Geodetic Imaging of Tectonic Deformation with InSAR

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UNIVERSITY OF MIAMI

GEODETIC IMAGING OF TECTONIC DEFORMATION WITH InSAR

By

Heresh Fattahi

A DISSERTATION

Submitted to the Faculty
of the University of Miami
in partial fulfillment of the requirements for
the degree of Doctor of Philosophy

Coral Gables, Florida

August 2015
UNIVERSITY OF MIAMI

A dissertation submitted in partial fulfillment of
the requirements for the degree of
Doctor of Philosophy

GEODETIC IMAGING OF TECTONIC DEFORMATION WITH InSAR

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Precise measurements of ground deformation across the plate boundaries are crucial observations to evaluate the location of strain localization and to understand the pattern of strain accumulation at depth. Such information can be used to evaluate the possible location and magnitude of future earthquakes. Interferometric Synthetic Aperture Radar (InSAR) potentially can deliver small-scale (few mm/yr) ground displacement over long distances (hundreds of kilometers) across the plate boundaries and over continents. However, Given the ground displacement as our signal of interest, the InSAR observations of ground deformation are usually affected by several sources of systematic and random noises. In this dissertation I identify several sources of systematic and random noise, develop new methods to model and mitigate the systematic noise and to evaluate the uncertainty of the ground displacement measured with InSAR. I use the developed approach to characterize the tectonic deformation and evaluate the rate of strain accumulation along the Chaman fault system, the western boundary of the India with Eurasia tectonic plates.

I evaluate the bias due to the topographic residuals in the InSAR range-change time-series and develope a new method to estimate the topographic residuals and mitigate the effect from the InSAR range-change time-series (Chapter 2). I develop a new method to
evaluate the uncertainty of the InSAR velocity field due to the uncertainty of the satellite orbits (Chapter 3) and a new algorithm to automatically detect and correct the phase unwrapping errors in a dense network of interferograms (Chapter 4). I develop a new approach to evaluate the impact of systematic and stochastic components of the tropospheric delay on the InSAR displacement time-series and its uncertainty (Chapter 5). Using the new InSAR time-series approach developed in the previous chapters, I study the tectonic deformation across the western boundary of the India plate with Eurasia and evaluated the rate of strain accumulation along the Chaman fault system (Chapter 5). I also evaluate the co-seismic and post-seismic displacement of a moderate M5.5 earthquake on the Ghazaband fault (Chapter 6).

The developed methods to mitigate the systematic noise from InSAR time-series, significantly improve the accuracy of the InSAR displacement time-series and velocity. The approaches to evaluate the effect of the stochastic components of noise in InSAR displacement time-series enable us to obtain the variance-covariance matrix of the InSAR displacement time-series and to express their uncertainties. The effect of the topographic residuals in the InSAR range-change time-series is proportional to the perpendicular baseline history of the set of SAR acquisitions. The proposed method for topographic residual correction, efficiently corrects the displacement time-series. Evaluation of the uncertainty of velocity due to the orbital errors shows that for modern SAR satellites with precise orbits such as TerraSAR-X and Sentinel-1, the uncertainty of 0.2 mm/yr per 100 km and for older satellites with less accurate orbits such as ERS and Envisat, the uncertainty of 1.5 and 0.5 mm/yr per 100 km, respectively are achievable. However, the
uncertainty due to the orbital errors depends on the orbital uncertainties, the number and
time span of SAR acquisitions.

Contribution of the tropospheric delay to the InSAR range-change time-series can be
subdivided to systematic (seasonal delay) and stochastic components. The systematic
component biases the displacement times-series and velocity field as a function of the
acquisition time and the non-seasonal component significantly contributes to the InSAR
uncertainty. Both components are spatially correlated and therefore the covariance of
noise between pixels should be considered for evaluating the uncertainty due to the
random tropospheric delay. The relative velocity uncertainty due to the random
tropospheric delay depends on the scatter of the random tropospheric delay, and is
inversely proportional to the number of acquisitions, and the total time span covered by
the SAR acquisitions.

InSAR observations across the Chaman fault system shows that relative motion between
India and Eurasia in the western boundary is distributed among different faults. The
InSAR velocity field indicates strain localization on the Chaman fault and Ghazaband
fault with slip rates of ~8 and ~16 mm/yr, respectively. High rate of strain accumulation
on the Ghazaband fault and lack of evidence for rupturing the fault during the 1935
Quetta earthquake indicates that enough strain has been accumulated for large (M>7)
earthquake, which threatens Balochistan and the City of Quetta. Chaman fault from
latitudes ~29.5 N to ~32.5 N is creeping with a maximum surface creep rate of 8 mm/yr,
which indicates that Chaman fault is only partially locked and therefore moderate
earthquakes (M<7) similar to what has been recorded in last 100 years are expected.
Acknowledgement

Many people have helped me during this dissertation, and I would like to thank them for their supports and assistance. First and foremost I thank my advisor, Falk Amelung, who supported and guided me relentlessly during my PhD. I’m also grateful to my committee members, Timothy H. Dixon, Shimon Wdowinski and Guoqing Lin, for their support, time, and many advices. I would like to thank Petar Marinkovic for his helps with correcting the local oscillator drift of the Envisat ASAR data, Michiel Otten who helped to better understand the uncertainty of ERS and Envisat orbits, Eric Fielding for his helps to use the MODIS products for troposphere precipitable water vapor and Brian Mapes for nice discussions on troposphere. I would like to thank Corne Kreemer for his support and for the GPS velocities across the southern San Andreas fault. I would like to owe my special thanks to my friend and colleague, Estelle Chaussard who has been extremely supportive and helpful specially in last two years of my study.

I would like to thank my friends and colleagues in the InSAR-lab, Fernando Green for useful discussions on long-wavelength noise in InSAR data, Marco Bagnardi for nice discussion and collaboration on the topographic residual in the InSAR time-series, Scott Baker who inspired me to learn python, Batuhan Osmanoglu and Sang-wan Kim for nice discussion on InSAR time-series analysis, Hyung-Sup Jung for his kind support with focusing of SAR data and my friend and officemate Qiong Zhang and all my friends and colleagues in the Geodesy and Seismology Group, Emanuelle Feliciano, Anieri Morales, Yunjun Zhang, Talib Oliver, Emre Havazali, Dario Solano, Wenliang Zhao amd Peng Li for their company, support and InSAR discussions at lunch time at the weekly Geodey
meetings and at wetlab. I woul like to thank my friend Saeed Roshan, the only chemist
who learnt InSAR principles by having lunch with the Geodesy group.
This journey would not have been possible without support from my family and friends. I
am specially grateful to my parents, sisters and brothers, and my in-laws for their
continuous unconditional support and encouragement. And last but not the least I would
like to owe my special thanks to my wife, Yalda Zarnegarnia. Without her patience and
sacrifice I could not have completed this dissertation.
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Chapter 1. Introduction

To reduce the societal effects of geo-hazards such as earthquakes, volcanic eruptions, land-subsidence, landslides, sinkholes, climate change and sea level rise, we need to study the processes that cause them. Many of these natural and anthropogenic processes deform the ground surface with variable spatial and temporal characteristics. Precise measurements of ground surface deformations are the key observations to characterize and understand the processes involved in different geo-hazards.

1.1 Location and magnitude of future earthquakes

In plate tectonics, the earth’s lithosphere, which is referred to the outermost shell of the planet consisting of crust and uppermost mantle, is divided to several rigid tectonic plates, which are moving relative to each other. The relative motions between tectonic plates are mostly accommodated by slipping on faults located in the boundary between the plates. Two central assumptions of plate tectonics including the rigidity of tectonic plates and localized deformation at narrow plate boundaries have been contradicted with evidences for internal deformation of the plates and for distributed and diffuse deformation across wide plate boundary zones [Gordon, 1998].

Shear deformation in localized transform boundaries may be: a) accommodated by a major fault such as the central San Andreas fault [Jolivet et al., 2014a], b) distributed among different faults roughly parallel to the plate motion direction such as the southern San Andreas fault system where relative plate motion between pacific and north America tectonic plates is distributed among San Andreas, San Jacinto and Elsinore faults [Fialko, 2006], or c) by bookshelf faulting such as the southern Iceland where shear motion is
accommodated by an array of faults trending perpendicular to the shear direction [Sigmundsson et al., 1995].

Although at deeper parts, faults generally slip aseismically, at shallower seismogenic depths some regions on a fault surface may slip aseismically, with a rate less than or equal to the long term slip rate at depth, while other regions may be fully locked accumulating elastic strain that can drive future earthquakes. An earthquake is the result of sudden release of the accumulated strain and can be explained with stick-slip friction model [Brace and Byerlee, 1966]. In this simple model, when the ratio of shear to normal stress overcomes the static friction co-efficient, the fault slips and the frictional resistance drops to a lower dynamic friction co-efficient. Parts of the accumulated stress releases during the co-seismic rupture and parts may release days to decades later during a post-seismic period [Thatcher and Rundle, 1979]. This mechanical model of earthquake has been widely revised by the rate and state variable constitutive law [Scholz, 1998].

In rate- and state- dependent friction laws, shear stress supported by a frictional surface is a function of the slip rate and state variables characterizing the state of asperity contacts as [Scholz, 1998]:

\[
\tau = \mu_0 + \left[ a \ln\left( \frac{v}{v_0} \right) + b \ln\left( \frac{v_0 \theta}{L} \right) \sigma \right] 
\] (1.1)

Where \( \tau \) is the shear stress and \( \sigma \) is effective normal stress (applied normal stress minus pore pressure), \( v \) is slip rate and, \( \mu_0 \) is the friction coefficient at a steady-state slip rate of \( v_0 \), \( a \) and \( b \) are empirical constants related to material properties, \( L \) is a critical slip distance and \( \theta \) is a state variable that evolves with time. If \((a-b)<0\) the fault zone is governed by a velocity weackening behavior where friction decreases with sliding
velocity. Earthquake usually nucleate at regions of fault surface with velocity weakening frictional behavior. If (a-b)>0 frictional behavior is velocity strengthening, which allows for aseismic slipping of faults. A fault region could be also “conditionally stable” in which the system may become unstable under sufficiently strong dynamic loading. Earthquakes may nucleate in the unstable field, but may propagate into the conditionally stable field.

Regardless of the mechanical model of the rupture, characterizing the areas where strains are accumulated and the rate of strain accumulation are crucial to understand the location and magnitude of the future earthquakes. Such analysis is possible through precise measurements of ground deformation across the locked faults.

The derivative of the rate of ground displacement over distance gives the strain rate. Across a locked fault the peak strain rate occurs above the fault given as [Segall, 2010]

$$\dot{\varepsilon} = \frac{V}{\pi D}$$  \hspace{1cm} (1.2)

The size of an earthquake depends on the amount of accumulated strain and is quantified with the seismic moment, which its rate of accumulation is given as [Segall, 2010]

$$\dot{M} = \mu LDV$$  \hspace{1cm} (1.3)

where $\mu$ is the shear modulus in Pa representing the elastic properties of crust, L is the length of a fault, D is the depth at which the fault is locked accumulating strain with a rate of V. Therefore the unit of $\dot{M}$ is N.m. Multiplication of $\dot{M}$ by the time since the last major earthquake gives the first order estimate of the accumulated seismic moment (M) which can be used to calculate the moment magnitude of the earthquake as:
\[ M_w = \frac{2}{3} \log_{10}(M) - 6.07 \] (1.4)

For example, a 150 km segment of a strike-slip fault locked from surface to a depth of 15 km, slipping with a rate of 13 mm/yr below the locking depth, corresponds to Mw 7.3 earthquake every 300 years.

Measuring ground deformation across the plate boundaries reveal where the strain is localized (maximum strain rate) which helps to understand the location of future earthquakes. The surface displacements can be also used to estimate the slip rates (V) and locking depths of faults (D), which can be used to evaluate the magnitude of future earthquakes.

1.2 Models of deformation for crust and mantle

The deformation of rocks in response to forces in Earth’s interior is governed by rock rheology, which is the study of the flow and deformation of all forms of matter [Bürgmann and Dresen, 2008]. In plate tectonics the stronger lithospheric plates move over weaker underlying asthenospheric mantle. Several models have been proposed to describe the strength of lithosphere including the Jelly sandwich, crème brûlée and split banana. In the jelly sandwich model, a weak middle and lower crust is sandwiched between stronger upper crust and mantle. In the crème brûlée model the strong crust is overlaid on the significantly weaker upper mantle. In the banana split model there is a lateral variation of the strength of lithosphere with a reduction of strength along the plate boundaries due to various weakening processes involving thermal, fluid and strain rate effects [Bürgmann and Dresen, 2008].

To quantify the rheology of crust and mantle several constitutive laws including Hooke law, Maxwell and Burgers bodies have been proposed to relate the stress to strain
and strain rate. In Hooke law the rheology of the upper crust is explained by linear elastic relation between stress and strain. Deformation of Maxwell fluid is modeled with the sum of the viscous and elastic responses. Burger body can be used to represent material responses with more than one relaxation time. The Burgers body exhibits early Kelvin solid behavior and a long-term Maxwell-fluid response [Bürgmann and Dresen, 2008].

1.3 Space geodesy to understand the tectonic deformation

Space geodesy in last few decades has been widely used to characterize the ground deformation and to evaluate the rate of strain accumulation and release across and along the plate boundaries. Areas characterized with high rate of strain accumulation from Global strain rate map obtained from GPS observations are well consistent with recorded and historical earthquakes [Bird & Kreemer, 2014]. Wright et al, [2013] estimated the global seismogenic thickness of $14 \pm 5$ and $14 \pm 7$ km from coseismic and interseismic geodetic observations of crustal displacement based on 78 earthquake source mechanisms for continental earthquakes and 187 estimates of interseismic locking depths from space geodetic observations reported in the literature.

