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On the Dynamics of Mesoscale Eddies and Jets in the Presence of Topography

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UNIVERSITY OF MIAMI

ON THE DYNAMICS OF MESOSCALE EDDIES AND JETS IN THE PRESENCE OF TOPOGRAPHY

By
Changheng Chen

A DISSERTATION

Submitted to the Faculty
of the University of Miami
in partial fulfillment of the requirements for the degree of Doctor of Philosophy

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A dissertation submitted in partial fulfillment of
the requirements for the degree of
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ON THE DYNAMICS OF MESOSCALE EDDIES AND JETS IN THE
PRESENCE OF TOPOGRAPHY

Changheng Chen

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The discovery of multiple nearly zonal oceanic jets enriches the classical view of mid-latitude ocean circulation, which was thought to be dominated by turbulent eddies and large-scale oceanic gyres. These jets emerge as a result of baroclinic instability and are maintained by baroclinic eddies. Potential importance of interactions of these ocean currents with major topographic features, such as the mid-Atlantic ridge, calls for the studies of the effects of bottom topography on the dynamics of eddies and the eddy-driven jets.

The importance of bottom topography in the linear baroclinic instability of zonal flows on the $\beta$-plane is examined by using analytical calculations and a quasi-geostrophic (QG) eddy-resolving numerical model. The particular focus is on the effects of a zonal topographic slope, which are compared with the effects of a meridional slope. A zonal slope always destabilizes background zonal flows that are otherwise stable in the absence of topography, regardless of the slope magnitude, whereas the meridional slopes stabilize/destabilize zonal flows only through changing the lower-level background potential vorticity (PV) gradient beyond a known critical value. Growth rates, phase speeds and vertical structure of the growing solutions strongly depend on the slope magnitude. In the numerical simulations configured with an isolated meridional ridge, unstable modes develop on both sides of the ridge and propagate eastward of the ridge, in agreement with our analytical results.
In the second part of this study, we describe a novel mechanism for the generation of oceanic alternating jets by topographic ridges. The dynamics of these jets is examined using a baroclinic QG model configured with an isolated meridional ridge. Zonal topographic slopes of the ridge lead to the formation of a system of currents, consisting of mesoscale eddies, meridional currents over the ridge, and multiple zonal jets in the far field. Dynamical analysis shows that transient eddies are vital in sustaining the deep meridional currents, which in turn play a key role in the upper-layer PV balance. The zonal jets owe their existence to the eddy forcing over the ridge but are maintained by the Reynolds and form stresses in the far field. Locally nonzonal mean currents are shown to produce zonal jets in the far field, but the presence of local vorticity source appears to be the truly fundamental cause of jet existence. We conclude that local nonzonality of the mean PV contours, due to either a topographic ridge or a nonzonal background flow, can generate multiple zonal jets through a remote mechanism.

Lastly, we explore the relationship between eddies and striations in a baroclinic double gyre ocean. Results show that alternating vertically coherent striations exist in the eastern part of the double gyre in short time mean field. Both the baroclinic QG model simulation and the analysis of altimetry sea level anomalies suggest that striations are intrinsically associated with eddy trains. Eddy vorticity forcing is closely associated with the striations and tends to drive the striations to drift meridionally. The concept of eddies being intrinsically related to striations helps to come up with a better parameterization scheme to represent the effects of eddies/striations in comprehensive models.
To my parents, Qiju Han and Shujing Chen.
I owe my sincere gratitude to many people for helping me complete this dissertation.

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Chapter 1

Introduction

Ocean circulation is like a photographic mosaic. When viewed as a whole, it appears to be delineations of large-scale gyres, while close examination reveals that the small sections of the mosaic are depictions of mesoscale and submesoscale eddy turbulence. However, physical processes, whose spatial scale is between that of the large-scale circulation and turbulent eddies, remain elusive. A careful examination of the elusive features becomes possible with the advent of long time series of satellite data. A significant discovery is the ubiquitous existence of multiple alternating jets in the oceans (Maximenko et al. 2005; Maximenko et al. 2008).

The oceanic jets are localized, intense, energetic flows that often preferentially channel most of the kinetic energy in the oceans into zonal direction. An important implication of this comes from the adjustments that must be made to the ways of parameterizing unresolved motions in coarse resolution ocean models. Meanwhile, high horizontal shear associated with the jets has profound implications for tracer transport and mixing. The oceanic jets can act as flexible mixing barriers, inhibiting tracer transport across their axes while enhancing it along them.

A number of mechanisms for jet formation have been proposed, most of them rely on the presence of a large-scale vorticity gradient. The vorticity gradient precon-
ditions large-scale flows for instability (e.g., Pedlosky 1987) and controls nonlinear evolution of eddying flow and the formation of eddy-driven flows, such as jets. In some regions of the ocean, such as the Mid-Atlantic Ridge, potential vorticity (PV) variation produced by topographic effects outweighs the planetary contribution. The vorticity gradient due to seafloor topography can induce jet flows and potentially influence jet characteristics and tracer transport properties. Indeed, topography has been shown to affect jet formation and to alter jet orientation, jet width and tracer transport (Thompson 2010; Thompson and Sallée 2012; Boland et al. 2012).

The goals of this study are to expand and enrich our understanding of quasi-geostrophic (QG) baroclinic instability theory by considering the effects of bottom topography, and to investigate the dynamics of mesoscale eddies and jets in the present of topography. The rest of this chapter discusses our current understanding of oceanic jets, highlights the effects of topography on jet dynamics from previous studies, and states opens questions to be addressed in this study.

1.1 Discovery of jets in the oceans

Two distinct types of oceanic jet-like features are detected from satellite altimetry sea level anomalies (SLA) and mean dynamic topography (MDT). Maximenko et al. (2005) discovered ubiquitous time-varying jets in the oceans from SLA. Specifically, they assume that three components of motion exist in SLA: large-scale time-mean flow, eddies, and structures hidden behind SLA. The authors removed eddy noise by taking average of the SLA over a period of 18 weeks. The resultant SLA is thus “eddy-free” and only contains the hidden structure and large scale features, such as ENSO. The hidden structure emerges when taking differentiations along horizontal coordinates of the “eddy-free” SLA and appears zonally elongated in the zonal geostrophic velocity (Fig. 1.1) and geostrophic vorticity fields. The structure is de-
fined as zonal jets, which are time-dependent. The second type of jets, as shown in Fig. 1.2, emerge in zonal velocity fields derived from a 10-year MDT after removing signals with a two-dimensional filter with a half-width of 4° (Maximenko et al. 2008). Due to their persistence in time they are referred to as stationary jets.

**Figure 1.1:** 18-week average around the period centered on September 10, 1997 of zonal geostrophic velocity (in cm s\(^{-1}\)), derived from Aviso 10-year anomalies of sea level (Maximenko et al. 2005).

Little is known about the connection between these two types of jet-like features (Tab. 1.1). As illustrated in Fig. 1.1, the strength of the time-dependent zonal jets is as large as \(O(10 \text{ cm s}^{-1})\), in contrast to \(O(1 \text{ cm s}^{-1})\) for the stationary jets. The temporal scale of the stationary jets is comparable to that of the large-scale gyre circulation, considering their robustness in the 10-year MDT. The time-dependent jets are intermediate in temporal scale between mesoscale eddies and large-scale gyre circulation (Richards et al. 2006). These two types of jets, therefore, may represent different physical phenomena and could arise from disparate mechanisms.

Jet-like structures are also found in *in situ* datasets. Maximenko et al. (2008) used XBT and float data of the World Ocean Database to validate the existence of
Table 1.1: Non-stationary jets vs stationary jets

<table>
<thead>
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<th></th>
<th>Signal Source</th>
<th>Strength (cm s(^{-1}))</th>
<th>Temporal Scale</th>
</tr>
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<tr>
<td>Non-Stationary Jets</td>
<td>18-week SLA</td>
<td>10</td>
<td>weeks to months</td>
</tr>
<tr>
<td>Stationary Jets</td>
<td>12-year MDT</td>
<td>1</td>
<td>at least a few years</td>
</tr>
</tbody>
</table>

stationary jets in MDT. The authors showed that jet-like features in XBT profiles are coherent with those in MDT in the eastern parts of the North and South Pacific. van Sebille et al. (2011) detected alternating quasi-zonal jets in the North Atlantic from time-mean zonal velocities estimated from Argo float data. They found that the jets tilt in the horizontal plane, but show no significant tilt in the vertical; velocities and widths are similar for westward and eastward jets. By analyzing upper ocean temperature from Argo floating profiles, Buckingham et al. (2014) found quasi-zonal band fronts in the subtropics, which validates their finding of similar flow patterns from multiyear (2002-2011) microwave SST.
1.2 Features of oceanic jets

In general, oceanic jets are between mesoscale eddies and the long time-mean flow in spatial and temporal scale and have an intensity up to $O(10 \text{ cm s}^{-1})$, which is strong enough to dominate the circulation (Richards et al. 2006). The following description of the features of oceanic jets is based on the results of analysis of observational data and numerical studies.

In the inner ocean basins, jets extend zonally for a few hundred kilometers or even across ocean basins; their meridional width is about 300 km and decreases poleward. The jets are vertically coherent at middle latitudes and most energetic at the surface; their signatures are still distinct at the depth of 1 km (Maximenko et al. 2005). They are less vertically coherent in the tropics (Richards et al. 2006). Temporally, the jets are robust and persistent for up to a few years (Richards et al. 2006) but without obvious seasonal variations (Maximenko et al. 2005). In addition, jets in the interior ocean basins are latent in the sense that they only appear in time-averaged zonal geostrophic velocity and geostrophic vorticity fields.

Jets in the Southern Ocean exhibit peculiar properties. The jets manifest themselves as multiple discontinuous filaments that are narrower and less latent than those in the inner basins. They appear explicitly in instantaneous velocity fields computed from SLA (Sokolov and Rintoul 2007) and in eddy-permitting numerical models (Lee and Coward 2003).

These filamentary jets are easily confused with the Antarctic Circumpolar Current fronts, traditionally viewed as circumpolar features manifested in the regions of great horizontal gradients in water mass properties (e.g. Orsi et al., 1995). Sokolov and Rintoul (2007) tried to make a connection between the filamentary jets and the Antarctic Circumpolar Current (ACC) fronts by explaining that the narrow jets are branches
of the major ACC fronts. However, the jets and fronts are probably coexisting flow features with different temporal and spatial scales.

Oceanic jets are non-zonal in certain regions of the ocean. Maximenko et al. (2005) revealed that the time-averaged jets from SLA in the Gulf Stream extension are tilted. Maximenko et al. (2008) also found stationary jets emanating from the eastern Pacific tilt toward the equator. The tilt may be due to the meridional advection by the wind driven gyre (Maximenko and Niiler 2006). On the contrary, off the eastern North Pacific, Ivanov et al. (2009) apply a discrete wavelet transform for a 16-year SLA, revealing that robust quasi-zonal jets over that region do not show apparent tilt.

1.3 Are these jet-like structures real jets or artifacts of propagating eddies?

It is a universal phenomenon that turbulent flows on rotating planets spontaneously organize themselves into zonal jet-like features, with the “stripes” at the cloud level on Jupiter being a well-documented example (Kondratyev and Hunt 1982). Turbulent flows in the Earth’s oceans are no exception. Nevertheless, the detection of jets in the ocean is challenging.

Oceanic jets are latent in nature. They emerge in satellite altimetry data only when appropriate temporal and spatial filtering are applied. It is the lack of the knowledge of their spacial and temporal scale makes the detection difficult, and explains why two distinct types of jets are discovered when different filtering procedures in time and space are chosen. Maximenko et al. (2005) discovered time-varying jets from SLA. SLA is the high-pass filtered absolute dynamic topography (ADT) in space; time-varying jets become explicit after low-pass filtering in time—time-average over 18 weeks—is applied to SLA. The second type is stationary jets (Maximenko et al.
2008). They are the result of high-pass filtered MDT in space; MDT itself is the result of low-pass filtered ADT in time.

Also, mesoscale eddies may mask jet signatures. Mesoscale eddies are known to move westward. Their pathway is consistent with the jet orientation and can be mistakenly recognized as zonal jets. Schlax and Chelton (2008) argued that zonal jets discovered in satellite altimetry data are likely to be artifacts of propagating mesoscale eddies. They use a model for westward propagating Gaussian eddies with random starting times and locations, which has successfully reproduced zonally elongated features that resemble the zonal jets found in Maximenko et al. (2005). However, Ivanov et al. (2012) applied a discrete wavelet transform on satellite altimetry data and found jets. The authors argued that these jets are not artifacts of eddies based on three criteria: the strength of the zonal jets is stronger than that of the artifacts; jet axes are not always parallel to the paths of propagating mesoscale features; and generation of zonal jets results from phase synchronization, which does not exist in the artifacts. Buckingham and Cornillon (2013) showed that eddies are a dominant source to zonal velocity of striation in the South Pacific derived from 4-year mean ADT.

1.4 Jets in numerical simulations

Numerical simulations have successfully reproduced the observed oceanic jets. Distinct jets readily form in QG models of a single unstable current. Barotropic zonal jets alternating in direction with latitude are observed in a double periodic QG model forced by a vertically sheared eastward zonal background flow; and their meridional widths coincide with the Rhines scale (Panetta 1993). Berloff et al. (2009a) took a step forward to study the mechanism of jet formation in a QG model close to that of Panetta (1993). The simulated jets have strong baroclinic signature, and they arise
from the instabilities of basin modes and the interactions between these modes. The correspondence of the jets’ width to the Rhines scale, however, is not confirmed. Using a barotropic QG model of an unstable eastern boundary current in a meridional channel, Wang et al. (2012) demonstrate that the radiating instability of the eastern boundary current could generate quasi-zonal features reminiscent of jets derived from observations (e.g. Maximenko et al., 2008).

Jets also form but are latent in double-gyre QG models. For the first time in a barotropic QG box model of a mid-latitude basin, Nadiga (2006) showed that alternating zonal jets arise from the arrest of the inverse energy cascade by free Rossby waves and the subsequent channeling of energy into zonal modes, in agreement with the mechanism of jet formation proposed by Rhines (1975). The zonal jets are stronger in the west of the basin. The underlying mechanism for the enhanced jet signature in the western basin is not clear. In a two-layer, wind-driven, shallow-water model, Tanaka and Akitomo (2010) found the ubiquitous presence of alternating zonal flows in the double-gyre circulation and found that in the subpolar western boundary region the zonal flows are essentially Rhines jets, and that the zonal flows in the eastern basin the zonal flows arise from the nonlinear effect of Rossby basin modes. O’Reilly et al. (2012) used a two-layer QG basin model to simulate jet formation in a double gyre. They show that stochastic wind forcing resembling the North Atlantic Oscillation can excite Rossby waves at the east side of the basin, and these waves are unstable and able to generate zonal jets throughout the basin.

Eddy-permitting models are able to reproduce oceanic jets close in nature to those existing in the real ocean. Attempts have been made to examine the three-dimensional structure of oceanic jets in eddy-permitting models. Nakano and Hasumi (2005), in an eddy-permitting ocean general circulation model of the Pacific Ocean, found zonal jets with the meridional scale of $3^\circ-5^\circ$ below the main thermocline. Similar zonal
jets with strong vertical coherence are also observed in Maximenko et al. (2005) and Richards et al. (2006). The dynamics and tracer transport properties of the zonal jets also have been studied. Kamenkovich et al. (2009b) demonstrated that multiple zonal jets form in an eddy-resolving primitive equation model and eddies play a central role in supporting these jets. In an accompanying paper, Kamenkovich et al. (2009a) showed that tracer transport in the zonal direction far exceeds that in the meridional direction, and this anisotropy is primarily due to transient eddies and not to the shear dispersion associated with zonal jets. Despite this, jets can serve as flexible material transport barriers, favoring the transport of tracers along their axes. Due to their propensity to advect zonal temperature gradients, the zonal jets, simulated in a coupled atmosphere-ocean general circulation model, can generate sea surface temperature anomaly, which induces wind stress anomaly; the meridional gradient of this wind stress in turn can reinforce the original zonal jets (Taguchi et al. 2012).

1.5 Mechanisms of the formation of oceanic jets

A number of mechanisms have been proposed for jet formation.

Turbulence theory. This theory of zonal jet formation in geostrophic turbulence was first proposed by Rhines (1975) and further expounded in later studies (e.g. Vallis and Maltrud 1993, Danilov and Gryani...
jets emerge subsequently. The characteristic scale is called the Rhines scale, $L_R = (2U/\beta)^{1/2}$, where $U$ is the rms velocity of the flow and $\beta$ is a meridional gradient of planetary vorticity.

Studies have applied this theory to explain the formation of oceanic jets (Galperin 2004, Nadiga 2006). The correspondence of the Rhines scale to meridional width of oceanic jets is not conclusive. Panetta (1993) showed that the meridional widths of jets observed in a double periodic QG model coincide with the Rhines scale; other studies (Maximenko et al. 2005, Berloff et al. 2009a) do not observe such a relation. Galperin (2004) found the energy spectrum of oceanic jets obeys the same power law as that of zonal flows on the outer planets and argue that it is plausible that oceanic jets are dynamically similar to those observed in the atmospheres of giant gas planets, such as Jupiter. It is important to acknowledge that the atmosphere of Jupiter is weakly stratified and highly rotating, while the Earth’s oceans undergo a strong stratification and a weak rotation effect as measured by the Burger number being far greater than one. This difference may be attributable to the fact that oceanic jets are latent while those on Jupiter are explicit. The oceanic jets may arise from different mechanisms.

“PV Phillips effect". Baldwin et al. (2007) and other studies (Dritschel and McIntyre 2008, Wood 2009) related jet formation to the central role of PV as a tracer. Planetary rotation serves as a vast reservoir of PV, with a north-south gradient that supports Rossby wave propagation mechanism. Due to the PV gradient, small amplitude disturbances of appropriate wavelengths tend to generate Rossby waves instead of irreversibly mixing the PV. This resilience of PV to being mixed is analogous to the response of a cluster of particles in an elastic medium, in which the particles vibrate back and forth instead of irreversibly deforming. This phenomenon is called “Rossby wave elasticity".
Strong mixing can weaken PV gradient, which is followed by weaker Rossby wave propagation mechanism and reduced Rossby wave elasticity, which in turn facilitates further mixing. Since the large-scale PV gradient is conserved, in the less mixed regions the PV gradient becomes larger and hence greater Rossby wave elasticity. And these regions are less susceptible to further mixing. This inhomogeneous mixing creates a profile in latitude of alternating steep and weak PV gradients, called PV staircase. The velocity profile deducted from this PV field through PV inversion corresponds to thin, fast eastward jets located at the latitudes of the steepest PV gradients.

Mechanisms constraining the staircase structure are not clear, neither are those setting the meridional width of these staircases. Berloff et al. (2009a) obtained zonal jets in a QG channel model, instead of observing a PV staircase, they got a “PV washboard” pattern.

**Modulational instability.** Zonal jets occur for the modulation of a primary Rossby wave, which transfers energy into zonal flows. Berloff et al. (2009a) showed that meridional Rossby waves arising from baroclinically unstable background flows in a two-layer QG model go through a secondary instability, in which the most unstable modes are perpendicular to the primary Rossby wave fronts. These secondary zonal modes correspond to zonal jets. Connaughton et al. (2010) further showed that if the primary meridional Rossby waves are weak, the most unstable secondary modes are non-zonal. This off-zonal inclined modulation may help explain the discovery of slanted jets in observations (Maximenko et al. 2008).

**Rossby wave instability.** O’Reilly et al. (2012) attributed the emergence of zonal jets in the ocean interior to triad interactions between baroclinic and barotropic Rossby waves. They investigate the response of a baroclinic double-gyre QG model to stochastic wind forcing. The primary phenomenon observed is the excitement of
both barotropic and baroclinic Rossby waves, emanating from the eastern boundary. A triad interaction between a baroclinic wave, a second baroclinic wave and a barotropic wave follows; the latter waves grow at the expense of the potential energy of the first wave. The barotropic wave has an asymmetric horizontal structure—it is zonally elongated. As the waves propagate westward, this zonal signature or zonal perturbation intensifies, eventually manifesting themselves as zonal jets on top of the double gyre circulation.

