacity, even when expanding capacity is cost free.

Lemma 2.1 notes the counter-intuitive result that having higher capacity may in fact lead to lower profits for both firms even when there is no cost of capacity. As its capacity increases, the entrant sells more quantity by lowering its price and its profits peak at \( k = k^s \). It would then prefer not to use more capacity. However, it cannot credibly convey to the incumbent firm that it would not release more quantity into its market. Consequently, the incumbent would believe that the entrant, in order to get higher profit, would deviate at \( k = k^s \) resulting in more intense price competition and lower profits for both firms for \( k > k^s \). This result is related to the ‘judo economics’ of Gelman and Salop (1983). They show that the entrant may be able to encourage accommodation from a cost-advantage incumbent by committing to limited capacity. We differ in the analysis by showing that even in the Ineffectively Impeded Entry region limited capacity can serve as a device to alleviate price competition and the entrant is thus able to get higher profit. In addition, Our model doesn’t make assumption that the incumbent has cost advantage.

When the entrant’s capacity is larger than \( k^s \), if the incumbent continues to lower the price to compete with the entrant, its profit will be lower than the profit it can get by setting its price at the level in the uncapacitated scenario. Thus the incumbent will set the price at \( p^u_2 \) when the entrant has unlimited capacity and the game switches to the uncapacitated case. At \( k = k^s \), the incumbent’s profit in both scenarios are equal. However, our analysis shows that the entrant’s profit at \( k = k^s \) in the two scenarios are not equal. This leads to the next proposition.

**Proposition 2.2.** At the critical threshold \( k^s \) dividing the uncapacitated and capacitated regions, (a) the entrant in capacitated scenario sells less in its own market than it does in uncapacitated scenario, that is, \( q^s_1(k^s) < q^u_1 = \frac{\alpha_1 - c_1}{2} \); However, the entrant
in the capacitated scenario sells more in the two markets together than it does in uncapacitated case, that is: \( k_s^3 > q_1^{us} + q_t^{us} \); (b) the entrant’s profit in the capacitated scenario is higher than the profit in the unconstrained case, that is, \( \pi_s^i(k_s^3) > \pi_1^{us} \).

The intuition behind Proposition 2.2(a) is that because of limited capacity, the entrant faces a trade-off between obtaining higher profit in the incumbent’s market and sacrificing some of the monopoly profits in its own market. As long as the entrant can obtain higher total profits, the entrant is willing to sell more in the incumbent’s market and less in its own market. Proposition 2.2(b) notes the interesting result that the entrant with limited capacity (slightly less than \( k_s^3 \)) is able to make higher profit than the entrant with unlimited capacity (\( k = k_s^3 \)). This is a consequence of the price competition between the two firms. In the case of unconstrained capacity for the entrant, the incumbent firm faces risk of larger supply quantity of the entrant firms’ product, and consequently, price competition becomes more intense leading to lower profits. However, in the limited capacity region, the supply is reduced resulting in higher prices and higher profits for both firms.

2.4. Nash Competition

In this section, we consider the simultaneous price setting competition game. The demand and profit functions are exactly the same as in the Stackelberg game but market dynamics are different. The sequence of events in the Nash game are as follows: the incumbent (firm 2) and the entrant (firm 1) simultaneously choose price \( p_2^n \) and \( p_1^n \) and the entrant also chooses price \( p_1^n \) in its own market to maximize their profits.

Since the structure in Nash competition is similar to that in Stackelberg competition, we only give a brief explanation in this section. All details are in the Appendix. With the market dynamics in Nash competition, we solve the following problems simultaneously:

\[
\max_{p_1^n, p_2^n} \pi_1^n = (p_1^n - c_1)(\alpha_1 - p_1^n) + (p_1^n - t - c_1)[(1 - \mu)\alpha_2 - p_2^n + \theta p_2^n] \tag{2.16}
\]
\[ s.t. \quad [\alpha_1 - p_1^n] + [(1 - \mu)\alpha_2 - p_t^n + \theta p_2^n] \leq k \quad (2.17) \]

and

\[ \max_{p_2^n} : \pi_2^n = (p_2^n - c_2)(\mu\alpha_2 - p_2^n + \theta p_2^n). \quad (2.18) \]

We define the following and list all the expressions in Table 2.3:

\[ D = 2\alpha_1 - 2\alpha_2 - c_2\theta - \alpha_1\theta^2 + t(2 - \theta^2) + 2\alpha_2\mu - \alpha_2\theta\mu \]

\[ E = 2\alpha_2 + c_2\theta - c_1(2 - \theta^2) - t(2 - \theta^2) - 2\alpha_2\mu + \alpha_2\theta\mu \]

Table 2.3: Equilibrium Expressions in Nash Game

<table>
<thead>
<tr>
<th>Blockaded and Effectively Impeded Entry</th>
<th>Ineffectively Impeded Entry</th>
<th>Unlimited</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_1^n )</td>
<td>( D + k(4 - \theta^2) )</td>
<td>( \alpha_1 + c_1 )</td>
</tr>
<tr>
<td>( p_1^n )</td>
<td>( 4k - 2t - 2\alpha_2 - 3c_2\theta + 6\alpha_2\mu - 3\alpha_2\theta )</td>
<td>( 2k + 2t + 2\alpha_2 - 2c_2\theta - 2\alpha_2\mu + 2\alpha_2\theta )</td>
</tr>
<tr>
<td>( p_1^n )</td>
<td>( -2k + \theta )</td>
<td>( 4 - \theta^2 )</td>
</tr>
<tr>
<td>( q_1^n )</td>
<td>( D + k(4 - \theta^2) )</td>
<td>( \alpha_1 + c_1 )</td>
</tr>
<tr>
<td>( q_1^n )</td>
<td>( 4k - 2t - 2\alpha_2 - 3c_2\theta + 6\alpha_2\mu - 3\alpha_2\theta )</td>
<td>( 2k + 2t + 2\alpha_2 - 2c_2\theta - 2\alpha_2\mu + 2\alpha_2\theta )</td>
</tr>
<tr>
<td>( q_1^n )</td>
<td>( -2k + \theta )</td>
<td>( 4 - \theta^2 )</td>
</tr>
<tr>
<td>( \pi_1^n )</td>
<td>( A - 2k\theta )</td>
<td>( E )</td>
</tr>
<tr>
<td>( \pi_2^n )</td>
<td>( A - 2k\theta )</td>
<td>( E )</td>
</tr>
</tbody>
</table>

* in Appendix A

The results are similar to those of the Stackelberg competition and are given in Proposition 2.3.

**Proposition 2.3.** In equilibrium, there exist three thresholds (\( k_1^n < k_2^n < k_3^n \)) in capacity size with four regions such that:

(a) **Blockaded Entry region** (\( k \leq k_1^n \)): the entrant would voluntarily not enter the incumbent’s market.

(b) **Effectively Impeded Entry region** (\( k_1^n < k \leq k_2^n \)): the entrant is interested in entering the incumbent’s market but the incumbent can lower its price to stop entry.

(c) **Ineffectively Impeded Entry region** (\( k_2^n < k \leq k_3^n \)): Both players compete in the incumbent’s market and the entrant will optimally allocate its limited capacity into the two markets to maximize its total profits.

(d) **Unlimited Capacity region** (\( k_3^n < k \)): The capacitated problem switches to the uncapacitated case.

See Figure 2.3 for an illustration of the results (the numerical parameters are the same as in Figure 2.2). Similar to the case of Stackelberg competition, the entrant
firm’s profits are again not monotone increasing in its capacity. Although the structure of thresholds of the Nash game are similar to those of Stackelberg competition, the next proposition shows some important differences.

**Figure 2.3: Nash Competition**

**Proposition 2.4.** At the threshold capacity (\( k^n_3 \)) in Nash competition,

(a) the entrant in capacitated scenario sells the same quantities in each market as it does in the uncapacitated scenario, that is: \( q^n_1(k^n_3) = q^{un}_1 \) and \( q^n_t(k^n_3) = q^{un}_t \);

(b) The entrant gets the same profit in each market in both scenarios, that is, \( \pi^n_1(k^n_3) = \pi^{un}_1 \) and \( \pi^n_t(k^n_3) = \pi^{un}_t \).
The results above are in contrast to the findings in Proposition 2.2 for the Stackelberg game. Essentially with simultaneous pricing in the Nash game, the equilibrium prices are continuous in the entrant’s capacity size and therefore, the profits are also continuous at the critical threshold level.

2.4.1 Comparison Between Competition Models

In this section, we compare the results from the two competition models. We focus our analysis on the impact of different market dynamics on both firms’ profits, incumbent’s strategic regions, and the entrant’s capacity allocation behavior.

**Proposition 2.5.** (a) The region for Blockaded Entry in both Stackelberg and Nash competition is the same, that is, \( k_s^1 = k_n^1 \); However, the region for Effectively Impeded Entry is larger in the Nash game, that is, \( k_s^2 < k_n^2 \). Finally, the region with equilibrium decisions under uncapacitated case occurs later in Stackelberg competition, that is \( k_n^3 < k_s^3 \); (b) When competition occurs, both the entrant’s and the incumbent’s profit are higher in Stackelberg competition than in Nash competition, that is: when \( k > k_s^2 \), \( \pi_s^1(k) > \pi_n^1(k) \) and \( \pi_s^2(k) > \pi_n^2(k) \).

Proposition 2.5(a) is an interesting result as the incumbent firm, as Stackelberg leader, is willing to compete with the entrant for lower values of the entrant’s capacity size (as compared to the Nash game). This is again attributed to the higher prices for both firms in the Stackelberg game.

In Stackelberg competition, as expected, the incumbent is better off since it has the first-mover advantage. However, it is counter-intuitive that the entrant’s profit is also higher in Stackelberg competition as compared to Nash competition. In the Stackelberg game, the incumbent, as leader, is able to affect lower price competition by setting a higher price which induces a high price response from the entrant as well. In contrast, in the simultaneous game, it is not possible to make a credible high price commitment leading to more intense price competition and lower profits for both firms as compared to the Stackelberg game. As such, the first-mover advantage for the incumbent firm benefits both players. In a different problem of supplier
encroachment, Arya et al. (2007) show a similar result in that both the supplier and the retailer (and consumers as well) are better off under sequential (leader-follower) encroachment than under simultaneous moves due to lower wholesale price charged by the supplier. In contrast, in our research, prices are higher in the Stackelberg game which would make consumers worse off. We also note that in the Stackelberg game, the quantity sold by the two firms in the incumbent’s market are higher even though prices are higher.

**Lemma 2.2.** In the Stackelberg game, the leader (incumbent firm) and the follower (entrant firm) sell less in their own markets, whereas the follower (entrant firm) sells more in the incumbent’s market compared to the Nash game, that is, \( q_s^1(k) \leq q_n^1(k) \), \( q_s^2(k) \leq q_n^2(k) \) and \( q_s^t(k) \geq q_n^t(k) \), \( \forall k \).

These results are quite surprising as in a standard Cournot competition model, the leader sells higher quantity due to its first-mover advantage. However, as we note above, this is not the case in the Bertrand game as prices (quantities) are higher (lower) in the Stackelberg game compared to the simultaneous Nash game.

**Lemma 2.3.** At the threshold capacity where the capacitated case switches to uncapacitated case, the entrant sells less in its own market but more in the incumbent’s market in Stackelberg competition than in Nash competition, that is, \( q_1^a(k_3^s) < q_1^a(k_3^n) \) and \( q_t^a(k_3^s) > q_t^a(k_3^n) \).

Lemma 2.3 notes that the entrant strategically allocates more of its limited capacity into the incumbent’s market in Stackelberg competition than in Nash competition because of the higher profit it can get in the incumbent’s market. Consequently, at the threshold capacity \( k_3^a \) in Stackelberg competition, the limited capacity has some impact on the entrant’s own market, that is, the entrant will sell less than its monopoly quantity in Stackelberg competition. However, at the threshold capacity \( k_3^n \) in Nash competition, the entrant will sell the monopoly quantity in its own market, and therefore, limited capacity has no impact on the entrant’s own market.
2.5. Branding

In this section, we consider the case when the incumbent firm can use an additional mechanism (besides price) to stop the entrant from potential entry or to better compete with it. We make the consumers' market preference ($\mu$), a given exogenous parameter in previous sections, as a decision variable. That is, in addition to its pricing strategy, the incumbent can invest in product branding to change consumer's preference for its product. While the total market size $\alpha_2$ is fixed, using branding, the incumbent firm can build stronger preference towards its product. We use an additional superscript $b$ to denote decisions in the branding model. Let $s$ be the coefficient in the incumbent’s investment function, that is, the cost of branding, $c(\mu) = \frac{1}{2}s\mu^2$.

Without loss of generosity, we will analyze the branding scenario based on the Stackelberg competition model in section 2.3.

We start with the case of monopoly in branding model. From equation (2.5), we obtain the incumbent’s profit function as a monopoly with branding as follows:

$$\max_{p_2^m, \mu_2^m} \pi_2^m = \left[ (\mu_2^m + \theta(1 - \mu_2^m))\alpha_2 - (1 - \theta^2)p_2^m \right] (p_2^m - c_2) - \frac{s(\mu_2^m)^2}{2}. \quad (2.19)$$

It can be shown that if $s > \alpha_2^2(1 - \theta)/2(1 + \theta)$, the profit function is jointly concave and therefore the first-order conditions are sufficient to characterize the optimal decisions:

$$\mu_2^m = \frac{\alpha_2[\alpha_2\theta - c_2(1 - \theta^2)]}{2s(1 + \theta) - \alpha_2^2(1 - \theta)}, \quad p_2^m = \frac{s\alpha_2\theta + c_2(1 - \theta)[s(1 + \theta) - \alpha_2^2(1 - \theta)]}{(1 - \theta)[2s(1 + \theta) - \alpha_2^2(1 - \theta)]} \quad (2.20)$$

and the incumbent’s monopoly profit is

$$\pi_2^m = \frac{s[\alpha_2\theta - c_2(1 - \theta^2)]^2}{2(1 - \theta)[2s(1 + \theta) - \alpha_2^2(1 - \theta)]}. \quad (2.21)$$
2.5.1 Stackelberg Competition Model with Branding

In the case of entry, the sequence of events are as follows: (1) The incumbent invests in branding $\mu^b$ and sets price $p_2^b$, (2) The capacitated entrant decides whether to enter the incumbent’s market and accordingly, sets its price and allocates capacity.

We proceed by backward induction to solve this problem. As in the previous sections, we show that the optimal strategy is divided into four regions based on the entrant firm’s capacity. But now, the incumbent uses both branding investment as well as pricing strategy to thwart entry or to better compete with the entrant.

**Proposition 2.6.** In equilibrium, there exist three thresholds ($k_1^b < k_2^b < k_3^b$) in capacity size with four regions such that:

(a) **Blocked Entry region** ($k \leq k_1^b$): the incumbent will choose monopoly decisions $\mu^m$ and $p_2^m$ and get monopoly profits $\pi^m_2$.

(b) **Effectively Impeded Entry region** ($k_1^b < k \leq k_2^b$): the incumbent will choose $\mu^m'$ and $p_2^m'$ to prevent entry and earn modified monopoly profits $\pi^m_2'$.

(c) **Ineffectively Impeded Entry region** ($k_2^b < k \leq k_3^b$): the incumbent will allow entry and choose optimal branding investment $\mu^b(k)$ and get profits $\pi^b_2(k)$.

(d) **Unlimited Capacity region** ($k_3^b < k$): The constrained-capacity problem switches to the unconstrained capacity case.

There are interesting differences between the branding model in Figure 2.4 and the price-only model as shown in Figure 2.1. In the *Effectively Impeded Entry region*, the incumbent invests in increasing $\mu$ and may also increases its price to successfully stop entry. This counter-intuitive results are in fact consistent with Thomas (1999)'s empirical evidence. The paper finds that over the sample period, which is similar to the *Effectively Impeded Entry* region in our model, the ready-to-eat cereal industry is characterized by rapidly increasing real prices but high advertising. By investing in the consumer preference and consequently increasing in the marketing share, the incumbent is closer to be a monopoly and therefore can take the advantage of being a monopoly to increase its price to cover its investment cost. However, if the coefficient $s$ in the cost function of branding is sufficient high ($s > \frac{\alpha^2(1-\theta)}{\theta}$), then the incumbent
will in fact lower its price to stop entry while still invest in increasing $\mu$. But when $s$ is sufficient high then the incumbent’s ability to keep its monopoly and to deter entry is very weak. For example, based on the same numerical parameters ($\theta = 0.5$ and $\alpha_2 = 100$), $s$ should be at least greater than 10000. As a result, if $s = 10050$, then the Blockaded Entry region in fact does not exist and the Effectively Impeded Entry region is so tiny that a very capacitated firm would lead to the region not existing.

To summarize, using branding as an additional lever, as long as within a reasonable range, the incumbent firm is able to deter entry by increasing price and increase its investment in consumer’s preference for its product.
But the investment in $\mu$ becomes increasingly costly when the entrant’s capacity is sufficiently large as it can also lower its price to gain entry. Consequently, at some threshold, it becomes optimal for the incumbent to allow entry and to compete. In this region of Ineffectively Impeded Entry, investment in $\mu$ and the price $p^*_2(k)$ are both decreasing in the entrant’s capacity size.

In the branding model, by making the consumer preference ($\mu$) endogenous as the additional lever for the incumbent to deter entry, we find that the branding decision $\mu$ has some impact on the threshold value. But the variation of the threshold values is dependent on the coefficient $s$ in the incumbent’s investment function $s\mu^2/2$. If the value of $s$ is not too high, then the threshold values $k^b_1$ and $k^b_2$ is higher than those in pricing model. That is, the incumbent is more able to blockade and effectively impede entry by using both price and non-price strategy than by using pricing only strategy. As $s$ increases, the values of $k^b_1$ and $k^b_2$ decrease. In fact $k^b_1$ and $k^b_2$ is negatively related to the value of $s$ and beyond to some point, $k^b_1$ and $k^b_2$ will be lower than $k^s_1$ and $k^s_2$.

2.6. Conclusions

In this chapter, we first propose a Stackelberg price competition model where we study the entry-deterrence problem of an incumbent firm who faces competition from a capacititated entrant firm. The key idea of the model is to characterize the different pricing strategies to block, deter, or accommodate the entrant based on the entrant’s capacity size. When the entrant’s capacity is higher than a critical threshold, the incumbent has to lower price to stop entry. Allowing entry is optimal strategy when the entrant’s capacity size is sufficiently high. Finally, beyond another threshold value, the capacititated problem switches to the uncapacitated case.

We also propose a Nash price competition model to study the the entry-deterrence problem. We then compare the two price competition model. We show that profits for both firms are higher in Stackelberg competition than in Nash competition. Our analysis also indicates the surprising result that profits for the entrant firm are not
monotone in its capacity. We also study, in addition to the pricing strategy, a branding model where the incumbent has an additional mechanism to compete with the entrant. Interestingly, the model shows that the incumbent would invest in improving consumer’s preference for its product while also increasing the price in order to deter the potential entrant.
Chapter 3

Optimal Design of Channel Effort

3.1. Introduction and Literature Review

3.1.1 Motivation

To survive in today’s more intense competition, firms are constantly developing new technologies to improve their product design and production process. Consumers, while welcoming these new products, often find themselves lack the knowledge to fully enjoy these more sophisticated products and are in need of customer support more than ever. Kelly, a friend of one of the authors, was in such a situation recently. After moving into a new apartment, Kelly needed to buy some furniture to fill in her new cozy rooms. Kelly was considering buying furniture at Wal-Mart, Home Depot, or Target. Although price is a big factor, Kelly was also concerned about assembling the furniture after she bought it. Even if she could find the instructions from the manufacturer’s website, Kelly expected that she might need a few phone calls or even visit the store again for assistance after her purchase.

Seeking after-sales support not only is a hassle for consumers and reduces consumer’s valuation for the product, but also adds burden to firms’ operations as more resource needs to be allocated to the after-sale customer support function. Recognizing this, many firms are trying to educate their consumers by providing more detailed product information before purchase. Home Depot has thousands of how-to videos on its website to demonstrate how to use, assemble, or install its products. These
videos are made and managed by Home Depot and complement manufacturer’s user manuals by providing audio and video introduction and illustration of products sold in the store. In Kelly’s case, after watching some of the videos, her concern about the possible hassle was largely alleviated because she had a better idea now about how complex the assembly job would be. She finally decided to purchase the product carried by Home Depot.

Other examples abound in firms’ marketing activity to educate and support consumers. The Micro Center, a large computer and electronics retailer, offers free walk-in technical support for a wide range of topics and issues such as hardware troubleshooting, software configuration, and upgrade. Samsung sets up Samsung mini-stores in Best Buy. Samsung employees are sent out there to assist with product demonstration, basic product services, Samsung account set-up, warranty registration and after-sales support (Faulkner 2013).

As the above examples show, pre-sales effort to educate consumers can be provided either by retailers or by manufacturers. The pre-sales effort of making and uploading thousands of how-to videos by Home Depot is a good example of the retailer investing in the pre-sales effort. Samsung employees working in Best Buy is an example of manufacturer providing pre-sales efforts to educate consumers. On the other hand, the after-sales support can also be provided either by retailers, as in the example of Micro Center, or by manufacturers, as in the example of Samsung.

Motivated by these examples, in this chapter, we explore different service channel structures in providing pre-sales effort (to educate consumers) and after-sales support (to assist consumers in using products properly). We consider a single manufacturer-retailer dyad selling a single product to consumers. Based on the anecdotal evidence, we consider three different service channels: (a) the retailer investing in pre-sales effort and providing after-sales support (Model RR), (b) the manufacturer investing in pre-sales effort and providing after-sales support (Model MM), and (c) the retailer investing in pre-sales effort but the manufacturer providing the after-sales support (Model RM). We model the hassle experienced by consumers when seeking help with
using the product by an inconvenience cost. The probability of incurring the inconvenience cost is assumed to be dependent on the service provider’s pre-sales effort level. We model the operational cost of providing after-sales support by a handling cost. These allow us to capture the negative effect of selling to less-informed consumers on both the consumers’ valuation as well as firms’ profit margin. We are interested in the following research questions: How does the service channel structure affect the performance of the channel members and the whole channel? Does the retailer or the manufacturer have more incentive to invest in pre-sales effort? How are the prices and consumer demand affected?

The results of our study demonstrate that among the three service channels, RR is the best design in terms of the channel members’ profit as well as consumer demand. In the RR model, since the manufacturer is not responsible for providing either service, he is willing to set a low wholesale price. This allows the retailer to charge a low selling price and invest more in the pre-sales effort. The double marginalization problem is minimal in this model. Our analysis also shows that the performance of the RM model can be the worst under some condition. The misalignment of incentives to reduce the amount of consumers seeking after-sales support put the RM channel at a disadvantage in creating consumer demand and investing in pre-sales effort.

In the above three service channel structures, the pre-sales effort is executed by different channel members, i.e., either by the manufacturer in the MM model or by the retailer in the RR and RM models. However, anecdotal evidence shows that manufacturers and retailers may also collaborate in pre-sales effort. For example, more than 1,200 Best Buy employees staffed Microsoft’s stores in Best Buy and received training to sell Microsoft products (Microsoft.com 2013). In an extension to the basic models, we consider a cost-sharing arrangement between the manufacturer and retailer, with the manufacturer agreeing to share a fraction of the retailer’s investment in pre-sales effort. Surprisingly, we find that the retailer, although with some of the pre-sales effort cost off her shoulder, can end up being worse off if she has to enter the cost-sharing arrangement.

The rest of this chapter is organized as follows. In section 3.1.2, we briefly discuss
the related literature. In section 3.2, we introduce the model, and in section 3.3, we analyze three different channel models and compare the performance of the three basic models. In section 3.4, we extend the analysis to consider a model of cost-sharing in pres-sales effort. And finally, in section 3.5, we conclude and discuss future research. All proofs are presented in the Appendix B.

3.1.2 Literature Review

Three streams of literature are relevant to our study: the literature on demand-enhancing pre-sales effort, the literature on service provision in channels, and the literature on collaboration between channel members. We will discuss each stream in detail and highlight the differences and contribution of our research in this section.

The first stream investigates the impact of selling effort on consumer’s demand, a topic that has been widely studied in the marketing and operations management literature. The selling effort that has been looked at ranges from activities taken by sales force in general (see for example Bhardwaj 2001, Gurnani et al. 2010) to specific forms such as franchisee’s service input (Desai and Srinivasan 1995), in-store merchandizing, advertising, and using point-of-sale information (Krishnan et al. 2004). In our models, the specific selling effort we focus on is in the form of educating consumers by providing product-related information, product demonstration and etc. More importantly, most papers in this literature assume a direct effect of selling effort on demand and model the effect by an aggregate measure. The demand function is usually a defined function that is decreasing in the selling effort. Different from these papers, consumer demand in our models is derived from utility theory. Selling effort (pre-sales effort specifically) affects demand indirectly through its impact on consumer’s utility. In particular, we consider the situations where activities taken by channel members in educating consumers can lower their likelihood of seeking support after the purchase. This would result in a lower expected inconvenience cost for consumers when they make the purchase decision, and therefore would boost the demand. Our research enriches the literature by modeling this more fundamental effect of selling effort. In
this aspect, our work is closely related to Ofek et al. (2011). In their model, consumers may make costly product returns after their purchase. Similar to our model, consumer’s valuation is affected by the probability of making a return and retailers can invest in store assistance effort to reduce this probability. However, there are significant differences in the model setting and research focus. First, we consider a manufacturer-retailer dyad in which the wholesale price is endogenously chosen by the manufacturer. Therefore, in our model the manufacturer may influence the effort level indirectly through wholesale price. Ofek et al. (2011) does not include upstream seller in their model and cannot capture this interaction. Second, their paper focuses on competing retailers that can operate both a physical store and an online store and examines how the presence of online stores affects retailers’ pricing strategy and effort levels. Our research has a very different focus. We study various service channel designs that lead to different ways of providing pre-sales and after-sales service. Our focus is on comparing the effectiveness of these service channel structures.

The second stream of literature are the papers that study service provision in marketing channels. The concept of service is captured as a generalization of all kinds of non-price factors. Such services include provision of product information, free repair, advertising (Iyer 1998), pre-sales advice, and after-sales service (Desiraju and Moorthy 1997; Tsay and Agrawal 2000). This literature can be divided into two groups. In one group, researchers focus on either services that are provided prior to consumer’s purchase (papers reviewed in the first stream) or those after the purchase (Kim et al. 2007). Our work is more closely related to papers in the other group that consider both types of services. The biggest distinction between our research and those in this literature is that in our model the cost of providing after-sales services is dependent on the effort level put in the pre-sales service. Therefore, unlike most other papers that model the two types of services as independent to each other, our model captures the interaction between them.

