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Behavioral Game Theory for Smart Grid Energy Management

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BEHAVIORAL GAME THEORY FOR SMART GRID ENERGY MANAGEMENT

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ENERGY MANAGEMENT

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The next generation power system, known as the smart grid, is viewed as a key enabler for effectively delivering electricity to customers in a sustainable manner. Given the increasing growth in the demand for energy, both in terms of quality and quantity, and the limited energy resources, the development of smart energy management mechanisms is necessary in order to pave the way toward the deployment of smart grid features such as demand-side management or large-scale, energy exchange. In particular, energy management mechanisms in the smart grid must deal with a complex and dynamic power grid composed of utility companies, traditional power sources and loads, as well as new customer-owned devices such as storage units or electric vehicles. Modeling, analyzing, and better understanding the operation and economics of energy management under such a heterogeneous environment is an important step toward realistically deploying the smart grid.

The primary goal of this research is to develop new energy management mechanisms that properly account for the decisions and control processes at the side of both customers and utility companies in the smart grid. While there has been considerable recent works on demand-side management, storage units integration, and related ideas, most existing

works are based on classical game-theoretic concepts such as static noncooperative games and do not account for the real-world user behavior and its impact on the operation of the smart grid and of energy markets. Indeed, most of the existing models proposed so far are based on the ideal assumption that grid customers can make rational and perfect decisions. For instance, even if it is technologically beneficial for power companies to offer dynamic pricing mechanisms aimed at reducing peak-hour demand, customers may not subscribe to such features. For example, on a hot summer day, customers may not allow the power company to reduce their air conditioning usage during peak hours, despite the associated benefits to the grid and/or the lower offered prices. Accordingly, there is a need to develop a novel framework that captures realistic user behavior during energy management while taking into account the associated benefits and costs on both customers and the grid.

Studying realistic user behavior in energy management faces numerous challenges. First, introducing storage units for energy trading between customers and the grid can lead to both competition and cooperation at different levels that can involve the complex interactions between customers, power companies, and energy providers. Second, due to the large-scale and heterogeneous nature of the smart grid, it is necessary to deploy distributed energy management protocols, in which the control center only provides a small amount of control information without operating at customers' sides. Finally, the involvement of customer-owned devices leads to uncertainties that stem from the unpredictable behavior of end-users.

In this thesis, we propose a novel approach based on *behavioral game theory* that can serve as a framework for capturing realistic user behavior in smart grid energy management. Taking such realistic decision-making settings into account allows us to go beyond classical game-theoretic concepts in order to explore how a user perceives the actions of its opponents and how this user evaluates its own utility function. In particular, we adopt the mathematical tools expounded by the Nobel-prize winning framework of prospect theory (PT), to study the decisions made by grid users and their impact on energy management.

Our primary results include the development of four game-theoretic models for studying user behavior in energy management. In the first model, we introduce a two-level approach that combines both auction theory and game theory to model and analyze energy trading markets between customers and the grid. For this first game, we study the various properties of the equilibrium and assess the performance of adopting a game-theoretic approach as opposed to conventional heuristic schemes. Then, in the second scenario, we study the use of PT as a tool for understanding the charging and discharging behavior of customer-owned storage units. In particular, we respectively develop two frameworks that enable us to consider the case in which a customer can have weighted observation on its opponents' operation as well as the case in which a customer can have its own, subjective evaluation on the gains and losses achieved from utilizing the storage unit. Our results show that ignoring user behavior during charging or discharging of storage units can lead to undesirable or unexpected performance in terms of both the load of the grid and the power company's revenues. Third, we study a demand-side management scenario in which customers can decide on whether or not to subscribe to demand-side management. For this application, we show that PT can provide a novel insight on the load reduction over hours, and due to different rationality settings, customers might take a more positive/negative participation in practice. Finally, we study how the use of PT can provide interesting insights on security problems such as the problem of hardware trojan detection.

To my family

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Chapter 1

Introduction

The next-generation power grid, known as the smart grid, is envisioned to be a large-scale, distributed system composed of many new elements such as renewable energy sources, storage units, electric vehicles, and a two-way communication infrastructure [1]. More importantly, the smart grid will become a largely user-centric system, in which customer participation is an important feature [2]. Examples of customer participation in the grid are numerous. On the one hand, customers may be required to sign up to certain demand-side management pricing schemes that allow the power company to control their electricity usage for the purpose of reducing peak hour grid load. On the other hand, customers may be allowed to actively trade energy with the grid via the use of electric vehicles or more generally, customer-owned storage devices. For instance, an energy market allows customers to exchange the electricity that is available in their storage units, thus impacting the overall technical and economic aspects of the grid. Due to the importance of customer participation and energy trading in the smart grid, the goal of this research is to develop new analytical tools that allow to model various problems related to user-assisted energy trading in the smart grid [3–5].

One key byproduct of the smart grid evolution is an ability to deliver innovative *energy management* services [6]. For example, embedded controls in smart meters and programmable thermostats can allow automated energy monitoring at customer premises, whose energy usage and behavior can be recorded and conveyed to the utility company's control center. Such automation allows the power company to respond to electricity information, using a variety of day-ahead and real-time pricing signals. These decisions will depend on the actions taken by all customers. Such a complex eco-system that includes customers, automated devices, and power companies makes energy delivery in the smart grid more challenging than in classical power systems.

In this first chapter, we will provide an introduction to energy management and trading in the smart grid while shedding light on the key challenges and opportunities.

1.1 Motivation

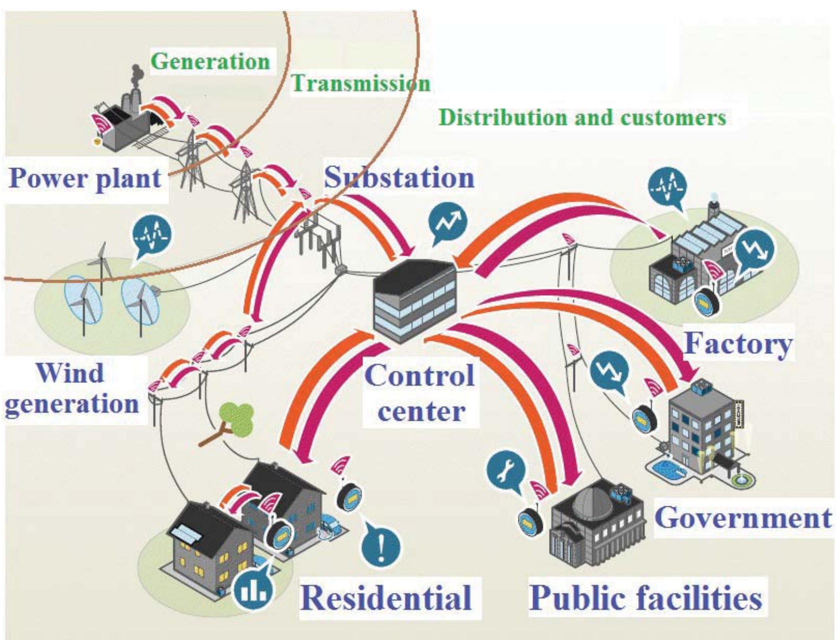

Figure 1.1: An illustration of a smart grid.

The smart grid is a power system composed of intelligent nodes that can operate, communicate, and interact autonomously to efficiently deliver electricity to customers. Fig. 1.1 shows an example of a smart grid. The heterogeneous nature of the smart grid motivates the adoption of advanced techniques for overcoming the various technical challenges at different levels such as design, control, and implementation [7]. In general, a smart grid involves transmission lines and distribution lines, using which electricity could be transmitted to users. Transmission lines consists of copper wires that carry electric power from generating plants to local substations. At the power plants, an operator could use a step-up transformer so as to increase the voltage and decrease the transmission loss. At the local substations, another transformer steps down the power to a lower voltage and then, the electricity goes into distribution lines. In particular, a distribution line carries less than $70,000$ volts and connects high-voltage transmission systems to end-use customers.

Energy management relates to the processes using which the overall load, generation, and energy exchange occur in a smart grid. In particular, we are mainly concerned with energy management mechanisms that can improve the overall energy usage and consumption at customers. For optimizing energy consumption, there is a need for intelligently monitoring, controlling, and conserving energy in a building or organization. For example, a user can monitor its energy consumption and compare it with the data provided by power company on historical energy usage. This might provide the customer with an opportunity to better understand how to optimize its future energy usage and how to actively participate in energy management. In this thesis, our goal is to study how the decisions of customers can impact the overall energy management process in the smart grid.

1.2 Energy Management in Smart Grid: Challenges and Opportunities

Deploying efficient energy management and trading mechanisms in future smart grid systems faces many challenges. One challenge at the distribution side is to allocate the energy based on different natural resources, i.e., wind and fossil. In [8], the authors considered clustered generation and associated loads as a grid resource. The clustered sources and loads can operate in parallel to the grid, or in an island mode, in which case the resources could be disconnected depending on the requirements of utilities/customers or when the quality of power falls below certain standards. Another challenge for smart grid energy management is assessing potential of using customer-owned storage units. The authors in [9] addressed the problem of intermittent renewable energy generation by using energy storage to deal with dynamic loads and sources. Such energy storage units can gradually replace fossil fuels as the primary sources of energy via the energy stored from renewable resources and this process is likely to become more common over the next few decades. Moreover, the introduction of information and communication is another important enabler for smart energy management. The work in [10] proposed an energy management system based on power line communication and described smart meters to communicate in local area. This energy management system managed the peak energy consumption of all devices and combined a home network with Internet.

In general, energy management in the smart grid involves the planning and operation of energy-related production and consumption units. Next, we summarize some of the main research topics related to energy management in the smart grid.

1.2.1 Demand-Side Management

Demand-side management is an essential feature of smart grid systems using which a power company is able to control the energy consumption at the consumers' premises. For example, demand-side management at residential homes aims at reducing consumption especially for mitigating the power supply during peak hour. Also, demand-side management can be used to provide incentives to consumers to shift their consumption to hours during which the energy grid is less loaded (e.g., by providing lower prices during night hours) [11–13]. Demand-side management often entails interactions between two main players: power companies and consumers. Consumers can be residences, businesses, or even electric cars. These interactions can be at technical levels (i.e., interactions between smart meters and power company control centers) as well as at social levels (i.e., service agreements between the power companies and their consumers). Enabling the interconnections of consumers and power companies can only be made possible with efficient demand-side management techniques. The essence of demand-side management revolves around the interactions between various entities with specific objectives that are often modeled via game-theoretic techniques [2, 14–16]. For example, to study various energy-related interactions between customers of a smart grid, game-theoretic approaches are particularly suitable as they allow to devise distributed mechanisms to handle the individual objective of every player in a demand-side management game. Moreover, game-theoretic approaches can naturally incorporate pricing issues which are an integral part of demand-side management.

1.2.2 Integration and Operation of Storage Units and Electric Vehicles (EVs)

A power grid system could be generally divided into two main components: electric power transmission and electric power distribution. Electric power transmission or high-voltage transmission deals with the transmission of the energy generated at power plants, over transmission lines, to substations that service some geographical areas. In contrast, electric power distribution is the last stage for delivering electricity in which the distribution network carries the electricity received at a substation and, subsequently, delivers it to the consumers premises. In this context, the concept of storage units can encompass battery banks, electric vehicles, or other customer-owned storage components which can connect to the grid for providing/storing energy to small geograph-

Figure 1.2: An example of a smart grid with storage units.

ical areas or for acting as load and receiving energy from the grid. Fig. 1.2 shows an example of a smart grid integrating smart storage units, such as electric vehicles (left) and renewables, such as solar panels (right).

Storage units provide many opportunities for smart energy management. For example, the power company could announce a lower electricity price and encourage the customer to store energy in a battery (e.g., electric vehicle), and then, the stored electricity can be sold at peak hour. Due to the increase in the number of storage units, that foreseen in the next decade [6], there will be a need to optimize the way in which energy is managed in a smart grid so as to handle this new load, particularly in peak hours.

In classical power grids, it is common to optimize the system by defining and solving a system-level optimization problem based on a centralized objective function. However, in the presence of customer-owned devices, it is of interest to define a specific objective function for each device. This is mainly due to the heterogeneous nature of the smart grid, which often consists of different components such as electric vehicles, energy storage devices (e.g., batteries), wind turbines, and solar farms. To this end, it is only natural to adopt distributed analytical techniques, such as game theory [2, 9, 14–41], so as to control and optimize smart grid systems that encompass a distribution network. For example, the users in local area must observe the electricity price, and their operations, or electricity usage, are independent. Moreover, the actions taken by one customer will impact the price and the actions of other customers, thus motivating the need for mathematical tools that can capture such interdependence. Thus, there is a need to develop a generic framework that can capture such competition that arises between the customers as well as the competition that arises between the power companies.

Another important element in the smart grid is the aggregator. In general, an aggregator is a smart grid component that can purchase energy from the primary power plant outside of the microgrid or the distributed generation units. In particular, the distributed generation units and the customers in the certain area are considered to be equipped with smart agents (i.e., smart meters), which receive signals from the aggregator. The objective of the aggregator is to maximize its revenues while taking into account the individual objectives in the grid and customers in its service area. Clearly, optimizing energy usage in the smart grid in the presence of customer-owned devices will require handling the objective of multiple players, or decision makers, whose goals are largely interdependent and strongly impact the grid's performance.

1.2.3 Reactive Power Compensation

Reactive power compensation is another technical challenge of energy management to improve the performance of AC power systems. What we discussed so far is mainly dependent on the customer storage units or their load requirement. In particular, the electricity bills will rely on such DC energy or real power, i.e., *watt*. For the reactive power, the term "compensation" can witness a wide and diverse field of both system and customer problems, especially for improving relative power quality services. In general, reactive power compensation can be classified into two groups: load compensation and voltage support [42]. On the one hand, the purpose of load compensation is to increase the value of power factor, which is a parameter to measure the angle between real power and apparent power, and to balance the real power drawn from the initial AC supply. On the other hand, voltage support can help reducing voltage fluctuation over transmission lines. Many works studied how to attenuate an adequate control of reactive power [43–45].

For the transmission systems, reactive power improves the stability of the AC system via increasing the maximum value of active power that can be delivered into distributed network. Current practical reactive power compensation in control centers is often based on power flow using limited execution time and available data from the real-world power system. For example, voltage support can be generally formulated as an optimization problem, the objective of whose function is counting for the power system conditions. Such function can be a minimization function of the power transmission loss to reduce the targeted power system under the normal operation. Thus, there might need the technology of energy management, i.e., optimization, to analyze reactive power compensation for maintaining system stability.

For the load compensation (mostly at the level of the distribution system), the objective of reactive power compensation is to chase an effective control strategy for the three-phase nonlinear loads via the active power line conditioner in practice. There might also need some technologies, such as $p-q$ original theory, $d-q$ transformation, to efficiently complete relative target in power system. However, since 1980s, most of these technologies were depending on the formulation that has been applied in a systematic way for the three-phase nonlinear loads. Thus, current algorithms, such as control methods, mainly focus on how to balance three-phase, or three-wire, systems with the nonsinusoidal voltage, which increases the power factor.

In a nutshell, reactive power compensation allows to improve the efficiency of delivering energy in power systems by increasing the power factor at end-user side. Due to the aging devices and the varying energy requirements from end nodes, investigating how to control and manage reactive power in a smart grid system is challenging. In a community, it might not be efficient or economic for updating or installing new compensation devices. Thus, optimization technologies, such as game theory, would provide a theoretic analysis tool to study the interaction between end nodes and to coordinate neighbor user's reactive compensation, falling into the scope of energy management.

1.2.4 Other Challenges

Another important component of the smart grid is the communication infrastructure. On the one hand, the smart grid elements must be able to communicate information such as outage management to the power companys control center. On the other hand, smart meters need to communicate with nearby control centers so as to exchange information such as meter readings, pricing, or other control data. Recently, enabling two-way communications between the grid and PHEVs (or electric vehicles) has also received considerable attention in the research community [21, 46, 47]. For example, power line communication is one of the candidate technologies that can be used to ensure data communication between the different smart grid elements such as sensors that are used to collect data (e.g., household loads, monitoring data, maintenance, and prices inquiry) and transmit it to a control node. Therefore, there is a need to propose and introduce new communication architectures that are designed for efficient operation in a heterogeneous smart grid environment.

Beyond the above-mentioned challenges, energy management in the smart grid also faces many security challenges. In order to guarantee the stability of power system systems without jeopardizing the privacy of customers, new privacy-preserving energy management mechanisms must be deployed. Moreover, resilience to failures and malicious attacks are also important challenges facing the deployment of the smart grid infrastructure.

1.3 Contributions and Limitations

To manage the energy distribution in the smart grid, we have discussed the current challenges and the potential of customers' participation in grid energy management. The main goal of this thesis is to develop a new framework for optimizing energy management in the smart grid. The proposed framework will have three key features: *a)* ability to capture the heterogeneous nature of the grid, *b)* ability to integrate customer behavior in smart grid energy management, and *c)* ability to optimize the overall grid performance in a distributed way, under a variety of energy management and demand-side management scenarios. To this end, we will introduce notions from game theory and behavioral games theory to optimize the smart grid operation and study the behavior of customers in energy management scenarios. In this section, we discuss the main contributions of this research. The main challenges of applying game-theoretic mechanisms to application will be also discussed and, we provide a few limitations from various perspectives based on the proposed framework. In what follows, we summarize the contributions and limitations.

The main contributions of this thesis are:

1. In the process of energy trading, we proposed a novel double-auction market model that allows the incorporation of power markets with multiple buyers and multiple sellers. Compared to related works on smart grid markets [22, 23, 48–

50], the proposed model combines a double auction with a noncooperative game allowing the sellers (customers that own storage devices or EVs) to strategically decide on the amounts they exchange with the grid depending on the current market state, thus, yielding a dynamic pricing mechanism;

2. We develop new results on the existence of a Nash equilibrium for smart grid energy trading games that exhibit a discontinuity in the utility function due to the presence of an underlying auction model that captures realistic grid pricing schemes, unlike the classical models that often assume continuous utilities;

3. We propose a new learning algorithm that is guaranteed to reach an equilibrium for the proposed two-level energy trading game. In particular, we are interested in overcoming two key challenges: a) allowing storage units to intelligently decide on the energy amount to sell while taking into account the effect of these decisions on both their utilities and the energy trading price in the market; and b) developing and analyzing a mechanism to characterize the trading price of the energy trading market that involves the storage units and the potential energy buyers in the grid;

4. To incorporate user behavior into the decision making processes of the smart grid customers, we introduce a new approach based on the framework of *prospect theory*. The proposed approach allows proper modeling of realistic user behavior in energy management. Instead of studying a noncooperative game with rational players, or seeking a statistical representation of the gains from energy management via conventional expected utility theory, we develop a framework that can capture the customer behaviors in practice while pinpointing the impact of such behavioral factors on the performance of energy charging and discharging;

5. Using storage units, we build two new models to optimize their usage based

on prospect theory. In the first model, we studied the customers' charging and discharging operations via a "weighting effect". Compared to the classical approaches, this case allow a customer to have a realistic weighted observation on others' behavior. In the second scenario, we explore customers' behaviors via the so called "framing effect". We allow each customer to evaluate the utility based on its subjective perception on the monetary gains and losses that are related to energy management. Compared to the classical approaches, this scenarios allows us to understand how individual perceptions on energy management mechanisms can change the operating point of the smart grid system's features.

6. We developed a novel approach for studying and analyzing demand-side management in the smart grid while explicitly factoring in user behavior. In particular, the proposed model allows that each customer can decide whether or not to participate in a load shifting program, using which the power company seeks to reduce the total peak hour demand so as to maintain a desirable target load on the grid. In this game, all customers minimize a cost function that captures their one-day payment. Our results show that the participation in demand-side management can be strongly affected by the customers' choices, which explains some of the empirical results that show a low penetration of demand-side management in practical grids.

7. We proposed a learning algorithm that can be used to find an equilibrium for the demand-side management game under prospect-theoretic considerations. For such an algorithm, we were able to prove convergence to suitable equilibrium points, thus providing a practical way for implementing prospect-theoretic algorithms in real-world smart grids.

8. We studied a hardware trojan detection game between an attacker and a defender. In this game, manufacturers of hardware for critical infrastructures such as the smart grid can have an incentive to act maliciously and introduce hardware trojans into its integrated circuits (ICs). For instance, such a malicious attacker can select a certain trojan type to insert into the integrated circuits in order to maximize the damage that it inflicts on the defender via the trojan-infected IC, while the defender attempts to detect the trojan in the IC. In particular, we formulate a zero-sum game and study how the attacker and defender can make their decisions based on subjective perceptions on each others' possible strategies and the accompanying gains and losses.

The limitations of the proposed framework are listed below.

1. For future work, it is of interest to develop a dynamic game model in which all players could time-dependently observe each others' strategies as well as the grid's state so as to dynamically determine their underlying actions. In this respect, the work done in seeking a pure Nash equilibrium serves as a basis for developing such a more elaborate dynamic game model in which players can make long-term decisions with regard to their energy trading processes;

2. The current models and scenarios have focused on storage units and other relatively passive grid components. In contrast, there is a need to expand the developed prospect theoretic algorithms to account for stochastic grid components such as renewable energy sources, whose impact on the grid is still not well-understood.

1.4 Outline of the Work

The rest of this thesis is organized as follows. Chapter 2 introduces the basics of game theory. Chapter 3 reviews the important and related smart grid energy management literature. Chapter 4 proposes a novel game-theoretic approach for energy trading in smart grid. Chapter 5 develops a prospect theoretic framework for capturing charging/discharging behavior in the smart grid. Moreover, Chapter 6 studies the same grid model in Chapter 5 via framing effect. Chapter 7 investigates the deviation of customer load requirement under prospect theory. We also briefly explore the security problem in Chapter 8. Chapter 9 concludes the thesis and discusses possible future work.

Chapter 2

Game theory and Related Techniques

2.1 Introduction and Examples

Game theory can be viewed as a branch of applied mathematics as well as applied sciences [51]. It has been used in the social sciences, most notably in economics, but has also penetrated into a variety of other disciplines such as political science, biology, computer science, philosophy, engineering, and more importantly, smart grid. Even though game theory is a relatively young discipline, the ideas underlying it have appeared in various forms, throughout history and in numerous sources, such as in the Bible, the Talmud, the works of Descartes and Sun Tzu, and the writings of Charles Darwin [52, 53]. During the past decade, there has been a surge in research activities that employ game theory to model and analyze a variety of problems in the smart grid, particularly, for energy management [37, 50, 54–57].

One classical example of game theory is the so-called "Prisoner's Dilemma". This game captures a scenario in which a conflict of interest arises due to independent decision making. The Prisoner's dilemma pertains to analyzing the decision making process in the following hypothetical setting. Two criminals are arrested after being suspected of a crime in unison, but the authorities do not have enough proof and evidence to con-

Table 2.1: Prisoner's Dilemma

	Cooperate	Defect
Cooperate	$(3,3)$	$(0,5)$
Defect	$(5,0)$	$(1,1)$

vict either. Thus, two prisoners are isolated and offered, separately, the following deal: if one testifies to convict the other, he will get a reduced sentence or go free. Here, the prisoners do not have information about each other's "move". The payoff if they both say nothing (and thus cooperate with each other) is good, since neither can be convicted without further proof. If one of them betrays and the other one does not, then the betrayer benefits because he goes free while the other one is imprisoned, since there is now sufficient evidence to convict the silent one. If they both confess, they both get a reduced sentences, which can be viewed as a null result. The obvious dilemma is the choice between two options, where a good decision, acceptable to both, cannot be made without cooperation which is not possible here due to the prisoners being in isolated cells with no means to communicate.

A representative Prisoner's Dilemma is depicted in Table 2.1. One player acts as the row player and the other one plays as the column player, and both have the action options of cooperating (C) or defecting (D). Thus, there are four possible outcomes to the game: $\{(C,C),(D,D),(C,D),(D,C)\}$. Under mutual cooperation, $\{(C,C)\}$, both players will receive the reward payoff of 3. Under mutual defection, $\{(D,D)\}$, both players receive the punishment of defection, 1. When one player cooperates while the other one defects, $\{(C,D),(D,C)\}$, the cooperating player receives the payoff, 0, and the defecting player receives the temptation to defect, 5. If one player cooperates, the other player will have a better payoff (5 instead of 3) if he or she defects; if one player defects, the other player will still have a better payoff (1 instead of 0) if he or she defects. Regardless of the other player's strategy, a player in the Prisoner's Dilemma has an incentive to always select defection and $\{(D,D)\}$ is known to be an *equilibrium* – a state in which no prisoner can improve its utility, given the fixed strategy of its opponent. Although cooperation will give each player a better payoff of 3, greediness and lack of trust leads to an inefficient outcome.

To sum up, the Prisoners' Dilemma can help to explain the interaction between players and the key point that game theory provides is an insight into the interdependent decision-making. For competitive players, i.e. the users in smart grid, there will be little communication between the players and most smart grid customers will be self-interested, thus seeking to optimize their own benefits. Therefore, noocoperative models, as explained next, would be of interest to develop for the smart grid.

2.2 Noncooperative Games

Noncooperative game theory is one of the most important branches of game theory focusing on the study and analysis of competitive decision making involving several players, such as in the example of a the Prisoner's Dilemma. It provides an analytical framework suited for characterizing the interactions and decision making process involving several players that have partially or totally conflicting interests over the outcome of a decision process which is affected by their actions. For instance, firms operating in the same market compete over pricing strategies, market control, trading of goods, and the like. Such a game involves a competitive situation where each player needs to take its decision independent of the other players, given the possible choices of the other players and their effect on the player's objectives or utilities. Note that the term noncooperative does not always imply that the players do not cooperate, but it means that, any cooperation that might arise must be self-enforcing with no communication or coordination of strategic choices among the players. For example, a smart meter can show the real-time electricity price in the smart grid. As a customer, one must make a demand-side management decision without asking “neighbors” whether they close the washing machines or not at a certain time.

We have studied Prisoner’s Dilemma using a so-called matrix game, in which the strategies of players constitute the rows and columns in a matrix. In Prisoner’s Dilemma, we see that, the equilibrium $\{(D,D)\}$ is a strategy profile that no player has an incentive to *unilaterally* deviate to another strategy, given that other players’ strategies remain fixed. In particular, such equilibrium is called *Nash equilibrium* (NE). In essence, the NE provides a characterization of the outcome of a game, given that the players are acting noncooperatively. In an engineering system, this translates into predicting and analyzing the operating point of the system, under distributed decision making.

In this thesis, we mainly focus on *noncooperative game* to study energy management in the smart grid, such as a user-centric energy trading mechanisms and demand side management. In particular, the interactions between smart grid elements must not have a large communication overhead, due to the size and real-time nature of the grid. Moreover, smart grid users might neither make an agreement with each other before they buy/sell electricity, nor ask the power company whether they can use certain loads during certain times. Therefore, noncooperative games are a useful approach to optimize the system while minimizing communication overhead. Example problems that can be tackled via noncooperative game theory include allocation of resources, choices of charging/discharging or transmit power, demand-side management, and pricing, and many others. In particular, we will introduce several noncooperative game models to study smart grid energy management scenarios in which smart grid customers and power companies interact and compete with one another for various goals, such as energy allocation, energy trading, and pricing.

2.3 Prospect Theory (PT)

One of the main drawbacks of classical game theory is that it assumes that players are rational. A rational player is a player which will always choose the action that is most beneficial to him/her. However, in practice, empirical studies [58] have shown that real-world decision making may deviate from this rational path of conventional game theory. The presence of human players, such as the customers in the smart grid, further motivates the relaxation of this rational assumption.

In this respect, we will study the decision-making processes of smart grid users via a novel *behavioral game-theoretic frameworks* such as prospect theory which extend game-theoretic mechanisms to cases in which users may act irrationally under risk and uncertainty and, to cases when users have subjective evaluation on utility. Instead of viewing each others' actions objectively, players could have different subjective assessments about their own utilities and each others actions. For example, in demand-side management, even though rational behavior dictates that users follow the load shifting mechanisms of the power company, some customers may turn on certain appliances at unexpected times thus hindering the performance of demand-side management. In such situations, prospect theory directly addresses how these choices are framed and evaluated, given the subjective observation of players in the decision-making process.

We will propose to use prospect theory to study the behavior of the users in energy management, scenarios and, then, compare it to classical game theory in which decisions are often guided by the strictly rational notions of expected utility theory (EUT). In EUT, the players will attempt to maximize an objective expected utility (expected with respect to probabilistic decision parameters) in which users are assumed to act rationally and objectively. However, as previously mentioned, PT studies [58–60] have shown that, in real life, users often act irrationally when faced with risk and uncertainty of outcome, as is the case in the smart grid, where the decisions of the customers are largely interdependent leading to risky outcomes. For example, a power company might announce a high peak-hour price and encourage local customers to store energy before the peak-hour, i.e. 2pm. In this case, 2pm might have a high electricity price, or be a new peak hour, if every customer stores energy. As a result, if customers do not behave rationally and, for example, over-purchase or over-store energy, the overall result of this simple demand-side management scheme might not be in line with what the power company expects. These irrational decisions can stem not only from the users' behavior but also from computational errors occurring at the smart grid devices that are often resource-constrained.

One important PT notion is the so-called *weighting effect*. In particular, in PT [61], it is observed that in real-life decision-making, people tend to subjectively weight uncertain outcomes. This weighting effect allows to capture each user's subjective evaluation on the mixed strategy of its opponents. Thus, under PT, instead of objectively observing the information given by the other players, each user perceives a weighted version of its observation. Here, the observation under PT maps an objective probability to a subjective one. PT studies have shown that most people could often overweight low probability outcomes and underweight high probability outcomes [58]. For example, we can choose the popular Prelec function as an PT observation (for a given probability σ that statistically represents the customer's choices in smart grid) [61]:

$$w(\sigma) = \exp(-(-\ln \sigma)^{\alpha}),\ 0 < \alpha \leq 1, \tag{2.1}$$

where α is a parameter used to characterize the distortion between subjective and objective probability. Note that when $\alpha = 1$, (2.1) is reduced to the conventional EUT probability. Fig. 2.1 illustrates the probability weighting effect. In this figure, we can see that, the objective probability and subjective estimation insect at $\sigma = 1/e$ and the curve approximates to $w = 1/e$ as α increases.

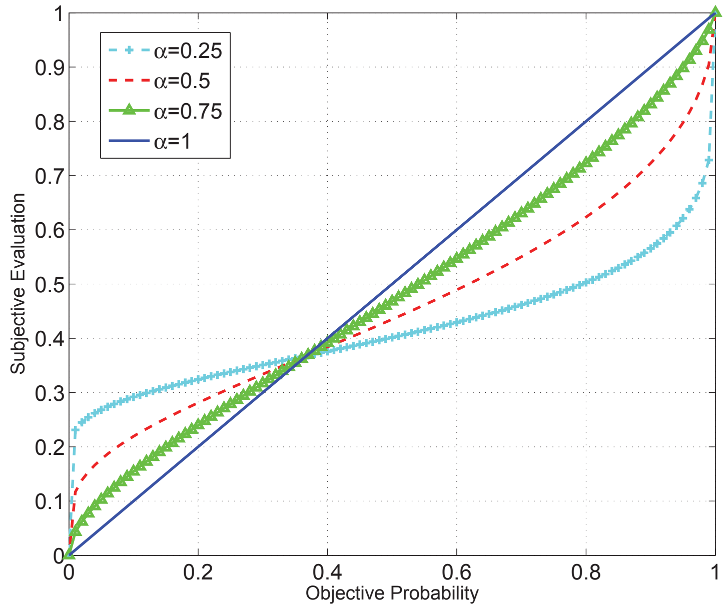

Figure 2.1: The effect of weight as the objective probability varies.