**Radiating instability.** This theory simply states that zonal jets close to the oceanic eastern boundary can occur for nonlinear radiating instability of an eastern boundary current. Hristova et al. (2008) explore the radiating instability of both western and eastern meridional boundary current. They find an unstable eastern boundary current, unlike a western boundary currents, supports a great number of persistent radiating wave modes. The fact that these radiating wave modes have large zonal extent may account for the emergence of zonal jets extending from the eastern boundary current to the interior of the ocean.

Wang et al. (2012) explain the persistence of these radiating wave modes in terms of triad resonance. In a barotropic QG model of an eastern boundary current, they find a short-wave mode (SM) trapped at the eastern boundary current region and a long-wave mode (LM) radiating from the current into the interior. LM is a decaying mode and can not support persistent zonal jets. These two modes satisfy a triad resonance condition that $l_{SM} + 2l_{LM} = 0$ and $\omega_{SM} + 2\omega_{LM} = 0$, where $l$ is meridional wavenumber and $\omega$ is frequency. As such, LM is sustained by interacting nonlinearly with SM via triad resonance and consequently sustains zonal jets.

**The $\beta$-plume mechanism.** A $\beta$-plume is a zonally-elongated, gyre-like response to a localized vortices source or sink and is established by the emission of low-frequency Rossby waves (Stommel 1982). The dominant zonal currents, found
west of the source or sink, are interpreted as zonal jets. Stommel (1982) first introduced this mechanism to explain the formation of a mid-depth, westward extending circulation west of the South Pacific Rise. Two well-known examples of $\beta$-plume induced flows are the Azores Current and the Hawaiian Lee Countercurrent. The outflow and sinking of Mediterranean water from Gibraltar acts as weak local vorticity forcing in the east, which drives a steady cyclonic jet-like circulation, with its south branch resembling the Azores Current (Özgökmen et al. 2001, Maximenko et al. 2012). The Hawaiian Lee Countercurrent originates from a similar mechanism, but the vorticity forcing is the wind stress curl dipole in the lee of Big Island of Hawaii (Maximenko et al. 2012; Belmadani et al. 2013; Davis et al. 2014). Afanasyev et al. (2012) extended this mechanism to explain the ubiquitous presence of zonal jets throughout the ocean shown in observations.

In conclusion, these mechanisms are unlikely to be exclusive in practice, and it is plausible that more than one are at play in the ocean under certain conditions.

1.6 Why oceanic jets are important?

The existence of oceanic jets is important at least in three ways. The first relates to the parameterization of unresolved motions. Unresolved motions in the ocean is usually parameterized based on the assumption that the surface eddy kinetic energy is isotropic. The fact that eddying flows prefer channelling energy into zonal direction to form zonal jets may prove this assumption inaccurate. The intertwined relation between mesoscale eddies and jets complicates the way to parameterize the unresolved motions. Jets owe their existence and persistence to mesoscale eddies in most of the proposed mechanisms; this fact renders an assumption that if mesoscale eddies are parameterized accurately, consequently, jets will be resolved.
The second one concerns tracer transport and mixing. The meridional variance of zonal velocity of jets is far greater than the zonal variance. The direct effect of this difference leads to inhibited mixing across the jets and enhanced mixing along jet axes. A summary of the dynamical systems theory on the core of a jet acting as a barrier for the cross-jet transport is given in (Kamenkovich et al. 2014). This anisotropic mixing influences the large scale distributions of salinity, temperature, and other properties in the ocean. Smith (2005) found tracer transport along coherent jets in two-dimensional turbulent flow is much larger that the corresponding across-jet transport. Other studies (e.g. Dritschel and McIntyre 2008) showed that in barotropic flows, the cores of the eastward jets act as transport barriers, while the westward jets often correspond to broad stirring zones. Berloff et al. (2009a) showed that for baroclinic system, whether eastward (westward) jets correspond to mixing barriers (stirring zones) depends on the direction of the background flow and the vertical layer in which the jets reside. In eddy-resolving numerical model, Kamenkovich et al. (2009a) also found the material transport in the zonal direction far exceeds that in the meridional direction, this anisotropy, however, is related to transient eddies but not to zonal jets. The authors separated the effects of eddies and jets by removing the time-mean advection (eddy-only simulations), revealing that it is eddies not jets that induce the anisotropy in tracer transport.

Lastly, oceanic jets have important implications for climate predictions. In coarse ocean models used for climate prediction, mesoscale eddies and jets are not resolved. It is expected that the next generation climate models that possess the capability to resolve jets, whose impact may be crucial in determining the climate.
1.7 Effects of topography on oceanic jet properties and formation

In this section a brief illustration of the topographic steering effect on westerly and easterly flows will be presented, and a summary of studies about topographic influence on oceanic jet characteristics and formation is given.

Topography steers ocean flows. In the ocean flows encountering topographic obstacles are subject to constraint by conservation of absolute vorticity, \((\zeta + f)/h\), where \(\zeta\) is relative vorticity of the flow, \(f\) is the Coriolis parameter and \(h\) is depth of the fluid column. To the extent that oceanic flows are barotropic, they tend to follow contours of constant \(f/h\) (Pedlosky 1987).

Topographic steering effect is well illustrated in Holton (2004) with a simple example of a westerly/easterly airflow impinging on an infinitely long meridional ridge. A westerly uniform flow will lead to an alternating cyclonic and anticyclonic rotations downstream of the ridge; as a consequence, the flow leaves a wave-like trajectory. In contrast, an easterly wind does not show any deviation from its original trajectory after it crosses the ridge due to \(\beta\)-effect. In reality, since the ocean is stratified and velocities tend to be faster near the ocean surface than at mid-depth, oceanic flows do not literally follow contours of \(f/h\). Even so, topography can determine the path of surface intensified flows, since ocean currents tend to be vertically coherent.

Jet formation in geostrophic turbulence relies on the presence of a large-scale vorticity gradient. In some regions of the ocean the PV variation produced by topographic effects can dominate the planetary contribution. The existence of the Mid-Atlantic Ridge proves this point (Fig. 1.3). The Ridge elevation is 1–3 km, and its width is about 1500 km; therefore, its slopes, \(s\), can be roughly estimated to be \(\pm 0.7–2.0 \times 10^{-3}\) on its two sides. Assuming a two-layer ocean with an upper layer,
Figure 1.3: Topography of the ocean floor from 2-minute Gridded Global Relief Data (ETOP02) v2 (National Geophysical Data Center, 2006).

$H_1$ of 1 km and a motionless lower layer, $H_2$ of 3 km at midlatitude of 45°N, the PV gradient due to the slope of the Mid-Atlantic Ridge, $s$, is then $fs/H_2 = 1.03 \times 10^{-4} s^{-1} \times 10^{-3}/3000 \text{ m}=3.43 \text{ m}^{-1} s^{-1}$, which is about two times of the planetary one at this latitude, which is $1.62 \text{ m}^{-1} s^{-1}$.

Topography is a significant factor in the dynamics of oceanic jets and eddies. By analyzing observations, Sokolov and Rintoul (2007) showed that bottom topography greatly influences the path, width, and intensity of jets in the Southern Ocean. Idealized studies further demonstrated the effects of topography: Multiple topographic bumps affect jet spacing, variability, and meridional transport properties (Thompson 2010); the number of jets varies along a zonally asymmetric ridge (Thompson and Sallée 2012); baroclinic jets are tilted in the presence of a zonal topographic slope and are nearly perpendicular to the barotropic PV gradient (Boland et al. 2012); eddy kinetic energy is enhanced downstream of isolated topographic features and zonally asymmetric ridges (Witter and Chelton 1998; Thompson and Sallée 2012).
Meridional ridges also can strongly influence the mean circulation. Vallis and Maltrud (1993) found multiple jets along meridional topographic ridges on the $f$-plane and concluded that these jets result from the anisotropic turbulent inverse energy cascade due to topographic $\beta$-effect. In an idealized ACC model configured with meridional ridges, Treguier and Panetta (1994) observed two zonal jets with enhanced structure and quasi-stationary meanders leeward of the ridges and found that the jets transport is reduced due to the ridges. MacCready and Rhines (2001) studied meridional eddy transport in the presence of a meridional ridge and found that the transport increases at and downstream of the ridge. They also observed multiple zonal jets downstream of the ridge but did not explore them.

1.8 Open questions in jet dynamics in the presence of topography

As discussed above, two kinds of jet-like feature have been discovered in satellite altimetry data; however, little is know about the connection or discrepancy between the two. Studies debated whether jets are artifacts of insufficient averaging of eddies (Schlax and Chelton 2008; Ivanov et al. 2012); while Buckingham and Cornillon (2013) showed that eddies contribute greatly to the existence of striations. A necessary clarification on the definition of jets requires a better understanding of the relation between eddies and jets.

Thus, we devote part of chapter 4 to investigate whether propagating eddies can induce jet-like flows with an idealized kinematic model. We introduce westward propagating random eddies into the model to validate the study of Schlax and Chelton (2008); resultant flows in this experiment dose not show jets. In a contrast experiment, we introduce eddy trains—a set of eddies propagating along certain latitudes—and
do successfully produce jets. These results helps to bridge between the two distinct jets from altimetry data and suggests that they are likely to be essentially the same phenomena.

Oceanic jet formation in the presence of topography also poses questions. Generation of mesoscale motions, such as oceanic jets, in the World Ocean is characterized by several stages, each involving interactions and energy transfer between motions on various spatial scales. The initial stage of eddy development in a large-scale background flow is traditionally described through the rapid growth of infinitesimal normal-mode perturbations, which can, due to their spatial structure, efficiently extract energy from large-scale background state (e.g. Pedlosky, 1987). The corresponding linear instability theory can provide important insights into the spatial structure of the most rapidly growing (“most unstable”) modes. This initial stage is followed by nonlinear development, during which interactions between linear modes become important. These interactions and the corresponding energy transfer are in large part determined by properties of dominant linear modes. For this reason, linear analysis is an important stepping stone for understanding the dynamics of fully nonlinear eddying flows and eddy-driven jets.

The above mechanism of eddy generation relies on the presence of a mean PV gradient, which preconditions large-scale currents for instability (e.g. Pedlosky, 1987) and controls nonlinear evolution of eddying flow. Bottom topography can modify background PV gradient and thus play an important role in the dynamics of eddy formation, as well as the stability of background flows. Several previous studies recognized the importance of the combined effect of a background flow and topographic slopes for baroclinically unstable flows (Blumsack and Gierasch 1972; Mechoso 1980; Isachsen 2011; Pennel et al. 2012). Hart (1975a,b), in a two-layer QG model on the $f$-plane, showed that a unidirectional zonal slope has a destabilizing effect on a circu-
lar gyre and generates an asymmetric mean current. A meridional slope can produce small-scale fluctuations in vertically sheared zonal flows (Steinsaltz, 1987). More complex topographic configurations lead to additional dynamical effects. In particular, Samelson and Pedlosky (1990) demonstrated that a zonal flow can become locally unstable due to the presence of a localized meridional slope. Small-scale topography can also significantly impact stability of zonal currents (Reznik and Tsybaneva 1999; Benilov 2001). In the absence of background flows, Samelson (1992) found that free wave modes become surface-intensified in a two-layer QG model with a small-scale topography, whose horizontal scale is similar to that of the waves. Hallberg (1997) demonstrated that strong topography significantly impacts vertical structure of linear waves and effectively eliminates the barotropic component of the flow, and that the motions in the two layers are coupled when the topographic and planetary vorticity gradients are parallel or anti-parallel.

Nonlinear development of eddying flows in the presence of topography can lead to the formation of multiple zonal jets. The dynamics of these multiple zonal jets strongly rely on the action of mesoscale eddies (e.g. Panetta 1993; Rhines 1994; Berloff et al. 2009b; Kamenkovich et al. 2009b; Melnichenko et al. 2010). In particular, Berloff et al. (2009a) demonstrated that jet formation is triggered and controlled by the emergence of linear, meridionally oriented (“noodle”) modes, whose structure determines several important properties of final nonlinear solution.

Thus, to expand the theory of QG baroclinic instability with topography and to follow the same line of previous studies, chapter 2 employes the classical Phillips model to investigate the effects of bottom topography on linear baroclinic instability, as a stepping stone to interpret the formation of zonal jets in the presence of topography. The initial linear stage is an essential element in the process of eddy and jet formation. The particular focus of this chapter is on the effects of a zonal topographic slope,
which are compared with the effects of a meridional slope. The effects are examined in terms of growth rates, phase speeds and vertical structure of unstable modes.

Chapter 3 proceeds to study nonlinear dynamics of eddies and jets in the presence of topography. It describes the formation of a system of currents in the same baroclinic QG model configured with an isolated meridional ridge. A novel mechanism for the generation of alternating oceanic jets by topographic ridge is proposed. In addition to the idealized kinematic experiments to examine the relation between eddies and jets, chapter 4 also investigates the dynamics of eddies and jets in the circulation in a baroclinic, wind-driven, double-gyre, QG model.

The dissertation concludes with chapter 5, giving a summary of this study and proposing possible questions for future work.
Chapter 2

Effects of Topography on Baroclinic Instability

In this chapter, we examine the influence of topographic variations on the linear baroclinic instability of a uniform zonal flow. Our particular focus here is on the effects of zonal topographic slopes; an analysis of meridional slopes is carried out for comparison. This initial linear stage is an essential element in the process of eddy and jet formation, and linearized models have been proven to be useful in describing complex nonlinear interactions (Berloff et al. 2009a).

This chapter is organized as follows: The model is described in section 2.1; a necessary instability condition and the dispersion relation for unstable modes in the presence of constant topographic slopes (zonal and meridional) are derived in section 2.2, where we also discuss the dependence of the most unstable mode on the direction of the mean potential vorticity (PV) gradient; in section 2.3, we use a numerical version of the linear model to study the relevance of the constant-slope results of section 2.2 to more realistic configurations with a meridional topographic ridge; conclusions are drawn in section 2.4.
2.1 The model

We consider a two-layer quasi-geostrophic (QG) model with bottom topography on the $\beta$-plane. PV, $q_n$, in each of the two dynamically active isopycnal layers is governed by

$$\frac{\partial q_n}{\partial t} + J(\psi_n, q_n) = 0 \ (n = 1, 2), \quad (2.1)$$

where the layer index starts from the top and $\psi_n$ is the streamfunction in the $n^{th}$ layer. $J(,) \; \text{is the Jacobian operator.}$

We are interested in the baroclinic instability of large-scale zonal ocean currents with a vertical shear, and consider a horizontally uniform flow $U$ in the upper layer and a motionless lower layer:

$$\psi_1 = \varphi_1 - U y, \quad \psi_2 = \varphi_2, \quad (2.2)$$

where $\varphi_1, \varphi_2$ describe disturbances.

The isopycnal potential vorticities consist of several components: relative vorticity of disturbances, $\beta$-term, and the stretching terms due to the mean flow, disturbances and topography. They are presented in sequence in the following equation:

$$q_n = \nabla^2 \varphi_n + [\beta_0 - (-1)^n F_n U] y + (-1)^n F_n (\varphi_1 - \varphi_2) + \delta_{n2} f_0 \frac{\eta_b(x, y)}{H_2}, \quad (2.3)$$

where $\beta_0$ is the planetary vorticity gradient, $H_n$ are the depths of the upper and lower layers, $\eta_b(x, y)$ is the spatially varying elevation of bottom topography, $F_n = f_0^2/(g' H_n)$, $f_0$ is the Coriolis parameter, $g'$ is reduced-gravity coefficient associated with the density jump between the isopycnal layers, and $\delta_{n2}$ is the Kronecker delta.

It needs to be pointed out that Eq. (2.3) is simplified by applying the rigid-lid approximation. For convenience, we also define $F$ as the inverse of the square of the
internal Rossby deformation radius: \( F = f_0^2 (H_1 + H_2)/(g'H_1 H_2) \); and introduce two coefficients \( \alpha_1 = H_2/(H_1 + H_2) \) and \( \alpha_2 = H_1/(H_1 + H_2) \). It follows that

\[
F_1 = \alpha_1 F, \quad F_2 = \alpha_2 F.
\] (2.4)

\[ \text{2.2 Linear stability analysis: Analytical study} \]

\[ \text{2.2.1 Necessary instability condition} \]

For tractability of the analytical analysis, we consider an inviscid flow over a constant slope. First, by linearizing the PV equations, we obtain

\[
\left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \left[ \nabla^2 \varphi_1 - \alpha_1 F (\varphi_1 - \varphi_2) \right] + \frac{\partial \varphi_1}{\partial x} (\beta_0 + \alpha_1 FU) = 0,
\] (2.5)

\[
\frac{\partial}{\partial t} \left[ \nabla^2 \varphi_2 + \alpha_2 F (\varphi_1 - \varphi_2) \right] + \frac{\partial \varphi_2}{\partial x} (\beta_0 - \alpha_2 FU - S_y) - \frac{\partial \varphi_2}{\partial y} S_x = 0,
\] (2.6)

where \( S_x \equiv s_x f_0/H_2 = \partial \eta_b/\partial x f_0/H_2 \), \( S_y \equiv s_y f_0/H_2 = \partial \eta_b/\partial y f_0/H_2 \), \( s_x \) and \( s_y \) are zonal and meridional topographic slopes, respectively. The normal-mode solutions can be sought in the form \( \varphi_n = A_ne^{i(kx + ly - \omega t)} \), which upon substitution yields two coupled algebraic equations for \( A_1 \) and \( A_2 \):

\[
[(\omega - Uk)(k^2 + l^2 + \alpha_1 F) + k(\beta_0 + \alpha_1 FU)]A_1 + \alpha_1 F(Uk - \omega)A_2 = 0,
\] (2.7)

\[
\omega \alpha_2 FA_1 + [-\omega(k^2 + l^2 + \alpha_2 F) - k(\beta_0 - \alpha_2 FU) - \kappa]A_2 = 0,
\] (2.8)

where \( \kappa = f_0/H_2 (K \times \nabla \eta_b) \), which is proportional to the cross product of wavevector \( K = (k, l) \) and topographic slope \( \nabla \eta_b \). The corresponding terms in the PV equation represent vortex tube stretching in the lower layer, due to the motion over topography.
In the following section, we derive a necessary instability condition. Multiplying Eqs. (2.7) and (2.8) by $\alpha_2 F A_1^*/(U - c)$ and $\alpha_1 F A_2^*/(-c)$, respectively, we get

$$
\alpha_2 F K^2|A_1|^2 + \alpha_1 F \alpha_2 F (|A_1|^2 - A_1^* A_2) - \frac{\alpha_2 F |A_1|^2 (\beta_0 + \alpha_1 F U)}{U - c} = 0, \quad (2.9)
$$

$$
\alpha_1 F K^2|A_2|^2 + \alpha_1 F \alpha_2 F (|A_2|^2 - A_1 A_2^*) - \frac{\alpha_1 F |A_2|^2 (\beta_0 - \alpha_2 F U + \kappa/k)}{-c} = 0, \quad (2.10)
$$

where $K^2 = k^2 + l^2$, $c = \omega/k$ and $A_n^*$ are the complex conjugates of $A_n$. Summing up Eqs. (2.9) and (2.10), we have

$$
K^2 F (\alpha_2 |A_1|^2 + \alpha_1 |A_2|^2) + \alpha_1 \alpha_2 F^2 (|A_1|^2 + |A_2|^2 - (A_1^* A_2 + A_1 A_2^*))
- \frac{\alpha_2 F |A_1|^2 (\beta_0 + \alpha_1 F U)}{U - c} - \frac{\alpha_1 F |A_2|^2 (\beta_0 - \alpha_2 F U + \kappa/k)}{-c} = 0. \quad (2.11)
$$

Taking the imaginary part of Eq. (2.11) leads to

$$
\left[ \frac{\alpha_2 F |A_1|^2 (\beta_0 + \alpha_1 F U)}{|U - c|^2} + \frac{\alpha_1 F |A_2|^2 (\beta_0 - \alpha_2 F U + \kappa/k)}{|-c|^2} \right] \omega_i = 0. \quad (2.12)
$$

where $\omega_i$ is the imaginary part of $\omega$. Growing solutions for $k \neq 0$ can exist only if

$$
\left( U + \frac{\beta_0}{\alpha_1 F} \right) \left( U - \frac{\beta_0 + \kappa/k}{\alpha_2 F} \right) > 0. \quad (2.13)
$$

Perturbations with an infinitely long zonal scale, namely $k = 0$, are always stable. It is obvious that such perturbations propagate perpendicular to the mean interface slope between the upper and lower layers, and cannot feed on the available potential energy stored in the background stratification.