Although dense networks of permanent GPS observations in few plate boundaries, such as western United States and Japan have provided long time-series (decadal) and accurate observations of ground deformation, such observations are not available for many other regions on earth such as the Chaman fault system, which forms the western boundary of the India plate with Eurasia.

Interferometric Synthetic Aperture Radar (InSAR) is the most promising technique to fill the gaps of geodetic measurements across different plate boundaries and to evaluate strain across continents. However, this requires precise measurements of long-
wavelength (e.g., >50 km) ground displacement with high spatial resolution. Although InSAR has been widely used to study different natural and anthropogenic processes deforming the solid earth with short wavelength ground displacement, the application of the technique to study the long-wavelength ground displacement such as those due to strain accumulation across locked faults is limited due to the fact that InSAR measurements of ground displacements are usually affected by contributions other than ground displacement. Given ground displacement as our signal of interest, InSAR observations are usually affected by several sources of noise and high-resolution deformation fields are only achievable if the signal (ground displacement) is properly separated from noise. In particular, the main obstacle to measure long-wavelength displacement (e.g., due to tectonic deformation) with InSAR is the spatially correlated long-wavelength noise with similar spatial pattern and frequency as the signals.

Therefore, the main objectives of this dissertation are to:

a) develop a methodology to measure long-wavelength ground displacement with InSAR and to quantify the uncertainty of the measurements.

b) apply the developed method to characterize the tectonic deformation and evaluate the rate of strain accumulation along the Chaman fault system, the western boundary of the India tectonic plate with Eurasia.

Achieving the goals of this dissertation requires a proper definition of signal, random and systematic noise in different steps of the InSAR time-series analysis.

1.4 Signal, random noise and systematic noise in InSAR time-series analysis

A linear model of observation equations, \( y = Ax + e \), can be characterized with Gauss-Markov model as
\[ E\{y\} = Ax \quad D\{y\} = Cy \]  

(1.5)

where \( y \) is a \( m \times 1 \) real stochastic vector of observations, \( A \) is the \( m \times n \) design matrix, \( x \) is the \( n \times 1 \) vector of real and non-stochastic un-known parameters, \( E \) and \( D \) denote the expectation and dispersion operators respectively, \( e \) is the observation error with \( E\{e\}=0 \), and \( Cy \) is the variance-covariance matrix. The first part of the equation is the functional model and the second part is the stochastic model. The Gauss-Markov model enables us to evaluate the uncertainty of the estimated unknown parameters given the uncertainty of the measurements expressed by the variance-covariance matrix (\( Cy \)).

The variance-covariance matrix of the geodetic observations such as GPS coordinate time-series have been commonly modeled with white noise, random-walk and flicker noise. The first is temporally uncorrelated and the two others are correlated noise. Large number of epochs in GPS coordinate time-series (thousands of epochs) has provided the opportunity to evaluate the noise from the GPS observations [Zhang et al., 1997; Mao et al., 1999; Dmitrieva et al., 2015]. In contrast, limited number of epochs of the InSAR range-change time-series (eg: ~40 to ~50 for Envisat and ERS and ~20 to ~30 for ALOS-1), together with the relative observations between pixels and spatial correlation of different components of noise in InSAR observations have prevented proper expression of the variance-covariance matrix of the InSAR measurements based on the InSAR range-change time-series observations.

In this dissertation, I tackle this problem by distinguishing between random noise and systematic noise in the InSAR time-series observations. The random noise should be evaluated with the stochastic model and the systematic noise, which bises the signal, with
the functional model. I use the Gauss-Markov model in two main steps of the InSAR time-series analysis:

1) to invert the interferometric phases for estimating the range-change time-series and afterwards to obtain the displacement time-series

2) to estimate the velocity fields of linear ground displacement from InSAR displacement time-series and express the uncertainty of the InSAR velocity fields with variance-covariance matrix.

The definition of signal, random and systematic noise for the two steps above is different for estimating the range-change time-series, displacement time-series and the velocity of the linear ground displacement.

In chapter 2 I evaluate the contribution of the topographic residual to the InSAR range-change time-series and develop a functional model to estimate and remove the bias due to the topographic residual from the range-change time-series.

In chapter 3 I evaluate the contribution due to random orbital errors to the InSAR range-change time-series using a stochastic model.

In chapter 4 I develop a method to estimate the unwrapping error and eliminate the bias due to the error from the InSAR range-change time-series.

In chapter 5 I apply the developed methodology to real InSAR data across the Chaman fault system and study the tectonic deformation across the fault system.

In chapter 6 I study a M5.5 earthquake on the southern end of the Ghazaband fault.

In Chapter 7 I present the conclusions of this dissertation and the direction of the future research.
Chapter 2. DEM error correction in InSAR time-series

2.1 Summary

We present a mathematical formulation for the phase due to the errors in Digital Elevation Models (DEMs) in InSAR time-series obtained by the Small Baseline (SB) or Small BAseline Subset (SBAS) method. We show that the effect of the DEM error in the estimated displacement is proportional to the perpendicular baseline history of the set of SAR acquisitions. This effect at a given epoch is proportional to the perpendicular baseline between that epoch and the reference acquisition. Therefore the DEM error can significantly affect the time-series results even if small-baseline interferograms are used. We propose a new method for DEM error correction of InSAR time-series, which operates in the time domain after inversion of the network of interferograms for the displacement time-series. This is in contrast to the method of Beradino et al. (2002) in which the DEM error is estimated in the interferogram domain. We show the effectiveness of this method using simulated InSAR data. We apply the new method to Fernandina volcano in the Galapagos Islands and show that the proposed DEM error correction improves the estimated displacement significantly.

2.2 Background

Differential Synthetic Aperture Radar Interferometry (InSAR) is a powerful technique to estimate the land surface deformation caused by natural and anthropogenic processes including earthquakes, volcanic eruptions, landslides, hydrological subsidence, etc [Gabriel et al., 1989; Massonnet et al., 1993; Massonnet and Feigl, 1998; Amelung et al., 1999]. The temporal evolution of surface deformation can be reconstructed from a set
of SAR acquisitions using InSAR time-series techniques, such as Small Baseline (SB) [Berardino et al., 2002; Lanari et al., 2004, 2007; Lauknes et al., 2011] or Persistent Scatterer (PS) [Ferretti et al., 2001, 2011; Hooper et al., 2004; Shanker and Zebker, 2007]. In the SB technique the phase history with respect to the first acquisition is reconstructed from a network of interferograms with small spatial and temporal baselines, which ensures maximum interferometric coherence. SBAS (Small BAseline Subset) is a well-known SB technique in which, the independent subsets of interferograms are combined to estimate the displacement history using the minimum-length solution of the phase velocity obtained by singular value decomposition [Berardino et al., 2002].

Estimation of ground displacement using InSAR requires the removal of phase contributions due to the Earth’s topography from the interferograms. The topography is approximated using a Digital Elevation Model (DEM) and then the topographic phase is estimated using information about the imaging geometry. In practice this is usually accomplished along with removing the phase due to the earth curvature in a processing step commonly referred to as flattening [Pepe et al., 2011a], carried out before interferogram filtering [Fattahi et al., 2009]. The topographic phase is proportional to the perpendicular baseline separation between the two SAR images forming the interferogram. Errors in the DEM lead to baseline-dependent phase residuals in the interferograms, which, if not removed, leads to errors in the displacement history [Berardino et al., 2002].

Here we present a mathematical formulation for the phase due to the DEM error in the InSAR time domain. We show that the phase due to the DEM error at each epoch is proportional to the perpendicular baseline between SAR acquisition at that epoch and the
reference acquisition (and not on the perpendicular baseline of individual interferograms). This explains the unexpectedly large effects of DEM errors in SBAS results noted by [Berardino et al., 2002] and why the DEM error correction is necessary despite the use of small baseline interferograms. Thus the use of small baseline interferograms does not necessarily minimize the effect of DEM error. We propose a new method to correct for DEM errors in the time domain, which is under certain conditions equivalent to the method of [Berardino et al., 2002] in the interferogram. Original SBAS uses interferometric phases to estimate the DEM error and then inverts the corrected interferograms to estimate the phase velocities [Berardino et al., 2002]. An alternative has been presented to estimate the DEM error in the ALOS Palasar data by inverting interferometric phases to jointly estimate the DEM error and phase velocities [Samsonov, 2010]. Here we present a new algorithm to estimate the DEM error after the time-series inversion of interferometric phases. Our proposed method for DEM error correction in the time-domain is different from the method of [Pepe et al., 2011b] in that we estimate the parameters of low-pass displacement instead of applying low-pass filtering. Also our method uses phase velocity history instead of phase history, which can have significant effect on DEM error estimation in case of time-variable displacement.

There are several potential advantages of the DEM error correction in the time domain compared to the interferogram domain. First, it is more efficient because design matrixes are smaller. Second, the employment of alternative temporal deformation models is straightforward (such as the offset due to an earthquake). Third, it facilitates the identification and correction of orbital errors as well as atmospheric contributions, which are most efficiently applied in the time domain [Ferretti et al., 2001; Gourmelen et al.,
Finally it is independent of the network of interferograms. As we show below, in the interferogram domain the network may affect the DEM error estimation if the displacements are not well approximated by the assumed deformation model.

### 2.3 The phase due to the DEM error in the InSAR time-series

We start with the differential interferometric phase corrected for flat earth and topographic effects. Considering an interferogram formed from two SAR acquisitions at times $t_A$ and $t_B$, we can write the following formula for each pixel [Berardino et al., 2002; Pepe et al., 2011b]:

$$
\Delta \phi(t_A,t_B) = \Delta \phi_{\text{def}}(t_A,t_B) + \Delta \phi_{\text{topo}}(t_A,t_B) + \Delta \phi_{\text{atm}}(t_A,t_B) + \Delta \phi_{\text{orb}}(t_A,t_B) + \Delta \phi_{\text{noise}}(t_A,t_B)
$$

(2.1)

where $\Delta \phi(t_A,t_B)$ represents the measured interferometric phase, $\Delta \phi_{\text{def}}(t_A,t_B)$ the phase due to the ground displacement in radar line-of-sight direction between times $t_A$ and $t_B$, $\Delta \phi_{\text{topo}}$ the interferometric phase due to the error in the DEM used for topographic phase estimation, $\Delta \phi_{\text{atm}}(t_A,t_B)$ the phase due to the differences in atmospheric delay between $t_A$ and $t_B$, $\Delta \phi_{\text{orb}}$ the phase due to the orbital error and $\Delta \phi_{\text{noise}}$ the phase noise due to different decorrelation phenomena and thermal noise.

The accuracy of the DEM, as an estimation of Earth’s topography, is limited by the method and instruments used for generating the DEM. Also the topography of earth may change by time. The height of the Earth’s topography ($z_{\text{topo}}$) is approximated by a DEM such that; $z_{\text{topo}} = z_{\text{DEM}} + z^\varepsilon$, with $z_{\text{DEM}}$ the height of the DEM and $z^\varepsilon$ the error of the DEM. After the topographic phase estimation, the phase due to the DEM error remains in
the differential interferograms and can be expressed for each pixel as [Massonnet and Feigl, 1998]:

\[ \Delta \phi_{\text{topo}}(t_A, t_B) = \frac{4 \pi}{\lambda} \frac{B(t_A, t_B)}{r \sin(\theta)} z \epsilon \] \quad (2.2)

where \( B_{\perp}(t_A, t_B) \) is the perpendicular baseline between the two SAR acquisitions, \( r \) is the range between target and SAR antenna, \( \theta \) is the look angle and \( \lambda \) is the transmitted signal central wavelength.

InSAR is a relative measurement technique measuring displacement at the time of the slave image since the master acquisition. Accordingly, InSAR time-series resolve displacement relative to a reference time. In the SBAS the first acquisition is commonly used as the reference acquisition. We substitute \( B_{\perp}(t_A, t_B) \) in (2) by the difference of perpendicular baselines between the SAR images acquired at times \( t_B \) and \( t_A \) and the reference acquisition, \( B_{\perp}(t_A, t_B) = B_{\perp}(t_B) - B_{\perp}(t_A) \), (see Appendix 2.1), and rewrite (1):

\[
\Delta \phi(t_A, t_B) = \frac{4 \pi}{\lambda} [d(t_B) - d(t_A)] + \frac{4 \pi}{\lambda} \left[ \frac{B_{\perp}(t_B) - B_{\perp}(t_A)}{r \sin(\theta)} \right] z \epsilon \\
+ [\phi_{\text{atm}}(t_B) - \phi_{\text{atm}}(t_A)] + \Delta \phi_{\text{orb}}(t_A, t_B) + \Delta \phi_{\text{noise}}(t_A, t_B) \] \quad (2.3)

where \( B_{\perp}(t_B) \) is the perpendicular baseline between the SAR acquisition at \( t_B \) and the reference acquisition. Similarly \( B_{\perp}(t_A) \) is the perpendicular baseline between the acquisition at \( t_A \) and the reference acquisition. \( d(t_A) \) and \( d(t_B) \) are the line-of-sight displacement at \( t_A \) and \( t_B \) with respect to the reference acquisition. \( \phi_{\text{atm}}(t_A) \) and \( \phi_{\text{atm}}(t_B) \) are the phase due to the atmospheric delay at times \( t_A \) and \( t_B \) respectively.