In the above function, the observation under PT is different from that of EUT due to the weighting effect; and in practice, the realistic results under subjective observations might deviate from that under rational choices, as we model the gains/losses of customers in smart grid.

Although we gave an example of weighting effect and illustrated the difference between PT and EUT, weighting a human natural attitudes of possible outcomes can follow different models, i.e., different functions. In addition, PT and behavioral game theory [62–64] will offer a large set of mathematical tools that can be leveraged to design and optimize realistic smart grid energy management schemes. Therefore, in this thesis, we will explore the use of PT in energy management so as deploy new smart grid energy management schemes, under different scenarios, which allow to capture realistic decision-making parameters and behaviors.

2.4 Framing Effect

In Section 2.3, we have discussed with an example how weighting effect modifies the rationality of players. In this section, we will introduce another important idea brought forward by PT, the so-called *framing effect* [61, 65–69]. Indeed, framing effect allows players making decisions via a subjective evaluation utility, while the work of weighting effect stands for a subjective observation on others' action.

When dealing with the energy management problems, the assumption of objective metric for assessing utility function might not be reasonable. For example, each customer will have an individual perception on the economic gains and losses in energy trading. A saving of $10 per month may not seem like an attractable gain for a relatively wealthy customer. However, for a poor customer, such saving amount might be viewed as a highly significant reduction. Thus, the objective measure of $10, can be viewed differently by different customers.

For evaluating the payoff, a player can have its perceptions of utility functions under the idea of framing or reference points [6]. In essence, each individual frames its gains or losses with respect to a possibly different reference point. Back to the aforementioned example, the wealthy customer will frame the $10 with respect to its initial wealth which could be close to millions and, thus, this customer views the $10 as insignificant. In contrast, the poor customer might have a wealth close to 0 and, thus, when framing the $10 with respect to this reference point, the gains are viewed as significant. These gains and losses are measured with respect to a reference point that needs not be 0 and that may be different between players. Furthermore, the magnitudes of both gains and losses will be shaped due to the fact that, each player will observe and evaluate the utility based on its benefit-cost. In this case, one popular way to evaluate utility is by framing the gains and losses [58–60].

To capture the effect of such utility framing in a game-theoretic setting, we will use the behavioral framework of prospect theory [58]. In particular, we will study the following key features of utility framing: *1) Reference point:* a player can evaluate its utility using its own individual reference point and such evaluation represents how players act differently via a possibly similar utility value; *2) Gain/loss aversion:* a player has different attitudes on evaluating the gain and loss values; and *3) Diminishing sensitivity:* a player is risk averse in gains and risk seeking in losses. By taking these three framing characters, for each player k, we can write the individual utility function as a function of a reference point which allows to evaluate the individual gains and losses of the players [60]:

$$u_k^{\mathrm{PT}}(a) = \begin{cases} \left(u_k(a) - u_k^0\right)^{\alpha_k} & \text{if } u_k(a) \geq u_k^0, \\ -\gamma_k \left(u_k^0 - u_k(a)\right)^{\beta_k} & \text{otherwise,} \end{cases} \tag{2.2}$$

where u_k^0 is the reference point, $\alpha_k, \beta_k \in (0,1]$ are the weighting factors for capturing the gain and loss distortions of player k, respectively, and $\gamma_k \in [0,\infty)$ is an aversion parameter. Here, we note that the framing utility has a desired S-shape function and customers can have different reference points. Based on the distortion parameters α_k and β_k, the proposed utility function is *concave for gains* and *convex for losses*. Moreover, when $\gamma > 1$ and $\alpha_k = \beta_k$, player k's loss is larger than its gain, defined as the case "loss aversion" in [70]. Fig. 2.2 provide a typical PT value function assuming a zero reference point for gains/losses.

In smart grid, when one decision maker (a customer) changes the way in which it evaluates its objective function (the payoff of energy exchanging), the overall operation of any optimization mechanisms will be significantly affected. By introducing the framing effects, the utility envisioned by a customer does not only cover purely economical payoff. For example, during summer times, at night, users may perceive the prospective

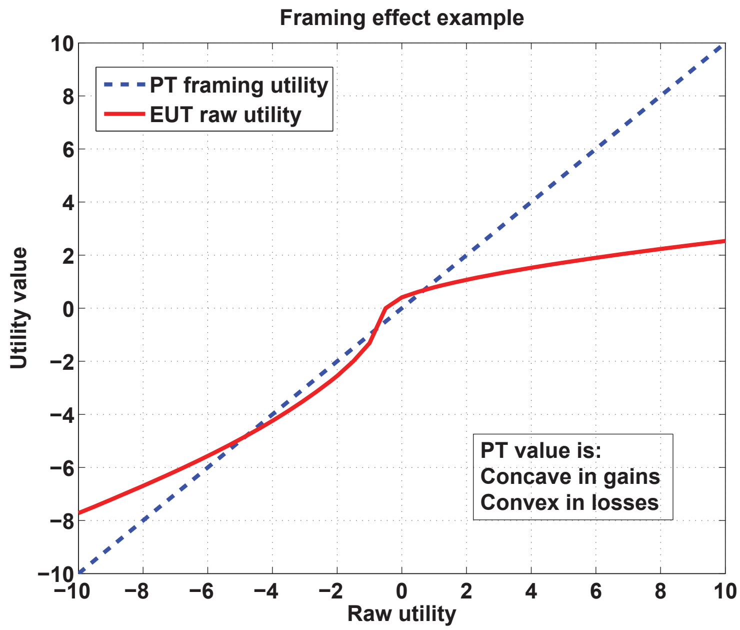

Figure 2.2: Illustration of the prospect-theoretic framing effect.

gains from turning off high-capacity loads (such as an air conditioning unit) for the sake of feeding energy back to the grid much lower than they perceive them during winter days. How this "frame of reference" transforms the utility will fundamentally change the outcome of an energy management mechanism that is based on classical objective notions. Thus, properly designing and developing framing notions is interested in energy management and must be investigated to better understand its impact, compared to classical game-theoretic formulation.

Based on the weighting and framing effect, in the thesis, we will study a charging/discharging operation problem and another load shifting case to highlight, as an example, the impact of PT on smart grid protocols.

Chapter 3

Literature Review

In this chapter, we provide a thorough review on related works in the area of smart grid energy management. Section 3.1 focuses on discussing representative techniques of integrating energy storages and resources in power systems. In addition, Section provides some works in the context of demand-side management, the aim of which mainly depends on shift demands and balance the load curve. Finally, Section 3.3 discusses recent important studies from the consideration of users' real decisions.

3.1 Energy Trading in the Smart Grid

There has been considerable work that studies how users in a smart grid may be able to trade and exchange energy [9, 17–21]. The authors in [9] addressed the problem of intermittent renewable energy generation by using energy storage to deal with dynamic loads and sources. This energy storage can be used to mitigate the requirement of generating power in peak hour and can smooth out the variations of energy utility due to random power demand and uncertain energy supply. They exploited the dispatchable functionalities of energy storages via the round-trip efficiency. Deploying energy storage units was also explored in [17]. The authors studied the economic aspects of deploying storage units such as electric vehicles. Their generic model was introduced to analyze the feasibility and benefits of using electric vehicles as energy storages in distribution networks, in which distribution system planners could estimate the preliminary feasibility of energy storages. In [18], distributed resources are allocated by the provision of two-way energy flow and a unified operational value proposition of energy storage is presented. The authors proposed this unified operational value index of energy storage in the smart grid electricity market environment, given the heterogeneity of many storage service providers.

The work in [19] proposed a new scheme to improve system reliability and encouraged energy efficiency from demand-side management. An energy management system was designed with an aggregator, using which customers could participate in the energy market and reduce the consumption costs. The authors in [20] studied the impact of energy storage systems in the grid. They also summarized different kinds of energy storage devices while specifying technical requirements, such as grid voltage, peak shaving and reliability, and financial benefits, such as reducing the need of extra generation capacity and increasing revenue from renewable sources. In [21], the storage units placed in buildings, i.e. photovoltaic sources, were transformed into a microgrid. Based on the DC-bus voltage of photovoltaic sources, an energy management harvested the maximum renewable power. Other related works have assessed the advantages of deploying and maintaining storage units such as in [22–31].

Combining the above works in energy management with considering customers' decisions on their trading amount, one main challenge is to study the complex interactions between the storage units (and their owners) and the various smart grid elements. The authors in [57] modeled the strategic behavior of buses that are connected to renewable energy resources and compared a game-theoretic framework with a classical Newton-Raphson algorithm for power transmission. The model in [57] explored both economic factors and system stability parameters, using an iterative algorithm that can find the Nash operating point of the system. A game-theoretic approach to control individual sources/loads was proposed in [55], to optimize the reliability and robustness of a power system without using central control. This work proposed to formulate different grid components so as to integrate and combine supply and demand-side management into a single framework, and, thus, avoiding the need for a central or supervisory control. Demand-side management for smart grid was studied in [71]. The authors demonstrated that both users and power companies could potentially benefit from the economical and environmental advantages of demand-side management features. Their proposed pricing scheme captured the maximum social benefits, in which each customer provided information of energy consumption and gird operator determined the bill payment or social objectives. In [50], a new technique based on cooperative game theory is proposed to allow wind turbines to aggregate their generated energy and optimize their profits. The work in [50] studied how power producers can form cooperative coalitions to pool their variable power to jointly offer their aggregate power output as single entity into a forward energy market. Using tools from coalitional game theory, the super-additivity of underlying game was shown and a mechanism that minimizes the worst-case dissatisfaction was developed. For providing reliable power supply to small communities, the work in [72] used an islanded operation mode of a microgrid to maintain power balance, in which a power consumer presented its discrete power demands and a bargaining approach was applied for their load-shedding schemes.

Reactive power compensation was studied in [73] using game theory with the objective of optimizing wind farm generation. A strategic game model was developed in [74] to analyze an oligopoly within an energy market with various grid-level constraints. Transmission expansion planning and generation expansion planning were studied through a dominant strategy using three game-theoretic levels in [75]. Using an IEEE 30-bus test systems, a comprehensive approach to evaluate electricity markets was presented in [48] to study the impact of various constraints on the market equilibrium. Developing a distributed energy storage system for transferring photovoltaic power to electric vehicles as well as introducing efficient power management schemes between storage units and the smart grid have been studied in [76] and [22], respectively.

3.2 Load Shifting for Demand-side Management (DSM)

There has been an abundant body of work dealing with demand-side management [2, 14, 32–35]. The authors in [2] proposed a distributed DSM system, in which each user optimized its best operation to minimize the energy costs. For the autonomous and distributed energy management, they assumed that the grid customers can schedule the daily consumption of the household appliances, while the power company can adopt a pricing scheme at different times and areas. In particular, this paper considered a two-way digital communication infrastructure and its relative formulation had been widely studied for energy consumption scheduling.

The work in [14] proposed a home energy management algorithm to help in decreasing customers energy bills. In order to meet the increasing power demand and solving the integration problems of renewable energy sources, a home energy management can provide a number of benefits such as savings in the electricity bill, reduction in peak demand and meeting the demand side requirements. For example, the battery state of charge can enable multi-customer to optimize and to efficiently utilize electricity. By dividing the electrical appliances into two house groups, the load demand types were classified as critical loads and non-critical loads. The critical loads were not preferred to shift, such as TV and PC, and the non-critical loads allowed to be shift to anytime, such as electric vehicles, washing machine. Thus, their load requirement can be partly moved from the peak hour to a low-demand time. In [32], the authors presented an architecture for a DSM system and implemented an online scheduling algorithm for peak-load shaving, allowing the separation of domestic power load control from grid dynamics. In the proposed architecture of home power management systems, domestic power load control was allowed to separated from grid and thus, the operation of appliances can be modeled as a finite state machine, or several operation times. For example, each state machine can be easily adapted to represent a specific appliance. Depending on the intrinsic characteristics of this appliance, it can be easily to determine its usage preference (i.e., working hours) in a certain time slot. Thus, this paper viewed the load shifting problem via allocating the resources.

The authors in [33] presented an optimal control model for load shifting. They optimized load shifting and improved energy efficiency through the control of "conveyor belts". The conveyor belt system could be divided into several groups and, a time-of-use electricity tariff was used for the input of the objective function so as to minimize electricity payments and thus maximize load shifting. The work in [34] dealt with the load shifting problem in a household equipped with smart appliances and an energy storage unit with conversion losses. The proposed framework established a discrete time, open loop, optimal control system that can be solved by the battery level of initial charging amount. This open loop system did not need to perform the optimization task at each sampling instant and, instead, minimized the impact of controller for the energy consumption. In [35], the authors proposed a load management approach for the coordination between multiple electric vehicles. Due to the impact of the unpredictable load from electric vehicles, an accident from overloads, stresses, voltage deviations and power losses might occur in the distribution system. Instead of considering the competition in energy market, this paper mainly built a coordination problem for peak demand shaving so as to improve voltage profile and minimize power losses. In particular, the proposed approach allowed the owners to charge their vehicles based on a priority selection of time zones.

3.3 Game Theory and Related Ideas in Energy Management

All the above-mentioned works were assuming that, users in smart grid can make decisions rationally and objectively. This assumption does not involve individual experience/characteristics in reality, especially for a large-scale system. Due to the promising outlook of introducing the tool of game theory in energy management, devising new schemes to study the competition under realistic interactions between customers and power companies is both challenging and desirable in energy markets. In this section, we summary some related works on realistic behaviors from many fields so as to explore the subjective determinations in smart grid energy management.

Rational decisions in energy management. Existing studies in energy management assumed that, customers could rationally choose their actions as the power companies provide relevant information on the grid, such as pricing information [15, 16, 36–41]. The authors in [36] developed a day-ahead price-based scheduling strategy for the coordination of wind and storage units in a power company. They proposed an approach that integrated the sum of wind and storage unit generation in an hour, and mitigated potential wind energy imbalance charges in electricity markets. As a means to achieve a desired feedback control strategy, pricing mechanisms among selfish agents in the context of heating, ventilation and air conditioning (HVAC) systems was investigated in [37]. The authors proposed a linear-quadratic game, in which the social planner influences the game by choosing the quadratic dependence on control actions for each agents cost function. In [38], an energy price competition was formulated as an N-person game among power companies using two different model. The authors in [39] proposed a game model for demand-side management, in which an isolated microgrid with one wind turbine was built. Using dynamic potential game theory, the participating users could utilize the available renewable and conventional energy resources to minimize the total energy cost in the system. The work in [40] considered integrating electricity elements for balancing massive energy production from renewable sources in demand-side management. Using a noncooperative game, a distributed algorithm was proposed to reduce the loss of producing or storing energy rather than purchase energy from grids. In [15], DSM is studied using a congestion game, which allows the utility company to control the load with little signaling overhead. The work in [16] proposed different game-theoretic approaches to optimize a day-ahead DSM mechanism by using storage units. The overall value of implementing DSM and demand response schemes was studied in [41] via a Stackelberg game formulation. Other related game-theoretic solutions for smart grid pricing, DSM and energy management are discussed in [55, 77–85].

Applications of behavioral game theory. None of the above works incorporates the realistic behaviors of the customers which, in practice, can deviate from the conventional, rational norm set by game theory as previously discussed. The authors in [86] defined a new notion of bounded rationality, in which players' payoffs are discounted by the computation time they take to produce their actions. They studied asymmetric discounting in the proposed model that enables different players to subjectively consider a discount factor. In [87], bounded rationality was defined by human cognitive and emotional architecture, resulting in some failure of rational, occasionally in important decisions. The work in [88] formulated a duopoly game with delay via bounded rationality. Deriving the Nash equilibrium using bifurcation diagrams (i.e. chaotic theory), a local stability analysis had been provided and their results showed that, the users/firms with a delayed bounded rationality would have a higher chance to reach an NE than those without delay. The authors in [89] modeled a cloud price competition problem with Bertrand game. Although their work were based on the realistic electricity prices under a smart grid environment, the achieved NE in their proposed dynamic game was still based on EUT. In [56], the interactions between a single supplier and two retailers were studied using an allocation game. Based on bounded rationality, the players were not perfect optimizers but faced some uncertainties in their opponents' actions. In this work, the authors proved that the retailers might not converge to perfect rationality as assumed by the Nash equilibrium. The work in [90] studied some general classes of optimal control problems, using path integral methods from a decision-theoretic point of view. Because the a decision-maker, i.e. an optimal controller, might has only limited resources, an information-theoretic measure was proposed to explore the bounded rationality of a decision-maker in both continuous and discrete domain.

Compared to the existing literature on energy management in the smart grid, there has a need to established significant technological, economic, and environmental benefits for features such as DSM and energy trading. Yet, the real-world deployment of such mechanisms requires a mathematical and empirical framework that can capture the realistic behavioral patterns of customers and power companies. In essence, the behavioral game framework studies how decision making, in real life, can deviate from the tenets of expected utility theory, a compared conventional game-theoretic notion in Section 2.3, player rationality, conformity to pre-determined decision making rules that are unaffected by real-life perceptions of benefits and risk.

The weighted observation on actions. While interesting, most of this body of existing work [36, 38–40, 56, 86–90], either does not address smart grid problems or is still reliant on traditional, rational game-theoretic notions. Moreover, although there are many studies that have applied PT to solve problems in the social sciences [58, 91, 92] and, more recently, wireless networks [93–97], most of this work cannot be directly extended to smart grid models, as proposed here. Indeed, our work here aims to break new ground in using realistic behavioral game-theoretic models to study end-user influence on the energy management processes of the smart grid. In particular, we will develop novel game-theoretic models for smart grid energy management that explicitly account for the decision making processes of the involved human players such as the smart grid customers. This approach, based on PT in Section 2.3 and Section 2.4, will provide a mathematical tool for analyzing the realistic actions of customers in smart grid energy management. In a nutshell, this thesis introduces a behavioral game theory into smart grid energy management.

Our first behavioral notion is based on the weighted observation on actions. The authors in [98] studied the decision made by risky prospects and uncertain prospects. Risky prospects can be the probabilities associated with the possible outcomes are assumed to be known while the uncertain prospects can represent the probabilities are not assumed to be known. They chose a nonlinear transformation of the probability scale that can overweight low probabilities and underweight moderate and high probabilities. They gave a series of examples and showed that people were less sensitive to uncertainty than to risk. The work in [99] modeled the payoff of New York cab drivers so as to predict a positive labor supply response to transitory fluctuations in wages. Because of human cognitive biases, drivers can misapply or ignore apparently straightforward decision frameworks such as utility maximization. In [100], the authors developed a family structure, such as gender, age, and income, to predict variability in individuals' subjective financial risk tolerance. Their results shown that people with children could not prefer choices with low risk and the age did not impact the requirement of children financial education. The authors in [94] envisioned a scenario when users' actions can interfere the wireless network and they can adjust their transmission probabilities over a random access channel under throughput rewards, delay penalties and energy costs. In particular, deviation of users would degrade the system throughput performance and impact the service provider's revenues. In particular, in PT [94, 98–100], it is observed that in real-life decision-making, people tend to subjectively weight uncertain outcomes, implying which the decision of a user subsequently depends on the decision of others. Under such consideration, each user perceives a weighted version of its observation on the other actions, compared to the classical observation given by the other players (i.e., the information reported in energy market). Thus, the weighing represents a deviation, or a distorted viewpoint, that a certain player or the agent in smart grid can have on the actions of others.

The viewed evaluation on utility. Another important notion brought forward by PT is utility framing as discussed in Section 2.4. In classical game theory, we assume that the utility function that must be optimized is based on an objective metric, as in the case in energy management, a smart grid system must find the maximum energy output that can meet or match the demand. However, when dealing with smart energy management mechanisms consisting of human players, the idea of an objective metric for evaluating utility functions might not be a reasonable assumption. The authors in [101] compared the decision made under rational theory, in terms of expected utility theory, with the descriptive psychological analysis using prospect theory, so as to study political candidates and public referendum issues. Their results have shown that risk aversion in the domain of gains, risk seeking in the domain of losses, and a greater sensitivity to losses than to gains. Also, the results explained how a shift in the reference point could lead to reversals of preferences in the evaluation of political and economic options, contrary to the assumption of invariance. The work in [68] proposed an autonomous network and system based on the usage of psychology impact. Through determining consumer preference (or evaluation of certain resource), psychological factors could have effect on the evolution of autonomous networks and systems. In [102], the authors analyzed the outcomes and assessed the persuasive impact of framed messages in health communication research (i.e., attitudes, intentions, or behavior). They concluded that gain-framed messages were more likely than loss-framed messages to encourage prevention behaviors, especially for skin cancer prevention, smoking cessation, and physical activity. The work [103] investigated the making choices between risky options under framing effect. By exploring whether non-human primates exhibited a similar reflection effect, monkeys had been exhibiting an analogous reversal of risk preferences depending on whether outcomes were presented as gains or losses, suggesting that similar framing effects also influence choice in nonhuman primates. The review paper in [104] summarized prospect theory and the difficulties inherent in applying it. Most of the applications were discussed and used to make sense of observed behavior.

Compared to the above-mentioned deviating behavior, we can envision the evaluation viewed by customer own perceptions. Transferring utility from the objective measurement to a viewed value will fundamentally change the outcome, i.e., payment and demand that are based on classical objective notions. Clearly, the utility evaluation is different from the deviating behavior, and we need to properly design and develop relative models in this thesis, so as to study the impact of customer behavior in energy management.

Chapter 4

Multi-Customer Energy Trading in Smart Grid

In this chapter, we propose a baseline game-theoretic framework to study the interaction between energy sellers and buyers in smart grid, in which the energy market is depending on auction theory and a noncooperative game.

4.1 System Model

Consider a smart grid system having a number of nodes that are in need of energy. These nodes could represent substations and/or distributed energy sources that are servicing an area or group of consumers (e.g., loads, pumped-storage in hydro plants, etc.). Here, we consider that a certain number K, ,of the smart grid elements is unable to meet their demand due to factors such as intermittent generation and varying consumption levels at the grids loads. In this respect, such K grid elements must find alternative sources of energy by acquiring this energy from other elements that have excess energy stored in energy storage units. Thus, we consider that a number, N ,of storage units are deployed in the grid. In particular, all these units belong to customers that have excess energy that they wish to sell. We let N and K denote, respectively, the sets of all N sellers and all buyers. In what follows, we use *seller* to imply any storage unit $i \in \mathcal{N}$ and *buyer* to imply any smart grid element $k \in \mathscr{K}$. Our model generally involves several types of electricity sellers and buyers.

Each buyer $k \in \mathscr{K}$ has a maximum unit price or reservation bid b_k at which it is willing to participate in an energy trade with a seller. Since we focus on the storage units' perspective of the market, we assume that the buyers wish to buy a fixed amount of energy x_k. This models a scenario in which, over a certain given time period, x_k is imposed on the buyers from the practical energy requirements of the users and customers. For each seller i, we define a reservation price s_i per unit energy sold, under which seller i will not trade energy. Given these buying and selling profiles of the various grid elements, an energy exchange market is set up in which the buyers seek to acquire energy so as to meet their demand while the sellers, i.e., the storage units and their owners, seek to collect revenues from selling their extra energy surplus. Here, all N sellers and K buyers will interact so as to determine various energy trading properties that include the quantities exchanged and the price at which energy is traded. Unlike conventional markets in which the sellers only control the reservation prices, in our model, the storage units can also strategically choose the maximum quantity of energy a_i that they want to put up for sale in the market. The choice of a proper a_i is directly dependent on an inherent tradeoff between the potential profits that the sellers foresee and the accompanying costs that relate to the physical characteristics of the storage devices. Indeed, such a tradeoff is a byproduct of the fact that frequently charging or discharging of storage devices is costly as it can lead to a reduction in the storage device's lifespan as well as to other practical costs [49, 105, 106]. Hence, given the buyers' set of bids and energy requirements, the maximum energy amount a_i that any seller i decides to trade strongly affects both the gains/revenues and cost of every storage unit in $\mathscr{N}$ as well as the trading price. Fig. 6.1 provides an illustrative example of the model considered. To analyze such an energy exchange market, we next propose a new framework that builds on the powerful analytical tools of game theory and auction theory.

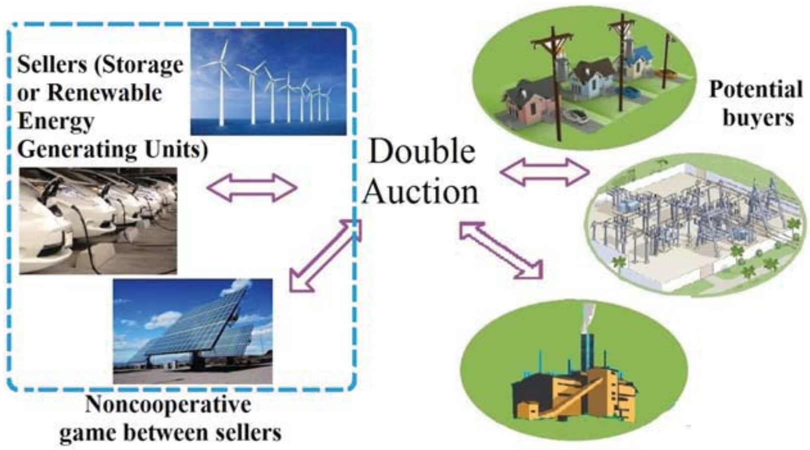

Figure 4.1: An illustrative example of the model studied.

4.2 A Game-Theoretic Approach to Energy Trading

In this section, we first formulate a noncooperative game between the sellers, and then study the proposed energy trading mechanism using a double auction, also discussing its various properties.

4.2.1 Noncooperative Game Model

The complex interactions and decision making processes of the storage units are analyzed using the analytical tools of noncooperative game theory [107]. In particular, we formulate a noncooperative game in normal form, $\Xi = \{\mathscr{N}, \{\mathscr{A}_i\}_{i\in\mathscr{N}}, \{U_i\}_{i\in\mathscr{N}}\}$, which is characterized by three main elements: (a) a set $\mathscr{N}$ of sellers or *players*, (b) the action or *strategy* of each player $i \in \mathscr{N}$ which maps to an amount of energy, $a_i \in \mathscr{A}_i := [0, B_i]$, that will be sold, and (c) a *utility function* U_i of each seller $i \in \mathscr{N}$ which reflects the gains and costs from trading and selling energy. Before defining the utility functions, we note that, in the game Ξ, the reservation price s_i is not included as part of seller i's strategy space. This implies that the sellers must reveal their correct reservation prices when participating in the game. This consideration is motivated by the fact that, when we determine the market's trading price, as explained in the next section, we will develop a *truthful* and strategy-proof double auction mechanism that guarantees that no buyer or seller can benefit by cheating or changing its true reservation price or bid.

Given a certain strategy choice a_i by any storage unit $i \in \mathscr{N}$, the utility function can be characterized by

$$U_i(a_i, a_{-i}) = \sum_{k \in \mathscr{K}} (p_{ik}(a) - s_i) q_{ik}(a) - f\left(\sum_{k \in \mathscr{K}} q_{ik}(a)\right), \tag{4.1}$$

where a is the $N \times 1$ vector of all strategy selections, $a_{-i} := [a_1, a_2, \ldots, a_{i-1}, a_{i+1}, \ldots, a_N]^T$ is the vector of actions selected by the opponents of storage unit i, $p_{ik}(a)$ is the price at which energy is traded between seller i and buyer k, q_{ik} is the quantity of energy exchanged from seller i to buyer k, and $f(\cdot)$ is a function that reflects the cost of selling energy. As previously mentioned, these costs depend on numerous factors such as the physical type of the storage unit or the amount of time the unit is put into charging or discharging modes. Moreover, we note that f must be an increasing function of the amount, $\sum_{k \in \mathscr{K}} q_{ik}(a)$, sold in total by storage unit i. Here, the utility in (4.1) is also a function of the amount x_k of energy that every buyer $k \in \mathscr{K}$ must buy and of the buyers' reservation bids. However, for notational convenience, we have not explicitly written dependence.

The goal of each storage unit i is to choose a strategy $a_i \in \mathscr{A}_i$ in order to maximize its utility as given in (4.1). For characterizing a desirable outcome for the studied game Ξ, one must derive a suitable solution for all N optimization problems that the sellers need to solve. We can first see that, in (4.1), every vector of strategies a selected by the sellers will yield different trading prices $p_{ik}(a)$. These prices are also functions of the reservation prices of the sellers, the quantity bought, and the reservation bids of the buyers or grid elements. Thus, prior to finding a solution for the energy exchange game, we will first introduce a scheme for characterizing the trading price.

4.2.2 Double Auction Mechanism for Market Analysis

The formulated game is useful to study the sellers' interactions. However, in order to find the prices at which energy is traded, we must define suitable mechanisms using the rich tools of double auctions [108] and [109]. Inherently, a double auction is a suitable representation for a trading market that involves multiple sellers and multiple buyers. For the proposed game Ξ, applying a double auction is needed so as to derive the trading prices, the quantities of energy traded, as well as the number of involved sellers and buyers, given the chosen strategy vector a (maximum quantities offered for sale), the reservation prices s_i, $\forall i \in \mathscr{N}$, the quantities x_k to be bought, and the bids b_k, $\forall k \in \mathscr{K}$.

When dealing with a double auction, the buyers and sellers have to decide on whether to be truthful about their reservation bids and prices, given: (i) the potential utility that they will obtain as captured by the first term of (4.1), and (ii) the buyers' potential savings $\sum_{i \in \mathscr{N}} (b_k - p_{ik}) q_{ik}$ with q_{ik} being the quantity bought by k from i. Here, our emphasis is on having a double auction mechanism that yields, for any a, a solution that is *truthful and strategy-proof*. A truthful auction is a scheme in which no seller $i \in \mathscr{N}$ can benefit by cheating about its reservation price such as by misreporting it to and no buyer $k \in \mathscr{K}$ will gain by under-bidding or over-bidding. A strategy-proof solution is of interest as it guarantees truthful reporting by all buyers and sellers.

With this in mind, for the proposed system, we develop a double auction scheme that follows from [108] and [109]. In this scheme, the first step is to sort the sellers in *increasing order* of their reservation prices such that, without loss of generality, we have

$$s_1 < s_2 < \ldots < s_N. \tag{4.2}$$

The next step is to arrange the buyers in a decreasing order of their reservation bids, as follows:

$$b_1 > b_2 > \ldots > b_K. \tag{4.3}$$

We note that the orderings in (4.2) and (4.3) assume that whenever two buyers or sellers have equal reservation prices or bids, one can group them into a single, virtual buyer or seller.

Therefore, in order to match the supply and demand, all sellers whose indices are such that $i < L$ and all buyers such that $k < M$ will be part of the double auction trade. To determine the trading price, once the intersection is identified, one can select any suitable point within the interval $[s_L, b_M]$ [108]. For our energy market, given a seller's strategy vector a, we assume that all sellers $i < L$ and buyers $k < M$ will exchange energy at a price $\bar{p}(a)$ such that

$$\bar{p}(a) = \frac{s_L + b_M}{2}. \tag{4.4}$$

Here, the price depends on a since, for every maximum energy to sell vector a, the intersection point of demand and supply may occur at different M and L.

Once the trading price is found, we need to find the amount of energy that is traded between the $L-1$ sellers and the $M-1$ buyers. First, given the unified trading price in (4.4), the $L-1$ sellers will be indifferent between buyers. This implies that, at the double auction solution, each seller's utility in (4.1) depends only on the quantity sold but not on the identity of the buyer who bought this amount. Hence, assuming that the cost function $f(\cdot)$ is quadratic, by using the proposed double auction, we can derive (4.1) as

$$U_i(a_i, a_{-i}) = (\bar{p}(a) - s_i)Q_i(a) - \tau_i Q_i^2(a), \tag{4.5}$$

with $Q_i(a)$ being the *total quantity* of energy sold by i and τ_i being a penalty factor that weighs the costs reaped by storage unit i when discharging/selling energy. Here, we must stress that our analysis can accommodate any type of cost function f.