In the absence of topography, whether or not baroclinic instability can develop is determined by such background parameters as the velocity shear ($U$), stratification ($\alpha_1 F$ and $\alpha_2 F$) and planetary vorticity gradient ($\beta_0$) (Pedlosky 1987). In the presence
of topography, the stability properties also strongly depend on the magnitudes of
the topographic slope ($\nabla \eta_b$) and the wavevector ($k, l$), as well as on the relative
orientation of these two vectors.

When the topographic slope is purely meridional, $\kappa/k$ reduces to $S_y$ and is in-
dependent of the wavevector magnitude and orientation. The slope exerts a stabi-
lizing/destabilizing effect on the zonal background flow through changing the back-
ground PV gradient in the lower layer. As in the classical Phillips model with a flat
bottom, the flow remains stable for all perturbations as long as $U$ does not exceed
critical values:

$$\min \left( -\frac{\beta_0}{\alpha_1 F}, \frac{\beta_0 + S_y}{\alpha_2 F} \right) < U < \max \left( -\frac{\beta_0}{\alpha_1 F}, \frac{\beta_0 + S_y}{\alpha_2 F} \right).$$

(2.14)

Note that the above condition implies that the background PV gradient does not
change sign in the vertical.

In the presence of a zonal slope, the stability properties are fundamentally dif-
ferent, and become strongly dependent on the wavevector of the perturbation itself.
Most importantly, for any combination of background parameters, one can always find
a perturbation that satisfies (2.13) and can, therefore, grow. In particular, even a
small zonal slope can destabilize an otherwise stable zonal background flow; a similar
situation exists for the background currents that are slightly nonzonal (Kamenkovich
and Pedlosky 1996). On the other hand, the stability properties of the perturba-
tions whose wavevectors are parallel to the direction of the topographic slope, are
not affected by topography. In other words, if the flow is stable over a flat bottom,
the introduction of topography can destabilize only those modes whose wavevectors
have across-slope components; for a purely zonal slope, that means $l \neq 0$. In the
study of the effects of short-scale sinusoidal topography on baroclinic instability,
Benilov (2001) demonstrated similar results: the perturbations are affected the most
if their wavevectors are parallel to the isobaths; while those, whose wavevectors are perpendicular to the isobaths, are not affected by the topography.

In summary, a topographic slope changes the necessary instability condition through modifying the mean PV gradient in the lower layer. Meridional slope can stabilize a zonal background flow by preventing the meridional PV gradient from changing sign between the layers. A zonal slope generates a zonal component of the background PV gradient, which makes the concept of critical shear largely irrelevant, since any zonal background flow over a zonal slope can potentially become unstable.

### 2.2.2 The dispersion relation

The necessary condition for instability (2.13) can only indicate when a zonal background flow can be potentially unstable; an actual solution is needed for determining if growing solutions indeed exist. Nontrivial solutions for $A_1$ and $A_2$ of Eqs. (2.7) and (2.8) exist only if the determinant of the coefficients is zero. This condition leads to the dispersion relation:

\[
\begin{align*}
[(k^2 + l^2 + \alpha_1 F)(k^2 + l^2 + \alpha_2 F) - \alpha_1 \alpha_2 F^2]\omega^2 & - [U(k^2 + l^2) - 2k(\beta_0 - \alpha_2 FU)(k^2 + l^2) - \kappa(k^2 + l^2 + \alpha_1 F) - k\beta_0 F(\alpha_1 + \alpha_2)]\omega \\
- k[U(k^2 + l^2) - \beta_0](k(\beta_0 - \alpha_2 FU) + \kappa) & = 0.
\end{align*}
\]

(2.15)

To obtain a quantitative perspective on the effects of topography in the rest of this paper, we set the Coriolis parameter $f_0 = 0.83 \times 10^{-4}$ s$^{-1}$, the background planetary vorticity gradient $\beta_0 = 2 \times 10^{-11}$ m$^{-1}$ s$^{-1}$ and internal Rossby deformation radius equal to 25 km. The isopycnal layers thicknesses are set to $H_1 = 1$ km and $H_2 = 3$ km, unless stated otherwise. In the following we examine the effects of meridional and zonal slopes first on the $f$-plane, which allows us to focus on the effects of topography,
and then on the $\beta$-plane, where we investigate an interplay between the effects of the planetary vorticity gradient and topography.

### 2.2.2.1 The $f$-plane

The topographic effects are most pronounced in the absence of the planetary vorticity gradient $\beta$. We, therefore, analyze these effects on the $f$-plane first. By setting $\beta = 0$ and solving the dispersion relation, we can get the growth rate of unstable waves on the $f$-plane as:

$$\omega_i = |k| \frac{\sqrt{-\Delta}}{2K^2(K^2 + F)}, \quad (2.16)$$

where $\Delta = (K^2 + \alpha_1 F)^2 S^2 + 2UK^2(K^4 + \alpha_1 F K^2 - 2\alpha_1 \alpha_2 F^2)S + U^2 K^4(K^4 - 4\alpha_1 \alpha_2 F^2) < 0$, $S = \kappa/k$. In the flat-bottom case, the background zonal flow is always unstable for sufficiently long wavelengths. The corresponding phase speed, the real part of $c$ is

$$c_r = \frac{K^2 + 2\alpha_2 F}{2(K^2 + F)} U - \frac{S(K^2 + \alpha_1 F)}{2K^2(K^2 + F)}, \quad (2.17)$$

The equations reveal at least two interesting properties. First, the waves over a flat bottom propagate in the same direction as the mean current but with $c_r < U$; Eq. (2.17) suggests that the presence of bottom topography can potentially change both direction and magnitude of the phase speed. However, as we will see in the following analysis, the most unstable mode always has a phase speed that is slower than the mean current in the top layer. Secondly, two pairs of oppositely signed $U$ and $S$ result in the same $\omega_i$. In other words, the parameter of importance is the relative orientation of the mean isopycnal slope (which determines the sign of $U$) with respect to the topographic slope. In the analysis of this section, therefore, it is sufficient to consider an eastward background (EB) flow only.
a. **Meridional slope:** \( \beta_0 = 0, \ s_x = 0 \) and \( s_y \neq 0 \)

When the slope is purely meridional, \( S \) reduces to \( S_y \). Figure 2.1a shows the growth rate as a function of \((k, l)\) for \( U = 4 \text{ cm s}^{-1} \) over three negative (northward deepening) meridional slopes: weak \(-10^{-3}\), intermediate \(-5 \times 10^{-3}\) and strong \(-10^{-2}\). Note that the growth rate depends only on the magnitude of \( k \) and \( l \), and the unstable waves are shown here within an incomplete annulus in the \((k, l)\) plane. The greatest growth rate is found at \( l = 0 \); in other words, the most unstable mode is a meridional “noodle” mode. As the meridional slope gets steeper, the unstable wavenumber range moves away from the origin in the \((k, l)\)-plane, indicating shorter zonal and meridional wavelengths of the unstable modes, and becomes narrower. This inverse relationship between the steepness of the meridional slope and the zonal wavelength of the most unstable mode is further illustrated Fig. 2.2.1c. In contrast, the zonal wavelength of the most unstable mode increases with \( U \).

Figure 2.2 shows the maximum growth rate and the corresponding phase speed as a function of the meridional slope for EB flows, \( U = 4, 5 \) and \( 6 \text{ cm s}^{-1} \). Interestingly, the maximum growth rate is found in the flat bottom case \( (S_y = 0) \). Consistent with (2.14), positive meridional slope (deepening southward) exerts a stabilizing effect on the EB flows, resulting in the decrease of the maximum growth rate and a complete elimination of the unstable modes as the slope becomes sufficiently steep. The corresponding “cutoff” value of the slope is determined by the necessary instability condition (2.13) and satisfies \( S_y = UF_2 \). As a negative (deepening northward) meridional slope becomes steeper, the maximum growth rate also decreases, but remains positive (unstable). The corresponding phase speeds, as shown in Fig. 2.2b, increase in magnitude with steepening of the negative meridional slope but remain smaller than \( U \). The most unstable mode always propagates eastward, for both positive and negative zonal wavenumbers.
Figure 2.1: Spatial structure of unstable modes in the presence of a meridional topographic slope. The greatest growth rates of the unstable modes in the EB flow of 4 cm s$^{-1}$ as functions of wavenumbers for three different meridional slopes $-10^{-3}$, $-5 	imes 10^{-3}$ and $-10^{-2}$ on the (a) $f$-plane and (b) $\beta$-plane; the units are year$^{-1}$. (c) The wavenumber of the most unstable mode as a function of the meridional slope for three different EB flows: 4, 5 and 6 cm s$^{-1}$ on the $f$-plane (solid lines) and $\beta$-plane (dash lines).

b. Zonal slope: $\beta_0 = 0$, $s_x \neq 0$ and $s_y = 0$

The effects of a zonal slope on the growth rates are similar to those of the meridional one (Fig. 2.3). In the flat-bottom case, unstable modes are found within a symmetric circle in the $(k, l)$ plane, with the exception of $k = 0$. Zonal slope distorts
the symmetry, and the unstable zonal wavelength becomes shorter. Additionally, slowly growing nearly zonally oriented modes with large meridional wave numbers emerge at larger values of the slope. As in the case of a meridional slope, the unstable wavenumber interval (the shaded area in Fig. 2.3) shrinks as the slope becomes steeper, so in this sense the effects of the zonal slope can be interpreted as being stabilizing.

The zonal slope does not significantly influence the wavelengths, growth rate and corresponding phase speed of the most unstable mode (figure not shown). The most unstable mode has a nearly meridional orientation with its zonal wavenumber exceeding its meridional one by an order of magnitude for several values of the main parameters ($U = 4, 5, 6 \text{ cm s}^{-1}$ and $H_2 = 3, 5 \text{ km}$). The corresponding zonal wavelengths and phase speeds do not vary much, regardless of the magnitude of the zonal slope. This is consistent with the fact that the “noodle” mode is unaffected by the

Figure 2.2: Time dependence of the most unstable mode in the presence of a meridional topographic slope. (a) The greatest growth rate and (b) the corresponding phase speed for the EB flows with speeds of 4, 5, and 6 cm s$^{-1}$ on the $f$-plane (solid lines) and $\beta$-plane (dash lines).
zonally sloping topography, as discussed earlier (see section 2.3.1). The presence of both the meridional and zonal slopes, as well as the zonal slopes on the $\beta$-plane (see the next section) can, however, significantly change the orientation of the most unstable mode.

In summary, as the topographic slopes become stronger on the $f$-plane, the unstable wavenumber interval shrinks and the unstable wavelengths tend to become shorter, especially for meridional slopes. The most unstable mode has a purely zonal wavevector for meridional slopes and a nearly zonal wavevector for purely zonal slopes. In the next section, we will see that the latter property will change on the $\beta$-plane.

2.2.2.2 $\beta$-plane

We next examine the effects of the interplay between the planetary vorticity gradient and topography on the linear baroclinic instability of a zonal flow. The $\beta$-effect has generally stabilizing influence on zonal currents. In particular, a flow over a flat bottom is no longer unstable for all values of $U$ and can only support growing modes if its vertical shear exceeds a critical threshold. These critical shears, for our choice of parameters, are $-1.6$ cm s$^{-1}$ and $5.0$ cm s$^{-1}$. In the following, “supercritical” refers to shears with $U > 5.0$ cm s$^{-1}$ and $U < -1.6$ cm s$^{-1}$, while “subcritical” refers to all other values of $U$. We will demonstrate that the subcritical values of the velocity shear can be destabilized by topography.

a. Meridional slope: $\beta_0 \neq 0$, $s_x = 0$ and $s_y \neq 0$

As is discussed above, a meridional slope can stabilize a current by affecting the meridional PV gradient in the lower layer and preventing it from changing sign in the vertical. Alternatively, subcritical flows can become unstable due to the presence of a meridional slope. The analysis of an EB flow on the $f$-plane is repeated here on the $\beta$-plane and the results are plotted by the dashed lines in Figs. 1 and 2. Overall, the
dependence of the wavelengths, growth rate and phase speed of the most unstable mode on a meridional slope is very similar between the $\beta$- and $f$-planes. The results for westward background (WB) flows are also similar and not discussed here.
b. Zonal slope: \( \beta_0 \neq 0, s_x \neq 0 \) and \( s_y = 0 \)

The effects of a zonal slope are considered next for three values of EB flows: subcritical (4 cm s\(^{-1}\)), critical (5 cm s\(^{-1}\)) and supercritical (6 cm s\(^{-1}\)), and WB flows for three supercritical values (\(-2, -3\), and \(-4\) cm s\(^{-1}\)). Positive (deepening westward) and negative (deepening eastward) zonal slopes with the same magnitude have the same effects on the greatest growth rate and the corresponding phase speed; this is because two pairs of oppositely signed \( l/k \) and \( s_x \) correspond to the same \( \kappa \) and, consequently, the same solution to the dispersion relation. Thus, we only consider positive zonal slopes in the following analysis.

Even a small zonal slope can destabilize a subcritical zonal background flow. For example, a small zonal slope of \( 10^{-4} \) makes a current with \( U = 4 \) cm s\(^{-1}\) unstable in a limited range of wavenumbers (red contours in Fig. 2.4a). The unstable wavenumber range increases substantially, when the zonal slope is as large as \( 10^{-3} \) (black contours). As the zonal slope gets even larger, unstable wavenumbers are squeezed into a narrow wavenumber interval similar to that observed on the \( f \)-plane. The unstable wavenumber range changes in a similar manner for the supercritical flow of \( U = 6 \) cm s\(^{-1}\) and large slopes. In contrast, smaller values of \( U \) correspond to very narrow unstable wavenumber ranges (figure not shown).

Unlike a zonal/meridional topographic slope on the \( f \)-plane or a meridional topographic slope on the \( \beta \)-plane, a zonal slope on the \( \beta \)-plane modifies the shape of the most unstable mode, which is no longer the meridionally oriented “noodle” mode. Figure 2.4b shows the zonal and meridional wavenumbers of the most unstable mode as the zonal slope increases from 0 to \( 10^{-2} \) and for \( U = 4, 5 \) and 6 cm s\(^{-1}\). As expected, only the supercritical flow \( U = 6 \) cm s\(^{-1}\) is unstable at a zonal slope of 0, where the most unstable mode has a shape of the meridional “noodle” mode (\( l = 0 \)). The wavevector of the most unstable mode for \( U = 6 \) cm s\(^{-1}\) (blue curve) rotates
first counterclockwise in the \((k, l)\) plane as the zonal slope increases, changing from a meridional “noodle” mode \((l = 0)\) to a slanted mode \((l \approx k)\), and then clockwise for very large slopes. For the subcritical/critical shears \(U = 4\) and \(5\) cm s\(^{-1}\), unstable modes start emerging for a zonal slope as small as \(10^{-4}\) and the most unstable wavevector rotates clockwise in the \((k, l)\) plane, changing its orientation from nearly meridional to nearly zonal.

In contrast to the \(f\)-plane, as shown in Figs. 4c and 4d, the growth rate of the most unstable mode increases sharply with the zonal slope, before approaching a nearly constant value at the slope of approximately \(5 \times 10^{-3}\). The corresponding phase speed exhibits similar behavior and remains eastward for both negative and positive values of \(k\).

Several effects of a zonal slope on WB flows on \(\beta\)-plane are qualitatively similar to those of supercritical EB flows (Fig. 2.5). In particular, the range of unstable wavenumbers shrinks as the slope magnitude increases (Fig. 2.5). The growth rate increases, and the wavevector of the most unstable mode rotates counterclockwise as the slope gets larger (Fig. 2.5d). The most unstable mode of a WB flow also propagates downstream (westward), but its phase speed magnitude decreases with the zonal slope (figure not shown).

c. Vertical structure

The vertical structure of the most unstable mode shows a strong dependence on meridional and zonal slopes. The most unstable mode becomes more surface-intensified as a negative meridional slope becomes steeper. As shown in Fig. 2.6a the ratios between the amplitudes of the most unstable mode in the top \((A_1)\) and bottom \((A_2)\) layers for \(U = 4\) and \(6\) cm s\(^{-1}\) increase sharply with the slope magnitude. These changes are explained by the increase in the relative importance of the baroclinic component of the mode. The corresponding ratio between the barotropic mode \(A_0 = \)
Figure 2.4: Unstable modes in the presence of a zonal topographic slope and EB flows on the β-plane. (a) The greatest growth rates for the EB flow with a speed of 4 cm s⁻¹ as a function of zonal and meridional wavenumber for three zonal slopes $10^{-4}$, $10^{-3}$ and $10^{-2}$; the units are year⁻¹. (b) The wavenumber of the most unstable mode for $U = 4$, 5 and 6 cm s⁻¹ as a functions of the zonal slope, which increases along the curves and three values are shown: $10^{-4}$ (open circles), $10^{-3}$ (stars) and $10^{-2}$ (diamonds); the dots being the results from numerical simulations for $s_x = 5 \times 10^{-4}$ and $10^{-3}$ and the three different EB flows. (c) The greatest growth rate in curves and the numerical results (dots with error bars) for $s_x = 5 \times 10^{-4}$ and $10^{-3}$ and the three different EB flows and (d) the corresponding phase speed and the numerical results (dots with error bars) for $s_x = 5 \times 10^{-4}$ and $10^{-3}$ and for the same values of $U$. 
\( (A_1 H_1 + A_2 H_2)/(H_1 + H_2) \) and the residual baroclinic one in the upper layer \( A_z = A_1 - A_b \) is shown in Fig. 2.6b. Although this ratio is about 1 when the meridional slope is very small, it decreases to less than 0.5 as the slope becomes steeper.

A zonal slope exerts distinct effects on subcritical and supercritical background flows. For the subcritical EB flow of 4 cm s\(^{-1}\) \( A_1 \) and \( A_2 \) are nearly the same when the zonal slope is very small (Fig. 2.7a). As in the case of a meridional slope, the most unstable mode becomes more surface-intensified as the zonal slope gets larger, but this effect is weaker than in the case of a meridional slope, and \( A_1/A_2 \) reaches a nearly constant value of about 4.5. These changes are attributed to the increase in the relative importance of the baroclinic component of the mode; although the barotropic component tends to dominate over the baroclinic one for all slopes, \( A_b/A_z \) decreases sharply from more than 5 to almost 1, as the zonal slope increases (Fig. 2.7b). For the supercritical EB flow of 6 cm s\(^{-1}\), as in the subcritical case, the most unstable mode is surface-intensified for all zonal slopes. \( A_1/A_2 \) decreases slightly in the beginning, but increases afterwards as the zonal slope increases. In contrast to the subcritical case, however, \( A_b/A_z \) for \( U = 6 \) cm s\(^{-1}\) is around 1 for all zonal slopes, indicating a mix of barotropic and baroclinic components of the most unstable mode. Changes in a negative zonal slope lead to a similar dependence.