In the SBAS algorithm a set of phase-unwrapped interferograms formed from \( N + 1 \) SAR acquisitions is inverted to reconstruct the phase history, \( \phi(t) \), at \( N \) epochs (\( t_1, \ldots, t_N \)) with respect to the reference acquisition at \( t_0 \), implying \( \phi(t_0) = 0 \):
\[ \phi(t_i) = \frac{4\pi}{\lambda} d(t_i) + \phi^{\text{topo}}(t_i) + \phi^{\text{atm}}(t_i) + \phi^{\text{orb}}(t_i) + \phi^{\text{noise}}(t_i) \]  

wherein \( d(t_i) \) is the cumulative line-of-sight displacement at time \( t_i \), and \( \phi^{\text{topo}}(t_i) \), \( \phi^{\text{atm}}(t_i) \), \( \phi^{\text{orb}}(t_i) \) and \( \phi^{\text{noise}}(t_i) \) are the phase histories due to the DEM error, atmospheric delay, orbital error and noise respectively, with respect to the reference acquisition. Considering (3), the phase due to the DEM error can be expressed as:

\[ \phi^{\text{topo}}(t_i) = \frac{4\pi}{\lambda} \frac{B(t_i)}{r \sin(\theta)} z^e \]  

where \( B(t_i) \) represents the baseline history of SAR acquisitions. \( B(t_i) \) at each time epoch \( t_i \), is the perpendicular baseline between the SAR image acquired at that time and the reference acquisition (see Appendix 2.1). According to (5), it’s clear that the phase history due to the DEM error is proportional to the baseline history. This means that while selecting small spatial baseline interferograms in the SBAS method minimizes the phase due to the DEM error in individual interferograms, but it does not reduce this effect in the estimated phase history. The phase due to the DEM error in the estimated phase history is minimized only when a set of SAR acquisitions with small range of perpendicular baselines exist. This is why SBAS requires a DEM error correction although small baseline interferograms are used.

### 2.4 Proposed method for the DEM error correction

Considering (4), the estimated phase history, includes displacement and the effects of DEM error, atmospheric delay, remaining orbital errors and noise. To estimate the DEM error (\( z^e \)), following [Berardino et al., 2002] we consider a cubic temporal deformation model as follows:

\[ \phi_{\text{def}}(t_i) = \bar{v}(t_i - t_0) + \frac{1}{2} \bar{a}(t_i - t_0)^2 + \frac{1}{6} \Delta \bar{a} (t_i - t_0)^3 \]  

(2.6)
where $\bar{v}$ is the mean velocity, $\bar{a}$ is the mean acceleration and $\Delta\bar{a}$ is the mean acceleration variation. Also other assumptions including simpler linear models or more complicated models considering specific events like earthquake or volcanic eruptions could be used in (6). Accordingly the design matrix, which will follow in (9) should change too. By substituting (5) and (6) into (4) we obtain:

$$\phi(t_i) = \bar{v}(t_i - t_0) + \frac{1}{2} \bar{a}(t_i - t_0)^2 + \frac{1}{6} \Delta \bar{a}(t_i - t_0)^3 + \frac{4\pi}{\lambda} \frac{B(t_i)}{r \sin(\theta)} z^{\varepsilon} + \psi(t_i)$$  \hspace{1cm} (2.7)$$

wherein $\psi(t_i)$ represents different high frequency components including phase due to the atmospheric delay, remaining effects of orbital error, noise and high-frequency components of the displacement which was not considered in the deformation model.

Following the original SBAS method, we now rewrite (7) in terms of the phase velocity history between two consecutive epochs, $v_i = \frac{\phi(t_i) - \phi(t_{i-1})}{t_i - t_{i-1}}$, with $i = 1,...,N$ (see Appendix 2.2):

$$v_i = \bar{v} + \frac{1}{2}\bar{a}(t_i + t_{i-1} - 2t_0) + \frac{(t_i - t_0)^3 - (t_{i-1} - t_0)^3}{6(t_i - t_{i-1})} \Delta \bar{a} + \frac{4\pi}{\lambda} \frac{\dot{B}_i}{r \sin(\theta)} z^{\varepsilon} + \dot{\psi}_i$$  \hspace{1cm} (2.8)$$

where $\dot{B}_i$ is the baseline velocity history defined as $\dot{B}_i = \frac{B(t_i) - B(t_{i-1})}{t_i - t_{i-1}}$ with $i = 1,...,N$ and $\dot{\psi}_i$ is the temporal gradient of $\psi(t_i)$, considered as noise in the DEM error estimation process. To estimate the DEM error and the parameters of the assumed deformation model, a linear system with $N$ equations is formed for each pixel as follows:

$$V = AX + n$$  \hspace{1cm} (2.9)$$

wherein $V^T = [v_1,v_2,...,v_N]$ is a vector of estimated phase velocities obtained from the inversion of uncorrected interferometric phases using regular SBAS, $X^T = [\bar{v},\bar{a},\Delta \bar{a},z^{\varepsilon}]$ is
the vector of unknown parameters, \( n \) is the residual vector and \( A \) is an \((N \times 4)\) design matrix as follows:

\[
A = \begin{bmatrix}
1 & \frac{(t_1 - t_0)}{2} & \frac{1}{6}(t_1 - t_0)^2 & \frac{4\pi \hat{B}_{l1}}{\lambda r \sin(\theta)} \\
1 & \frac{(t_2 + t_1 - 2t_0)}{2} & (t_2 - t_0)^3 - (t_1 - t_0)^3 & \frac{4\pi \hat{B}_{l2}}{\lambda r \sin(\theta)} \\
& \ddots & \ddots & \ddots \\
1 & \frac{(t_N + t_{N-1} - 2t_0)}{2} & (t_N - t_0)^3 - (t_{N-1} - t_0)^3 & \frac{4\pi \hat{B}_{lN}}{\lambda r \sin(\theta)}
\end{bmatrix}
\]

(2.10)

The solution of this linear system of equations generally can be obtained by minimizing the \( L_p \) norm of residuals, which can be written as follows:

\[
\hat{X} = \arg \min \| n \|_p = \arg \min \| V - AX \|_p = \arg \min \left( \sum_{i=1}^{N} |V - AX|_p \right)^{1/p}, \quad p \geq 1
\]

(2.11)

where \( \hat{X} \) is the estimated vector of unknown parameters and \( p \) is the power. In this paper we use \( L_2 \)-norm minimization (un-weighted least-squares) to obtain the solution:

\[
\hat{X} = (A^\top A)^{-1}A^\top V.
\]

(2.12)

After the estimation of \( z^t \), the contribution to the phase is calculated using (5) and removed from \( \phi(t_i) \). The estimated phase history after DEM error correction may still be affected by the other sources of error like atmospheric delay. Thus the next step is to remove the atmospheric delays, which is beyond the scope of this paper. Usually spatial-temporal filtering is used for this purpose [Ferretti et al., 2001; Berardino et al., 2002].

### 2.5 Simulated data

We demonstrate the effect of DEM errors in InSAR time-series using simulations. We specify a displacement history, the time and perpendicular baseline of a set of interferograms (i.e. the baseline history), and a DEM error. We then generate a set of
simulated interferograms and invert it for the displacement history without DEM error correction and using different DEM error correction methods. We assess the quality of the DEM error correction method by comparison of the estimated with the original simulated displacement history.

2.5.1 Illustration of the DEM error
To illustrate the DEM error in InSAR time series we assume no ground deformation (zero displacement), a DEM error of 50 m and the network of interferograms as shown in Figure 2.1(a). The network consists of two branches of zero baseline interferograms, which connect to one single branch for the last few acquisitions. The 16 interferograms of the network have small perpendicular baseline (<200 m) with 12 of them with zero baseline. The baseline history varies by 400 m between the first 13 acquisitions and is constant for the last few acquisitions (see Figure 2.1(b)). The phase due to the DEM error is calculated using (2) employing typical parameters of the Envisat SAR sensor. The estimated displacement history after time-series inversion without considering any DEM error correction (see Figure 2.1(c)) varies by 9 cm between acquisitions although the simulated displacement history is zero. The estimated displacement in Figure 2.1(c), which is actually the time-series effect of the DEM error, is proportional to the baseline history of simulated data shown in Figure 2.1(b), as we expect from (5).
Figure 2.1 Simulated data illustrating the effect of the DEM error in InSAR time-series: (a) Baseline-time plot of network of interferograms, (b) perpendicular baseline history, (c) simulated displacement history (dashed line) and estimated displacement history without DEM error correction (solid line). The estimated displacement is affected by the DEM error and proportional to the baseline history.

2.5.2 DEM error correction

We now simulate a more realistic set of interferograms based on the real baseline history of 59 Envisat SAR acquisitions covering Fernandina volcano used in section V of
this paper. The network of simulated interferograms (perpendicular baselines less than 400 m and temporal separation less than 900 days) and the perpendicular baseline history is shown in baseline-time plots in Figure 2.2(a) and (b). The range of the baseline history of the whole data set is around 1400 m. For the interferogram simulation, we consider a DEM error of 20 m and four different displacement histories: (i) zero displacement, (ii) constant velocity typical for tectonic processes, (iii) exponential temporal displacement, and (iv) complex time-variable displacement including rapid uplift and subsidence typical for volcanic processes.

**Figure 2.2** Application of the new DEM error correction algorithm on the simulated data considering different temporal variations of ground displacement: (a) Baseline-time plot of network of interferograms, (b) baseline history. (c-f) Simulated displacement history
(dashed lines), estimated displacement history without DEM error correction (filled circles), DEM error corrected using the new algorithm (triangles) and using the original SBAS method (plus signs) for a pixel with (c) zero, (d) linear, (e) exponential, and (f) complex time-variable displacement. The baseline history is taken from real Envisat SAR data acquired over Fernandina volcano (see section V). Note that the proposed and original SBAS methods achieve identical results for cases presented in c, d and e.

The simulated and estimated displacements time-series are shown in Figure 2.2(c) to (f). The uncorrected displacements in this figure are the result of time-series inversion of simulated interferograms using SB method without DEM error correction. The estimated displacement histories without DEM error correction (filled circles) differ from the simulated ones (dashed lines) in all cases because of the DEM error effect. In the absence of deformation (see Figure 2.2(c)) it can be clearly seen that the estimated displacement history without DEM error correction is proportional to the baseline history of Fig.2 (b). Figure 2.3 shows this correlation clearly. In the three cases Figure 2.2(c) to (e), the estimated displacement histories after the DEM error correction (triangles) equal the simulated displacement histories. In these cases the displacement histories are simple and well described by the polynomial deformation model of (6). For the last case (Figure 2.2(f)) the estimated displacement history differs from the simulated one by 1 cm (or less) at each epoch. This difference is the result of the inadequate temporal model representation of the surface deformation.
Figure 2.3 Baseline history of Figure 2.2(b) and uncorrected displacement of Figure 2.2(c). Correlation between two histories is due to the effect of DEM error.

We also compare our new method for DEM error correction with the original SBAS method (plus signs). For the simple cases the original SBAS method retrieves the simulated displacement history perfectly as does the new method (see Figure 2.2(c) to (e)). For the complex displacement history the difference for the original SBAS method is more significant than for the new method [Figure 2.2(f)].

To better understand the difference between the new and the original DEM error correction method, we consider different networks of interferograms for the same set of SAR acquisitions. In addition to the SB network of Figure 2.2(a), we consider a Delaunay triangulated network (see Figure 2.4(a)), and another network with thresholds of 900 days and 300 meters for the temporal separation and spatial baseline of the interferograms (see Figure 2.4(b)). The latter is the same network of the Envisat SAR data used in the section V.

For the three cases the displacement histories estimated using the new method are nearly identical and close to the simulated displacement whereas the original SBAS method leads to different results (see Figure 2.2(f), Figure 2.4(c) and (d)). These differences occur because the original SBAS method estimates the DEM error and parameters of the deformation model in the interferogram domain. When the actual ground displacement can’t be approximated by the deformation model, the DEM error estimation is biased and different interferogram networks lead to different estimates. In contrast, in our algorithm, which operates in the time domain, the estimation is independent of the network of interferograms.
Figure 2.4 Effect of interferogram network on DEM error correction (simulated data). (a) Delaunay-triangulated network, (b) network consisting of all interferograms with spatial and temporal baseline thresholds of 300 m and 900 days with incoherent interferograms removed manually. (c, d) Same as Figure 2.2(f). See Figure 2.2(a) for the network with thresholds of 400 m and 900 days (without removing interferograms) and Figure 2.2(f) for the corresponding displacement histories. For complex time-variable displacement, the new method is independent of the interferogram network but not the original SBAS method.

Another experiment demonstrates how the estimated DEM error depends on the network of interferograms. We test four different networks (Figure 2.5). The first network is a sequential network in which interferograms are formed from two consequent acquisitions (Figure 2.5(a)). This network is unrealistic because some interferograms have too long baseline but it is useful for this demonstration. The second and third networks are identical to the first except of one additional interferogram, which covers a time period with significant displacement (Figure 2.5(c)) and a time period without
significant displacement (Figure 2.5(e)), respectively. The fourth network is a tree-like network commonly used in PS time-series algorithms (Figure 2.5(g)).