Furthermore, most existing papers in this group usually assume that these services are provided by a single channel member. Thus they have not studied the problem of service channel design which is the focus of this research. By explicitly modeling these
two types of services, our model allows different channel members to take different roles in providing these services. In particular, motivated by anecdotal evidence, we consider three service channel designs, one in which the retailer provides both types of service, one in which the manufacturer provides both, and the third one in which the retailer provides pre-sales services and the manufacturer provides after-sales services. Furthermore, in our model, these two types of services are related to each other in the sense that the pre-sales service affects the need for after-sales service, and therefore has an impact on the cost incurred for the latter.

Lastly, our research is also related to a stream of literature on cooperation between channel members. Various contract forms have been studied, for example buyback (Pasternack 1985; Padmanabhan and Png 1997; Taylor 2002), revenue sharing (Cachon and Lariviere 2005), quantity discount (Jeuland and Shugan 1983, 1988; Ingene and Parry 1995; Raju and Zhang 2005), and quantity flexibility contract (Tsay and Lovejoy 1999; Milner and Rosenblatt 2002). In this research, a cost-sharing contract, in which the manufacturer shares a portion of the retailer’s cost in providing pre-sales effort, is considered. Other papers have also looked at similar mechanisms, for example, Chao et al. (2009) studies a model of cost-sharing in total recall cost, Tang et al. (2014) considers a model of cost-sharing in supply reliability improvement. The emphasis of this literature has been largely focused on designing contracts that can achieve channel coordination and lead to various channel profit allocation between channel members. The contract terms, for example, the cost-sharing portion, are chosen by a central planner (or negotiated between the channel members). In our model, however, the cost-sharing decision is endogenously set by the Stackelberg leader, the manufacturer. To the opposite of what is known in the coordination literature, we show that in such a setting the cost-sharing contract may actually hurt the retailer and the whole channel when the manufacturer is very powerful, because the retailer may over-invest in the selling effort. We also point out that a simple two-part tariff can help achieve channel coordination in our model.
3.2. Model Formulation

We assume that the market under consideration has a two-echelon channel structure and we model it as a bilateral monopoly between a single manufacturer and a single retailer. The manufacturer produces and sells a single product through the retailer. The manufacturer incurs a unit production cost \( c \) and charges a wholesale price \( w \). The retailer sells the product at a unit price \( p \). The consumers are heterogeneous in their valuation of the product. We denote the consumption value by \( v \), and assume that it is uniformly distributed within the consumer population between 0 to 1.

We model an information-intensive product. As motivated by the examples in Section 1, for such a product, consumers may need some after-sales support to use it properly. Seeking after-sales support often takes some time and resource, for example, calling customer support, traveling to the store, or reaching out to the manufacturer. In some cases, consumers may even feel frustrated when they are in such a situation because they cannot enjoy the product immediately and contacting customer support may not always be smooth. Due to these factors, we introduce an inconvenience cost \( r \) that captures any disutility to consumers when seeking after-sales support. We assume that prior to purchase, all consumers have the same information about the product, and form an expectation of the likelihood of seeking after-sales support after purchase. We denote this as the base probability \( \alpha \), which is also known to both the manufacturer and retailer. Therefore, the effect of seeking after-sales support on consumers’ valuation is an expected disutility of \( \alpha r \).

Providing after-sales support and dealing with frustrated customers also requires certain resource from the service provider, such as assigning workforce and allocating store space to this function. We introduce a handling cost \( k \) that is incurred by the service provider for helping a consumer. The expected handling cost, therefore, is \( \alpha k \). In practice, the handling cost is usually different for different service providers. Since the focus of this research is to compare the performance of different service channel structures, we assume the same handling cost for both the manufacturer and retailer.
However, the equilibrium results can be easily generalized to allow different handling costs.

The manufacturer or retailer can reduce the likelihood of consumers seeking after-sales support by investing in some pre-sales effort, such as providing more detailed product information and having in-store demonstration. We denote the effort level by \( \lambda \) and assume that \( \lambda \in [0, 1] \). With the effort, the probability that consumers form prior to purchase is then \( \alpha(1 - \lambda) \). Specifically, we assume that when the effort level \( \lambda = 1 \), consumers are fully informed about the product, and hence do not need any additional support when using the product. Taking into account the effort \( \lambda \), consumers’ expected disutility becomes \( \alpha(1 - \lambda)r \), and the after-sales service provider’s expected handling cost becomes \( \alpha(1 - \lambda)k \). We assume that the investment in pre-sales effort has a diminishing impact and we capture this by assuming a convex effort cost function \( h\lambda^2/2 \). Similar to the handling cost, for comparison purpose, we assume the same coefficient \( h \) in the investment function, that is, the cost of pre-sales effort, for both the manufacturer and retailer.

The market demand, \( D \), for the product can be derived from consumers’ utility function. Prior to purchase, a consumer with valuation \( v \) would derive a net consumer surplus of \( v - p - \alpha(1 - \lambda)r \). Thus, all consumers with positive net surplus will purchase the product, and \( D = 1 - p - \alpha(1 - \lambda)r \). Following the rational expectation theory, we assume that consumers’ expected probability of seeking after-sales support equals the fraction of consumers that actually need such support after purchase. Denote the demand for after-sales support by \( T \), and it is given by \( D(1 - \lambda)\alpha \).

In this chapter, we collectively call pre-sales effort and after-sales support as services. Motivated by anecdotal evidence, we consider three different service channel designs: the service channel with the retailer providing both pre-sales effort and after-sales support (Model RR in Figure 3.1), the service channel with the manufacturer providing both pre-sales effort and after-sales support (Model MM in Figure 3.1), and the service channel with the retailer providing pre-sales effort and the manufacturer providing the after-sales support (Model RM in Figure 3.1). We refer to the first two channels as pure service channel and the third one as mixed service channel, since
in the former both pre-sales effort and after-sale support are provided by the same channel member while in the latter they are provided by different channel members. The notations are summarized in Table 3.1.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Explanation</th>
</tr>
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<tbody>
<tr>
<td>$\alpha \in [0, 1]$</td>
<td>Base probability of consumers seeking after-sales support</td>
</tr>
<tr>
<td>$v, v \sim U[0, 1]$</td>
<td>Consumer valuation</td>
</tr>
<tr>
<td>$c \geq 0$</td>
<td>Manufacturer’s unit production cost</td>
</tr>
<tr>
<td>$k$</td>
<td>Cost of providing after-sales support</td>
</tr>
<tr>
<td>$r$</td>
<td>Consumers’ inconvenience cost</td>
</tr>
<tr>
<td>$p$</td>
<td>Unit selling price</td>
</tr>
<tr>
<td>$w$</td>
<td>Unit wholesale price</td>
</tr>
<tr>
<td>$\lambda_R, \lambda_M$</td>
<td>Pre-sales effort level chosen by R or M</td>
</tr>
<tr>
<td>$\frac{h \lambda^2}{2}$</td>
<td>Cost of pre-sales effort</td>
</tr>
<tr>
<td>$D$</td>
<td>Market demand ($D = 1 - p - \alpha(1 - \lambda)r$)</td>
</tr>
<tr>
<td>$T$</td>
<td>The market demand for after-sales support ($T = D(1 - \lambda)\alpha$)</td>
</tr>
</tbody>
</table>

Throughout this chapter, we use superscript $c, RR, MM, \text{and } RM$ to denote
the centralized system, RR model, MM model, and RM model, respectively. We use subscript $R$ and $M$ to denote the retailer and manufacturer, respectively.

### 3.3. Model Analysis

As a benchmark, we first analyze the centralized channel where a single firm produces and sells the product. In this case, both pre-sales and after-sales service are provided by the firm. The centralized channel chooses the selling price and effort level to maximize its expected profit:

$$
\max_{p, \lambda \in [0, 1]} \pi_c(p, \lambda) = \left[1 - p - \alpha(1 - \lambda)r\right] \left[p - c - k\alpha(1 - \lambda)\right] - \frac{h\lambda^2}{2},
$$

(3.1)

where $1 - p - \alpha(1 - \lambda)r$ is the market demand derived from consumers’ utility, $p - c - k\alpha(1 - \lambda)$ is the firm’s profit margin after taking into account the cost of after-sales support, and the last term is the cost of pre-sales effort.

It can be shown that if $2h > (k + r)^2\alpha^2$, the profit function is jointly concave and therefore the first-order conditions are sufficient to characterize the optimal decisions:

$$
p_c = \frac{h(1 + c + k\alpha - r\alpha) - (k + r)(k + cr)\alpha^2}{2h - (k + r)^2\alpha^2}, \quad \lambda_c = \frac{(k + r)\alpha(1 - c - k\alpha - r\alpha)}{2h - (k + r)^2\alpha^2}
$$

(3.2)

The centralized system’s optimal profit is $\pi_c = \frac{h(1-c-k\alpha-r\alpha)^2}{4h-2(k+r)^2\alpha^2}$.

To guarantee the feasibility of the solution in (3.2), we impose the following assumptions throughout this chapter:

**Assumption 1:** $2h > (1 - c)(k + r)\alpha$.

**Assumption 2:** $1 - c > (k + r)\alpha$.

Assumption 1 and 2 collectively ensure the joint concavity of the profit function and that the optimal effort level $\lambda_c$, selling price $p_c$ and market demand are within 0 and 1.
3.3.1 Retailer Service Model (RR Model)

In this service model, the retailer is fully responsible for both the pre-sales service and after-sales support. An example of this service channel structure is RadioShack. RadioShack uploads on its website videos such as instruction of using a cell phone, review of a camera’s feature or some product’s test. They also have free expert for after-sales support when you buy a phone or tablet from RadioShack.

We model the interaction between the two firms as a two-stage game, with the manufacturer selecting the wholesale price $w$ in the first stage, and the retailer choosing the selling price $p$ and $\lambda$ in the second stage. We characterize the equilibrium of the game by solving the retailer’s problem first. For a given $w$, the retailer’s optimization problem is given as follows.

$$\max_{p, \lambda \in [0, 1]} : \pi^{RR}_{R}(p, \lambda) = [1 - p - \alpha(1 - \lambda)r][p - w - \alpha(1 - \lambda)k] - \frac{h\lambda^2}{2}.$$  (3.3)

The profit function is jointly concave in $p$ and $\lambda$ by Assumption 1 and 2, and therefore the first-order conditions are sufficient to characterize the optimal best response functions, $p^{RR}(w)$ and $\lambda^{RR}_{R}(w)$. The manufacturer, by taking into account the retailer’s reaction $p^{RR}(w)$ and $\lambda^{RR}_{R}(w)$, solves the following problem:

$$\max_{w} : \pi^{RR}_{M}(w) = (1 - p^{RR}(w) - \alpha(1 - \lambda^{RR}_{R}(w)r)(w - c).$$  (3.4)

Again, by Assumption 1 and 2, the objective function is concave in $w$, and the first order condition yields the optimal wholesale price $w^{RR*} = (1 + c - k\alpha - r\alpha)/2$. We summarize the resulting equilibrium results in Table 3.2. Proofs of the results in Table 3.2 are included in the Appendix B.

3.3.2 Manufacturer Service Model (MM Model)

In this service model, the manufacturer is fully responsible for both the the pre-sales service and after-sales support. This is what Samsung does. Samsung launched its own mini stores within more than 1,400 Best Buy locations. They are staffed by
trained Samsung employees to “assist with product demonstrations, basic product services, Samsung account set-up, warranty registration and post-purchase support”, according to the company.

We study a two-stage game similar to that in the RR model. The leader, the manufacturer, now needs to decide on both the wholesale price $w$ and the pre-sales effort level $\lambda$, and the follower, the retailer selects the selling price accordingly. The retailer’s problem is simply given as:

$$\max_p : \pi_{MM}^R(p) = [1 - p - \alpha(1 - \lambda)r] (p - w) \quad (3.5)$$

and the first-order condition gives the best responses $p_{MM}(w, \lambda) = \frac{[1 + w - r\alpha(1 - \lambda)]}{2}$.

Given $p_{MM}(w, \lambda)$, the manufacturer solves the following problem:

$$\max_{w, \lambda \in [0,1]} : \pi_{MM}^M(w, \lambda) = [1 - p_{MM}(w, \lambda) - \alpha(1 - \lambda)r] \left[ w - c - \alpha(1 - \lambda)k \right] - \frac{h\lambda^2}{2}.$$  \quad (3.6)

The joint concavity of the profit function follows from Assumption 1 and 2. The optimal solution is given by:

$$w_{MM}^* = \frac{2h(1 + c + k\alpha - r\alpha) - (k + r)(k + cr)\alpha^2}{4h - (k + r)^2\alpha^2}, \quad \lambda_{MM}^* = \frac{(k + r)\alpha(1 - c - k\alpha - r\alpha)}{4h - (k + r)^2\alpha^2}.$$

Other equilibrium results are listed in Table 3.2.

In the two decentralized service channels we studied above, both the pre-sales effort and after-sales support are provided by the same channel member. Although these are the most common service channel designs observed in practice, there does exist some situations where the two types of service are provided by different channel members. We will consider such a mixed service channel design next.

### 3.3.3 Mixed Service Model (RM Model)

Mixed service channel designs are observed less often in practice. Between the two possible mixed service channel designs, anecdotal evidence suggests that the RM
structure, in which the retailer invests in the pre-sales effort and the manufacturer provides the after-sales support, is more common than the MR structure (in which the pre-sales effort and after-sales support are provided by the manufacturer and retailer, respectively). For example, some RTA (Ready to Assemble) furniture manufacturers sell furniture through retailers such as Target, BJ’s or ToysRus. The retailers usually demonstrate fully-assembled furniture in the store. This requires the retailers allocate certain store space, have skilled employees to complete the assembling work, and assign knowledgeable sales agent to help potential customers. All these come at the retailer’s cost. However, after customers purchase the furniture, the retailers will not provide any additional help with assembling. Customers have to call the manufacturers’ service center for support.

The two-stage game is the same as that for the RR model, with one change. Here, the cost of providing after-sales support is incurred by the manufacturer, not by the retailer. Therefore, the retailer’s optimization problem is given as:

$$\max_{p, \lambda \in [0,1]} \pi_{RM}^R(p, \lambda) = (1 - p - \alpha (1 - \lambda)r) (p - w) - \frac{h\lambda^2}{2}. \quad (3.7)$$

From the joint concavity of the profit function in $p$ and $\lambda$ (by Assumption 1 and 2), the retailer’s best response function is given as:

$$p_{RM}^R(w) = \frac{h + hw - hr\alpha - r^2 \omega^2}{2h - r^2 \omega^2}, \quad \lambda_{RM}^R(w) = \frac{r\alpha(1 - w - r\alpha)}{2h - r^2 \omega^2}.$$  

Given $p_{RM}^R(w)$ and $\lambda_{RM}^R(w)$, the manufacturer selects the optimal $w$ to maximize its expected profit, i.e.,

$$\max_{w} \pi_{M}^R(w) = [1 - p_{RM}^R(w) - \alpha (1 - \lambda_{RM}^R(w))r] [w - c - \alpha (1 - \lambda_{RM}^R(w)k)] . \quad (3.8)$$

It can be shown that the profit function is concave in $w$ if Assumption 1 and 2 hold. The first-order condition yields the equilibrium wholesale price:

$$w_{RM}^R = \frac{2h(1 + c + k\alpha - r\alpha) - r\alpha^2(k(2 - r\alpha) + r(1 + c - r\alpha))}{4h - 2r(k + r)\alpha^2}.$$
Other equilibrium results are summarized in Table 3.2.

### 3.3.4 Comparison of different service channels

The three decentralized service channels are dominated by the centralized channel in terms of channel profits due to the double marginalization effect. The firm in the centralized channel puts more pre-sales effort and is able to create more demand than does the manufacturer or the retailer in the decentralized channels. However, we focus on the comparison of the three different decentralized channels. Specifically, we examine the impact of different service channel structures on the performance of the channel members as well as on consumer welfare. We first compare the two pure service channels, RR and MM.

**Proposition 3.1.** For the pure service channels RR and MM, the equilibrium prices and effort levels are related as: a) \( p^{RR^*} < p^{MM^*} \) when the effort cost is high (when \( h > \alpha^2 r (k+r) \)), and \( p^{RR^*} > p^{MM^*} \) otherwise; b) \( w^{RR^*} < w^{MM^*} \); and c) \( \lambda^{MM^*}_M < \lambda^{RR^*}_R \).

In the RR model, the manufacturer is responsible for neither pre-sales effort nor after-sales support and thus the manufacturer incurs no cost other than the production cost. Notice that, in this setting, wholesale price is the manufacturer’s only lever. Therefore the manufacturer, with a low cost burden, is able to set a lower wholesale price to boost the demand. As a result, the retailer, with the low wholesale price, is able to respond by setting a lower retail price and at the same time to put more effort into the pre-sales effort to increase the demand. Notice that when the effort cost \( h \) is low, the retailer heavily invests in the pre-sales service. The effect of increasing demand is big enough such that the retailer can take further advantage by charging a higher selling price.

In the MM model, on the other hand, the manufacturer is responsible for providing both pre-sales effort and after-sales support. Therefore it needs to set a higher wholesale price to cover some cost of providing both services, leading to a higher retail price and thus lower demand than the RR model. Also, we see that the MM model has a lower pre-sales effort level than the RR model. Due to double marginalization,
Table 3.2: Equilibrium Expressions in Service Channels

<table>
<thead>
<tr>
<th></th>
<th>Pure Service Channel</th>
<th>Mixed Service Channel</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RR</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w )</td>
<td>( \frac{1}{2} (1+c-k\alpha-r\alpha) )</td>
<td>( 2h(1+c+k\alpha-r\alpha)-(k+r)(k+cr)\alpha^2 )</td>
<td>( {MM, RM} &gt; RR )</td>
</tr>
<tr>
<td>( p )</td>
<td>( \frac{(k+r)\alpha A}{2(2h-B)} )</td>
<td>( \frac{hA^2}{4h-B} )</td>
<td>No perfect sequencing</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>( \frac{1}{2(2h-B)} )</td>
<td>( \frac{hA}{4h-B} )</td>
<td>( RR &gt; {MM, RM} )</td>
</tr>
<tr>
<td>( D )</td>
<td>( \frac{hA\lambda}{4h-(k+r)\alpha(1-c+k\alpha+r\alpha)} )</td>
<td>( \frac{4h-B}{4h-B} )</td>
<td>( RR &gt; {MM, RM} )</td>
</tr>
<tr>
<td>( T )</td>
<td>( \frac{4h\lambda}{4h-B} )</td>
<td>( \frac{hA^2(4h-B)}{4(2h-B)^2} )</td>
<td>No perfect sequencing</td>
</tr>
</tbody>
</table>

| **MM**             |                      |                       |                          |
| \( \pi_M \)        | \( \frac{hA^2}{4(2h-B)} \) | \( \frac{hA^2}{8h-2B} \) | \( RR > \{MM, RM\} \)  |
| \( \pi_R \)        | \( \frac{hA^2}{8(2h-B)} \) | \( \frac{hA^2}{8h-4C} \) | \( RR > \{MM, RM\} \)  |
| \( \pi_T \)        | \( \frac{3hA^2}{8(2h-B)} \) | \( \frac{hA^2}{hA^2(2h-r^2\alpha^2)} \) | \( RR > \{MM, RM\} \)  |

| **RM**             |                      |                       |                          |
| \( w \)            | \( \frac{1}{2} (1+c-k\alpha-r\alpha) \) | \( 2h(1+c+k\alpha-r\alpha)-(k+r)(k+cr)\alpha^2 \) | \( \{MM, RM\} > RR \) |
| \( p \)            | \( \frac{(k+r)\alpha A}{2(2h-B)} \) | \( \frac{hA^2}{4h-B} \) | No perfect sequencing    |
| \( \lambda \)      | \( \frac{1}{2(2h-B)} \) | \( \frac{hA}{4h-B} \) | \( RR > \{MM, RM\} \)  |
| \( D \)            | \( \frac{hA\lambda}{4h-(k+r)\alpha(1-c+k\alpha+r\alpha)} \) | \( \frac{4h-B}{4h-B} \) | \( RR > \{MM, RM\} \)  |
| \( T \)            | \( \frac{4h\lambda}{4h-B} \) | \( \frac{hA^2(4h-B)}{4(2h-B)^2} \) | No perfect sequencing    |

\( A = 1-c-k\alpha-r\alpha \)
\( B = (k+r)^2\alpha^2 \)
\( C = r(k+r)\alpha^2 \)
the manufacturer’s pre-sales effort is only partially reflected in the selling price of the product, thus it reduces the incentive to invest in such effort.

A higher pre-sales effort level in the RR model leads to a higher market demand for the product, as summarized in the following result:

**Proposition 3.2.** For the pure service channels RR and MM, the demand for the product and demand for after-sales support are related as $D_{MM}^* < D_{RR}^*$ and $T_{MM}^* < T_{RR}^*$.

It is interesting to point out that a higher pre-sales effort level in the RR model does not lead to less consumers seeking after-sales support. Although the likelihood of a consumer encountering difficulty and looking for support after purchase is lower in the RR model (as a result of a higher pre-sales effort level), there are more consumers purchasing the product. The joint effect is a higher demand for after-sales support.

**Proposition 3.3.** For the pure service channels RR and MM, the manufacturer’s and the retailer’s profits are related as $\pi_{MM}^* < \pi_{RR}^*$ and $\pi_{MM}^* < \pi_{RR}^*$.

Proposition 3.3 shows that the ranking of the channel member’s profit is the same, both being higher in the RR model and lower in the MM model. This implies that the retailer, who is closer to the end consumers, is more efficient in providing the pre-sales service. Since the retailer is able to act more directly, by both adjusting selling price and investing in pre-sales effort, to affect the underlying demand, the effect of double marginalization is less severe in the RR model.

Although our analysis points to the RR model as a superior service channel design, in terms of both channel members’ higher profit as well as higher consumer demand, the MM channel structure is still observed in practice. Some are used because retailers do not have the required technology know-how to provide the needed services. In some other situations, manufacturers have certain technical or cost advantages in providing these services. As in the example of Samsung, its employees have more expertise knowledge of its products than Best Buy employees. Furniture manufacturers can provide better after-sales support than their retailers, since the retailer do not have detailed information about individual components. Our model can be
easily generalized to capture such differences in service efficiency, by allowing \( h \) and \( k \) to be different for the retailer and manufacturer. As one would expect, the above comparison result may reverse when the manufacturer becomes much more efficient.

Next, we compare the pure service channels with the mix service channel. It turns out that the RR model still stands out as the best service channel structure, as shown in the next result.

**Proposition 3.4.** In comparison of the results between the pure and mixed service channels, we can conclude that: (a) The wholesale price is the lowest in the RR model; (b) The level of pre-sales effort and consumer demand are the highest in the RR model; (c) The manufacturer’s and retailer’s profits are also the highest in the RR model.

We have seen that when the retailer is able to use both the selling price and pre-sales effort to affect the underlying demand, the effect of double marginalization is less severe and the RR model outperforms the MM model. Proposition 3.4 suggests that when the after-sales support is provided by the manufacturer, however, the retailer cannot fully enjoy the benefit. The effect of pre-sales effort in reducing the expected cost of handling troubled customers is retained by the manufacturer. This dampens the retailer’s incentive in investing in the pre-sales service.

**Proposition 3.5.** Comparing the service channels RM and MM, we can conclude that: (a) The manufacturer’s profit is higher in the MM model if and only if \( k > r \); b) The retailer’s profit is higher in the MM model unless both \( k \) and \( h \) are small (when \( k < 2r/3 \) and \( h < h_2 \) where \( h_2 \) is a threshold given in the proof).

Given that the after-sales service is provided by the manufacturer, the manufacturer has a clear preference regarding the pre-sales service: it will take charge only when the inconvenience cost for customers seeking after-sales support is relatively low. It can be shown that both demand and manufacturer’s net margin \((w - c - k(1 - \lambda)\alpha)\) are higher in the MM model, as a result of manufacturer directly investing in the pre-sales effort. However, when the inconvenience cost \( r \) is relatively high, the manufacturer’s incentive in investing in the pre-sales effort also increases, and the investment
cost starts to offset the benefit of higher demand higher return. Giving up the control of the pre-sales service may be a better option in this case.

For the retailer, it can be shown that its net margin \((p - w)\) is higher in the MM model when the inconvenience cost is relatively low (when \(r < k\)). In this case, the retailer has no incentive in taking charge of the pre-sales service because it does not bring in any benefit. Only when the inconvenience cost is sufficiently high, in other words, when the handling cost is sufficiently low, as the manufacturer lowers the wholesale price, the benefit of pre-sales service starts to show up as the net margin increases. However, as the required effort level also increases, the retailer is willing to take charge of the pre-sales service only when the investment cost \(h\) is not very high.

Proposition 3.5 has an interesting implication. Given that the after-sales support is provided by the manufacturer, the two channel members’ preference regarding the pre-sales service may not be aligned. When the inconvenience cost is relatively low compared to the handling cost, both channel members prefer the manufacturer taking the responsibility for the pre-sales service. When handling the after-sales service and providing the pre-sales service are both relatively inexpensive, both channel members prefer the retailer taking charge of the pre-sales service. However, in situations where the handling cost is neither too high nor too low \((2r/3 < k < r)\), or where both the inconvenience cost and investment cost are high \((k < 2r/3 \text{ and } h > h_2)\), neither channel member is willing to provide the pre-sales service.

### 3.4. Collaboration on Pre-sales Effort

In the models studied in §3.3, the pre-sales effort is provided by either the manufacturer or the retailer, and the cost of such effort is born by the service provider alone. As in the Microsoft example mentioned in §3.1.1, it is not uncommon to see in practice that the two channel members collaborate on educating consumers prior to their purchase. Similar examples abound in other industries. In the auto industry because sales associates are key to customer education, manufacturers send trainers to dealerships to keep the sales force informed about the latest models’ features and
specifications. For example, according to reports from J.D.Power Asia Pacific, dealers in India are increasingly collaborating with the automakers toward enhancing the effectiveness of their sales-related efforts, such as product training, guidance in test drives and vehicle display (J.D. Power 2014). In this section, we extend our basic models to analyze such a situation. In particular, we study models in which the pre-sales effort is still determined by the retailer, but the manufacturer agrees to share a portion of the retailer’s cost for such effort. We refer to this as the cost-sharing model in this section.