### 2.2.3 Effects of the orientation of the background PV gradients

The largest dynamical effect of the introduction of a zonal topographic slope is to make the background PV gradient in the lower layer non-meridional. The effects of this rotation on the most unstable mode are explicitly explored in this section, by keeping the magnitude of the PV gradient fixed, but changing its orientation. This is readily achieved by changing \( S_x \) and \( S_y \) simultaneously. The spatial structure of the
Figure 2.5: Unstable modes in the presence of a zonal topographic slope and WB flows, on the $\beta$-plane. Growth rates (yr$^{-1}$) as a function of zonal and meridional wavenumber are shown for different zonal slopes: (a) $1 \times 10^{-4}$, (b) $1 \times 10^{-3}$, (c) $1 \times 10^{-2}$ for the WB flow with a speed of $-3$ cm s$^{-1}$; the units are year$^{-1}$. (d) The wavenumbers of the most unstable mode for $U = -4, -3$ and $-2$ cm s$^{-1}$ — the zonal slope increases along the curves and four values are shown: $5 \times 10^{-4}$ (circles), $7 \times 10^{-4}$ (stars), $1.1 \times 10^{-3}$ (diamonds) and $1.3 \times 10^{-3}$ (crosses); beyond the slope value of $\sim 10^{-3}$, it becomes very difficult to accurately calculate the wavelength of the most unstable mode, because unstable modes are found in the increasingly narrow interval near the origin in the $(k, l)$ plane and the corresponding growth rates are too close to be distinguished within the computation accuracy.
Figure 2.6: Vertical structure of the most unstable modes in the case of EB flows over meridional topographic slopes. (a) The ratios between the magnitudes of the upper and lower layer streamfunctions for $U = 4$ and $6$ cm s$^{-1}$, as a function of the meridional slope. (b) The corresponding ratios between the barotropic and the baroclinic modes in the upper layer.

Figure 2.7: Vertical structure of the most unstable modes in the case of EB flows over zonal topographic slopes. (a) The ratios between the magnitudes of the upper and lower layer streamfunctions for $U = 4$ and $6$ cm s$^{-1}$, as a function of the zonal slope. (b) The corresponding ratios between the barotropic and the baroclinic modes in the upper layer.
most unstable mode is characterized by the orientation of the wavevector \((k, l)\) with respect to the \(y\)-axis and denoted as angle \(\theta_{kl}\). For example, \(\theta_{kl} = 90^\circ\) corresponds to a meridional “noodle” mode.

We follow Flierl (1978) and define barotropic and baroclinic PV, \(q_{BT}\) and \(q_{BC}\) in the following way:

\[
q_{BT} = \alpha_2 q_1 + \alpha_1 q_2, \tag{2.18}
\]
\[
q_{BC} = \sqrt{\alpha_1 \alpha_2} (q_1 - q_2). \tag{2.19}
\]

The barotropic and baroclinic PV gradients, \(\nabla q_{BT}\) and \(\nabla q_{BC}\) are

\[
\nabla q_{BT} = \alpha_1 S_x \mathbf{i} + (\beta_0 + \alpha_1 S_y) \mathbf{j}, \tag{2.20}
\]
\[
\nabla q_{BC} = -\sqrt{\alpha_1 \alpha_2} S_x \mathbf{i} + \sqrt{\alpha_1 \alpha_2} (FU - S_y) \mathbf{j}. \tag{2.21}
\]

We begin by examining the relationship between \(\theta_{kl}\) and the orientation of the barotropic PV gradient, quantified by \(\theta_{\nabla q_{BT}}\), the angle between this gradient and the \(x\)-axis \((\theta_{\nabla q_{BT}} = 90^\circ\) in the absence of topography). Note that \(\theta_{\nabla q_{BT}} = \theta_{kl}\) means that the wavevector is perpendicular and the velocity vector is parallel to the direction of the barotropic PV gradient. We are also interested in how the relationship between \(\theta_{kl}\) and \(\theta_{\nabla q_{BT}}\) is affected by the vertical shear and stratification. For this purpose, we will also vary \(U\) and \(H_2\).

First, we consider three EB flows: weak (1 cm s\(^{-1}\)), intermediate (6 cm s\(^{-1}\)) and strong (12 cm s\(^{-1}\)), while keeping the depths of the two isopycnal layers \(H_1 = 1\) km and \(H_2 = 3\) km. We maintain the same magnitude of \(\nabla q_{BT}\) as \(2\beta_0\) and rotate it from the northward (90°) to the eastward (0°) directions by changing the bottom slopes from \((S_x, S_y) = (0, \beta_0/\alpha_1)\) to \((S_x, S_y) = (2\beta_0/\alpha_1, -\beta_0/\alpha_1)\).
Both EB flows of 1 cm s\(^{-1}\) and 6 cm s\(^{-1}\) are stable when \(\nabla q_{BT}\) points to the north with a purely meridional slope. A very small zonal component of the PV gradient (\(\theta \nabla q_{BT}\) is slightly less than 90°), however, destabilizes these flows, and the most unstable mode is nearly zonally oriented and has an almost meridional wavevector (\(\theta_{kl}\) is close to 0°). For the EB flow of 1 cm s\(^{-1}\), as \(\nabla q_{BT}\) becomes more zonal, \(\theta_{kl}\) is always shifted by 90° with respect to \(\theta \nabla q_{BT}\) (red line in Fig. 2.8a). This indicates that the most unstable wavevector is parallel to the barotropic PV gradient for a background flow as weak as 1 cm s\(^{-1}\). A similar situation is observed for \(U = 4\) cm s\(^{-1}\) and small zonal slopes in the previous section. For a stronger background velocity shear of 6 cm s\(^{-1}\), this parallel orientation of the most unstable wavevector and the barotropic PV gradient is less obvious.

The EB flow of 12 cm s\(^{-1}\) is, in contrast, baroclinically unstable when the barotropic PV gradient is purely meridional (\(\theta \nabla q_{BT} = 90°\)), and \(\theta_{kl}\) is also 90° (the meridional “noodle” mode); the velocity vector is parallel to \(\nabla q_{BT}\). This is a typical situation for the modes growing in the baroclinic shear, as this orientation enables these modes to most efficiently extract energy from the background state (Pedlosky 1987). For nonzero zonal slopes, the difference between \(\theta_{kl}\) and \(\theta \nabla q_{BT}\) increases. When the PV gradient is zonal (\(\theta \nabla q_{BT} = 0\)), this difference is 90° and the wavevector is parallel to the PV gradient.

The relationship between the barotropic PV gradient and the most unstable wavevector is sensitive to the mean stratification. We consider here the EB flow of 6 cm s\(^{-1}\) and three different values of \(H_2\): 1, 3 and 5 km. The upper layer is 1 km for all the three cases. These values can be interpreted as cases with the thermoclines that are deep, intermediate and shallow in comparison to the total depth of the ocean. Note that, when the bottom is flat, a larger \(H_2\) (and smaller \(\alpha_2\)) implies an increased stability of the background flow. For a deep lower layer of 5 km, \(\theta_{kl}\) is always nearly
parallel to $\nabla q_{BT}$ (Fig. 2.8b). The difference between $\theta_{\nabla q_{BT}}$ and $\theta_{kl}$ shifts away from 90° as the lower layer becomes thinner (black and red curves).

The relationship between $\theta_{kl}$ and $\theta_{\nabla q_{BC}}$ has also been examined, but the former angle does not appear to be sensitive to the orientation of the baroclinic PV gradient and is not discussed here.

![Graph](image)

**Figure 2.8:** Orientation of the wavevectors and the barotropic PV gradient (a) three EB flows, and $H_2 = 3$ km; and for (b) the EB flow with a speed of 6 cm s$^{-1}$ and three different values of $H_2$.

### 2.3 Meridional ridge: Numerical results

The linear baroclinic instability properties of zonal flows over topography are studied next in a linearized numerical QG model. The model is exactly the one described in section 2.2, except it is confined in a zonally re-entrant channel, centered at 45°N. Its meridional width is $L_y = 3600$ km; its zonal length is $L_x = 4L_y$. Grid resolution is 7km. This numerical code is adapted from Berloff et al. (2009a) with nonlinear advection terms turned off. All parameters are the same as those used in section 2.3.
The model is initialized with random perturbations, among which some wavenumbers become unstable and grow exponentially.

By assuming that the most unstable mode tends to dominate the flow evolution in the model at long times, we can estimate the growth rate, phase speed and spatial structure of the most unstable mode. Specifically, we calculate the growth rate from the last 100 days of each simulation (before the model becomes numerically unstable), by fitting an exponent into the energy time evolution. The phase speed is estimated from the slope of Hovmöller diagrams during the same time period. Errors in both growth rate and phase speed are estimated by bootstrapping. The zonal and meridional wavelengths of the most unstable mode are determined by eye, by counting the number of maxima (as in Fig. 2.9b). Note that, the model domain allows only a discrete set of wavenumbers, consequently, the model may not permit the analytically most unstable mode. This bias is, however, found to be not significant in the cases considered below. The ability of these methods to capture the main properties of the most unstable modes is first verified in the case of a constant topographic slope. The corresponding linear numerical solutions are very close to those from the analytical dispersion relation of Eq. (2.15). Specifically, the growth rate from the model is only 4% smaller than the analytical one; the wavelength and phase speed of the most unstable mode are also nearly the same as the analytical values.

We next proceed with a numerical analysis of the linear solutions over a topographic ridge and examine the extent to which the unstable modes developing in the presence of the ridge can be described using our analytical results with a constant zonal slope. We need to point out that the numerical results, even in the vicinity of the ridge itself can, in general, be very different from the analytical ones due to the more complicated topography and boundary conditions in the former case. We, however, find that the numerical solutions are generally consistent with the local sta-
bility properties from theory. The numerical results for several parameters are shown in Figs. 4b-4d by solid dots with error bars.

We begin with the EB flow of 4 cm s$^{-1}$ over a ridge with the slopes of $\pm 10^{-3}$, which is subcritical over flat bottom. The meridional ridge is shown in Fig. 2.9a; it has a height of 1500 m and a width of 3000 km. The numerical results exhibit slanted orientations on both sides of this symmetric ridge and very small amplitudes over the flat bottom section of the channel (Fig. 2.9b). The spatial structure of this numerical solution closely matches that of the most unstable mode from the analytical model with a constant zonal slope of $10^{-3}$. Specifically, the wavevector (scaled by the internal Rossby deformation radius) of the most unstable mode from the numerical simulation is $(k, l) = (0.69, 0.48)$, which is very close to the analytical result $(k, l) = (0.68, 0.43)$ for the positive (eastward deepening) slope of $10^{-3}$ (Fig. 2.4b). The negative slope corresponds to an oppositely signed $l$. The calculated growth rate is 4.89 year$^{-1}$, which is somewhat lower than the analytical one, 5.91 year$^{-1}$ (Fig. 2.4c). This can also be partly due to the fact that several unstable modes grow in this simulation together with the most unstable mode, but at lower rates. Nearly negligible motions away from the ridge are also consistent with local stability properties of this subcritical current which is stable over the flat bottom. The propagating speed of the most unstable mode is 0.0065 m s$^{-1}$ and eastward, which is slightly lower than the analytical result of 0.0076 m s$^{-1}$. We remind the reader that the most unstable wave modes over the negative and positive zonal slopes of the ridge propagate eastward with the same phase speed.

The results for the meridional ridges with slopes $\pm 5 \times 10^{-4}$ and $\pm 10^{-3}$ and the EB flows $U = 4, 5$ and 6 cm s$^{-1}$ are also generally consistent with local stability properties. In particular, unstable modes exhibit slanted structures over the ridges; away from the ridges, the amplitudes are very small, and the meridional “noodle”
mode in the flat-bottom case is not visible. In other words, the disturbances are strongly trapped to the ridges. This is generally consistent with the growth rates over a flat bottom being significantly smaller than those over a zonal slope (Fig. 2.4). The growth rates of the numerical solutions are $13 - 19\%$ lower than the analytical ones, and the phase speeds are within $20\%$ of the analytical ones. Other experiments configured with different topographic features, such as a double ridge and a trough, exhibit a similar agreement with the local analytical results (figures not shown).

![Figure 2.9](image)

**Figure 2.9:** Linear numerical results. (a) Schematics of the meridional ridge and (b) the disturbance streamfunction for the EB flow with a speed of 4 cm s$^{-1}$, scaled to the streamfunction magnitude; black lines denote the positions of the edges and center of the ridge. These most unstable mode are patchy instead of being uniform due to the boundary effects of the channel walls.

### 2.4 Summary and conclusions

This study demonstrates the importance of bottom topography in the linear baroclinic instability of oceanic currents. Constant topographic slopes are shown to significantly modify stability properties of zonal flows, through the changes in the background PV gradient. A zonal slope introduces a zonal component in the PV gradient, which has
a strong destabilizing effect on the flow; this is also consistent with the enhanced eddy kinetic energy over zonal slopes found by Boland et al. (2012). In particular, even a small zonal slope can destabilize a flow that is otherwise stable in the absence of topography, and the growth rates of the most unstable modes increase with the magnitude of the slope. In contrast, a meridional slope can stabilize/destabilize a zonal flow only through changing the background PV gradient in the lower layer beyond a known threshold.

The spatial structure of the fastest growing modes is also sensitive to the orientation of the topographic slope. The importance of the baroclinic component of the most unstable mode increases with the increase of zonal slope magnitude, and the mode becomes surface-intensified. Surface intensified modes are also found in a linear two-layer QG model in Samelson (1992), in which the horizontal scales of the topography are comparable to that of the waves, and in Hallberg (1997), where they become coupled with bottom-intensified flows when the topographic gradient is meridional. Northward- or zonally sloping topography in our study also leads to the shortening of the most unstable wavelength, in comparison with the flat bottom.

When the slope is purely meridional or zero, the most unstable mode has the shape of a meridional “noodle” mode and the wavevector has a purely zonal orientation. This orientation allows the most efficient extraction of energy from the background stratification (Pedlosky 1987). The introduction of a zonal slope causes the most unstable modes to become slanted horizontally. The relationship between the PV gradient and the orientation of the most unstable wavenumber depends on how stable (subcritical) the zonal flow is. For a weak zonal flow or a thick lower layer, the orientation of the wavevector is parallel (and the velocity vector is perpendicular) to the barotropic PV gradient, which indicates a strong topographic control on the mode orientation. A similar and strong relationship between eddy-driven jets and the
barotropic PV gradient is reported by Boland et al. (2012). This is consistent with the mechanism of jet formation proposed by Berloff et al. (2009a), in which jets form as a result of the secondary instability of the primary unstable modes; in their model with a flat bottom the jets are perpendicular to the most unstable “noodle” modes of the linear problem. Most unstable modes in strong (supercritical) currents, in contrast, exhibit a weak relation between the orientations of their wavevector and the barotropic PV gradient. These wavevectors instead appear to be primarily controlled by the direction of the velocity shear.

The results of the linearized numerical simulations with meridional ridges/troughs are generally consistent with local stability properties. In particular, the dominant growing solution is localized to the ridge and has the spatial structure and growth rate consistent with those predicted by the linear theory over a constant zonal slope. This “trapping” of the modes to the ridge can be explained by the enhanced instability due to the zonal slopes on the sides of the ridge. The entire growing anomaly propagates to the east in the eastward background current and to the west in the westward one. During the nonlinear evolution stage, one can therefore expect coherent structures to form downstream of the ridge. This result may help explain the highly energetic eddy activity downstream of topography observed in Thompson and Sallée (2012) and our nonlinear simulation. This property can also have important implications for the formation of the eddy-driven jets next to the meridional ridges, which is observed in our nonlinear study.
Chapter 3

On the Dynamics of Flows Induced by Topographic Ridges

In this chapter, we investigate the mechanism of the formation of quasi-stationary jets over zonally varying bottom topography, with focus on an isolated meridional ridge. We demonstrate that such topographic features are an essential factor for the jet formation in the otherwise stable flows, and that the mechanism is nonlocal.

This chapter is structured in the following way. The model is introduced in section 3.1. Section 3.2 demonstrates how an otherwise stable flow becomes destabilized by a meridional ridge, thus, leading to the formation of stationary meridional currents over the ridge, of stationary and migrating zonal jets downstream of the ridge, and of transient eddies in the adjacent regions. Section 3.3 explores the dynamics, and in section 3.4 we show that a locally nonzonal background flow can induce zonal jets in the far field and then argue that this process is very similar to the jet formation due to a topographic ridge. Conclusions are given in section 3.5.

3.1 The model

We consider a two-layer quasi-geostrophic (QG) model with a bottom topography on the $\beta$-plane (Fig. 3.1). The potential vorticity (PV), $Q_n$, in each of the two layers is
governed by
\[
\frac{\partial Q_n}{\partial t} + J(\psi_n, Q_n) = \nu \nabla^4 \psi_n - \delta_{n2} \gamma \nabla^2 \psi_n,
\]  (3.1)
where the layer index, \( n = 1, 2 \), starts from the top, \( \psi_n \) is the streamfunction in the \( n \)th layer, \( J(\cdot) \) is the Jacobian operator, and \( \delta_{n2} \) is the Kronecker delta. The terms with \( \nu \) and \( \gamma \) are lateral and bottom friction.

We are interested in the dynamics of a large-scale zonal ocean current with a vertical shear and, thus, consider a horizontally uniform background flow, \( U \), in the upper layer and no flow in the deep layer so that
\[
\psi_1 = \varphi_1 - Uy, \\
\psi_2 = \varphi_2,
\]  (3.2)
where \( \varphi_1, \varphi_2 \) describe disturbances around the background flow.

The PV consists of several components: the relative vorticity of disturbances, the \( \beta \)-term, and the stretching terms due to the mean flow, disturbances, and topography,
\[
Q_n = \nabla^2 \varphi_n + [\beta_0 - (-1)^n F_n U] y + (-1)^n F_n (\varphi_1 - \varphi_2) + \delta_{n2} f_0 \frac{\eta_b(x,y)}{H_2},
\]  (3.3)
where \( \beta_0 = 2 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1} \) is the planetary vorticity gradient, \( H_n \) are the depths of the layers, \( \eta_b(x,y) \) is the spatially varying elevation of bottom topography, \( F_n = f_0^2 / (g'H_n) \) are stratification parameters, \( f_0 = 0.83 \times 10^{-4} \text{ s}^{-1} \) is the Coriolis parameter, and \( g' \) is the reduced-gravity coefficient associated with the density jump between the isopycnal layers. We define \( F \) as the inverse of the square of the internal Rossby deformation radius, \( F = f_0^2 (H_1 + H_2) / (g'H_1H_2) \), and introduce
\[ \alpha_1 = \frac{H_2}{(H_1 + H_2)} \quad \text{and} \quad \alpha_2 = \frac{H_1}{(H_1 + H_2)} \] so that

\[ F_1 = \alpha_1 F, \]
\[ F_2 = \alpha_2 F. \]  

(3.4)

The computational domain is doubly periodic, with a meridional width, \( L_y = 3600 \) km, and a zonal width, \( L_x = 4L_y \). Stratification is defined by \( H_1 = 1 \) km, \( H_2 = 3 \) km and the internal Rossby deformation radius, \( F^{-1/2} = 25 \) km. The bottom friction is \( \gamma = 10^{-7} \) s\(^{-1}\). The numerical model allows large Reynolds numbers on relatively coarse grids (Karabasov and Goloviznin 2009), thus, we use \( \nu = 10 \) m\(^2\) s\(^{-1}\). Our convergence tests confirm that the model solutions are nearly the same at increased resolution.

\[ \text{(Figure 3.1: Sketch of the two-layer model with a topographic ridge at the bottom.)} \]
3.2 Phenomenology: Jets and eddies

Jet formation in geostrophic turbulence relies on a large-scale PV. In some regions of the ocean, PV gradients induced by topography can dominate over the planetary PV gradient. The Mid-Atlantic Ridge illustrates this point. The Ridge elevation is 1–3 km, and its width is about 1500 km; therefore, its slopes, $s$, can be roughly estimated to be $\pm 0.7–2.0 \times 10^{-3}$ on its two sides. In our model, the PV gradient due to the slopes are $f_0s/H_2 = \pm 2.0–5.5 \times 10^{-11}$ m$^{-1}$ s$^{-1}$, which are larger than $\beta_0$. Consequently, broad baroclinic currents that are stable over a flat bottom can become unstable over such topographic slopes (Chen and Kamenkovich 2013).

We consider here two types of topography: constant meridional slopes and isolated meridional ridges, and demonstrate that a background flow that is stable over flat bottom can become unstable and develop eddies and jets due to the presence of the topography.