The estimated displacement histories with DEM error correction using the original SBAS method, with DEM error correction using the new method and without DEM error correction are shown in the right panel of Figure 2.5. In the first and in the third cases the estimated displacement histories using the original SBAS method and the new method are identical. In the second case the original SBAS method produces a biased solution. This shows that in the interferogram domain one additional interferogram can change the least squares solution for the DEM error if it covers a time period of displacement not described by the assumed deformation model. This also explains the biased estimation of the DEM error using the original SBAS method using the tree-like network in Figure 2.5(h) and using the SB networks in Figure 2.4 and Figure 2.2(f).

To understand why the original SBAS method produces more biased results than the new method it is instructive to compare the solution in the time domain from the phase velocity history (the new method) with that obtained from the phase history. Figure 2.6 shows that the phase velocity gives a less biased solution than the phase history.

This can be explained as follows. The phase history consists at each epoch of values relative to the reference epoch. Thus, at each epoch it can be considered as a hypothetical interferogram between that epoch and the reference epoch. The entire phase history can then be considered as a tree-like network of interferograms. On the other hand, the phase velocity history can be considered as a sequential network of interferograms. In other words, time-domain estimation of DEM error from phase history is equivalent to interferogram-domain estimation using a tree-like network. Time-domain DEM error
estimation from phase velocity history is equivalent to interferogram-domain estimation using a sequential network (see Appendix 2.3). As we have shown above, the latter is less affected by deficiencies of the assumed deformation model compared to the tree-like network. Accordingly, the DEM error is best estimated in the time domain from the phase velocity history rather than from the phase history.

Figure 2.5 Simulated data demonstrating the difference between DEM error estimation in time and interferogram domain. Left panels are different networks of interferograms and right panels are corresponding estimated displacements before and after DEM error correction. DEM error estimation in interferogram domain depends on the network of interferogram. The estimation in the time domain using the phase velocity history is equivalent to the estimation in the interferogram domain using a sequential network of interferograms.
25

Figure 2.6 Comparison of DEM error correction in time-domain using phase velocity history and phase history. The DEM error is best estimated in the time domain from the phase velocity.

We quantify the effectiveness of the DEM error correction methods using the Root Mean Square Error (RMSE) between the simulated and the estimated displacement histories:

\[
RMSE(\hat{d}) = \left\{ \frac{1}{N} \sum_{t_i=1}^{t_f=N} (d(t_i) - \hat{d}(t_i))^2 \right\}^{1/2}
\] (13)

where \(d\) and \(\hat{d}\) are the simulated and estimated displacements respectively.

The RMSE for the different displacement histories and interferogram networks of Figure 2.2, Figure 2.4 and Figure 2.5, with DEM error correction using the new method, the original SBAS method, and without DEM error correction, are listed in Table I. The estimated DEM error is also listed. The original simulated DEM error was 20 m in all cases.

The RMSE of the estimated displacement history without DEM error correction is 53 mm in all cases. This parameter is zero or nearly zero using both, the original SBAS and
the new methods for the first two cases with simple displacement histories (linear and exponential).

| Table 2.1 | RMSE for estimated displacement history with and without DEM error correction. Simulated DEM error is 20m in all cases. SB¹ is small baseline network with perpendicular and temporal baseline thresholds of 400 m and 900 days, SB² with thresholds of 300 m and 900 days and incoherent interferograms removed. Sequential¹ is a network with each interferogram formed from two consequent acquisitions. Sequential² and Sequential³ have two additional interferograms as shown in Figure 2.5(c) and Figure 2.5(e) respectively. |
|---|---|---|---|---|---|
| Displacement history | Network | RMSE (mm) | Estimated DEM error (m) | Figure |
| | | Uncorrected | New algorithm (time-domain) | Original SBAS (interferogram domain) | New algorithm | Original SBAS |
| Linear | SB¹ | 53 | 0 | 0 | 20 | 20 | Figure 2.2(c) and (d) |
| Exponential | SB¹ | 53 | 0.2 | 0.9 | 20.07 | 20.4 | Figure 2.2(e) |
| | SB¹ | 53 | 3 | 43 | 18.7 | 36.2 | Figure 2.2(f) |
| | Delaunay | 53 | 3 | 10 | 18.7 | 16.2 | Figure 2.4(c) |
| Time-variable | SB² | 53 | 3 | 31 | 18.7 | 31.7 | Figure 2.4(d) |
| | Sequential¹ | 53 | 3 | 3 | 18.7 | 18.7 | Figure 2.5(b) |
| | Sequential² | 53 | 3 | 5 | 18.7 | 17.9 | Figure 2.5(d) |
| | Sequential³ | 53 | 3 | 3 | 18.7 | 18.7 | Figure 2.5(f) |
| | Tree-like | 53 | 3 | 16 | 18.7 | 13.9 | Figure 2.5(h) |

The deviation from zero for the exponential case occurs because this displacement history is not fully approximated by our assumed deformation model. For the cases with complex time-variable displacement history the new method leads to more accurate results compared to the original SBAS method for most networks. Only for the sequential network and the second modified sequential network (mod. sequential²) the results are identical for both methods. The table clearly shows that the DEM error estimated with the
original SBAS method can depend on the network of interferograms used. In the above analysis we only have considered connected networks. Any discontinuity in the network would decrease the rank of the matrix $B$ of [Berardino et al., 2002] and to additional bias of the estimated displacement history and DEM error.

2.5.3 Effect of atmospheric delays on DEM error estimation

The effect of atmospheric delays on the DEM error estimation is demonstrated in Figure 2.7. We use the exponential displacement history of Figure 2.2(d). We simulate the atmospheric phase contribution as additive random Gaussian noise. We consider a DEM error of 20 m. Thus phase histories have contributions from displacement, DEM error and atmospheric delay. We compare the estimated phase histories before and after DEM error correction (converted into displacements for simplicity, filled circles and triangles in Figure 2.7) with the simulated phase histories (dashed lines). For small atmospheric noise (standard deviation of 7 mm) the simulated phase history is well retrieved (see Figure 2.7(a)). For significant atmospheric noise (standard deviation of 30 mm) the estimated phase history differs from the simulated one (see Figure 2.7(b)). This occurs because strong atmospheric delays cause the noise term $\dot{\psi}_i$ in (8) to be large and result in a biased DEM error estimation. Orbital errors and thermal noise have similar effects.
Figure 2.7 Effect of atmospheric delay on DEM error correction (simulated data). Filled circles are estimated phase histories (converted to displacements) before DEM error correction, which has contributions from an exponential displacement (Same as Figure 2.2(e)), a 20 m of DEM error and atmospheric delay with standard deviation of (a) 7 mm and (b) 30 mm. The estimated signals after the DEM error correction (triangles) have been compared with the simulated displacement plus atmospheric effect (dashed line) and RMSE has been calculated. Significant atmospheric delays can bias the DEM error estimation.

Figure 2.8 Baseline history of Alos PALSAR data sets at different latitudes: (a) 1.5 S covering Tungurahua volcano, Equador (b) 36.5 N covering Cranfield, Ms., USA.

2.5.1 Effect of temporal behavior of baseline history on DEM error estimation

A characteristic of the cases discussed above is that the baseline and displacement histories are not correlated, which is favorable for DEM error estimation. Such baseline histories are typical for the Envisat and ERS satellites. When the displacement and baseline histories are correlated it is not possible to estimate the DEM error. This can be
the case for Alos-PALSAR [Samsonov, 2010]. Typical Alos baseline histories are shown in Figure 2.8. The baseline history has irregular temporal behavior at the Equator (Figure 2.8(a)) and saw-tooth type behavior at mid- and high-latitudes (period about 2 years, Figure 2.8(b)).

In the first case the baseline and displacement histories are unlikely to be correlated and the DEM error can be estimated. The problems associated with the second case are demonstrated in Figure 2.9. We assume zero deformation and a DEM error of 20 m. For a linear baseline history (e.g. SAR acquisitions from 2009 to 2011 from Figure 2.8(b)) the simulated displacement history is not retrieved (Fig. 9 (a) and (c)). The estimated displacement histories before and after DEM error correction are both proportional to the baseline history with the slope representing the DEM error. If the linear baseline increase is followed by a discontinuity (Figure 2.9(b)), the simulated displacement history is retrieved, indicating that the DEM error can be properly estimated (Figure 2.9(d)).

**Figure 2.9** Effect of the baseline history on DEM error estimation (simulated data). (a) Linear, (b) sawtooth baseline history. (c,d) Simulated displacement history (dashed line) and estimated displacement history before (circles) and after DEM error correction.
(triangles). For correlating displacement and baseline histories the DEM error can’t be resolved resulting in a bias of the estimated displacement history.

![Figure 2.10 Location map of Fernandina volcano, Galapagos Islands, Ecuador.](image)

2.6 Real data

Fernandina is one of the Galapagos Islands located around 1000 km west of Ecuador (see Figure 2.10) [Jónsson et al., 1999; Amelung et al., 2000]. This volcanic island is considered as one of the most active volcanoes in the world with at least 15 eruptions during 1950-2010. Two major eruptions occurred in May 2005 and April 2009 and two seismic swarms in Dec 2006 and Aug 2007 [Bagnardi and Amelung, 2012].

We used a data set of 59 Envisat-ASAR images from 2003/03/11 to 2010/09/07 acquired along the descending satellite orbit (Track= 412, Frame=3609, Beam=12). During the InSAR processing, we estimate the topographic phase using an SRTM DEM with 90 m pixel size down sampled to 30 m. We took 5 looks in azimuth direction to obtain roughly square pixels. The perpendicular baseline history of SAR images and the network of interferograms are shown in the baseline-time plots in Figure 2.2(b) and Figure 2.4(b) respectively (perpendicular baselines less than 300 m, time separation less than 910 days, manual removal of incoherent interferograms).
Figure 2.11 (a) Estimated displacement history for a pixel in the center of Fernandina caldera before (circles) and after (triangles) DEM error correction. (b) Uncorrected displacement history (filled circles, dashed line) and baseline history (open circles, solid line) for the 2003-2005.5 time period. This period is characterized by rapid baseline variation (see Figure 2.2(b)). The histories are proportional because of a DEM error. Please note that the total offset due to the 2009 eruption may have been underestimated in C-band ASAR data because of high phase gradient problem, since recent results from L-band ALOS PALSAR data shows more than 1 meter offset for this eruption [21].

Figure 2.11(a) shows the estimated displacement history for a pixel inside the caldera with and without DEM error correction. Without DEM error correction the estimated displacement history at the beginning of the time period is rough, varying by up to 25 cm.
between epochs (until 2005). A zoom into this time period (see Figure 2.11(b)) shows that the displacement history correlates with the baseline history indicating a DEM error.

Figure 2.11(a) also shows that the uncorrected displacement history does not resolve the subsidence associated with an early-2007 seismic swarm but does resolve deformation associated with a late-2007 seismic swarm and a 2009 eruption. This is consistent with our expectation about the effect of the DEM error. Over our test area Envisat had a stable orbit after June 2007 with a range in baseline of less than 400 m (day 1400 in Figure 2.2(b)). Consequently, for this time period the DEM error has only little effect and the displacement histories estimated with and without DEM error correction are nearly identical. The shift between them is because of a perpendicular baseline of averaged 900 m with respect to the first acquisition.

2.7 Conclusion

We have presented the mathematical formulation for the phase due to the DEM error in InSAR time domain. We have shown that the phase due to the DEM error at each epoch is proportional to the perpendicular baseline between the corresponding SAR acquisition and the temporal reference acquisition. Selecting small spatial baseline interferograms typical for the SB time-series approach minimizes the phase due to the DEM error in the interferograms but not in the estimated displacement history. The effect of the DEM error depends on the total range in perpendicular baselines of an interferogram network and not on the baseline of individual interferograms. This explains the need for DEM error correction even if only small baseline interferograms are used.

We have proposed a new algorithm for DEM error correction. In this algorithm the DEM error correction is estimated in the time domain from the phase velocity history.
This is in contrast to the original SBAS method of [Berardino et al., 2002] in, which the DEM error is estimated in the interferogram domain from the interferometric phases.

For simple ground displacement histories, which are well approximated by the assumed deformation model, the new method is equivalent to the original SBAS method. For complex, time-variable displacement histories (typical for volcanic unrest or including earthquakes) the new method yields more accurate estimations of the DEM error and of the displacement history. This occurs because the new method is applied in the time domain and therefore independent of the interferogram network. In contrast, in the original SBAS method the estimated DEM error may depend on the interferogram network.
Chapter 3. InSAR uncertainty due to Orbital errors

3.1 Summary

Errors in the satellite orbits are considered to be a limitation for InSAR time-series techniques to accurately measure long-wavelength (> 50 km) ground displacements. Here we examine how orbital errors propagate into relative InSAR line-of-sight (LOS) velocity fields and evaluate the contribution of orbital errors to the InSAR uncertainty. We express the InSAR uncertainty due to the orbital errors in terms of the standard deviations of the velocity gradients in range and azimuth directions (range and azimuth uncertainties). The range uncertainty depends on the magnitude of the orbital errors, the number and time span of acquisitions. Using reported orbital uncertainties we find range uncertainties of less than 1.5 mm/yr/100 km for ERS, less than 0.5 mm/yr/100 km for Envisat and ~0.2 mm/yr/100 km for TerraSAR-X and Sentinel-1. Under a conservative scenario, we find azimuth uncertainties of better than 1.5 mm/yr/100 km for older satellites (ERS and Envisat) and better than 0.5 mm/yr/100 km for modern satellites (TerraSAR-X and Sentinel-1). We validate the expected uncertainties using LOS velocity fields obtained from Envisat SAR imagery. We find residual gradients of 0.8 mm/yr/100 km or less in range and of 0.95 mm/yr/100 km or less in azimuth direction, which fall within the one-sigma to two-sigma uncertainties. The InSAR uncertainties due to the orbital errors are significantly smaller than generally expected. This shows the potential of InSAR systems to constrain long-wavelength geodynamic processes, such as continent-scale deformation across entire plate boundary zones.
3.2 Background

Interferometric Synthetic Aperture Radar (InSAR) time-series methods such as Small Baseline (SB) [Berardino et al., 2002] and Persistent Scatterer (PS) [Ferretti et al., 2001, 2011; Hooper et al., 2012] methods are well-established techniques to measure and study short-wavelength (<50 km) crustal displacements such as at volcanoes [Lu et al., 2010; Samsonov and d’Oreye, 2012; Chaussard et al., 2013a; Pritchard et al., 2013], along creeping faults [Lyons and Sandwell, 2003; Champenois et al., 2012], land subsidence [Osmanoglu et al., 2011; Chaussard et al., 2013b], and slumping associated with slow landslides [Lauknes et al., 2010; Liu et al., 2013; Motagh et al., 2013]. Many geodynamic processes that cause long-wavelength crustal deformation (> 50 km) have also been studied using InSAR. Examples include strain accumulation along locked continental faults [Elliott et al., 2008; Walters et al., 2011; Garthwaite et al., 2013] and post-seismic deformation due to flow in the lower part of the lithosphere and uppermost mantle [Pollitz et al., 2001; Gourmelen and Amelung, 2005; Ryder et al., 2011].