### 3.4.1 Cost-sharing Model for the RR channel

We first consider a scenario in which the cost of the after-sales support is still born by the retailer in the cost-sharing model. The two-stage game is similar to the one studied in §3.3.1 for the RR model, except that now the manufacturer has an additional decision to make. Specifically, the manufacturer first chooses the wholesale price \( w \) and the cost-sharing portion \( \phi \) (\( \phi \in [0,1] \)). Given these contract terms, the retailer then selects the selling price \( p \) and the pre-sales effort level \( \lambda \). We first restrict ourselves to situations where the manufacturer is very powerful and can enforce a cost-sharing arrangement. In other words, we first ignore the participation constraint that the retailer makes at least her reservation profit, \( \pi^{RR}_R \) (her optimal profit without cost-sharing). Throughout this section, we use the notation (\( \hat{\cdot} \)) to denote variables for the cost-sharing model.

The retailer’s problem is given as:

\[
\max_{p,\lambda \in [0,1]} : \hat{\pi}^{RR}_R(p,\lambda) = [1 - p - \alpha(1 - \lambda)r][p - w - k\alpha(1 - \lambda)] - (1 - \phi)h\lambda^2/2 \quad (3.9)
\]

where the last term represents the retailer’s shared portion of the pre-sales effort cost. Since the joint concavity of the retailer’s profit function cannot be guaranteed and depends on the manufacturer’s choice of \( \phi \), we analyze two sub-problems, one in which retailer’s problem is concave, the other convex. The manufacturer, in choosing the optimal \( w \) and \( \phi \), therefore needs to solve two sub-problems, each corresponding to
a different retailer solution. The ultimate optimal solution is obtained by comparing the manufacturer’s optimal profit in each sub-problem. We refer the detailed analysis to the Appendix B and summarize the results below.

**Proposition 3.6.** In the RR model with cost-sharing,

(a) if $2h > \frac{3(k+r)^2\alpha^2}{2}$: the manufacturer’s equilibrium wholesale price, cost-sharing portion, and profit are:

\[
\hat{w}_1^* = \frac{[8h - 3(k + r)^2\alpha^2][1 - c - (k + r)\alpha]}{16h - 9(k + r)^2\alpha^2} + c, \quad \phi_1^* = \frac{1}{3},
\]

and $\hat{\pi}_{RR,1} = \frac{2h[1 - c - (k + r)\alpha]^2}{16h - 9(k + r)^2\alpha^2}$.

The retailer’s equilibrium selling price, pre-sales effort level, and profit are:

\[
\hat{p}_1^* = 4h(3 + c + k\alpha - 3\alpha) - 3(k + r)\alpha^2[3k + r + 2cr - r(k + r)\alpha],
\]

\[
\hat{\lambda}_{RR,1} = \frac{6(k + r)\alpha[1 - c - (k + r)\alpha]}{16h - 9(k + r)^2\alpha^2},
\]

and $\hat{\pi}_{RR,1}^* = \frac{4h[1 - c - (k + r)\alpha]^2[4h - 3(k + r)^2\alpha^2]}{[16h - 9(k + r)^2\alpha^2]^2}$.

(b) if \( \frac{[1 - c + 2(k + r)\alpha]^2}{8} \leq 2h \leq \frac{3(k + r)^2\alpha^2}{2} \): the manufacturer’s equilibrium wholesale price, cost-sharing portion, and profit are:

\[
\hat{w}_2^* = 1 - (k + r)\alpha, \quad \phi_2^* = 1 - \frac{(k + r)^2\alpha^2}{2h}, \quad \text{and} \quad \hat{\pi}_{RR,2}^* = \frac{[1 - c - (k + r)\alpha]^2(k + r)^2\alpha^2}{8h - 4(k + r)^2\alpha^2}.
\]

The retailer’s equilibrium selling price, pre-sales effort level, and profit are:

\[
\hat{p}_2^* = \frac{4h(1 - r\alpha) - (k + r)\alpha^2[(3 - c)k + (1 + c)r - (k + r)^2\alpha]}{4h - 2(k + r)^2\alpha^2},
\]

\[
\hat{\lambda}_{RR,2}^* = \frac{(k + r)\alpha[1 - c - (k + r)\alpha]}{2h - (k + r)^2\alpha^2}, \quad \text{and} \quad \hat{\pi}_{RR,2}^* = 0.
\]

Proposition (3.6) indicates that the consumer’s inconvenience cost $(r)$ and the cost of providing after-sales support $(k)$ together affect both the retailer’s and the manufacturer’s optimal decisions. We can re-write the threshold value $2h > \frac{3(k+r)^2\alpha^2}{2}$
into \((k+r) < \frac{2\sqrt{3}h_3}{3\alpha}\). When \((k+r)\) is relatively high, the manufacturer has less incentive to share the burden of cost of providing pre-sales effort with the retailer and thus reduces the portion \((\phi)\), that is, \(\phi_2^* < \phi_1^*\). As a result, the retailer has less incentive to put pre-sales effort to increase demand and reduce the number of consumers coming back for after-sales support, that is \(\hat{\lambda}_{R,2}^{RR^*} < \hat{\lambda}_{R,1}^{RR^*}\). With higher portion of the cost of providing pre-sales effort, together with the increase in \(k + r\), the retailer has to increase its selling price. Both effects causes the demand \(D = 1 - p - \alpha(1 - \lambda)r\) decreases.

With the cost-sharing mechanism, the retailer’s cost burden for pre-sales effort is relieved. One would expect that consequently the manufacturer would offer a portion of cost-sharing inducing the retailer would set a higher effort level which may benefit both parties in the channel. Although the manufacturer can indeed benefit from the cost-sharing contract, it is surprising to see that, the retailer will never be better off if the manufacturer enforces such a cost-sharing arrangement.

**Lemma 3.1.** In the RR model with cost-sharing, (a) In equilibrium, the manufacturer is better off but the retailer is worse off, that is, \(\hat{\pi}_M^{RR^*} > \pi_M^{RR^*}\), \(\hat{\pi}_R^{RR^*} < \pi_R^{RR^*}\); (b) The retailer’s profit is the highest when \(\phi = 0\).

It can be shown that when the manufacturer shares a portion of the cost of providing pre-sales service, the retailer indeed is willing to put more effort in such services, i.e., \(\hat{\lambda}_{R}^{RR^*} > \lambda_{R}^{RR^*}\). However, the pre-sales effort acts like a double-edged sword to the retailer. On one hand, a higher effort level increases the revenue for the retailer, as it leads to a higher demand \((\hat{D}^* > D^*)\) and higher effective margin. On the other hand, it also costs more to provide a higher level of service. We see that even when the manufacturer shares part of the cost, the retailer still incurs a higher cost for providing the pre-sales service than before (in RR model without cost-sharing), i.e., \((1 - \phi^*)\frac{h(\lambda^*)^2}{2} > \frac{h(\lambda^*)^2}{2}\). Our result implies that the latter effect dominates the former one, and therefore the retailer is worse off than without cost-sharing. In fact, as in part (b) of Lemma 3.1 points out, this is the case not only at manufacturer’s optimal choice of \(\phi\), but also in any cost-sharing setting.
Our analysis, based on part (b) of Lemma 3.1, implies that in industries where manufacturers (or sellers) are very powerful, the cost-sharing arrangement should be avoided. When manufacturers are able to enforce such an arrangement, retailers will get hurt. As the retailer gets more negotiation power, it will certainly refuse to enter the cost-sharing contract due to Lemma 3.1. In other words, when facing such a retailer, the manufacturer needs to ensure that the retailer gets at least its reservation profit, $\pi_{RR}^*$, the optimal profit it will make without cost-sharing.

**Lemma 3.2.** In the RR model with cost-sharing, the decentralized channel is better off if $2h < \frac{4(k+r)^2\alpha^2}{3}$ and worse off if $2h > \frac{4(k+r)^2\alpha^2}{3}$.

Lemma 3.2 implies that the cost-sharing mechanism benefits the entire channel only when $h$ is relatively low (i.e., when $2h < \frac{4(k+r)^2\alpha^2}{3}$) and the problem of retailer over-investing in pre-sales effort is less severe. In this case, a Pareto improvement outcome can be achieved, i.e., each party makes a higher profit than without cost-sharing. A fixed payment can be used to allocate the channel surplus between the two parties.

When $h$ is relatively high, i.e., $2h > \frac{4(k+r)^2\alpha^2}{3}$, however, the cost-sharing mechanism actually hurts the entire channel. This is somewhat counter to our intuition, as one would expect that the cost-sharing mechanism is most helpful when the effort cost is high. Our results indicate that as the manufacturer takes a dominant position in the channel and is utilizing the cost-sharing mechanism to optimize his own profit, his gain is always at a cost of his channel partner, the retailer. When the problem of retailer over-investing is more severe, the manufacturer’s gain cannot offset the retailer’s loss, and as a result the entire channel is worse off.

Figure 3.2 is based on a numerical example with parameters given as $\alpha = 0.25$, $c = 0.2$, $k = 0.2$, and $r = 0.1$. When $h = 0.0375$, $\pi_{RR}^* = \hat{\pi}_{RR}^*$ at $\phi' = 0.3316$. If it were to have an optimal central planner, it would make the manufacturer choose some value of $\bar{\phi}$ between 0 and 0.3316 and the channel profit would be maximized with this portion of cost-sharing.
3.4.2 Cost-sharing Model for the RM channel

In our cost-sharing model, the cost of the after-sales support could also be born by the manufacturer. In this scenario, the two-stage game is similar to the one studied in §3.3.3 for the RM model, except that the manufacturer has an additional decision to make. Specifically, the manufacturer first chooses the wholesale price $w$ and the cost-sharing portion $\phi$ ($\phi \in [0, 1]$). The retailer’s objective function is given by:

$$\max_{p, \lambda \in [0, 1]} : \hat{\pi}_R^{RM}(p, \lambda) = [1 - p - \alpha(1 - \lambda)r](p - w) - (1 - \phi)\frac{h\lambda^2}{2}. \quad (3.10)$$

Since the retailer does not bear the cost of after-sales support, its margin is adjusted to $p - w$. Similar to the case where the retailer bears the cost of the after-sales support, the joint concavity of the retailer’s profit function depends on the manufacturer’s
choice of $\phi$. The manufacturer, by choosing the optimal $w$ and $\phi$, solves the following objective function:

$$\max_{w, \phi} \hat{\pi}_{RM}^{M}(w, \phi) = [1 - p - \alpha(1 - \lambda)r][w - c - k\alpha(1 - \lambda)] - \phi \frac{h(\lambda)^2}{2}.$$  \hfill (3.11)

It can be shown that, by eliminating the boundary solutions, two interior solution are feasible and are given in the following result.

**Proposition 3.7.** In the RM model with cost-sharing,

(a) if $2h > \frac{r(2k + 3)\alpha^2}{2}$: the manufacturer’s equilibrium wholesale price, cost-sharing portion, and profit are:

$$\hat{w}^* = \frac{8h[1 + c + (k - r)\alpha] - [2k + r(1 + 2c - \alpha r)](2k + 3r)\alpha^2}{16h - (2k + 3r)^2\alpha^2}, \quad \phi^* = \frac{2k + r}{2k + 3r},$$

and $\hat{\pi}_{RM}^{M,1} = \frac{2h[1 - c - (k + r)\alpha]^2}{16h - 9(2k + 3r)^2\alpha^2}$.

The retailer’s equilibrium selling price, pre-sales effort level, and profit are:

$$\hat{p}^* = \frac{4h(3 + c + k\alpha - 3r\alpha) - (2k + 3r)\alpha^2[2k + r(1 + 2c - \alpha r)]}{16h - (2k + 3r)^2\alpha^2};$$

$$\hat{\lambda}_{R}^* = \frac{2(2k + 3r)\alpha[1 - c - (k + r)\alpha]}{16h - (2k + 3r)^2\alpha^2},$$

and $\hat{\pi}_{RM}^{R,1} = \frac{4h[1 - c - (k + r)\alpha]^2[4h - r(2k + 3r)\alpha^2]}{[16h - \alpha^2(2k + 3r)^2]^2}$.

(b) if $\frac{|1 - c + (2k + r)\alpha|^2}{8} \leq 2h \leq \frac{r(2k + 3)\alpha^2}{2}$: the manufacturer’s equilibrium wholesale price, cost-sharing portion, and profit are:

$$\hat{w}^* = 1 - r\alpha, \quad \phi^* = 1 - \frac{r^2\alpha^2}{2h}, \quad \text{and} \quad \hat{\pi}_{RM}^{M,2} = \frac{r^2\alpha^2[1 - c - (k + r)\alpha]^2}{8h - 4r(2k + r)\alpha^2}.$$

The retailer’s equilibrium selling price, pre-sales effort level, and profit are:

$$\hat{p}^* = \frac{4h(1 - \alpha r) + \alpha^2 r[3k + r\alpha - (1 + c) r - 4k]}{4h - 2\alpha^2 r(2k + r)};$$
\[ \hat{\lambda}_R = \frac{r\alpha[1 - c - (k + r)\alpha]}{2h - r(2k + r)\alpha^2}, \quad \text{and} \quad \hat{\pi}_{RM,2} = 0. \]

The structure in the RM model with cost-sharing is similar to the RR model with cost-sharing. We have seen that in the RR model, under certain circumstances the cost-sharing mechanism may not bring benefit to the entire channel members. It is worthwhile to point out that this is not the case for the RM model. When the after-sales support is provided by the manufacturer, collaborating in the pre-sales service may indeed create a win-win situation for both channel members, even with a powerful manufacturer. Specifically, we find that the manufacturer’s optimal choices of \( w \) and \( \phi \) may lead to a higher profit for the retailer and the entire channel. Therefore, the retailer is willing to accept such a contract when she has a choice.

**Lemma 3.3.** In the RM model with cost-sharing, (a) In equilibrium, the manufacturer is always better off, that is, \( \hat{\pi}_{RM}^* > \pi_{RM}^* \); (b) The retailer is better off, as long as the cost of after-sales support, compared to the inconvenience cost \( r \), is high enough, that is, if \( k > \frac{r}{2} \), then \( \hat{\pi}_{R}^* > \pi_{R}^* \).

Since the retailer is not responsible for providing after-sales service, she does not see the benefit of a reduced effective handling cost as a result of her pre-sales effort, and therefore does not have as much incentive to set a high effort level. Therefore, with a cost-sharing arrangement, it is possible that the retailer does not over-invest like in the RR channel.

However, when the cost of after-sales support \( k \) is small, the optimal cost-sharing portion \( \phi_{RM}^* = \frac{2k + r}{2k + 3r} \), which is close to \( \frac{1}{3} \). So in the RM model with cost-sharing, the benefit of the retailer from relieving the burden of after-sales support is not salient and thus the retailer might be worse off. The following lemma shows that unlike the RR model with cost-sharing, the retailer in the RM model might be better off.

**Lemma 3.4.** In the RM model with cost-sharing, the retailer’s profit is not the highest when \( \phi = 0 \).

Unlike Lemma 3.1, which shows that the retailer will get hurt in the cost-sharing setting, Lemma 3.4 indicates that the retailer can be better off with the cost-sharing
in the RM channel. Compared to the cost-sharing portion \( \phi^{RR^*} = \frac{1}{3} \) in the RR channel, the cost-sharing portion \( \phi^{RM^*} = \frac{2k+r}{2k+3r} \) in RM channel is higher, that is, the manufacturer in the RM channel is more willing to share the pre-sales effort cost with the retailer than in the RR channel. The incentive of the manufacturer to be more willing to share with the retailer the portion of the pre-sales effort is that the manufacturer bears the cost of after-sale support and therefore it wants to encourage the retailer to put more pre-sales effort to reduce the number of people coming back for after-sales support. As a result, the retailer in the RM channel has more incentive to put pre-sales effort compared to the retailer in the RR channel, that is, \( \hat{\lambda}^{RR^*} < \hat{\lambda}^{RM^*} \). Therefore, unlike the retailer in the RR channel, where it incurs a higher cost for providing the pre-sales service with cost-sharing than without cost-sharing, the retailer in the RM channel, is possible to incur a lower cost for providing the pre-sales service than without cost-sharing, i.e., \( (1-\phi^*) \frac{h(\hat{\lambda}^*)^2}{2} < \frac{h(\hat{\lambda}^*)^2}{2} \) under some conditions. As a result, the retailer in the RM channel, getting higher demand and higher margin with cost-sharing compared to without cost-sharing, might be better off.

However, the retailer can still be worse off. As we can see from Proposition 3.7, under some condition, the retailer with the cost-sharing is worse off with zero profit.

**Remark 1:** a) If \( k < \frac{(\sqrt{3}-1)r}{2} \) and \( 2h < \frac{r(2k+3r)a^2}{2} \), then the retailer is worse off; b) If \( \frac{(\sqrt{3}-1)r}{2} < k < \frac{r}{2} \), the retailer is better off when \( h \) is low and is worse off when \( h \) is high.

### 3.5. Concluding Remarks

Building on a single manufacturer-retailer dyad selling a single product to consumers, we model three different service channel structures in providing pre-sales effort and after-sales support. Our results show that RR channel is the best design in terms of the channel member’s profit. This is because in the RR channel, the retailer, who sets the selling price and effort level, is closer to the consumers and thus the impact of the double marginalization on the profit is less.
We also extend our study to cost-sharing models and show that in the RR model, the retailer is always worse off with the cost-sharing contract. However, in the RM model, the retailer can be better off with the cost-sharing contract. We find that using a cost-sharing contract studied above, the manufacturer can never induce the retailer to choose an effort level and selling price that maximizes total channel profit. To achieve coordination, a simple two-part tariff can be used by the manufacturer. In the RR model, the manufacturer can set the wholesale price at cost and a fixed payment to be made by the retailer, i.e., \( w = c \) and \( F = \pi^c - \pi^{RR^*}_R \). In the RM model, the manufacturer needs to offer a wholesale price that is contingent on the retailer’s effort level as well as a fixed payment, i.e., \( w = c + k\alpha(1 - \lambda) \) and \( F = \pi^c - \pi^{RM^*}_R \). In both cases, the fixed payment \( F \) is used to distribute the efficiency gains. Since the manufacturer is the Stackelberg leader, it takes all the gains and only leave to the retailer its reservation profit.
Chapter 4

Product Differentiation through Information Provision

4.1. Introduction and Literature Review

4.1.1 Problem Background

From the time she started the application for MBA programs in graduate school, Ella has been thinking about her new adventure in the schools. After going through the long application procedure, Ella has received several offers from a few top MBA schools. However, getting into MBA programs to build up her career was just the beginning. But there, sitting at the top of her to-do list after all these months, is one important decision she has to make: accepting which offer. Getting an MBA from the right school is a way of landing a better job and of accelerating a career but finding the right school is not just about looking at rankings, or choosing one considered as “prestigious”. The key to make the right choice is to find a school that is most fit to Ella in such factors as classroom atmosphere, relevance of the curriculum, course style (case method vs. lecture method), student community and so forth. Her father has narrowed the list down to two alternatives. The tuitions of the two schools are pretty close. If she could just visit each school, she would have some of the information she needs to make her decision. Fortunately, one of the two schools indeed asks her a campus visit and she can even attend some MBA classes to experience the real MBA program.
Ella’s decision is obviously not only affected by the schools’ tuitions, but also by
the probability of her fitting into the schools. In particular, Ella’s decision is also
affected by whether the school provides Ella some information by allowing an campus
visit.

This chapter presents a model of price competition in which a buyer is uncertain
about a product, and the two sellers, after price commitment, may allow the buyer to
learn about their value through a pre-purchase information revelation. Many other
examples abound in this setting of the game.

The T-Mobile allows consumers to experience the nation-wide 4G LTE network for
7 days by sending them an iPhone 5s. If a consumer doesn’t like the network, he or she
just needs to return the iPhone to his or her nearest T-Mobile retail store. Verizon’s
Take Me Home program allows customers borrow smartphone such as Galaxy S3 to
try out on Verizon’s network free for a limited time. Like the T-Mobile program,
Verizon customers unhappy with the service are free to return their smartphones to
the company without any fees.

Both Sony and Samsung open home-theater shops inside the Best Buy stores to
allow consumers to try their products. The “Sony Experience at Best Buy” areas
will showcase Sony televisions, including its new 4K Ultra HD TVs, as well as media
players, and the Sony PlayStation 4. Samsung ’s home theater shops will highlight
the company’s Curved UHD TVs.

In this chapter we incorporate strategic information provision into a model of si-
multaneous price competition. As in the simplest case of Bertrand’s classic model,
two ex ante undifferentiated sellers make simultaneous price offers to a buyer, inter-
ested in purchasing at most one of the sellers’ products. Unlike Bertrand’s model,
however, the buyer is uncertain about her valuation for each product: if it matches
her style, taste or needs she is willing to pay up to a known value, but if it is a
mismatch she would not purchase the product at any price. The sellers and buyer
share a commonly known prior that each product is a match; because the buyer’s
uncertainty is about the match between her tastes and the products, neither firms
nor buyers initially have private information about the buyer’s valuation. However,
after making price commitments, each seller can allow the buyer to inspect his good, thereby endowing her with private information about her valuation for his product. These decisions are made independently and simultaneously by sellers. After acquiring the information offered by the seller, the buyer decides which good, if any, to purchase. We characterize the sub-game perfect equilibria of this game.

The rest of this chapter is organized as follows. In section 4.1.2, we briefly discuss the related literature. In section 4.2 and 4.3, we introduce the basic setting of the model and model analysis. And finally, in section 4.4, we briefly conclude this chapter. All proofs are presented in the Appendix C.

4.1.2 Literature Review

Two streams of literature are relevant to our study: the literature on price and information provision strategy of a monopoly, and the literature on the interaction of information provision in competition setting.

The first stream of literature investigate the affect of information provision from a monopoly in consumer’s purchase decision. Lewis and Sappington (1994) study a model to show when it is optimal for the seller to provide no additional information versus fully informed information to consumers. In their model, the seller allows buyers to get information about their value for a product prior to purchase and then use the information for price discrimination. Ottaviani and Prat (2001) consider the incentive for a monopolist to reveal information to the buyer, and they conclude that a full information revelation is the seller’s optimal solution. Bergemann and Pesendorfer (2007) study a model of optimal information provision for a monopolistic seller and show that the seller has no incentive to provide information with a single buyer. We build on this literature stream by studying two firms’ information provision decision within the setting of price competition. We show that in the competitive setting, information provision takes place with non-zero probability either in a form of mixed strategy or pure strategy.

There is also a growing literature on the strategic information provision and interaction in competition setting (see for example Shaked and Sutton 1982, Iyer et al.
2005, Meruer and Stahl 1994 and Davis et al. 1995). The literature closest to ours is Moscarini and Ottaviani (2001). They consider price competition when buyers learn about their value for a product prior to purchase. However, anecdotal examples in introduction and many other cases show that the firms decision is usually first to set prices before provide information to consumers. As a result, in contrast to Moscarini and Ottaviani’s analysis, we study a game in which two firms first commit to price and then decide whether they will provide their product information to the buyer. To our best knowledge, ours is the first model to consider such a setting based on many real world examples. Kuksov and Lin (2010) examine the optimality of information provision in a competitive setting. Unlike the assumption in their paper, however, our assumption is that one firm’s information provision would not resolve the uncertainty about the buyer’s valuation for the other firm’s product.

4.2. Model Formulation

We consider a market interaction between two firms, indexed by \( j \in \{A, B\} \) and a single buyer, \( B \). The firms’ products are in direct competition: the buyer would be willing to purchase at most one of them. The buyer is initially uncertain about how well each product matches her taste or needs. Her valuation for each product, denoted \( v_j \), is therefore uncertain, and neither she nor the firms have private information about it. Firms and the buyer believe that \( v_j \) is an independent realization of a Bernoulli random variable, where \( Pr(v_j = 1) = q \), and \( Pr(v_j = 0) = 1 - q \).

The game takes place in three stages. In the first stage, firms simultaneously post prices \( (p_0, p_1) \). In the second stage, firms observe prices and then choose whether to permit the buyer to inspect the good. If firm \( j \) provides information, then the buyer learns her value for good \( j \). If no information is provided, the buyer acquires no additional information about her value for good \( j \). ¹ Her beliefs are therefore unchanged. In the third stage, the buyer chooses which good, if any, to purchase.

¹Because firms do not possess private information about the buyer’s value, not providing information does not signal private information to the buyer.
Her *ex post* utility from purchasing good \( j \) is simply \( u_j = v_j - p_j \). If she is uncertain about her value for good \( j \) (because the firm did not allow her to inspect the good) her expected utility from purchasing good \( j \) is \( E[u_j] = q - p_j \). If she purchases neither good, the buyer’s payoff is 0. If indifferent between buying and not buying, we assume that the consumer chooses to buy the product; if indifferent between buying product A and product B, we assume the consumer randomizes fairly.

**4.3. Model Analysis**

As is standard we proceed by backwards induction.

**4.3.1 Stage Three: Purchase Decision**

The buyer purchases whichever good offers her a higher expected surplus, provided this surplus is positive. Thus, if \( V_j \in \{0, 1, q\} \) represents her expected valuation for product \( j \) at the purchase stage, she will purchase good \( j \) for certain if and only if

\[
V_j - p_j > V_i - p_i \quad \text{and} \quad V_j - p_j \geq 0. \tag{4.1}
\]

She will randomize fairly between products if and only if

\[
V_j - p_j = V_i - p_i \quad \text{and} \quad V_j - p_j \geq 0 \tag{4.2}
\]

and will buy neither product otherwise. Obviously the buyer will never purchase at a price greater than 1; hence, only prices in the unit square are relevant for the analysis.

It is most straightforward to understand the buyer’s behavior using a series of figures. Figure 4.1 summarizes the buyer’s equilibrium purchase decision if both firms allow the buyer to learn her match quality before purchase. In this case, the price of both goods is relevant only if both goods match. In this instance the buyer will purchase the good that is less expensive. If only one good matches, the buyer purchases it provided the price does not exceed one.
Figure 4.2 describes the buyer’s purchase decision if A reveals but B does not. If $\rho_B \leq q$ then the buyer is willing to purchase $B$, rather than forego it. Thus if
product A is a mismatch she will purchase B under this condition. In addition, if A is significantly more expensive, she will prefer to purchase B even if she matches with product A. Thus will happen whenever $1 - p_A < q - p_B \Rightarrow p_A - p_B > 1 - q$. If this condition fails, then the buyer purchases A if she matches, and if she doesn’t, evaluates B in isolation.

Figure 4.3 shows the buyer’s purchase decision if neither firm reveals. The buyer purchases the product from the firm charging the lower price, provided this price is below $q$.