3.2.1 Jets over a meridional topographic slope

In our model, an eastward background flow of 4 cm s$^{-1}$ is stable over a flat bottom, since the critical $U$ is 5 cm s$^{-1}$. If the meridional slopes are large enough, it can lower the critical $U$ so that the same background flow is baroclinically unstable, and the equilibrated nonlinear flow contains eddies and alternating zonal jets (Fig. 3.2a). In contrast, even small zonal slopes can destabilize background flows (Chen and Kamenkovich 2013).

Jet width strongly depends on $\beta_0$, eddy viscosity, bottom friction and $U$ (Berloff et al. 2009b). For a sensitivity study, we fix $\beta_0$, eddy viscosity and bottom friction and examine the dependence of the jet scale on $U$ and the meridional slope, $s_y$; we choose 33 sets of these parameters that satisfy the necessary condition for baroclinic instability (Chen and Kamenkovich 2013). Alternating zonal jets form in all of these
Figure 3.2: (a) A 100-year mean barotropic streamfunction and the corresponding zonally averaged zonal velocity in the case of a meridional slope of $-5 \times 10^{-4}$. (b) The Rhines scale, $L_R$, versus the actual width, $L$, of the zonal jets from 33 experiments with different values of the meridional slope, $s_y$, and the background zonal flow, $U$. The dashed line is the linear fit to the experimental points.

Simulations. We determine the number of equilibrated zonal jets by counting the maxima of the zonally and time-averaged velocity and calculate the mean jet widths by dividing the width of the channel by the number of the jets. The jet widths are in a good agreement with the Rhines scale (Fig. 3.2b), defined here as $L_R = \sqrt{u_{rms}/\beta^*}$, where $u_{rms}$ is the rms of barotropic eddy velocity, and $\beta^* = \beta_0 + \alpha_1 f_0 s_y / H_2$.

Boland et al. (2012) found a similar agreement for the case of zonal slope in both barotropic and baroclinic QG models. Berloff et al. (2009b) showed that zonal jets do not follow the Rhines scaling for a wider range of parameters. We need to emphasize that we consider here only a relatively narrow range of parameters relevant to this study. We shall see later that the Rhines scale can also predicate jet width in flows over a meridional ridge, and it can be used to explain how the spatial variability in the eddy energy can render jet merging and splitting.
3.2.2 Jet formation due to a topographic ridge

An isolated meridionally uniform ridge (Fig. 3.1) in this study can be considered as an idealized representation of a segment of the Mid-Atlantic Ridge or the Kerguelen Plateau. The ridge has a strong destabilizing effect on zonal flows, regardless of how weak they are (Chen and Kamenkovich 2013). We illustrate this effect with an eastward background zonal flow, \( U = 4 \text{ cm s}^{-1} \), which is stable over a flat bottom.

The reference solution has a moderately sloped meridional ridge of \( \pm 10^{-3} \), which satisfies the constraint of the QG theory that the slope of flow particle trajectory should be much smaller than the depth-to-width aspect ratio. The base of the ridge extends 1500 km in longitude and the height is 750 m at the highest point.

The model is spun up for 5 years until a statistically steady state is reached. In addition to the constant background flow, \( U \), in the top layer, the flow has a rich structure consisting of mesoscale eddies, stationary meridional currents on the top of the ridge, migrating quasi-zonal jets, and stationary alternating zonal jets (Fig. 3.3). Three geographical regions with distinct flow regimes can be identified (Fig. 3.3): the ridge itself (Region I), the area downstream of the ridge with strong transient currents (Region II), and the far field with stationary alternating jets (Region III). In what follows, we describe flow properties in these regions.

3.2.2.1 Stationary meridional currents in Region I

First, we focus on the flow in Region I. As the background flow crosses the ridge, it generates stationary meridional currents over and downstream of the ridge (Fig. 3c and Fig. 3.4). The strongest meridional currents are near the ridge peak: a southward meridional current on the eastern side of the ridge in the deep layer and a northward meridional current above it. We will investigate the dynamical balance of the meridional currents in section 3.4.1.
Adding the background zonal flow, $U$, to the upper-layer meridional currents results in a total flow with stationary meanders over and downstream of the ridge. Away from the ridge, the meanders transform into purely zonal motions. In the deep layer, the mean flow follows the ridge. Abernathey and Cessi (2014) observed similar standing meanders downstream a meridional ridge configured in a QG model, which are responsible for the local heat flux intensification. Such meanders are consistent with conservation of the PV, which demands creation of anticyclonic circulation due to the squeezing of a water column as it climbs up the ridge. A detailed explanation of the meandering response of eastward flows encountering a meridional ridge is given in Holton (2004).

Characteristics of the topographic meridional currents are closely linked to the steepness of the ridge. We carry out two additional experiments, in which we halve and double the width of the ridge but maintain its height unchanged. Thus, these cases correspond to steep and gentle ridge slopes of $\pm 2 \times 10^{-3}$ and $\pm 5 \times 10^{-4}$, respectively. Baroclinic meridional currents are present in both cases, although their properties are different. The jets are more upper-ocean intensified in the steep-ridge case; similar property was observed in the linear solutions of Chen and Kamenkovich (2013). Over the steep ridge, the northward meridional current in the upper layer (thick solid line in Fig. 3.4) is stronger and narrower than the corresponding current over the gentle ridge (thin solid line in Fig. 3.4). In section 3.4.1, we shall see that the surface intensification of the currents is linked to their width and strength through a dynamical balance. Moreover, the meridional currents over the gentle ridge are less coherent and entail several standing circular loops (Fig. 3.5a).

The topographic currents remain parallel to the ridge even if the orientation of the ridge on the horizontal plane changes. We rotate the meridional ridge with slopes of $\pm 10^{-3}$ in clockwise direction by $30^\circ$ and $45^\circ$. The upper-layer time-averaged flow for
Figure 3.3: The system of currents in the model: transient eddies, migrating zonal jets, and stationary meridional and zonal jets. Shown is the kinetic energy (cm$^2$ s$^{-2}$) of (a) an instantaneous flow, (b) a 5-year mean flow, and (c) a 100-year mean flow in the upper layer of the reference solution. The vertical lines show the position of the ridge. (d) The partition of the domain into three regions, based on the flow characteristics. The solid black curve shows the profile of the meridional ridge in the longitude-depth plane.

the 45° case exhibits topographic currents parallel to the bathymetry (Fig. 3.5b) and stronger than in the reference solution. The 30° case reveals similar characteristics.
In the rest of this paper, only the case with a north-south oriented and \( \pm 10^{-3} \) sloped ridge will be considered.

**Figure 3.4:** Meridional currents for three cases with differently sloped meridional ridges. The curves show the meridional average of time-averaged meridional velocities. The zonal background flow is 4 cm s\(^{-1}\) in all the cases.

**Figure 3.5:** Time-averaged streamfunction (contours) and velocity (arrows) in the upper layer for the cases of (a) a meridional ridge with slopes of \( \pm 5 \times 10^{-4} \) and (b) a slanted ridge with the slope of \( \pm 10^{-3} \) and a 45\(^\circ\) orientation. The solid lines show the position of the ridges. The zonal background flow is 4 cm s\(^{-1}\) in both cases.
3.2.2.2 Jets in Regions II and III

Away from the ridge, the flow consists of transient eddies and alternating zonal jets (Fig. 3.3a). To isolate zonal jets from the transient eddies, we apply a low-pass filter in time (10-year average) and obtain alternating zonal jets in Regions II and III (Fig. 3.3b). A very long average over 100 years does not significantly change jets in Region III but results in very weak circulation in Region II. The latter fact indicates that the jets in Region II are time-dependent.

Width of the jets in regions II and III varies with the eddy energy along the channel, and the jets merge and split accordingly (Fig. 3.3b). Consistent with the Rhines scaling, zonal jets are wider in Region II, since the eddy rms velocity, $u_{rms}$, there is larger (Fig. 3.3a). Specifically, the rms velocity in Region II is 29 per cent higher than in Region III, and the zonal jets in Region II are about 14 per cent wider (82 vs. 72 km) (Fig. 3.3b). A transition zone exists between Regions II and III, where 22 jets in Region II evolve into 25 jets in Region III. In particular, four zonal jets split into six jets around $y = 900$ km (Fig. 3.3b).

Figure 3.6 shows the Hovmöller diagram of time- (10-day) and zonally averaged barotropic zonal velocities in Region II—this indicates southward migration of the jets with a speed of 0.04 cm s$^{-1}$. The drift cannot, however, be explained by the meridional advection by the meridional currents over Region II, since the mean meridional current in Region II in the upper layer is about 50 times faster than the southward drift, and the lower-layer current is northward. Boland et al. (2012) observed drifting jets over topographic slopes and argued that the drift is a consequence of PV conservation. However, the drift that we observe occurs downstream of the meridional ridge, and it is over the flat bottom. In section 3.4.2, we shall see that this drift is due to the offset between the eddy forcing and the PV of these jets, and the drift can be also explained by the off-core eddy acceleration on the jets.
Note also that the width of Region II is consistent with the zonal decay scale of the meridional currents (Fig. 3.4). In particular, both the decay scale and the Region II width in the narrow-ridge/gentle-ridge case are smaller/larger than those in the reference solution. It is, therefore, possible that the drift of the jets in Region II is tied to the presence of the mean meridional currents and the resulting non-zonal orientation of the PV contours.

![Figure 3.6: Drift of the jets as shown by the time series of zonal averages of 10-day mean, barotropic, zonal velocities (cm s\(^{-1}\)) in Region II. The jets drift southward with a speed of 0.04 cm s\(^{-1}\).](image)

### 3.2.2.3 Coherent vortices

Okubo (1970) and Weiss (1991) to detect and describe eddying flow introduced the Okubo-Weiss parameter:

\[
Q^2 = S^2 - \omega^2,
\]

where \(S^2 = (\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y})^2 + (\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y})^2\) is the squared horizontal strain rate, and \(\omega^2 = (\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y})^2\) is the squared horizontal vorticity. Negative values of the parameter correspond to a vorticity dominated flow, such as eddy interiors;
while positive values suggest that the flow is strain-dominated, such as the flow in between eddies.

For the purpose of this study, we define here a new parameter based on the Okubo-Weiss parameter to detect coherent vortices:

\[ E^2 = \frac{\omega^2}{2|\omega|} (|Q|^2 - Q^2), \]  

(3.6)

which returns nonzero values only when the Okubo-Weiss parameter is negative. Positive values of \( E^2 \) represent the cores of cyclones, and negative values represent the cores of anticyclones. We need to emphasize here that \( E^2 \) captures only the strong vortices, but excludes other types of transient eddies. In the following, we present only the results for barotropic coherent vortices, since our analysis reveals similar characteristics for baroclinic coherent vortices.

Vortices originate from the local baroclinic instability over the topographic ridge and propagate eastward. This can be seen from Fig. 3.7a, which shows the instantaneous distributions of barotropic coherent vortices in our reference solution. Vortices are the strongest over and downstream of the ridge as indicated by the corresponding largest magnitudes of \( E^2 \). When the ridge is steep, vortices do not show the propensity to propagate along any steady pathway (Fig. 3.7a). In contrast, barotropic coherent vortices over a gentle ridge follow the \( f/h \) contours, \( 2f_0/[H_1 + (H_2 - \eta_b)] \), and thus conserve the barotropic PV. This is demonstrated by Fig. 3.7b, which shows instantaneous paths (dashed lines) of two chains of vortices across a gentle ridge with slopes of \( 5 \times 10^{-4} \), parallel to the reference \( f/h \) contour of \( 4.05 \times 10^{-8} \) m\(^{-1}\) s\(^{-1}\).

These results suggest that the meridional ridge effectively serves as a vorticity source, and the vortices generated over the ridge produce relative vorticity anomalies downstream. To examine the propagation of relative vorticity anomalies, we introduce a new parameter, “vortex concentration”, which is the meridional average of \(|E^2|\).
Figure 3.7: (a) Snapshot of the coherent vortices as shown by the parameter $E^2$ (s$^{-2}$) for the reference solution; the black curves show $f/h$ contours with an interval of $5.00 \times 10^{-9}$ m$^{-1}$ s$^{-1}$. (b) Same property but for the case of a gentle ridge with $\pm 5 \times 10^{-4}$ slopes; the two dash lines denote the paths of two chains of vortices, which are parallel to the reference $f/h$ contour. The vertical solid lines show the position of the ridge, and the background flow is 4 cm s$^{-1}$ in both cases.

This property characterizes the number and intensity of vortices at each longitude. Hovmöller diagrams of the vortex concentration over 5 years show intense relative-vorticity generation events over the ridge and the subsequent vortices propagating eastward with a speed of approximately 1 cm s$^{-1}$ (Fig. 3.8). These vortices can be linear normal modes of the system as suggested by Berloff and Kamenkovich (2013a).

These propagating vortices are highly correlated with the jet axes. Figure 3.9a shows a 10-year mean $|E^2|$, whose spatial distribution is similar to the zonal jets structure in Fig. 3.3b. As further demonstrated by Figs. 3.9b and c, the coherent barotropic vortex paths almost perfectly coincide with the cores of the eastward barotropic zonal jets. In particular, the correlation coefficients of the zonally averaged 10-year mean $|E^2|$ and the zonal velocity of the jets are 0.90 in Region II and 0.96 in Region III.
Figure 3.8: Propagation of coherent vortices as shown by the Hovmöller diagram of meridionally averaged vortex concentration (s$^{-2}$). The coherent vortices propagate eastward at a speed about 1.14 cm s$^{-1}$. The vertical lines show the position of the ridge, and the solid bold line indicates the propagation in the space-time plane.

This is in contrast to the absence of coherent barotropic vortices along the westward barotropic zonal jets.

Cyclones and anticyclones propagate along preferred tracks, and cyclones tend to occupy the north flank of zonal jets and anticyclones occupy the south flank. The spatial distribution of a 10-year mean zonal velocity of the jets has a clear correlation to that of the $E^2$ parameter averaged over the same time period (Figs. 3.10a and 3.10b). Zonal average of the two parameters over flat bottom of the channel shows a 90° phase shift (Fig. 3.10c). The cyclones/anticyclones reside at the north/south flank of the eastward zonal jets; in contrast, no cyclone or anticyclone exist at the flanks of westward jets.

Berloff and Kamenkovich (2013b) demonstrated that some of the eddies preferentially straddle eastward jets and maintain jet existence via eddy forcing. The coherent vortices in this model may also play an important role in maintaining the jets, but isolation of this role is not straightforward. As we discuss jet dynamics in section 3.4.2, we will not differentiate between coherent vortices and other types of transient eddies.
3.3 The importance of eddies in the jet dynamics

In this study, eddies have an impact on the jets by redistributing PV. One way eddies affect the jets is through the eddy PV flux convergence ("eddy PV forcing"), which is an internally generated property. In the baroclinic system, eddy PV forcing comprises
Figure 3.10: Zonal jets and eddy preferred tracks in a two-layer, double-periodic, QG channel model with a meridional ridge with slopes of $\pm 10^{-3}$ in the bottom. (a) 10-year mean barotropic zonal velocity, $u$, showing zonal jets. (b) Preferred tracks of coherent eddies as indicated by parameter $E^2$. (c) Zonally averaged $u$ over the flat bottom overlaid on the corresponding zonally averaged $E^2$. The vertical lines in (a) and (b) denotes the position of the meridional ridge.

Convergences of the Reynolds and form stresses. Another way to describe the effects of eddies on the jets is via eddy acceleration. Specifically, the eddy acceleration component, $(u' \partial / \partial x + v' \partial / \partial y)v'$, accelerates zonal mean flow; and the other eddy acceleration component, $(u' \partial / \partial x + v' \partial / \partial y)u'$, accelerates meridional mean flow. Here, $u'$ and $v'$ are zonal and meridional eddy velocities, which are deviations from the mean state. The two quantities are inherently related: The curl of the eddy acceleration is equivalent to the convergence of the eddy relative vorticity flux or of the Reynolds stress (Pedlosky 1987). Here, to illustrate the essential dynamical effects of eddies on the zonal jets, we study the roles of both eddy PV forcing and eddy acceleration.
3.3.1 Role of eddies over the ridge

3.3.1.1 The role of eddies in the dynamical balance of the topographic meridional currents

First, we focus on the role of eddies in the dynamical balance of the stationary, topographic, meridional currents. To do so, we take the time average of the QG equations and obtain:

\[ \overline{J(\varphi_1, q_1)} + \overline{Uq_{1x}} + (\beta_0 + F_1U)\overline{\varphi_{1x}} = \nu \nabla^4 \overline{\varphi_1}, \]  
(3.7)

\[ \overline{J(\varphi_2, q_2)} + (\beta_0 - F_2U)\overline{\varphi_{2x}} - f_0 \frac{\eta H}{H^2} \overline{\varphi_{2y}} = \nu \nabla^4 \overline{\varphi_2} + \gamma \nabla^2 \overline{\varphi_2}, \]  
(3.8)

where \( q_1 = \nabla^2 \varphi_1 - F_1(\varphi_1 - \varphi_2) \), \( q_2 = \nabla^2 \varphi_2 + F_2(\varphi_1 - \varphi_2) \), and the overbars indicate the time averages. We further separate the flow into the time-averaged and eddying components: \( \varphi_n = \overline{\varphi_n} + \varphi'_n \), where \( \overline{\varphi_n} \) stands for the time-averaged streamfunction, and \( \varphi'_n \) is the residual perturbation streamfunction describing transient eddies. Note, \( \varphi'_n \) represents all transient eddies, including the coherent vortices discussed in section 3.3.2.

Examination of each of the terms in the above equations leads to the following balances over Region I, within 1% accuracy. In the upper layer, there is a linear balance between the advection of meridional currents PV by \( U \) and the advection of the planetary vorticity by \( \overline{\varphi_1} \),

\[ U\overline{\varphi_{1xx}} + UF_1\overline{\varphi_2} + \beta_0\overline{\varphi_1} = 0. \]  
(3.9)

In the deep layer, the nonlinear advection is important, and the advection term, \( \overline{J(\varphi_2, q_2)} = \overline{J(\varphi_2, q_2)} + \overline{J(\varphi'_2, q_2)} \), is dominated by the “eddy PV forcing” component (second term). Stationary flow over the ridge is predominantly meridional; hence,
$J(\varphi_2', q_2')$ is negligible, and to the leading order, we have a balance between $J(\varphi_2', q_2')$ and the meridional advection of the background PV by $\bar{v}_2$,

$$J(\varphi_2', q_2') + (\beta_0 - F_2 U)\bar{v}_2 = 0. \tag{3.10}$$

The topographic terms do not enter the balance explicitly, and their role is in destabilizing the flow and generating the eddy PV forcing. The essential role of the eddy PV forcing is consistent with Holloway (1992), which showed that eddy forcing over a meridional ridge supports the formation of topographic mean flow via the so-called Neptune effect. Here, the deep topographic currents play a key role in maintaining the upper-layer meridional currents via tilting the layer interface.

The entire meridional currents have a strong baroclinic component. This baroclinicity is further outlined by the strong negative correlation between $\tau_1$ and $\bar{v}_2$ of -0.84. This high correlation enables us to find an analytical solution of Eq. (3.9) by assuming that $\bar{v}_2 = \alpha \bar{v}_1$ ($\alpha < 0$ is a constant):

$$U \bar{v}_{1xx} + (\beta_0 + \alpha UF_1)\bar{v}_1 = 0, \tag{3.11}$$

and the solution is

$$v_1(x) = \begin{cases} C_1 \cos \left( \sqrt{\frac{\beta_0 + \alpha UF_1}{U}} x \right) + C_2 \sin \left( \sqrt{\frac{\beta_0 + \alpha UF_1}{U}} x \right), & \alpha > -\frac{\beta_0}{UF_1}, \\ C_1 e^{-\sqrt{\frac{\beta_0 + \alpha UF_1}{U}} x} + C_2 e^{-\sqrt{\frac{\beta_0 + \alpha UF_1}{U}} x}, & \alpha < -\frac{\beta_0}{UF_1}, \end{cases} \tag{3.12}$$

where $C_1$ and $C_2$ are constants. In the reference solution, $\alpha \approx -0.21$, which is larger than $-\beta_0/UF_1 \approx -0.42$; thus, the first analytical solution in Eq. (3.12) applies, and it helps to interpret some properties of the numerical solution. Firstly, the alternating meridional currents are not trapped to the ridge. Secondly, the eddy PV forcing in the
deep layer has a strong influence on the structure of these jets. Particularly, if $v_2 = 0$ (or $\alpha = 0$) due to a hypothetical absence of eddy PV forcing in the deep layer, $\overline{v_1}$ will have alternating currents of minimum widths. In contrast, our solution suggests that strong (relative to the upper-layer linear terms) eddy forcing corresponds to meridional currents that are trapped near the ridge peak (large negative $\alpha$), but we do not obtain this regime in the reference solution.