Once the auction is concluded, different approaches can be applied to find the quantity of energy traded between each of the $L-1$ participating sellers and $M-1$ participating buyers [108]. For our work, we will apply the technique of [109] in which the entire volume traded is divided in a way to maintain the truthfulness of the auction. Using this approach, the total amount $Q_i(a)$ that is sold by any storage unit i, for a given strategy vector a is:

$$Q_i(a) = \begin{cases} a_i & \text{if } \sum_{k=1}^{M-1} x_k \geq \sum_{j=1}^{L-1} a_j, \\ (a_i - \beta_i)^+ & \text{if } \sum_{k=1}^{M-1} x_k \leq \sum_{j=1}^{L-1} a_j, \end{cases} \tag{4.6}$$

where $(\alpha)^+ := \max(0, \alpha)$ and β_i represents the fraction of the oversupply $\sum_{j=1}^{L-1} a_j - \sum_{k=1}^{M-1} x_k$ that is allotted to seller i. The mechanism in (4.6) implies that whenever the total demand at the auction's outcome exceeds the supply, then every seller i would sell all of the energy a_i that it introduced into the market. However, when the total supply exceeds the total demand, then all sellers get an *equal* share of the oversupply's burden. Here, $\beta_i = \frac{\sum_{j=1}^{L-1} a_j - \sum_{k=1}^{M-1} x_k}{L-1}$. Nonetheless, if, for a seller i, we have $\frac{(\sum_{j=1}^{L-1} a_j - \sum_{k=1}^{M-1} x_k)}{L-1} > a_i$, then, seller i does not sell any energy as per the second case in (4.6). The remaining "oversupply" $\frac{(\sum_{j=1}^{L-1} a_j - \sum_{k=1}^{M-1} x_k)}{L-1} - a_i$ of this seller is subsequently divided equally between the other $L-2$ sellers and the result is added to their share $\beta_j,\ j < L,\ j \neq i$. This scheme will be repeated as long as each seller sells a nonnegative quantity. Here, for the "oversupply" case, the total energy put into the market by the participating sellers, $\sum_{j=1}^{L-1} a_j$, is greater than or equal to that requested by the buyers, $\sum_{k=1}^{M-1} x_k$, but the real energy obtained by seller k is the expected energy, and thus mathematically, $\sum_i q_{ik} = x_k$. For the "over-demand" case, the amount of energy requested by the buyers exceeds the amount put into the market by the sellers, that is, $\sum_i q_{ik} < x_k$. An analogous process can be carried out to find the amount bought by the grid's elements or buyers. Using (4.6), as shown in [108] and [109], we will have the following:

Lemma 1 *In the proposed game* Ξ*, by using (4.6), no seller or buyer benefits by cheating about its reservation price* s_i, $\forall i \in \mathcal{N}$ *or reservation bid* b_k, $\forall k \in \mathcal{K}$*. The double auction is thus* strategy-proof or truthful.

4.3 Proposed Solution and Algorithm

Any storage unit $i \in \mathcal{N}$ can use the proposed double auction in order to estimate its utility, as per (4.5), for every a_i given the strategy choices a_{-i} of its opposing players. Each seller seeks to maximize its utility by selecting the proper strategy $a_i \in \mathcal{A}_i$. In order to solve a noncooperative game in normal form such as Ξ , one popular solution is that of a *Nash equilibrium* [107]. A Nash equilibrium is a state of the game such that no player can increase its utility by *unilaterally* deviating from this equilibrium state. Formally, the Nash equilibrium is defined as follows [107]:

Definition 1 *Consider the proposed noncooperative game in normal form* $\Xi = \{\mathcal{N}, \{\mathcal{A}_i\}_{i\in\mathcal{N}}, \{U_i\}_{i\in\mathcal{N}}\}$*, with* U_i *given by (4.5) given the underlying double auction. A vector of strategies* a^* *is said to be at a* Nash equilibrium (NE)*, if and only if, it satisfies the following set of inequalities:*

$$U_i(a_i^*, a_{-i}^*) \geq U_i(a_i, a_{-i}^*),\ \ \forall a_i \in \mathcal{A}_i,\ i \in \mathcal{N}. \tag{4.7}$$

Next, we first prove the existence of an NE for the proposed game and, then, we propose an algorithm that could find an NE for our model.

Theorem 1 *For the noncooperative game* $\Xi = \{\mathscr{N}, \{\mathscr{A}_i\}_{i\in\mathscr{N}}, \{U_i\}_{i\in\mathscr{N}}\}$, *there exists at least one pure-strategy Nash equilibrium.*

Proof: Assume $\mathscr{A}_i \subseteq \mathbb{R}^m (i = 1, \ldots, \mathscr{N})$, *is a non-empty, convex and compact set. As shown in [110], if* $\forall i$, $U_i : \mathscr{A} \to \mathbb{R}^1$ *is* 1) *graph-continuous* 2) *upper semi-continuous in* a 3) *quasi-concave in* a_i, *then the game* $\Xi = \{\mathscr{N}, \{\mathscr{A}_i\}_{i\in\mathscr{N}}, \{U_i\}_{i\in\mathscr{N}}\}$ *possesses a pure-strategy Nash equilibrium.*

A function U_i *is said to be graph continuous if there exists a function* $A_i = F_i(A_{-i})$, $\forall a \in A$ *such that* $U_i(F_i(a_{-i}), a_{-i})$ *is continuous in* a_{-i}. *In particular, as shown in [110], a piecewise continuous function is graph continuous if the strategy space is compact. Due to the discontinuity of our utility function as* a_{-i} *varies, we define*

$$\begin{aligned} \bar{p}(a_i, a_{-i}) &= p_1, \\ \bar{p}(a_i, a_{-i} + \Delta a) &= p_2. \end{aligned} \tag{4.8}$$

at a jump point. Thus, we can assume that in a given range Δa,

$$\bar{p}(a_i, a_{-i}) = \frac{p_2 - p_1}{\Delta a} \Delta a_{-i}, \tag{4.9}$$

and when Δa *is a small value, the slope becomes infinite, which implies that the function is a piecewise continuous function on* a_{-i}.

Mathematically, $U_i(a_i, a_{-i})$ *is upper semi-continuous at* a_{i0} *if there exists a neighborhood* a_i *such that*

$$\lim_{a_i \to a_{i0}} \sup U_i(a_i, a_{-i}) \le U_i(a_{i0}, a_{-i}). \tag{4.10}$$

Similarly, for a jump point of utility function, we define $a_{i0} = a_i + \Delta a$ *such that* $p_3 \le p_4$,

$$\begin{aligned} \bar{p}(a_i, a_{-i}) &= p_3, \\ \bar{p}(a_{i0}, a_{-i}) &= p_4. \end{aligned} \tag{4.11}$$

Clearly, the utility function is upper semi-continuous because rational players seek a higher profit around the jump point. Here, we only need to prove that the utility function is quasi-concave.

In a given range of constant price, by simplifying (4.5) and (4.6), we have:

$$f(Q) = (\bar{p} - s_i)Q - Q^2,$$
$$Q(a) = \begin{cases} a_i, & \text{if } \sum_{k=1}^{M-1} x_k \geq \sum_{j=1}^{L-1} a_j, \\ (a_i - \beta_i)^+, & \text{if } \sum_{k=1}^{M-1} x_k \leq \sum_{j=1}^{L-1} a_j, \end{cases} \tag{4.12}$$
$$U_i = f(Q(a)).$$

Before proceeding further with the proof, we need to state the following Lemma from [111]:

Lemma 2 *Suppose $g : X \to R$ is quasilinear and $h : g(X) \to R$ is a quasi-concave function. Then $h \circ g : X \to R$ is quasi-concave.*

This result has been extended to concave functions with strict conditions in [111]. Now,

$$\frac{\partial Q(a)}{\partial a_i} \geq 0,$$
$$Q(\lambda a_i^x + (1-\lambda)a_i^y, a_{-i}) \geq \min[Q(a_i^x), Q(a_i^y), a_{-i}],$$
$$Q(\lambda a_i^x + (1-\lambda)a_i^y, a_{-i}) \leq \max[Q(a_i^x), Q(a_i^y), a_{-i}], \tag{4.13}$$
$$\forall\, a_i^x \neq a_i^y, \lambda \in (0,1),$$

where a_i^x and a_i^y belong to the action set $\mathscr{A}_i$ of seller i. Thus, $Q(a)$ is both quasi-concave and quasi-convex, and hence it is a quasi-linear function. Subsequently we can obtain the partial derivative with respect to Q from (4.12):

$$\frac{\partial U(Q)}{\partial Q} = \bar{p} - s_i - 2Q,$$
$$U(\lambda Q_x + (1-\lambda)Q_y) \geq \lambda U(Q_x) + (1-\lambda)U(Q_y), \tag{4.14}$$
$$\forall\, Q_x \neq Q_y, \lambda \in (0,1).$$

Thus, $U(Q)$ is a concave function (which is quasi-concave). Following Lemma 2, we substitute (4.13) into (4.14),

$$\begin{aligned} & U[Q(\lambda a_i^x + (1-\lambda)a_i^y, a_{-i})] \\ \geq & U[\min\{Q(a_i^x, a_{-i}), Q(a_{i_y},, a_{-i})\}], \\ \geq & \min\{U[Q(a_i^x, a_{-i})], U[Q(a_{i_y}, a_{-i})]\}. \end{aligned} \tag{4.15}$$

Thus, U is a quasi-concave function of a_i.

Intuitively, the partial derivative of U on Q is positive before the local maximum of U and negative after it. The partial derivative of Q on a_i is 1 or a nonnegative number depending on the auction except at the inflection point when $a_i = \beta_i$. Around this inflection point, $U(a_i, a_{-i})$ firstly increases then might decrease as a_i varies in a price-holding graph-continuous range. Therefore, $U(a)$ is a quasi-concave function in a, and the game $\Xi = \{\mathcal{N}, \{\mathcal{A}_i\}_{i\in\mathcal{N}}, \{U_i\}_{i\in\mathcal{N}}\}$ possesses a pure-strategy Nash equilibrium as it satisfies all required conditions.

At any NE of the proposed game, no storage unit can improve its utility by *unilaterally* changing the maximum quantity of energy that it wishes to sell, given the equilibrium strategies of the other storage units. Having established existence, we must develop a scheme that allows the game Ξ to reach an NE. To do so, we must first define the notion of a *best response*:

Definition 2 *The* best response *$r(a_{-i})$ of any storage unit $i \in \mathcal{N}$ to the vector of strategies a_{-i} is a set of strategies for seller i such that*

$$r(a_{-i}) = \{a_i \in \mathcal{A}_i | U_i(a_i, a_{-i}) \geq U_i(a_i', a_{-i}),\ \forall a_i' \in \mathcal{A}_i\}. \tag{4.16}$$

Hence, for any storage unit $i \in \mathcal{N}$, when the other storage units' strategies are chosen as given by a_{-i}, any best response strategy in $r(a_{-i})$ is at least as good as any other strategy in $\mathcal{A}_i$. Using the concept of a best response, we can subsequently define a novel algorithm that can be used by the storage units and buyers so as to exchange energy. In particular, we propose the following iterative algorithm that is guaranteed to converge to a Nash equilibrium of the game:

Theorem 2 *There exists a searching inertia weight w, $0 < w < 1$, such that, the iterative algorithm*

$$a_i^{(n+1)} = (1-w)r(a_{-i}^{(n)}) + wa_i^{(n)}, \tag{4.17}$$

converges to an NE.

Proof: In the proposed model, the classical best response dynamics may not converge due to the underlying auction mechanism. Depending on the different amounts between sellers and buyers, the price is a piecewise continuous function. From (4.5) and (4.16), we have:

$$\begin{aligned} r(a_{-i}) &= \arg\max_{a_i}[(\bar{p}(a)-s_i)Q_i(a) - \tau_i Q_i^2(a)], \\ &= \begin{cases} \frac{\bar{p}(a)-s_i}{2\tau_i}, \\ \quad \text{if } \sum_{j=1}^{L-1} a_j^{(n)} \le \sum_{k=1}^{M-1} x_k, \text{ (sell} \le \text{buy)}; \\ \frac{(\bar{p}(a)-s_i)(L-1)+2\tau_i(\sum_{j=1,j\neq i}^{L-1} a_j - \sum_{k=1}^{M-1} x_k)}{2\tau_i(L-2)}, \\ \quad \text{if } \sum_{j=1}^{L-1} a_j^{(n)} \ge \sum_{k=1}^{M-1} x_k, \text{ (sell} \ge \text{buy)}. \end{cases} \end{aligned} \tag{4.18}$$

When selling amounts are less than buying, the best response of each seller is a constant. When selling amounts are greater than buying, and, we only use best response for iterations, we have

$$a_i^{(n+1)} = \frac{1}{L-2} \sum_{j=1,j\neq i}^{L-1} a_j^{(n)} + G_i, \tag{4.19}$$

where $G_i = \frac{(\bar{p}(a)-s_i)(L-1)}{2\tau_i(L-2)} - \frac{1}{L-2}\sum_{k=1}^{M-1} x_k$. To sum all $L-1$ sellers,

$$\sum_{1}^{L-1} a_i^{(n+1)} = \sum_{1}^{L-1} a_i^{(n)} + \sum_{1}^{L-1} G_i. \tag{4.20}$$

The second term on right hand side is not 0 *and this might lead a price change. In other words, in iteration* γ*,*

$$\begin{cases} \text{if } \sum_{k=1}^{M-2} x_k \leq \sum_{j=1}^{L-1} a_j^{(\gamma)} \leq \sum_{k=1}^{M-1} x_k, & \bar{p}(a) = p_1(\text{sell} \leq \text{buy}), \\ \text{if } \sum_{k=1}^{M-1} x_k \leq \sum_{j=1}^{L-1} a_j^{(\gamma)} \leq \sum_{k=1}^{M} x_k, & \bar{p}(a) = p_2(\text{sell} \geq \text{buy}). \end{cases} \tag{4.21}$$

It is possible that, in iteration $\gamma+1$*, the best response changes the total amount* $\sum a_i^{(\gamma+1)}$ *in (4.20) and this can lead to a price changing loop.*

However, from (4.5) and (4.16), our proposed algorithm in (4.16) has

$$a_i^{(n+1)} = wa_i^{(n)} + (1-w)\frac{1}{L-2}\sum_{j=1, j\neq i}^{L-1} a_j^{(n)} + (1-w)G_i. \tag{4.22}$$

Similarly, to sum over all $L-1$ *sellers,*

$$\sum_{1}^{L-1} a_i^{(n+1)} = \sum_{1}^{L-1} a_i^{(n)} + (1-w)\sum_{1}^{L-1} G_i. \tag{4.23}$$

In particular, when $n > \gamma$*, we have (4.23). Thus, there must exist a weight w, such that*

$$\begin{cases} \text{if } \sum_{k=1}^{M-2} x_k \leq \sum_{j=1}^{L-1} a_j^{(\gamma+1)} \leq \sum_{k=1}^{M-1} x_k, & \bar{p}(a) = p_1(\text{sell} \leq \text{buy}), \\ \text{if } \sum_{k=1}^{M-1} x_k \leq \sum_{j=1}^{L-1} a_j^{(\gamma+1)} \leq \sum_{k=1}^{M} x_k, & \bar{p}(a) = p_2(\text{sell} \geq \text{buy}). \end{cases} \tag{4.24}$$

Remark 1 *We note that the above result is generated for the case in which the* $(L-1)$ *sellers are able to sustain the oversupply. Similar results can easily be generated for the cases in which the oversupply cannot be split among all sellers; however, this case is omitted due to space limitations.*

Remark 2 *With a given price range, the utility in (4.5) can be viewed as a concave function. More precisely, there exists a weight, such that no player could arbitrarily approach its current best response, which might change the price and pull some participating players out of the auction.*

In a given price range,

$$\begin{aligned}
&\text{if } \frac{\partial U_i}{\partial a_i} > 0, \quad \frac{U(a_i^{(n+1)}, a_{-i}^{(n)}) - U(a_i^{(n)}, a_{-i}^{(n)})}{a_i^{(n+1)} - a_i^{(n)}} > 0, \\
&\text{if } \frac{\partial U_i}{\partial a_i} < 0, \quad \frac{U(a_i^{(n+1)}, a_{-i}^{(n)}) - U(a_i^{(n)}, a_{-i}^{(n)})}{a_i^{(n+1)} - a_i^{(n)}} < 0.
\end{aligned} \tag{4.25}$$

An increasing/decreasing monotonic function would approach to its upper/lower boundaries. It is not difficult to obtain the upper boundary from concavity:

$$U(a_i^{(n+1)}, a_{-i}^{(n)}) \leq U(a_i^{(n)}, a_{-i}^{(n)}) + \frac{\partial U_i}{\partial a_i}(a_i^{(n+1)} - a_i^{(n)}). \tag{4.26}$$

For the lower boundary,

$$\begin{aligned}
&U(a_i^{(n+1)}, a_{-i}^{(n)}) \\
=&U(a_i^{(n)}, a_{-i}^{(n)}) + \frac{\partial U_i}{\partial a_i}(a_i^{(n+1)} - a_i^{(n)}) \\
&- \int_0^{a_i^{(n+1)} - a_i^{(n)}} \frac{\partial U_i(a_i^{(n)}, a_{-i}^{(n)})}{\partial a_i^{(n)}} t - \frac{\partial U_i(a_i^{(n)} + t, a_{-i}^{(n)})}{\partial (a_i^{(n)} + t)} t \, \mathrm{d}t.
\end{aligned} \tag{4.27}$$

In particular, because $\frac{\partial U_i}{\partial a_i}$ is Lipschitz continuous when the weight holds the price in a range,

$$\left|\left| \frac{\partial U_i(a_i^{(n)}, a_{-i}^{(n)})}{\partial a_i^{(n)}} - \frac{\partial U_i(a_i^{(n)} + t, a_{-i}^{(n)})}{\partial (a_i^{(n)} + t)} \right|\right| \leq L||a_i^{(n)} - (a_i^{(n)} + t)||. \tag{4.28}$$

Thus, we have

$$\begin{aligned}
U(a_i^{(n+1)}, a_{-i}^{(n)}) \geq &U(a_i^{(n)}, a_{-i}^{(n)}) + \frac{\partial U_i}{\partial a_i}(a_i^{(n+1)} - a_i^{(n)}) \\
&- \frac{1}{2}L(a_i^{(n+1)} - a_i^{(n)})^2.
\end{aligned} \tag{4.29}$$

Remark 3 *The upper and lower boundary of $U(a_i^{(n+1)}, a_{-i}^{(n)})$ are also bounded by (4.21).*

Due to the above-mentioned price and boundary analysis, we can conclude that $|a_i^{(n+1)} - a_i^{(n)}| < \varepsilon$ after some iterations γ, where ε is a small value. Substituting in (4.17), we obtain (at the final iteration):

$$\begin{aligned}
(1-w)a_i^{(T_f+1)} =&(1-w)r(a_{-i}^{(T_f)}) \\
a_i^{(T_f+1)} =&r(a_{-i}^{(T_f)}),
\end{aligned} \tag{4.30}$$

Table 4.1: Proposed Energy Trading Solution

Phase 1 - Proposed Dynamics:
Each storage unit $i \in \mathcal{N}$ chooses a starting strategy $a_i^{\text{init}} = B_i$
repeat,
a) Each seller $i \in \mathcal{N}$ observes its best response strategy $r_i(a_{-i})$
b) Each seller $i \in \mathcal{N}$ randomly selects the better response strategy between the current strategy and best response strategy in (4.17):
$wa_i + (1-w)r_i(a_{-i})$, where $0 \leq w \leq 1$. As using the method:
a) An auctioneer (utility operator) communicates with the buyers and sellers using the grid's two-way communication architecture (see [112] or [106] and references therein).
b) The price and amounts of energy to be traded are found via the double auction of Section 4.2.2.
Auction
a) The auctioneer advertises s_i, $\forall i \in \mathcal{N}$ and b_k $\forall k \in \mathcal{K}$.
b) Each seller publishes its expected price, and the auctioneer orders the sellers as required.
c) After ordering, the auctioneer tells seller i, during its turn, of the current vector of strategies a_{-i}.
d) Seller i computes and submits its strategic response in (4.17) using (4.16).
until convergence to an NE strategy vector a^*.
Phase 2 - Market and Trading
a) The auctioneer performs the double auction mechanism given the equilibrium choices as per a^*.
b) Actual energy exchange occurs and revenues are collected.

which is the best response *of $a_i^{(n)}$. Consequently, our algorithm converges to an NE.*

The sellers and buyers in the proposed noncooperative game can interact using a novel algorithm composed of two phases: a strategic dynamics phase and an actual market and energy trading stage. Our strategic dynamics stage begins with every seller choosing an initial strategy a_i^{init}. While this initial strategy can be chosen arbitrarily by each seller, the most intuitive choice is that each seller starts by trying to sell all of its available surplus of stored energy. Therefore, we let $a_i^{\text{init}} = B_i,\ \forall i \in \mathcal{N}$. Subsequently, an iterative process begins in which the sellers can take turns in choosing their maximum amounts of energy to sell (i.e., strategies). To this end, at any iteration θ, any storage unit i, during its turn to act, will choose a strategy that approaches its best response strategy, as given by (4.16) and (4.17). This iterative algorithm is executed until guaranteed convergence to an NE. In particular, the proposed algorithm has been shown to always converge to a Nash equilibrium in Theorem 2. A summary of the proposed algorithm is given in Table 4.1. Here, we note that, although in Table 4.1 we present a sequential implementation of the algorithm, the players may also utilize a parallel approach. In a sequential implementation, the players act sequentially, in an arbitrary order such that each player is able to observe (or is notified by the auctioneer of) the actions taken by the previous players. In contrast, in a parallel approach, at an iteration t, all players respond, using (4.17), to the actions observed by the other players at iteration $t-1$. Once an NE of the game is reached, the last phase of the algorithm is the practical market operation. During this final phase, given the equilibrium strategies, all sellers and buyers submit their bids and then engage in an actual double auction in which each storage unit discharges (sells) the desired energy amount and is rewarded accordingly.

We finally note that, in the presence of an elaborate communication infrastructure, the sellers and the buyers can interact directly without the need for a control center. In this case, each seller can, individually, decide on the amount of energy it wants to sell at each iteration of the proposed approach, while directly notifying the other players of its choice. The rest of the operation would still follow the iterative process discussed earlier.

4.4 Simulation Results and Analysis

For simulating the proposed system, we consider a geographical region in which a number of storage units have a surplus of stored energy that they wish to sell to existing customers (e.g. loads, substations, etc.) in a smart grid. Each unit has a surplus between 75 MWh and 220MWh that can be sold. The reservation prices of the sellers are chosen randomly from a range of $[10,50]$ dollars per MWh while reservation bids of the buyers are chosen randomly from a range of $[15,60]$ dollars per MWh. The demand of each buyer is chosen randomly from within a range of $[20,60]$ MWh. Unless stated otherwise, the cost per energy sold is set to $\tau_i = 0.5$, $\forall i \in \mathcal{N}$. All statistical results are averaged over all possible random values for the different parameters (prices, bids, demand, etc.) using a large number of independent simulation runs.

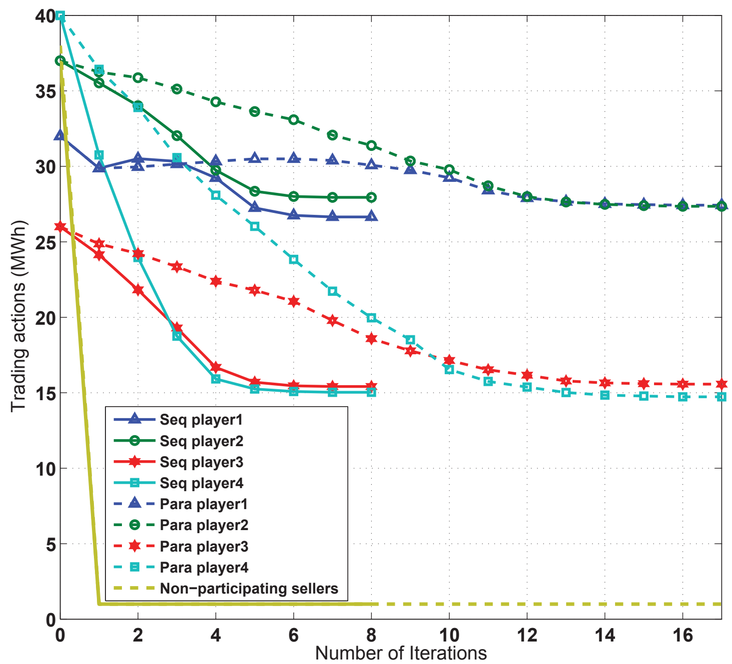

Figure 4.2: Average action per seller (storage unit) resulting from the proposed game approach and from the number of storage units $K = 5$ buyers, $N = 6$ sellers.

In Fig. 4.2, we show, for a smart grid with $K = 5$ buyers and $N = 6$ sellers, the average action per seller (storage unit) resulting from the proposed game approach at the equilibrium using both sequential and parallel approaches. The sequential algorithm's performance is compared with that of the parallel algorithm in which the sellers, simultaneously, attempt to sell energy depending on their previous actions. Here, in particular, we choose the same weight $w = 0.3$ for both the sequential and the parallel algorithms. In Fig. 4.2, we can see that, for the proposed algorithm, the trading action per storage unit converges to different values with increasing iterative steps. Fig. 4.2 shows that two players out of six decide not to participate in the market. The dashed line relates to those sellers who do not participate in the market due to the fact that would the trading price would then lead to a negative utility. However, these sellers would still have some energy in hand and they can offer it for sale at a later time instant in which the trading price in another auction or area might give them an opportunity to obtain positive utility. Thus, although they do not trade at the current market price, they will maintain their available energy and eventually participate in a future market. In particular, we use the "dashed line" to indicate a baseline action value of 1 to represent those sellers that do not participate in the market, but rather prefer to wait for future trading opportunities. We can also observe that, for the sequential algorithm (solid line), the action of player 1 increases a little at the beginning. This is due to the fact that, the player who plays first in one iteration has a higher opportunity to sell energy than others. In general, as seen in Fig. 4.2, because of the competition over the resources, the actions are essentially decreasing, which means that at the equilibrium, not all players will sell their maximum available energy.

Fig. 4.3 presents, for a smart grid with $K = 3,5$ buyers, the average achieved utility per seller resulting from the proposed game as the number of storage units N varies. Here, we set $w = 0.5$ for the sequential algorithm and $w = 0.1$ for the parallel algorithm. For comparison purposes, we develop a conventional, baseline greedy algorithm using which, iteratively, each seller tries to sell the maximum amount that it could sell (while accounting for the changes of the utility in (4.5)) while first choosing the highest-bid buyers. The greedy process continues until no additional energy trade is possible. In this greedy scheme, the trading price is selected as the middle point between the concerned buyer's reservation bid and the concerned seller's reservation price. In Fig. 4.3, we can see that the average utility per storage unit is decreasing with N. The reason behind this mainly involves two issues. First, the increase in sellers can lead to an increased competition and, thus, a decrease in the overall trading price. Second, the number of sellers $L-1<N$ that will actually participate in the final energy exchange market reaches a certain maximum that no longer increases with N due to the fixed demand (i.e., the number of buyers). This figure demonstrates that, for all N, the proposed noncooperative game approach yields a significant performance improvement, in terms of the average utility achieved per storage unit. In particular, this advantage of the proposed approach reaches up to 130.2% (the maximum at $K=5, N=4$) relative to a conventional greedy approach.

Fig. 4.4 shows, for $K=5$ buyers, the average number of iterations needed before convergence of the different algorithms as the number of storage units N increases. In this figure, we can see that the average number of iterations of the proposed sequential algorithm is similar to that of the classical best response algorithm (whenever this algorithm converges, recall from Theorem 2 that a best response dynamics may not converge). As expected, Fig. 4.4 shows that the parallel algorithm requires a much higher number of iterations. In particular, the average number of iterations resulting from the sequential algorithm varies from 7.7 at $N=6$ to 8.2 at $N=7$, in contrast, for the parallel case, it varies from 25.8 at $N=4$ to 32.5 at $N=10$. This result indicates that the proposed algorithm, particularly with a sequential implementation, has a reasonably fast convergence speed.

Fig. 4.5 shows, for different buyers and sellers, the average utility per seller as the penalty factor τ varies. From (4.5), we can see that the utility would decrease with increasing τ and this is corroborated in Fig. 4.5. In particular, when τ is equal to 1, the utility of each player is dramatically influenced by the penalty part in (4.5). The first revenue term in (4.5) the sellers obtained in the auction remains the same as the second penalty term increases, even though the total utility is positive.

Fig. 4.6 shows the average utility from the different proposed approaches as the

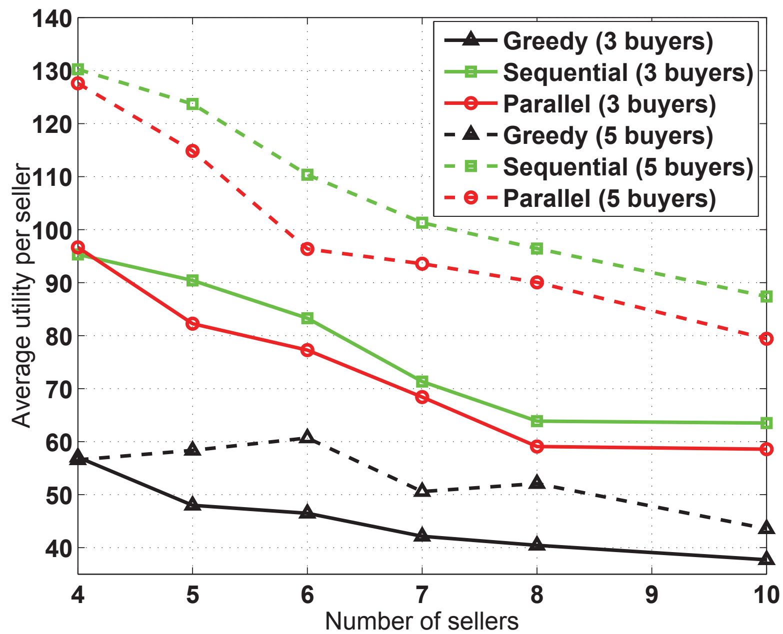

Figure 4.3: Performance assessment in terms of average utility per seller as the number of storage units N varies for $K = 3, K = 5$ buyers.

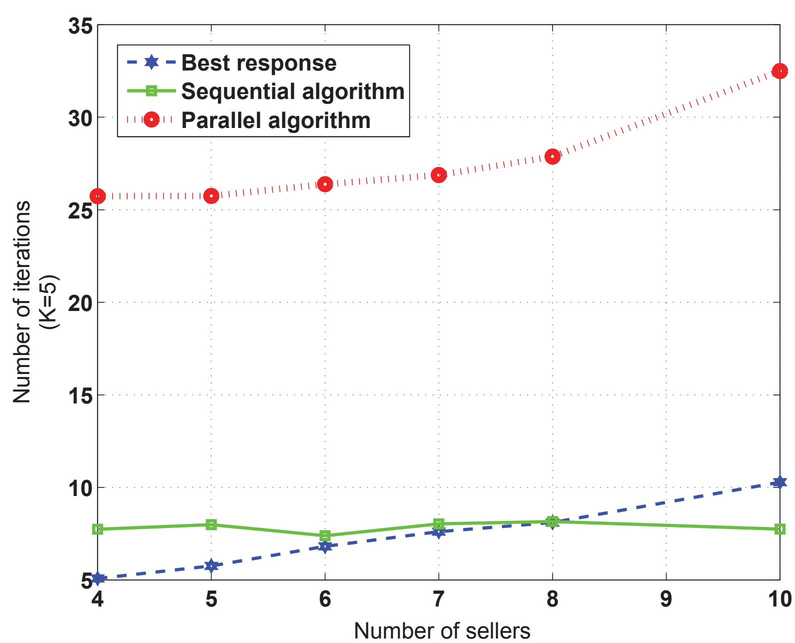

Figure 4.4: Average number of iterations per seller as the number of storage units N varies for $K = 5$ buyers.