4.3.2 Stage Two: Information Provision

At the information provision stage firms take the previous period’s prices as given, and anticipate an optimal purchase decision by the buyer following the outcome of information provision. Because prices are inherited from the first stage, in the second stage they are treated as exogenous. Therefore, let these prices be $(p_H,p_L)$ and consider first the case $p_H > p_L$. Two results are immediate. First, if $p_H - p_L > 1 - q$ then information provision is irrelevant, and the buyer purchases the less expensive
product.

\[ p_H - p_L > 1 - q \Rightarrow \pi_H = 0 \text{ and } \pi_L = p_L \quad (4.3) \]

Second, if a firm’s price is above expected quality, then it has a dominant strategy to provide information for certain. This implies that whenever \( p_L > q \) both firms provide information for certain. The low price firm makes the sale if it matches; otherwise the high price firm makes the sale if it matches.

\[ p_L > q \Rightarrow \pi_H = q(1 - q)p_H \text{ and } \pi_L = qp_L \quad (4.4) \]

Two possible combinations remain. In both \( p_H - p_L < 1 - q \) so that information provision are relevant to the buyer’s decision. In the first \( p_H < q \), so that both prices are low. In the second, one price is low and the other high \( p_L < q < p_H \). In case one \( (p_H < q \text{ and } p_H - p_L < 1 - q) \), the information provision game can be represented by the following matrix game:

\[
\begin{array}{cc}
R & N \\
\hline
R & q(1-q)p_H,qp_L & qp_H,(1-q)p_L \\
N & (1-q)p_H,qp_L & 0,p_L \\
\end{array}
\quad (4.5)
\]

In this game, firm H is the row player while firm L is the column player. Obvious best responses are marked by stars in the payoff matrix. It is trivial to verify the following equilibrium for Case I:

- If \( q \leq \frac{1}{2} \) then the unique pure strategy equilibrium is firm H reveals and firm L does not reveal; \( \pi_H = qp_H \) and \( \pi_L = (1-q)p_L \)

- If \( q > \frac{1}{2} \) then no pure strategy equilibrium exists. The unique mixed strategy equilibrium is for the high price firm to reveal with probability \( r_H = \frac{1-q}{q} \) and for the low price firm to reveal with probability \( r_L = \frac{q}{1-1+q^2} \); \( \pi_H = \frac{q^2-q^3}{1-1+q^2}p_H \) and \( \pi_L = qp_L \)

Next, consider the second case, in which \( p_L < q < p_H \) and \( p_H - p_L < 1 - q \). Here
the inspections game is described by the following matrix game:

\[
\begin{array}{cc}
R & N \\
R & q(1 - q)p_H, qp_L & qp_H, (1 - q)p_L \\
N & 0, qp_L & 0, p_L
\end{array}
\] (4.6)

As described previously, the high price firm has a dominant strategy to reveal. The low price firm’s best response thus depends on the probability that the product is a match. It is trivial to verify the following equilibrium for Case II:

- If \( q \leq \frac{1}{2} \) then the unique pure strategy equilibrium is for firm H to reveal and for firm L to not reveal; \( \pi_H = qp_H \) and \( \pi_L = (1 - q)p_L \).
- If \( q > \frac{1}{2} \) then the unique pure strategy equilibrium is for both firms to reveal; \( \pi_H = q(1 - q)p_H \) and \( \pi_L = qp_L \).

One significant result immediately arises: in equilibrium, information provision take place with non-zero probability. When the option to differentiate products through information provision exists, firms anticipate utilizing this option with positive probability for any combination of first period prices. Thus, while firms may be undifferentiated ex ante, at the purchase stage a non-zero probability of differentiation always exists. Thus, the assumption that firms that are undifferentiated ex ante remain so after setting pricing is inconsistent with a model in which firms can provide information to differentiate their product from a competitor’s.

By then reflecting the payoffs across the diagonal, we find the Figure 4.4 and Figure 4.5, which represent the expected payoff of the firm with price on the \( x \) axis, as a function of both prices. We summarize the firm payoffs in the Figure 4.4 and Figure 4.5.

4.3.3 Stage One: Prices Decisions

In this section we describe equilibrium price setting. The lemma 4.1 helps us to characterize mixed strategy equilibrium of the pricing, in which we find that no symmetric pure strategy equilibrium exists.
Lemma 4.1. No symmetric pure strategy equilibrium exists in pricing stage.

By slightly undercutting the anticipated price a firm’s market share increases
discontinuously. Such deviations are always profitable. A symmetric equilibrium, if it exists, must be in mixed strategies. Despite the discontinuous nature of the payoff function in prices, we characterize the mixed strategy equilibrium of the pricing stage in closed form.

The qualitative nature of the equilibrium is in four cases depending on the value of \( q \). Define \( q_{ML} \) as the unique real root of the following polynomial:

\[
t_{ML}(q) \equiv 1 - 2q + q^2 - q^3
\]

Although \( q_{ML} \) can be expressed analytically, the expression is not illuminating. Numerically \( q_{ML} \approx 0.56984 \). Also \( t_{ML}(q) > 0 \iff q < q_{ML} \). Similarly, define \( q_{MH} \) as the unique real root of the following polynomial:

\[
t_{MH}(q) \equiv 1 - 2q + 2q^2 - 2q^3
\]

Although \( q_{MH} \) can be expressed analytically, the expression is not illuminating. Numerically \( q_{MH} \approx 0.64780 \). Also \( t_{MH}(q) > 0 \iff q < q_{MH} \).

The four cases for the equilibrium are

- Case L : \( q \in (0, 1/2] \)
- Case ML : \( q \in (1/2, q_{ML}] \)
- Case MH : \( q \in (q_{ML}, q_{MH}] \)
- Case H : \( q \in (q_{MH}, 1] \)

We describe each case in turn.

**Proposition 4.1.** *(Equilibrium L).* In case L, each firm setting price equal to the realization of random variable \( P_L \) with cumulative distribution function \( F(p) = \frac{p-(1-q)}{pq} \) with support on \([1-q, 1] \) constitutes a symmetric mixed strategy Nash equilibrium. No other symmetric mixed equilibrium with connected support exists. Equilibrium firm profit is equal to \( q - q^2 \).
Figure 4.6 shows the case of Equilibrium L when $q \in (0, 1/2]$. When the probability of a match is low, firms play a ‘go for broke’ strategy, always revealing information and charging high prices. With a low match probability, a firm has an opportunity to make a sale only if its product is a match, thus when a firm sets prices it is only concerned with the case in which the consumer matches its product. Because the probability of a match is low, it is unlikely that the consumer will also match the other product. Hence, each firm anticipates having significant, but not perfect market power. In order to combat this strategy by its competitor, a firm might want to drop its price below $q$ while withholding information, hoping to cater to consumers that do not match with the competitor or even all consumers. However, because $q$ is low, this strategy is not profitable. Paradoxically, when the probability that the consumer likes the product is low, both firms always reveal information and set high prices. Firms earn significant positive profits in this equilibrium. The profits are greater than the firms’ profits in the absence of information provision but with a perfect ability to coordinate on price (which would generate a profit of $q/2$).
As for the consumer and social surplus, we observe that because prices are always greater than $1 - q$ (and thus both are greater than $q$) both firms always reveal information on the equilibrium path. Hence, the consumer will purchase one of the products, unless both are a mismatch. Thus expected social surplus is equal to the probability that the consumer matches at least one of the products

$$S_L = 2q - q^2. \tag{4.9}$$

Because prices act only as transfers between the buyer and the firm, realized social surplus is always the sum of realized consumer surplus and firm profits. As a result, expected social surplus is also the sum of expected consumer surplus and expected firm profit. Therefore, expected consumer surplus is simply the difference between expected social surplus and firm profit:

$$C_L = 2q - q^2 - 2(q - q^2) = q^2 \tag{4.10}$$

**Proposition 4.2.** (Equilibrium ML) In case ML, each firm setting price equal to the realization of random variable $P_{ML}$, a mixture of two random variables, is an equilibrium. With probability $\theta = (1 - q)^{1 - q/\theta} \frac{1 - q}{q}$ the cumulative distribution function of $P_{ML}$ is $F_T(p) = (1 - q)^{\frac{p - pq - q^2}{p - ML(q)}}$ with support on $[\frac{q^2}{1 - q}, \frac{1 - q}{q}]$. With probability $1 - \theta$, the cumulative distribution function of $P_{ML}$ is equal to $F_B(p) = \frac{pq^2 - q + 2q^2 - q^3}{(2q - 1)p}$ with support on $[\frac{(1 - q)^2}{q}, q]$. Equilibrium firm profit is equal to $(1 - q)^2$.

When the probability of match is greater than $1/2$ but not significantly greater, the ‘go for broke’ strategy described above is no longer profitable. Downward deviations to prices below $q$ become profitable when played against the ‘go for broke’ strategy. In equilibrium, firms set prices according to a mixed strategy with support in two disconnected intervals: one strictly above $q$ and the other from something less than $q$ up to $q$. The two mixed pricing strategy in the two disconnected intervals represent the firms’ attitude toward their competition strategy. On the one hand, since the probability of match is not too high, firms still consider themselves having some
market powers and thus want to charge a high price and at the same time to reveal information to the buyer as they do when probability of match is low (less than 1/2).

On the other hand, however, the two firms are not as confident in their market power as they do in the case of Equilibrium L and thus start to be willing to charge a relatively lower price and hold some information with each other.

As for the expected social surplus, we observe that when at least one firm sets a price in the upper interval, which happens with probability $2\theta - \theta^2$, both firms always provide information and expected social surplus is $2q - q^2$, as before. When both firms price are in the lower interval, information provision takes place according to a mixed strategy: the low price firm provides information with probability $r_L = \frac{q}{1 - q + q^2}$ while the high price firm provides information with probability $r_H$. However, when both firms’ price are in the lower interval, expected social surplus is thus given by the following expression:

$$r_H r_L (2q - q^2) + r_H (1 - r_L) (q + (1 - q)q) + r_L (1 - r_H) (q + (1 - q)q) + (1 - r_H)(1 - r_L)q,$$

Figure 4.7: Case ML: $q \in (1/2, q_{ML}]$
which gives us
\[ \frac{(1 - q)^4 + q}{1 - q + q^2}. \] (4.11)

Hence, the expected social surplus and consumer surplus in this equilibrium are given by
\[ S_{ML} = (2\theta - \theta^2)(2q - q^2) + (1 - \theta)^2 \frac{(1 - q)^4 + q}{1 - q + q^2}, \] (4.12)
and
\[ C_{ML} = S_{ML} - 2(1 - q)^2 \] (4.13)

**Proposition 4.3.** (Equilibrium MH). In case MH, each firm setting price equal to the realization of random variable \( P_{MH} \) with cumulative distribution function \( F(p) = \frac{p(1-q+q^2)-(q-q^2)}{pq} \) with support on \([\frac{q-q^2}{1-q+q^2}, q]\) constitutes a symmetric mixed strategy Nash equilibrium. Equilibrium firm profit is equal to \( q \frac{q-q^2}{1-q+q^2} \).

Figure 4.8: Case MH: \( q \in (q_{ML}, q_{MH}] \)

When the probability of a match is even higher, both firms give up the strategy of charging a high price. They start to engage into a tougher price competition and be more willing to hold information. The game is starting to collapse into the
Bertrand price competition. The expected social surplus are \( S_{MH} = \frac{(1-q)^4 + q}{1-q+q^2} \) and 
\[ C_{MH} = \frac{(1-q)^4 + q}{1-q+q^2} - 2q \frac{q-q^2}{1-q+q^2}. \]

**Proposition 4.4.** *(Equilibrium H).* In case H, each firm setting price equal to the realization of random variable \( P_H \) with cumulative distribution function \( F(p) = \frac{(1-q) + q^2 - (1-q)^2}{pq^2} \) with support on \([\frac{(1-q)^2}{q^2}, t_{MH}(q)+q^2] \) constitutes a symmetric mixed strategy Nash equilibrium. Equilibrium firm profit is equal to \( \frac{(1-q)^2}{q} \).

Figure 4.9: Case H: \( q \in (q_{MH}, 1] \)

With a high match probability, a firm has very weak market power and thus has an opportunity to make a sale only if its price is cheaper. Therefore when a firm sets prices it is more concerned with the possibility of having a lower price. Firms will hold their information and charge a very low prices to compete with each other. The expected social surplus are \( S_H = \frac{(1-q)^4 + q}{1-q+q^2} \) and 
\[ C_H = \frac{(1-q)^4 + q}{1-q+q^2} - 2 \frac{(1-q)^2}{q}. \]
4.4. Conclusions

This chapter studies a simultaneous price competition model where two ex-ante undifferentiated sellers who want to differentiate themselves by information provision make simultaneous price offers to a buyer. The buyer is uncertain about her valuation for each product and that each seller can provide information of the product to the buyer. We characterize four mixed strategy equilibrium of pricing. We find that when the probability of a match is low, firms will adopt a ‘go for broke’ strategy, that is, they always provide information and charge high prices. We also find that when the probability of a match increases, the game is more collapsing to the simplest case of Bertrand classic model. In addition, we show that while firms may be undifferentiated ex ante, at the purchase stage a non-zero probability of information provision always exists.
Chapter 5

Conclusions

In this dissertation, we first propose a price competition model where we study the entry-deterrence problem of an incumbent firm who faces competition from a capacitated entrant firm. The incumbent must choose its pricing and branding strategies in order to block, deter, or accommodate the entrant. When the entrant’s capacity is low, the threat of entry is low. However, when the entrant’s capacity is higher than a critical threshold, the incumbent has to lower price to stop entry. Finally, allowing entry is optimal strategy when the entrant’s capacity size is sufficiently high, and beyond another threshold value, the capacitated problem switches to the uncapacitated case.

The study considers two pricing competition models and we show that profits for both firms is higher in Stackelberg competition than in Nash competition. Surprisingly, the entrant, as follower, is also better off in Stackelberg competition because of reduced price competition resulting in higher prices and higher quantities sold. We show that in Stackelberg competition, the capacitated entrant sells fewer products in its own market but more in the incumbent’s market than an uncapacitated entrant firm. Our analysis also indicates the surprising result that profits for the entrant firm are not monotone in its capacity. While higher capacity for the entrant increases potential for competition for the incumbent and lower profits for it, as the entrant cannot credibly commit to selling less quantity in the incumbent’s market, the price competition becomes intense leading to lower profits for the entrant firm as well.
In the branding model, in addition to the pricing strategy, the incumbent has an additional mechanism to better compete with the entrant. Interestingly, our results show that in order to deter the potential entrant, the incumbent would invest in improving consumer’s preference for its product while also increasing the price. This is in contrast to the results when only the pricing strategy is available.

In future study, researchers can extend our models to a broader setting where the incumbent also has limited capacity. Such a setting requires a two-dimensional analysis of capacity thresholds for both firms. Another extension to our work is the case when the entrant also has the option of investing in consumers’ preference in the incumbent’s market. In this case, consumer’s preference is a decision variable for both firms. Finally, the problem when the entrant firm’s capacity size is private information would be another interesting problem for future research.

In the second part of this dissertation, by building on a single manufacturer-retailer dyad selling a single product to consumers, we model three different service channel structures in providing pre-sales effort and after-sales support. Our results show that RR channel is the best design in terms of the channel member’s profit. This is because in the RR channel, the retailer, who sets the selling price and effort level, is closer to the consumers and thus the impact of the double marginalization on the profit is less.

Based on anecdotal examples, we further extend our research to cost-sharing models. We show that in the RR model, counter-intuitively, the retailer is always worse off with the cost-sharing contract. However, in the RM model, the retailer can be better off with the cost-sharing contract. We find that using a cost-sharing contract, the manufacturer can never induce the retailer to choose an effort level and selling price that maximizes total channel profit. To achieve coordination, a simple two-part tariff can be used by the manufacturer.

In future study, researchers can extend our models to an asymmetric information setting where the manufacturer cannot observe the retailer’s pre-sales effort level in the RR model and vice versa in the RM model. Another interesting problem is to
study the impact of competition on the channel by allowing two manufacturers who compete each other for a retailer.

In the last part of this dissertation, we study a price competition model in a information revelation setting. Two ex-ante undifferentiated sellers want to differentiate themselves by revealing product information and make simultaneous price commitment to a buyer. The buyer is uncertain about her valuation for each product. We show that while firms may be undifferentiated ex ante, at the purchase stage a non-zero probability of information provision always exists. We characterize four mixed strategy equilibrium of pricing and we find that when the probability of a match is low, firms will adopt a ‘go for broke’ strategy. In addition, we also find that when the probability of a match increases, the price competition is more intense and our model is collapsing to the simplest case of Bertrand model. In future study, researchers can consider a setting where there are two sellers with two substitute products. Another extension to our model is to consider a sequential pricing game where one of the two sellers first sets its price and then the other seller post its price before revealing information to the buyer.
Appendix A

Proofs for Chapter 2

Proofs of Results in Stackelberg and Nash Competition in Uncapacitated Case

Since the results of the unlimited capacity case will be used to obtain the threshold values \( k_3^u \) and \( k_3^s \) and to prove the other conclusions, we first derive some results of the uncapacitated case which will be used in the rest of the proofs.

(a) Uncapacitated Case under the Stackelberg Competition

In the Stackelberg competition, we solve the following objective function with respect to \( p_1^{us} \) and \( p_t^{us} \):

\[
\max_{p_1^{us}, p_t^{us}}: \pi_1^{us} = (p_1^{us} - c_1)(a_1 - p_1^{us}) + (p_t^{us} - t - c_1)((1 - \mu)a_2 - p_t^{us} + \theta p_2^{us}) \tag{A.1}
\]

\[
p_1^{us} = \frac{\alpha_1 + c_1}{2}, \quad p_t^{us}(p_2^{us}) = \frac{c_1 + t + \alpha_2 + p_2^{us}\theta - \alpha_2\mu}{2} \tag{A.2}
\]

Then substituting \( p_t^{us} \) into \( \pi_2^{us} = (p_2^{us} - c_2)(\mu\alpha_2 - p_2^{us} + \theta p_t^{us}) \), we get:

\[
\max_{p_2^{us}}: \pi_2^{us} = (p_2^{us} - c_2)(-p_2^{us} + \alpha_2\mu + \frac{1}{2}\theta(c_1 + t + \alpha_2 + p_2^{us}\theta - \alpha_2\mu)) \tag{A.3}
\]

From the first order condition for \( p_2^{us} \), we get

\[
p_2^{us} = \frac{-2c_2 - c_1\theta - t\theta - \alpha_2\theta + c_2\theta^2 - 2\alpha_2\mu + \alpha_2\theta\mu}{2(-2 + \theta^2)}.
\]
Then we can obtain \( p_t^{us} \), \( p^{us}_2 \), \( q_t^{us} \), \( q^{us}_2 \), \( \pi_1^{us} \) and \( \pi_2^{us} \) respectively (see Table 2.2) with
\[
\pi_1^{us} = \frac{(4c_1 + 4t - 4\alpha_2 - 2c_2\theta - 3c_1\theta^2 - 3t\theta^2 + \alpha_2\theta^2 + c_2\theta^2 + 4\alpha_2\mu - 2\alpha_2\theta\mu - \alpha_2\theta^2\mu)^2}{16(2 - \theta^2)^2} + \frac{(\alpha_1 - c_1)^2}{4} \quad (A.4)
\]

(b) Uncapacitated Case under the Nash Competition

In the Nash competition, we solve the following objective function \( \pi_1^{un} = (p_1^{un} - c_1)(\alpha_1 - p_1^{un}) + (p_2^{un} - t - c_1)((1 - \mu)\alpha_2 - p_2^{un} + \theta p_2^{un}) \) with respect to \( p_1^{un} \) and \( p_2^{un} \). (Notice that \( p_1^{un} \) is the constant monopoly price). We solve the first order condition of \( \pi_2^{un} = (p_2^{un} - c_2)(\mu\alpha_2 - p_2^{un} + \theta p_2^{un}) \) with respect to \( p_2^{un} \). Simultaneously, we solve \( p_2^{un}(p_1^{un}) \) and \( p_1^{un}(p_2^{un}) \) to get \( p_1^{un} \) and \( p_2^{un} \). Therefore, we obtain \( \pi_1^{un} \)
\[
\pi_1^{un} = \frac{(2\alpha_2 + c_2\theta + c_1(-2 + \theta^2) + t(-2 + \theta^2) - 2\alpha_2\mu + \alpha_2\theta\mu)^2}{(4 - \theta^2)^2} + \frac{(\alpha_1 - c_1)^2}{4} \quad (A.5)
\]
and \( \pi_2^{un} \) as given in Table 2.3.

(c) Profit Comparison between Stackelberg and Nash Competition with Unlimited Capacity

From parts (a) and (b), \( p_t^{us} - p_t^{un} = q_t^{us} - q_t^{un} = \frac{\theta^3(c_1\theta + t\theta + \alpha_2\theta + c_2(-2 + \theta^2) + 2\alpha_2\mu - \alpha_2\theta\mu)}{4(8 - 6\theta^2 + \theta^4)} \). As such, the sales amounts for the entrant in the incumbent’s market is higher under the Stackelberg competition. Interestingly, \( p_t^{us} - p_t^{un} > 0 \) is also true for \((c_1\theta + t\theta + \alpha_2\theta + c_2(-2 + \theta^2) + 2\alpha_2\mu - \alpha_2\theta\mu) > 0\), which must hold so that \( q_t^{us} > 0 \). Since in both models, the entrant sells the same quantities at same prices in its own market, we conclude that \( \pi_1^{us} > \pi_1^{un} \). That is, with unlimited capacity, the entrant is better off in the Stackelberg competition.

For the incumbent,
\[
\pi_2^{us} = \frac{(c_1\theta + t\theta + \alpha_2\theta + c_2(-2 + \theta^2) + 2\alpha_2\mu - \alpha_2\theta\mu)^2}{8(2 - \theta^2)} \quad (A.6)
\]
and
\[
\pi_2^{un} = \frac{(c_1\theta + t\theta + \alpha_2\theta + c_2(-2 + \theta^2) + 2\alpha_2\mu - \alpha_2\theta\mu)^2}{(4 - \theta^2)^2} \quad (A.7)
\]
Since \( 8(2 - \theta^2) - (4 - \theta^2)^2 = -\theta^4 < 0 \), we can conclude that \( \pi_2^{us} > \pi_2^{un} \). That is, the incumbent is also better off with the Stackelberg game under unlimited capacity.
Proof of Proposition 2.1

Let us first define capacity threshold values as:
\[ k_1^s = \frac{2\alpha_1(1-\theta^2)-2\alpha_2+\alpha_2\theta^2+2\alpha_2\mu-\alpha_2\theta\mu-\alpha_2^2\mu-\alpha_2\theta(1-\theta^2)+2(1-\theta^2)}{4(1-\theta^2)} \]
\[ k_2^s = \frac{\alpha_1(8-7\theta^2)-8\alpha_2+3\alpha_2\theta^2+8\alpha_2\mu-4\alpha_2\theta\mu-3\alpha_2^2\mu-\alpha_2\theta(4-3\theta^2)+t(8-7\theta^2)}{2(8-7\theta^2)} \]
\[ k_3^s = \frac{(2-\theta^2)(t+\alpha_1+3\alpha_2\theta+c_2(-4+3\theta^2)+4\alpha_2\mu-3\alpha_2\theta\mu)-4\sqrt{(8-10\theta^2+3\theta^4)(-2\alpha_2+c_1\theta+t\theta+c_2\theta^2+2\alpha_2\mu-\alpha_2\theta\mu)}\sqrt{(-2\alpha_2+c_1\theta+t\theta+c_2\theta^2+2\alpha_2\mu-\alpha_2\theta\mu)^2}}{2\theta(2-\theta^2)} \]

To begin our proof, we first solve the best response of the entrant to incumbent’s price \( p^*_2 \). The entrant will set its prices according to the following objective function under capacity constraint:

\[
\max_{p_1^*, p_2^*} \pi_1^e = (p_1^* - c_1)(\alpha_1 - p_1^*) + (p_2^* - t - c_1)((1 - \mu)\alpha_2 - p_2^* + \theta p_2^*) , \tag{A.8}
\]

\[ s.t. \ [\alpha_1 - p_1^*] + [(1 - \mu)\alpha_2 - p_2^* + \theta p_2^*] \leq k. \tag{A.9} \]

Using Lagrange relaxation and solving the problem with respect to \( p_1^* \) and \( p_2^* \), we get

\[ p_1^*(p_2^*) = \frac{-2k - t + 3\alpha_1 + \alpha_2 + p_2^*\theta - \alpha_2\mu}{4} , \tag{A.10} \]

\[ p_2^*(p_2^*) = \frac{-2k - t + \alpha_1 + 3\alpha_2 + 3p_2^*\theta - 3\alpha_2\mu}{4} . \tag{A.11} \]

Substituting \( p_1^*(p_2^*) \) and \( p_2^*(p_2^*) \) into \( q_1^e \) given in (2), we get

\[ q_1^e(p_2^*) = \frac{2k - t - \alpha_1 + \alpha_2 + p_2^*\theta - \alpha_2\mu}{4} . \tag{A.12} \]

Clearly, the entrant will sell any product in the incumbent’s market if and only if the above equation returns a strictly positive value. Otherwise, the entrant will only operate in its own local market. Next, we turn to the incumbents pricing problem and find its monopoly price. By maximizing equation (2.5) with respect to \( p_2^m \), we get the optimal price and profit as \( p_2^m = \frac{c_2-c_2\theta^2+\alpha_2(\theta+\mu-\theta\mu)}{2-\theta^2} \) and \( \pi_2^m = \frac{(c_2(-1+\theta^2)+\alpha_2(\theta+\mu-\theta\mu))^2}{4(1-\theta^2)} \).

We note that the condition \( p_2^m - c_2 > 0 \) should be satisfied, which implies that \( \alpha_2 > \frac{c_2-c_2\theta^2}{\theta+\mu-\theta\mu} \) or \( c_2(-1+\theta^2)+\alpha_2(\theta+\mu-\theta\mu) > 0 \) (This condition will also be used for proofs of other propositions). Clearly, at this price if \( q_1^e(p_2) \leq 0 \) then the incumbent
does enjoy monopoly in its market. It is straightforward to deduce that for $p_2 = p_2^m$, this condition holds if and only if $k \leq k_1^s$, where $k_1^s$ is given above.