Forthcoming satellites and satellite constellations with frequent image acquisitions [Sansosti et al., 2014] have the potential to deliver coherent measurements of the long-wavelength deformation within and across entire plate boundary zones. Other geodynamic processes producing long-wavelength crustal deformation include strain accumulation and release along subduction faults [Béjar-Pizarro et al., 2013], subsidence of river deltas [Dixon et al., 2006; Dokka et al., 2006], and glacial isostatic adjustment [Jiang et al., 2010; Bevis et al., 2012], and InSAR studies have been conducted [Liu et al., 2012; Auriac et al., 2013; Zhao et al., 2014]. However, long-wavelength artifacts with similar spatial pattern may bias the estimated long-wavelength ground deformation [Lohman and Simons, 2005b; Biggs et al., 2007]. None of the studies give uncertainties
including the error due to long-wavelength artifacts. Measuring long-wavelength displacement with InSAR requires a better understanding of the error budget of the technique.

Errors in the satellite state vectors, commonly called orbital errors, are traditionally considered as InSAR’s main limitation for measuring long-wavelength displacement. Orbital errors cause long-wavelength phase contributions to interferograms [Massonnet and Feigl, 1998]. Another known source for long-wavelength phase contributions are atmospheric delays, consisting of ionospheric and tropospheric components. Ionospheric delays are more significant for L-band [Meyer, 2011] than for C and X-band [Hanssen, 2001]. Tropospheric delays significantly contribute to the interferometric phase, resulting in short and long-wavelength phase patterns [Li, 2005; Doin et al., 2009; Fournier et al., 2011; Jolivet et al., 2011; Gong et al., 2013; Walters et al., 2013].

A variety of strategies are available to mitigate the effect of orbital errors and other long-wavelength phase contributions. A simple method is the estimation of a linear or quadratic surface that fits to the interferometric phases [Massonnet and Feigl, 1998] or the estimation of baseline components corrections in a processing step commonly referred to as baseline re-estimation [Rosen et al., 2004]. Another approach estimates the components of baseline error using the number of residual fringes resulting from orbital errors [Kohlhase et al., 2003]. More accurate methods use a network of interferograms for a consistent estimation of surfaces fitted to the interferometric phases [Biggs et al., 2007], or the amount of corrections required to compensate the orbit of each acquisition [Pepe et al., 2011a; Bähr and Hanssen, 2012]. Long-wavelength phase contributions are conveniently removed in the time-domain after the time-series inversion [Gourmelen et
The general drawback of these methods is that all long-wavelength phase patterns are treated as orbital errors and therefore not only orbital effects but also the long-wavelength displacement signal is removed.

The long-wavelength displacement signal can be separated from long-wavelength artifacts using GPS measurements [Lundgren et al., 2009; Gourmelen et al., 2010; Wei et al., 2010; Manzo et al., 2012; Wang and Wright, 2012; Kaneko et al., 2013; Tong et al., 2013]. This, however, makes the InSAR results dependent on GPS. This is in contrast to independent InSAR and GPS estimates, which can be combined to reduce the uncertainty, or to infer the vertical displacements, which are notoriously difficult to measure with GPS.

Two methods have been proposed to separate long-wavelength deformation from orbital effects exploiting the different spatial-temporal characteristics of tectonic displacement and orbital errors. Biggs et al. [2007] and Wang et al. [2009] iteratively estimate deformation and orbital errors from phase-unwrapped data using a model assumption about the deformation, whereas Zhang et al. [2013] simultaneously estimate the two components from the wrapped interferograms.

All these approaches assume that the contributions from orbital errors are significant. However, most satellites are precisely tracked using laser ranging, DORIS or GPS, resulting in uncertainties of the orbits of 2-10 cm [Yoon et al., 2009; Eineder et al., 2011; Rudenko et al., 2012]. InSAR time-series methods use several tens of SAR acquisitions. If the errors in the satellite orbits cancel out in products derived from multiple acquisitions, they should not have significant impact on the ability of the InSAR
technique to resolve long-wavelength deformation. However, we need to evaluate the uncertainty of InSAR measurements of long-wavelength deformation.

In this paper we investigate how the InSAR uncertainty depends on orbital errors. This paper is organized as follows. First we develop a formulation to express the uncertainty of InSAR velocity fields in terms of baseline uncertainties, which is directly related to the orbital uncertainties (section 2). We then use these expressions to evaluate the actual uncertainties given the orbital uncertainties of different SAR satellites and typical image acquisition scenarios (section 3). Next, we use real InSAR data acquired over non-deforming and deforming areas in the southwestern U.S. to compare observed velocity gradients with the expected uncertainties, and with GPS observations (section 4). Finally, we discuss the uncertainty of InSAR velocity field for measuring long-wavelength deformation (section 5).

3.3 Propagation of orbital errors to the InSAR data

InSAR time-series techniques, such as SB and PS, generate range-change histories relative to a given reference point and epoch in the Line Of Sight (LOS) direction of the radar. We assume that estimated range-change histories are not biased for example due to the unwrapping errors or discontinuity in the network of interferograms. Therefore at each epoch, the measured range change contains contributions from ground displacement, orbital errors, atmospheric delays, systematic errors and random noise. The LOS velocity, here after called velocity, for each pixel is the slope of the linear fit to the range-change history; It is always relative to a reference point, which is commonly chosen to be located in a non-deforming area. At each pixel, the components of velocity, $v$, are

$$v = v_{\text{dis}} + v_{\text{orb}} + v_{\text{atm}} + v_{\text{sys}} + v_{\text{noise}}$$  (3.1)
where $v_{\text{dis}}$, $v_{\text{orb}}$, $v_{\text{atm}}$ and $v_{\text{noise}}$ are the contributions from the ground displacement, orbital errors, atmospheric delay and random noise. $v_{\text{sys}}$ includes systematic contributions due to DEM errors, instrument drift and approximations in the processing software. At each pixel, we express the uncertainty of the InSAR velocity as the variance of $v$, $\sigma^2$, as,

$$\sigma^2 = \sigma_{\text{orb}}^2 + \sigma_{\text{atm}}^2 + \sigma_{\text{noise}}^2 \tag{3.2}$$

where $\sigma_{\text{orb}}^2$, $\sigma_{\text{atm}}^2$ and $\sigma_{\text{noise}}^2$ are the variances of the velocity due to the orbital errors, atmospheric delay, and noise, respectively. The variation of $\sigma_{\text{atm}}^2$ with distance is an important topic of research [Emardson, 2003; Li, 2005; Meyer et al., 2008; Barnhart and Lohman, 2013]. The focus of this paper is to evaluate $\sigma_{\text{orb}}^2$, which increases with distance from the reference point. We first derive the phase contribution due to the orbital errors for both individual interferograms and the time-series epochs, and then express the contribution from orbital errors to the InSAR velocity fields.

**Figure 3.1** Geometry for repeat orbit SAR interferometry. $H_s$ represents the satellite height relative to the reference ellipsoid at master acquisition. See text for the other symbols.
3.3.1 Individual interferograms

The interferometric phase due to the imaging geometry is the result of the spatial separation of the radar antenna during the acquisition of two SAR images, known as the spatial baseline. Considering the acquisition geometry of Figure 3.1, the interferometric phase, \( \varphi \), which contains phase contributions from the earth curvature and topography can be expressed as [Bürgmann et al., 2000; Simons and Rosen, 2007]

\[
\varphi = \frac{4\pi}{\lambda} (r_M - r_S) = \frac{4\pi}{\lambda} r_M \left( 1 - \sqrt{1 + \frac{\bar{B}^2}{r_M^2} - \frac{2\bar{B}\bar{l}}{r_M}} \right)
\]  

(3.3)

wherein \( \lambda \) is the radar carrier wavelength, \( r_M \) and \( r_S \) are the ranges from the SAR antenna at master and slave positions to the target on the ground, \( \bar{B} \) is the baseline vector with the length of \( B \), \( \bar{l} \) is the unit vector from the master antenna toward the target, and \( \bar{B}\bar{l} \) is the dot product of \( \bar{B} \) and \( \bar{l} \) [Rosen et al., 2000]. Assuming all SAR images are focused to the zero Doppler geometry, \( \beta = 0 \), in case of space-borne SAR systems, parallel and perpendicular components of baseline, \( B_h \) and \( B_\perp \), are related to horizontal and vertical components, \( B_h \) and \( B_v \), as

\[
B_h = B_h \sin(\theta) - B_v \cos(\theta)
\]

(3.4)

\[
B_\perp = B_h \cos(\theta) + B_v \sin(\theta)
\]

(3.5)

and equation (3.3) can be approximated with [Hanssen, 2001]

\[
\varphi \approx \frac{4\pi}{\lambda} B_h
\]

(3.6)
The effect of this parallel ray approximation depends on the baseline length. For a typical Envisat interferogram with $B_h$ and $B_v$ of 400 m and 100 m respectively, the effect of the approximation is ~ 1.3 radians. However, for evaluating orbital errors, which translates to small baseline errors ($< 1$m), the difference between equations (3.3) and (3.6) is less than 1 milliradian, i.e. it is negligible.

We evaluate the spatial variation of the interferometric phase using the Taylor expansion of equation (3.6) around a pixel at range and azimuth coordinates of $(r_0, s_0)$, which can be any pixel such as the scene center or the first pixel of the interferograms [Pepe et al., 2011a; Bähr and Hanssen, 2012]

$$d\phi(r,s) = \phi(r,s) - \phi(r_0, s_0) = \frac{\partial \phi}{\partial \vartheta} \, d\vartheta \bigg|_{r_0,s_0} + \frac{\partial \phi}{\partial s} \, ds \bigg|_{r_0,s_0} + ...$$

(3.7)

where $d\phi(r,s)$ is the variation of the interferometric phase between pixels at range and azimuth coordinates $(r, s)$ and $(r_0, s_0)$. The first order terms, $\frac{\partial \phi}{\partial \vartheta} \, d\vartheta$ and $\frac{\partial \phi}{\partial s} \, ds$, express the linear phase variation in range and azimuth directions respectively, commonly referred to as linear phase ramps. The slopes of these linear terms are expressed by the phase gradients $\frac{\partial \phi}{\partial \vartheta}$ and $\frac{\partial \phi}{\partial s}$ in range and azimuth directions, respectively. The second and higher order terms in equation (3.7) are negligible for evaluating the phase variation due to the orbital errors in range direction [Bähr & Hanssen, 2012]; These terms in azimuth direction are functions of the error in the baseline curvature, which is expected to be small or negligible (Appendix 3.1). It worth noting that second and higher order terms express non-linearity (curvature) of the phase
variation, and therefore ignoring those terms does not affect our evaluation of phase
gradients of the linear terms.

To express the phase gradients in terms of the baseline components, we consider a
linear model of the baseline in azimuth direction as

\[ B_h(s) = B_{h0} + B'_h s \]  \hfill (3.8)

\[ B_v(s) = B_{v0} + B'_v s \]  \hfill (3.9)

where \( s \) is the along-track distance from the beginning of the master scene (in meters)
equivalent to the acquisition time of radar echoes at the master acquisition, \( B_{h0} \) and \( B_{v0} \)
are the horizontal and vertical baselines at \( s = s_0 \) which is usually the beginning of the
master acquisition, \( B'_h \) and \( B'_v \) (with unit meters per meters) are slopes of \( B_h \) and \( B_v \)
respectively. From equations (3.4) to (3.9), it follows that the phase gradients can be
expressed as [Pepe et al., 2011a; Bähr and Hanssen, 2012]

\[ \frac{\partial \phi}{\partial \vartheta} = \frac{4\pi}{\lambda} B_{\perp} \]  \hfill (3.10)

\[ \frac{\partial \phi}{\partial s} = \frac{4\pi}{\lambda} B'_{\parallel} \]  \hfill (3.11)

where \( B'_{\parallel} \) is the slope of \( B_{\parallel} \). In obtaining equations (3.9) and (3.10) we use the fact that
\( B_h \) and \( B_v \) only change in azimuth direction and assume that \( \vartheta \) changes only in range
direction [Pepe et al. 2011], i.e. \( \frac{\partial B_h}{\partial \vartheta} = \frac{\partial B_v}{\partial \vartheta} = \frac{\partial \vartheta}{\partial s} = 0 \).