Eddy acceleration, $(u'\partial/\partial x + v'\partial/\partial y)u'$, exerts opposite effects on the meridional currents in the two layers. In the upper later, it decelerates the meridional current, as manifested by the correlation coefficient of -0.46 between the meridional averages of the eddy acceleration and $\overline{v_1}$. In the lower layer, we find an accelerating effect, and the corresponding correlation coefficient is 0.50.

### 3.3.1.2 Can the eddy PV forcing over the ridge induce zonal jets?

In this section, we examine the role of the deep eddy PV forcing over the ridge in generating zonal jets in the far field. For this purpose, we replace the ridge with a flat bottom and introduce PV source into Region I in the deep layer. The PV equations become:

$$\frac{\partial Q_n}{\partial t} + J(\psi_n, Q_n) = \delta_n \mathcal{F}_n + \nu \nabla^4 \psi_n - \delta_n \gamma \nabla^2 \psi_n,$$  \hspace{1cm} (3.13)

where $Q_n$ is given by Eq. (3.3) without the topographic term, and $\mathcal{F}_n$ is the eddy source, which equals the time-averaged eddy PV forcing over Region I from our reference solution (Fig. 3.10a). Despite the importance of this eddy forcing in maintaining the meridional currents in the reference solution, introducing it into the model with a flat bottom does not lead to the formation of meridional currents. Instead, new eddies are formed, and they act to cancel the PV source locally in the time-mean sense. This implies that the formation of the meridional currents relies on the topographic $\beta$-effect due to the meridional ridge.
Figure 3.11: Zonal jets induced by the localized eddy PV forcing. (a) The eddy PV forcing, $J(\varphi'_2, q'_2)$, in the deep layer over the ridge region. (b) Time- and zonally averaged barotropic zonal velocity from the eddy PV forcing case (blue curve) overlaid on its counterpart from the reference solution (red curve). (c) Time-averaged barotropic zonal velocity for the eddy PV forcing case. The solid lines denote the forcing region.

The deep-layer eddy PV forcing generates alternating zonal jets in the far field, confirming the key role of the ridge-induced vorticity source in the jet dynamics. These zonal jets (Figs. 3.11b and c) closely resemble those in our reference solution. Firstly, the zonal jets downstream of the eddy forcing region are stronger and wider than those in the far field; consequently, the zonal jets undergo splitting and merging events. These properties are consistent with the Rhines scaling as in the reference solution. Secondly, the zonal jets downstream of the forcing region also drift in latitudinal direction, and this is why their signature is smeared in time mean field. The reason for the jet drift is essentially the same as in the reference solution, and we will discuss it in the following section.
3.3.1.3 Zonal jet formation due to a localized stochastic forcing

Is the exact spatial structure of the vorticity source \( \delta_n \mathcal{F}_n \) used in the last section important for the jet formation? To answer this question, we carried out a second experiment with a localized stochastic forcing in the deep layer in Region I. The stochastic forcing is white in time, and its spatial correlations are below the grid resolution. The strength of the stochastic forcing is comparable to that of the eddy forcing in the last section.

Multiple alternating zonal jets form in the far field, demonstrating that a localized vorticity source can effectively generate zonal jets, but its spatial structure is not critical for the jet formation. The jets in this experiment are similar to those in the last section. One distinction is that they are wider than those in the last section: 16 jets form in the channel in the last section in contrast to 26 in the last section.

The characteristics of the zonal jets, however, depend on the strength of the stochastic forcing. In a separate experiment, we choose a stochastic forcing, whose strength is 10 times weaker and observe noticeable changes in the properties of the corresponding zonal jets. In particular, 26 jets form in the domain, which are less latent and narrower than those from the original stochastic forcing. A detailed investigation of the dependence of the zonal jets on the strength of the stochastic forcing is beyond the scope of this study. Berloff (2005) observed jet formation due to a homogeneous stochastic forcing; our results show that a localized stochastic forcing can induce jets in the far field.

3.3.2 Role of eddies in maintaining zonal jets over the flat bottom

In this section, the nonlinear dynamics of the zonal jets will be illuminated by the analysis of the role of eddy forcing in supporting the jets in Regions II and III. Over
the flat-bottom part of the channel, the leading-order time-averaged PV balance is between the Reynolds stress forcing (RSF), form stress forcing (FSF), dissipation, and bottom friction:

\[
J(\phi_n', \nabla^2 \phi_n') + J(\phi_n', (-1)^n F_n (\phi_1' - \phi_2')) = \nu \nabla^4 \phi_n - \delta_{n2} \gamma \nabla^2 \phi_n, \tag{3.14}
\]

where the left-hand side terms are the two components of the eddy forcing, and the right-hand side ones are the dissipation and bottom friction associated with the steady zonal jets. The Reynolds stress can be interpreted as an effective turbulent viscosity and can be a source or sink of PV. The form stress here represents the transfer of zonal momentum between the two layers, and it is proportional to the meridional flux of density.

Since dissipation always tends to resist the formation of the jets, a positive (negative) correlation of the RSF/FSF with the dissipation term means that the RSF/FSF supports (resists) the jets. Figures 3.11a, b show the upper-layer zonally averaged dynamical terms in Regions II and III. In the upper layer, the RSF supports, while the FSF resists the jets. In the deep layer (figure not shown), both the RSF and FSF support the zonal jets. The spatial correlation coefficients between the zonally averaged RSF/FSF and dissipation are shown in Tab. 3.1.

| Table 3.1: Correlation coefficient between RSF/FSF and dissipation |
|-------------------|-------------------|-------------------|-------------------|
|                   | Upper Layer        | Lower Layer       |
|                   | Region II | Region III | Region II | Region III |
| RSF               | 0.60       | 0.66       | 0.70       | 0.87       |
| FSF               | -0.54      | -0.55      | 0.58       | 0.66       |

The RSF and FSF compensate each other in Region III, while the forcing terms are not in phase in Region II (Fig. 3.12). The resultant total eddy PV forcing coincides
with the PV anomalies (associated with the zonal jets) in Region III (Fig. 3.13b), where the zonal jets are stationary. In contrast, the total eddy PV forcing in Region II is shifted southward relative to the zonal jet PV (Fig. 3.13a). This is further indicated by the low spatial correlation of -0.12 between the two terms in Region II in contrast to 0.73 in Region III. The off-core eddy PV forcing on the zonal jets in Region II, therefore, acts to push the jets southward, resulting in the drift. Both components of the eddy PV forcing are important in this regard. Jet drift is a universal consequence of breaking reflection symmetry by either topography or the spatial structure of eddy forcing; if the phase lines of the eddy forcing are not parallel or perpendicular to the lines of the constant background potential vorticity, jets will drift (Srinivasan, UCLA, 2014, personal communication). We hypothesize that the formation of the meridional currents over and downstream of the ridge renders nonzonal PV contours and induces asymmetries in the spatial structure of eddy forcing, leading to the jet drift.

We further interpret the eddy effects using the concept of eddy acceleration. A 10-year mean of the eddy acceleration, \( (u' \partial / \partial x + v' \partial / \partial y)v' \), of the upper-layer zonal jets is shown in Fig. 3.14a. The eddy acceleration is only robust over and downstream of the ridge, where the eddy activity is most intense, as shown in Fig. 3.3a. The eddies provide an eastward/westward acceleration to the eastward/westward zonal jets and also act to shift the cores of the jets southward. The latter effect is further shown in Fig. 3.14b, in which the zonal average of the eddy acceleration is clearly shifted southward to the zonal velocity of the jets. The situation is very similar in the deep layer (figure not shown).
3.4 Zonal jet formation in a locally nonzonal background flow

Since the background zonal flow is stable over a flat bottom, the jets in the far field are caused by the zonal topographic component of the mean PV gradient in the ridge region. In this section, we examine the effects of the nonmeridional PV gradient but due to a locally nonzonal background flow over flat bottom. Note that this setting can also help to identify the role of meridional currents in the reference simulation.

For this purpose, we impose a locally-nonzonal flow in the upper layer—the mean flow is purely zonal and has a velocity of U everywhere, except over the region where
Figure 3.13: Correlation between the eddy PV forcing and jets. Zonal averages of a 10-year mean total eddy PV forcing ($s^{-2}$) overlaid on the corresponding PV anomalies (arbitrary units) in (a) Region II and (b) Region III of the upper layer. The correlation coefficients between the two are -0.12 in (a) and 0.73 in (b).

Figure 3.14: Eddy acceleration ($cm s^{-2}$). (a) A 10-year mean eddy acceleration of zonal jets, $\left< \frac{u'}{x} \frac{\partial x}{\partial x} + \frac{v'}{y} \frac{\partial y}{\partial y} \right>v'$, in the upper layer of the reference solution. (b) Zonal average of the 10-year mean eddy acceleration overlaid on the zonally averaged zonal velocity of the jets, $\bar{u}$ ($cm s^{-1}$), in the upper layer in Region II.
it is nonzonal with \((U,V)\) velocities. The governing PV equations become:

\[
\frac{\partial q_1}{\partial t} + J(\varphi_1, q_1) + (\beta_0 + F_1 U) \frac{\partial \varphi_1}{\partial x} - \frac{\partial Z}{\partial x} \frac{\partial \varphi_1}{\partial y} + U \frac{\partial q_1}{\partial x} + V \frac{\partial q_1}{\partial y} = \nu \nabla^4 \varphi_1 + \mathcal{F}^*, \tag{3.15}
\]

\[
\frac{\partial q_2}{\partial t} + J(\varphi_2, q_2) + (\beta_0 - F_2 U) \frac{\partial \varphi_2}{\partial x} = \nu \nabla^4 \varphi_2 - \gamma \nabla^2 \varphi_2, \tag{3.16}
\]

where \(V\) is the meridional flow in the upper layer, \(Z = \partial V/\partial x - F_1 \int V \, dx\) is the PV gradient associated with the meridional flow, and

\[
\mathcal{F}^* = - (\beta_0 + F_1 U) V_1 - U \partial Z/\partial x + \nu \nabla^4 (\int V \, dx) \tag{3.17}
\]

is the additional forcing term required to maintain the meridional flow.

We choose a subcritical background zonal flow, \(U = 4 \text{ cm s}^{-1}\), and a meridional flow,

\[
V(x) = A \text{sech}^2 \left( \frac{x - C}{W} \right), \tag{3.18}
\]

where \(A\) is the magnitude of the meridional flow, \(C\) determines the position of its peak, and \(W\) defines its width. We set \(A = 4 \text{ cm s}^{-1}\) and choose \(W = 100 \text{ km}\) (Fig. 3.14b).

Started from the initial state of rest with small-magnitude perturbations, the system reaches a statistical equilibrium characterized by baroclinic meridional currents near the region of the mean-flow nonzonality (Fig. 3.15b) and by alternating zonal jets in most of the domain (Fig. 3.15a). Secondary meridional currents emerge in both layers as a result of the rectification of the flow (Fig. 3.15b), but the mean flow remains nonzonal in the background meridional flow region. Zonal jets downstream of the unstable region are stronger than those over the rest of the channel, and, unlike the jets in Region II of the reference solution, they do not drift meridionally. Also, the positions of the zonal jets upstream and downstream of the background meridional
Figure 3.15: Solution with a locally nonzonal background flow and flat bottom. (a) A 20-year mean barotropic velocity and streamfunction showing zonal jets. (b) The background meridional flow, $V$, and the corresponding 20-year mean meridional velocities $\overline{v}_1$ and $\overline{v}_2$, respectively.

flow are shifted, thus, the zonal jets are slightly slanted but continuous across the periodic channel.

Alternating zonal jets also form in the model, if $V$ is replaced with the strongest baroclinic meridional current taken from the reference solution. These zonal jets closely resemble those in the reference solution. In general, without topography, a subcritical but locally nonzonal flow is able to generate a system of currents similar to the one that exists in the presence of a meridional ridge. One can, therefore, expect a similar mechanism for jet formation in these two configurations.

Nonlocal radiating mechanism can be invoked to explain the zonal jet formation in this section. Locally nonzonal currents are inherently unstable and can support unstable growth of waves with a long oscillating tail (Kamenkovich and Pedlosky 1994). These radiating waves of nonzonal flows extend into the far field (Kamenkovich
and Pedlosky 1996; Hristova et al. 2008), and triad interactions between the radiating and trapped waves can render zonal jets (Wang et al. 2012). The radiating waves carry into the far field the energy and PV anomalies and feed the zonal jets. We hypothesize that fundamentally similar mechanism acts in our study.

When the meridional ridge is gentle, standing circular loops form over the ridge instead of coherent meridional currents (section 3.3). The locally nonzonal flow is very complex and, in the bottom layer, resembles boundary currents flowing along irregular coasts. Nevertheless, zonal jets still form in the far field. We hypothesize here, however, that such a flow generates zonal jets via the same nonlocal mechanism.

### 3.5 Summary and conclusions

In this study, we explore the effects of bottom topography on the formation of eddies and eddy-driven currents in baroclinic oceanic flows. We demonstrate that a single meridional topographic ridge can destabilize an otherwise stable eastward background flow, and that this instability leads to the formation of a system of currents, consisting of mesoscale eddies, stationary meridional currents over the ridge, migrating zonal jets downstream of the ridge, and stationary alternating zonal jets in the far field. Eddy PV forcing is essential for the dynamics of these currents. In particular, the meridional currents owe their existence to the local action of the eddies over the ridge, but the zonal jets arise from a novel nonlocal eddy-forced mechanism. Zonal component of the mean PV gradient, whether due to the presence of a meridional ridge or due to a locally nonzonal background flow, is essential for this process.

Large-scale topographic slopes can destabilize a broad flow even with a weak vertical shear. Meridional slopes can do this by changing the meridional PV gradient beyond a stability threshold (Chen and Kamenkovich 2013), and zonal slopes are even more effective in this regard (Boland et al. 2012). This jet-formation mecha-
nism over topographic slopes relies on basin-scale PV gradients, and it could involve transformation of basin-scale unstable modes (Berloff et al. 2009a); the dynamics is therefore most likely similar to that over the flat bottom. This mechanism is locally-controlled in the sense that the formation of eddy-driven jets is determined by the local properties of the background state.

The novel nonlocal mechanism of jet formation in this study involves destabilization of a subcritical (stable) baroclinic current flowing over an isolated ridge, which leads to the creation of an energetic eddy field. The eddy generation over the ridge serves as a vorticity source for eddies and eddy-driven zonal jets in the flat-bottom part of the domain. These zonal jets cannot form unless such a vorticity source is present. This is demonstrated by a numerical simulation, in which the meridional ridge is replaced by a steady eddy forcing term. Coherent vortices (defined by a modified Okubo-Weiss parameter) form over the ridge, propagate downstream, and transport vorticity anomalies into the far field. These coherent vortices tend to follow $f/h$ contours only when the ridge is sufficiently gentle. In the far field, coherent vortices straddle the eastward zonal jets, suggesting a potential role in maintaining these zonal jets (see also Berloff and Kamenkovich 2013a,b). Potential contribution of the coherent vortices to this nonlocal mechanism of jet generation has not, however, been isolated in this study and should be a subject of future research.

Eddy PV forcing is also vital for maintaining the meridional currents over the ridge. Such currents are capable of inducing zonal jets in the far field, as demonstrated by an experiment with a locally nonzonal background flow over flat bottom. Flows with a localized meridional component can support radiating normal modes, which transfer energy from the local unstable region into the far field (Kamenkovich and Pedlosky 1994; Kamenkovich and Pedlosky 1996; Hristova et al. 2008). We hy-
pothesize that triad interactions between the radiating and trapped modes generate multiple zonal jets, as in the case of an eastern boundary current (Wang et al. 2012).

More complicated situations are also possible. In particular, broad and gently sloped ridges result in strong stationary meanders but also generate zonal jets in the far field. It is tempting to conclude that any locally nonzonal flow is capable of exciting zonal jets. The presence of the local vorticity source related to the region of enhanced instability seems to be, as discussed above, a fundamental element of the nonlocal mechanism of jet generation in all cases. Another interesting aspect of the flow near the ridge is the drift in zonal jets. The drift is explained by the eddy PV forcing being out of phase with the positions of the zonal jets at the lee of the ridge. We hypothesize that the formation of the meridional currents over and downstream of the ridge renders nonzonal PV contours and induces asymmetry in the spatial structure of eddy PV forcing, leading to the jet drift (Srinivasan UCLA, 2014, personal communication).
Chapter 4

Eddies and Striations in a Baroclinic Double-Gyre Ocean

Due to the lack of a thorough understanding of the nature of the zonally elongated structures discovered from altimetry, both “jets” and “striations” are used to describe these structures in literature. The choice of “jets” or “striations” is somehow arbitrary and sometimes interchangeable. Some studies suggested that these structures are coherent jet flows, either stationary or non-stationary, and an integral component of the flow. Stationary jets are visible in long-term means. They are known to exist in quasi-geostrophic (QG) channel models (Berloff et al. 2011). Non-stationary jets can exhibit low-frequency variability and/or migration, and they are challenging to define (Maximenko et al. 2005). Other studies defined these structures as striations, which are a broader term for elongated structures visible in low-pass filtered data (Maximenko et al. 2008). The striations can be stationary or non-stationary as well. A brief summary of the definitions of “jets” and “striations” is given in Tab. 4.1. In this study, we use the more general term “striations” to describe these elongated structures.

The striations are intrinsically related to eddies. Schlax and Chelton (2008) showed that the artifacts of time averaging of westward propagating random eddies resemble the striations from altimetry data. Buckingham and Cornillon (2013)
found that eddies contribute considerably to the striations from the 4-year mean absolute dynamic topography, and disapproved the model of random eddies proposed by Schlax and Chelton (2008) since the time-mean zonal velocity of eddies, a dominate source of the zonal velocity of striations, decays with averaging period with a different rate from the inverse relationship in Schlax and Chelton (2008). Chen et al. (2014) suggested that the low-frequency component of non-dispersively propagating eddies manifest as striations.

**Table 4.1:** Two disparate views of the zonally elongated structure observed in the ocean

<table>
<thead>
<tr>
<th></th>
<th>Definition</th>
<th>Relationship with eddies</th>
</tr>
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<tbody>
<tr>
<td>Jets</td>
<td>Coherent component of the flow, including stationary and non-stationary jets</td>
<td>Jets regulate formation of eddies; eddies maintain jets.</td>
</tr>
<tr>
<td>Striations</td>
<td>Broader term for elongated structures visible in low-pass filtered data</td>
<td>Artifacts of time averaging of random eddies</td>
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<td></td>
<td></td>
<td>Manifestation of systematically propagating eddy trains</td>
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</tbody>
</table>

To shed light on the nature of the striations, we study their formation in a nonzonal background flow—a baroclinic double gyre circulation, and examine their relationship with eddies in this chapter. Previous studies (Nadiga 2006; Tanaka and Akitomo 2010; O’Reilly et al. 2012) investigated jet formation and its dynamics in double-gyre circulation models; however, most of the studies focused on barotropic systems or showed that striations only emerge from additional forcing in baroclinic systems. In our study, we shall see that striations are intrinsic feature in the wind-driven double-gyre circulation and that eddies propagate in a systematic way in the eastern part of the domain, manifesting as eddy trains and leading to the emergence of quasi-zonal striations.
This chapter is structured in the following way. Section 4.1 shows the detection of striations from a double-gyre circulation, studies the characteristics of the striations, investigates effects of a meridional ridge on the general circulation and the striations. Section 4.2 explores the relationship between eddies and striations by employing an idealized kinematic model and examines the dynamical role of eddies in sustaining striations in the double-gyre model. Section 4.3 interprets the relationship of coherent eddies and striations in the real ocean based on the results from the kinematic model and the double gyre model. Conclusions are given in section 4.4.