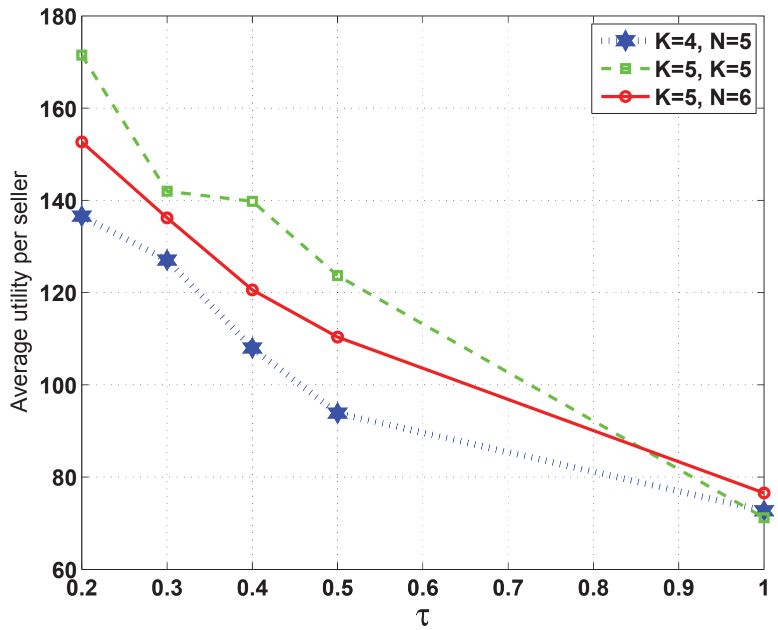

Figure 4.5: Performance assessment in terms of the penalty factor τ resulting from the proposed game approach for different number of buyers and sellers.

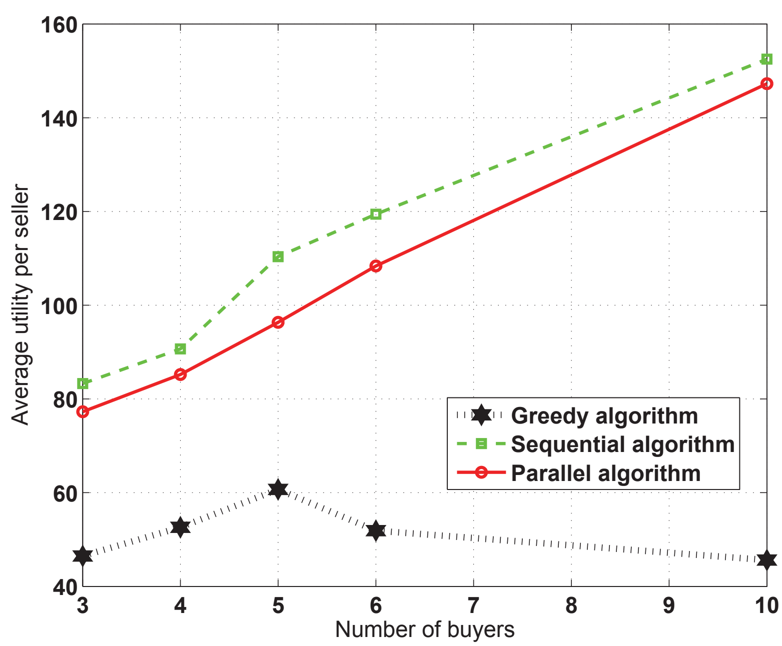

Figure 4.6: Performance assessment in terms of average utility per seller as the number of buyers K varies, for $N = 6$ sellers.

number of buyers K varies, for $N = 6$ sellers. Each iteration consists of a series of choices by the sellers, using the same initial information. In Fig. 4.6, we can also see that, as the number of buyers, K, increases the average utility per seller increases due to the availability of additional buyers that are willing to participate in the market. In fact, Fig. 4.6 shows that, as K increases, the sellers have a larger utilities due to the availability of more buyers. In particular, our proposed algorithm yields a performance improvement ranging between 72.3% (for $K = 4, N = 6$) to 234.4% (for $K = 10, N = 6$) relative to the greedy scheme. Further inspection of Fig. 4.6 reveals that any change in the numbers of buyers does not impact the increasing average utility rate of our algorithm, while the greedy algorithm reaches a maximum when the number of buyers is similar to that of sellers.

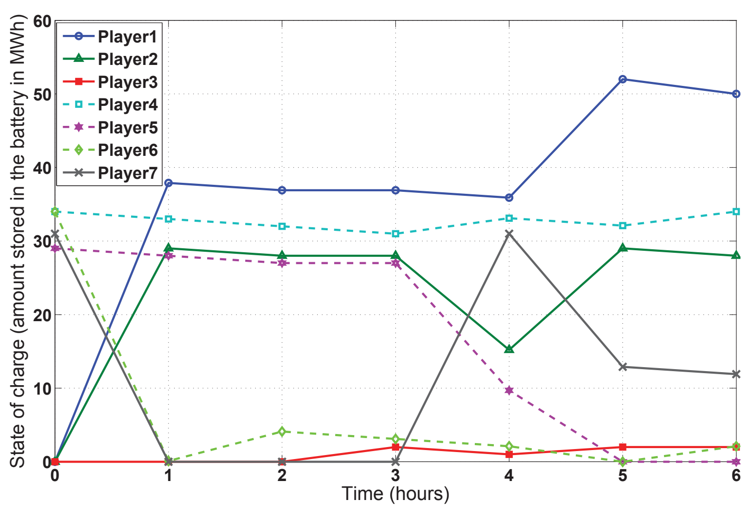

Figure 4.7: Performance assessment in terms of the amount per player as the number of runs varies for initial $K = 3$ buyers and $N = 4$ sellers.

Fig. 4.7 shows, for a smart grid with $K = 3$ buyers and $N = 4$ sellers, the state of charge represented by the battery amount per player resulting from a time-dependent game as time evolves. Here, we show the results for the proposed sequential algorithm with a weight $w = 0.5$. For our simulations, we assume that the period corresponds to 1 hour as is typical in a residential community [113]. In Fig. 4.7, we can see that, for the proposed game, four sellers would sell their energy into the market during the first time instant. Then, after the first run, all players reconsider their roles and still participate in the market, especially for those that did not sell/buy enough in the previous time slots. The iteration would then lead to a new trading price and this process continues, as previous users are still in the market and no new players join in this group. In Fig. 4.7, we can see that all players have an opportunity to act as sellers or buyers every hour. Player 1, for example, acts as a buyer during the first hour. After the second hour, this player changes from a buyer to a seller, and then acts as a buyer at the fifth hour. Player 4 sells a small amount in the first three hours and becomes a seller at the fourth and sixth time instants. During the second and third hours, Player 2 does not sell a large amount despite the fact that it had acted as a buyer and fully charged at the first time instant. After 4 hours have elapsed, Player 2 sells 12.8 MWh. This player tends to sell the energy because it reaches its battery limitation and has an incentive to become a seller after the first hour.

4.5 Conclusions

In this chapter, we have introduced a novel approach for studying the complex interactions between a number of storage units seeking to sell part of their stored energy surplus to smart grid elements. We have formulated a noncooperative game between the storage units in which each unit strategically chooses the maximum amount of energy surplus that it is willing to sell so as to optimize a utility function that captures the benefits from energy selling as well as the associated costs. To determine the trading price that governs the energy trade market between storage units and smart grid elements, we have proposed an approach based on double auctions, which leads to a strategy-proof outcome. We have shown the existence of a Nash equilibrium and solve the underlying game via a novel algorithm using which the storage units can reach a Nash equilibrium point. Simulation results have shown that the proposed approach enables the storage units to act strategically while improving their average utilities.

Chapter 5

Prospect Theory for User-Centric Charging

In this chapter, we introduce a prospect-theoretic model to study uses' behaviors in smart grid, as they charge/discharge energy in their storage units.

5.1 System Model

Consider a smart grid in which N customers are present. Let $\mathscr{N}$ be the set of all N customers. Under normal operating conditions, we assume that each customer $i \in \mathscr{N}$ constitutes a constant load D_i on the grid. Among all N customers, a subset $\mathscr{K} \subseteq \mathscr{N}$ of K customers is assumed to be "active". Here, an active customer refers to a user equipped with a smart home and able to actively participate in the energy management of the smart grid, as allowed by the power company. Every customer $k \in \mathscr{K}$ owns a storage unit that initially stores an amount of energy $S_k < D_k$. At a given period of time, we assume that the participation of each customer $k \in \mathscr{K}$ is restricted to one of two actions: a) charge the needed amount D_k (act as load) or b) discharge/sell the surplus S_k to the other customers (act as source).

Naturally, any given action by a customer $k \in \mathscr{K}$ will affect both the power system (needed generation, losses, etc.) and the market economics (prices). We assume that the power company allows the customers to charge or discharge, but it requires that the total generation power remains within a nominal, desirable value to maintain the power system's stability [114]. In this studied scenario, all K participating users use storage units to charge and discharge so as to optimize their overall monetary benefits. The decisions of the customers are, however, largely coupled, which leads to a noncooperative game-theoretic.

5.2 A Prospect Theoretic Approach for Storage Optimization

In this section, we first formulate a noncooperative game, in which the participated users could assess the effect of discharging/charging energy. Then, we study the proposed game using classical expected utility and prospect theory, respectively.

5.2.1 Noncooperative Game Model

We analyze the interactions between the active customers using noncooperative game theory [107]. As the strategy choices of the customers are largely *interdependent*, we can formulate a strategic noncooperative game $\Xi = (\mathscr{K}, \{\mathscr{A}_k\}_{k\in\mathscr{K}}, \{u_k\}_{k\in\mathscr{K}})$, that is characterized by three main elements: *a)* the players are the active customers in the set $\mathscr{K}$, *b)* the action $a_k \in \mathscr{A}_k := \{D_k, S_k\}$ of each player is to either charge/buy a total amount of energy D_k ($a_k = D_k$) or discharge/sell the available surplus S_k ($a_k = S_k$), and *c)* the utility function u_k of each player k which captures the benefit-cost tradeoffs associated with the different choices. Each customer k is assumed to have enough storage capacity to handle an amount $D_k + S_k$. Here, we note that, although the customers may have other demands, our model is solely focused on the discharge/charge actions and their impact on the grid and customers. The utility function achieved by a player $k \in \mathscr{K}$ that chooses an action a_k is given by

$$\begin{aligned} u_k(a_k, a_{-k}) = &-\alpha(a_k, a_{-k})\Big(D_k + L_k(a_k, a_{-k})\Big) \\ &+ \gamma(a_k, a_{-k})S_k - \beta\Big(G(a_k, a_{-k}) - \hat{G}\Big)^2, \end{aligned} \tag{5.1}$$

where $a_{-k} = [a_1, a_2, \dots, a_{k-1}, a_{k+1}, \dots, a_K]$ is the vector of action choices of all players other than k, $L_k(a_k, a_{-k})$ are the total losses over the distribution/transmission lines which depend on the total demand and are computed using conventional optimal power flow algorithms [114], $G(a_k, a_{-k})$ denotes the total generation by *the power company* (not the customers) under current action choices, and β is a regulation penalty factor, that allows the power company to maintain a regulated power supply, i.e. $\hat{G}$. Maintaining such a regulation is important for many operational aspects of the grid, such as the conversion between AC and DC. We note that, in our game, the actions are positive and we have positively/negatively defined charging/discharging unit payments α and γ in (5.1). Here, we define the charging price and the discharging price, respectively, set by the power company and participating users as follows:

$$\alpha(a_k, a_{-k}) = \begin{cases} c(a_k, a_{-k}) & \text{if } a_k = D_k, \\ 0 & \text{otherwise,} \end{cases} \tag{5.2}$$

with $c(a_k, a_{-k})$ being the unit price in the energy market which follows the pricing strategy of the power company. Moreover,

$$\gamma(a_k, a_{-k}) = \begin{cases} b_k & \text{if } a_k = S_k, \\ 0 & \text{otherwise,} \end{cases} \tag{5.3}$$

with b_k being the unit price at which a certain customer k would sell its surplus S_k. We assume that each customer can set its own price, but the power company will impose a pricing restriction B, such that $b_k < B$, $\forall k \in \mathscr{K}$.

The utility function in (5.1) captures both the economic benefits of customer participation as well as the impact on the power system (via the regulation term). Here, while the power company allows the K active customers to actively decide on whether to buy or sell energy, it mandates that the generated power in the considered geographical area remains within desired, stable operating conditions. Also, we note that both demand

and line loss determine the total generation level and, excessive charging or discharging might damage the generator due to a frequency variation thus requiring regulation [115]. Without loss of generality, we assume that the normal, stable operating conditions correspond to the case in which all N customers act as loads and we let $\hat{G} = \sum_{i \in \mathscr{N}} (D_i + L_i)$ denote the total generated power required for this distribution area during normal operation. L_i represents the losses incurred over the distribution/transmission lines for delivering D_i to customer i which depend on the total demand and are computed using power flow algorithms. Therefore, for the case in which $a_k = D_k, \forall k \in \mathscr{K}$, we have $G(a) = \hat{G}$ (with a being the vector of all strategies). Consequently, any actions taken by a certain customer that shifts the generated power from its nominal value $\hat{G}$ will require the power company to regulate the generation. The need for this regulation indirectly yields a cost penalty on the active participants as captured in (5.1).

5.2.2 Expected Utility Theory

In a smart grid, owing to uncertainty in power generation as well as the fact that the customers can make certain decisions (such as whether to allow the use of their storage device or not) with different frequency over time, it is reasonable to assume that customers make probabilistic choices. Therefore, we are interested in studying the game under *mixed strategies* [107]. As customers are often uncertain when presented with different choices in practice, a mixed-strategy solution can better capture their realistic behavior. Let $p = [p_1, \dots, p_k]$ be the vector of all mixed strategies, where, for every customer $k \in \mathscr{K}$, $p_k(a_k)$ is the probability distribution over the pure strategies $a_k \in \mathscr{A}_k$.

Under the conventional EUT model, the utility of each user is simply the expected value over its mixed strategies. Thus, the EUT utility of a player k is given by

$$U_k^{\text{EUT}}(p) = \sum_{a \in \mathscr{A}} \left(\prod_{l=1}^{K} p_l(a_l) \right) u_k(a_k, a_{-k}), \tag{5.4}$$

where a is the vector of all players' strategies and $\mathscr{A} = \mathscr{A}_1 \times \mathscr{A}_2 \times \cdots \times \mathscr{A}_K$.

5.2.3 Prospect Theory

As previously mentioned, EUT evaluates an objective expected utility in which users are assumed to act rationally and objectively. However, it has been observed that, in real life, users' behavior deviates considerably from the rational path predicted by EUT. For the proposed game, a customer k has to decide on its action, in the face of uncertainty induced by the mixed strategies of its opponents, which impact directly its utility as in (5.4). In order to capture such behavioral factors in the proposed energy trading game, we turn to the framework of prospect theory [58].

As in Section 2.3, we have defined a *weighting effect* in (2.1) as

$$w(\sigma) = \exp(-(-\ln\sigma)^{\alpha}),\ 0 < \alpha \leq 1. \tag{5.5}$$

Under PT, the expected utility achieved by a player k, given the weighting effect, is

$$U_k^{\mathrm{PT}}(p) = \sum_{a\in\mathscr{A}} \left(p_k(a_k) \prod_{l\in\mathscr{K}\setminus\{k\}}^{K} w(p_l(a_l)) \right) u_k(a_k, a_{-k}). \tag{5.6}$$

Here, we assume that a player uses a subjective evaluation only on the other players' strategy probabilities. Thus, customer k's subjective evaluation of its own probability is equal to its objective probability. Given the set of probability distributions $\mathscr{P}_k$ over its set of strategies $\mathscr{A}_k$, the solution of the game can be found via the notion of a mixed-strategy Nash equilibrium:

Definition 3 *A mixed strategy profile $p^* \in \mathscr{P} = \prod_{k=1}^{K} P_k$ is a mixed strategy Nash equilibrium if, for each player $k \in \{1, 2, \ldots, K\}$, we have (for either PT or EUT)*

$$U_k(p_k^*, p_{-k}^*) \geq U_k(p_k, p_{-k}^*),\ \forall p_k \in \mathscr{P}_k. \tag{5.7}$$

5.3 Solution: The Two-Player Case

To gain greater insight into the solution of the proposed game, we analyze a case study for the scenario in which only $K = 2$ customers are active. In particular, we are interested in analyzing the *proper mixed Nash equilibrium* of the game. A proper mixed-strategy Nash equilibrium is the solution in which each player chooses a certain action a_k with probability $0 < p_k < 1$. While the existence of a mixed-strategy Nash equilibrium is well-known for conventional EUT games [107], it is of interest to study whether the PT game admits such an equilibrium. Moreover, for both the EUT and PT games, we are interested in guaranteeing a *proper* mixed strategy Nash equilibrium, in which the users will indeed mix between their strategies. With this in mind, we can state the following result:

Theorem 3 *For the proposed two-player smart grid game* $\Xi = (\mathscr{K}, \{\mathscr{A}_k\}_{k\in\mathscr{K}}, \{u_k\}_{k\in\mathscr{K}})$, *there exists a unique, proper mixed Nash equilibrium for both the EUT and PT games if* $-c(D_k, D_{-k})D_k + \beta(D_k+S_k)^2 < b_k S_k < -c(D_k, S_{-k})D_k + \beta(D_k+S_k)^2 + 2\beta\prod_{l=1}^{2}(D_l + S_l)$, *where* $k = \{1, 2\}$.

Proof: In the proposed model, there always exists at least one mixed NE under EUT as guaranteed by Nash's result [107]. Thus, our proof mainly focus on finding a condition to guarantee 1) *there exists a* proper *mixed NE under EUT and PT, and* 2) *such a proper mixed NE is unique. By using the indifference principle under EUT, a proper mixed-strategy Nash equilibrium,* (p_1^*, p_2^*), *exists when the average charging utility is equal to the average discharging utility. For example, computing customer* 1*'s average utility by* p_2^*, *we have* $p_2^* u_1(D_1, D_2) + (1-p_2^*)u_1(D_1, S_2) = p_2^* u_1(S_1, D_2) + (1 - p_2^*)u_1(S_1, S_2)$; *that is,*

$$p_2^* = \frac{u_1(S_1,S_2) - u_1(D_1,S_2)}{u_1(D_1,D_2) - u_1(S_1,D_2) + u_1(S_1,S_2) - u_1(D_1,S_2)}. \quad (5.8)$$

A sufficient condition to have a proper mixed strategy Nash equilibrium, such that $0 < p_2^* < 1$, *is to have:*

$$\text{sgn}\bigg(u_1(S_1,S_2) - u_1(D_1,S_2)\bigg) = \text{sgn}\bigg(u_1(D_1,D_2) - u_1(S_1,D_2)\bigg), \quad (5.9)$$

where $\mathrm{sgn}(\cdot)$ *denotes the sign of a function and*

$$\begin{cases} u_1(D_1,D_2) = -c_{11}(D_1+L_1(D_1,D_2)), \\ u_1(D_1,S_2) = -c_{12}(D_1+L_1(D_1,S_2)) - \beta(G(D_1,S_2)-\hat{G})^2, \\ u_1(S_1,D_2) = b_1 S_1 - \beta(G(S_1,D_2)-\hat{G})^2, \\ u_1(S_1,S_2) = b_1 S_1 - \beta(G(S_1,S_2)-\hat{G})^2. \end{cases} \tag{5.10}$$

On the other hand, we assume that player i's subjective evaluation of its own probability is equal to its objective probability, such that $w_1(p_1) = p_1, w_2(p_2) = p_2$*. Then, using the indifference principle under PT, the Player* 1*'s average utility of charging* $w_1(p_2^*)u_1(D_1,D_2) + w_1(1-p_2^*)u_1(D_1,S_2)$ *is equal to its average discharging utility* $w_1(p_2^*)u_1(S_1,D_2) + w_1(1-p_2^*)u_1(S_1,S_2)$*; that is,*

$$\frac{w_1(p_2^*)}{w_1(1-p_2^*)} = \frac{u_1(S_1,S_2)-u_1(D_1,S_2)}{u_1(D_1,D_2)-u_1(S_1,D_2)} > 0, \tag{5.11}$$

as which is analogous to the condition (5.9) under EUT. Computing player 2*'s average utility by* p_1^**, we also have the condition*

$$\mathrm{sgn}\Big(u_2(S_1,S_2)-u_2(S_1,D_2)\Big) = \mathrm{sgn}\Big(u_2(D_1,D_2)-u_2(D_1,S_2)\Big).$$

To solve (5.9), we need to simply

$$\begin{aligned} & u_1(D_1,D_2)-u_1(S_1,D_2) \\ =& -c_{11}(D_1+L_1(D_1,D_2)) - b_1S_1 + \beta(G(S_1,D_2)-\hat{G})^2, \\ =& -c_{11}(D_1+L_1(D_1,D_2)) - b_1S_1 + \beta\{[D_2 - S_1 + D_{others} \\ & + L(S_1,D_2)] - [D_1+D_2+D_{others}+L(D_1,D_2)]\}^2, \\ =& -c_{11}D_1 - b_1S_1 + \beta(D_1+S_1)^2, \end{aligned} \tag{5.12}$$

where D_{others} *represents the total constant demand of non-participating users. Here, we assumed that the losses* $L(\cdot)$ *are negligible with respect to the demand, which is a reasonable assumption when dealing with two players only, i.e.,* $L_k << D_k,\ k = 1,2$*. Similarly,*

$$\begin{aligned} & u_1(S_1,S_2)-u_1(D_1,S_2) \\ =& b_1S_1 - \beta(D_1+S_1+D_2+S_2)^2 + c_{12}D_1 + \beta(D_2+S_2)^2, \\ =& c_{12}D_1 + b_1S_1 - \beta(D_1+S_1)^2 - 2\beta(D_1+S_1)(D_2+S_2). \end{aligned} \tag{5.13}$$

If (5.12) is greater than 0, (5.13) cannot be greater than 0 due to the fact that, in practice, as the LMP price increases with the generated power, the price at a lower generation level cannot exceed that charged at a higher level, thus, mathematically, $c_{12} \leq c_{11}$. *Thus, both sides of (5.9) have to be negative and then, we obtain the range of* $b_k S_k$ *in Theorem 3.*

Under given loads and surpluses, Theorem 3 provides a relationship between the unit selling price b_k of each player, the LMP price $c(a_1, a_2)$, and the penalty factor for regulation β, such that we could obtain a proper mixed strategy equilibrium. From the utility functions in (5.4) and (5.6), we can mathematically see the difference between EUT and PT. Here, given the players' mixed strategies, we define the company's expected revenues under the equilibrium probabilities, (p_1^*, p_2^*) for EUT and $(p_1^{*,\mathrm{PT}}, p_2^{*,\mathrm{PT}})$ for PT. The power company generates a revenue depending on the energy sold to the two customers, although the customers' probability of charging or discharging can be different between EUT and PT. Thus, the power company revenues obtained from customers 1 and 2 are as follows:

$$\begin{aligned} R_{EUT} =& p_1^* p_2^* c_{11}(D_1 + D_2 + L_{1,2}) + p_1^*(1 - p_2^*) c_{12}(D_1 + L_1) \\ &+ (1 - p_1^*) p_2^* c_{21}(D_2 + L_2), \\ R_{PT} =& p_1^{*,\mathrm{PT}} p_2^{*,\mathrm{PT}} c_{11}(D_1 + D_2 + L_{1,2}) + p_1^{*,\mathrm{PT}}(1 - p_2^{*,\mathrm{PT}}) c_{12} \\ &(D_1 + L_1) + (1 - p_1^{*,\mathrm{PT}}) p_2^{*,\mathrm{PT}} c_{21}(D_2 + L_2), \end{aligned} \tag{5.14}$$

where $L(\cdot)$ is the loss in power flow (5.1). R_{EUT} is the expected revenue obtained by the power company. And R_{PT} is the PT revenue obtained by the power company, in which player 1 and player 2 use their subjective perspectives.

5.4 Simulation Results and Analysis

For simulating the proposed system, we consider a geographical region in which two active customers equipped with storage units exist. We choose typical values for the demand and surplus: $D_1 = 20$ kWh, $D_2 = 15$ kWh, $S_1 = 10$ kWh, $S_2 = 5$ kWh, $\alpha = 0.25, \beta = 0.0018$. The constant load is set as 200 kWh, and power line parameters are set from a typical 4-bus system [116]. The following examples assume that the generation power (kW) is numerically equal to the energy (kWh) in a one-hour time unit. For pricing, we assume that $c(a_1, a_2)$ follows a conventional LMP scheme, such as the following:

$$c = \begin{cases} \$0.05/\text{kWh} & \text{power company generation is} \leq 200 \text{ kWh}, \\ \$0.10/\text{kWh} & \text{power company generation between } 200-250 \text{ kWh}, \\ \$0.15/\text{kWh} & \text{power company generation between } 250-300 \text{ kWh}, \\ \$0.20/\text{kWh} & \text{power company generation is} > 300 \text{ kWh}. \end{cases}$$

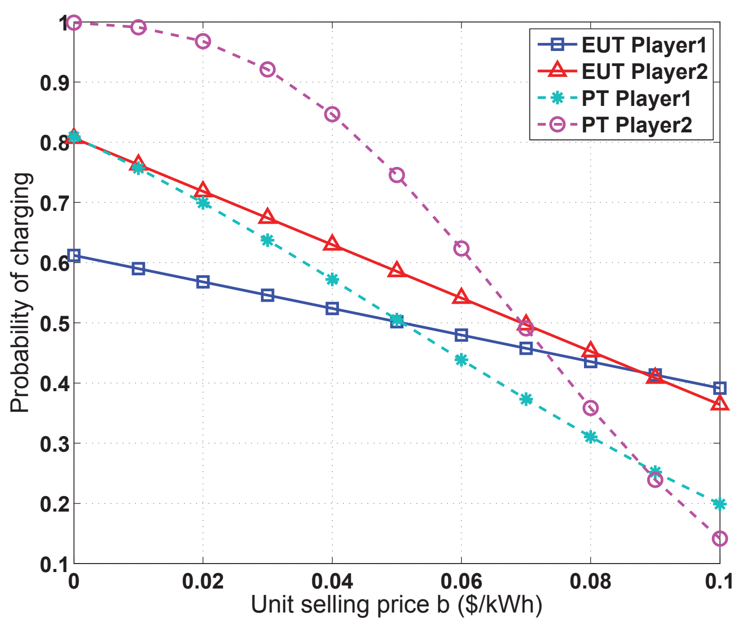

Figure 5.1: Probability of charging under EUT and PT as b varies.

In Fig. 5.1, we depict the impact of the unit selling price on the behavior of the customers. Without loss of generality, we assume that both customers use the same price $b_1 = b_2 = b$ and we vary the price within the range in which the equilibrium exist as per Theorem 3. Fig. 5.1 shows how the probability of charging for both players varies as b increases, for both the EUT and PT cases. Clearly, as the selling price increases, both players would have more incentive to discharge than to charge, as the benefit would start outweighing the regulation penalty. More interestingly, Fig. 5.1 shows that, for both customers, the PT behavior significantly differs from the EUT behavior. For example, for customer 2, below a selling price of $b = \$0.07$ per kWh, the probability of charging at the equilibrium for PT is much higher than EUT. This implies that for low gains, each customer follows a more *conservative, risk-averse* strategy under PT and is less interested in reaping the benefits of selling energy than in the EUT case. However, as the selling price crosses the threshold, the probability of charging for customer 2 under PT becomes much smaller than under EUT. This implies that once the selling benefits are significant (and the risks decrease), customer 2 starts selling more aggressively under PT than under EUT. A similar behavior can be observed for customer 1, although the benefit threshold of customer 1 is smaller ($b = \$0.05$), since customer 1 has more energy to sell/buy.

In Fig. 5.2, we show that the expected load on the grid significantly differs between PT and EUT. For PT, when the unit price for buying energy is small, the customers are less interested (compared to EUT) in selling energy now. However, as the unit price crosses a threshold, the customers will sell more aggressively and, thus, the overall load on the grid will be smaller than expected. Fig. 5.2 can provide important guidelines for demand-side management in the smart grid. For example, assume the power company wants to increase its price to drive customers to sell more and reduce their average load to about 10 kWh while keeping the generation regulation within limits. Based on EUT, the company would have to increase the minimum LMP price to roughly \$0.077 per kWh. In reality, because users behave subjectively when faced with risk, the company does not need to introduce such a high price increase. In contrast, it can increase it to about \$0.06 per kWh and obtain the desired load reduction. On the other hand, if the company wants to reduce its price to sustain up to 23 kWh of load (from the two customers in question), based on EUT, it would have to offer a relatively low price of \$0.035 per kWh. In contrast, based on PT, a price of about \$0.047 per kWh can achieve the same impact yet yield more profits. Clearly, ignoring the fact that users' behavior can deviate from the rational EUT path can yield undesirable loads on the grid which further motivates the need for PT analysis.

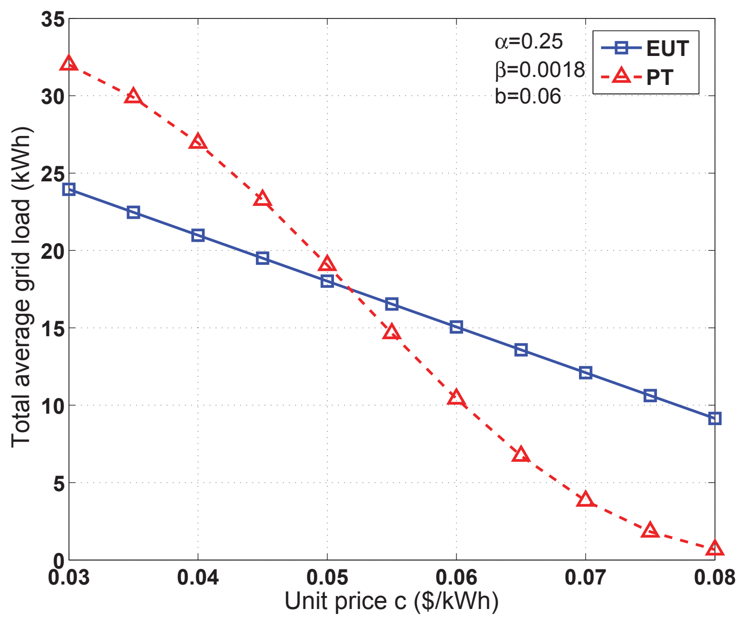

Figure 5.2: The total average grid load for participating customers under EUT and PT as the power company's unit price c varies.