We are interested in the phase contribution from the baseline error \( \tilde{B}^\varepsilon \), which relates
to the actual baseline, \( \tilde{B} \), as \( \tilde{B} = \tilde{B}^\varnothing + \tilde{B}^\varepsilon \), where \( \tilde{B}^\varnothing \) is the baseline from the satellite state
vectors of two orbits. For simplicity, in the following we omit the superscript $\varepsilon$ and use baseline components to refer to the baseline error components.

### 3.3.2 Time-series

Similarly as for individual interferograms, the phase contribution from orbital errors at each epoch is equivalent to the error of the baseline between that epoch and the reference epoch. Therefore we use equations (3.10) and (3.11) to express the phase gradients at each epoch as

$$\frac{\partial \phi(t_i)}{\partial \theta} = \frac{4\pi}{\lambda} B_{\perp}(t_i)$$

$$\frac{\partial \phi(t_i)}{\partial s} = \frac{4\pi}{\lambda} B_{s}'(t_i)$$

where $\frac{\partial \phi(t_i)}{\partial \theta}$ and $\frac{\partial \phi(t_i)}{\partial s}$ are the phase gradients in range and azimuth directions at epoch $t_i$, with $i = 1, ..., N$, referenced to the first epoch. Here $B_{\perp}(t_i)$ and $B_{s}'(t_i)$ are the errors of the baseline components between the epoch $t_i$ and the reference epoch.

### 3.3.3 Velocity fields

To evaluate the contribution from orbital errors to InSAR velocity field, we express the variation of $v_{orb}$ between two given pixels at $(r, s)$ and $(r_0, s_0)$. For this we obtain the Taylor expansion about $(r_0, s_0)$ as

$$dv_{orb} = v_{orb}(r, s) - v_{orb}(r_0, s_0) = \frac{\partial v_{orb}}{\partial \theta} d\theta \bigg|_{r_0, s_0} + \frac{\partial v_{orb}}{\partial s} ds \bigg|_{r_0, s_0} + ... = \Re d\theta + \alpha ds + ...$$

where $dv_{orb}$ is the variation of the velocity field due to the orbital errors, $\Re d\theta$ and $\alpha ds$ are the linear components of the velocity variations in range and azimuth directions.
respectively. The slope of these linear terms are expressed by the velocity gradients in range direction, $\mathcal{R}$, and in azimuth direction, $\alpha$.

In order to express $\mathcal{R}$ and $\alpha$ in terms of the baseline error components, let’s assume that the phase gradients of N epochs are known with respect to the first epoch. In practice these phase gradients can be estimated by fitting planes to the phase histories in range and azimuth directions. In order to estimate the velocity gradient, we consider a linear model as

$$d = Am$$

(3.15)

where $d$ is a vector of $N$ range-change gradients (phase gradients converted to the range-change) in range or azimuth directions with elements $d_i = (\lambda/(4\pi)) \frac{\partial\phi(t_i)}{\partial\theta}$ or $d_i = (\lambda/(4\pi)) \frac{\partial\phi(t_i)}{\partial s}$, and $A$ is the design matrix as $A = [(t_1, t_2, ..., t_N)^T, [1, 1, ..., 1]^T]$. The vector of model parameters, $m$, has the form of $m = [\mathcal{R}, c]^T$ or $m = [\alpha, c]^T$. The intercept, $c$, is not considered in the following. We obtain the least squares solution, $m = (A^TA)^{-1}A^Td$, as

$$\mathcal{R} = \frac{\sum_{i=1}^{N} t_i B_\perp(t_i) - \bar{t} \sum_{i=1}^{N} B_\perp(t_i)}{\|\bar{\Delta}t\|^2}$$

(3.16)

$$\alpha = \frac{\sum_{i=1}^{N} t_i B'_\parallel(t_i) - \bar{t} \sum_{i=1}^{N} B'_\parallel(t_i)}{\|\bar{\Delta}t\|^2}$$

(3.17)

where $\bar{\Delta}t$ with elements $\Delta t_i = t_i - \bar{t}$, is the vector of temporal distance of acquisition dates from the mean of SAR acquisition dates defined as $\bar{t} = \sum_{i=1}^{N} t_i / N$ (with unit years);
\|\Delta t\| \text{ represents the Euclidian length of } \Delta t. \text{ Equations (16) and (17) show that the velocity gradients are linear in } B_\perp \text{ and } B'_\perp.

From equation (15) and given the covariance matrix of the velocity gradients in range direction as \( C_d = \sigma_{B_\perp}^2 I \), where \( \sigma_{B_\perp}^2 \) represents the variance of the perpendicular baseline error, we obtain the covariance matrix of the unknown vector as \( C_x = \sigma_{B_\perp}^2 (A^T A)^{-1} \). We are interested in \( \sigma_{\Re}^2 \), which is the first element of \( C_x \) expressed as

\[
\sigma_{\Re} = \frac{\sigma_{B_\perp}}{\|\Delta t\|} \tag{3.18}
\]

A similar approach can be followed to obtain the standard deviation of the velocity gradient in azimuth direction, \( \sigma_\alpha \), expressed as

\[
\sigma_\alpha = \frac{\sigma_{B_\parallel}}{\|\Delta t\|} \tag{3.19}
\]

where \( \sigma_{B_\parallel} \) represents the standard deviation of the parallel baseline slope error. Equations (18) and (19) can be also expressed using horizontal and vertical baseline representations (Appendix 3.2). We obtain the velocity uncertainty due to orbital errors at a given pixel as

\[
\sigma_{\text{orb}}^2 = \Delta \theta^2 \sigma_{\Re}^2 + \Delta s^2 \sigma_\alpha^2 \tag{3.20}
\]

where \( \Delta \theta \) and \( \Delta s \) are the range and azimuth distances from the reference point respectively. We refer to \( \sigma_{\Re} \) and \( \sigma_\alpha \) as the range and azimuth uncertainties. The uncertainty for a given pixel increases with distance from the reference point whereas the uncertainties of the gradients are constant over the swath.
From equations (18) and (19) it can be seen that $\sigma_{Rl}$ and $\sigma_\alpha$ are functions of $\sigma_{B\perp}$, $\sigma_{B\parallel}$, number and time-span of acquisitions. More precise orbits, more SAR acquisitions (larger $N$) and longer time span of SAR acquisitions decrease $\sigma_{Rl}$ and $\sigma_\alpha$, resulting in more precise velocity fields.

In the next section we first infer the baseline uncertainty from the reported or expected orbital uncertainties of different SAR satellites and then use equations above to obtain the uncertainty of the velocity gradients.

### 3.4 Uncertainty of the velocity gradients

In order to evaluate $\sigma_{Rl}$ and $\sigma_\alpha$ based on orbital uncertainty, we need to infer the standard deviation of the baseline error components from the standard deviation of the orbital errors. To this end, we consider the orbital parameters in the along-track, across-track and vertical coordinate system of Figure 3.1. Based on this coordinate system, $B_h$ and $B_v$ at a specific azimuth line can be related to the horizontal and vertical components of the orbits as

$$B_h = O_{h2} - O_{h1}$$
$$B_v = O_{v2} - O_{v1}$$

where $O_{h1,2}$ and $O_{v1,2}$ are the horizontal and vertical components of orbits 1 and 2, respectively. Assuming that the orbits are independent with identical error distributions, the standard deviations of the baseline error components can be written as

$$\sigma_{Bh}^2 = 2\sigma_{Oh}^2$$
$$\sigma_{Bv}^2 = 2\sigma_{Ov}^2$$
where $\sigma_{Bh}$ and $\sigma_{Bv}$ are the standard deviations of the horizontal and vertical baseline error components, and $\sigma_{Oh}$ and $\sigma_{Ov}$ are the standard deviations for the horizontal and vertical orbit error components.

We use the reported root-mean-squared (RMS) orbital errors of SAR satellites as the estimations of $\sigma_{Oh}$ and $\sigma_{Ov}$ and refer to them as orbital uncertainty in the following. Uncertainty of satellite orbits is usually expressed based on the altimeter crossover differences, and RMS of differences of independent orbit solutions using different gravity models and processing approaches. If the solutions are not fully independent, then the reported RMS may only show a lower bound of the actual orbital uncertainty.

The uncertainty of ERS orbits in vertical (radial) direction is 2-3 cm [Rudenko et al., 2012] and the uncertainty of horizontal component varies from 11-18 cm and 6-11 cm for ERS-1 and ERS-2 respectively [Rudenko et al., 2012]. Orbits of ENVISAT are more precise with uncertainty of ~2 cm in vertical direction [Otten et al, 2012; Rudenko et al, 2012] and 3-6 cm in Horizontal direction [Otten et al, 2012; Michiel Otten, personal communication]. The uncertainty of ENVISAT orbits in horizontal direction varies between 4-6 cm for the period before October 2004 and 3-5 cm after October 2004 [Otten et al, 2012; Michiel Otten, personal communication]. The orbits are less precise before that date because of a smaller number of DORIS observations [Michiel Otten, personal communication]. Newer generation of SAR satellites use onboard GPS receivers and thus have more precise orbits. The uncertainty of TerraSAR-X orbits have been reported to be ~2 cm in total [Yoon et al., 2009], which is significantly better than the mission requirement of 10 cm.
In order to evaluate the velocity gradients from different SAR satellites, we use orbital uncertainties in vertical and horizontal directions of 2.5 cm and 12 cm for ERS1/2, of 2 cm and 4 cm for ENVISAT and 1 cm and 3 cm for TerraSAR-X and Sentinel-1.

**Table 3.1** Standard deviation of velocity gradients in range direction as a function of orbital uncertainty calculated using equations (27), (28) and (B.1). \(d\theta\) is the look angle variation equivalent to 100 km ground range.

<table>
<thead>
<tr>
<th>Similar Instrument</th>
<th>(\sigma_{Oh}) (cm)</th>
<th>(\sigma_{Ov}) (cm)</th>
<th>Acquisition/ year</th>
<th>Total time (years)</th>
<th>(\theta_0) (degrees)</th>
<th>(d\theta) (degrees)</th>
<th>(\sigma_{|}) (mm/yr/100 km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ERS -1/2</td>
<td>12</td>
<td>2</td>
<td>6</td>
<td>8</td>
<td>16</td>
<td>8</td>
<td>1.44</td>
</tr>
<tr>
<td>ENVISAT (IS2)</td>
<td>4</td>
<td>2</td>
<td>6</td>
<td>8</td>
<td>16</td>
<td>8</td>
<td>0.48</td>
</tr>
<tr>
<td>TSX (Stripmap-strip_009)</td>
<td>3</td>
<td>1</td>
<td>15</td>
<td>8</td>
<td>33.7</td>
<td>10</td>
<td>0.2</td>
</tr>
<tr>
<td>Sentinel-1(IW)</td>
<td>3</td>
<td>1</td>
<td>15</td>
<td>8</td>
<td>29</td>
<td>7</td>
<td>0.2</td>
</tr>
</tbody>
</table>

### 3.4.1 Uncertainty of velocity gradient in range direction

Table 1 summarizes the standard deviation of the range gradients for different SAR satellites, given their orbital uncertainty, revisiting time and typical imaging geometry. We use 6 acquisitions per year for ERS and Envisat. This number better reflects the archives than the nominal revisiting capability of these satellites, around 10-11 acquisitions per year. Similarly, we use 15 acquisitions per year, half of the actual revisiting cycle, for TerraSAR-X and Sentinel-1. We assume a total time-span of 8 years for all satellites. For Envisat we use the imaging specification of the IS2 mode, which is similar to ERS. For TerraSAR-X we use the specifications of strip-map (strip-009) and for Sentinel-1 we consider the Interferometric Wide swath mode (IW). We use equation (3.18) to obtain \(\sigma_{\|}\). It can also be obtained using equation (B.1). For each satellite, we
use the look angle in near range for $\vartheta_0$. To simplify the comparison between different satellites, we express $\sigma_{g}$ over 100 km ground range although the actual ground range may be different. To do this we multiply the result from equation (18) by $d\vartheta$ in Table 1. Table 3.1 shows that $\sigma_{g}$ is $\sim$1.4 mm/yr/100 km for ERS-1/2, $\sim$0.48 mm/yr/100 km for Envisat and $\sim$0.2 mm/yr/100 km for TerraSAR-X and Sentinel-1.

### 3.4.2 Uncertainty of velocity gradient in azimuth direction

In order to evaluate $\sigma_a$, we need to infer the uncertainty of baseline slopes from the standard deviation of orbital errors. Given $B_{h1}$ and $B_{h2}$ as the horizontal baseline errors at the first and last line of SAR acquisitions with swath length of $\Delta s$, $B_h'$ can be written as

$$B_h' = \frac{B_{h2} - B_{h1}}{\Delta s} \quad (3.25)$$

Using error propagation principles, $\sigma_{B_h'}$ can be expressed as

$$\sigma_{B_h'}^2 = \frac{1}{\Delta s^2} (\sigma_{B_{h1}}^2 + \sigma_{B_{h2}}^2 - 2\sigma_{B_{h1},B_{h2}}) \quad (3.26)$$

where $\sigma_{B_{h1},B_{h2}}$ is the covariance of $B_{h1}$ and $B_{h2}$. In practice the state vectors of a satellite orbit are provided in discretized time steps and therefore interpolation of state vectors is required to calculate the baseline components. Because the state vectors of the same orbit are highly correlated and also because usually the same set of state vectors is used to calculate $B_{h1}$ and $B_{h2}$, they become dependent variables with non-zero
correlation and covariance. Therefore, the correlation of $B_{h_1}$ and $B_{h_2}$ is required to calculate $\sigma_{Bh'}$. Considering the relationship of covariance and correlation coefficient as

$$\sigma_{Bh1,Bh2} = R \sigma_{Bh1} \sigma_{Bh2}$$

where $R$ is the correlation of $B_{h_1}$ and $B_{h_2}$ such that $-1 \leq R \leq 1$, assuming $\sigma_{Bh1} = \sigma_{Bh2} = \sigma_{Bh}$ we conclude that $\sigma_{Bh'}$ varies from 0 to $\frac{2 \sigma_{Bh}}{\Delta s}$ and $\sigma_{Bv'}$ from 0 to $\frac{2 \sigma_{Bv}}{\Delta s}$.