### 4.1 Eddies and striations in double gyre circulation

In this section, we examine the formation of striations and study their possible relation to coherent eddies in a baroclinic, wind-driven, double-gyre, QG model. Effects of a meridional topographic ridge is also studied.

#### 4.1.1 The model

The three-layer, double-gyre, QG model represents a closed middle ocean basin on the $\beta$-plane (Fig. 4.1). A wind stress curl provides the energy source and drives the general flow within the basin into a double-gyre. Potential vorticity (PV), $Q_n$, in each of the three dynamically active isopycnal layers are governed by

$$\frac{\partial Q_n}{\partial t} + J(\phi_n, Q_n) = \delta_n \mathcal{F} + \nu \nabla^4 \phi_n - \delta_n \gamma \nabla^2 \phi_n \quad (n = 1, 2, 3),$$

where the layer index starts from the top, $\phi_n$ are streamfunction disturbances, the terms with $\nu$ and $\gamma$ are lateral viscosity and bottom friction, respectively, $J(,)$ is the
The stratification parameters of this model are as follows:

\[
F_1 = \frac{f_0^2}{g_1' H_1}, \quad F_{21} = \frac{f_0^2}{g_1' H_2}, \quad F_{22} = \frac{f_0^2}{g_2' H_2}, \quad F_3 = \frac{f_0^2}{g_2' H_3},
\]

where \( f_0 \) is the Coriolis parameter, \( g_\prime_n \) are reduced-gravity coefficients associated with the density jump between the isopycnal layers, and \( H_n \) are depths of the three layers. The isopycnal PV anomalies, \( Q_n \), are related to the streamfunction perturbations via
PV inversion:

\[ Q_n = \nabla^2 \varphi_n + \beta_0 y + \delta_{n1} F_1(\varphi_2 - \varphi_1) + \delta_{n2} [F_{21}(\varphi_1 - \varphi_2) + F_{22}(\varphi_3 - \varphi_2)] + \delta_{n3} F_3(\varphi_2 - \varphi_3) + \delta_{n3} \frac{\eta_b}{H_3}, \]  

where \( \eta_b \) is the spatially varying elevation of bottom topography.

The basin scale is 3840 × 3840 km, and the grid size is 7.5 km. The thicknesses of the layers are 250, 750, and 3000 m from top to bottom. The Rossby deformation radii are 30.65 and 14.23 km. The reason of choosing the depths of the layers and the Rossby deformation radii is to closely approximate the sharp thermocline. The wind stress on the top layer is supposed to produce a mixing layer, the middle layer represents the sharp thermocline, and the deep layer is an ideal, relatively calm, deep ocean layer. This model can be considered as an idealized North Atlantic Ocean. This model is adapted from Karabasov et al. (2009) and allows large Reynolds numbers on relatively coarse grids.

### 4.1.2 Detection of striations in the double-gyre circulation

A steady double-gyre circulation is clearly shown in the upper layer with a flat bottom (Fig. 4.2a). The asymmetry in the wind stress curl along middle latitudes leads to the formation of a meandering “Gulf Stream”, which diminishes in strength as it penetrates northeastward into the eastern basin. The double gyre shrinks below the upper layer, while the eddies shedding off the Gulf Stream leave a strong signature all across the vertical layers. The barotropic component of the steady flow in the basin shows a double gyre, the Gulf Stream and eddies associated with it.

Another distinct flow feature—alternating striations within the gyres—is clearly shown in a 1-year mean barotropic streamfunction and its corresponding velocity field (Fig. 4.3). The striations are most distinct in the eastern parts of the double gyre,
Figure 4.2: Solution of the wind-driven model with a flat bottom. Long term mean streamfunction in (a) the upper, (b) middle and (c) deep layers, as well as (d) the barotropic component.

where the background flow is weak and dominantly nonzonal. The absence of the striations in the long-term mean field (Fig. 4.1) may indicate unsteady nature of the striations, due to which that long term mean may smear their signature. Section 4.1.3 will show that the striations are indeed unsteady.

4.1.2.1 Spatial filtering

The total flow shown in Fig. 4.3 consists of a double gyre, the Gulf Stream and eddies associated with it, and striations. We first calculate the 2-D wavenumber spectrum
Figure 4.3: Short time mean flow in the basin with a flat bottom. (a) 1-year mean barotropic streamfunction and (b) the corresponding velocity field.

of the 1-year mean barotropic streamfunction. It is emphasized that the wavenumber spectrum only defines the power of the 1-year mean flow in the wavenumber space, it cannot distinguish if a flow feature associated with any peak in the spectrum is from eddy trains or random eddies, neither can temporal filtering in the following section.

A dominating peak around $k = 0$ and $l = \pm 1$ shows up in the 2-D wavenumber spectrum (Fig. 4.4a). This peak corresponds to the double gyre. Another two peaks, $l = 5$ and 7, are clearly seen in the profile of the two-dimensional spectrum at $k = 0$. The corresponding scales are 384 and 274 km, which are larger than the scale of the striations shown in Fig. 4.3. These peaks are associated with the Gulf Stream and the countercurrents around it, whose width are around 350 km. Another peak at $l = 12$ is also distinct in Fig. 4.4b, and it represents the striations; the corresponding scale of $l = 12$ is 160 km, which is about the average scale of those striations.

We do a similar spectrum analysis of the 1-year barotropic streamfunction within a rectangular box of $1505 \times 1505$ km, which contain the zonally elongates structures in the subtropical gyre. The results further confirms the existence of the striations.
Figure 4.4: The 2-D wavenumber and its profile at $k = 0$ for the 1-year mean barotropic streamfunction in the basin as shown in Fig. 4.3a.

...centrates around $k, l = 0$ and corresponds to the time mean flow and the second one concentrates around $l = 10$ and corresponds to the striations. The scale of the striations vary in different parts of the basin Fig. 4.3, this is likely why the average scale of the striations is around 192 km ($l = 10$) in the selected box in contrast to 160 km as shown in Fig. 4.4.

To isolate the striations from the rest of the flow, we choose a filter that excludes the first peak in Fig. 4.4a, essentially the double gyre, and apply it to the 1-year mean barotropic streamfunction. This filter is essentially a 2-D wavenumber filter with the values of 0 at $-2 < l < 2$ and of 1 elsewhere. By timing this 2-D filter to the 2-D wavenumber spectrum shown in Fig. 4.3a and doing an inverse fast Fourier transform, we obtain a residual flow field excluding the double gyre. The residual flow is shown in Fig. 4.6a. The Gulf Stream and its countercurrents are clearly seen, and striations with width around 160 km also show up in the eastern basin.
Figure 4.5: The 2-D wavenumber spectrum and its profile at \( k = 0 \) for 1-year mean barotropic streamfunction within a rectangular box of 1505 \( \times \) 1505 km containing the zonally elongated structures as shown in Fig. 4.3a.

4.1.2.2 Temporal filtering

The other way to isolate the striations is using temporal filtering. We follow the literature and reason that oceanic striations are intermediate in temporal scale between mesoscale eddies and long time-mean flow (Richards et al. 2006), and isolate the striations by taking time average of the anomalous streamfunction, which is the residual of the snapshot streamfunction subtracting the long term mean.

The choice of the time averaging period is arbitrary. In this study, we adapt the degree of anisotropy parameter, \( \alpha \), to make the choice. The degree of anisotropy has been used to study zonality of the flow field (Huang et al. 2007). We multiply it with the magnitude of area-averaged zonal velocity and define a new parameter, \( \delta \):

\[
\delta = \frac{\overline{|u|^2}}{u_0} \cdot \frac{<u^2> - <v^2>}{<u^2> + <v^2>}, \tag{4.4}
\]

where the second factor on the right hand side is \( \alpha \), \( u \) and \( v \) are time-averaged zonal and meridional velocities, and \( u_0 \) is a reference velocity. The overbars denotes zonal...
Figure 4.6: Barotropic striations detected from two filtering methods applied to 1-year mean streamfunction. 1-year mean striations derived from (a) the spatial filtering method and (b) the temporal filtering method.

and meridional averages, and \(< \cdot >\) stands for area average. The larger \(\delta\) is, the more zonal and more robust the flows are.

To determine the best filter to get the most robust zonal striations, we calculate \(\delta\) for temporal filters of 1 week to 10 years for the flows in a subpolar box and a subtropical box (Fig. 4.6b). The results are shown in Fig. 4.7. In general, \(|\overline{u^y}|/u_0\) and \(\alpha\) increase with the increase in the temporal filter in the beginning; in other words, as the time averaging period increases, the resulting flow becomes more anisotropic and more zonal. While sufficiently large time averaging period yields weak zonal flows. The maximum value of \(\delta\) occurs for the temporal filter of 27 weeks for the subtropical box and of 53 weeks for the subpolar box. Thus, we choose the temporal filter of 1 year as the filter to identify striations in the following analysis.

Both the spatial and temporal filtering methods yield the presence of striations. The striations derived from the two methods have strikingly similar spatial distribution and strength (Fig. 4.6). The only difference is within the flow over the Gulf Stream region. The temporal filtering filters out long term mean steady Gulf Stream;
Figure 4.7: Degree of anisotropy and meridional mean of the magnitude of zonal mean zonal velocity in the subtropical box (upper panel) and subpolar box (lower panel) for striations detected from the temporal filtering method.

thus, the result shows circular meanders instead of a coherent Gulf Stream flow system as from the spatial filtering method. The results also show that 1 year is likely to be a proper time average span to get robust striations. In the following, we study the characteristics of the striations based on the results of the temporal filtering with a filter of 1 year.

4.1.3 Characteristics of the striations

Distinct barotropic striations show up within the eastern part of the double gyre limbs. The velocity of the striations alternate with latitude. The strength of the westward and eastward striations are comparable and are about 1 cm s$^{-1}$ as shown by the zonal average of zonal velocity (Fig. 4.8). The widths of the eastward and westward striations are comparable as well. The averaged width is around 160 km
for the striations in the subtropical gyre and 158 km in the subpolar gyre. Striations also exist in the westward branch of the subtropical gyre, but are less coherent.

![Figure 4.8: Zonal average of the zonal velocity of the striations in the eastern basin shown in Fig. 4.6b.](image)

The striations are vertically coherent. They reach into the deep layer and become weaker (Fig. 4.8). Figure 4.8 plots the zonally averaged zonal velocity of the striations in the eastern part of the domain. The profile of the striations in the upper layer (blue curve) shows the influence of the background double-gyre: Both the striations of in the northern and southern domain are influenced by the local westward gyre currents. This influence does not show up in the middle and deep layers, where the background double-gyre flow is much weaker; also, the striations in the two layers align parallel, showing a coherent structure. Figure 4.9 shows the baroclinic components. The striations are strongest in the upper layer, where their velocity is on the scale
of 10 cm s\(^{-1}\); in the deep layer, it decreases to about 2 cm s\(^{-1}\). Figure 4.9a shows that the flow around the striations in the upper layer is less coherent compared to that below the upper layer. It is likely that the high-pass filtered field (Fig. 4.9a) still contains eddying motions that make the striations less coherent.

![Baroclinic Jets (Upper Layer)](image1)

![Baroclinic Jets (Middle Layer)](image2)

![Baroclinic Jets (Deep Layer)](image3)

**Figure 4.9:** Baroclinic components of the striations: (a) Upper layer, (b) middle layer, and (c) deep layer. The corresponding barotropic component is shown in Fig. 4.8b.

The barotropic striations tend to drift latitudinally away from the inter-gyre boundary. The Hovmöller diagram of zonal velocity of the striations in the subtropical gyre shows at least two properties (Fig. 4.10b). First, the striations strengthen
and weaken with time. Secondly, they drift southward with a speed as large as \(0.4\) cm s\(^{-1}\). In contrast, the striations in the subpolar gyre (Fig. 4.10a) are less coherent and have a tendency to drift northward with a speed as large as \(0.6\) cm s\(^{-1}\). To examine if the drift is purely numerical, we run another simulation by doubling the model resolution; our results show that the striations in the new simulation also drift in a similar manner and at approximately the same speeds. The gyre flow direction in the two regions of the striations is consistent with the direction of the striation drift; the meridional average of the meridional gyre flows is about \(0.1\) and \(-0.1\) cm s\(^{-1}\) in the subpolar and subtropical boxes, respectively. Thus, the gyre flow may be partly responsible for the striation drift.

![Figure 4.10: Hovmöller diagram of zonal velocity of the barotropic striations in (a) the subpolar and (b) subtropical boxes as shown in Fig. 4.8b.](image)

4.1.4 Effects of a topographic ridge on the circulation

In this section, we examine the effects of a meridional ridge on the circulation. We introduce a ridge with a height of 750 m and a zonal span of 1500 km; thus, the
slope of the ridge is $\pm 10^{-3}$ on its two sides. The south and north parts of the ridge are smoothed to prevent formation of non-dynamical flows at the boundaries. The configuration of the ridge is shown in Fig. 4.1.

\begin{figure}
\centering
\begin{subfigure}{0.4\textwidth}
\centering
\includegraphics[width=\textwidth]{fig41a.png}
\caption{Upper Layer}
\end{subfigure}
\begin{subfigure}{0.4\textwidth}
\centering
\includegraphics[width=\textwidth]{fig41b.png}
\caption{Middle Layer}
\end{subfigure}
\begin{subfigure}{0.4\textwidth}
\centering
\includegraphics[width=\textwidth]{fig41c.png}
\caption{Deep Layer}
\end{subfigure}
\begin{subfigure}{0.4\textwidth}
\centering
\includegraphics[width=\textwidth]{fig41d.png}
\caption{Barotropic Component}
\end{subfigure}
\caption{Solution of streamfunction for the wind-driven model with a meridional ridge with slopes of $\pm 10^{-3}$ in the bottom of the basin. Long time averages of (a) streamfunction in the upper layer, (b) the middle layer, and (c) the deep layer. (d) The barotropic streamfunction. The contours in the figures show the position of the ridge.}
\end{figure}

One direct effect of the meridional ridge on the general flow is from topographic steering and blocking. The ridge tends to confine the general circulation in the western basin (Fig. 4.11). The double-gyre is the most robust in the upper layer but is slightly distorted by the ridge compared to the flat bottom case. It also seems that the Gulf
Figure 4.12: Solution of velocity for the wind-driven model with a meridional ridge with slopes of $\pm 10^{-3}$ in the bottom of the basin. A 1-year mean (a) barotropic flow, (b) baroclinic flow in the upper layer, (c) baroclinic flow in the middle layer, and (d) baroclinic flow in the deep layer. The contours in the figures show the position of the ridge.

Stream is blocked by the ridge and cannot shoot across ridge into the eastern basin as seen over a flat bottom.

More interestingly, the Gulf Stream flows southward once it encounters the ridge instead of extending northeast as in the flat bottom case. The conservation of vorticity can be applied here to explain this new phenomenon. The Gulf Stream is coherent vertically and has a significant barotropic component. As it climbs up the ridge, the
depth of water column decreases, the flow is subject to anticyclonic motion, acquires negative relative vorticity and steers southward, resulting in a decrease in planetary vorticity $f$. The strong steering effect eventually leads to the blocking of the Gulf Stream by the ridge.

Striations become weaker and shorter in zonal extent (Fig. 4.12). Alternating striations still exist at the eastern parts of the double-gyre, but their zonal extension is shortened by the ridge. This can be explained by the conservation of vorticity as well. In the presence of the ridge, eddy trains cannot propagate along latitudes, instead, they are steered by the ridge. Coherent eddies are weak in the eastern basin away from the inter-gyre boundary, and they do not possess the vorticity intensity to cross over the ridge and thus form coherent eddy trains. However, eddies shedding off the Gulf Stream over the top of ridge are able to propagate downstream and organize themselves into slanted striations downstream of the ridge (Fig. 12a). This phenomenon is very similar to the channel model simulation presented in chapter 3.

4.2 Interpretation of the striations as eddy trains

In chapter 3, we showed that coherent eddies in the channel model propagate along preferred tracks in latitude, creating alternate cyclonic and anticyclonic eddy trains. Zonal striations form between the eddy trains. In this section, we will prove that eddies trains are also responsible for the formation of the striations in the double gyre model.

Cyclones and anticyclones in the model tend to propagate along alternate latitudes at the flanks of the striations. A 1-year mean of $E^2$ shows a clear correspondence of the preferred tracks of eddies to the striations (Fig. 4.13a). The solid contours indicate eastward zonal velocity, and the dash contours indicate westward zonal velocity. It is distinct that cyclonic eddy tracks are to the north of eastward velocity, while
anticyclonic eddy tracks are to the south. The adjacent cyclonic and anticyclonic motions, at least, partly contribute to the striations between them.

Figure 4.13: 1-year mean preferred tracks of coherent eddies overlaid on the zonal velocity of striations in the subtropical box. (b) Trajectories of 10 coherent eddies in the subtropical box; shaded color stands for coherent eddies at their starting positions.

Figure 4.13b shows a snapshot of distribution of coherent eddies in the subtropical box. The 5 blues lines denote the trajectories of 5 selected anticyclones, indicating westward propagation of an eddy train. The trajectories stop when the eddies dissipate completely, merge into other eddies, or propagate out of the subtropical box. The red lines denote the trajectories of 5 cyclones. Thought these sparse eddy trajectories cannot show whether the eddies in this domain are random eddies or systematically propagating eddy trains, the trajectories still show that the selected eddies propagate westward preferentially along certain latitude bands: anticyclonic eddies in the latitude band north of the latitude band of cyclonic eddies.

Are the tracks are from eddies trains or just random eddies, and what are their relationships with the striations? Three possible scenarios exist as shown in Tab. 4.1. First, random single eddies propagate westward and leave tracks, which show up as striations in short term mean field (Fig. 4.14a). This scenario is similar to the one proposed by Schlax and Chelton (2008) and is not important in regards of the effects of the striations as material transport barriers, since isotropic distribution of cyclones
and anticyclones yields zero mean flow in long term mean. The second scenario concerns eddy trains. Coherent eddies move as eddy trains and produce striations (Fig. 4.14b), which can act as material transport barriers and consequently affect the general circulation. In the third scenario, eddies trains and coherent striations coexist (Fig. 4.14c). Striations can affect the formation of new eddies, in return, eddies can support or resist the striation formation. The difference between the second and third scenarios is the strength of jets; separating these two cases is challenging and not attempted in this study.

4.2.1 An idealized kinematic model for propagating eddies

We first design an idealized kinematic model to study how propagating eddies can induce mean flows, with a focus on the first two scenarios: random eddies and eddy trains. The model has a double periodic domain with the size of $100 \times 100$. We introduce into the domain anticyclones and cyclones, which alternate in latitude. The eddies are represented by negative (positive) sea surface height anomalies as shown by blue (red) shades in Figs. 4.15 and 4.16. The eddies have slightly different zonal propagating speeds, $C_x$, and their meridional propagating speeds, $C_y$ are 100 times smaller than the zonal ones.

In the first experiment, we introduce 3 cyclones and 3 anticyclones into the domain. Their initial positions are shown in Fig. 4.15a. The eddies propagate mainly westward and leave zonal tracks across the domain in time-averaged field (Fig. 4.15b), showing up as zonal striations. This scenario is very similar to the generation of striations in Schlax and Chelton (2008). The EOF analysis of the propagating eddies shows a field full of cyclonic and anticyclonic flows (Figs. 4.15c and 4.15d), suggesting no mode of variability that look like striations. These two modes show the variations of or deviations from the time-mean zonal striations. Moreover, the widths and
Figure 4.14: Three scenarios of the relationship between eddies and striations. (a) Striations being the artifacts of time-averaging of propagating random eddies. (b) Propagating eddy trains leading to the emergence of coherent striations. (c) Coexistence of eddies and coherent striations, and eddies striding the striations.