In Fig. 5.3, we show how the power company revenues under EUT and PT vary as the regulation parameter β increases. In particular, we vary β from 0.0014 to 0.0024 while satisfying the existence of a proper mixed Nash equilibrium. First, the solid

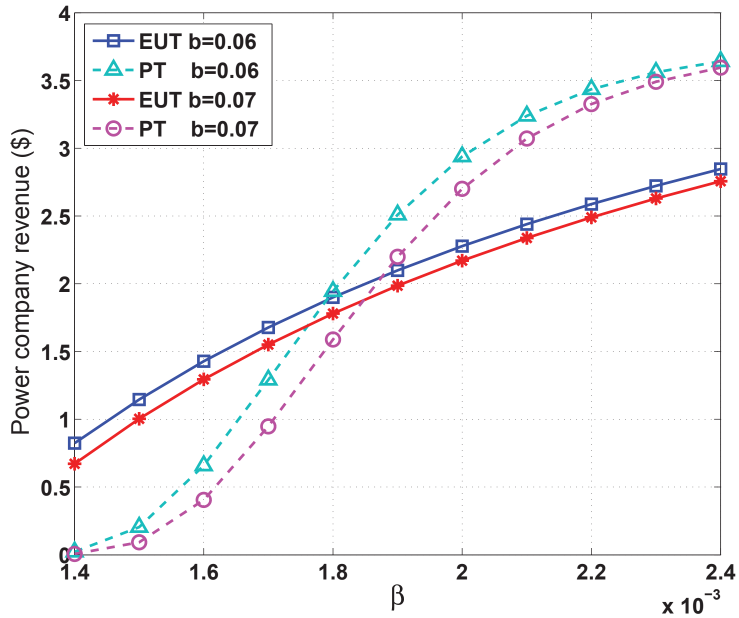

Figure 5.3: Total revenue for the power company under EUT and PT as the regulation parameter β varies.

lines show that the revenue under EUT is concave. This is due to the fact that the objective probability of charging is computed from a nonlinear utility that integrates power regulation. As the parameter β increases, both players want to store/charge more, since discharging increases the penalty of power regulation. Also, we can see that, after the crossing point (i.e $\beta = 0.0018$ when $b = 0.06$), the power company would obtain a high revenue from the PT actions of players. This is because players are more likely to charge (act more conservatively) at a high β comparable to their objective action. Thus, the power company must choose an optimal β while balancing the tradeoff between its own revenues and effective customer participation via discharging.

5.5 Conclusions

In this chapter, we have introduced a novel approach for studying the problem of customer-owned energy storage integration in the smart grid. We have developed a novel game-theoretic approach, based on prospect theory, using which each player subjectively observes and determines its actions so as to optimize a utility function that captures the benefit from selling energy as well as the associated regulation penalty. For the two-player scenario, we have shown the existence of an equilibrium for both EUT and PT. Simulation results have shown that prospect theory enables the power company to better decide on its pricing parameters, given realistic behavior of the users which deviate considerably from conventional EUT behavior. This chapter mainly scratches the weighting effect of prospect theory, which is expected to become a key technique in the design and analysis of a user-centric smart grid.

Chapter 6

Framing in Grid Energy Storage Management

In this chapter, we further apply prospect theory to investigate the impact of the framing effect on storage management. In particular, we extend the model of Section 5 to allow the customers to subjectively evaluate their utility in the charging/discharging game.

6.1 System Model and Problem Formulation

Consider the same model in Section 5. Fig. 6.1 provides an illustrative 4-bus example of the model considered. This figure depicts a possible micro-grid in local area including one generator, one constant load and two potential consumers.

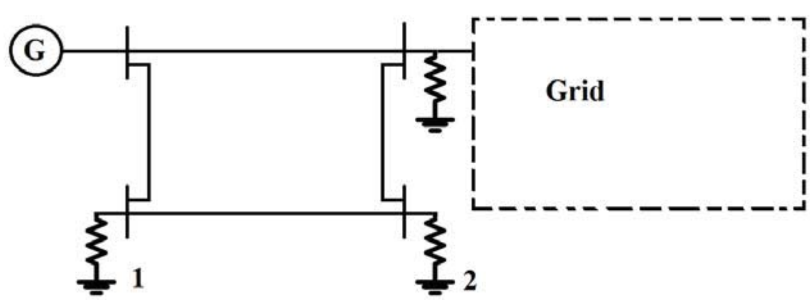

Figure 6.1: An illustrative example of the model studied.

6.1.1 Problem Formulation

We analyze the competitions and interactions between the active customers using non-cooperative game theory [107]. In particular, the utility function achieved by a player owning a storage unit, $k \in \mathscr{K}$, when choosing a certain action a_k is given by:

$$\begin{aligned} u_k(a_k, a_{-k}) = & -\eta_d(a_k, a_{-k})\Big(D_k + L_k(a_k, a_{-k})\Big) \\ & + \eta_s(a_k, a_{-k})S_k - \tau\Big(G(a_k, a_{-k}) - \hat{G}\Big)^2, \end{aligned} \tag{6.1}$$

where $a_{-k} = [a_1, a_2, \ldots, a_{k-1}, a_{k+1}, \ldots, a_K]$ is the vector of strategies chosen by the opponents of storage unit k, $L_k(a_k, a_{-k})$ denotes the transmission losses over the power lines which rely on the total demand and can be computed using optimal power flow algorithms [114], $G(a_k, a_{-k})$ represents the total supply by the power company under current discharging/charging operations, and τ is the penalty factor for power regulation. In particular, we define the charging (demand) price and the discharging (supply) price, respectively set by the power company and participating customers:

$$\eta_d(a_k, a_{-k}) = \begin{cases} c(a_k, a_{-k}) & \text{if } a_k = D_k, \\ 0 & \text{otherwise,} \end{cases} \tag{6.2}$$

where $c(a_k, a_{-k})$ is the unit of electricity price in the energy market that announced by the power company. Moreover,

$$\eta_s(a_k, a_{-k}) = \begin{cases} b_k & \text{if } a_k = S_k, \\ 0 & \text{otherwise,} \end{cases} \tag{6.3}$$

where b_k is the unit of electricity price set by a given customer k for selling its available surplus S_k. In (6.2) and (6.3), each customer can either bid a buying price or announce a selling price. Here, we note that the utility in (6.1) also depends on the power requirements from the "inactive" customers in $\mathscr{N} \setminus \mathscr{K}$, i.e., the constant loads and transmission losses, however, this dependence is dropped for notational convenience.

The utility function in (6.1) captures both the monetary benefits of the operation of storage units and the underlying impact on supply-demand relationship. For instance, when the power company allows the K active customers to charge or discharge energy, it also requires the generation in the considered geographical area to be maintained within a desired operation range. In this case, both the demand and line loss determine the total generation level and, excessive charging or discharging might damage the generator due to a frequency variation thus requiring power regulation [115]. Without loss of generality, a normal, suitable condition of operation range corresponds to the case that, all N customers act as a load and the active customers use the same unit price in energy market as its charging/discharging reference. For the operating condition, let $\hat{G} = \sum_{i\in\mathcal{N}}(D_i + L_i)$ be the total generation to meet the power requirement under the normal operation. L_i is the transmission losses incurred over power lines for delivering D_i to customer i. Thus, for the normal case under $a_k = D_k, \forall k \in \mathcal{K}$, we have $G(a) = \hat{G}$, where a is the vector of all strategies. Consequently, any actions taken by an active customer k will shifts the generated power from its nominal value $\hat{G}$ and, the power company yielding needs to regulate the generation via charge a cost penalty on the active customers as captured in (6.1).

6.1.2 Expected Utility Theory

Due to the variation in power generation and demand, it is reasonable to assume that customers can make probabilistic choices over their inherent decisions, i.e., charging/discharging operations. In particular, the probability set can represent the frequency over the finite number of action choices, using which a customer charges or discharges energy. Moreover, by analyzing the frequency with which a customer will discharge or charge energy, one can better understand how such operations will occur over a large period of time. Therefore, we mainly study the proposed game under *mixed strate-*

gies [107]. In this proposed game, an active customer/participant can both obtain an individual benefit from the operation of storages and pay back to the power company in maintaining the supply-demand relationship by selling/discharging energy to the grid. Then, a mixed, probabilistic choices in such game can *1)* reflect the frequency of charging/discharging operation, and *2)* represent the possibility of user choice in operating the storage units, over a pre-determined time horizon. In this respect, we let $p = [p_1, \ldots, p_k]$ denote the vector of all mixed strategies, while $p_k(a_k), \forall k \in \mathscr{K}$ is the probability distribution over customer k's pure strategies $a_k \in \mathscr{A}_k$.

In this game, each customer will objectively choose a mixed strategy vector so as to optimize its respective utility in (6.1). Under the conventional expected utility theory (EUT), each player aims at maximizing its expected value by combining the mixed strategies and respective utilities. In this respect, for any player $k \in \mathscr{K}$, its EUT utility is given by:

$$U_k^{\text{EUT}}(p) = \sum_{a \in \mathscr{A}} \left(\prod_{l=1}^{K} p_l(a_l) \right) u_k(a_k, a_{-k}), \tag{6.4}$$

where a is the vector of all players' strategies and the strategy set $\mathscr{A} = \mathscr{A}_1 \times \mathscr{A}_2 \times \cdots \times \mathscr{A}_K$.

6.1.3 Prospect Theory

In a classical noncooperative game, a player evaluates an objective expected utility in (6.4). However, such evaluation may not truly reflect the exact way in which a customer views its own utility from a given action. For example, if a decision to charge or discharge leads to a benefit of \$100, this \$100 amount may be viewed differently by a wealthy customer as opposed to a poor customer. Indeed, a wealthy customer may view this savings as irrelevant and thus not worth the effort, while a poor customer may in fact be motivated to adopt a strategy that can yield such a reduction in costs. Clearly, in practice, customers might make subjective decisions due to their individual viewpoint on the utility/payoff. This process implies that, based on a weighted payoff consisting of both gains and losses such as in (6.4), the customers will act differently depending on their subjective evaluation of their objective utility. Thus, the difference between such subjective decisions and classical, objective game-theoretic constructs require one to develop a novel framework for analyzing the problem of energy charging and discharging in the smart grid.

To this end, many empirical studies [61, 65, 66] have shown that subjective utility evaluations can indeed change the outcomes of a noncooperative game. In general, a player will have its real-life evaluation of its utility based on a reference level which will change the "raw" utility value into a subjective one measured with respect to that reference. When operating customer-owned storage units, the decision made by a customer on whether to charge or discharge energy depends on customers' perception on the utility (costs and benefits) achieved from such a decision. In the decision-making process, a customer has a reference value, or criterion, that will impact its utility and thus its charging/discharging decision. Clearly, to account for a reference point, the overall computation of the utility will differ from the EUT case in (6.1).

As shown in Section 2.4, we have defined a *framing effect* in (6.5) as

$$u_k^{\mathrm{PT}}(a) = \begin{cases} \left(u_k(a) - u_k^0(a^0)\right)^{\alpha_k} & \text{if } u_k(a) \geq u_k^0(a^0), \\ -\gamma_k\left(u_k^0(a^0) - u_k(a)\right)^{\beta_k} & \text{otherwise.} \end{cases} \tag{6.5}$$

Compared to the EUT utility function in (6.4), the expected utility under PT framing is:

$$U_k^{\mathrm{PT}}(p) = \sum_{a\in\mathscr{A}} \left(\prod_{l=1}^{K} p_l(a_l)\right) u_k^{\mathrm{PT}}(a_k, a_{-k}). \tag{6.6}$$

Given the each player's mixed strategy set over its action set $\mathscr{A}_k$, we next define the concept of a *mixed-strategy Nash equilibrium* (NE), as the game-theoretic solution under both EUT and PT:

Definition 4 *A mixed strategy profile p^* is said to be a* mixed strategy Nash equilibrium *if, for each player $k \in \mathscr{K}$, we have:*

$$U_k(p_k^*, p_{-k}^*) \geq U_k(p_k, p_{-k}^*),\ \forall p_k \in \mathscr{P}_k, \tag{6.7}$$

where $\mathscr{P}_k$ is the mixed strategy set of all available strategies observed by player k. In this respect, the mixed-strategy Nash equilibrium notion in (6.7) is applicable for both EUT and PT, the difference would be in whether one is using (6.4) or (6.6), respectively.

6.2 Game Solution

In this section, we first show that the existence of a mixed-strategy Nash equilibrium, which is known to hold for classical EUT games, will also hold for the proposed PT game. Then, we study some additional properties of the proposed game.

Lemma 3 *For the proposed charging/discharging game, there exists at least one mixed strategy Nash equilibrium for both PT and EUT.*

Proof: In the proposed game, a player will follow an EUT strategy using (6.1) and (6.4), while it has a PT-based behavior using (6.1), (6.5) and (6.6). Since the PT utility of a pure strategy only changes underlying EUT value, there exists at least one mixed NE in the PT game, as well as its existence in EUT.

Based on the existence of a mixed-strategy Nash equilibrium, we first prove a property for customer sensitivity using the aversion parameter γ_k.

Theorem 4 *For the proposed charging/discharging game, there exists a threshold γ_0, such that, when $\gamma_k < \gamma_0$, $U_k^{PT} > U_k^{EUT}$, and when $\gamma_k > \gamma_0$, $U_k^{PT} < U_k^{EUT}$.*

Proof: In the proposed game, the utility derivative on γ can be obtain by (6.4) and (6.6):

$$\begin{aligned}\frac{\partial U_k^{EUT}}{\partial \gamma_k} =&0,\\ \frac{\partial U_k^{PT}}{\partial \gamma_k} =&\frac{\partial U_k^{PT}(u_k > u_k^0)}{\partial \gamma_k} + \frac{\partial U_k^{PT}(u_k < u_k^0)}{\partial \gamma_k}\\ =&0 - \left(u_k^0(a^0) - u_k(a)\right)^{\beta_k}\\ <&0.\end{aligned} \tag{6.8}$$

At a mixed NE p^{EUT*}, the rational utility will be a constant value. For PT cases, we can obtain the expected utility via p^{PT*} and U_k^{PT} is a strictly decreasing function as γ increases. Then, U_k^{PT} and U_k^{EUT} will intersect at a point. In particular, we can computer γ_0 using the parameters at the intersected point (i.e., $\alpha, \beta, U_k^{EUT}, u_k^0$). Hence, when $\gamma_k < \gamma_0$, $U_k^{PT} > U_k^{EUT}$, and when $\gamma_k > \gamma_0$, $U_k^{PT} < U_k^{EUT}$. This conclusion implies that, a small/large γ will decrease/increase the loss evaluation, and then increase/decrease the expected value under PT.

Due to the framing effect in (6.6), most PT games will have a different mixed NE than its EUT case. The next theory thus studies a specific case when EUT and PT solutions are equal. In particular, we are interested in analyzing the *proper mixed Nash equilibrium* of the game. A proper mixed-strategy Nash equilibrium is the solution in which each player chooses a certain action a_k with probability limitation $0 < p_k < 1$. With this in mind, we can state the following result:

Theorem 5 *Consider the case in which for a customer, we only consider the impact of the reference point and we set $\alpha_k = \beta_k = \gamma_k = 1$. For a two-player smart grid game $\Xi = [\mathscr{K}, \{\mathscr{A}_k\}_{k\in\mathscr{K}}, \{u_k\}_{k\in\mathscr{K}}], k \in \{1,2\}$, if customer k chooses a reference point outside of its EUT range $[u_{k,\min}^{EUT}, u_{k,\max}^{EUT}]$, its proper mixed NE under PT will be equal to the EUT result.*

Proof: For a two-player game, we can use the indifference principle to solve for the NE under both EUT and PT. A proper mixed-strategy Nash equilibrium, (p_1^*, p_2^*)*, exists when the average charging utility is equal to the average discharging utility. For example, computing PT customer* 1*'s average utility by* p_2^**, we have* $p_2^{PT*}u_1^{PT}(D_1,D_2)+(1-p_2^{PT*})u_1^{PT}(D_1,S_2)=p_2^{PT*}u_1^{PT}(S_1,D_2)+(1-p_2^{PT*})u_1^{PT}(S_1,S_2)$*; that is,*

$$p_2^{PT*}=\frac{u_1^{PT}(S_1,S_2)-u_1^{PT}(D_1,S_2)}{u_1^{PT}(D_1,D_2)-u_1^{PT}(S_1,D_2)+u_1^{PT}(S_1,S_2)-u_1^{PT}(D_1,S_2)}, \tag{6.9}$$

where $u_1^{PT}(S_1,S_2)\le u_1^{PT}(D_1,S_2)$ *and* $u_1^{PT}(D_1,D_2)\le u_1^{PT}(S_1,D_2)$ *in the proposed game. When* $\alpha_k=\beta_k=\gamma_k=1$ *and* u_k^0 *is out of its EUT range,* $u_k^0\in(-\infty,u_{k,\min}^{EUT}]\bigcup[u_{k,\max}^{EUT},+\infty)$*, we have*

$$\begin{aligned}&u_1^{PT}(S_1,S_2)-u_1^{PT}(D_1,S_2)\\&=\begin{cases}\left(u_1^{EUT}(S_1,S_2)-u_1^0\right)-\left(u_1^{EUT}(D_1,S_2)-u_1^0\right), \text{ if } u_1^0\le u_{1,\min}^{EUT},\\ \left(u_1^0-u_1^{EUT}(S_1,S_2)\right)-\left(u_1^0-u_1^{EUT}(D_1,S_2)\right), \text{ if } u_1^0\ge u_{1,\max}^{EUT},\end{cases}\\&=u_1^{EUT}(S_1,S_2)-u_1^{EUT}(D_1,S_2),\end{aligned} \tag{6.10}$$

and similarly,

$$u_1^{PT}(D_1,D_2)-u_1^{PT}(S_1,D_2)=u_1^{EUT}(D_1,D_2)-u_1^{EUT}(S_1,D_2). \tag{6.11}$$

Also, at the mixed NE under EUT,

$$p_2^{EUT*}=\frac{u_1^{EUT}(S_1,S_2)-u_1^{EUT}(D_1,S_2)}{u_1^{EUT}(D_1,D_2)-u_1^{EUT}(S_1,D_2)+u_1^{EUT}(S_1,S_2)-u_1^{EUT}(D_1,S_2)}, \tag{6.12}$$

and thus, $p_2^{PT*}=p_2^{EUT*}$*. Similarly, we can obtain* $p_1^{PT*}=p_1^{EUT*}$ *if Customer* 2 *chooses an extreme reference point.*

In (6.4) and (6.6), we can mathematically see the difference between EUT and PT. However, if the customers choose an extreme reference point and do not distort utility values, PT result will be equal to EUT. Here, we remark that, solving most PT cases will obtain a different NE from EUT, unless under some specific conditions.

6.3 Simulation Results and Analysis

For simulating the proposed system, we study a geographical area consisting of two active customers equipped with storage units. In particular, we assume that each customer has fixed power for its charging and discharging operations, such that, $D_1 = 20$ kWh, $D_2 = 15$ kWh, $S_1 = 15$ kWh, $S_2 = 10$ kWh, $\tau = 0.0012$. Also, local constant load is assumed to set as 200 kWh, and other power line parameters follow a typical 4-bus system in [116]. Based on the total power generation, the power company will announce $c(a_1, a_2)$ as a conventional locational marginal pricing (LMP) [117] scheme, such as:

$$c = \begin{cases} \$0.03/\text{kWh} & \text{power company generation is} \leq 180 \text{ kWh}, \\ \$0.05/\text{kWh} & \text{power company generation between } 180-200 \text{ kWh}, \\ \$0.07/\text{kWh} & \text{power company generation between } 200-220 \text{ kWh}, \\ \$0.09/\text{kWh} & \text{power company generation between } 220-240 \text{ kWh}, \\ \$0.11/\text{kWh} & \text{power company generation is} > 240 \text{ kWh}. \end{cases}$$

Fig. 6.2 shows the charging strategies for the customers at both the EUT and PT equilibria and for different values of γ. In this figure, $b = [0.06\ 0.06]$, $\alpha = [0.7\ 0.7]^T$ and $\beta = [0.6\ 0.6]^T$. The reference point is chosen to coincide with the case in which customers discharge/sell energy at the same price that is announced by the power company. In particular, we can obtain the expected reference point from the pure utility in (6.1) and the probability production in (6.4), i.e., $u^0 = [-2.32\ -2.19]^T$. When $\gamma = 2$ for both customers, compared to EUT, both customers increase their charging probability under PT. To maximize the expected utility, customers evaluate their payoff based on the observation of a high-price utility reference (the PT reference price is \$0.03 while EUT reference price can be seen as \$0). Moreover, $\gamma > 1$ will render the customers averse to the losses (losses are larger than gains), and which will decrease the expected utility

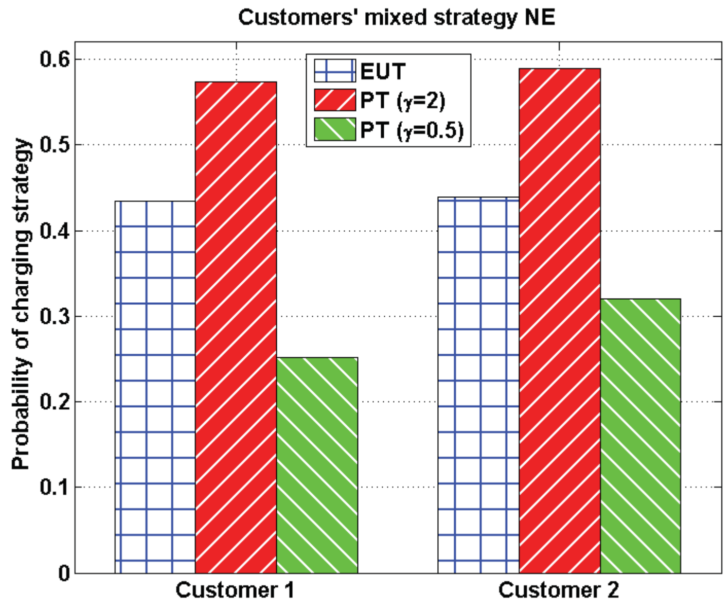

Figure 6.2: Customers' mixed-strategies at the equilibrium for both EUT and PT with $\alpha_i = 0.7, \beta_i = 0.6$.

for both customers. Thus, the customers will observe that, their PT opponents decrease their subjective PT utility and will increasingly take a conservative behavior, i.e., charging operation. In contrast, when $\gamma = 0.5$, PT evaluation of gains are larger than losses. Then, customers will observe an increasing PT utility from their opponents. Thus, they reduce the charging probability and adopt a more risk-seeking behavior.

Fig. 6.3 shows the total expected utility (sum for both customers) under both EUT and PT, as the reference point varies. In this figure, we maintain $\alpha = 0.7, \beta = 0.6$ so as to study the impact of $u_1^0 = u_2^0 = u^0$. First, we can see that, the expected PT utility will decrease when the reference point value increases. In essence, the reference point is subtracted from the EUT utility in (6.5), and in practice, such reference is based on how a customer evaluates its payoff. For a high reference or electricity price, PT customers will value their stored energy more than in cases in which the reference point is smaller.

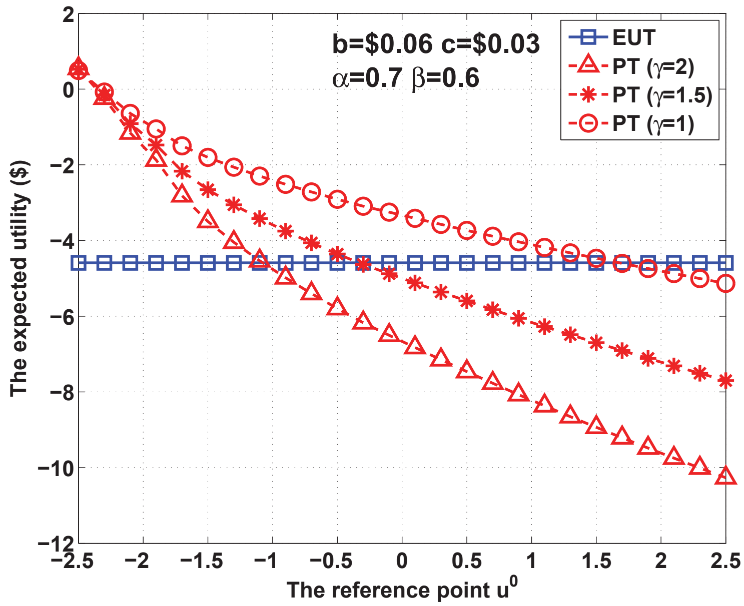

Figure 6.3: The total utility under both EUT and PT as the reference point u^0 varies.

Thus, using a same selling/discharging price under EUT and PT, the payoff sold by PT customers are smaller than EUT, due to the fact that, in PT, the reference point reduces the gains from selling energy. Indeed, the reference point changes the fraction of EUT value in (6.5) and then, changes the PT evaluation for each customer, which naturally translates the objective payoff into real-life utility value. Second, this figure also shows that, the framing aversion parameter, i.e., γ, would have different impacts on the PT utility. In particular, when γ increases from 1 to 2, the losses viewed by PT customers will increase due to (6.5). Thus, with an increasing γ, a PT customer will start valuing its gains less than in the EUT case, which leads to increasing its conservative, charging strategy.

In Fig. 6.4, we investigate the impact of the unit selling price on the behavior of the customers. Without loss of generality, we assume that both customers use a same price $b_1 = b_2 = b$ but different framing parameters $\alpha = [0.6\ 0.7]^T$, $\beta = [0.8\ 0.5]^T$ and

$\gamma = [0.9\ 1.1]^T$. Fig. 6.4 shows how the charging probabilities of both customers vary as b increases, for both the EUT and PT cases. Clearly, as the selling price increases, both customers would have increasing incentive to discharge than to charge. Moreover, Fig. 6.4 shows that, for both customers, the PT behavior significantly differs from the EUT behavior. On the one hand, for Customer 1, its PT charging probability is always less than its EUT charging probability, because of its opponent Customer 2's utility (each customer's utility depends on the action of others). For example, due to the Customer 2's framing parameters, the evaluation of its PT gains is always greater the e-valuation of losses in the selling price range. Then, Customer 2's PT utility increases in 6.5 and, Customer 1 decreases (increases) its PT charging (discharging) probability so as to take advantage from Customer 2's way of perceiving its utility. On the other hand, below a selling price of $b = \$0.04$ per kWh, Customer 2's probability of charging at the equilibrium for PT is smaller than EUT. Here, the framing parameters of Customer 1 do not yet provide a very beneficial situation for Customer 2. Then, we can see that, Customer 2 will decrease its PT charging probability when b is small, so as to compete with Customer 1. After $b = \$0.04$ per kWh, Customer 1's discharging benefit increases, however, its discharging penalty also increases due to the power regulation. Then, Customer 1's utility decreases in 6.5. This leads to a larger PT charging probability for Customer 2, compared to EUT, when b goes to a high price. In fact, there will exist a threshold for both customers if b is large enough, corresponding to Theorem 4.

In Fig. 6.5, we show that the expected load on the grid significantly differs between PT framing considerations and EUT results. First, as the selling price b increases, both PT and EUT customers are interested in discharging energy and then the total load will decrease. Moreover, when the unit selling price is small, PT load is less than EUT, while after crossing a threshold, i.e., \$0.07, PT load is more. Indeed, this result corresponds

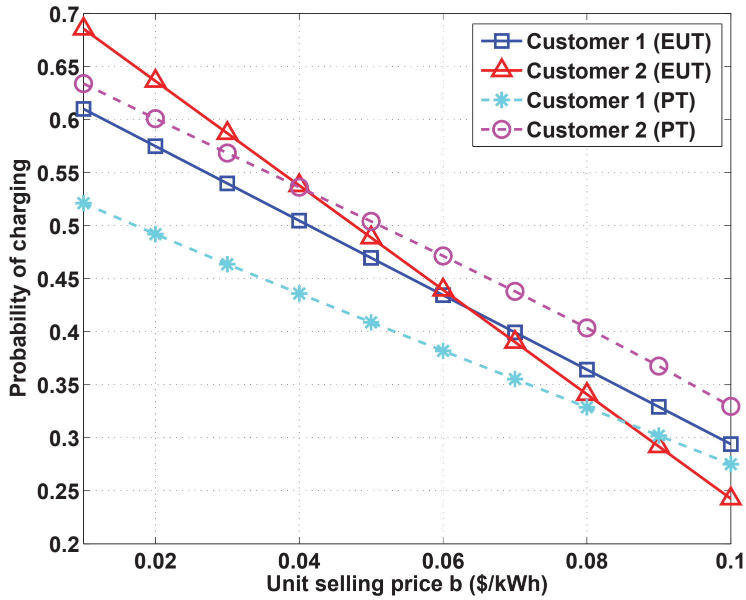

Figure 6.4: Probability of charging under both EUT and PT as the selling price b varies.

to the charging probability in Fig. 6.4, as the selling price increases. We note that, the reference point is a negative value due to the regulation in (6.1), as discussed in Fig. 6.2. For a small selling price, the impact of the regulation penalty is small. Thus, PT utility is larger than EUT for both customers. This implies that, a PT customer will decrease its charging behavior in response to its opponent's monetary increment in (6.5). Similarly, for a large selling price, the losses from the regulation are large and PT utility will be less than EUT (in this case, both PT and EUT utility are negative values). Thus, compared to EUT, PT customers will discharge less energy to reduce the payment. Fig. 6.5 can also provide important guidelines for a power company to control its local demands in the smart grid. For instance, based on EUT, if the power company wants to reduce the average load to about 8 kWh, it would allow the customers to increase the selling price to roughly \$0.05 per kWh. However, in reality, because customers frame their utilities in (6.5), they would concerns the impact of losses, or their payment from power regulation. Thus, customers sell energy at a low selling price, i.e., \$0.04 per kWh, to prevent from regulation, which has already controlled the total load via $\hat{G}$. On the other hand, if the company sustains down to 4 kWh of load, we can see that the customer will need to sell electricity around \$0.08 per kWh under EUT. In contrast, based on PT, customers observe a small utility from their opponents, and then, they will take the risk of selling electricity at a high price, i.e., \$0.09 per kWh.

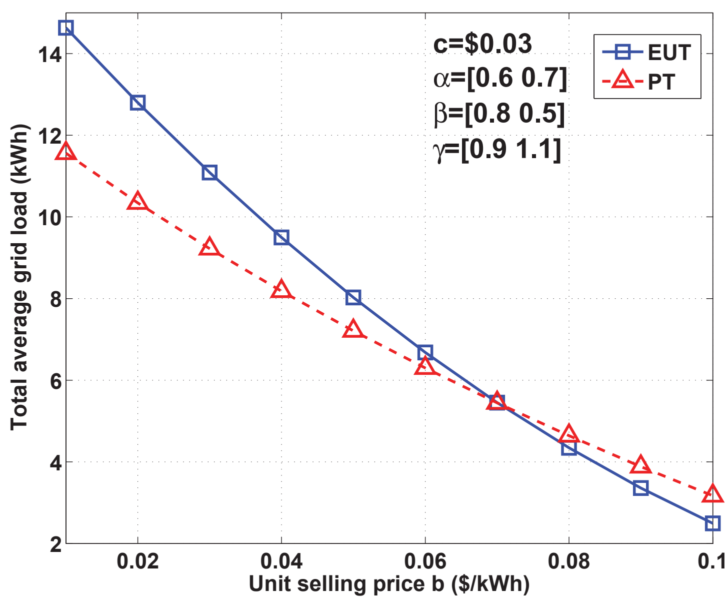

Figure 6.5: The total average grid load for participating customers under EUT and PT as b varies.

6.4 Conclusions

In this chapter, we have introduced a novel approach for studying the integration of charging/discharging operation equipped with the customer-owned energy storage in the smart grid. We have developed a behavioral framework, based on prospect theory, using which each player can frame its payoffs and can determine its actions so as to optimize a utility function that captures both benefit from energy selling and the associated regulation penalty. We have studied the properties of the Nash equilibrium of the game while accounting for the impact of utility framing. For the two-player scenari-o, our simulations have shown that prospect theory provides insightful results on how the customers' perceptions of their utility functions can affect the operation of energy storage units.

Chapter 7

The Application of Prospect Theory for Demand-Side Management

In this chapter, we study users' behaviors in the context of demand-side management. Compared to Chapter 5 and Chapter 6, this part in thesis mainly focuses on solving a multi-agent case in energy management, in which a load shifting mechanism can account for user decisions under the weighting effect of prospect theory.