When the entrant’s capacity is above $k_1^s$, it will be better off by entering the incumbent’s market under $p_2 = p_2^m$. Inferring, this strategy, the incumbent will be compelled to lower its price either to prevent the entrant from entering the market or to compete with the entrant in the market. To keep the entering off, the incumbent determines its price under the constraint $q_t^2(p_2) \leq 0$, which basically will be binding for $k > k_1^s$. After substituting $p_t^s$ given in (A.11) into (2.5), we maximize the incumbent’s objective function under this constraint and get the following optimal price:

$$p_2^{m'} = \frac{-2k + t + \alpha_1 - \alpha_2 + \alpha_2 \mu}{\theta}. \quad (A.13)$$

Then, substituting the above price into the incumbent’s monopoly profit function, we get the following optimal modified monopoly profit for the incumbent:

$$\pi_2^{m'}(k) = \frac{(2k - t - \alpha_1 + \alpha_2 + c_2 \theta - \alpha_2 \theta)(t + \alpha_1 - 2k)(1 - \theta^2) - \alpha_2 + \alpha_2 \mu - \alpha_2 \theta \mu}{\theta^2}. \quad (A.14)$$

On the other hand, to compete with the entrant post-entry, the incumbent maximizes the competition profit given by (2.9). In this case, after substituting $p_t^s$ given in (A.11) into (2.9), we maximize the incumbent’s objective function and get the following optimal price:

$$p_2^s = \frac{4c_2 - 2k \theta + t \theta + \alpha_1 \theta + 3\alpha_2 \theta - 3c_2 \theta^2 + 4\alpha_2 \mu - 3\alpha_2 \theta \mu}{2(4 - 3\theta^2)}. \quad (A.15)$$

Substitute $p_2^s$ back into equation $p_t^s$, $p_2^s$, $\pi_2^s$, and $\pi_1^s$ we get

$$\pi_2^s = \frac{(-2k \theta + t \theta + \alpha_1 + 3\alpha_2 \theta + c_2(-4 + 3\theta^2) + 4\alpha_2 \mu - 3\alpha_2 \theta \mu)^2}{64 - 48\theta^2}, \quad (A.16)$$

and $p_t^s$, $p_2^s$ and $\pi_1^s$ as given in Table A.1.

Next we compare the incumbent’s profits under modified monopoly in the Effectively Impeded Entry region and post-entry competition in Inffectively Impeded
Entry region. Noting that the condition $\alpha_2 > \frac{c_2 - c_2^2}{\theta + \mu - \theta \mu}$ should be satisfied, we get $[\pi_2^{m'}(k)]'' = 8(1 - \frac{1}{\theta}) < 0$ and $[\pi_2^s(k)]'' = \frac{\theta^2}{8 - 6\theta^2} > 0$. Therefore $\pi_2^{m'}(k)$ and $\pi_2^s(k)$ are concave quadratic function and convex quadratic function respectively as demonstrated in Figure A.1.

![Figure A.1: Incumbent’s Profit Curves](image)

It is obvious to see that $\pi_2^{m} > \pi_2^{m'}(k)$ if $k < k_1^s$, which is the monopoly region. Although, in general $\pi_2^{m'}(k) < \pi_2^s(k)$, from (A.12) and (A.15), we observe that competition leads to positive sales for the entrant only if $k > k_2^s$. Furthermore, we note that at $k = k_2^s$, $\pi_2^{m'}(k) = \pi_2^s(k)$ implying that above this threshold the incumbent lets the entrant enter into the market and switches from entry deterrence to the market competition strategy.

Letting $\pi_2^s(k) = \pi_2^{us}$ and solving for $k$, we get $k_2^s$ as defined above. Hence, we have the threshold value $k_3^s$, beyond which the capacitated case switches to the uncapacitated case because the incumbent’s profit would be lower than the profit from the uncapacitated case where it will set price at $p_2^{us}$. After some cumbersome mathematical operations we can show that $k_2^s < k < k_3^s$. Consequently, we conclude that
\[ k_2^* < k < k_3^* \] is the competition region with limited capacity and \( k > k_3^* \) with unlimited capacity.

### Table A.1: Equilibrium Expressions in Stackelberg Game in Competition Region

\[
\begin{align*}
\pi_1^s &= \frac{1}{32(4-3\theta)^2} \left[ 4(4k^s)^2 \right] + \frac{1}{32(4-3\theta)^2} \left( 2(4k^s)^2 \right) - (t_1 + \alpha_1) (8-3\theta^2) \left( (1-\mu) + 4\theta \mu \right) \\
\pi_2^s &= \frac{1}{32(4-3\theta)^2} \left[ 4(4k^s)^2 \right] + \frac{1}{32(4-3\theta)^2} \left( 2(4k^s)^2 \right) - (t_1 + \alpha_1) (8-3\theta^2) \left( (1-\mu) + 4\theta \mu \right) \\
q_1^s &= \frac{8(\alpha_1 - \alpha_2 + t) + \alpha_2(8 - \theta(4k^s + 4)) \mu - 4c - 2\theta(\theta - \theta)(\alpha_1 - \alpha_2 + \alpha_2) + t_1 + 2k(8 - \theta)}{8(4-3\theta)^2} \\
q_2^s &= \frac{8(\alpha_1 - \alpha_2 + t) + \alpha_2(8 - \theta(4k^s + 4)) \mu - 4c - 2\theta(\theta - \theta)(\alpha_1 - \alpha_2 + \alpha_2) + t_1 + 2k(8 - \theta)}{8(4-3\theta)^2} \\
p_1^s &= \frac{4(4\alpha_1 - 2\alpha_2 + 2\theta - 2t) - 2(8 - \theta) \left( (1 - \alpha_2 - 3\alpha_2 - \mu) \right) - \alpha_2(8 - \theta(4k^s + 4)) \mu}{8(4-3\theta)^2} \\
p_2^s &= \frac{4(4\alpha_1 - 2\alpha_2 + 2\theta - 2t) - 2(8 - \theta) \left( (1 - \alpha_2 - 3\alpha_2 - \mu) \right) - \alpha_2(8 - \theta(4k^s + 4)) \mu}{8(4-3\theta)^2} \\
\end{align*}
\]

**Proof of Lemma 2.1**

We first observe that the entrant’s equilibrium profit function is concave in \( k \) \((\pi_1^s(k))'' = -\frac{64 - 80\theta^2 + 23\theta^4}{4(4-3\theta)^2} < 0\). Letting \( \pi_1^s(k)' = 0 \) and solving for \( k \), we get \( k^* \). As such, at \( k^* \), \( \pi_1^s(k) \) reaches its maximum value. Furthermore, it is straightforward to observe that the equilibrium profit function is decreasing at \( k = k_3^* \), implying from concavity that \( k_3^* > k^* \). By substituting \( k^* \) into \( p_2^s(k) \), we can obtain \( p_2^s(k^*) \), which is the incumbent’s price at \( k^* \). Clearly, this result indicates that as the entrant uses more of its capacity beyond \( k^* \) it enjoys diminishing profits at equilibrium. So does the incumbent. As such, if the entrant would not deviate from \( k^* \) then it would keep its price at \( p_2^s(k^*) \) even if the entrant has additional available capacity. However, as its best response, the entrant would deviate by taking advantage of the incumbent’s decision and improve its profits by utilizing the additional capacity under the incumbent’s price fixed at \( p_2^s(k^*) \), which in turn actually degrades the incumbent’s profits as a result of lost sales. Consequently, the incumbent would be compelled to deviate from \( p_2^s(k^*) \) by decreasing its price eventually leading back to the equilibrium conditions given in Table A.1. Consequently, under perfect information, the entrant’s capacity beyond \( k^* \) will result in lower equilibrium profits for both players. This completes the proof.
Proof of Proposition 2.2

For \( k = k_3^s \), the entrant’s sales quantity can be calculated by substituting \( k_3^s \) into \( q_1^* \) given in Table A.1. Let \( q_1^*(k_3^s) \) denote this amount and let \( W = \frac{\alpha_1 - c_1}{2} - q_1^*(k_3^s) \). We can rewrite \( W \) as

\[
W = \frac{B}{2\theta} \left( \sqrt{(64 - 80\theta^2 + 25\theta^4)} - 1 \right)
\]

where \( B = c_1\theta + t\theta + \alpha_2\theta + c_2(-2 + \theta^2) + 2\alpha_2\mu - \alpha_2\theta\mu > 0 \). It is straightforward to see that \( W > 0 \) since \( \theta < 1 \). Consequently, we show that \( q_1^*(k_3^s) < \frac{\alpha_1 - c_1}{2} \), where the right hand side denotes the uncapacitated sales for the entrant in its own market.

Under the uncapacitated case where \( k > k_3^s \), the total quantity that the entrant sells is \( Q_k^u = q_k^u - q_k^u = 4t^u + \frac{\alpha_1 - c_1}{2} = \frac{-4\alpha_1 - 4\alpha_2 - 2\alpha_2\theta + 2\alpha_1\theta^2 + \alpha_2\theta^2 + c_2\theta^3 + c_1(8 - 5\theta^2) + t(4 - 3\theta^2) + 4\alpha_2\mu - 2\alpha_2\theta\mu - \alpha_2\theta^2\mu}{4(-2 + \theta^2)} \).

Letting \( V = k_3^s - Q_k^u \), we get \( V = \frac{B(8 - 5\theta^2) - \sqrt{(64 - 80\theta^2 + 24\theta^4)}}{8\theta - 4\theta^3} \), which also can be easily shown to be strictly positive. Therefore, total quantities that the entrant sells under the capacitated case is in fact higher than under the uncapacitated case. This concludes the proof of the first part of the proposition.

To prove the second part we first know from Lemma 2.1 that \( k^* > k_3^s \) and that the entrant’s equilibrium profit function is decreasing when \( k > k^* \).

Now suppose there exists a value \( k = k_\lambda \) that makes the \( q_1^*(k_\lambda) = \frac{\alpha_1 - c_1}{2} \) (solve \( q_1^*(k) = \frac{\alpha_1 - c_1}{2} \) for \( k \) and we have \( k_\lambda \)). It is easy to verify that \( q_1^*(k) \) is an increasing function. The first part of Proposition 2.2 shows that \( q_1^*(k_3^s) < \frac{\alpha_1 - c_1}{2} \) and therefore \( k_\lambda > k_3^s \). We thus get \( \pi_1^*(k_\lambda) < \pi_1^*(k_3^s) \).

Substituting \( k_\lambda \) into \( p_t^*(k) \) and \( q_t^*(k) \), we have \( p_t^*(k_\lambda) \) and \( q_t^*(k_\lambda) \). \( p_t^*(k_\lambda) - p_t^{us} = \frac{\theta^3(B)}{4(2 - \theta^2)(8 - 5\theta^2)} > 0 \) as \( B = (c_1\theta + t\theta + \alpha_2\theta + c_2(-2 + \theta^2) + 2\alpha_2\mu - \alpha_2\theta\mu) > 0 \). Therefore, \( \pi_1^{us} < \pi_1^*(k_\lambda) < \pi_1^*(k_3^s) \). This completes the proof of the second part of Proposition 2.2.

Proof of Proposition 2.3

The proof can be carried out using the similar approach used for the proof of
Proposition 2.1 and thus the details will be omitted. Under Nash competition the threshold capacity values are given below:

\[
k_1^n = \frac{2\alpha_1(1-\theta^2)-2\alpha_2+\alpha_2\theta+2\alpha_2\mu-\alpha_2\theta\mu-\alpha_2\theta^2\mu-c_2\theta(1-\theta^2)+2(1-\theta^2)}{4(1-\theta^2)}
\]

\[
k_2^n = \frac{2\alpha_2+c_2\theta+t(-2+\theta^2)+\alpha_1(-2+\theta^2)-2\alpha_2\mu+\alpha_2\theta\mu}{2(-2+\theta^2)}
\]

\[
k_3^n = \frac{-4\alpha_1+4\alpha_2-2\alpha_2\theta+\alpha_1\theta^2+c_1(8-3\theta^2)-2t(-2+\theta^2)+4\alpha_2\mu-2\alpha_2\theta\mu}{2(-2+\theta^2)}
\]

It is easy to verify that \(k_2^n - k_1^n = \frac{\theta_1^3(c_2(-1+\theta^2)+\alpha_2(\theta+\mu-\theta\mu))}{4\theta^4-3\theta^2+2} > 0\) as \(c_2(-1+\theta^2) + \alpha_2(\theta+\mu-\theta\mu) > 0\) (\(\alpha_2 > \frac{c_2-\theta^2}{\theta+\mu-\theta\mu}\)). Noting that \(q_{11}^n = \frac{E_1}{4-\theta^2} > 0\) (See Table 2.3), we get \(k_3^n - k_2^n = \frac{8-3\theta^2}{2(8-\theta^2+\theta^4)} > 0\) as \(E > 0\). Therefore, \(k_1^n < k_2^n < k_3^n\). The equilibrium outcomes are given in Table A.2.

| Table A.2: Equilibrium Expressions in Nash Game in Competition Region |
|------------------|------------------|
| \(\pi_1^1\) | \(\frac{2(2\alpha_2+c_2\theta+t(-2+\theta^2)+\alpha_1(-2+\theta^2)-2\alpha_2\mu+\alpha_2\theta\mu)^2}{(8-3\theta^2)^2} - c_2k(8-3\theta^2)^2+(k^2-k_1\theta)(32-16\theta^2+\theta^4)+k(8t(-2+\theta^2)+8(\theta^2-2)(c_2\theta+\alpha_2(2+\theta-2)\mu)) \) |
| \(\pi_2^n\) | \(-\frac{2\alpha_1-2\alpha_2+c_2\theta-\alpha_1\theta^2-k(-4+\theta^2)-t(-2+\theta^2)+2\alpha_2\mu-\alpha_2\theta\mu}{8-3\theta^2} \) |
| \(q_1^n\) | \(\frac{2\alpha_1-2\alpha_2+c_2\theta-\alpha_1\theta^2-k(-4+\theta^2)-t(-2+\theta^2)+2\alpha_2\mu-\alpha_2\theta\mu}{8-3\theta^2} \) |
| \(q_2^n\) | \(\frac{-2\alpha_1+2\alpha_2+c_2\theta+\alpha_1\theta^2-2k(-2+\theta^2)+t(-2+\theta^2)-2\alpha_2\mu+\alpha_2\theta\mu}{8-3\theta^2} \) |
| \(p_1^n\) | \(\frac{6\alpha_1+2\alpha_2+c_2\theta-2\alpha_1\theta^2+k(-4+\theta^2)+t(-2+\theta^2)-2\alpha_2\mu+\alpha_2\theta\mu}{8-3\theta^2} \) |
| \(p_2^n\) | \(\frac{4k-2k+2\alpha_2-2\alpha_2-4\alpha_2\theta+4\alpha_2\mu-3\alpha_2\theta\mu}{8-3\theta^2} \) |

**Proof of Proposition 2.4**

For the first part, it is easy to observe that by substituting \(k_3^n\) into \(q_1^n(k)\) and \(q_2^n(k)\) given in Table A.2, we get \(q_1^n(k_3^n) = \frac{\alpha_1-c_2}{2} = q_{11}^n\), and \(q_2^n(k_3^n) = \frac{E_1}{4-\theta^2} = q_{12}^n\). Hence the market sales are exactly equal to the uncapacitated case at the threshold capacity level. For the second part, we first point that the entrant’s equilibrium profit is concave also under Nash competition and moreover, existence of additional capacity beyond \(k^n\) lead to decreasing equilibrium profits for both parties which can be proven following the approach used in Lemma 2.1. From the equilibrium results given in Table A.2, and the profit functions under the capacitated and uncapacitated cases give the same values at \(k = k_3^n\). This completes the proof.

**Proof of Proposition 2.5**
It is easy to see that \( k_1^s \) and \( k_1^n \) are indeed the same from the proofs of Propositions 2.1 and 2.3. We see that \( k_2^s < k_2^n \) since \( k_2^s - k_2^n = -\frac{3\theta^3(c_2(-1+\theta^2)+\alpha_2(\theta+\mu-\theta\mu))}{2(2-\theta^4)(8-\theta^4)} < 0 \) as \( c_2(-1+\theta^2) + \alpha_2(\theta + \mu - \theta\mu) > 0 \). To see \( k_3^n < k_3^s \), we first recall that under the uncapacitated case, \( q_{t}^{un} > q_{t}^{us} \) and \( q_{1}^{us} = q_{1}^{un} = \frac{\alpha_1-c_1}{2} \). As such, it is true that \( Q_{1}^{us} = q_{1}^{us} + q_{t}^{us} > Q_{1}^{un} = q_{1}^{un} + q_{t}^{un} = k_3^n \). From Proposition 2.2, we know that \( k_3^s > q_{1}^{us} + q_{t}^{us} \), and hence, \( k_3^s > q_{1}^{us} + q_{t}^{us} > q_{1}^{un} + q_{t}^{un} = k_3^n \). This completes the proof of the first part.

To prove the second part, we first show that \( \pi_1^s(k) > \pi_1^n(k) \) when \( k > k_2^s \). Consider the case \( k_2^s < k \leq k_2^n \). In this region, the entrant is not yet in the incumbent’s market in the Nash competition but enters into the incumbent’s market in the Stackelberg competition. From Table 2.3, we know that the entrant’s equilibrium profit is \( k(\alpha_1 - c_1 - k) \) in this region in the Nash competition. The profit under the Stackelberg competition, \( \pi_1^s(k) \), is given in Table A.1. To compare, we take the difference between profits as follows:

\[
\pi_1^s(k) - k(\alpha_1 - c_1 - k) = \frac{(8(2k - t - \alpha_1 + \alpha_2 - \alpha_2\mu) + 4c_2\theta + 4\alpha_2\theta\mu + \theta^2(-14k + 7t + 7\alpha_1 - 3\alpha_2 - 3c_2\theta + 3\alpha_2\mu))^2}{32(4 - \theta^2)^2},
\]

which is clearly strictly positive implying that the Stackelberg profits are higher.

In \( k_2^n < k \leq k_3^n \), the entrant is in the incumbent’s market in both competition cases. In this region, the equilibrium profits \( \pi_1^s(k) \) and \( \pi_1^n(k) \) are given in Table A.1 and Table A.2 respectively. Observe that both functions are concave quadratic in \( k \). Therefore, there are two roots to \( \pi_1^s(k) = \pi_1^n(k) \). Let \( k_{\alpha} \) and \( k_{\beta} \) denote these roots. After several mathematical iterations we get \( k_{\alpha} = \)

\[
\frac{\alpha_2(\theta(3\theta(-3\theta^2 + (3\theta(\theta + 4) - 32\mu + 32) - 64\mu) + 128(\mu - 1)) - \theta^4 - 60\theta^2 + 64) + (45\theta^4 - 160\theta^2 + 128)(\alpha_1 + \theta)^2}{90\theta^4 - 320\theta^2 + 256}
\]

and \( k_{\beta} = \)

\[
\frac{t\theta + \alpha_1\theta + 3\alpha_2\theta + c_2(-4 + 3\theta^2) + 4\alpha_2\theta\mu - 3\alpha_2\theta\mu}{2\theta}.
\]

From this, we get \( k_{\beta} - k_{\alpha} = \frac{8(9\theta^4 - 36\theta^2 + 32)(c_2(-1 + \theta^2) + \alpha_2(\theta + \mu - \theta\mu))}{\theta(45\theta^4 - 160\theta^2 + 128)} > 0 \), that is, \( k_{\alpha} < k_{\beta} \). It can be shown that since \( \alpha_2 > \frac{c_2-c_2\theta^2}{\theta^2+\mu-\theta\mu} \), \( k_{\alpha} < k_2^n \) and \( k_3^s < k_{\beta} \) implying that the profit functions have no intersections in the interval \([k_2^n, k_3^n] \). Since \( \pi_1^n(k_2^n) < \pi_1^s(k_2^s) \) at \( k_2^s \), it must be true that \( \pi_1^s(k) < \pi_1^n(k) \) on \([k_2^n, k_3^n] \) as well (see the illustration in Figure A.2).
In $k_3^n < k \leq k_3^s$, the entrant is still in capacitated competition region in the Stackelberg game but is in the unlimited capacity competition region in the Nash game. The entrant’s profit function of Nash competition is thus constant $\pi_1^n(k) = \pi_1^{un}$ in this interval while the function of Stackelberg competition is still $\pi_1^s(k)$, which is concave and decreasing. At the boundaries of this region, $\pi_1^{un} = \pi_1^n(k_3^n) < \pi_1^s(k_3^n)$ and $\pi_1^n(k_3^n) = \pi_1^{un} < \pi_1^{us} < \pi_1^s(k_3^n)$. Consequently, $\pi_1^n(k) < \pi_1^s(k)$ must hold for the whole interval (See Figure A.2).

For $k > k_3^s$, the entrant reaches unlimited capacity region in both competition cases and therefore the proof here is the same as that of the uncapacitated case, that is, $\pi_1^{us} > \pi_1^{un}$. Consequently, we conclude that the entrant’s profits are higher in the Stackelberg competition than in the Nash competition in the region of $[k_2^s, \infty)$.

Next we show that $\pi_2^s(k) > \pi_2^n(k)$, that is, the incumbent is also better off under the Stackelberg competition. In $k_2^s < k \leq k_2^n$, the incumbent is still in the Effectively Impede Entry region in Nash competition but is in competition region in Stackelberg
competition. From Table A.1 and Table 2.2, we get $\pi^*_n(k) - \pi^*_n(k) = \frac{(8(2k-t-\alpha_1+\alpha_2-\alpha_2\mu)+4c_2\theta+4\alpha_2\theta\mu+\theta^2(-14k-7t+7\alpha_1-3\alpha_2-3c_2\theta+3\alpha_2\mu))^2}{16\theta^2(4-3\theta^2)}$, \hfill (A.22)

which is clearly strictly positive implying that the incumbent is better off with the Stackelberg game in the given region.

For $k^n_2 < k \leq k^n_3$, the incumbent is in the capacitated competition region in both Nash and Stackelberg cases. From Table A.1 and Table A.2

$$\pi^s_2(k) - \pi^u_2(k) = \frac{9\theta^4(-2k\theta + t\theta + \alpha_1\theta + 3\alpha_2\theta + c_2(-4 + 3\theta^2) + 4\alpha_2\mu - 3\alpha_2\theta\mu)^2}{16(8-3\theta^2)^2(4-3\theta^2)}, \hfill (A.23)$$

which is clearly strictly positive implying that the incumbent is better off with the Stackelberg game in this region as well.

When $k^n_3 < k \leq k^n_3$, the incumbent is in the unlimited capacity competition region in the Nash game but is still in the capacitated competition region in Stackelberg game. The incumbent’s profit is therefore constant at $\pi^u_2$ under the Nash game whereas its Stackelberg profits are as defined in Table A.1. It can be shown that $\pi^s_2(k^n_3) > \pi^u_2(k^n_3) = \pi^u_2$ and $\pi^s_2(k^n_3) > \pi^u_2(k^n_3) = \pi^u_2$, where $\pi^s_2(k)$ is monotone decreasing in the region. As a result, $\pi^s_2(k) > \pi^u_2(k)$ in the whole region.

For $k > k^n_3$, the incumbent reaches the unlimited capacity region in both competition cases, where $\pi^u_2 > \pi^u_2$. Consequently, we conclude that the incumbent’s profits are higher in the Stackelberg competition than in the Nash competition in the region of $[k^n_3, \infty)$.

**Proof of Lemma 2.2:**

The lemma follows from comparing the quantities in Table 2.2 and Table 2.3.

**Proof of Lemma 2.3:**

The lemma follows directly from the proofs of Proposition 2.2 and Proposition 2.4. Proposition 2.2 shows that $q^s_1(k^n_3) < q^u_1 = \frac{\alpha_1-c_1}{2}$ and $k^n_3 = q^s_1(k^n_3) + q^s_1(k^n_3) > q^u_1 + q^u_1$. As such, $q^u_1 < q^u_1(k^n_3)$. Proposition 2.4 shows that $q^u_1 = q^u_1(k^n_3)$. Since $q^u_1 = q^u_1 = \frac{\alpha_1-c_1}{2}$, we can conclude that $q^s_1(k^n_3) < q^u_1 = q^u_1(k^n_3)$.

Since $q^u_1 = q^u_1(k^n_3) < q^u_1 + q^u_1$ and $q^u_1 < q^u_1(k^n_3), q^u_1(k^n_3) > q^u_1(k^n_3)$ is true.

**Proofs of Results in Branding Model for the Uncapacitated Case**

Here we use $ub$ to denote uncapacitated branding case.
From $\pi_2^{us}$, we obtain the incumbent’s profit function with branding in uncapacitated case as follows:

$$\max_{\mu} \pi_2^{ub}(\mu) = \frac{(c_1\theta + t\theta + \alpha_2\theta + c_2(-2 + \theta^2) + 2\alpha_2\mu - \alpha_2\theta\mu)^2}{8(2 - \theta^2)} - \frac{s\mu^2}{2} \quad (A.24)$$

Solving the problem with respect to $\mu$ and then substituting it back into the objective function, we get

$$\mu^{ub} = \frac{\alpha_2(2 - \theta)((c_1 + t + \alpha_2)\theta - c_2(2 - \theta^2))}{4s(2 - \theta^2) - \alpha_2^2(2 - \theta)^2}, \quad (A.25)$$

$$\pi_2^{ub} = \frac{s((c_1 + t + \alpha_2)\theta - c_2(2 - \theta^2))^2}{2(4s(2 - \theta^2) - \alpha_2^2(2 - \theta)^2)} = \frac{s((c_1 + t + \alpha_2)\theta - c_2(2 - \theta^2))}{2\alpha_2(2 - \theta)} \mu^{ub} \quad (A.26)$$

### Proof of Proposition 2.6

Since the proof can be carried out using the similar approach employed in Proposition 2.1, we provide a general outline. The mathematical details are tedious and thus some of them are omitted. In the branding case, we replace the index $s$ that we use for the Stackelberg competition case with $b$. The capacity threshold values for the branding case are as follows:

$$k_1^b = \frac{\alpha_2^2(1-\theta)((c_2 + t + \alpha_1)(1-\theta) - \alpha_2) + s(-2(t + \alpha_1 - \alpha_2) + c_2\theta + 2(t + \alpha_1 - \alpha_2)\theta^2 - c_2\theta^3)}{2(1-\theta)(2s(1-\theta) - \alpha_2^2(1-\theta))},$$

$$k_2^b = \frac{\alpha_2^2(-4 + 3\theta)(-c_2 - t - \alpha_1 + \alpha_2 + c_2 + t + \alpha_1)\theta + s(-2(t + \alpha_1 - \alpha_2) + 4c_2\theta + 7(t + \alpha_1 - 3\alpha_2)\theta^2 - 3c_2\theta^3)}{2(\alpha_2^2(-1+\theta)(-4+3\theta) + s(-8+7\theta^2))},$$

$$k_3^b = \frac{N((t + \alpha_1 + 3\alpha_2)\theta - c_2(4 - \theta^2)) + \sqrt{O}(\alpha_2^2(8 - 10\theta + 3\theta^2)^2 + 32\theta^2(8 - 10\theta + 3\theta^2)^2 + 4\alpha_2(8 - 10\theta + 3\theta^2)^2 - 4\theta^4 + 15\theta^4))}{28N},$$

where $N = \alpha_2^2(-2 + \theta)^2 + 4s(-2 + \theta^2)$ and $O = (c_1 + t + \alpha_2)\theta + c_2(-2 + \theta^2)$.