Zonal extensions of the time-mean zonal striations depend on the size and initialized positions of the eddies.

In the second experiment, we introduce into the domain eddy trains—3 pairs of cyclones and anticyclones reentering the domain multiple times. Time average leads to the presence of multiple zonal striations. As suggested by the EOF analysis, the resultant flow manifests itself as meridionally drifting zonal striations (Fig. 4.16). The first EOF accounts for 6% of the total variation of the flow and shows 6 alternating
Figure 4.15: Solution of the kinematic experiment of propagating random eddies. (a) Initial positions of the eddies in a doubly periodic channel; the arrows show eddy propagation speeds and directions. (b) Time mean field showing zonal striations. The first two EOF modes of the propagating eddies are shown in (c) and (d). The scale of the shades is arbitrary. Blue (red) shades show negative (positive) sea surface height.

zonal striations; the second EOF accounts for 5% of the total variation of the flow and shows 6 alternating zonal striations shifted in position (Figs. 4.16c and 4.16d). The third EOF does not show any coherent variation.

One main conclusion from these two experiments is that low-pass filtering—time average in the experiments—cannot distinguish between the striations from random eddies and eddy trains. EOF analysis can achieve this goal. If the striations are
Figure 4.16: Solution of the statistical experiment of propagating eddy trains. (a) Initial positions of eddies in a doubly periodic channel; the arrows show eddy propagation speeds and directions. (b) Time mean field showing zonal striations. The first two EOF modes of the propagating eddies are shown in (c) and (d). The scale of the shades is arbitrary; the blue (red) shades show negative (positive) sea surface height.

due to eddy trains, EOF analysis of the eddying flow can capture the striations as variance of the mean flow; however, if the striations are artifacts of propagating random eddies, EOF analysis will not show striations as a coherent component of the total flow (Tab. 4.2). Moreover, meridional propagating speeds of the eddy trains are essential for the meridional migration of the striations as shown in the EOFs. We will apply EOF analysis to interpret the striations in the double-gyre ocean.
Table 4.2: Resultant flows from two kinds of propagating eddies

<table>
<thead>
<tr>
<th></th>
<th>Time Average</th>
<th>EOF Modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random eddies</td>
<td>Zonal striations</td>
<td>No mode of variation shows striations.</td>
</tr>
<tr>
<td>Eddy trains</td>
<td>Zonal striations</td>
<td>Modes of variation show drifting striations.</td>
</tr>
</tbody>
</table>

4.2.2 The relationship between coherent eddies and the striations in the double-gyre ocean

![Figure 4.17](image.png)

**Figure 4.17:** Reconstructed velocity fields from the first two EOF modes of 197 snapshots of yearly mean streamfunction. (a) The first EOF (16%). (b) The second EOF (12%).

The EOF analysis of the eddying flow in the double-gyre ocean shows striations (Fig. 4.17), confirming that eddy trains are likely to be responsible for the formation of striations as suggested by the kinematic model. To calculate the EOFs, we take simple moving averages of 50-year daily streamfunction data and obtain 197 yearly mean snapshots. The EOF analysis of the 197 yearly mean streamfunction shows striations as a integral variance of the total flow. The first two reconstructed velocity fields from the EOF modes have similar features, except that the spatial positions of the striations from the second EOF mode are shifted to those from the first EOF.
mode. Also, the first EOF mode shows robust alternating striations in the northern and southern boundaries of the basin.

A similar EOF analysis of the flow within the subtropical box (indicated by the rectangular box in Fig. 4.6b) further shows the migrating striations. Figure 4.18 shows the velocity fields reconstructed from the first two EOF modes, which clearly show the striations are shifted in the subdomain. The first EOF accounts for 29% of the total variance and the second 22%. The remaining EOF modes do not show quasi-zonal striations. Due to the limited snapshots of streamfunction for the EOF analysis, we can not get reliable analysis on the temporal scales of the EOF modes.

![Figure 4.18: Reconstructed velocity fields from the first two EOF modes of 197 snapshots of yearly mean streamfunction in the subtropical box. (a) The first EOF (29%). (b) The second EOF (22%).](image)

### 4.2.3 The dynamical role of eddies in the striation formation

Due to the high correlation between eddies and striations shown above, we expect a strong dynamical connection between the two. In this section, we study the role of eddy vorticity forcing on the striations without distinguishing coherent eddies from transient eddies. Coherent eddies are defined by Eq. 3.6 in chapter 3. Here, transient eddies are defined as deviations from time mean flow.
To get an explicit expression of the eddy forcing term, we first decompose the total $\varphi$:

$$\varphi = \varphi + \varphi_s(t) + \varphi'(t),$$  \hspace{1cm} (4.5)

where $\varphi$ represents the long term mean streamfunction, $\varphi_s$ represents the time-dependent streamfunction associated with striations, and $\varphi'$ represents transient eddies. Letting $\langle \cdot \rangle$ stand for low pass temporal filtering, we obtain:

$$\varphi' = \varphi - \langle \varphi \rangle,$$  \hspace{1cm} (4.6)

$$\varphi_s = \langle \varphi - \varphi \rangle.$$  \hspace{1cm} (4.7)

The nonlinear term related to the eddy forcing is $\langle J(\varphi, q) \rangle$. Expanding it we have:

$$\langle J(\varphi, q) \rangle = \langle J(\langle \varphi \rangle, \langle q \rangle) \rangle + \langle J(\varphi', q') \rangle + \langle J(\varphi', \langle q \rangle) + J(\langle \varphi \rangle, q') \rangle.$$  \hspace{1cm} (4.8)

Thus, the eddy vorticity forcing term, $\mathcal{F}_e$, is

$$\mathcal{F}_e = \langle J(\varphi, q) \rangle - J(\langle \varphi \rangle, \langle q \rangle).$$  \hspace{1cm} (4.9)

Eddy vorticity forcing is essential for maintaining the striations. Results show that eddy forcing alternates with latitude and has a high correlation with the alternating quasi-zonal striations (Fig. 4.19a). The zonal average of the eddy forcing term is highly correlated with the striation PV gradient; the correlation coefficient is 0.72 (Fig. 4.19b). The eddy forcing is slightly shifted southward of the striation PV gradient. This shift is likely responsible for the southward migration of the striations. Chapter 3 also discusses a similar jet drifting phenomenon and proposes that this shift in eddy forcing drives the striations to drift meridionally. Moreover, the spatial
distribution of the eddy forcing resembles that of the preferred tracks of coherent eddies. We speculate the coherent eddies can potentially contribute the eddy forcing.

Figure 4.19: Barotropic eddy vorticity forcing and striations in the subtropical box. (a) 1-year mean eddy forcing term (s$^{-2}$, in shades) overlaid on zonal velocity of the striations in contours; solid contours show eastward velocity, and dash ones show westward velocity. (b) Zonal averages of the eddy forcing term and striation PV, the correlation coefficient between the two is 0.72; the scale of two is arbitrary

4.3 Coherent eddies and striations in the ocean

In this section, we examine the relevance of the results from the double-gyre model and the kinematic model to the real ocean, and applying these results to interpret the relationship between coherent eddies and striations in the North Pacific Ocean.

Quasi-zonally elongated striations are pervasive in the ocean and are closely related to propagating coherent eddies. The average of anomalous, geostrophic, zonal velocity over 1 year shows ubiquitous present of zonal striations in the North Pacific (Fig. 4.20a). The striations can extend hundreds of kilometers zonally. This long extension can hardly be interpreted as the result of simple averages of single random eddies, as proposed by Schlax and Chelton (2008). One possible scenario is that eddies propagate along preferred zonal tracks, manifesting as eddy trains and contributing to these striations.
Figure 4.20: (a) A 1-year average of anomalous, geostrophic, zonal velocity calculated from altimetry sea level anomalies in the North Pacific. (b) The corresponding preferred tracks of coherent eddies over the same time period.

Figure 4.21: A zoomed-in figure of Fig. 1. The left two panels show the zonal velocity and the preferred tracks of coherent eddies at two latitude bands, respectively. The correlation coefficient between the two is 0.51. The right panel shows the profiles of the two quantities at 107°W; the correlation coefficient of the two profile is 0.81.
To investigate the relationship between eddy trains, if they exist in the ocean, and the striations, we first use the modified Okubo-Weiss parameter, $E^2$, as shown in Eq. 3.6, to identify coherent eddies. A high correspondence exists between the preferred tracks of eddies and the striations as suggested by the corresponding 1-year average of $E^2$ in Fig. 4.20b. Coherent eddies in the ocean propagate in a systematic way: Cyclones and anticyclones prefer to propagate as eddy trains, respectively. The spatial variation $E^2$ indicates a striking correlation to the striations’ distribution. A closer examination of the two figures at the latitude band of 10-25 $^\circ$N reveals that there is a 90° phase shift; the striations with cyclone trains to their north and anticyclone trains to their south are eastward, otherwise, they are westward (Fig. 4.21). This scenario is depicted in Fig. 4.14b. The results confirm that propagating eddy trains are a candidate mechanism for the formation of striations.

To further prove that the striations are associated with eddy trains instead of random eddies, we do an EOF analysis of the zonal velocity of the quasi-zonal striations. Figures 4.22 shows the reconstructed zonal velocity fields from the first two EOFs, which explained 29% and 23% of the total variance. These two modes exhibit non-stationary nature of the striations. However, in the case of random single eddies, EOF analysis will not show similar coherent variability of striations. Thus, the striations likely arise from the systematically propagating eddies trains as suggested by the kinematic model.

### 4.4 Summary and conclusions

Striations emerge from 1-year mean flow in a three-layer, QG model driven by wind-stress curl. Independence of the striations on additional forcing, which has been shown to be essential for the formation of striations in previous studies (e.g. O’Reilly et al. 2012), reveals that striations are intrinsic feature of wind-driven circulation.
The striations are most robust in the eastern part of the double-gyre circulation. They also exhibit coherent vertical structures, though their strength decreased with depth. The striations tend to drift away from inter-gyre boundary.

Eddies and striations are intrinsically related. Our analysis of the wind-driven flow and sea level anomalies shows high correlations between the tracks of coherent eddy trains and striations. Cyclones and anticyclones in the oceans propagate predominantly zonally and tend to align alternately. Time average of their propagating trajectories yields preferential tracks for the coherent eddies. Within the tracks of cy-
clones and anticyclones are alternating striations. Our kinematic model experiments confirm that striations can arise from systematically propagating eddy trains.

Nonlinear interactions of resonant basin modes can induce striations. Berloff (2005) showed that the interplay between resonant baroclinic modes and some secondary modes due to a homogeneous stochastic forcing in a closed basin can lead to the formation of zonal jets. Moreover, the instabilities of Rossby basin modes are likely to be responsible for the prominent eddies trains in the eastern basin. LaCasce and Pedlosky (2004) showed that the long baroclinic Rossby waves as components of low-frequency baroclinic Rossby basin modes are unstable at all wave amplitudes, and that this instability contributes largely to the midlatitude eddy field. With an idealized 1.5-layer basin model, Chen et al. (2014) showed that striations at the subtropical gyre are consistent with the non-dispersive eddies, the low-frequency component of which manifest as banded structures. However, the dynamics of the formation of non-dispersive eddies are not clear. Theories about the eddies “self-organize” into eddy trains remain to be done.

The presence of striations is dynamically associated with eddy vorticity forcing. A high correlation between eddy forcing and striation PV gradient suggests that eddy forcing is important for maintaining the striations. A phase shift between the two implies that eddy forcing is probably responsible for the drift of the striations. Similar mechanism is observed in chapter 3. The local meridional gyre flow is likely to be partly responsible for the drift.

A meridional ridge in the bottom layer exerts topographic steering effect on the flow. The zonal extension of striations decreases, largely due to the reduced occurrence of eddy trains in the presence of the ridge. This is probably explains why there does not exist sufficiently zonally elongated striations, such as those in the interior of the ocean, in the Souther Ocean, where topographic obstacles are prevalent.
Chapter 5

Conclusions

Stationary, multiple, zonal jets are clearly visible in planetary atmospheres, with the most well-known example being the banded winds on Jupiter (Kondratyev and Hunt 1982). The detection of the oceanic counterparts of these jets became possible with the advent of long time series of satellite data (Maximenko et al. 2005). The oceanic jets tend to be quasi-stationary, have complicated structure, and are masked by surrounding eddy field. These peculiarities are in part explained by the presence of the continents, topography, and non-stationary atmospheric forcing in the oceans.

Although a number of mechanisms for the jet formation have been proposed, most of them rely on a presence of a large-scale vorticity gradient. In some regions of the ocean, potential vorticity (PV) gradients induced by topography can dominate over the planetary PV gradient. In this study, we study the effects of bottom topography on the dynamics of eddies and jets in quasi-geostrophic (QG) framework. Generation of anisotropic motions, such as zonal jets, in the ocean is characterized by at least two stages: the linear stage of the rapid growth of infinitesimal perturbations and the nonlinear one involving interactions between modes. Using the classical Phillips model embedded with bottom topography, chapter 2 investigates the effects of bottom topography on the linear baroclinic instability as a stepping stone to interpret the
formation of zonal jets in the presence of topography; then the nonlinear dynamics of jet formation due to a meridional ridge is illuminated in chapter 3 by the analysis of the central role of eddy vorticity forcing in jet formation. Chapter 4 examines the relation of propagating eddies and jets with an idealized kinematic model and with the analysis of altimetry data, and studies dynamics of jets in a wind-driven, double-gyre, QG model.

5.1 Effects of topography on baroclinic instability

In chapter 2, we investigate the effects of bottom topography on linear baroclinic instability of a QG zonal flow, using analytical and numerical techniques. Idealized topography has a form of a constant slope in the meridional and zonal directions, as well as of an isolated meridional ridge. Analytical linear analysis demonstrates that even a small zonal slope can destabilize a flow that would be otherwise stable in the absence of topography. In contrast, the meridional slope can stabilize/destabilize a zonal background flow only through intensifying/weakening the background PV beyond a known critical value. The unstable modes have a shape of slanted noode modes, whose spatial structure is further discussed in relation to the shape of topography.

The results of this idealized study can help to interpret eddy generation over more complex topographic features in the real ocean and more sophisticated numerical models. The demonstrated importance of topography strongly suggests that the traditional stability analysis based entirely on the vertical shear in zonal ocean currents is insufficient for predicting the basic properties of growing disturbances. Topography, as well as such features of the mean current as its nonzonal orientation and spatial variability, can be expected to be critical factors in oceanic eddy generation.
5.2 Nonlocal mechanism of jet formation due to the presence of topography

The nonlinear study of chapter 3 describes a novel mechanism for generation of stationary coherent jets by topographic ridges. The dynamics of these jets is examined using the classical Phillips model configured with an isolated meridional ridge. The zonal topographic slopes and the corresponding zonal PV gradient lead to the formation of a system of currents, consisting of mesoscale eddies, a meridional jet on the top of the ridge and multiple zonal jets in the far field. Dynamical analysis shows that the transient eddies are vital in sustaining the deep meridional jet, which in turn plays a key role in balancing the PV gradient in the upper layer. The zonal jets own their existence to the eddy vorticity forcing over the ridge. In the far field, these zonal jets are maintained by the Reynolds and form stresses.

The nonlocal mechanism of this study is likely to be important in the real oceans, where nonzonal topographic ridges are ubiquitous. Our study suggests that these emerging eddy-driven jets drift and merge/split downstream of the ridges, and that stationary jets should exist only far away from topographic features. This property may render the existence of stationary jets nearly impossible. However, the essential elements of jet formation over flat bottom, such as the importance of secondary instabilities and eddy forcing, are likely to be universal even in realistic complex configurations. One shortcoming of this study on jet formation is that the choose of the QG framework may favor the inverse energy cascade and thus the formation of the jets. A more realistic study of jet formation requires more advanced models with continuous stratification and configured with realistic topography. It would also be interesting to explore the dynamics of topographic currents by analyzing satellite data and comprehensive numerical simulations.
5.3 Relationship between eddies and striations in complex background flows

Chapter 4 demonstrates that eddy trains are a plausible mechanism for the formation of striations with an idealized kinematic model, a wind-driven double gyre model, and the analysis of altimetry data. Coherent eddies in the models and altimetry data tend to align zonally and are parallel to zonal striations, indicating the intrinsic relation between eddy trains and striations. Quasi-zonal striations are an integral part of the wind-driven circulation and are most robust in the eastern parts of the double gyre. The prevalent eddy trains in this region may arise from the instabilities of Rossby basin modes. LaCasce and Pedlosky (2004) showed that the instability of low-frequency baroclinic Rossby basin modes contributes largely to the midlatitude eddy field. The dynamics of the formation of eddy trains remain unclear. Eddy vorticity forcing maintains the striations and tends to drive them to drift meridionally—a phenomenon also observed in the channel model in chapter 3. A meridional ridge in the bottom layer of the wind-driven double gyre model is shown to decrease the presence eddy trains due to the ridge results in shorter quasi-zonal striations.

Compared to a channel model of Antarctic Circumpolar Current (ACC) or a branch of the gyre circulation, the two-dimensional flow in the wind-driven double gyre model is more complicated. The striations from the double gyre model can represent the striations in a mid-latitude gyre. In both models, the formation of eddies from baroclinic instability are essential for the emergence of jets/striations. This study identifies two regimes of the eddying flow. The flow in the channel model consists of strong stationary jets co-existing with transient eddies, while the distinction between migrating striations and eddy trains in a two-dimensional gyre flow is less clear. Our analysis of the altimetry data in the North Pacific strongly suggests the
importance of the second regime and demonstrates that eddy trains play a key role in the formation of striations. Which of these two regimes is more applicable to other parts of the real ocean remains to be seen. Moreover, as mentioned in the section 5.2, the study of jet formation in the QG framework may bias the energy cascade toward the inverse direction.

The concept of striations being intrinsically related to eddies can potentially yield better ways for parameterizing the effects of eddies/striations in comprehensive models. The definition of striations proposed here is needed to be solidified by analyzing altimetry data. One key problem for the solidification is to find a better way to identify eddies and take the effects of transient eddies into account. One can expect a high anisotropic material transport due to the existence of eddy trains/striations in the oceans.

### 5.4 Open questions

To conclude this dissection, we propose a few possible questions, which are beyond this scope of this study, for future work.

The first question is about the relationship between stationary and non-stationary striations. Whether jets/striations are an integral part of the flow remains unclear. There is a discrepancy among the definitions of jets/striations from SLA (Maximenko et al. 2005), from MDT (Maximenko et al. 2008) and from ADT (Buckingham and Cornillon 2013). These studies show the existence of both stationary and non-stationary jets/striations. The connection, if any, between the two is unknown.

In the same line of research, the second question is on the importance of distinguishing eddies and jets/striations. A better understanding of the relationship between the two is essential for how to parameterize eddy/jet/striation effects in coarse resolution ocean models. Specifically, if the effects of jets and/or striations can
simply be represented by parameterizing eddies, the impacts of eddy/jet/striation on tracer transport, if any, can eventually be estimated with a proper parameterizing scheme, as well as the consequent effects on climate modeling.

Last but not least, the disparate views of the ACC: one being coherent continuous and circumpolar fronts, and the other narrow filamentary structures or jets, are worth studying as well. Though Sokolov and Rintoul (2007) have investigated this issue and proposed that those filamentary structures are branches of the major ACC fronts; the relationship between alternating quasi-zonal flow features and the ACC fronts is far from clear. One explanation is that they are different flow features with different spatio-temporal scales. Considering that hydrographic data are snapshots of water mass properties, such as temperature and salinity, fronts derived from this kind of data are explicit, instantaneous features. In this sense, fronts are probably flow patterns with larger temporal and horizontal spatial scales. While the quasi-stationary jets from satellite data and eddy-resolving models are features time-averaged over a few weeks to months, whose spatial scale is between that of the large scale time-mean flow (gyres) and mesoscale eddies.
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