This means that evaluation of velocity gradients in azimuth direction requires information about the correlation of baselines at the start and end of a scene. If baselines are fully positively correlated ($R=1$), then $\sigma_{Bh'} = \sigma_{Bv'} = 0$ and no gradient in azimuth direction is expected.

Table 2 summarizes $\sigma_\alpha$ for different SAR satellites, given the orbital uncertainties, acquisition and imaging parameters from Table 1. Since we don’t have exact information about the correlation coefficient then we evaluate $\sigma_\alpha$ assuming different values of $R$ for two different scenarios. We consider a conservative scenario ($R=0.9$), and an optimistic scenario ($R=0.99$). Table 3.2 shows that for the conservative scenario $\sigma_\alpha$ varies from 1.5 mm/yr/100 km for ERS, to 0.5 mm/yr/100 km for TerraSAR-X and Sentinel-1. For the optimistic scenario, $\sigma_\alpha$ varies from 0.5 mm/yr/100 km for ERS to better than 0.15 mm/yr/100 km for TerraSAR-X and Sentinel-1. For completeness, the table also shows $\sigma_\alpha$ for the unrealistic worst-case scenario of independent baseline errors at the beginning and end of the swath ($R=0$). The real data discussed in section 5 suggest that R is high and likely close to 1.
Table 3.2 Standard deviation of velocity gradients in azimuth direction as a function of orbital uncertainty and baseline correlation calculated using equations (27), (28), (30) and (B.2).

<table>
<thead>
<tr>
<th>Instrument</th>
<th>$\sigma_{Oh}$ (cm)</th>
<th>$\sigma_{Ov}$ (cm)</th>
<th>$\sigma_{\alpha}$ (mm/yr/100 km)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Worst case (R=0)</td>
</tr>
<tr>
<td>ERS-1/2</td>
<td>12</td>
<td>2</td>
<td>4.8</td>
</tr>
<tr>
<td>ENVISAT</td>
<td>4</td>
<td>2</td>
<td>2.7</td>
</tr>
<tr>
<td>TSX</td>
<td>3</td>
<td>1</td>
<td>1.47</td>
</tr>
<tr>
<td>Sentinel</td>
<td>3</td>
<td>1</td>
<td>1.34</td>
</tr>
</tbody>
</table>

3.5 Velocity gradients in real InSAR data

In this section we use 2003-2010 descending ENVISAT ASAR data (beam IS2) from four different tracks of the southwestern U.S. to investigate the velocity gradients (Figure 3.2, Table 3). The tracks 41, 270 and 499 cover non-deforming areas on the stable North American Plate with swaths consisting of 5, 2 and 2 frames with 33, 28 and 37 acquisitions, respectively. The track 356 (5 frames) covers the deforming plate boundary zone, including the San Andreas fault. The first frame located in the relatively non-deforming part of the North American plate represents one of the non-deforming areas. For the analysis of the first frame we use all acquisitions, but for the analysis of the whole swath we use only cloud-free acquisitions with cloud coverage less than 5%, because of the tropospheric delay correction discussed below (17 acquisitions out of a total of 42). These acquisitions cover the time span from Feb 2003 to March 2010, right before the April 2010 M7.2 El Mayor-Cucapah earthquake.

In the non-deforming areas, we expect to observe velocity gradients in range and azimuth direction within the range of uncertainties expressed by the standard deviation of the velocity gradients. Given the uncertainty of Envisat orbits (tables 2 and 3), the
number and time span of acquisitions (Table 3), the standard deviation of velocity gradients in range direction varies for the four data sets between 0.6 mm/yr/100 km for track 356 and 0.88 mm/yr/100 km for track 270 (Table 3). The standard deviation of velocity gradients in azimuth direction varies between 1.1 mm/yr/100 km and 1.6 mm/yr/100 km for R=0.9, and varies from ~0.3 to ~0.5 mm/yr/100 km for R=0.99. In the deforming area, we expect to retrieve the well-known deformation measured by GPS.

Table 3.3 Observed velocity gradients in range and azimuth directions across and along the swath for non-deforming areas of Figure 3.3 together with uncertainties of range and azimuth gradients. The observed gradients are without any correction (raw) and after the three corrections (local oscillator drift, topographic residuals and tropospheric delay; OD, topo and trop). The uncertainties are propagated from Envisat’s orbital uncertainty (see Table 2), given the number of acquisitions, time span of acquisitions and ASAR IS2 imaging beam mode configuration with $\vartheta_0 = 16^\circ$ and $d\vartheta = 8^\circ$ over 100 km ground range.

<table>
<thead>
<tr>
<th>Track number</th>
<th>N</th>
<th>Uncertainty of velocity gradients [mm/yr/100 km]</th>
<th>Observed velocity gradients [mm/yr/100 km]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\sigma_\mathcal{R}$</td>
<td>$\sigma_\alpha$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R=0.9</td>
<td>R=0.99</td>
</tr>
<tr>
<td>T41</td>
<td>33</td>
<td>0.67</td>
<td>1.2</td>
</tr>
<tr>
<td>T270</td>
<td>28</td>
<td>0.88</td>
<td>1.6</td>
</tr>
<tr>
<td>T499</td>
<td>37</td>
<td>0.71</td>
<td>1.3</td>
</tr>
<tr>
<td>T356</td>
<td>42</td>
<td>0.6</td>
<td>1.1</td>
</tr>
</tbody>
</table>
Figure 3.2 Location map of the four Envisat swaths analyzed. The first frame on track 356 is used as a non-deforming area (in Figure 2.3) and as part of the larger swath covering the deforming area (in Figure 2.4). Track 356 covers major faults in this area, including San Andreas Fault (SAF), San Jacinto Fault (SJF) and Elsinore Fault (EF).

3.5.1 Data analysis

We generate zero doppler Single Look Complex (SLC) data using Modular SAR Processor software (MSP) from Gamma Remote Sensing, except for autofocus and azimuth compression, for which we use an algorithm based on the pseudo inverse fourier transform [Hyung-Sup Jung, personal communication]. We use the JPL/Caltech ROI_PAC software [Rosen et al., 2004] for interferogram processing and the DORIS orbits and Shuttle Radar Topography Mission (SRTM) DEM [Farr and Kobrick, 2000] to remove the phase due to the imaging geometry from each interferogram. For each area, we coregister the wrapped interferograms to a master SAR image. We use the statistical-
cost network-flow algorithm for phase unwrapping (SNAPHU) [Chen and Zebker, 2001] to unwrap the interferograms and spatially reference all the phase-unwrapped interferograms to the same coherent pixel. We invert for the phase history at each epoch, which is then temporally referenced to the first epoch [Berardino et al., 2002]. The networks of interferograms are fully connected, so that the design matrices for the time-series inversion have full rank. We select the coherent pixels using a temporal coherence threshold [Pepe et al., 2006] of 0.9. This threshold, in a redundant network eliminates pixels affected by phase-unwrapping errors.

We use the empirical model of Marinkovic & Larsen [2013] to correct for the local oscillator drift of the ASAR instrument (Appendix 3.3) and the time-domain method of Fattahi & Amelung [2013] to correct for topographic residuals. After these corrections, the remaining phase histories in non-deforming areas contain contributions from orbital errors and atmospheric delay.

To correct the tropospheric delay, we use different approaches. In the non-deforming areas, we use the empirical phase-elevation approach [e.g: Doin et al., 2009]. In this approach the phase proportional to the topography for epochs at which the correlation coefficient between the two is larger than a threshold (e.g: 0.4) is removed. In the deforming area we use Envisat’s MEdium Resolution Imaging Spectrometer (MERIS) data to remove at each epoch the wet delay component of the tropospheric delay [Walters et al., 2013] and use the ERA-Interim numerical weather model, provided by the European Center for Medium-Range Weather Forecast (ECMWF), to calculate and remove the hydrostatic delay [Jolivet et al, 2011, 2012, 2014]. Thus, for this track we can use only SAR acquisitions with simultaneous, cloud-free MERIS acquisitions.
3.5.2 Velocity fields in non-deforming areas

Figure 3.3 shows the velocity fields for the four non-deforming areas obtained from the raw time-series without any correction (raw), with local oscillator drift correction (OD), with both local oscillator drift and topographic residual corrections (OD, topo), and with local oscillator drift, topographic residual and tropospheric delay corrections (OD, topo, trop). The figure shows that the local oscillator drift correction dramatically reduces the range gradient (Figure 2.3, second column compared to first column) demonstrating the importance of this correction for ENVISAT ASAR data. The topographic residual correction further reduces the range gradient (Figure 2.3, third column), because this correction does not only remove residuals from DEM errors but any residuals proportional to the perpendicular baseline history [Fattahi & Amelung, 2013]. The velocity gradients removed by the topographic residual correction are most likely caused by timing error [Wang & Jonsson, 2014] or processing simplifications, which introduce gradients proportional to the baseline into the interferograms. In the time domain, the range gradients are proportional to the perpendicular baseline history and therefore removed by the topographic residual correction (See Figure S3.1 for an example showing the effect of different corrections on a timeseries epoch for track 41).

For a quantitative investigation, we estimate the velocity gradients by fitting planes in range and azimuth directions to the velocity fields before any correction, and after the three corrections, summarized in Table 3. Before any correction, the gradients vary from 13 to 25 mm/yr/100 km in range direction and -0.53 to 3 mm/yr/100 km in azimuth direction. The local oscillator drift correction reduces the gradients in range direction by ~19 mm/yr/100 km to -6 to 6 mm/yr/100 km. The topographic residual correction reduces the magnitude of the range gradients to 0.6 mm/yr/100 km or less. These two corrections
affect only the gradient in range direction. The effect of the tropospheric delay correction in range is small, but it reduces the magnitude of the gradient in azimuth direction to 0.95 mm/yr/100 km or less. For the longest swath (track 41, 5 frames) the magnitude of the remaining azimuth gradient is less than 0.02 mm/yr/100 km.

**Figure 3.3** Velocity fields of four different tracks in non-deforming areas in radar coordinates obtained from the InSAR raw time-series (first column), from time-series corrected for OD (second column), corrected for OD and topographic residuals (third column) and corrected for OD, topographic residuals and tropospheric delay (fourth column). Black squares: reference points.
Table 3 compares the gradients observed after all corrections (residual gradients) with the gradient uncertainties inferred from the orbital uncertainty. In range direction, all the residual gradients lie within one standard deviation. In azimuth direction and assuming R=0.9, the residual gradients lie within one standard deviations (Table 3). Assuming R=0.99 the residual gradient for tracks 41 and 270 lies within one standard deviations (Table 3). The gradients for the other two tracks lie outside this interval.

3.5.3 Velocity field in deforming area

Figure 3.4a shows the velocity field for five frames of track 356 covering the Southern San Andreas Fault (including the frame shown in Figure 3.3) corrected for local oscillator drift, topographic residuals and tropospheric wet and hydrostatic delays. The velocity field for the non-deforming northern part does not show any feature or gradient, whereas the southern part shows a LOS velocity increase caused by the right-lateral motion along the San Andreas Fault system.

Figure 3.4b shows the gridded horizontal GPS velocity field of Kreemer et al. [2012], interpolated to the same grid as the InSAR data, projected to the InSAR line-of-sight and referenced to the same reference pixel as InSAR data. Overall, the InSAR and GPS velocities agree very well.

Figure 3.5 shows the differences between the InSAR and GPS velocity fields for three different ways of handling the tropospheric delays in the InSAR; without any tropospheric correction (Figure 3.5a), with wet delay correction using MERIS (Figure 3.5b), and with both wet (using MERIS) and hydrostatic delay (using ERA-I) corrections (Figure 3.5c). The difference between InSAR and GPS is of the order of ±1 mm/yr in the northern part but varies in the southern part between up to 5 mm/yr without
tropospheric correction and 3 mm/yr with wet and hydrostatic corrections (in the southwest centered at coordinates 33.2,-116.3).

**Figure 3.4** LOS velocity from (a) InSAR and (b) horizontal GPS of Kreemer et al. [2012] projected in LOS direction relative to the InSAR reference point, overlaid on grey-shaded topography (black represents low elevations). The difference between the velocity fields is shown in Figure 2.5c.

The differences are either due to (1) horizontal deformation not captured by GPS because of sparse station spacing, (2) vertical deformation not included in the GPS, or (3) tropospheric delays related to imperfect hydrostatic delay correction of the InSAR data.
Figure 3.5  Difference between InSAR and horizontal GPS for InSAR a) without tropospheric delay correction b) corrected for wet delay (using MERIS), c) corrected for wet delay (using MERIS) and hydrostatic delay (using ERA-I). The colorscale is saturated at 3 mm/yr.