Noting the incumbent’s price $p_1^b$ and branding $\mu^b$, the entrant sets its prices according to the following objective function:

$$\max_{p_1^b, p_t^b} \pi_1^b = (p_1^b - c_1)(\alpha_1 - p_t^b) + (p_t^b - t - c_1)((1 - \mu^b)\alpha_2 - p_t^b + \theta p_t^b) \quad (A.27)$$

$$\text{s.t.} \quad \alpha_1 - p_1^b + (1 - \mu^b)\alpha_2 - p_t^b + \theta p_t^b \leq k, \quad (A.28)$$

Using Lagrange relaxation and solving for $p_1^b, p_t^b$, we get

$$p_1^b(p_2^b) = \frac{-2k - t + 3\alpha_1 + \alpha_2 + p_2^b\theta - \alpha_2\mu^b}{4}, \quad (A.29)$$
Substituting $\mu$ using Lagrange relaxation to optimize the above problem with respect to $p_b$, we get:

$$p_b^*(p_2^b) = \frac{-2k + t + \alpha_1 + 3\alpha_2 + 3p_2^b \theta - 3\alpha_2 \mu^b}{4}.$$  \hfill (A.30)

Substituting $p_b^*(p_2^b)$ into $q_b^*(k)$, we get:

$$q_b^*(p_2^b, \mu^b) = \frac{2k - t - \alpha_1 + \alpha_2 + p_2^b \theta - \alpha_2 \mu^b}{4}.$$  \hfill (A.31)

When we substitute $p_2^b = p_2^m$ and $\mu^b = \mu^m$ as given in (2.20) into (A.31) and solve it for $k$ when $q_b^*(k) = 0$, we get $k = k_1^b$. If $k < k_1^b$, then $q_b^*(k) < 0$ implying the Monopoly region for this case.

When $k_1^b < k$, the incumbent employs some combination of price and branding strategy to stop the entrant. Letting $q_b^*(p_2^b, \mu^b) = 0$ and solve for $k$, we get $k = \frac{(t + \alpha_1 - \alpha_2 - p_2^b \theta + \alpha_2 \mu^b)}{2}$. The incumbent’s objective function, when it uses the combination of price and branding to stop the entry, is:

$$\max_{p_2, \mu} : \pi_2^m = ((\mu^b + \theta(1 - \mu^b))\alpha_2 - (1 - \theta^2)p_2^b)(p_2^b - c_2) - \frac{s(\mu^b)^2}{2}$$  \hfill (A.32)

s.t. $k = \frac{(t + \alpha_1 - \alpha_2 - p_2^b \theta + \alpha_2 \mu^b)}{2}$.

Using Lagrange relaxation to optimize the above problem with respect to $p_2$ and $\mu$, we get

$$\mu^{m'}(k) = \frac{\alpha_2 ((2 - \theta)(\alpha_1 - \alpha_2) - c_2 \theta (1 - \theta) - \alpha_1 \theta^2 - (2k - t)(2 - \theta - \theta^2))}{2\alpha_2^2(-1 + \theta) - s\theta^2},$$  \hfill (A.33)

$$p_2^{m'}(k) = \frac{\alpha_2^2(\alpha_2 - (\alpha_1 - c_2 + t)(1 - \theta)) + s\theta(\alpha_1 - \alpha_2 + t) + 2k(\alpha_2^2(1 - \theta) - s\theta)}{2\alpha_2^2(1 - \theta) + s\theta^2}.$$  \hfill (A.34)

Substituting $\mu^{m'}(k)$ and $p_2^{m'}(k)$ into $\pi_2^m(p_2, \mu)$, we obtain $\pi_2^{m'}(k)$ as the modified monopoly function of branding case.

On the other hand, the incumbent may choose to allow for entry and compete with the entrant. In this case, we calculate the incumbent’s profit in the branding case using Stackelberg competition. As such, we compute the equilibrium via backward induction. For that we use the profit function $\pi_2^b(k)$ given in Table A.1. The incumbent’s objective function is:

$$\max_{\mu} : \pi_2^b(\mu) = \frac{(-2k \theta + t \theta + \alpha_1 + 3\alpha_2 \theta + c_2(-4 + 3\theta^2) + 4\alpha_2 \mu - 3\alpha_2 \theta \mu)^2}{64 - 48\theta^2} - \frac{s\mu^2}{2}.$$  \hfill (A.35)
Maximizing this function with respect to \( \mu \), we get

\[
\mu^b(k) = \frac{\alpha_2(4-3\theta)[(-2k + t + \alpha_1 + 3\alpha_2)\theta - c_2(4-3\theta^2)]}{8s(4-3\theta^2) - \alpha_2^2(4-3\theta)^2}.
\]

(A.36)

Note that \( p_s^k \) in Table A.1 is also a function of \( \mu \), we thus substitute \( u^b(k) \) back into the \( p_s^k \), we get

\[
p^b(k) = \frac{4s(2k - t - \alpha_1 - 3\alpha_2)\theta + c_2[\alpha_2^2(4-3\theta)^2 - 4s(4-3\theta^2)]}{\alpha_2^2(4-3\theta)^2 - 8s(4-3\theta^2)}.
\]

(A.37)

Substituting \( u^b(k) \) back into the objective function, we get

\[
\pi^b(k) = s\frac{[(-2k + t + \alpha_1 + 3\alpha_2)\theta - c_2(4-3\theta^2)]^2}{16s(4-3\theta^2) - 2\alpha_2^2(4-3\theta)^2}.
\]

(A.38)

Next, letting \( \pi^{m'}_2(k) = \pi^b_2(k) \) and solving for \( k \), we get \( k^b_2 \) given above. We note that for \( k < k^b_2 \), equilibrium for market competition results with negative quantities for the entrant similar to Proposition 2.1. When \( k^b_1 < k < k^b_2 \), the incumbent prevents the entry using \( \mu^{m'}_2(k) \) and \( p^{m'}_2(k) \). Above this level, the entrant enters the incumbent’s market using the branding investment and price given in (A.36) and (A.37) since \( \pi^{m'}_2(k) < \pi^b_2(k) \).

Letting \( \pi^b_2(k) = \pi^{ub}_2 \) (\( \pi^{ub}_2 \) is derived from the proof of unconstrained-capacity branding model) and solving for \( k \), we get \( k^b_3 \). If \( k > k^b_3 \) then the constrained-capacity scenario switches to the unconstrained-capacity scenario.
Appendix B

Proofs for Chapter 3

B.1. Proofs for the expressions in Table 3.2

RR model

We can show that the retailer’s objective function (3.3) is jointly concave if \( 2h > (k + r)^2\alpha^2 \), which follows from Assumption 1 and 2. Then from the first-order conditions, we get:

\[
\frac{\partial \pi_{RR}}{\partial p} = 0 \Rightarrow p = \frac{1 + w + (k - r)\alpha(1 - \lambda)}{2}, \quad (B.1)
\]

and

\[
\frac{\partial \pi_{RR}}{\partial \lambda} = 0 \Rightarrow \lambda = \frac{\alpha [r(p - w) + k(1 - p - 2r\alpha)]}{h - 2kr\alpha^2}. \quad (B.2)
\]

Solving (B.1) and (B.2) we get:

\[
p(w) = \frac{h(1 + w + k\alpha - r\alpha) - (k + r)(k + wr)\alpha^2}{2h - (k + r)^2\alpha^2}, \quad (B.3)
\]

\[
\text{and } \lambda(w) = \frac{(k + r)\alpha(1 - c - k\alpha - r\alpha)}{2h - (k + r)^2\alpha^2}. \quad (B.4)
\]

Substituting into manufacturer’s objective function (3.4) and upon simplifying terms, we get:

\[
\max_w : \pi_{RR}^M(w) = \frac{h(w - c)(1 - w - k\alpha - r\alpha)}{2h - (k + r)^2\alpha^2}. \quad (B.5)
\]

The profit function in (B.5) is concave in \( w \) since \([\pi_{RR}^M(w)]'' = -\frac{2h}{2h - (k + r)^2\alpha^2} < 0\), the first-order condition yields the optimal wholesale price:

\[
w_{RR^*} = \frac{(1 + c - k\alpha - r\alpha)}{2}. \quad (B.6)
\]
Substituting (B.6) back into (B.3) and (B.4), we can get the equilibrium retail price \((p^{RR})\) and effort level \((\lambda^{RR}_R)\) in Table 3.2 column RR. The equilibrium profits can also be calculated as:

\[
\pi^{RR}_M = \frac{h(1 - c - k\alpha - r\alpha)^2}{4 [2h - (k + r)^2\alpha^2]},
\]

\[
\pi^{RR}_R = \frac{h(1 - c - k\alpha - r\alpha)^2}{8 [2h - (k + r)^2\alpha^2]},
\]

and \(\pi^{RR}_t = \frac{3h(1 - c - k\alpha - r\alpha)^2}{8 [2h - (k + r)^2\alpha^2]}\).

At the equilibrium, the demand is:

\[
D^{RR} = 1 - p^{RR} - \alpha(1 - \lambda^{RR}_R)r = \frac{h(1 - c - k\alpha - r\alpha)}{2 [2h - (k + r)^2\alpha^2]},
\]

and the demand for after-sales support is:

\[
T^{RR} = D(1 - \lambda^{RR}_R)\alpha = \frac{h\alpha(1 - c - k\alpha - r\alpha) [4h - (k + r)\alpha(1 - c + k\alpha + r\alpha)]}{4 [2h - (k + r)^2\alpha^2]^2}.
\]

**MM model**

The retailer’s objective function (3.5) is concave since \([\pi^{MM}_R(p)]'' = -2\) and therefore from the first-order condition, we get

\[
p^{MM}(w, \lambda) = \frac{1 + w - r\alpha(1 - \lambda)}{2}.
\]

Substituting (B.7) into the manufacturer’s objective function (3.6) and upon simplifying terms, we get:

\[
\max_{w,\lambda \in [0,1]} : \pi^{MM}_M(w, \lambda) = \frac{1}{2} \left[(w - c - k\alpha(1 - \lambda))(1 - w - r(\alpha(1 - \lambda)) - h\lambda^2)\right].
\]

We can show that the manufacturer’s objective function (B.8) is jointly concave if \(4h > (k + r)^2\alpha^2\), which follows from Assumption 1 and 2. Then from the first-order conditions, we get:

\[
w^{MM} = \frac{2h(1 + c + k\alpha - r\alpha) - (k + r)(k + cr)\alpha^2}{4h - (k + r)^2\alpha^2},
\]

and \(\lambda^{MM}_M = \frac{(k + r)\alpha(1 - c - k\alpha - r\alpha)}{4h - (k + r)^2\alpha^2}\).
Substituting (B.9) and (B.10) and back into (B.7), we can get the equilibrium retail price \(p_{MM}^{*}\) and manufacturer’s effort level \(\lambda_{M}^{MM*}\) in Table 3.2 column MM. The equilibrium profits can also be calculated as:

\[
\pi_{MM}^{*} = \frac{h [1 - c - (k + r)\alpha]^2}{2[4h - (k + r)^2\alpha^2]},
\]

\[
\pi_{R}^{MM*} = \frac{h^2 [1 - c - (k + r)\alpha]^2}{[4h - (k + r)^2\alpha^2]^2},
\]

and \(\pi_{t}^{MM*} = \frac{h [1 - c - (k + r)\alpha]^2 [6h - (k + r)^2\alpha^2]}{2[4h - (k + r)^2\alpha^2]^2}\).

At the equilibrium, the demand is:

\[
D_{MM} = 1 - p_{MM}^{*} - \alpha(1 - \lambda_{M}^{MM*})r = \frac{h(1 - c - k\alpha - r\alpha)}{4h - (k + r)^2\alpha^2},
\]

and the demand for after-sales support is:

\[
T_{MM} = D(1 - \lambda_{M}^{MM*})\alpha = \frac{h\alpha(1 - c - k\alpha - r\alpha)[4h - (1 - c)(k + r)\alpha]}{[4h - (k + r)^2\alpha^2]^2}.
\]

**RM model**

We can show that the retailer’s objective function (3.7) is jointly concave if \(2h > r^2\alpha^2\), which follows from Assumption 1 and 2. Then from the first-order conditions, we get:

\[
\frac{\partial\pi_{R}^{RM}}{\partial p} = 0 \Rightarrow p = \frac{1 + w - r\alpha(1 - \lambda)}{2}, \quad \text{(B.11)}
\]

and \(\frac{\partial\pi_{R}^{RM}}{\partial \lambda} = 0 \Rightarrow \lambda = \frac{(p - w)r\alpha}{h}. \quad \text{(B.12)}
\]

Solving (B.11) and (B.12) we get:

\[
p_{RM}(w) = \frac{h + hw - hr\alpha - r^2w\alpha^2}{2h - r^2\alpha^2}, \quad \text{(B.13)}
\]

and \(\lambda_{R}^{RM}(w) = \frac{r\alpha(1 - w - r\alpha)}{2h - r^2\alpha^2}. \quad \text{(B.14)}
\]

Substituting into manufacturer’s objective function (3.8) and upon simplifying terms, we get:

\[
\max_{w} : \pi_{M}^{RM}(w) = \frac{h(1 - w - \alpha r)[(w - c)(2h - \alpha^2r^2) - 2ahk + (1 - w)k\alpha^2]}{(2h - \alpha^2r^2)^2}. \quad \text{(B.15)}
\]
The profit function in (B.15) is concave in \( w \) since 
\[
\left[ \pi_M^{RM}(w) \right]'' = \frac{2h[r(k+r)\alpha^2-2h]}{(2h-\alpha^2r^2)^2} < 0,
\]
the first-order condition yields the optimal wholesale price:
\[
w_{RM}^* = \frac{2h(1+c+k\alpha-r\alpha)-r\alpha^2[k(2-r\alpha)+r(1+c-r\alpha)]}{4h-2r(k+r)\alpha^2}.
\]
(B.16)

Substituting (B.16) back into (B.13) and (B.14), we can get the equilibrium retail price \( p_{RM}^* \) and effort level \( \lambda_{RM}^* \) in Table 3.2 column RM. The equilibrium profits can also be calculated as:
\[
\pi_M^{RM*} = \frac{h[1-c-(k+r)\alpha]^2}{8h-4r(k+r)\alpha^2},
\]
\[
\pi_R^{RM*} = \frac{h[1-c-(k+r)\alpha]^2(2h-\alpha^2r^2)}{8[2h-r(k+r)\alpha^2]^2},
\]
and 
\[
\pi_t^{RM*} = \frac{h[1-c-(k+r)\alpha]^2(6h-r(2k+3r)\alpha^2)^2}{8[2h-r(k+r)\alpha^2]^2}.
\]

At the equilibrium, the demand is:
\[
D_{RM} = 1 - p_{RM}^* - \alpha(1 - \lambda_{RM}^*)r = \frac{h(1-c-k\alpha-r\alpha)}{2[2h-r(k+r)\alpha^2]},
\]
and the demand for after-sales support is:
\[
T_{RM} = D(1 - \lambda_{RM}^*)\alpha = \frac{h\alpha(1-c-k\alpha-r\alpha)[4h-r\alpha(1-c+k\alpha+r\alpha)]}{4[2h-r(k+r)\alpha^2]^2}.
\]

B.2. Proofs for the main results in the paper

Proof of Proposition 3.1

1. From Table 3.2, we have:
\[
p_{RR}^* - p_{MM}^* = \frac{\alpha^2(k+r)^2(1-c-k\alpha-r\alpha)[\alpha^2r(k+r)-h]}{2[2h-\alpha^2(k+r)^2][4h-\alpha^2(k+r)^2]}.
\]
Clearly, \( p_{RR}^* > (\leq) p_{MM}^* \) if \( h < (> ) \alpha^2r(k+r) \).

2. From Table 3.2, we have \( w_{RR}^* - w_{MM}^* \)
\[
= \frac{\alpha \{(k+r)\alpha[k^2\alpha-r(1-c-r\alpha)+k(1-c+2r\alpha)]-8hk\}}{2[4h-(k+r)^2\alpha^2]} \]
\[
\leq \frac{\alpha \{(k+r)\alpha[k^2\alpha-r(1-c-r\alpha)+k(1-c+2r\alpha)]-4(1-c)(k\alpha+r\alpha)k\}}{2[4h-(k+r)^2\alpha^2]}.
\]
\[- \frac{\alpha(k + r)\alpha [k(1 - c - k\alpha) + (2k + r)(1 - c - r\alpha)]}{2[4h - (k + r)^2\alpha^2]} \]

\[< 0,\]

where the first inequality follows from the assumption that \(2h > (1 - c)(k + r)\alpha\), and the second inequality follows from the assumption that \(4h > 2h > (k + r)^2\alpha^2\) and \(1 - c > k\alpha\) and \(1 - c > r\alpha\).

3. From Table 3.2, it is clear that \(\lambda_M^{MM^*} < \lambda_R^{RR^*}\).

**Proof of Proposition 3.2** Evaluate the market demand \(D = 1 - p - \alpha(1 - \lambda)r\) at the equilibrium \(p^*\) and \(\lambda^*\), we obtain the results for \(D^{RR^*}\) and \(D^{MM^*}\) in Table 3.2. It is clear that \(D^{RR^*} > D^{MM^*}\).

Evaluate the demand for after-sales support \(T = D(1 - \alpha)\) at the equilibrium \(p^*\) and \(\lambda^*\), we obtain the results for \(T^{RR^*}\) and \(T^{MM^*}\) in Table 3.2, and

\[T^{RR^*} - T^{MM^*} = -\frac{h\alpha [1 - c - (k + r)\alpha]}{4} \left( \frac{\alpha^2(k + r)^2 Q(h)}{[4h - (k + r)^2\alpha^2]^2 [2h - (k + r)^2\alpha^2]^2} \right),\]

where \(Q(h)\) is a quadratic function of \(h\) defined as:

\[Q(h) = -16h^2 + 4(k + r)\alpha [2 - 2c + (k + r)\alpha] h - (k + r)^3\alpha^3 [3 - 3c - (k + r)\alpha].\]

It can be shown that \(Q(h)\) has two roots \((\bar{h}, \tilde{h})\):

\[\bar{h}, \tilde{h} = \frac{\alpha(k + r)\left(\alpha(k + r) + 2(1 - c) \pm \sqrt{5\alpha^2(k + r)^2 - 8\alpha(k + r)(1 - c) + 4(1 - c)^2}\right)}{8}\]

It can be shown that \(\tilde{h} < (1 - c)(k + r)\alpha/2\). For parameters that satisfy Assumption 1, we have \(Q(h) < 0\), and therefore \(T^{RR^*} > T^{MM^*}\).
Proof of Proposition 3.3  The proof follows from comparing the manufacturer’s and retailer’s profit in the two pure service channels:

\[ \pi_{RR}^* - \pi_{MM}^* = \frac{h(k + r)^2 \alpha^2 (1 - c - k \alpha - r \alpha)^2}{4[2h - (k + r)^2 \alpha^2][4h - (k + r)^2 \alpha^2]} > 0, \]

and

\[ \pi_{RR}^* - \pi_{MM}^* = \frac{h(k + r)^4 \alpha^4 (1 - c - k \alpha - r \alpha)^2}{8[2h - (k + r)^2 \alpha^2][4h - (k + r)^2 \alpha^2]} > 0. \]

Proof of Proposition 3.4

1. Proposition 3.1 already shows that \( w^{RR^*} < w^{MM^*} \) and here we only prove \( w^{RR^*} < w^{RM^*} \).

\[ w^{RR^*} - w^{RM^*} = -\frac{k \alpha(4h - r \alpha(1 - c + k \alpha + r \alpha))}{4h - 2r(k + r)\alpha^2} \]

\[ = -\frac{k \alpha(4h - (1 - c)r \alpha - r(k + r)\alpha^2)}{4h - 2r(k + r)\alpha^2} \]

\[ < -\frac{k \alpha(2h - (1 - c)(k + r)\alpha + 2h - (k + r)^2 \alpha^2)}{4h - 2r(k + r)\alpha^2} \]

\[ < 0, \]

where the last inequality follows from the assumption that \( 2h > (1 - c)(k + r)\alpha \) and \( 2h > (k + r)^2 \alpha^2 \).

2. Proposition 3.1 already shows that \( \lambda^{RR^*} > \lambda^{MM^*} \) and Proposition 3.2 already shows that \( D^{RR^*} > D^{MM^*} \). Here we prove \( \lambda^{RR^*} > \lambda^{RM^*} \) and \( D^{RR^*} > D^{RM^*} \). The expression of the denominator of \( \lambda^{RR^*} \) is smaller since \( B > C \) but the expression of the nominator is larger than that of \( \lambda^{RM^*} \), thus it is clear \( \lambda^{RR^*} > \lambda^{RM^*} \). From Table 3.2, it is clear that \( D_M^{RR^*} > D_R^{RM^*} \).

3. Proposition 3.3 already shows that the manufacturer and retailer’s profits in the RR model are higher than those in the MM model. Here, we only compare RR and RM model. From Table 3.2, we have:

\[ \pi_{RR}^* - \pi_{RM}^* = \frac{h^2 k^2 \alpha^2 [1 - c - (k + r)\alpha]^2}{4[2h - r(k + r)\alpha^2]^2 [2h - (k + r)^2 \alpha^2]^2} > 0 \]
and
\[
\pi^{RR^*}_M - \pi^{RM^*}_M = \frac{hk(k + r)\alpha^2 [1 - c -(k + r)\alpha]^2}{4 [2h - r(k + r)\alpha^2] [2h - (k + r)^2\alpha^2]} > 0.
\]

**Proof of Proposition 3.5**

1. From Table 3.2, we have:
\[
\pi^{MM^*}_M - \pi^{RM^*}_M = \frac{h(k^2 - r^2)\alpha^2 [1 - c -(k + r)\alpha]^2}{4 [2h - r(k + r)\alpha^2] [2h - (k + r)^2\alpha^2]}
\]

By Assumption 1 and 2, we have \(4h > 2h > (k + r)^2\alpha^2 > r(k + r)\alpha^2\). Therefore, \(\pi^{MM^*}_M > \pi^{RM^*}_M\) if \(k > (r)\).

2. From Table 3.2, we have:
\[
\pi^{MM^*}_R - \pi^{RM^*}_R = \frac{\alpha^2 Q(h)}{8 [4h - \alpha^2(k + r)^2]^2 [2h - \alpha^2 r(k + r)]^2},
\]

where \(Q(h)\) is a quadratic function of \(h\) defined as:
\[
Q(h) = 16k^2h^2 - 2\alpha^2(k + r)^4h + r^2\alpha^4(k + r)^4.
\]

It can be shown that \(Q(h)\) has two roots \((h_1 < h_2)\):
\[
h_{1,2} = \left(\frac{(k + r)^2 \pm \sqrt{(k^2 + 6kr + r^2)(k - r)^2}}{16k^2}\right)(k + r)^2\alpha^2.
\]

It can be shown that when \(3k > 2r\), \(h_2 < \frac{(k+r)^2\alpha^2}{2}\). Therefore, for the parameters that satisfy Assumption 1 and 2, we have \(Q(h) > 0\) and \(\pi^{MM^*}_R > \pi^{RM^*}_R\). When \(3k < 2r\), we can show that \(h_1 < \frac{(k+r)^2\alpha^2}{2} < h_2\). Therefore, for the parameters that satisfy Assumption 1 and 2, we have \(Q(h) > 0\) and consequently \(\pi^{MM^*}_R > \pi^{RM^*}_R\) when \(h > h_2\), and \(Q(h) < 0\) and consequently \(\pi^{MM^*}_R < \pi^{RM^*}_R\) when \(h < h_2\).
Proof of Proposition 3.6  
In the cost sharing model, the retailer’s problem is given as:

\[
\max_{p, \lambda \in [0, 1]} \hat{\pi}^{RR}(p, \lambda) = \left[1 - p - \alpha(1 - \lambda)r\right] \left[p - w - k\alpha(1 - \lambda)\right] - (1 - \phi)\frac{h\lambda^2}{2}. \tag{B.17}
\]

We can show that the retailer’s objective function (B.17) is jointly concave if \(2h(1 - \phi) > (k + r)^2\alpha^2\) or equivalently, \(\phi < 1 - \frac{(k + r)^2\alpha^2}{2h}\). We thus consider three possible cases for the retailer’s problem and accordingly manufacturer’s sub-problems: (1) \(\phi < 1 - \frac{(k + r)^2\alpha^2}{2h}\), (2) \(\phi > 1 - \frac{(k + r)^2\alpha^2}{2h}\), (3) \(\phi = 1 - \frac{(k + r)^2\alpha^2}{2h}\).