7.1 System Model

Consider a smart grid consisting of a set $\mathscr{N}$ of customers and each customer $i \in \mathscr{N}$ consumes a certain amount of energy per hour. All customers are offered the opportunity to participate in a demand side management scheme provided by the utility company. For customer i, we define an hourly *energy consumption scheduling vector* in line with existing works such as [118],

$$x_i = [x_i^1, x_i^2, \ldots, x_i^H], \tag{7.1}$$

where $H = 24$ hours. For a certain hour $h \in \mathscr{H} = \{1, 2, \ldots, H\}$, a scalar x_i^h represents the energy consumption of customer i. The total energy demand from all customers at time h is thus

$$d^h = \sum_{i \in \mathscr{N}} x_i^h. \tag{7.2}$$

At a given time h, we assume that the price per energy unit charged to a customer i is dependent on its fraction of the total current load as follows:

$$c_i^h\left(x_{i\in\mathcal{N}}\right) = B \cdot \frac{x_i^h}{\sum_{i\in\mathcal{N}} x_i^h}, \tag{7.3}$$

where B could be designed as a typical locational marginal pricing (LMP) scheme [117]. Hence, the total cost of user i over H hours would be given by:

$$\sum_{h=1}^{H} c_i^h\left(x_{i\in\mathcal{N}}\right) \times x_i^h. \tag{7.4}$$

Given the price and local consumption, an energy market is set up in which all customers seek to minimize their costs while maintaining their energy consumption at a desired level. Here, all $N = |n|$ customers will interact so as to determine their demands under DSM. These demands include the quantities of energy required and impact the price at a certain time. Instead of fixed reservation prices announced by a utility company, in our model, each user can strategically change its demand, in which its demand fraction of the total requirement impacts the underlying electricity price. In this respect, the price in (7.3) might cause an increased payment required of a customer due to others behavior, even if the customer itself does not participate in the load management. Moreover, the demand delayed by DSM will cause a varying electricity price in subsequent hours. To analyze such a load management scheme, we next propose a new framework that builds on the analytical tools of classical game theory and prospect theory.

7.2 Proposed Demand-Side Management Game

In this section, we first formulate a noncooperative game between the customers, and then study the proposed load shifting model using expected utility theory and prospect theory, also discussing their various properties.

7.2.1 Noncooperative Game Model

In order to analyze the interactions between customers, we use noncooperative game theory [107], as the strategy choices of the customers are interdependent. We formulate a strategic noncooperative game $\Xi = (\mathcal{N}, \{\mathscr{A}_i\}_{i\in\mathcal{N}}, \{u_i\}_{i\in\mathcal{N}})$, where $\mathcal{N}$ is the set of players, the action $a_i \in \mathscr{A}_i := \{1, 2, \ldots, 24\}$ of each customer is the time (hour) at which a customer i would like to begin participation in DSM, and the cost function u_i of customer i captures the electricity payment to the company, given the current price. Here, we note that, although the customers participate in load management, the total individual consumption must remain the same in a day. Thus, the cost function achieved by a player $i \in \mathcal{N}$ that chooses an action a_i is given by

$$u_i(a_i, a_{-i}) = \sum_{h=1}^{H} c_i^h \left(\sum_{i\in\mathcal{N}} y_i^h(a_i, a_{-i}) \right) \times y_i^h(a_i, a_{-i}), \tag{7.5}$$

where $a_{-i} = [a_1, a_2, \ldots, a_{i-1}, a_{i+1}, \ldots, a_N]^T$ is the vector of action choices of all players other than i, and $y_i^h(a_i, a_{-i})$ is the DSM consumption of user i compared to the nonparticipating consumption x_i^h in (7.1).

The goal of each customer i is to choose a strategy $a_i \in A_i$ so as to minimize its payment as given in (7.5). For characterizing a desirable outcome for the studied game Ξ, one must derive a suitable solution for all N optimization problems that each customer must solve. Note, every vector of strategies a selected by the customers will yield a different electricity price $c_i^h(\cdot)$ in (7.3) and an extreme load shifting might cause an unexpected demand at another time slot. Thus, prior to finding a solution for the energy exchange game, we will first introduce a scheme for load management.

7.2.2 Load Management

In this subsection, we design a load shifting mechanism to analyze local demand over hours. Inherently, load shifting allows part of the peak hour load to be moved to an off-peak hour, in which such controlled demand is delayed to the following time slot in our formulation. For the proposed game Ξ, we assume that, for each participating customer, the demand can be adjusted so as to meet a *predefined/target energy demand* distribution set by the utility company, given by

$$G_d = [g_d^1, g_d^2, \ldots, g_d^H]. \tag{7.6}$$

On the one hand, a utility company wants to reduce peak hour consumption in order to decrease load on the grid. On the other hand, the company has to avoid an extreme load shifting in order to retain the stability of the power system and to avoid creating a new peak hour. For example, if the power company wants to reduce 10% of the load at a given hour $\hat{H}$, the target energy demand could be defined by

$$g_d^h = \begin{cases} g^h, & \text{if } h \neq \hat{H}, \\ \beta g^h, & \text{if } h = \hat{H}, \end{cases} \tag{7.7}$$

where $\beta = 0.9$ and $g_h \in [g^1, g^2, \ldots, g^H]$ can be the *historical demand* referenced by the utility company. Moreover, customer i could choose its strategy a_i so as to minimize its payment by observing the difference between the predefined demand per user and its daily requirement. In particular, we assume that a customer i would not leave DSM after choosing to participate over 24 hours and thus, the number of participating customers at a given time h is $I^h \triangleq |\mathscr{I}^h| = |\{a_i \geq h\}|$. In this case, the reduced consumption of participating customer i is given by:

$$r_i^h = \begin{cases} \gamma_i (x_i^h - \frac{g_d^h - \sum_{i \in \mathscr{N} \setminus \mathscr{I}} x_i^h}{I^h})^+, & \text{if } g_d^h < l^h, \\ 0, & \text{if } g_d^h \geq l^h, \end{cases} \tag{7.8}$$

where $(q)^+ := \max(0, q)$ and $0 < \gamma_i \leq 1$ is a factor by which customer i wants to reduce from exceeding its requirement, as $\frac{g_d^h - \sum_{i \in \mathscr{N} \setminus \mathscr{I}} x_i^h}{I^h}$ is the averaged requirement suggested by the utility company for all participating customers. In particular, if the requirement of participating customer i is less than the averaged requirement for participating customers, i.e., $x_i^h < \frac{g_d^h - \sum_{i\in\mathcal{N}\setminus\mathcal{I}} x_i^h}{I^h}$, its reduced requirement is $r_i^h = \gamma_i \cdot 0 = 0$.

Using (7.8), if customer i participates in DSM at time h, its consumption will be

$$y_i^h = x_i^h - r_i^h. \tag{7.9}$$

Then, the participating customer i moves its shifted load to the following hour, and thus, the consumption it participated DSM at time $h < t < H$ is

$$y_i^t = (x_i^t + r_i^{t-1}) - r_i^t, \tag{7.10}$$

and the consumption at the final hour H is

$$y_i^H = x_i^H + r_i^{H-1}. \tag{7.11}$$

7.2.3 Expected Utility Theory

In a smart grid, as the customers may, over a long time period, change their DSM preferences, we are interested in studying the frequency with which they choose a certain time to begin DSM participation. Therefore, we mainly study the proposed game under *mixed strategies* [107] so as to capture the customers long-term, probabilistic choices of a DSM start time. Let $p = [p_1, \ldots, p_N]$ be the vector of all mixed strategies, where, for every customer $i \in \mathcal{N}$, we have $p_i = [p_i^1, \ldots, p_i^H]^T$ and $p_i(a_i)$ is the probability distribution over the pure strategies $a_i \in \mathcal{A}_i$.

Under the conventional EUT model, the cost of customer i is captured via the expected value over its mixed strategies. Computing each user's utility requires the vector of all players' strategies. In particular, we assume that the smart grid communication infrastructure will make this information available to users who participate in DSM. Thus, the EUT cost of a player i will be given by

$$U_i^{\text{EUT}}(p) = \sum_{a\in\mathcal{A}} \left(\prod_{l=1}^{N} p_l(a_l) \right) u_i(a_i, a_{-i}), \tag{7.12}$$

where a is the vector of all players' strategies and $\mathcal{A} = \mathcal{A}_1 \times \mathcal{A}_2 \times \cdots \times \mathcal{A}_N$.

7.2.4 Prospect Theory

As previously mentioned, a player can evaluate its expected utility by using (7.12), in which case the customers are assumed to act rationally and objectively under EUT. However, in real life, individuals do not truly behave rationally nor do they trust the rationality of others' behavior. Thus, in order to develop a realistic model of DSM, one must account for the fact that, in practice, customers may not assess their utilities objectively. Indeed, it has been shown that, despite the benefits of DSM, its adoption in practice has remained slow, due to unexpected customer behavior [119].

To study such realistic/practical participation models, we will develop a DSM game model in which a customer may act irrationally due to the uncertainty it perceives on other players' actions and, thus, on the energy supply-demand relationship. Indeed, it is observed that in real-life decision-making, people tend to subjectively weight uncertain outcomes [61]. For example, a customer cannot be sure how much demand other customers can reduce in practice, or whether others care about the electricity price in general. Thus, this customer's EUT evaluation as per (7.12) might *overweight* or *underweight* the mixed-strategy vectors of others. Moreover, this customer may need to properly assess whether to shift its demand or not, as it is unsure of other customers' strategies. In this respect, a customer might have its own subjective perception on the participation of other customers in the DSM game (and on the actual price), thus deviating from the rational assumption of conventional game theory and EUT.

In order to translate user uncertainty and risk into an observation on others' strategy, we will use the behavioral framework of prospect theory [58]. In this studied model, we mainly focus on how a customer evaluates the strategies of its opponents and, thus, acts accordingly.

One important PT concept is the so-called *weighting effect*. In the proposed game, we use this weighting effect to incorporate a subjective evaluation for each user's observation on the mixed strategy of its opponents. Thus, under PT, instead of objectively observing the mixed strategy vector p_{-i} chosen by the other players, each user perceives a weighted version of it, $w_i(p_{-i})$. $w_i(\cdot)$ is a nonlinear transformation that maps an objective probability to a subjective one. PT studies have shown that most people will often overweight low probability outcomes and underweight high probability outcomes [58]. Without loss of generality, we assume that all players use a similar weighting approach, such that $w_i(\cdot) = w(\cdot)$, $\forall i \in \mathscr{N}$. Although many weighting functions exist, we choose the widely used Prelec function (for a given probability σ) [61],

$$w(\sigma) = \exp(-(-\ln\sigma)^{\alpha}),\ 0 < \alpha \leq 1, \tag{7.13}$$

where α is a parameter used to characterize the distortion between subjective and objective probability. Note that when $\alpha = 1$, the weighting effect will coincide with the conventional EUT probability.

Under PT, the expected utility achieved by a player i, given the weighting effect, is

$$U_i^{\mathrm{PT}}(p) = \sum_{a \in \mathscr{A}} \left(p_i(a_i) \prod_{l \in \mathscr{N} \setminus \{i\}}^{N} w(p_l(a_l)) \right) u_i(a_i, a_{-i}). \tag{7.14}$$

Here, we assume that a player has a subjective evaluation only of the other players' strategy probabilities. In this respect, customer i's subjective evaluation of its own probability is equal to its objective probability.

Having defined the cost functions under both EUT and PT, our next goal is to find a solution for the game. Given the set of probability distributions $\mathscr{P}_i$ over its set of strategies $\mathscr{A}_i$, a suitable solution of the game would be the mixed-strategy Nash equilibrium defined as follows:

Definition 5 *A mixed strategy profile* $p^* \in \mathscr{P} = \prod_{i=1}^{N} \mathscr{P}_i$ *is a* mixed strategy Nash equilibrium *(NE) if, for each customer* $i \in \{1, 2, \ldots, N\}$*, we have (for either PT or EUT)*

$$U_i(p_i^*, p_{-i}^*) \leq U_i(p_i, p_{-i}^*),\ \forall p_i \in \mathscr{P}_i. \tag{7.15}$$

In practice, to avoid slow convergence times and unnecessary overhead, we might use the so-called approximate equilibrium solutions that allow the players to reach the neighborhood of an equilibrium. Hence, we mainly focus on the so-called ε-Nash equilibrium which is given by the following definition:

Definition 6 *An* ε-Nash equilibrium *is a mixed strategy profile* p^* *if, for every player i, we have*

$$U_i(p_i^*, p_{-i}^*) \leq U_i(p_i, p_{-i}^*) + \varepsilon,\ \forall p_i \in \mathscr{P}_i, \tag{7.16}$$

where ε *is a small positive number.*

7.3 Game Solution and Algorithm

In this section, we propose a novel algorithm, under both EUT and PT, to solve the studied DSM game and find an equilibrium point. The proposed algorithm builds on classical fictitious play (FP) [120]. In order to find the solution for the game Ξ under both EUT and PT, we find a mixed ε-NE in strategic form which represents a point within a close neighborhood of the exact equilibrium [107]. The difference between EUT in (7.12) and PT in (7.14) mainly relates to the weighting effect in (7.13), in which the weighting function maps the mixed strategies of customers to their underlying observations. In this respect, there must exist a mapping relationship between customer strategy and observation. Thus, we propose the following iterative algorithm to find an ε-Nash equilibrium of the proposed game:

$$p_i^{(k+1)} = p_i^{(k)} + \frac{\lambda}{k+1}(v_i^{(k)} - p_i^{(k)}), \tag{7.17}$$

where $0 < \lambda < 1$ is an inertia weight, k is the number of iterations, and the vector of player i's strategies $v_i^{(k)} = [v_i^{(k)}(a_1), v_i^{(k)}(a_2), \ldots, v_i^{(k)}(a_H)]^T$ holds $v_i^{(k)}(a_l) = 1$ if player

i chooses the lth strategy and $v_i^{(k)}(a_l) = 0$ for updating the strategies excluding the lth strategy. The pure strategy i.e., the lth strategy, is the one that minimizes the expected utility with respect to the updated empirical frequencies. Thus, player i can repeatedly choose its strategy and update $v_i^{(k)}$ as follows:

$$v_i^{(k)}(a_l) = \begin{cases} 1, \text{if } a_l^{(k)} = \arg\min_{a_i \in \mathscr{A}_i} u_i(a_i, p_{-i}^{(k-1)}), \\ 0, \text{otherwise}, \end{cases} \tag{7.18}$$

where the utility here is the expected value obtained by player i with respect to the mixed strategy of the opponent, when player i chooses pure strategy a_l for EUT and its weighted mixed strategy $w(p_l(a_l))$ for PT. It is well known that our algorithm (as a simplified iterative approach of smooth fictitious play (SFP)) is guaranteed to converge to a mixed ε-Nash equilibrium [121], as the players' beliefs (probabilistic choices of strategies) converge to a fixed point. SFP can reach an ε-NE under EUT, in which the belief difference ε_p between two iterations is small when the number of iterations k goes to infinity. However, to our knowledge, such a result has not been extended to PT. Here, we first prove that the proposed algorithm in (7.17) will converge to a fixed point in beliefs, as follows:

Theorem 6 *There exists an inertia weight λ, $0 < \lambda < 1$, such that, the iterative algorithm in (7.17) converges to a fixed point in belief, as a mixed ε_p-equilibrium.*

Proof: For the proposed DSM game, the proposed SFP process is guaranteed to converge to a fixed point in beliefs [121] under EUT. Here, we mainly prove that, under PT, a player i's iterative sequence $\{p_i(k)\}$ converges to a mixed ε_p equilibrium in beliefs.

From (7.17), we have

$$\begin{aligned} p_i^{(k+1)} =& (1-\frac{\lambda}{k+1})p_i^{(k)} + \frac{\lambda}{k+1}v_i^{(k)} \\ =& (1-\frac{1}{k+1})p_i^{(k)} + \frac{1}{k+1}v_i^{(k)} + \frac{1-\lambda}{k+1}p_i^{(k)} \\ & - \frac{1-\lambda}{k+1}v_i^{(k)}, \end{aligned} \tag{7.19}$$

where the first two terms represent the iteration using FP [120] and, we define

$$\varepsilon_p = \left| \frac{1-\lambda}{k+1}p_i^{(k)} - \frac{1-\lambda}{k+1}v_i^{(k)} \right|. \tag{7.20}$$

In (7.19) and (7.20), we present the belief difference between the proposed algorithm and FP, where there exists an ε_p difference at the kth iteration. In particular, the distance boundary between FP belief and the approached belief can be bounded as follows:

$$\varepsilon_p = \frac{1-\lambda}{k+1}|(p_i^{(k)} - v_i^{(k)})| \leq \frac{1-\lambda}{k+1} \cdot \sqrt{2}, \tag{7.21}$$

where ε_p is a small value as k increases. Indeed, the Euclidean distance in (7.21) involves $v_i^{(k)}$ in (7.18), one of whose components is 1, and the mixed strategy set $p_i^{(k)}$, whose components will sum to 1. Thus, the maximum Euclidean distance between $p_i^{(k)}$ and $v_i^{(k)}$ is less than $\frac{\sqrt{2}}{k+1}$ at the kth iteration.

Within the given boundary, (7.19) can be rewritten as

$$
\begin{aligned}
p_i^{(k+1)} =& (1-\frac{\lambda}{k+1})p_i^{(k)} + \frac{\lambda}{k+1}v_i^{(k)} \\
=& (1-\frac{\lambda}{k+1})(1-\frac{\lambda}{k})p_i^{(k-1)} + \frac{\lambda}{k+1}v_i^{(k)} \\
& + (1-\frac{\lambda}{k+1})\frac{\lambda}{k}v_i^{(k-1)} \\
=& \cdots \\
=& (1-\frac{\lambda}{k+1})(1-\frac{\lambda}{k})\cdots(1-\frac{\lambda}{2})p_i^{(1)} \\
& + \frac{\lambda}{k+1}v_i^{(k)} + (1-\frac{\lambda}{k+1})\frac{\lambda}{k}v_i^{(k-1)} + \cdots \\
& + \left(\prod_{j=2}^{k}(1-\frac{\lambda}{j+1})\right)\frac{\lambda}{2}v_i^{(1)}.
\end{aligned}
\tag{7.22}
$$

When k becomes infinite, the first term in (7.22), $\left(\prod_{k=1}^{k=+\infty}(1-\frac{\lambda}{k+1})\right)p_i^{(1)} = 0$*, because all multiple coefficients are less than* 1 *and* $|p_i^{(1)}| \leq 1$*. For the remaining terms in (7.22), we define a sequence* $\{h_j\}$*, such that,*

$$
\begin{aligned}
h_j =& (1-\frac{\lambda}{k+1})(1-\frac{\lambda}{k})\cdots(1-\frac{\lambda}{j+2})\frac{\lambda}{j+1}v_i^{(j)} \\
=& \left(\prod_{j\leq k}(1-\frac{\lambda}{j+2})\right)\frac{\lambda}{j+1}v_i^{(j)}.
\end{aligned}
\tag{7.23}
$$

Since $|h_j|$ *is decreasing as k increases and bounded by* $|v_i^{(j)}| = 1$*,* $\sum_{j=1}^{k} h_j$ *is convergent as k goes to infinity. Thus, we conclude that,* $p_i^{(k+1)}$ *will converge to a fixed point, as a mixed* ε_p*-equilibrium in beliefs using (7.17).*

Remark 4 *When* $\lambda = 1$*, the proposed algorithm in (7.19) is reduced to FP. Hence (7.19) and (7.22) can be derived as* $p_i^{(k+1)} = \frac{1}{k+1}p_i^{(1)} + \frac{1}{k+1}(v_i^{(k)} + \cdots + v_i^{(1)})$*. Thus, FP might cycle if* $v_i^{(k)}$ *repeated after some iteration, i.e.,* $v_i^{(2)} = v_i^{(4)} = v_i^{(6)} = \cdots$*, and* $v_i^{(1)} = v_i^{(3)} = v_i^{(5)} = \cdots$*, while the proposed algorithm in (7.17) converges to a fixed point.*

Theorem 7 *For the proposed DSM game, the proposed algorithm in (7.17) is guaranteed to converge to a mixed ε-NE under both EUT and PT, as its beliefs converge to a mixed ε_p-equilibrium.*

Proof: The convergence of the beliefs to a mixed ε_p-equilibrium is shown in Theorem 6. Based on Theorem 6, the convergence of a mixed ε-NE under EUT is a known result for SFP in [121]. The following proof mainly focuses on PT and we prove this theorem by contradiction.

Suppose that $\{p_k\}$ is an iterative process resulting from the proposed algorithm that it converges to a mixed strategy p^* (an ε_p equilibrium) after k iterations. Also, we assume a mixed NE $\delta^* = \{\delta_i^*, \delta_{-i}^*\}$ is near to the fixed point p^*. Based on contradiction, if the point $p^* = \{p_i^*, p_{-i}^*\}$ is not an ε-NE, there must exists a strategy $p_i'(a_i') \in p_i^*$, such that 1) $p_i(a_i) > 0, p_i(a_i) \in p^*$ (at least one mixed strategy of player i is not zero), and 2)

$$\begin{aligned} u_i^{PT}\left(a_i, p_{-i}^*\right) >& u_i^{PT}\left(a_i', p_{-i}^*\right), \\ u_i^{PT}\left(a_i, \delta_{-i}^*\right) + \varepsilon_i\left(a_i, \delta_{-i}^*\right) >& u_i^{PT}\left(a_i', \delta_{-i}^*\right) + \varepsilon_i\left(a_i', \delta_{-i}^*\right), \end{aligned} \tag{7.24}$$

where $u_i(a_i, p_{-i}^*)$ is the expected utility of pure strategy a_i and $\varepsilon_i(a_i, \delta_{-i}^*)$ is the utility difference between NE and ε-NE. Here, since the iterative mixed strategy decreases as the number of iterations n increases ($n \leq k$), the utility distance ε_s of a pure strategy between two neighboring iterations/steps must be less than a value after a certain iteration k. In particular, if the proposed algorithm converge to a fixed ε_p equilibrium at iteration k and it is not an ε-NE in belief (utility), we can find an ε_s, such that

$$\begin{aligned} 0 < \varepsilon_s <& \frac{1}{2}\left| u_i^{PT}(a_i, p_{-i}^*) - u_i^{PT}(a_i', p_{-i}^*) \right|, \\ 0 < 2\varepsilon_s <& (u_i^{PT}\left(a_i, \delta_{-i}^*\right) + \varepsilon_i\left(a_i, \delta_{-i}^*\right)) \\ &- (u_i^{PT}\left(a_i', \delta_{-i}^*\right) + \varepsilon_i\left(a_i', \delta_{-i}^*\right)), \end{aligned} \tag{7.25}$$

where $u_i^{PT}\left(a_i, \delta_{-i}^*\right) = \sum_{a\in\mathscr{A}} u_i(a_i, a_{-i}^*) w_i(\delta_{-i}^*)$. *For* $n \geq k$, *the proposed algorithm process satisfies*

$$\begin{aligned}
u_i^{PT}(a_i, p_{-i}^n) =& \sum_{a\in\mathscr{A}} u_i(a_i, a_{-i}^n) w_i(p_{-i}^n) \\
\geq& \left(\sum_{a\in\mathscr{A}} u_i(a_i, a_{-i}^*) w_i(p_{-i}^*)\right) - \varepsilon_s \\
=& \left(\sum_{a\in\mathscr{A}} u_i(a_i, a_{-i}^*) w_i(\delta_{-i}^*)\right) + \varepsilon_i(a_i, \delta_{-i}^*) - \varepsilon_s \\
>& \left(\sum_{a\in\mathscr{A}} u_i(a_i', a_{-i}^*) w_i(\delta_{-i}^*)\right) + \varepsilon_i(a_i', \delta_{-i}^*) + \varepsilon_s \\
=& \left(\sum_{a\in\mathscr{A}} u_i(a_i', a_{-i}^*) w_i(\delta_{-i}^*)\right) + \varepsilon_s \\
\geq& \sum_{a\in\mathscr{A}} u_i(a_i', a_{-i}^n) w_i(p_{-i}^n) \\
=& u_i^{PT}(a_i', p_{-i}^n).
\end{aligned} \tag{7.26}$$

Thus, player i would not play a_i *but* a_i' *after the nth iteration, and we will have* $p_i(a_i) = 0$ *and* $w_{-i}(p_i(a_i)) = 0$ *(the other player's observation on* p_i*). Here, we have proved that,* $p_i(a_i) = 0$ *contradicts the initial assumption* $p_i(a_i) > 0$*; thus the theorem follows.*

We summarize the proposed DSM solution in Table 7.1. To find the solution of this game under both EUT and PT, we use the algorithm in (7.17) to solve for the mixed ε-NE. In the first phase of the algorithm, each customer will set an initial probability vector to represent its individual participation, while the non-participating customer will set its probability as 0 in the whole DSM process. At the beginning, each participating customer observes others' strategies and evaluates its expected costs of every strategy. In the evaluation process, a participating customer can receive an estimate of the expected costs from the grid operator under EUT, since the grid operator objectively collects information and provides required services/information. Then, the customers will overweight or underweight the information about others' strategies based on the in-

Table 7.1: Proposed Load Shifting Solution

Phase 1 - Proposed Dynamics:

Each customer $i \in \mathscr{N}$ chooses a starting strategy set p_i^{init} as its mixed strategy of participation.

repeat,

1) Each customer $i \in \mathscr{N}$ observes others' participation
2) Each customer $i \in \mathscr{N}$ updates its better response strategy between its current strategy and the fictitious play strategy using (7.17):
   - i) The utility operator communicates with the customer using the grid's two-way architecture communication (see [112] or [106] and references therein).
   - ii) Customers' loads can be shifted via DSM as in Section 7.2.2.

     Load Shedding

     a) The utility operator advertises the current participation $\{\mathscr{A}\}_{i\in\mathscr{N}}$ using the mixed strategy provided by customers.
     b) Each customer publishes its participation based on a mixed strategy, representing the subjective observation with underlying weight effect.
     c) After combining probabilities as in (7.12), customer i will receive an expected cost under EUT, sent by the operator, and observe the current vector of strategies p_{-i} so as to assess its subjective utility in (7.14) under PT.
     d) Customer i chooses its strategic response in (7.17).

until convergence to a mixed NE strategy vector P^*.

Phase 2 - Power Company Strategy

1) The operator receives the participation information given the mixed strategy set as per a^*.
2) Actual load shifting is implemented under realistic participation.

dividual weighting effect in (7.13). Moreover, each customer will subjectively estimate its expected costs of every strategy and then, frequently report its participation based on its mixed strategy. The communication between customers and grid operator will continue until the expected costs satisfy the setting ε in utility (7.12), (7.14) and (7.16). Once a mixed ε-NE is reached, the customers will frequently signal their participation decisions based on probabilities and participate in demand-side management in Phase 2. This phase of the proposed load shifting solution is the practical market operation. Given the submitted information, the grid operator will shift/reduce local loads. The actual process of Phase 2 is beyond the scope of this chapter and will follow economic and standard, real-world contract negotiations that could be interesting to study in future work.

7.4 Simulation Results and Analysis

For simulating the proposed system, we consider a geographical region in which a number of customers have normal load requirements that they wish to satisfy from the grid. We use the real-world load profile in [122] which represents customers' initial demands, i.e., the data between April 29th, 2013 and May 4th, 2013 from Miami International Airport, since the data of local customer houses, or groups, is large, private and confidential. In all simulations, each customer can choose a starting time to participate in DSM from the time period between 18:00 and 20:00. Alternatively, the customer can decide not to participate; that is, $\mathscr{A}_i = \{18, 19, 20, 24\}, \forall i \in \mathscr{N}$. Also, we set $\beta = 0.86$ and $\gamma = 0.6$ for simulation purposes.

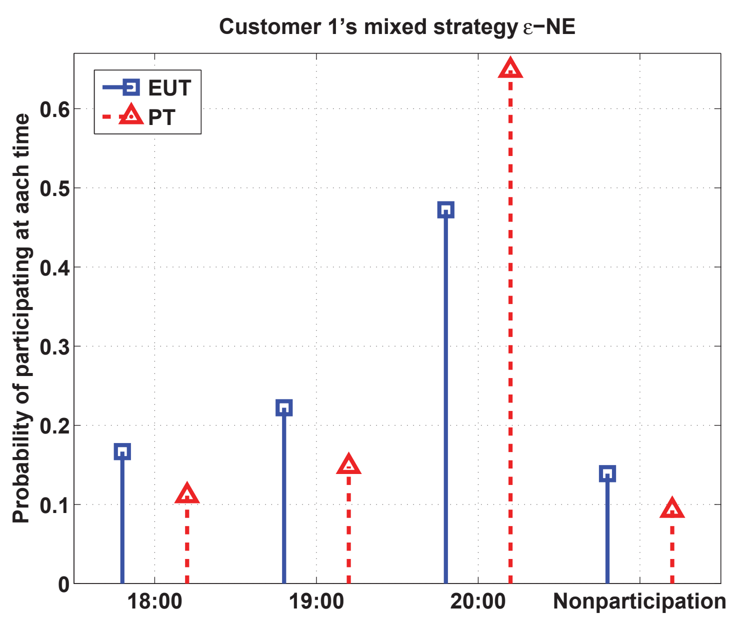

Figure 7.1: The probability performance of all mixed strategies for Customer 1 under both EUT and PT.

Fig. 7.1 shows, for a smart grid with 6 customers, all mixed strategies of a selected Customer 1 under both EUT and PT. In this figure, we choose $\alpha = 0.7$ to represent the distortion parameter for the weighting effect in (7.13). Clearly, the four mixed strategies of Customer 1's participation are different between EUT and PT. Compared to the EUT results (solid lines), the PT participating strategy (dash line) at 20:00 is larger, while the other PT mixed strategies are smaller. In essence, we observe that PT generally enhances (reduces) high (low) frequency EUT strategies. For instance, under EUT, Customer 1 will have the highest participation strategy at 20:00 due to load shifting as per in (7.8) and the varying price in (7.3). In particular, its largest mixed strategy is greater than the average mixed strategy, i.e., 0.25 in the proposed four-strategy game. Second, because of the weighting effect in (7.13), in Fig. 7.1, we can see how PT behavior deviates from EUT. In particular, we observe that a PT customer wants to participate more at the time when its hourly payment is low, which also corresponds to a case with high participation under EUT. In other words, for minimizing its payment, under PT, each customer tends to increase its participation at the hours with lower hourly payments and to decrease its participation at the hours with higher hourly payments. Accordingly, under PT, we can observe that higher EUT probabilities will become more pronounced. Thus, the largest mixed strategy using EUT, i.e., at 20:00, would be overweighted via PT observation, and vice versa. This is due to the fact that each PT customer takes a risk for participating when the hourly payments are low in practice.

Fig. 7.2 shows, using the same parameters as Fig. 7.1, the probability that each customer participates in DSM at 19:00 under both EUT and PT. In this figure, we can first see that the mixed strategy of each customer using PT is different from that resulting from EUT. Under EUT, the rational mixed strategies made by some customers, such as Customers 3-6, are higher than the average mixed strategy, because they have low payments at 19:00. In particular, given the price in (7.3), a customer's lowest demand between 18:00 and 20:00 would cause the lowest payment and the highest participation.

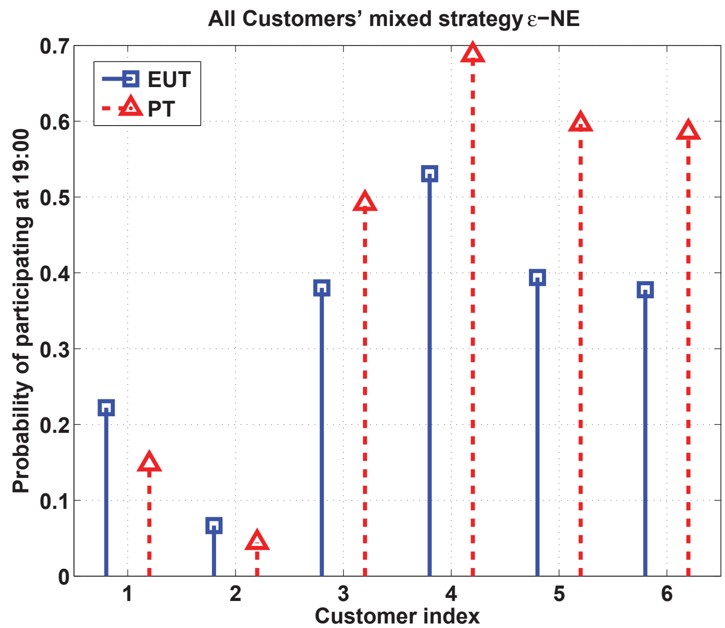

Figure 7.2: Mixed strategies for 6 customers using both EUT and PT at 19:00.

This implies that, a rational customer wants to participate in DSM when it does not require a lot of energy (or its demand is low) in practice, since the payment is low. Under PT, the customers' realistic decisions will impact the price in (7.3) and change their participations levels. In Fig. 7.2, some customers' mixed strategies using PT are greater than those of EUT, i.e., Customers 3-6, while Customers 1 and 2 have a lower participation probability using PT. If four PT customers, based on their loads between 18:00 and 24:00, decide to shift more load at 19:00, their PT demands will be less than EUT. Then, compared to EUT, the PT prices of Customers 1 and 2 will increase because of the changing load fraction in (7.3). Because of the increasing prices, Customers 1 and 2's payments increase and they will be less interested in participating at 19:00. Indeed, here will exist some customers, such as Customers 1 and 2, who want to decrease their participation at 19:00, under PT as discussed in Fig. 7.1. Then, the overall demand increases and Customers 3-6 obtain a decreasing pricing in (7.3) which will make these four customers more likely to participate at 19:00.

Fig. 7.3 shows the expected nonparticipating load profile in the proposed DSM game as time varies. We choose the same customers as in Fig. 7.1 with the distortion parameters set to $\alpha = 0.7$ for all customers. In Fig. 7.3, the nonparticipating load is the minimum expected load that the power company must supply, while the participating load represents the load that can be partly shifted based on each individual customer's action in (7.8). Here, all customers can start the game from 18:00 and the expected nonparticipating load between EUT and PT are different. On the one hand, the difference between EUT and PT during 18:00 and 20:00 is due to the change in the customers' decisions, as previously shown in Fig. 7.2. On the other hand, between 21:00 and 23:00, the customers nonparticipating load using PT is always less than that using EUT. Indeed, if we translate the proposed game into a "participate or not participate" game, 21:00 can be used to distinguish such two strategies and customer behaviors before 21:00 can impact their participations at a later time. However, the fact that the nonparticipation level for PT is less than EUT after 21:00 directly relates to the choice of a distortion parameter α. In particular, a small deviation from the rational strategy for $\alpha = 0.7$ leads to an increased competition between the customers due to the fact that, an irrational observation increases the costs in (7.5). Such small deviation represents a case in which a customer participates in DSM but does not trust its view of the opponents' strategy or the information received from the power company. Thus, a slight deviation from the rational path causes increasing costs and customers are more apt to shift their load under PT to decrease the impact of being non-rational, compared to EUT. As a result, after 21:00, as seen in Fig. 7.3, the PT nonparticipating load will be less than that for EUT. At 24:00, the power company needs to deal with the remaining load as defined in (7.11).

Compared to Fig. 7.3, Fig. 7.4 shows the expected nonparticipating load profile

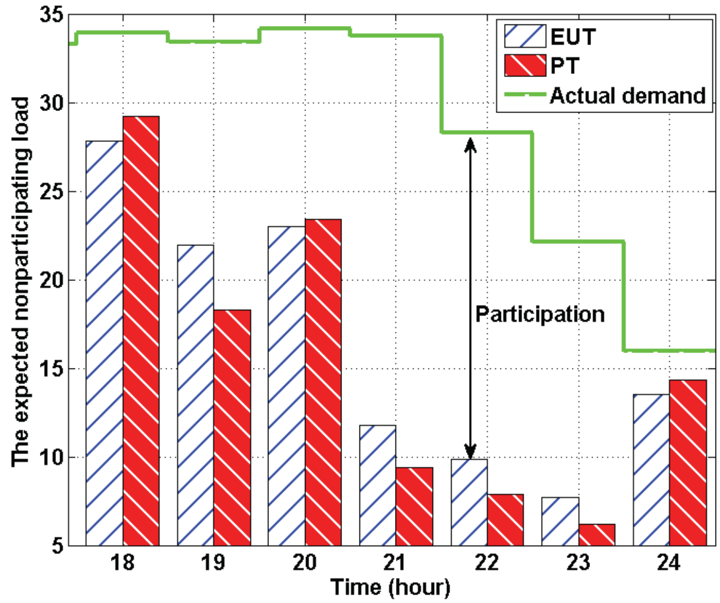

Figure 7.3: The expected nonparticipating load for the 6 customer game under the mixed strategy using both EUT and PT over 24 hours, when all customers have the same $\alpha = 0.7$.

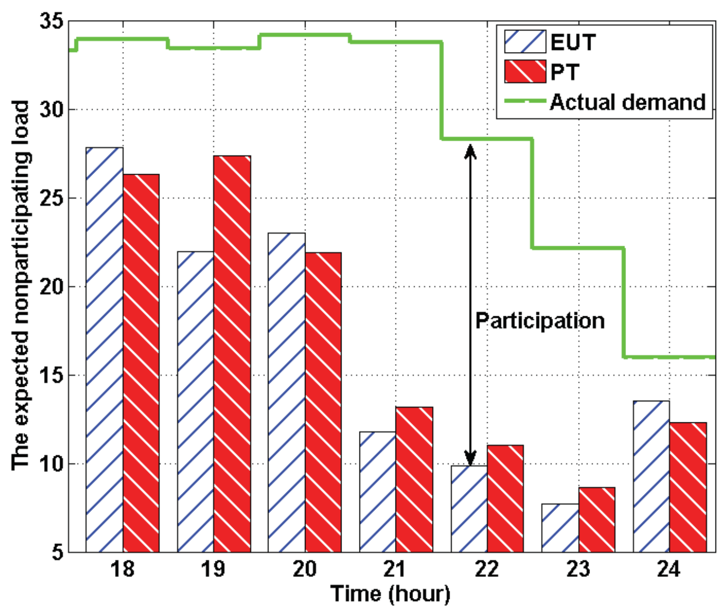

Figure 7.4: The expected nonparticipating load for the 6 customer game under the mixed strategy using both EUT and PT over 24 hours, when customers have different values of α.

using different distortion parameters, as time varies. In particular, we choose $\alpha = [0.5\ 0.5\ 0.2\ 0.1\ 0.1\ 0.1]^T$. In this figure, we can see that, when some customers have a very irrational observation on their opponents, the PT nonparticipating load between 21:00 and 23:00 will be higher than EUT. This implies that, in reality, if some customers deviate significantly from their rational strategies (for example, a customer forgets to assist the power company in load shifting), the power company will not be able to shift the total load predicted by the rational, objective model. Thus, the power company can use the distortion parameter to decide on how to design its DSM scheme.

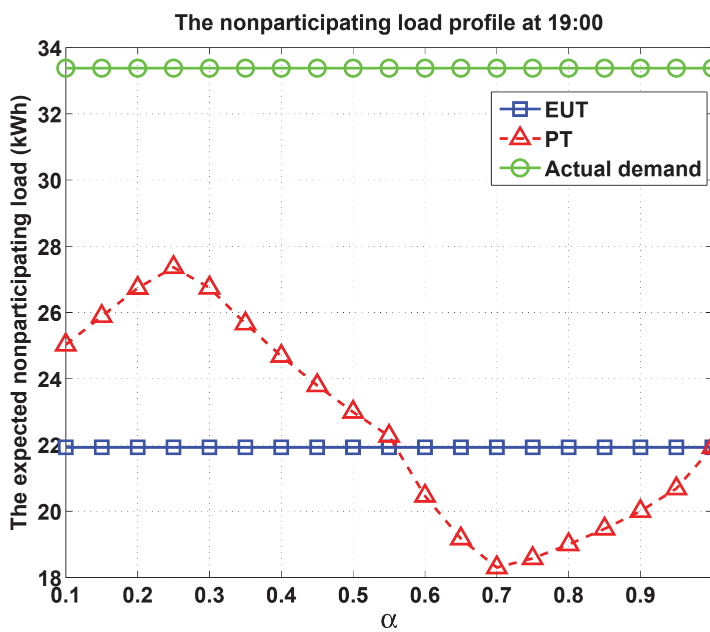

Figure 7.5: The expected nonparticipating load of all customers at 19:00 as α varies.

Fig. 7.5 shows the expected nonparticipating load at 19:00, as the weighting effect parameter α varies. In this figure, we can see that, the expected nonparticipating load under EUT is 65.7% of the total load and, the nonparticipating load under PT is less than EUT when $\alpha > 0.56$. This is because, the majority of PT customers have more interest in participating at 19:00, as shown in Fig. 7.2. Thus, the power company can shift more load in practice, compared to EUT. Also, this figure shows that, there exists a distortion threshold, such that, if α is greater (smaller) than the threshold, PT customers will have lower (higher) nonparticipating loads than EUT cases. A large distortion parameter, or a small deviation from EUT, yields an increased competition thus raising the costs to the customers. Consequently, the customers will become risk seeking and more apt to shift their loads and decrease their payments. Thus, the increasing PT costs will force the majority to shift more loads, compared to EUT. However, a small distortion parameter, or a large deviation from EUT, will lead to highly irrational behavior from the customers which will lead to increasingly high competition and decreasing participation, as customers become extremely risk averse and unwilling to participate in the DSM process. In a nutshell, Fig. 7.5 shows that a small deviation from EUT may be beneficial for the power company as it increases customers' participation. In contrast, a significant deviation from EUT will inevitably lead to highly risk averse behavior which will prevent most customers from participating; thus yielding detrimental results for the grid and preventing the operator from reaping the benefits of DSM.

7.5 Conclusions

In this chapter, we have introduced a novel approach for studying the problem of demand-side management. In particular, we have developed a game-theoretic approach, based on prospect theory, using which each player subjectively observes other players' actions and determines its own actions so as to minimize a cost function that captures the electricity cost over 24 hours. For the multi-player scenario, we have proposed an algorithm and have proved that it reaches a mixed ε-NE. Simulation results have shown that deviations from classical, objective game-theoretic DSM mechanisms can lead to unexpected results and loads on the grid, depending on the rationality level of the customers and their risk aversion. In a nutshell, the results of this chapter have provided important insights into the factors underlying the modest participation in DSM schemes observed in real-world smart grid systems.

Chapter 8

Hardware Trojan Detection

In this chapter, we explore the grid security in a hardware trojan detection game via an integrated circuits (IC).

8.1 System Model and Game Formulation

8.1.1 System Model

Consider an IC manufacturer who produces ICs for critical infrastructures such as the smart grid. This manufacturer, hereinafter referred to as an "attacker", has an incentive to introduce hardware trojans to maliciously impact the cyber-infrastructure that adopts the produced IC. Such a trojan, when activated, can lead to errors in the circuit, potentially damaging the underlying system. Here, we assume that the attacker can insert one trojan t from a set $\mathscr{T}$ of T trojan types. Each trojan $t \in \mathscr{T}$ can lead to a certain damage captured by a positive real-number $V_t > 0$.

Once the agency or company, hereinafter referred to as the "defender", receives the ICs, it can decide to test for one or more types of trojans. Due to the complexity of modern IC designs, it is challenging to develop test patterns that can be used to readily and quickly verify the validity of a circuit with respect to all possible trojan types. Particularly, the defender must spend ample resources if it chooses to test for all possible types of trojans. Such resources may be more costly than the actual damage caused by the trojans. Thus, we assume that the defender can only choose a certain subset $\mathscr{A} \subset \mathscr{T}$ of trojan types to test for, where the total number of trojans tested for is $|\mathscr{A}| < T$. The practical aspects for testing and verification of the circuit versus the subset of trojans $\mathscr{A}$ can follow existing approaches such as the scan chain approach developed in [123]. We assume that such testing techniques are reliable and, thus, if the defender tests for the accurate type of trojan, this trojan can then be properly detected.

In this model, if the defender tests for the right types of trojans that have been inserted in the circuit being tested, then, the attacker will be required to pay a certain fine F_t, if the trojan detected is of type t. This fine can be a monetary amount that the manufacturer is required to pay for the defender, depending on the seriousness of the threat. The fine can also be viewed as a renegotiation of contracts or other agreements between defender and attacker.

Our key goal is to understand the interactions between the defender and attacker in such a hardware trojan detection scenario. In particular, it is of interest to devise an approach using which one can understand how the defender and attacker can decide on the types of trojans that they will test for or insert, respectively, and how those actions impact the overall damage on the system and possible monetary fines on the attacker. Such an approach will provide insights on the optimal testing choices for the defender, given various possible actions taken by the attacker.

8.1.2 Noncooperative Game Formulation

For the studied hardware trojan detection model, the decisions of the attacker on which type of trojan to insert will impact the decisions of the defender on which trojans to test for and vice versa. Moreover, the choices by both attacker and defender will naturally determine whether any damages will be done to the system or whether any fine must be imposed. Due to this coupling in the actions and objectives of the attacker and defender, the framework of noncooperative game theory [107] provides suitable analytical tools for modeling, analyzing, and understanding the decision making processes involved in the studied attacker-defender hardware trojan detection scenario.

To this end, we formulate a static *zero-sum, noncooperative game* in strategic form $\Xi = \{\mathscr{N}, \{\mathscr{S}_i\}_{i\in\mathscr{N}}, \{u_i\}_{i\in\mathscr{N}}\}$, which is defined by its three main components: (i)- the *players* which are the attacker a and the defender d in the set $\mathscr{N} := \{a,d\}$, (ii)- the *strategy* space $\mathscr{S}_i$ of each player $i \in \mathscr{N}$, and (iii)- the *utility function* u_i of any player $i \in \mathscr{N}$.

For the attacker, the strategy space is simply the set of possible trojan types, i.e., $\mathscr{S}_a = \mathscr{T}$. Thus, an attacker can choose one type of trojans to insert in the circuit being designed or manufactured. For the defender, given the possibly large number of trojans that must be tested for, we assume that the defender can only choose to test for K trojan types simultaneously. The actual value of K would be determined exogeneously to the game via factors such as the resources available for the defender and the type of circuitry being tested. For a given K, the strategy space $\mathscr{S}_d$ of the defender will then be the set of possible subsets of $\mathscr{T}$ of size K. Therefore, each defender will have to choose one of such subsets, denoted $s_d \in \mathscr{S}_d$.

For each defender's choice of a size-K trojans set $s_d \in \mathscr{S}_d$ to test for and attacker's choice of trojan type $s_a \in \mathscr{S}_a$ to be inserted, the defender's utility function $u_d(s_d, s_a)$ will be:

$$u_d(s_d, s_a) = \begin{cases} F_{s_a} & \text{if } s_a \in s_d, \\ -V_{s_a} & \text{otherwise,} \end{cases} \tag{8.1}$$

where V_{s_a} is the damage done by trojan s_a if it goes undetected. Given the zero-sum nature of the game, the utility of the attacker is simply $u_a(s_d, s_a) = -u_d(s_d, s_a)$.

8.2 Prospect Theory for Hardware Trojan Detection: Uncertainty and Risk in Decision Making

8.2.1 Mixed Strategies and Expected Utility Theory

For the studied hardware trojan detection game, it is reasonable to assume that both defender and attacker make probabilistic choices over their strategies and therefore, we are interested in studying the game under *mixed strategies* [107] rather than under *pure, deterministic strategies*. The rationale for such mixed, probabilistic choices is two-fold: a) both attacker and defender must randomize between their strategies so as not to make it trivial for the opponent to guess their strategy and b) the hardware trojan detection game can be repeated over an infinite horizon and therefore, mixed strategies allow to capture the frequencies with which attacker or defender would use a certain strategy.

To this end, let $p = [p_d \; p_a]$ be the vector of mixed strategies of both players, where, for the defender, each element in p_a is the probability with which the defender chooses a certain size-K subset $s_d \in \mathscr{S}_d$ of trojans to test for and for the attacker, each element in p_a represents the probability with which the attacker chooses to insert a trojan $s_a \in \mathscr{S}_a$.

In traditional game theory [107], it is assumed that players act rationally. This rational assumption implies that each player, attacker or defender, will objectively choose its mixed strategy vector so as to optimize its expected utility. Indeed, under conventional expected utility theory (EUT), the utility of each player is simply the expected value over its mixed strategies which, for any of the two players $i \in \mathscr{N}$, is given by:

$$U_i^{\text{EUT}}(p_d, p_a) = \sum_{s \in \mathscr{S}} \Big(p_d(s_d) p_a(s_a) \Big) u_i(s), \tag{8.2}$$

where $s = [s_d \; s_a]$ is a vector of selected pure strategies and $\mathscr{S} = \mathscr{S}_d \times \mathscr{S}_a$.

8.2.2 Prospect Theory for Hardware Trojan Detection Game

In conventional game theory, EUT allows the players to evaluate an objective expected utility such as in (8.2) in which they are assumed to act rationally and to objectively assess their outcomes. However, in real-world experiments, it has been observed that users' behavior can deviate considerably from the rational behavior predicted by EUT. The reasons for these deviations are often attributed to the risk and uncertainty that players are often faced when making decision on certain game-theoretic outcomes.

In particular, several empirical studies [58, 61, 65–68] have demonstrated that, when faced with decisions that involve gains and losses, such as in the proposed hardware trojan detection game, players can have a subjective evaluation of their utilities, when faced with risks and uncertainty. In the studied game, both attacker and defender face several uncertainties. On the one hand, the defender can never be sure on which type of trojan the attacker will be inserting and, thus, when evaluating its outcomes using (8.2), it may overweight or underweight the mixed-strategy vector of the attacker p_a. Similarly, the attacker may also evaluate its utility given a distorted and uncertain view of the defender's possible strategies. On the other hand, the decisions of both attacker and defender involve humans (e.g., administrators at the governmental agency or hackers at the manufacturer) who might guide the way in which trojans are inserted or tested for. This human dimension will naturally lead to potentially irrational behavior that can be risk averse or risk seeking, thus deviating from the rational tenets of classical game theory and EUT.

For the proposed game, such considerations of risk and uncertainty in decision-making processes can translate into the fact that each player i must decide on its action, in the face of the uncertainty induced by the mixed strategies of its opponent, which impact directly the utility as in (8.2). In order to capture such risk and uncertainty factors in the proposed hardware detection game, we turn to the framework of prospect theory [58].

Given these PT-based uncertainty and risk considerations, as discussed in Section 2.3, the expected utility achieved by a player i will thus be:

$$U_i^{\mathrm{PT}}(p_d, p_a) = \sum_{s \in \mathscr{S}} \Big(p_i(s_i) w_{-i}(p_{-i}(s_{-i})) \Big) u_i(s_i, s_{-i}), \tag{8.3}$$

where the index $-i$ is used to indicate the opponent of i, i.e., the attacker if i is the defender and vice versa. Clearly, in (8.3), the uncertainty is captured via each player's weighting of its opponents strategy. This weighting depends on the rationality of the player and the uncertainty, which can be captured by α_i.

Given this re-definition of the game, our next step is to study and discuss the game solution under both EUT and PT.

8.3 Game Solution and Proposed Algorithm

8.3.1 Mixed-Strategy Nash Equilibrium

To solve the proposed game, under both EUT and PT, we seek to characterize the *mixed-strategy Nash equilibrium* of the game:

Definition 7 *A mixed strategy profile p^* is said to be a mixed strategy Nash equilibrium if, for each player $i \in \mathscr{N}$, we have:*

$$U_i(p_i^*, p_{-i}^*) \geq U_i(p_i, p_{-i}^*),\ \forall p_i \in \mathscr{P}_i, \tag{8.4}$$

where $\mathscr{P}_i$ is the set of all probability distributions available to player i. Note that, the mixed-strategy Nash equilibrium definition in (8.4) is applicable for both PT or EUT, the difference would be in whether one is using (8.2) or (8.3), respectively.

The mixed-strategy Nash equilibrium (MSNE) represents a state of the game in which neither the defender nor the attacker has an incentive to unilaterally deviate from its current mixed-strategy choice, given that the opposing player uses its MSNE strategies. Under EUT, this implies that, under a rational choice, the MSNE represents the case in which the defender has chosen its optimal randomization over its testing strategies and, thus, cannot improve its utility by changing these testing strategies, assuming that the attacker is also rational and utility maximizing as per EUT. Under PT, at the MSNE neither the attacker nor the defender can improve their perceived and subjective utility evaluation as per (8.3) by changing their MSNE strategies given their rationality levels captured by α_d and α_a. Thus, under PT the MSNE is a state of the game in which neither the defender nor the attacker can improve their utilities, under their current uncertain perception on one another.

Given the zero-sum, two-player nature of the game, finding closed-form solutions for the MSNE can follow the popular von Neumann indifference principle [107] under which, for each player, at the MSNE, the expected utilities of any pure strategy choice, under the mixed strategies played by the opponent, are equal. Such a principle can be trivially be shown to be applicable to both EUT and PT due to the one-to-one relationship between the probabilities and the weights. For the proposed game, given the large strategy space of both defender and attacker, it is challenging to solve the equations that stem from the indifference principle, for a general case. However, as will be shown for a numerical case study in Section 8.4, the game may admit multiple equilibrium points. Therefore, given an initial starting point of the system, one must develop learning algorithms [124] to characterize one of the MSNEs, as proposed next.

8.3.2 Proposed Algorithm: Fictitious Play and Convergence Results

To solve the studied hardware trojan detection game, under both EUT and PT, we propose a fictitious play-based algorithm summarized in Table 8.1.

In this algorithm, the first stage consists of a simple initialization phase in which

Table 8.1: Proposed Hardware Trojan Detection Game Algorithm

Stage 1 - Initialization
The defender and attacker choose a certain initial mixed strategy vectors p_d^{init} and p_a^{init}, respectively.
Stage 2 - Equilibrium Learning for both EUT and PT
repeat
Each player $i \in \mathscr{N}$ observes the actions of its opponent $s_{-i}(k-1)$ at time $(k-)$ and updates its knowledge of empirical frequencies $p_{-i}(k)$ as per (8.5).
Each player $i \in \mathscr{N}$ takes action $s_i(k)$ as per (8.6).
until convergence to a mixed-strategy Nash equilibrium.
Stage 3 - Negotiations between Defender and Attacker
The defender and attacker may negotiate the outcome of the game so as to transfer fines or assess damages, if any.

both attacker and defender set certain initial probability vectors p_d^{init} and p_a^{init}. In the second, MSNE learning stage, we propose an iterative process based on the fictitious play algorithm [107,124] for both PT and EUT. Here, the defender and attacker will use their observations on each others strategies at time $k-1$ to update their future strategies at time k. The time scale of the algorithm will depend on the actual time to market of IC manufacturing and the testing time. However, the proposed algorithm can accommodate any time scale, without loss of generality. To reach an equilibrium using fictitious play, the defender and attacker will update their beliefs about each others strategies by monitoring their actions. We let $s_i(k)$ be the strategy chosen by player i at time k. Let $p_i^{s_i}(k),\ s_i \in \mathscr{S}_i, i \in \mathscr{N}$, be the empirical frequency, defined as the frequency with which player i has chosen action s_i until time k. At any given iteration k, the following FP process is used by a player i to update its beliefs:

$$p_i^{s_i}(k) = \frac{k-1}{k} \cdot p_i^{s_i}(k-1) + \frac{1}{k} \cdot \mathbf{1}_{\{s_i(k-1)=s_i(k)\}}. \tag{8.5}$$

In essence, Stage 2 of the algorithm in Table 8.1 proceeds as follows. At iteration k, player i observes the actions of its opponent at time $k-1$, and then updates its knowledge of the frequencies using (8.5). In FP, the strategy chosen at time k is the ones that maximizes the expected utility with respect to the updated empirical frequencies. This expected utility would follow (8.2) for EUT and (8.3) for PT. Consequently, player i's strategy at each iteration can be chosen as:

$$s_i(k) = \arg\max_{s_i \in \mathscr{S}_i} u_i(s_i, p_{-i}(k-1)) \tag{8.6}$$

where the utility here is the expected value obtained by player i with respect to the mixed strategy of the opponent, when player i chooses pure strategy s_i. The expectation will be analogous to (8.2) and (8.3) for EUT and PT, respectively. Based on their observations, the two players first update their empirical frequencies using (8.5), and then choose their strategies as per (8.6). For a two-player, zero-sum game, it is well known [107] that FP is guaranteed to converge to an MSNE [124]. However, to our knowledge, such a result has not been extended to PT, as done in the following theorem:

Theorem 8 *For the proposed hardware trojan detection game, the proposed FP-based algorithm is guaranteed to converge to a mixed NE under both EUT and PT.*

Proof: The convergence of FP to an MSNE for EUT in a two-player zero-sum game is a known result [107, 124, 125]. For PT, one can easily verify that the convergence to a fixed point will directly follow from the EUT results in [107, 124, 125]. However, what remains to be shown is that this convergence will actually reach an MSNE for the case of PT. We prove this case using contradiction as follows.

Suppose that $\{p_k\}$ *is a fictitious play process that will converge to a fixed point and a mixed strategy* p^* *after k iterations. If the point* $p^* = \{p_i^*, p_{-i}^*\}$ *is not an MSNE, there must exist a strategy* $p_i'(s_i') \in p_i^*$*, such that* $p_i(s_i) > 0, p_i(s_i) \in p^*$ *(at least one mixed strategy of player i is not zero) and*

$$u_i^{PT}\left(s_i', p_{-i}^*\right) > u_i^{PT}\left(s_i, p_{-i}^*\right),$$

where $u_i(s_i, p_{-i}^*)$ *is the expected utility with respect to the mixed strategies of the opponents of player i, when player i chooses pure strategy* s_i*. Here, we can choose a value* ε

that satisfies $0 < \varepsilon < \frac{1}{2}|u_i^{PT}(s_i', p_{-i}^*) - u_i^{PT}(s_i, p_{-i}^*)|$ *as* p *converges to* p^* *at iteration* k*. Also, since the FP process decreases as the number of iterations* n *increases, the utility distance of a pure strategy between two neighboring iterations must be less than* ε *after a certain iteration* k*. For* $n \geq k$*, the FP process can be written as:*

$$\begin{aligned} u_i^{PT}(s_i, p_{-i}^n) &= \sum_{s \in \mathscr{S}} u_i(s_i, s_{-i}^n) w_i(p_{-i}^n) \\ &\leq \sum_{s \in \mathscr{S}} u_i(s_i, s_{-i}^*) w_i(p_{-i}^*) + \varepsilon \\ &< \sum_{s \in \mathscr{S}} u_i(s_i', s_{-i}^*) w_i(p_{-i}^*) - \varepsilon \\ &\leq \sum_{s \in \mathscr{S}} u_i(s_i', s_{-i}^n) w_i(p_{-i}^n) \\ &= u_i^{PT}(s_i', p_{-i}^n), \end{aligned} \tag{8.7}$$

Thus, player i *would not choose* s_i *but would rather choose* s_i' *after the nth iteration, mathematically, we will have* $p_i(s_i) = 0$ *and* $w_{-i}(p_i) = 0$ *(the other player's observation on* p_i*). Hence, we get* $p_i(s_i) = 0$ *which contradicts the initial assumption that* $p_i(s_i) > 0$*; thus the theorem is shown.*

Following the convergence to a mixed-strategy Nash equilibrium, the last stage in the Algorithm of Table 8.1 is the actual negotiation phase which can occur periodically between the defender and attacker. In this stage, after a period of time during which testing and hardware insertion occurs, the defender assesses its damages and the attacker assesses any possible fines and, subsequently, the two may directly interact or negotiate future trade or exchange based on past results. For example, after a certain period of time has passed, the defender might find that a certain manufacturer has been highly malicious and then it decides to severe all ties to it and/or replace it with another. The actual process of Stage 3 is beyond the scope of this chapter and will follow economic and real-world contract negotiations that could be interesting to study in future work.

8.4 Numerical Case Study: Analytical and Simulation Results

For simulating the hardware trojan detection game, we consider the scenario in which the attacker, denoted hereinafter by player 1, has four types of trojans (strategies) A, B, C, and D (i.e., $\mathscr{S}_a = \mathscr{T} = \{A, B, C, D\}$ whose damage values are $V_A = 1$, $V_B = 2$, $V_C = 4$ and $V_D = 12$. These numbers are used to illustrate different damage levels on the system. For example, these values can be viewed as monetary losses to the defender and attacking gains to the attacker. Given that there are no existing empirical data on the hardware detection game, we have chosen illustrative numbers that show 4 varying levels of damage. However, naturally, the subsequent analysis may be extended to analyze the game under other damage values. In this scenario, we assume that the defender, referred to as player 2, can test for $K = 2$ types of trojans at a time and, thus, it has 6 strategies. Without loss of generality, we assume that the fine is similar for all types of trojans, i.e., $F_{s_a} = F \; \forall s_a \in \mathscr{S}_a$.

For this numerical case study, we will first derive several analytical results that allow us to gain more insights on the proposed hardware detection game under both EUT and PT considerations. Then, we present several simulation result that provide additional insights and analysis on the proposed game and on the impact of PT considerations.