**Case 1: \(\phi < 1 - \frac{(k + r)^2\alpha^2}{2h}\)**  
Since (B.17) is joint concave, then from the first-order conditions, we get:

\[
\hat{p}^{RR}_R(w, \phi) = \frac{h(1 + w + k\alpha - r\alpha)(1 - \phi) - (k + r)(k + rw)\alpha^2}{2h(1 - \phi) - (k + r)^2\alpha^2}, \tag{B.18}
\]

\[
\hat{\lambda}^{RR}_R(w, \phi) = \frac{(k + r)\alpha(1 - w - k\alpha - r\alpha)}{2h(1 - \phi) - (k + r)^2\alpha^2}. \tag{B.19}
\]

Substitute (B.18) and (B.19) into (B.17) and we get the retailer’s profit function with respect to \(w\) and \(\phi\):

\[
\hat{\pi}^{RR}_R(w, \phi) = \frac{h \left[1 - w - (k + r)\alpha\right]^2 (1 - \phi)}{2 \left[2h(1 - \phi) - (k + r)^2\alpha^2\right]}. \tag{B.20}
\]

The manufacturer’s problem in this case can be expressed as:

\[
\max_{w, \phi} \hat{\pi}^{RR}_M(w, \phi) = \left[1 - \hat{p}^{RR}_R(w, \phi) - \alpha(1 - \hat{\lambda}^{RR}_R(w, \phi))\right] (w - c) - \phi \frac{h\hat{\lambda}^{RR}_R(w, \phi)^2}{2}. \tag{B.21}
\]

We first take partial derivative of (B.21) with respect to \(w\) and solving the first-order condition for \(w\), we can get:

\[
w(\phi) = \frac{2h(1 - \phi)^2 \left[1 + c - (k + r)\alpha\right] - \left[c - c\phi + (1 - (k + r)\alpha)(1 - 2\phi)\right] (k + r)^2\alpha^2}{4h(1 - \phi)^2 - (2 - 3\phi)(k + r)^2\alpha^2}, \tag{B.22}
\]

and the second-order condition yields:

\[
\left[\hat{\pi}^{RR}_M(w)\right]'' = -\frac{h \left[4h(1 - \phi)^2 - (2 - 3\phi)(k + r)^2\alpha^2\right]}{[2h(1 - \phi) - (k + r)^2\alpha^2]^2}. \tag{B.23}
\]
It can be shown that \( f(\phi) = 4h(1 - \phi)^2 - (2 - 3\phi)(k + r)^2\alpha^2 \) in the numerator is a convex quadratic function and if \( \phi < 1 - \frac{(k+r)^2\alpha^2}{2h} \) then \( f(\phi) > 0 \). Therefore, we can get \( [\hat{\pi}_M^{RR}(w)]'' < 0 \). Substituting equation (B.22) into (B.21) yields:

\[
\max_{\phi} \hat{\pi}_M^{RR}(\phi) = \frac{h[1-c-(k+r)\alpha]^2(1-\phi)^2}{24h(1-\phi)^2 - (2 - 3\phi)(k + r)^2\alpha^2}.
\] (B.24)

The first-order condition of (B.24) yields \( \phi = \frac{1}{3} \) and \( \phi = 1 \). But since \( \phi < 1 - \frac{(k+r)^2\alpha^2}{2h} \), \( \phi = 1/3 \) is the only possible extreme point on \( \phi \in [0, 1 - \frac{(k+r)^2\alpha^2}{2h}) \). As a result, we need to analyze two sub-cases, depending on whether \( \phi = \frac{1}{3} \) falls in the region of \( [0, 1 - \frac{(k+r)^2\alpha^2}{2h}) \) or not.

**Case 1.1:** \( 1 - \frac{(k+r)^2\alpha^2}{2h} > \frac{1}{3} \). i.e., \( (2h > \frac{3(k+r)^2\alpha^2}{2} ) \). If \( 1 - \frac{(k+r)^2\alpha^2}{2h} > \frac{1}{3} \) then \( \phi = \frac{1}{3} \) is the only extreme point and thus is the global maximum point. Substituting \( \phi = \frac{1}{3} \) into (B.24) yields the manufacturer’s profit in equilibrium:

\[
\hat{\pi}_{M,1}^{RR} = \frac{2h[1-c-(k+r)\alpha]^2}{16h - 9(k + r)^2\alpha^2}.
\] (B.25)

On substituting \( \phi = \frac{1}{3} \) into (B.22), and we can get the wholesale price \( \hat{w}^* \) given in part 1 of Proposition 3.6. Then substitute both \( \hat{w}^* \) and \( \phi^* = \frac{1}{3} \) back into (B.18) and (B.19) and we can get \( \hat{p}^* \), and \( \hat{\lambda}_{R}^* \) given in part 1 of Proposition 3.6.

**Case 1.2:** \( 1 - \frac{(k+r)^2\alpha^2}{2h} < \frac{1}{3} \). Since \( \phi = 1 - \frac{(k+r)^2\alpha^2}{2h} < \frac{1}{3} \), \( \phi = \frac{1}{3} \) is not a feasible solution. However, it can be shown that \( \hat{\pi}_M^{RR}(\phi) \) is an increasing function. As a result, the manufacturer’s equilibrium wholesale price and cost-sharing portion are \( \hat{w} = 1 - (k + r)\alpha \) and \( \phi = 1 - \frac{(k+r)^2\alpha^2}{2h} \). Therefore, the retailer’s equilibrium selling price and pre-sales effort level are:

\[
\hat{p}^{RR} = \frac{4h(1 - r\alpha) - (k + r)\alpha^2 [(3-c)k + (1 + c)r - (k + r)^2\alpha]}{4h - 2(k + r)^2\alpha^2}
\] (B.26)

and

\[
\hat{\lambda}_{R,2}^{RR} = \frac{(k + r)\alpha [1 - c - (k + r)\alpha]}{2h - (k + r)^2\alpha^2}.
\] (B.27)

The manufacturer’s and retailer’s equilibrium profit are

\[
\hat{\pi}_{M,2}^{RR} = \frac{[1-c-(k+r)\alpha]^2(k+r)^2\alpha^2}{8h - 4(k + r)^2\alpha^2} \quad \text{and} \quad \hat{\pi}_{R,2}^{RR} = 0.
\] (B.28)

Notice that for any given \( \phi \), the effort level \( \lambda \) should be within 0 and 1. And we can show that the lower bound of \( h \) should satisfy the condition given as \( 2h \geq \ldots \)
Case 2: $\phi = 1 - \frac{(k+r)^2\alpha^2}{2h}$ 

Substituting $\phi = 1 - \frac{(k+r)^2\alpha^2}{2h}$ into (B.17), we can get the retailer’s objective function as:

$$\max_p \pi_{RR}(p) = [1 - p - \alpha(1 - \lambda)r] [p - w - k\alpha(1 - \lambda)] - \frac{\lambda^2(k + r)^2\alpha^2}{4}. \quad (B.29)$$

It is a concave function with respect to $p$, thus from the first-order condition, we can get

$$\hat{p}_{RR} = \frac{1 + w + (1 - \lambda)(k - r)\alpha}{2}. \quad (B.30)$$

On substituting (B.30) into (B.29), we can get:

$$\max_{\lambda \in [0, 1]} \pi_{RR}(\lambda) = [1 - w - (1 - \lambda)(k + r)\alpha]^2 - \lambda^2(k + r)^2\alpha^2. \quad (B.31)$$

Since $[\hat{\pi}_{RR}^*(\lambda)]' = [1 - w - (k + r)\alpha]^2 / 2$, it is a linear function with respect to $\lambda$. The retailer chooses $\hat{\lambda}_{RR}^* = 1$ if $w < 1 - (k + r)\alpha$ and $\hat{\lambda}_{RR}^* = 0$ otherwise.

Case 2.1: $w \geq 1 - (k + r)\alpha$ When $\hat{\lambda}_{RR,1} = 0$, the manufacturer’s objective function is given by

$$\max_w \hat{\pi}_{M}^*(w) = \frac{(1 - w - (k + r)\alpha)(w - c)}{2}. \quad (B.32)$$

But since $w \geq 1 - (k + r)\alpha$, $\pi_{M}^*(w) \leq 0$, which is not feasible.

Case 2.2: $w < 1 - (k + r)\alpha$ When $\hat{\lambda}_{RR,2} = 1$, the manufacturer’s objective function is given by

$$\max_w \hat{\pi}_{M}^*(w) = \frac{(1 - w)(w - c)}{2} - \frac{h}{2} + \frac{(k + r)^2\alpha^2}{4}. \quad (B.33)$$

The manufacturer chooses: $\hat{w}_{RR^*} = \frac{(1 + c)(k + r)\alpha}{2}$ if $(k + r)\alpha < \frac{1 - c}{2}$, otherwise it chooses $\hat{w}_{RR^*} = \frac{(1 + c)}{2}$. Substituting $\hat{w}_{RR^*}$ into (B.33), we can get the manufacturer’s profit as:

$$\hat{\pi}_{M,3}^* = \frac{1}{8} [(1 - c)^2 - 4h + 2(k + r)^2\alpha^2] \text{ if } (k + r)\alpha < \frac{1 - c}{2}, \quad (B.34)$$

$$\hat{\pi}_{M,4}^* = \frac{(k + r)\alpha[2(1 - c) - (k + r)\alpha] - 2h}{4} \text{ if } (k + r)\alpha > \frac{1 - c}{2}. \quad (B.35)$$

And accordingly, the retailer’s profit as

$$\hat{\pi}_{R,3}^* = \frac{1}{16} [(1 - c)^2 - 4(k + r)^2\alpha^2] \text{ if } (k + r)\alpha < \frac{1 - c}{2}. \quad (B.36)$$
\[ \hat{\pi}_{RR}^* = 0 \text{ if } (k + r)\alpha > \frac{1-c}{2}. \] (B.37)

**Case 3:** \( \phi > 1 - \frac{(k+r)^2\alpha^2}{2h} \)  

We first solve the retailer’s objective function given in (B.17). Taking partial derivative of equation (B.17) with respect to \( p \) and since \([\hat{\pi}_R^R(p)]'' = -2\), from the first-order condition we can get

\[ p = \frac{1 + w + (1 - \lambda)(k - r)\alpha}{2}. \] (B.38)

On substituting (B.38) into (B.17), we can get:

\[
\max_{\lambda \in [0,1]} : \hat{\pi}_R^R(\lambda) = \frac{[1 - w - (1 - \lambda)(k + r)\alpha]^2 - 2h\lambda^2(1 - \phi)}{4}.
\] (B.39)

Since \( \phi > 1 - \frac{(k+r)^2\alpha^2}{2h} \), \([\hat{\pi}_R^R(\lambda)]'' > 0\). Therefore we check two end points, where \( \hat{\lambda} = 0 \) and \( \hat{\lambda} = 1 \). It is easy to verify that retailer’s solutions of the two cases are:

\[ \hat{\lambda} = 0, \quad \hat{p} = \frac{1 + w + (k - r)\alpha}{2}, \quad \text{and} \quad \hat{\pi}_R = \frac{[1 - w - (k + r)\alpha]^2}{4}, \] (B.40)

and

\[ \hat{\lambda} = 1, \quad \hat{p} = \frac{1 + w}{2}, \quad \text{and} \quad \hat{\pi}_R = \frac{(1 - w)^2 - 2h(1 - \phi)}{4}. \] (B.41)

**Case 3.1:** \( \hat{\lambda} = 0 \)  

If the retailer chooses \( \hat{\lambda} = 0 \), it should satisfy that \( \hat{\pi}_R \) given in (B.40) is greater than \( \hat{\pi}_R \) given in (B.41). However, if this was the case then \( \phi \leq 1 - \frac{(k+r)^2\alpha^2}{2h} \), which is contradictory to the condition of \( \phi > 1 - \frac{(k+r)^2\alpha^2}{2h} \). Therefore, \( \hat{\lambda}_R = 0 \) is not an optimal solution and we only need to consider the case of \( \hat{\lambda}_R = 1 \) in (B.41).

**Case 3.2:** \( \hat{\lambda} = 1 \)  

The manufacturer’s objective function in this case is thus given by:

\[
\max_{w,\phi} \pi_{RM}^R(w, \phi) = \frac{(1 - w)(w - c) - \frac{h\phi}{2}}{2} \quad \text{s.t.} \quad 1 - \frac{(k + r)^2\alpha^2}{2h} < \phi \leq 1.
\] (B.42)

It is a linear function in \( \phi \), and thus \( \phi = 1 - \frac{(k+r)^2\alpha^2}{2h} \). Substitute \( \phi = 1 - \frac{(k+r)^2\alpha^2}{2h} \) into (B.42), the manufacturer’s objective function goes back to the objective function given in (B.33). Therefore the manufacturer’s and the retailer’s optimal solution are the same as in case 2.2.
**Profit comparison**

If $1 - c > 2(k + r)\alpha$ then from the basic assumption $2h > (1 - c)((k + r)\alpha)$, it is true that $2h > \frac{3(k+r)^2\alpha^2}{2}$, which satisfies the concavity condition. We thus need to compare the manufacturer’s profit (when $\phi = \frac{1}{3}$), given by (B.25) in case 1.1 with (B.34) in case 2.2 (the profit from convex solution under the condition $1 - c > 2(k + r)\alpha$). After subtracting (B.34) from (B.25), we can see that the sign of the equation depends on the following convex quadratic function: $f(h) = 64h^2 - [32(1-c)(k+r)\alpha + 52(k+r)^2\alpha^2]h + 9(1-c)^2(k+r)^2\alpha^2 + 18(k+r)^4\alpha^4$. It can shown that $f(h)$ is a convex quadratic function of $h$ with no roots. Therefore, we can get $f(h) > 0$, which makes

\[
\hat{\pi}_{RR}^* \hat{\pi}_{M,1} = \frac{2h[1-c-(k+r)\alpha]^2}{16h-9(k+r)^2\alpha^2} > \hat{\pi}_{M,3} = \frac{(1-c)^2-4h+2(k+r)^2\alpha^2}{8}.
\]

Thus $\hat{\pi}_{M,3}$ is not optimal and we can rule out $\hat{\pi}_{M,3}^*$. If $1 - c < 2(k+r)\alpha$, then $w = 1 - (k+r)\alpha$, which makes the retailer’s profit $\hat{\pi}_{R}^* = 0$. The manufacturer will get $\hat{\pi}_{RR}^*$ given in (B.35). Thus we need to compare $\hat{\pi}_{RR}^*$ with $\hat{\pi}_{M,2}^*$. $\hat{\pi}_{M,2}^* - \hat{\pi}_{M,4}^*$ gives: $\frac{2h-(1-c)(k+r)\alpha}{8h-4(k+r)^2\alpha^2} > 0$. Thus the manufacturer cannot offer cost-sharing portion $1 - \frac{(k+r)^2\alpha^2}{2h}$. However, the manufacturer can choose $1 - \frac{(k+r)^2\alpha^2}{2h} - \epsilon$ to give the retailer $\epsilon > 0$ profit to make sure that the retailer would not response with the decision in ($\lambda = 1$ and $p = \frac{1+w}{2}$) since it makes no difference to the retailer with the same profit 0 under the contract with $w = 1 - (k+r)\alpha$ and $\phi = 1 - \frac{(k+r)^2\alpha^2}{2h}$.

**Proof of Lemma 3.1**

**Part a** The proof follows from comparing the manufacturer’s and retailer’s profit in the RR service channels. If $2h > \frac{3(k+r)^2\alpha^2}{2}$ then

\[
\hat{\pi}_{M,1}^* - \pi_{M}^* = \frac{h(k+r)^2\alpha^2[1-c-(k+r)\alpha]^2}{4[16h-9(k+r)^2\alpha^2][2h-(k+r)^2\alpha^2]} > 0,
\]

\[
\hat{\pi}_{R,1}^* - \pi_{R}^* = -\frac{h(k+r)^2\alpha^2[1-c-(k+r)\alpha]^2[32h-15(k+r)^2\alpha^2]}{8[16h-9(k+r)^2\alpha^2][2h-(k+r)^2\alpha^2]} < 0.
\]
And if \(2h < \frac{3(k+r)^2\alpha^2}{2}\), we know

\[
\hat{\pi}_{M,2}^{RR^*} = \frac{[1 - c - (k + r)\alpha]^2 (k + r)^2\alpha^2}{8h - 4(k + r)^2\alpha^2} \quad \text{and} \quad \pi_M^{RR^*} = \frac{h[1 - c - (k + r)\alpha]^2}{8h - 4(k + r)^2\alpha^2}.
\]

Since \(2h < \frac{3(k+r)^2\alpha^2}{2}\), it is true that \(h < (k + r)^2\alpha^2\). Hence, we can get \(\hat{\pi}_{M,2}^{RR^*} > \pi_M^{RR^*}\).

Since \(\hat{\pi}_{R,2}^{RR^*} = 0\), it is obvious that \(\hat{\pi}_{R,2}^{RR^*} < \pi_R^{RR^*}\).

**Part b**  
Substituting equation given in (B.22) into (B.20), we can get retailer’s profit function as function of \(\phi\):

\[
\hat{\pi}_R^{RR}(\phi) = \frac{h[1 - c - (k + r)\alpha]^2 [2h(1 - \phi) - (k + r)^2\alpha^2]}{2(4h(1 - \phi)^2 - (2 - 3\phi)(k + r)^2\alpha^2)^2} (1 - \phi)^3.
\]  

(B.44)

Since \(\phi \leq 1 - \frac{(k+r)^2\alpha^2}{2h}\), it can be shown that \(\phi = 0\) is the only extreme point between \([0, 1 - \frac{(k+r)^2\alpha^2}{2h}]\) with \(\hat{\pi}_R^{RR}(0) > \hat{\pi}_R^{RR}(1 - \frac{(k+r)^2\alpha^2}{2h}) = 0\). Also, \(\hat{\pi}_R^{RR''} < 0\) if \(\phi \in [0, 1 - \frac{(k+r)^2\alpha^2}{2h}]\). Therefore, the retailer profit with cost-sharing is highest when \(\phi = 0\).

**Proof of Lemma 3.2**  
The proof follows from comparing the manufacturer’s and retailer’s profit in the RR service channels: when \(2h > \frac{3(k+r)^2\alpha^2}{2}\):

\[
\hat{\pi}_{t,1}^{RR^*} - \pi_t^{RR^*} = -\frac{3h(k + r)^4\alpha^4 [1 - c - (k + r)\alpha]^2}{8 [16h - 9(k + r)^2\alpha^2]^2 [2h - (k + r)^2\alpha^2]} < 0.
\]

when \(2h < \frac{3(k+r)^2\alpha^2}{2}\), we know

\[
\hat{\pi}_{t,2}^{RR^*} - \pi_t^{RR^*} = -\frac{[1 - c - (k + r)\alpha]^2 [3h - 2(k + r)^2\alpha^2]}{16h - 8(k + r)^2\alpha^2}.
\]

It is obvious that if \(2h < \frac{4(k+r)^2\alpha^2}{3}\), then \(\hat{\pi}_{t,2}^{RR^*} > \pi_t^{RR^*}\) and if \(\frac{4(k+r)^2\alpha^2}{3} < 2h < \frac{3(k+r)^2\alpha^2}{2}\), then \(\hat{\pi}_{t,2}^{RR^*} < \pi_t^{RR^*}\)
Proof of Proposition 3.7  
Since the proof of Proposition 3.7 is very similar to the proof of Proposition 3.6, we omit the redundant details of the proof and focus on the difference.

In the cost sharing model, the retailer’s problem is given as:

$$\max_{p, \lambda \in [0, 1]}: \hat{\pi}_{RM}^R(p, \lambda) = [1 - p - \alpha(1 - \lambda)r](p - w) - (1 - \phi)\frac{h\lambda^2}{2}. \quad (B.45)$$

It can be shown that equation (B.45) is joint concave if

$$2h(1 - \phi) > r^2\alpha^2 \text{ or } \phi < 1 - \frac{r^2\alpha^2}{2h}. \quad (B.46)$$

**Case 1: \( \phi < 1 - \frac{r^2\alpha^2}{2h} \)**

If (B.46) is satisfied, we solve (B.45) for \( p \) and \( \lambda \) and substitute them into the manufacturer’s objective function, given as

$$\max_{w, \phi}: \hat{\pi}_{RM}^M(w, \phi) = [1 - p - \alpha(1 - \lambda)r][w - c - k\alpha(1 - \lambda)] - \phi\frac{h(\lambda)^2}{2}. \quad (B.47)$$

By the same method used in the proof of Proposition 3.6, we get the optimal decision in \( w^* \) and \( \phi^* \) for the manufacturer and \( p^* \) and \( \lambda^* \) for the retailer given in Proposition 3.7. Notice that \( \phi^* = \frac{2k + r}{2k + 3r} \), when \( 2h > \frac{r(2k + 3r)\alpha^2}{2} \) the concavity concavity condition holds. The manufacturer’s equilibrium wholesale price and portion of the pre-sales effort are:

$$w^* = \frac{8h(1 + c + k\alpha - 3r\alpha) - [2k + r(1 + 2c - \alpha\alpha)](2k + 3r)\alpha^2}{16h - (2k + 3r)^2\alpha^2}, \quad \phi^* = \frac{2k + r}{2k + 3r}. \quad (B.48)$$

The retailer’s equilibrium selling price and pre-sales effort level are:

$$p^* = \frac{4h(3 + c + k\alpha - 3r\alpha) - (2k + 3r)\alpha^2[2k + r(1 + 2c - \alpha\alpha)]}{16h - (2k + 3r)^2\alpha^2}, \quad \lambda^*_R = \frac{2(2k + 3r)\alpha[1 - c - (k + r)\alpha]}{16h - (2k + 3r)^2\alpha^2}. \quad (B.50)$$

We know from our basic assumption that \( 2h > (k + r)^2\alpha^2 \). It can be shown that \( (k + r)^2\alpha^2 > \frac{r(2k + 3r)\alpha^2}{2} \) if \( k > \frac{(\sqrt{3} - 1)r}{2} \). Thus, as long as \( k > \frac{(\sqrt{3} - 1)r}{2} \), we can make sure that the interior solutions given in Proposition 3.7 is available.

When \( k < \frac{(\sqrt{3} - 1)r}{2} \) and \( 2h < \frac{r(2k + 3r)\alpha^2}{2} \), we get \( \phi = 1 - \frac{r^2\alpha^2}{2k} \). The manufacturer’s profit is

$$\hat{\pi}_{RM, 2}^M = \frac{r^2\alpha^2[1 - c - (k + r)\alpha]^2}{8h - 4r(2k + r)^2\alpha^2}, \quad (B.51)$$
\(\pi_{M}^{RM} - \tilde{\pi}_{M}^{RM} \) is contingent on the condition \((2h - \alpha^2r^2)[4h - 4\alpha^2r(k + r)]\). Since \(4h < r(2k + 3r)\alpha^2\), but \(r(2k + 3r)\alpha^2 - 4\alpha^2r(k + r) = -r(2k + r)\alpha^2 < 0\). Therefore \(4h - 4\alpha^2r(k + r) < 0\) and \(\tilde{\pi}_{M,2}^{RM} > \pi_{M}^{RM}\).

**Case 2:** \(\phi \geq 1 - \frac{r^2\alpha^2}{2h}\)

When \(\phi \geq 1 - \frac{r^2\alpha^2}{2h}\), we can get two boundary solutions

\[
\tilde{\pi}_{M,3}^{RM} = \frac{(1 - c)^2 + 2r^2\alpha^2 - 4h}{8} \quad \text{if} \quad \frac{1 - c}{2} \geq r\alpha, \quad \text{(B.52)}
\]

and

\[
\tilde{\pi}_{M,4}^{RM} = \frac{2(1 - c)r\alpha - r^2\alpha^2 - 2h}{4} \quad \text{if} \quad \frac{1 - c}{2} < r\alpha. \quad \text{(B.53)}
\]

if \(\frac{1 - c}{2} > r\alpha\). Thus we compare \(\tilde{\pi}_{M,3}^{RM}\) with

\[
\tilde{\pi}_{M,1}^{RM} = \frac{2h[1 - c - (k + r)\alpha]^2}{16h - (2k + 3r)^2\alpha^2}. \quad \text{(B.54)}
\]

Let \(\tilde{\pi}_{M,1}^{RM} - \tilde{\pi}_{M,3}^{RM}\) and define \(g_1(h) = 16h[1 - c - (k + r)\alpha]^2 - [16h - (2k + 3r)^2\alpha^2][(1 - c)^2 - 4h + 2r^2\alpha^2]\). And upon simplifying the equation \(g_1(h)\), we can get:

\[
g_1(h) = 64h^2 - 4\alpha[8(1 - c)(k + r) + r(4k + 13r)\alpha]h + [(1 - c)^2 + 2\alpha^2r^2](2k + 3r)^2\alpha^2. \quad \text{(B.55)}
\]

It can be shown that \(g_1(h)\) is a convex function has two roots: \((\bar{h} < \bar{h})\):

\[
\bar{h} = \frac{8(1 - c)(k + r)\alpha + r(4k + 13r)\alpha^2 \pm \sqrt{\alpha^2(4 - 4c + 7r\alpha)[\alpha(16k^2 + 40kr + 17r^2) - 4(1 - c)(4k + 5r)]}}{32}. \quad \text{(B.56)}
\]

We can prove that \(\bar{h} < (1 - c)(k + r)\alpha/2\). Let \(g_2(h) = 16(1 - c)(k + r)\alpha - 32\bar{h}\) and upon simplifying it, we can get \(g_2(h) = \)

\[
8(1 - c)(k + r)\alpha - r(4k + 13r)\alpha^2 - \sqrt{\alpha^2(4 - 4c + 7r\alpha)[\alpha(16k^2 + 40kr + 17r^2) - 4(1 - c)(4k + 5r)]}. \quad \text{(B.57)}
\]

Therefore, we check if \(g_2(h) > 0\). First we prove \(8(1 - c)(k + r)\alpha - r(4k + 13r)\alpha^2 > 0\).

Since \(8(k + r)^2\alpha^2 - r(4k + 13r)\alpha^2 = (8k^2 + 12kr - 5r^2)\alpha^2\). But \((8k^2 + 12kr - 5r^2)\alpha^2 > 0\) if \(k > 4(\sqrt{3} - 3)r\). But we know that for the interior solution, \(k > \frac{(\sqrt{3} - 1)r}{2}\). But \(\frac{(\sqrt{3} - 1)r}{2} > \frac{1}{4} (\sqrt{19} - 3) r\), therefore \(8(k + r)^2\alpha^2 - r(4k + 13r)\alpha^2 > 0\), which proves \(8(1 - c)(k + r)\alpha - r(4k + 13r)\alpha^2 > 0\).

Next, we prove that \(g_2(h) > 0\). Let \(g_3(h) = [8(1 - c)(k + r)\alpha - r(4k + 13r)\alpha^2]^2 - r\alpha^2(4 - 4c + 7r\alpha)[\alpha(16k^2 + 40kr + 17r^2) - 4(1 - c)(4k + 5r)]\). Upon simplifying the
equation and letting $A = 1 - c$, we can get:

$$g_3(A) = 16\alpha^2 \left[ A^2(2k + 3r)^2 - 2\alpha r(k + r)(4k + 13r)A + 2\alpha^2 r^2(2k + 3r)^2 \right]. \quad (B.58)$$

If $k > (\sqrt{2} - 1)r$ and thus $-k^2 - 2kr + r^2 < 0$, then the condition of $B^2 - 4AC < 0$, therefore $g_3(A) > 0$, which makes $g_2(h) > 0$. Thus, $\bar{h} < (1 - c)(k + r)\alpha/2$ and $h > \bar{h}$, which makes $g_1(h) > 0$ and thus $\hat{\pi}_{RM}^M > \hat{\pi}_{RM}^A$.