8.4.1 Analytical Results

In this subsection, we derive a series of results to gain more insights on the Nash equilibria of the game as well as on the possible values of the fine and how they impact the game under both PT and EUT. First, we can state the following theorem with regard to the Nash equilibria of the game under both EUT and PT:

Theorem 9 *When $F > 0$, under both EUT and PT, the proposed game can admit multiple equilibria. However, in all of these equilibria, the attacker has the same mixed-strategy Nash equilibrium strategies.*

Proof: To capture the pure strategy payoffs of defender given the attacker's mixed strategy, we use the indifference principle as per the following equation:

$$
\begin{aligned}
u_d(s_d, p_a^*) =& M_a \cdot p_a^* \\
\begin{bmatrix} u_d(AB, p_a^*) \\ u_d(AC, p_a^*) \\ u_d(AD, p_a^*) \\ u_d(BC, p_a^*) \\ u_d(BD, p_a^*) \\ u_d(CD, p_a^*) \end{bmatrix} =& \begin{bmatrix} F & F & -4 & -12 \\ F & -2 & F & -12 \\ F & -2 & -4 & F \\ -1 & F & F & -12 \\ -1 & F & -4 & F \\ -1 & -2 & F & F \end{bmatrix} \cdot \begin{bmatrix} p_a^*(A) \\ p_a^*(B) \\ p_a^*(C) \\ p_a^*(D) \end{bmatrix},
\end{aligned} \tag{8.8}
$$

where $p_a^*(A) + p_a^*(B) + p_a^*(C) + p_a^*(D) = 1$. M_a is the utility matrix of the attacker which we can use to obtain the solution of attacker p_a^*. Using the indifference principle, for the defender, an MSNE must satisfy $u_d(AB, p_a^*) = u_d(AC, p_a^*) = u_d(AD, p_a^*) = u_d(BC, p_a^*) = u_d(BD, p_a^*) = u_d(CD, p_a^*)$. In particular,

$$
rank(M_a) = rank\left(\begin{bmatrix} F & F & -4 & -12 \\ 0 & -2-F & F+4 & 0 \\ 0 & -2-F & 0 & F+12 \\ -1 & F & F & -12 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right) = 4. \tag{8.9}
$$

Thus, the attacker has only one solution p_a^* since: 1) the rank of attacker's utility matrix is equal to the number of its mixed strategy and 2) the auxiliary equation $\sum_{s_a} p_a^*(s_a) = 1$ balances the requirement of $u_d(s_d, p_a^*)$ in (8.8). Similarly, we also could capture the pure strategy payoffs of attacker via defender's mixed strategy:

$$
u_a(s_a, p_d^*) = M_d \cdot p_d^*, \tag{8.10}
$$

where $M_d = -M_a^T$. *In particular,* $\sum_{s_d} p_d^*(s_d) = 1$ *and* $u_a(A, p_d^*) = u_a(B, p_d^*) = u_a(C, p_d^*) = u_a(D, p_d^*)$ *at the MSNE for the attacker. Since the rank of the defender's utility matrix,* $rank(M_d) = rank(M_a) = 4$, *is less than the number of defender's strategies, we will get multiple MSNE strategies for the defender.*

As an example, when $F = 8$ *in (8.8) and (8.10), we could obtain the only NE for attacker under EUT,* $p_a^* = [0.32, 0.29, 0.24, 0.16]^T$. *Also, we can compute the multiple NEs of defender under EUT:*

$$\begin{aligned} p_d^*(AB) =& -0.2259 + p_d(CD), \\ p_d^*(AC) =& -0.1290 + p_d(BD), \\ p_d^*(AD) =& 0.7097 - p_d(BD) - p_d(CD), \\ p_d^*(BC) =& 0.6452 - p_d(BD) - p_d(CD). \end{aligned} \tag{8.11}$$

Under PT, the auxiliary equation is equivalent to $\sum_p \exp(-(-\ln w(p))^{\frac{1}{\alpha}}) = 1$. *This equation does not change the ranks of neither* M_a *nor* M_d *nor the number of eigenvalues. Thus, based on Cayley-Hamilton theorem, PT and EUT have the same number of eigenvalues and then, have the same number of MSNEs. This is applicable for any value of the fine.*

Next, we show that, for both EUT and PT, there exists a value F^v for the fine at which neither the attacker nor the defender will win, i.e., the value of the game is zero:

Theorem 10 *For EUT, at the MSNE, there exists a fine value* F_{EUT}^v, *such that neither the attacker nor the defender wins.*

Proof: Since, for the case in which no player is a winner, the expected utility of attacker is equal to that of defender and the MSNE strategies of attacker are unique, we solve for F from the perspective of attacker's MSNE. In particular,

$$\begin{cases} U_a^{EUT}(p_d^{*EUT}, p_a^{*EUT}) = p_a^{*EUT'} \cdot M_d \cdot p_d^{*EUT} = 0, \\ U_d^{EUT}(p_d^{*EUT}, p_a^{*EUT}) = p_d^{*EUT'} \cdot M_a \cdot p_a^{*EUT} = 0, \end{cases} \tag{8.12}$$

where $p_a^{*EUT'}$ is the transpose of p_a^{*EUT}. Here, the expected utility of the defender requires one to first compute the MSNE of the attacker using M_a. Based on the indifference principle, at the defender's MSNE, we have $u_d(AB, p_a^*) = u_d(AC, p_a^*) = \cdots = u_d(CD, p_a^*)$. Moreover, we have:

$$[u_d(AB, p_a^*)\ u_d(AC, p_a^*)\ \cdots u_d(CD, p_a^*)]^T = M_a \cdot p_a^{*EUT} \tag{8.13}$$

Because the mixed strategy of the defender is nonnegative, i.e. $p_d^{*EUT} \geq 0$, we have

$$\begin{aligned} p_d^{*EUT'} \cdot u_d(s_d, p_a^*) =& 0 \\ u_d(s_d, p_a^*) =& 0 \\ \therefore \quad M_a \cdot p_a^{*EUT} =& 0. \end{aligned} \tag{8.14}$$

In particular, for $u_d(AB, p_a^*)$,

$$\begin{aligned} & F p_a^*(A) + F p_a^*(B) - 4p_a^*(C) - 12p_a^*(D) = 0, \\ & F_{EUT}^v = \frac{4p_a^*(C) + 12p_a^*(D)}{p_a^*(A) + p_a^*(B)}. \end{aligned} \tag{8.15}$$

Theorem 11 *For PT, at the MSNE, there exists a fine value F_{PT}^v, such that neither the attacker or defender wins.*

Proof: Similarly to Theorem 10, we have

$$\begin{cases} U_a^{PT}(p_d^{*PT}, p_a^{*PT}) = p_a^{*PT'} \cdot M_d \cdot p_d^{*PT} = 0, \\ U_d^{PT}(p_d^{*PT}, p_a^{*PT}) = p_d^{*PT'} \cdot M_a \cdot p_a^{*PT} = 0. \end{cases} \tag{8.16}$$

Although, at the mixed NE the indifference principle holds, $M_a \cdot p_a^{*PT} \neq 0$ *due to the nonlinear weighting effect. Thus,*

$$F_{PT}^{v} = \frac{p_a^{*PT'} \cdot \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 2 & 2 & 0 & 0 & 2 \\ 4 & 0 & 4 & 0 & 4 & 0 \\ 12 & 12 & 0 & 12 & 0 & 0 \end{bmatrix} \cdot p_d^{*PT}}{p_a^{*PT'} \cdot \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \cdot p_d^{*PT}} \tag{8.17}$$

Since the denominator is not 0*, then* F_{PT}^{v} *can be computed.*

Given Theorems 10 and 11, we can show the following result:

Corollary 1 *There exists a minimum fine value* F_{min}*, such that, the utility of attacker will be positive (the attacker wins overall) under both EUT and PT, i.e.* $U_a > 0, U_d < 0$*.*

Proof: Based on Theorem 10 and Theorem 11, it can be shown that the utilities of attacker and defender intersect at 0*. Also, the derivative of the utility with respect to F can be easily seen to be monotonic. Thus, there exists a fine* F_{min}*, such that* $U_a > 0$ *and* $U_d < 0$*.*

8.4.2 Numerical Results

In this subsection, we run extensive simulations for understanding how PT and EUT considerations impact the hardware trojan detection game. To obtain the mixed Nash equilibrium under EUT and PT, we use the proposed algorithm in Table 8.1. The initial strategies are chosen as follows: we choose the attacker's initial strategy set as $p_a = [0.2083\ 0.1667\ 0.3333\ 0.2917]^T$ and the defender's initial strategy set as

$p_d = [0.2051\ 0.2564\ 0.2564\ 0.0513\ 0.0513\ 0.1795]^T$. In the subsequent simulations, we assume that the fine for all trojans is equal to $F_{s_a} = F = 8$, $\forall s_a \in \mathscr{S}_a$, unless stated otherwise. We vary the values of the rationality parameters α_a (for the attacker) and α_d (for the defender).

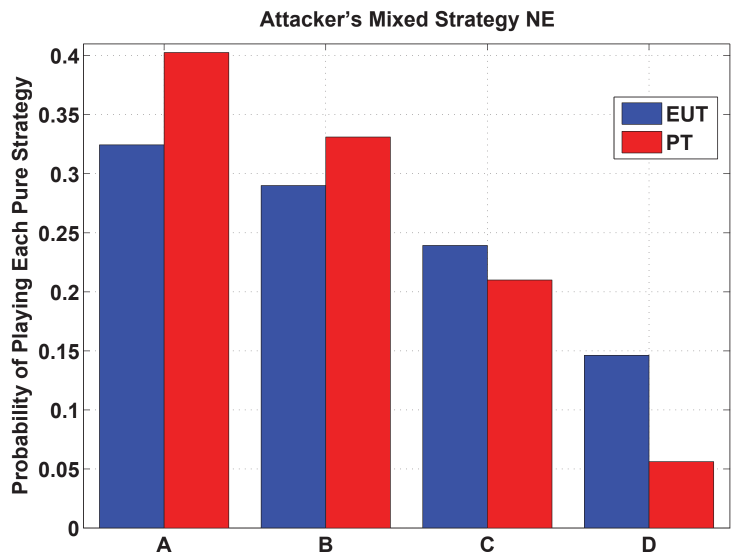

Figure 8.1: Attacker mixed-strategies at the equilibrium for both EUT and PT with $\alpha_a = \alpha_d = 0.5$.

Fig. 8.1 shows the four mixed strategies for the attacker at both the EUT and PT equilibria reached via fictitious play. In this figure, we choose $\alpha_a = \alpha_d = 0.5$ for both attacker and defender under PT. Here, we can first see that, the equilibrium mixed s-trategies of the attacker are differen between PT and EUT. Under PT, the attacker is more likely to insert trojans such as A or B whose value is less than C and D. This result shows that the attacker becomes more risk averse under PT and, thus, it will aim to insert low-valued trojans, rather than focus on higher valued trojans who are more likely to be detected due to their prospective damage. The impact of such risk aversion on the defender's behavior at the equilibrium is more pronounced as seen in Fig. 8.2.

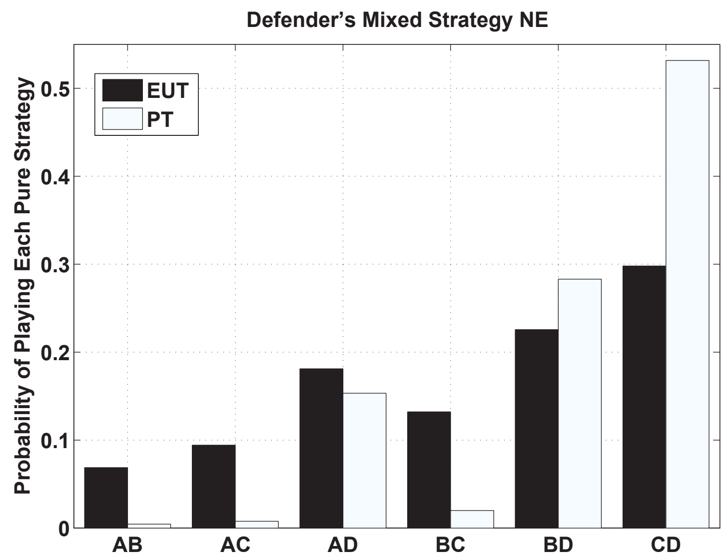

Figure 8.2: Defender mixed-strategies at the equilibrium for both EUT and PT with $\alpha_a = \alpha_d = 0.5$.

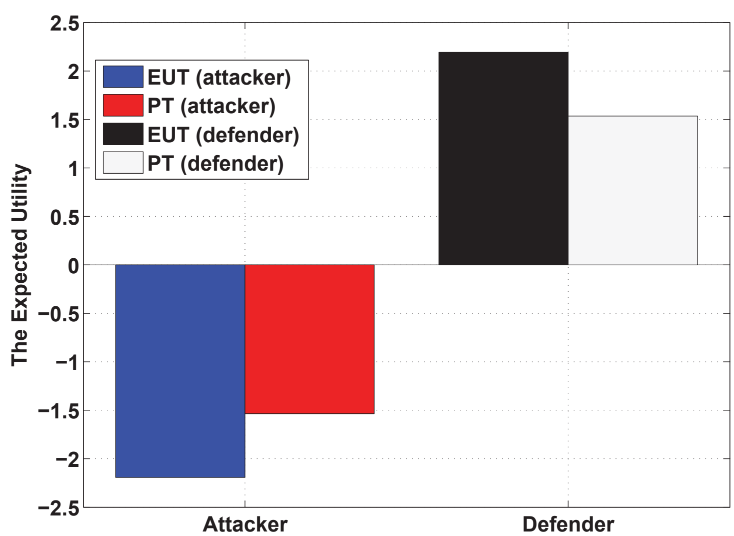

Figure 8.3: Expected utility at the equilibrium for the attacker and the defender under both EUT and PT with $\alpha_a = \alpha_d = 0.5$.

Under PT, the defender will more aggressively attempt to test for the trojan with the highest damage. In this respect, we can see that, under PT, the defender will have a 55% likelihood to test for the two most damaging trojans while ignoring the tests that pertain to trojans *A* and *B*.

A conservative PT-based defense approach coupled with a risk-averse attacker will naturally leads to a lower overall probability detection and, thus, will compromise the system more, when compared with the fully rational path of EUT. In other words, compared to rational EUT, the attacker is more likely to win in the PT scenario in which both attacker and defender deviate from the rational behavior. This result is corroborated in Fig. 8.3. In this figure, we show the expected utility of the attacker and defender, at both the EUT and PT equilibria. Clearly, under PT, the attacker is able to incur more damage, as opposed to EUT, and thus, the overall value of the game decreases from 2.1930 to 1.5356; a 30% decrease in utility!

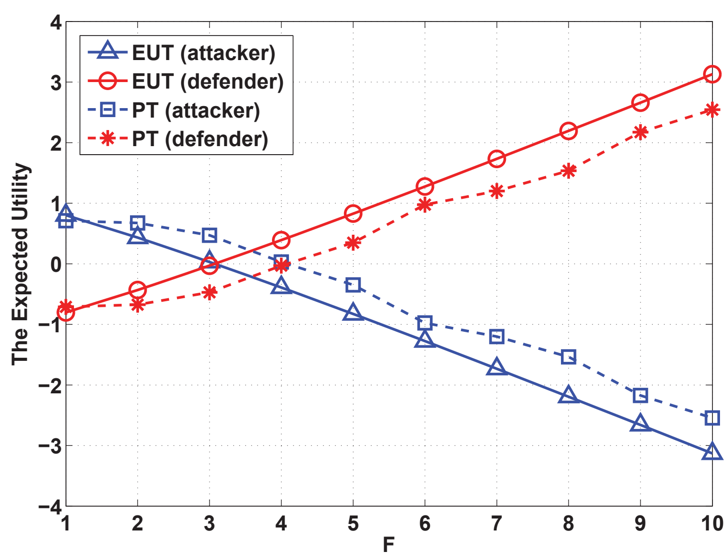

Figure 8.4: The utility performance as the value of the fine F varies for both EUT and PT with $\alpha_a = \alpha_d = 0.5$.

In Fig. 8.4, we show the expected utility for both PT and EUT, as the value of the fine varies for $\alpha_a = \alpha_d = 0.5$. This result is used to highlight the impact of the value of the fine and corroborates some of the insights of Theorems in Subsection 8.4.1. First, Fig. 8.4 shows the expected result that, as the fine value increases, the overall utility achieved by the defender will increase while that of the attacker will decrease, for both EUT and PT. In this figure, we can see that, under EUT, based on Theorem 10, the value of the fine for which neither the attacker nor defender wins is 3. In particular, at the crossing point, the attacker's MSNE is $p_a^* = [0.3818\ 0.3022\ 0.2133\ 0.1027]$ and we could compute $F_{\text{EUT}}^{v} = 3.0491$ in (8.15). For higher values, Fig. 8.4 shows that, under EUT the defender starts to win. More interestingly, we can see via Fig. 8.4 that, for PT, the value of the fine for which the utilities are 0 is 4 which is greater than EUT. This implies that, under irrational behavior and uncertainty, the defender must set higher fines in order to start gaining over the attacker. Moreover, Fig. 8.4 shows that, for this choice of α_a and α_d, the defender is better off under EUT rather than PT.

In Fig. 8.5, we study the case in which both attacker and defender have an equal rationality parameter, i.e., $\alpha_a = \alpha_d = \alpha$. In this figure, we show the equilibrium mixed-strategy probability for the most damaging strategy D for the attacker and the most defensive strategy CD, for the defender. Fig. 8.5 shows very interesting insights on the trojan hardware detection game. First, for games in which both defender and attacker significantly deviate from the rational path ($\alpha < 0.3$), the outcome of the game leads to both players using their most conservative strategies, with probability 1. This directly implies that, for this highly irrational case, the defender will always emerge as a winner. In contrast, for the regime $0.3 \leq \alpha \leq 0.7$, under which both attacker and defender are not completely rational (but have equal rationality level), the attacker becomes less likely to use its most damaging strategy D while the defender becomes more likely to

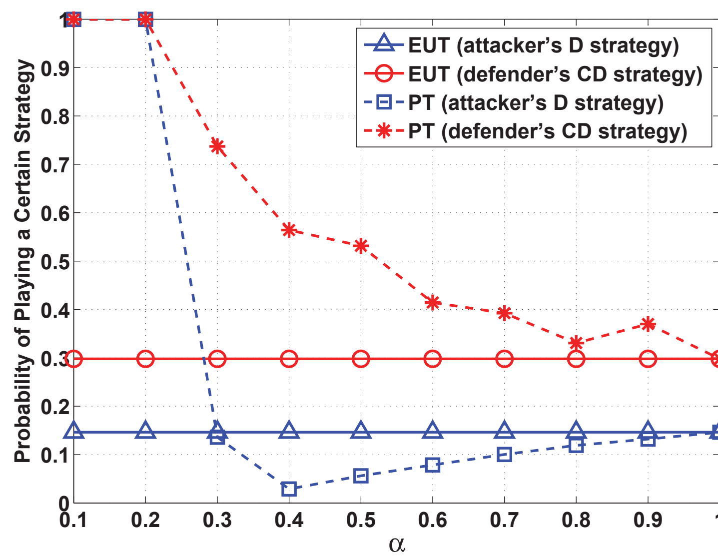

Figure 8.5: Equilibrium mixed strategies under PT and EUT for the most conservative defender and attacker options as the rationality of both players $\alpha_a = \alpha_d = \alpha$ varies.

use its most protective strategy *CD*.

Fig. 8.6 shows the expected utility at both the PT and EUT equilibria for a scenario in which the defender is completely rational ($\alpha_d = 1$) while the attacker has a varying rationality parameter. Fig. 8.6 shows that, under a completely rational defense strategy, the EUT performance will upper bound the attacker's performance. In other words, the attacker cannot do better than by behaving somewhat in line with the rational path, as the two utiliies coincide for $\alpha_a > 0.3$. Moreover, under a perfectly rational defense strategy, the attacker will immediately be detected if it deviates significantly from the EUT behavior, as evidenced in Fig. 8.6 by the expected utility achieved for $\alpha < 0.3$.

In Fig. 8.7, we consider the case in which the attacker is completely rational $\alpha_a = 1$ while the defender has varying rationality level. Fig. 8.7 shows that, as the rationality of the defender increases, its defense mechanism will perform better. Indeed, by avoiding extremely conservative and irrational perceptions on the attack strategy, i.e., for $\alpha_d \geq$

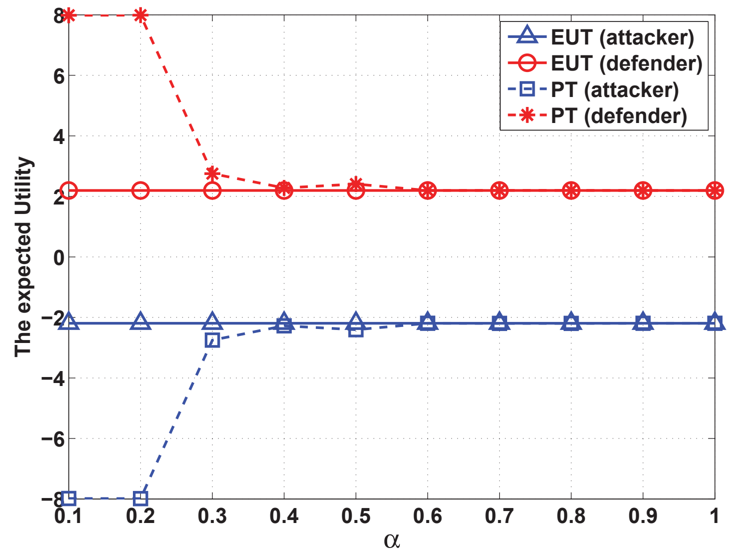

Figure 8.6: The expected utility at the equilibrium as the rationality of the attacker varies, under a completely rational defender with $\alpha_d = 1$.

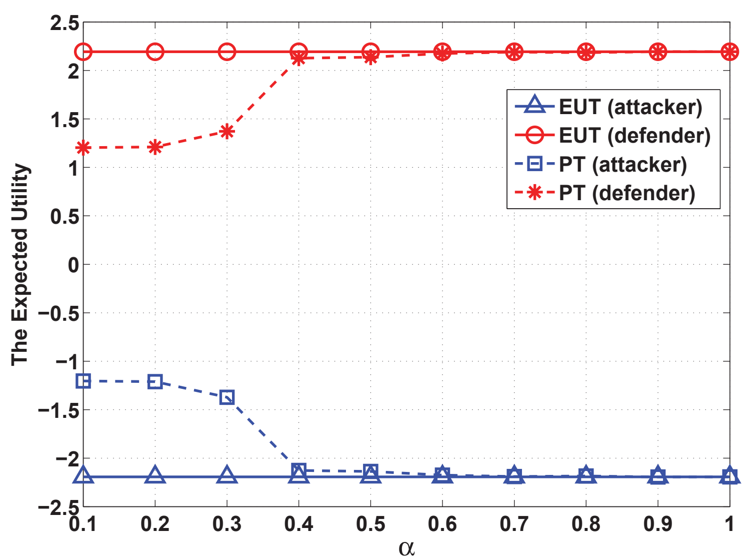

Figure 8.7: The expected utility utility at the equilibrium as the rationality of the defender varies, under a completely rational attacker with $\alpha_a = 1$.

0.4, the defender can maintain the performance of the system within the bounds of the fully rational, EUT behavior, even if its own rationality is below that of the attacker. In contrast, for $\alpha_d < 0.4$, the fully rational attacker will be able to exploit its rationality advantage and will thus have better chances of damaging the system. This damage increases with decreasing α_d. The worst-case system operation in this scenario occurs when the defender has a rationality parameter of $\alpha_d \leq 0.2$.

8.5 Conclusions

In this chapter, we have proposed a novel game-theoretic approach for modeling the interactions between hardware manufacturers, who can act as attackers by inserting hardware trojans and companies or agencies that act as defenders that test the circuits for hardware torjans. We have formulated the problem as a noncooperative game between the attacker and the defender, in which the attacker chooses the optimal trojan type to insert while the defender chooses the best testing strategy, depending on the trojan types. To account for the uncertainty and risk in the decision making processes, we have proposed a novel framework, based on the emerging tools of prospect theory, for analyzing the proposed game. To solve the game for both conventional game theory and for prospect theory, we have proposed a fictitious play-based algorithm and shown its guaranteed convergence to an equilibrium point. Thorough analytical and simulation results have been derived to assess the outcomes of the proposed games. Our results have shown that the use of prospect-theoretic considerations can provide insightful information on how irrational behavior, uncertainty, and risk can impact the interactions between attacker and defender in a hardware trojan detection game. In a nutshell, this chapter mainly analyzes uncertainty and user-centric decision making in security scenarios.

Chapter 9

Summary and Future Works

In this chapter, our developed approaches are summarized in Section 9.1 while discussing practical time requirements in Section 9.2. At the end, we outline the future works in Section 9.3.

9.1 Summary

The major results of this thesis are summarized as follows:

9.1.1 Energy Trading between Multiple Customers in the Smart Grid

Our work has focused on how to optimize the usage of storage units, under a variety of pricing schemes, while also integrating user behavior. First, a noncooperative game coupled with a double auction was proposed to study the complex interactions between a number of storage units seeking to sell part of their stored energy surplus to other smart grid elements. Here, the storage units can be either home-owned storage devices or electric vehicles. In this game, each storage unit could choose the maximum amount of energy surplus that its owner is expecting to sell so as to optimize a utility function that captures the benefits from energy selling as well as the associated costs. For this work, we have shown the existence of a Nash equilibrium and studied underlying properties as discussed in Chapter 4. Simulation results have shown that the proposed approach enables the storage units to act strategically while improving their average utilities.

9.1.2 Prospect Theory for Smart Grid Energy Storage Management

To further study customers' behaviors in smart grid, we have introduced a novel approach for dealing with the problem and integration of customer-owned energy storage in the smart grid. We have developed a novel game-theoretic approach, based on prospect theory, using which each player subjectively observes and determines its actions so as to optimize a utility function that captures the benefit from selling energy as well as the associated regulation penalty. In this case, we introduced a two-player game in Chapter 5, in which both users seek to optimize a utility function that captures the benefits from discharging as well as the payment of charging. We have shown the existence of an equilibrium for both EUT and PT. Simulation results have shown that prospect theory enables the power company to better decide on its pricing parameters, given realistic behavior of the users which can deviate considerably from the conventional rational behavior.

Compared to the subjective actions, we have also introduced another framework of prospect theory, and have studied how customer evaluates its utility using the same instance of charging/discharging operation equipped with the customer-owned energy storage. As discussed in Chapter 6, each player can frame its payoffs and can determine its actions for capturing the maximum value of a viewed utility, instead of the objective utility function. In particular, we have stated the properties of the Nash equilibrium of the game while accounting for the impact of utility framing. For the two-player scenario, our simulations have shown that prospect theory provides insightful results on how the customers' perceptions of their utility functions can affect the operation of energy storage units.

9.1.3 Prospect Theory for Demand-Side Management

In Chapter 7, we have introduced a novel approach for studying the problem of demand-side management. In particular, we have focused on customer behavior, in terms of the possible participating DSM hours, via the prospect-theoretical weighting effect. We have developed a game-theoretic approach, using which each player subjectively observes other players' actions and determines its own participating time so as to minimize a cost function that captures the electricity cost over 24 hours. For this multi-player scenario, we have proposed an algorithm and have proved that it reaches to a mixed ε-NE. Simulation results have shown that deviations from classical, objective game-theoretic DSM mechanisms can lead to unexpected results and loads on the grid, depending on the rationality level of the customers and their risk aversion.

9.1.4 Hardware Trojan Detection

We have proposed a novel game-theoretic approach for modeling the problem of testing for hardware trojans in Chapter 8. The attackers can insert hardware trojans while companies or agencies that act as defenders can test the circuits for hardware torjans. To account for the uncertainty and risk in the decision making processes, we have proposed a novel framework, based on prospect theory, for analyzing the proposed game. To solve the game for both conventional game theory and for prospect theory, we have proposed a fictitious play-based algorithm and shown its guaranteed convergence to an equilibrium point. Our results have shown that the use of prospect-theoretic considerations can provide insightful information on how irrational behavior, uncertainty, and risk can impact the interactions between agents in energy management.

9.2 Discussions on Game-theoretic Convergence time

For a multi-player game, the focus of this section is on the time convergence of a learning algorithm so as to provide some hints for practical deployments of the proposed mechanisms. Learning in a multi-agent system, such as the smart grid, is a challenging problem due to three main characteristics. First, designing an algorithm should adapt to changes in the environment, such as the departure or arrival of users. Second, due to the increasing number of users, it is necessary to ensure the scalability of the grid functions. Last but not the least, any energy management algorithm might be required to operate within a certain time limitations in practice.

The first challenge can generally be addressed by repeating the proposed algorithms, periodically, over time, so as to adapt to environmental variations such as when the number of users changes, or by using a dynamical process that a user can adopt to optimize its action via observing what other users have done in the past. On the one hand, strategic form games can be solved by repeating the studied game over time. For example, when a given number of users come in or leave from the multi-agent grid, the new game will depend on the current number of users with respective actions. Thus, the learning algorithm can be executed repeatedly to deal with this case. On the other hand, one can extend the results to a stochastic game model in which the actions themselves are directly function of time. This order might have been assumed before the "coming in" and "leaving out" happen in practice. In this thesis, we have focused on designing an algorithm in a static game and then, extended it to the case of varying users via repeating the proposed algorithm as shown in Fig. 4.7.

To enable the proposed approaches to handle a large population of users, one can develop heuristic approaches to convert the required number of agents in large-scale system into a small-scale system. For example, for a large number of agents, the first step using game-theoretic analyzing tools is to group those who have similar actions, such as the trading amounts in energy market, into a single, larger player, so as to decrease the effective number of players in the game. Moreover, based on the iteration step, some players will choose their dominant strategy for all possible cases in a game. Excluding such players can help us to handle large user populations. Last, for our proposed algorithm, there exists a inertia rate to tune the time to iterate as well as response to the scale of grid.

To meet a certain time limitation and deal with thousands of users, the convergence time should be manageable for most energy management centers. Based on our worst cases in energy trading, running a 20×100 different initial game settings on our laptop requires about 150 minutes. To the best of our knowledge, the computers, or server-s, in energy management center have more powerful computational capabilities than a personal laptop, and we can assume that the center can deal with such data based on an acceptable computational complexity. For example, our paper [126] related to Chapter 4 has provided the computational complexity for the proposed algorithm. For a sequential algorithm, the worst-case computational complexity is $O\bigg((L-1)(L+M)\bigg)$, where L, M are the number of sellers and buyers, respectively. For the parallel algorith-m, the proposed approach would require a lower worst-case computational complexity $O(L+M)$. This result shows that, the energy management centers can use a parallel algorithm for thousands of users, because for thousands of users, parallel algorithm has more advantages than the sequential one. Here, we need to stress that, for small number of users, the sequential algorithm needs less time than the parallel algorithm. However, for the practical problems, one can choose the parallel algorithm since it scales better with the number of users.

9.3 Future Works

The developed techniques have a lot of potential for extension. For example, it is of interest to extend the model of energy trading to a dynamic game model in which all players could time-dependently observe each others' strategies as well as the grid's state and dynamically determine their underlying actions. Another potential work is to study the use of behavioral PT-based games to investigate continuous strategy state for smart grid energy management problems. Next, the thesis will discuss some problems and remaining research tasks.

9.3.1 Building a General Model to Extend the Static Game to a Dynamic Game Model

We have proposed a game-theoretic algorithm to solve a double auction game in energy trading using a utility function that captures the benefits from energy selling as well as the associated costs. This work can be extended to a dynamic game model, when the participating customers (and possibly the power company) must make their energy management decisions based on a time-dependent observation of each others' strategies and corresponding grid's states. In this respect, the work done so far will serve as a basis for developing such a more elaborate dynamic game model in which players can make long-term decisions with regard to their energy trading processes.

Extending our static game to a dynamic game model, the customers can make decisions after observing dynamic information that can include customers' observation on past moves that may contingent on their previous actions. Due to this observation on past actions, the strategy might not be the final action and players can be impacted by dynamic information. For example, the dynamic game might include a "time" information that captures historical gains and losses of the system, in which the current strategy chosen by a certain player might depend on previous system situation as well as its opponents' choices. The dynamic game, or the proposed static game, can be studied from both the seller and buyer sides. Also, we will investigate such dynamics model under prospect-theoretic considerations.

9.3.2 Cumulative Prospect Theory in Smart Grid Problems

One important observation in prospect theory is the fact that users, in real life, may evaluate their utility or have a weighted observation on their opponents' actions [58–60]. In this respect, prospect theory maps all viewed evaluations using a framing effect function and changes all weighted probabilities using a weighting effect function. However, these two effects violate stochastic dominance. Stochastic dominance depends on preferences regarding outcomes, in which a preference might be a simple ranking of outcomes from favorite to least favored. Cumulative prospect theory satisfies stochastic dominance and, compared to original prospect theory, weighting is applied to the cumulative probability distribution function, as in rank-dependent expected utility theory [127] but not applied to the probabilities of individual outcomes.

For energy management, the current models and scenarios have focused on storage units and other relatively passive grid components. However, there exist some grid components, such as renewable energy, whose impact on the grid depends on natural resources and its characteristics. For example, in a wind farm, the energy generation depends on natural wind, in which its supply can be observed as a continuous curve. Then, for user, its decision in practice will correspond to the wind generation, instead of separable decisions. This stochastic grid components can be studied via cumulative prospect theory, which adopts a rank-dependent method for transforming probabilities [127]. Thus, there is a need to take such stochastic grid components into account via the advanced cumulative prospect theory.

9.3.3 Beyond Prospect Theory

Prospect theory is one approach to study the bounded rationality of participants in the smart grid. In this respect, other game-theoretic approaches might be introduced to study smart grid energy management problems. For example, notions such as the quantal equilibrium in which users make errors during their decision making processes can be explored for the smart grid. In addition, the approaches like cognitive hierarchy theory can improve the accuracy of predictions made by standard analytic methods and their applications are expected into energy management. Moreover, the exploration of prospect theory on grid can be continuously developed.

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