If $k < (\sqrt{2} - 1)r$ and thus $-k^2 - 2kr + r^2 > 0$, then the condition of $B^2 - 4AC > 0$.

Thus the convex function $g_3(A)$ has two roots: $(A < \bar{A})$:

$$A = \frac{r(k + r)(4k + 13r)\alpha \pm \sqrt{(-k^2 - 2kr + r^2)(16k^2 + 24kr + 7r^2)\alpha^2 r^2}}{(2k + 3r)^2} \quad (B.59)$$

Since $\hat{\pi}_{RM}^M = \frac{(1-c)^2 - 4h + 2r^2\alpha^2}{8}$ when $\frac{1-c}{2} > r\alpha$, thus $1 - c > 2r\alpha$ is a stronger condition when $k < r$. Since $A = 1 - c$, we check if $\bar{A}$ satisfies the condition or not. $\bar{A} - 2r\alpha = \frac{\alpha r\sqrt{(-k^2 - 2kr + r^2)(16k^2 + 24kr + 7r^2) - 4k^2 - 7kr - 5r^2}}{(2k + 3r)^2}$. But $(4k^2 + 7kr + 5r^2)^2 - (-k^2 - 2kr + r^2)(16k^2 + 24kr + 7r^2) = 2(2k + 3r)^2(4k^2 + 2kr + r^2) > 0$. Therefore, $\bar{A} = 1 - c < 2r\alpha$. Thus we need $A = 1 - c > \bar{A}$. As a result, when $A = 1 - c > \bar{A}$, $g_3(A) > 0$, which gives us the result $\hat{\pi}_{RM}^M > \hat{\pi}_{RM}^A$.

Proof of Lemma 3.3

Part a The proof follows from comparing the manufacturer’s and retailer’s profit in the RM channels: $\hat{\pi}_{RM}^M,1 - \pi_{RM}^* = \frac{h^2(2k + 3r)^2\alpha^2[1 - c - (k + r)\alpha]}{4(2k + r)(k + r)^2(16k^2 - 2k + 3r)^2\alpha^2} > 0$, given that $(2k + 3r)^2\alpha^2$ is less than $(k + r)^2\alpha^2$ if $k > \sqrt{\frac{2}{3}}r$.

$\hat{\pi}_{RM}^M,2$ is contingent on the condition $(2h - \alpha^2 r^2)[4h - 4r(k + r)\alpha^2]$. Since $4h < r(2k + 3r)\alpha^2$, but $r(2k + 3r)\alpha^2 - 4r(k + r)\alpha^2 = -r(2k + r)\alpha^2 < 0$. Therefore $4h - 4r(k + r)\alpha^2 < 0$ and $\hat{\pi}_{RM}^M,2 > \pi_{RM}^*$. Thus we can conclude that the manufacturer’s profit is better off, i.e., $\hat{\pi}_{RM}^M,2 > \pi_{RM}^*$. Part b It also can be shown that when $k > \frac{r}{2}$, $\hat{\pi}_R^M - \pi_R^M = \frac{h}{8} \left[ 1 - c - (k + r)\alpha \right]^2 \left( \frac{(4h - \alpha^2 r^2)(2k + 3r)}{16(k + 3r)^2} - \frac{2h\alpha^2 r^2}{2k + 3r} \right) > 0$. Since the proof is similar to the proof given in Proposition 3.2 and case 2 in Proposition 3.7, we thus omit the details here.
Proof of Lemma 3.2 As in the proof of Lemma 3.3 part (b), we can get the retailer’s profit function as function of $\phi$:

$$
\hat{\pi}_{RM}^R(\phi) = \frac{h [1 - c - (k + r)\phi]^2 [2h(1 - \phi) - r^2\phi^2] (1 - \phi)^3}{2 [4h(1 - \phi)^2 - r\phi^2(2k(1 - \phi) + r(2 - 3\phi))]^2} .
$$

(B.60)

It can be shown that $\phi = 1$ is one extreme point, with the other two extreme points between $[0, 1]$. And $\hat{\pi}_{RM}^R(0) > \hat{\pi}_{RM}^R(1) = 0$. Also, $[\hat{\pi}_{RM}^R(\phi)]' > 0$ at $\phi = 0$. Therefore, the retailer’s profit with cost-sharing is increasing when $\phi = 0$. 

Appendix C

Proofs for Chapter 4

Proof of Lemma 4.1

If $q \leq 1/2$, there is no symmetric pure strategy equilibrium exists. No symmetric pure strategy equilibrium can exist with $p_A = p_B = p > q$ because choosing this $p$ gives payoff $qp(1 - \frac{q^2}{2})$, which is strictly less than $qp$. Thus marginally undercutting the anticipated price is a profitable deviation. Similarly, no symmetric pure strategy equilibrium exists with $p < q$ because $(1 - q)p > p/2$. Hence, undercutting is a beneficial deviation. If $q > 1/2$, which is in cases ML, MH, H, no symmetric pure strategy Nash equilibrium exists. The payoff of selecting the same price as the other firm is, at most $p/2$. However, by undercutting, the firm can guarantee a payoff arbitrarily close to $qp$. As $q > 1/2$, the payoff of undercutting is greater.

Proof of Proposition 4.1

First consider a potential equilibrium in which all probability mass is below $q$. If this were an equilibrium, then the highest element of the support must be equal to $q$. Otherwise, a pure strategy just slightly above the highest element of the support gives a higher payoff. The payoff in this mixed strategy equilibrium must therefore be $q^2$. However, if a player chooses to deviate by playing $p = 1 - q$, his expected payoff is $q(1 - q)$, which is greater than $q^2$ whenever $q < 1/2$.

Next consider a potential equilibrium with connected support that spans $q$. Because the payoff function is discontinuous at $q$, in order for the expected payoff of a
pure strategy above to be the same as below $q$, a mass point must exist at $q$. However, in a symmetric equilibrium no common mass point can exist, otherwise a mixed strategy with marginally smaller mass point gives a higher payoff.

![Figure C.1: Case L](image)

Hence, only possibility for an equilibrium with connected support is one in which all mass is strictly above $q$. Given the other firm’s mixed strategy, a firm must be indifferent between all elements of the support of its equilibrium mixed strategy. This indifference condition is:

$$u = p((1 - F(p))q + F(p)q(1 - q)) \Rightarrow F(p) = \frac{pq - u}{pq^2} \quad (C.1)$$

It is straightforward to verify $F(.)$ is increasing. Because $F(.)$ is a cumulative distribution function, its support is $\left[\frac{u}{q}, \frac{u}{q-q^2}\right]$. In equilibrium, the top of the support of
the mixed strategy must be 1; otherwise a marginal deviation up improves the firm’s payoff. Hence \( u = q - q^2 \). Thus, the candidate equilibrium is

\[
F(p) = \frac{p - (1 - q)}{pq}
\]  

(C.2)

with support on \([1 - q, 1]\). To complete the proof, profitable downward deviations must be ruled out. Obviously any price \( p \in (q, 1-q) \) is dominated by the mixed strategy, as such a deviation gives payoff \( qp \) while the mixed strategy gives payoff \( q(1-q) \). Figure C.1 illustrates a downward deviation to some price below \( q \) for the player depicted on the x axis. The support of the proposed mixed strategies is given by the red shaded region. By deviating to a price below \( q \), the deviating player expects

\[
p((1 - F(1 - q + p)) + F(1 - q + p)(1 - q)) = \frac{p(1-q)}{p + 1 - q}
\]  

(C.3)

Observe that

\[
\frac{p(1-q)}{p + 1 - q} - q(1-q) = \frac{(p-q)(1-q)^2}{p + 1 - q} < 0
\]  

(C.4)

Hence, profitable downward deviations are also ruled out.

For the remaining cases, as demonstrated in lemma 4.1, no symmetric pure strategy equilibrium exists. We construct a symmetric mixed strategy Nash equilibrium.

**Proof of Proposition 4.2**

Observe first that \( q \in (1/2, q_{ML}] \Rightarrow \frac{q^2}{1-q} \leq \frac{1-q}{q} < 1 \) and \( \frac{(1-q)^2}{q} < q \) and \( \theta \in [0, 1) \). Furthermore, \( q \in (1/2, q_{ML}] \) also ensures that both distribution functions are increasing over the support. Taken together these conditions ensure that the distributions are well-defined. Also observe that the range of random variable \( P_{ML} = \frac{1-q}{q} - \frac{(1-q)^2}{q} = 1 - q \). We first demonstrate that all prices inside the support of \( P_{ML} \) give the same expected payoff when played for certain against \( P_{ML} \). The expected payoff of playing price \( p \in [\frac{(1-q)^2}{q}, q] \) for certain against \( P_{ML} \) is given by:

\[
(1 - \theta)((1 - F_B(p))qp + F_B(p)\frac{q - q^2}{1 - q + q^2 p}) + \theta qp = (1 - q)^2
\]  

(C.5)
The expected payoff of playing price $p \in \left[ \frac{q^2}{1-q}, \frac{1-q}{q} \right]$ for certain against $P_{ML}$ is given by:

$$(1 - \theta)(q - q^2)p + \theta((1 - F_T(p))qp + F_T(p)(q - q^2)p) = (1 - q)^2$$  \quad (C.6)

Hence, all pure strategies inside the support of $P_{ML}$ give expected payoff $(1 - q)^2$ when played against $P_{ML}$. Next, we demonstrate that no pure strategy outside of the support gives higher expected payoff if played for certain against mixed strategy $P_{ML}$. We do this with the aid of the Figure C.2: As clearly illustrated, a deviation to pure strategies in five distinct intervals must be ruled out.

1. D1: $[0, 2q - 1]$

2. D2: $(q - (1 - q), \frac{q^2}{1-q} - (1 - q)) = (2q - 1, \frac{2q-1}{1-q})$

3. D3: $\left[ \frac{2q-1}{1-q}, \frac{(1-q)^2}{q} \right)$
4. D4: \((q, \frac{q^2}{1-q})\)

5. D5: \((\frac{1-q}{q}, 1]\)

**D1:** Let \(p = 2q - 1 - x\). If \(p \in D1\) then \(x \in [0, 2q - 1]\). Thus,

\[
\hat{u} = \theta p + (1 - \theta)[(1 - F_B(q - x))p + F_B(q - x)qp] \quad (C.7)
\]

\[
\hat{u} = \frac{1}{q^3(q - x)}[q(2q - q^2 + q^3 - 1)x^2 - (2q^5 + 2q^2 - 3q + 1)x + (2q - 1)(1 - 4q + 6q^2 - 5q^3 + 3q^4)] . \quad (C.8)
\]

Hence, \(\hat{u} - (1 - q)^2 = \)

\[
\frac{1}{q^3(q - x)}[(q - 2q^2 + q^3 - q^4)x^2 - (q^5 + 2q^4 - q^3 + 2q^2 - 3q + 1)x + (q^6 - 8q^5 + 14q^4 - 17q^3 + 14q^2 - 6q + 1)] . \quad (C.9)
\]

Because \(x \leq 2q - 1 < q\), the denominator is positive. Thus, provided the term inside the brackets is positive for \(x \in [0, 2q - 1]\), the difference \(\hat{u} - (1 - q)^2 \leq 0\), as required. It is possible to show (numerically) that for \(q \in (1/2, q_{ML}]\) the term in brackets has no root inside \([0, 2q - 1]\), and therefore does not change sign over this interval. The sign at either endpoint therefore gives the sign over the entire interval. Substituting \(x = 0\) yields \(q^6 - 8q^5 + 14q^4 - 17q^3 + 14q^2 - 6q + 1\). A numerical method or plot then establishes that for \(q \in (1/2, q_{ML}]\), the expression is positive.

**D2:** Consulting the graph immediately shows that inside interval \((2q - 1, \frac{2q-1}{1-q})\) the payoff is increasing, and is therefore bounded by the payoff associated with \(\frac{2q-1}{1-q}\). This deviation is shown unprofitable (in the next step) when discussing interval D3.

**D3:** Let \(p = \frac{(1-q)^2}{q} - x\). In order for \(p \in D3\), it must be that \(x \in [0, \frac{(1-q)^2}{q} + \frac{2q-1}{1-q}]\). Among all such \(x\), the highest payoff must be attained by \(x = 0\), because \(\frac{(1-q)^2}{q}\) is a part of the support of \(P_{ML}\) (and gives expected payoff \((1-q)^2\)). The expected payoff associated with such \(p\) is

\[
(1 - \theta)qp + \theta[(1 - F_T(\frac{1-q}{q} - x))p + F_T(\frac{1-q}{q} - x)qp] \quad (C.10)
\]
Substitution and simplification gives

\[-\frac{1}{q(q + qx - 1)}((2q^2 - q)x^2 + (-q^3 + 4q^2 - 4q + 1)x + (-q^4 + 3q^3 - 3q^2 + q))\] (C.11)

The derivative of this payoff with respect to \(x\) is

\[-\frac{1}{q(q + qx - 1)^2}[(2q^3 - q^2)x^2 + (4q^3 - 6q^2 + 2q)x + (q^5 - 4q^4 + 8q^3 - 9q^2 + 5q - 1)]\] (C.12)

Thus, if the expression inside the brackets is positive for \(q \in (1/2, q_{ML}]\), then the deviation payoff is decreasing in \(x\), so that the optimal value of \(x\) is zero, as required.

The discriminant of the quadratic term inside brackets is

\[(4q^3 - 6q^2 + 2q)^2 - 4(2q^3 - q^2)(q^5 - 4q^4 + 8q^3 - 9q^2 + 5q - 1) = -4q^3(2q - 1)(1 - q)^4 < 0\]

Because the discriminant is zero, this expression never changes sign. Therefore, if the expression is positive for \(x = 0\), it is always positive. The derivative of this payoff with respect to \(x\) is

\[q^5 - 4q^4 + 8q^3 - 9q^2 + 5q - 1 = (1 - q)^2(3q - 2q^2 + q^3 - 1)\] (C.13)

Analytical or numerical solution clearly shows that this expression is positive over the required interval. Hence, all pure strategies in \(D3\) give smaller payoff than \((1 - q)^2\).

\[D4: \text{ The payoff of any } p \in (q, \frac{q^2}{1-q}) \text{ is equal to } \theta qp + (1 - \theta)q(1 - q)p \text{ but this payoff is smaller than the payoff of choosing the smallest element of the top region of support, which is just } \theta q(\frac{q^2}{1-q}) + (1 - \theta)q(1 - q)(\frac{q^2}{1-q}) = (1 - q)^2. \text{ Hence, deviations in this interval are unprofitable.} \]

\[D5: \text{ Let } p = \frac{(1-q)}{q} + x. \text{ In order for } p \in D5, \text{ it must be that } x \in [0, 1 - \frac{(1-q)}{q} + \frac{2p-1}{1-q}]. \text{ Among all such } x, \text{ the highest payoff must be attained by } x = 0, \text{ because } \frac{(1-q)}{q} \text{ is a part of the support of } P_{ML} \text{ (and gives expected payoff } (1 - q)^2). \text{ The expected payoff} \]
associated with such \( p \) is

\[
\theta(q - q^2)p + (1 - \theta)(1 - F_B\left(\frac{(1 - q)^2}{q} + x\right))(q - q^2)p
\]

(C.14)

Substituting and simplifying yields

\[
\frac{(1 - q)^2}{q(qx - (1 - q)^2)} \left[(-q^2)x^2 + (-q^3 + q^2 + q - 1)x + q^3 - 2q^2 + q\right]
\]

(C.15)

The derivative of this payoff with respect to \( x \) is

\[
- \frac{(1 - q)^2}{q(qx - (1 - q)^2)^2} \left[q^2x^2 + (2q^3 - 4q^2 + 2q)x + q^5 - 2q^4 + 3q^2 - 3q + 1\right]
\]

(C.16)

If the term in brackets is positive, for the required range of \( x \) then the payoff is decreasing in \( x \), which means \( x = 0 \) is optimal, as required. The discriminant of the quadratic term in brackets is

\[
(2q^3 - 4q^2 + 2q)^2 - 4q^2(q^5 - 2q^4 + 3q^2 - 3q + 1) = -4q^3(1 - q)^2(1 - q + q^3) < 0
\]

Because the discriminant is negative, the quadratic doesn’t change signs on the real line. Hence the sign at \( x = 0 \) is the sign for all real \( x \). At \( x = 0 \) the expression is

\[
q^5 - 2q^4 + 3q^2 - 3q + 1 = (1 - q)^2(1 - q + q^3) > 0
\]

Hence the bracketed term is positive, and the derivative of the payoff is negative for all \( x \). The optimal value of \( x \) is zero, which gives payoff \((1 - q)^2\). Thus, no deviation in \( D5 \) is profitable.

**Proof of Proposition 4.3**

Given the other firm’s mixed strategy, a firm must be indifferent between all elements of the support of its equilibrium mixed strategy. Because \( q \leq q_{MH} \) \( \Rightarrow \)
\[
\frac{q-q^2}{1-q+q^2} \geq 2q - 1,
\]
and as can be seen from the figure below, this payoff is:
\[
p((1 - F(p))q + F(p) \frac{q-q^2}{1-q+q^2}) = q \frac{q-q^2}{1-q+q^2}
\]  \hspace{1cm} (C.17)

Next, we verify that no pure strategy outside of the support gives a higher expected payoff if played against this mixed strategy. As can be seen in the Figure C.3, four types of deviations must be considered:

1. **D1:** \( p \in [0, 2q - 1) \)

2. **D2:** \( p \in [2q - 1, \frac{q-q^2}{1-q+q^2}) \)

3. **D3:** \( p \in (q, \frac{q-q^2}{1-q+q^2} + 1 - q) = (q, \frac{1-q+q^2-q^3}{1-q+q^2}) \)

4. **D4:** \( p \in [\frac{1-q+q^2-q^3}{1-q+q^2}, 1] \)

Figure C.3: Case MH
We rule out profitable deviations in each interval below.

**D1:** Let \( p = 2q - 1 - x \). If \( p \in D1 \) then \( x \in [0, 2q - 1) \). We can get:

\[
\hat{u} = (1 - F(q - x))p + F(q - x)qp \tag{C.18}
\]

Upon simplifying (C.18), we can get

\[
\hat{u} = \frac{1}{q^2(x - q)}\left[(-q^3 + q^2 - 2q + 1)x^2 + (3q^4 - 3q^3 + 5q^2 - 4q + 1)x + (q^4 - 2q^5)\right] \tag{C.19}
\]

The derivative of this expression is given by

\[
\frac{1}{q^2(x - q)^2}\left[(q^3 - q^2 + 2q - 1)x^2 + (-2q^4 + 2q^3 - 4q^2 + 2q)x + (q^5 - 2q^4 + 5q^3 - 4q^2 + q)\right]
\]

If the term in brackets is positive, then the derivative of the deviation payoff is negative, and no deviation inside \([0, 2q - 1)\) is better than choosing \(2q - 1\). The discriminant of the bracketed term is

\[
(-2q^4 + 2q^3 - 4q^2 + 2q)^2 - 4(q^3 - q^2 + 2q - 1)(q^5 - 2q^4 + 5q^3 - 4q^2 + q)]
\]

\[
= 4q(1 - q)^3(1 - 2q + q^2 - q^3) = 4q(1 - q)^3t_{ML}(q) < 0
\]

The final inequality follows because \( q > q_{ML} \Rightarrow t_{ML}(q) < 0 \). Hence the bracketed expression has no real roots. Thus for any value of \( x \) the sign is the same as for \( x = 0 \), which is positive for \( x \in [q_{ML}, q_{MH}] \). Hence the deviation payoff is decreasing in \( x \), and all deviations in \([0, 2q - 1)\) are dominated by \(2q - 1\). The deviation payoff of \( p = 2q - 1 \) is equal to \( q(2q - 1) \), which is less than \( \leq q\frac{q-q^2}{1-q+q} \) for \( q \leq q_{MH} \) (as noted above).

**D2:** All deviations inside \( D2 \) are obviously dominated by playing the smallest pure strategy inside of the support.
**D3:** All deviations inside $D3$ are obviously dominated by playing the maximum value inside $D3$, given by $\frac{1-q+q^2-q^3}{1-q+q^2}$. The payoff of playing this strategy is equal to $\frac{1-q+q^2-q^3}{1-q+q^2}(q - q^2)$. The difference between the deviation payoff and the payoff of the mixed strategy is given by

$$\frac{1-q+q^2-q^3}{1-q+q^2} (q - q^2) - q \frac{q-q^2}{1-q+q^2} = \frac{q-q^2}{1-q+q^2} (1-2q+q^2-q^3) = \frac{q-q^2}{1-q+q^2} t_{ML}(q) \leq 0$$

(C.20)

The final inequality follows because $q \geq q_{ML}$.

**D4:** Let $p = \frac{1-q+q^2-q^3}{1-q+q^2} + x$. We will show that the expected payoff of playing this pure strategy is decreasing in $x$. The optimal value of $x$ is therefore zero. This means that all pure strategies inside $D4$ are dominated by $\frac{1-q+q^2-q^3}{1-q+q^2}$, which is shown to give a smaller payoff than the mixed strategy in the analysis of $D3$, just above. The expected payoff of selecting $p = \frac{1-q+q^2-q^3}{1-q+q^2} + x$ is

$$\hat{u} = (1 - F(\frac{q - q^2}{1-q+q^2} + x))(q - q^2)(\frac{1-q+q^2-q^3}{1-q+q^2} + x).$$

(C.21)

Upon simplifying (C.21), we get

$$\hat{u} = \frac{(1-q)^2}{q(1-q+q^2)} \frac{q^3 - (1-q+q^2)x}{(1-q+q^2)x + q - q^2}((1-q+q^2)x + 1 - q + q^2 - q^3).$$

The derivative of this expected payoff with respect to $x$ is given by the following:

$$-\frac{(1-q)^2[(q^4 - 2q^3 + 3q^2 - 2q + 1)x^2 + (-2q^4 + 4q^3 - 4q^2 + 2q)x + (-q^6 + 3q^5 - 4q^4 + 3q^3 - 2q^2 + q)]}{q((1-q+q^2)x + q - q^2)^2}.$$ (C.22)

Hence, if the bracketed expression is positive, then the derivative of the deviation payoff is negative, and the optimal value of $x$ is zero. As $x = 0$ corresponds to $p = \frac{1-q+q^2-q^3}{1-q+q^2}$, and this deviation has already beed ruled out, establishing that the bracketed terms is positive is all that remains. The discriminant of the bracketed term after simplification is thus given by:

$$-4q(1-q)(1-q+q^2)^4 < 0.$$ (C.23)
Hence, the expression never changes sign. Substituting \( x = 0 \) gives

\[
(-q^6 + 3q^5 - 4q^4 + 3q^3 - 2q^2 + q) = q(1 - q)(t_{MH}(q) + q^4) > 0 \tag{C.24}
\]

The last inequality follows because \( q \leq q_{MH} \Rightarrow t_{MH}(q) \geq 0 \). Therefore, the expected payoff of any pure strategy outside of the support of \( P_{MH} \) is less than the expected payoff of \( P_{MH} \).

**Proof of Proposition 4.4**

Observe that \( q \geq q_{MH} \Rightarrow \frac{t_{MH}(q) + q^4}{q^2} \leq q \). Also observe that the range of random variable \( P_H \) is \( (1 - q^2, t_{MH}(q) + q^3) = 1 - q \). We first show that any pure strategy inside the support of \( P_H \) gives the same expected payoff when played against \( P_H \). As can be seen in the Figure C.4, this expected payoff is given by

\[
p((1 - F(p))q + F(p)\frac{q - q^2}{1 - q + q^2}) = \frac{(1 - q)^2}{q} \tag{C.25}
\]

Next, we show that all pure strategies outside of the support give smaller expected payoff than the mixed strategy. As illustrated in the figure above, two types of deviations are relevant:

1. **D1**: \( p \in [0, (1 - q)^2/q^2) \)

2. **D2**: \( p \in (\frac{t_{MH}(q) + q^3}{q^2}, 1] \)

We eliminate profitable deviations in both cases.

**D1**: Let \( p = \frac{(1-q)^2}{q^2} - x \). The expected payoff of playing \( p \) against \( P_H \) is given by

\[
\hat{u} = (1 - F\left(\frac{(1-q)^2}{q^2} + (1 - q) - x\right)p + F\left(\frac{(1-q)^2}{q^2} + (1 - q) - x\right)qp . \tag{C.26}
\]

Upon simplifying above equation, we get

\[
\hat{u} = \frac{(-q^6 + 4q^5 - 5q^4 + 6q^3 - 4q^2 + q)x + (q^6 - 3q^5 + 5q^4 - 6q^3 + 6q^2 - 4q + 1)x + (-q^6 + 4q^5 - 7q^4 + 7q^3 - 4q^2 + q)}{q^2(q^2p + q^3 - 2q^2 + 2q - 1)}
\]

and the derivative of this payoff is given by the following expression:
Thus if the numerator is positive, then the derivative is always negative, and $x = 0$ dominates all smaller values, as required. The discriminant of the numerator is given by

$$4q^6(1 - q)^4(1 - q + q^2)t_{ML}(q) < 0$$

the last inequality follows from the fact that $q > q_{MH} > q_{ML}$. Thus numerator doesn’t change sign. Evaluated at $x = 0$, the expression is positive. Hence, $x = 0$ is optimal, and the deviation payoff is less than the payoff of playing any value inside of the support.
D2: Let \( p = \frac{t_{MH}(q) + q^3}{q^2} + x \). The expected payoff of playing \( p \) against \( P_H \) is given by

\[
\hat{u} = (1 - F\left(\frac{t_{MH}(q) + q^3}{q^2} + x + 1 - q\right)) \frac{q - q^2}{1 - q + q^2} p.
\]  (C.27)

Simplifying (C.27), we get

\[
\hat{u} = \frac{2q^6 - 7q^5 - q^4x + 11q^4 + 2q^3x - 12q^3 - q^2x + 10q^2 - 5q + 1}{q^3(q^2 - q + 1)}. 
\]  (C.28)

The derivative of this payoff is given by \(-\frac{(q-1)^2}{q(q^2-q+1)}\), which is negative. Thus, Evaluated at \( x = 0 \), we have

\[
\frac{(q - 1)^3(2q^3 - q^2 + 2q - 1)}{q^3(q^2 - q + 1)} > 0
\]

Hence, \( x = 0 \) is optimal, and the deviation payoff is less than the payoff of playing any value inside of the support.
References

References for Chapter 2


Ding, Q., L. Dong, and P. Kouvelis. 2007. On the integration of production and


**References for Chapter 3**


**References for Chapter 4**