Examples for the first includes fault creep (yellow-red stripe in Figure 2.5c just north of the decorrelated area centered at coordinates 33.5,-116.0 and elongated dark blue area at 33.0,-115.8; San Andreas and Superstition Hills faults, respectively and for the second includes a subsiding area (red saturated area in Figure 2.5c at 32.8,-115.3). Some of the features in the northern part are due to imperfect hydrostatic delay correction. The larger residual in the southwest of up to 2 mm/yr could be due to both large-scale subsidence and imperfect hydrostatic delay correction. As there is a topographic relief of up to 2 km
and the uncorrected data show an even higher residual, we consider tropospheric delay the most likely cause.

3.6 Discussion

We have expressed the uncertainty of the InSAR velocity fields in terms of the uncertainties of the velocity gradients in range and azimuth directions (range and azimuth uncertainties). We found that these uncertainties depend on the orbital uncertainties, the number and time span of SAR acquisitions. For modern SAR satellites with very precise orbits such as TerraSAR-X and Sentinel-1, the range uncertainty is ~0.2 mm/yr/100 km (Table 1). For older satellites with less accurate orbits such as ERS and Envisat, for the same time span, the uncertainty is about 1.5 and 0.5 mm/yr/100 km respectively. The analysis of four Envisat InSAR data sets shows that the magnitude of the residual velocity gradients in range direction of ~0.7 mm/yr/100 km or less fall within one standard deviation of the range uncertainty (Table 3).

The azimuth uncertainty depends on the uncertainty of baseline slope, which is not known. We thus formulated the azimuth uncertainty as a function of the same parameters as the range uncertainty and the correlation coefficient between the baselines at the beginning and end of the swath. This parameter is not well constrained but is expected to be high (close to 1). Therefore we evaluated the azimuth gradient for different scenarios of R. In the worst case scenario of independent baselines (R=0 in Table 2), the azimuth uncertainty is ~5 mm/yr/100 km for ERS, ~3 mm/yr/100 km for Envisat, and ~1.5 mm/yr/100 km for TerraSAR-X and Sentinel-1. Such large azimuth uncertainties are unlikely because of two reasons. First the baselines along the swath are not independent due to the high correlation of the state vectors of each orbit and also due to the
requirement of orbit interpolation using the same set of state vectors to estimate the baseline components along the swath. Second, observed velocity gradients in real InSAR data for Envisat satellite in this paper are less than 1 mm/yr/100 km, significantly less than ~3 mm/yr/100 km azimuth uncertainty. Also, a recent study reports velocity gradients in azimuth direction of less than 1.5 mm/yr/100 km for 6 tracks of ERS data (each consisting of 6-7 frames) [Greene et al., 2014; in review]. Although a major portion of the observed gradients can be attributed to the atmospheric delay, the reported gradients are significantly less than the azimuth uncertainty for the worst case scenario.

The along-track baseline correlation and with it the azimuth uncertainty could in principle be estimated from the observed azimuth gradients (by estimating the standard deviation of the sample and substituting equation (27) into equation (26) and then into equation (19)). However, the sample of four analyzed data sets is not enough to estimate a standard deviation. Furthermore, the observed azimuth gradients do not necessarily reflect orbital errors but also could be due to imperfect compensation for tropospheric delays. If this is the case, it would suggest a baseline correlation close to 1 and very small azimuth uncertainty, consistent with the velocity gradient of 0.01 mm/yr/100 km for the five-frame swath of track 41 (Table 3).

The linear relationship between baseline errors and velocity gradients in equations (16) and (17) implies that for random baseline errors with zero mean (of independent orbits), the velocity gradients have zero mean. In other words, in InSAR time-series the phase contributions from orbital errors to the velocity field tend to cancel out as is generally expected. For satellites with precise orbits, precise InSAR velocity fields can be obtained without correcting orbital errors if long-wavelength artifacts from other sources
are identified and corrected for. In a previous study orbital errors were overestimated because other sources of long-wavelength artifacts were not properly identified [Gourmelen et al., 2010].

Other sources of long-wavelength artifacts include the topographic residuals, tropospheric delay and contributions from hardware issues. The topographic residual correction removes range gradients due to the processing approximations, which cause artifacts proportional to the perpendicular baseline. For Envisat, the most significant correction is for the local oscillator drift, which removes a range gradient of ~19 mm/yr/100 km for IS2 imaging geometry.

Tropospheric delays cause phase patterns at a variety of scales, which can significantly affect the estimated velocity fields. We found that MERIS imagery, acquired by Envisat simultaneously with the SAR imagery, is very efficient in mitigating the wet delay component, confirming the results of previous studies [Walters et al., 2013; Li et al., 2006]. The MERIS correction works only on cloud free days and imagery is available only for Envisat descending orbits, although NASA’s Moderate Resolution Imaging Spectrometer (MODIS) could be used for other satellites. Therefore, improved tropospheric correction using more accurate numerical weather models is required to improve the InSAR’s ability to resolve ground displacements over large areas.

Although the observed velocity gradients (section 4, Table 3) are small and fall within the uncertainty due to orbital errors, they also include contributions from residual atmospheric delay, ocean tidal loading and possibly unmodeled reference frame motion. The last two generate systematic long-wavelength phase patterns, which can be predicted and removed from InSAR data [DiCaprio & Simons, 2008; Bähr et al., 2011].
3.7 Conclusion

1. We have developed formulas for the uncertainty of InSAR velocity fields as a function of the orbital uncertainties. The standard deviation of the range gradient depends on the number of acquisitions, the time span of acquisitions, the imaging geometry, and the standard deviation of the baseline errors. The standard deviation of the azimuth gradient depends on the same parameters except the last, but instead on the standard deviation of the baseline slope error. Although there is a lack of knowledge about the standard deviation of the baseline slope error, they can be expressed in terms of the correlation coefficient between the baseline errors at the beginning and at the end of the swath.

2. The uncertainty in range direction is \(\sim1.5 \text{ mm/yr/100 km}\) for ERS, \(\sim0.5 \text{ mm/yr/100 km}\) for Envisat, and \(\sim0.2 \text{ mm/yr/100km}\) for TerraSAR-X and Sentinel-1. These uncertainties apply for general data acquisition scenarios. For specific datasets, the uncertainties can be calculated using equations (18) and (19). For Envisat data discussed in this paper, the observed velocity gradients in range direction are less than 0.8 mm/yr/100 km, falling within the one-sigma to two-sigma uncertainty.

3. Evaluation of the velocity uncertainty in azimuth direction requires information either about baseline slope errors or the correlation coefficient between baseline components at the beginning and end of the swath. The observations of velocity gradients in azimuth direction reported in this paper and those of Greene et al, [2014] suggest high correlation coefficient (R>0.9). Additional measurements of velocity gradients are required to better constrain R. Assuming R=0.9, the velocity uncertainty in azimuth direction is better than 1.5 mm/yr/100 km for older satellites (ERS and Envisat) and better than 0.5 mm/yr/100 km for modern satellites (TerraSAR-X and Sentinel-1). A
More optimistic scenario (R=0.99) suggests azimuth uncertainty better than 0.5 mm/yr/100 km for older satellites and better than 0.15 mm/yr/100 km for modern satellites. The uncertainty increases with swath length, but an exact number for the increase of the uncertainty with distance can’t be given because the dependence of R with distance is not known. For Envisat data discussed in this paper, the observed velocity gradients in azimuth direction are less than 1 mm/yr/100 km, falling within the one-sigma uncertainty, given R=0.9.

4. In practice the InSAR measurements can be biased by sensor hardware and by processing approximations. For Envisat an important effect is the drift of the local oscillator. The accuracies quoted above can only be achieved if systematic errors are identified and corrected for. The topographic residual correction of Fattahi and Amelung [2013] is an efficient way to correct for systematic effects reflected in biased perpendicular baseline. The InSAR uncertainty is dominated by atmospheric delays and not by orbital errors.
Chapter 4. Phase unwrapping error correction

4.1 Summary

InSAR range change time-series estimated from the inversion of a network of interferograms can be potentially biased by wrong phase jumps added to the Interferometric phases during the phase unwrapping, called unwrapping errors. We develop an automatic algorithm to detect and correct unwrapping errors in a dense network of interferograms. Using a simulated network of interferograms we evaluate the efficiency of the algorithm for three different scenarios with 10%, 20% and 50% unwrapping errors in the network. The algorithm successfully detects and corrects 100% of the interferograms with unwrapping errors in the first case, 78% in the second case and 67% in the third case. We quantify the efficiency of the algorithm using a parameter called temporal coherence, which varies from 0 to 1, where for a given pixel temporal coherence of 1 means no unwrapping errors in the network. The algorithm significantly improves the temporal coherence of a network of real interferograms from ~0.5 to ~0.9.

4.2 Background

Interferometric Synthetic Aperture Radar (InSAR) time-series analysis using Small Baseline (SB) [Berardino et al., 2002] or Permanent Scatterer (PS) [Ferretti et al., 2001] techniques have been successfully applied to study ground displacement due to the natural and anthropogenic processes. The principal observations of the InSAR are wrapped Interferometric phases (modulo 2pi) and integration of the wrapped phase,
commonly called phase unwrapping, is required to obtain a field of relative phase with respect to a given pixel. Several algorithms have been developed to unwrap individual interferograms commonly called two-dimensional algorithms [Goldstein et al., 1988]. These algorithms have been extended to three dimensions, in space and time, by using the multi-temporal InSAR observations [Pepe et al., 2006; Osmanoglu et al., 2014]. Despite existence of several three-dimensional unwrapping algorithms, the two-dimensional algorithms seem to be more popular and commonly used in the InSAR processing because of two reasons. First the two dimensional algorithms are simple to implement and the efficiency of some two-dimensional algorithms such as SNAPHU [Chen and Zebker, 2001] has been shown through years. In opposite, the three-dimensional phase unwrapping usually requires expensive processing, their application has been limited and the efficiency of the algorithms has not been widely shown. Therefore it’s relevant to develop algorithms to evaluate the Interferometric phases unwrapped with two-dimensional algorithms and possibly detect and correct the unwrapping errors (wrong 2pi phase jumps), using constraints from time domain.

Several algorithms have been developed to evaluate the phase unwrapping and correct the unwrapping errors. Yang et al., [2013] Used a region-growing algorithm to detect and correct the unwrapping errors, Lópe-Quiroz et al.,[2009] used the information from residual of time-series inversion of unwrapped interferograms. Biggs et al, [2007] used the phase consistency among three interferograms formed from three SAR images to visually identify the unwrapping errors and manually correct the phase jumps. Based on this idea we develop a mathematical frame work to automatically detect and correct the phase unwrapping errors in network of interferograms. Our definition of phase closure in
this paper is different from [Lopez-Quiroz et al, 2009] in which they refer to the residual vector of the time-series inversion as the phase misclosure.

4.3 Time-series inversion of Small Baseline interferograms

In the small baseline InSAR time-series algorithms M phase-unwrapped interferograms (\( \delta \varphi \)) formed from \( N+1 \) SAR acquisitions is inverted to estimate the phase velocities, which can be used to reconstruct the phase history, \( \phi(t) \), at \( N \) epochs (\( t_1, ..., t_N \)) with respect to the reference acquisition at \( t_0 \), implying \( \phi(t_0) = 0 \). From the phase histories, one can reconstruct the interferometric phases as \( \delta \hat{\varphi}_{ij} = \phi(t_j) - \phi(t_i) \) and calculate the residual vector for each interferometric phase as \( r_{ij} = \delta \varphi_{ij} - \delta \hat{\varphi}_{ij} \).

Non-zero residual values indicate rank deficiency of the design matrix used for the time-series inversion or noise in the observation vector (\( \delta \varphi \)). We assume connected network of interferograms, which results in full rank design matrix. The phase unwrapping error is an important component of noise, which results in biased estimation of phase histories and leads to non-zero residuals of the inversion. In the following we evaluate the unwrapping errors in more details.

4.4 Phase closure for the unwrapping error detection and correction

A redundant network of interferograms allows evaluating the noise in the observation vector (\( \delta \varphi \)) by checking the consistency of triplets of interferometric phases formed from common SAR acquisitions. For a given triplet of unwrapped interferometric phases \( \delta \varphi_{mn}, \delta \varphi_{nk}, \delta \varphi_{mk} \) formed from three SAR acquisitions acquired at \( t_m, t_n \) and \( t_k \), we
define the closure over the circuit for the unwrapped phase \( C_{mnk} \), referred to as phase closure, as

\[
C_{mnk} = \delta\varphi_{mn} + \delta\varphi_{nk} - \delta\varphi_{mk} \quad (4.1)
\]

Non-zero phase closures indicate noise in the unwrapped interferometric phases. The noise components include noise due to the geometrical or temporal decorrelation, inconsistency caused by processing steps such as filtering and multi-looking, imperfect geometry for focusing the SAR images and imperfect geometry for removing phase due to the non-zero spatial baseline between the SAR acquisitions or unwrapping errors. Since we are interested in unwrapping errors we isolate this component in the phase closure by removing the phase closure of wrapped interferograms from the phase closure of unwrapped interferograms as

\[
C_{mnk} = (\delta\varphi_{mn} + \delta\varphi_{nk} - \delta\varphi_{mk}) - \text{wrap}(\delta\varphi_{mn} + \delta\varphi_{nk} - \delta\varphi_{mk}) \quad (4.2)
\]

where \( \text{wrap} \) is a wrapping operator. Phase components from ground displacement, atmospheric delay, orbital errors, topography or any other systematic components such as the local oscillator drift of the radar instruments do not result in non-zero residuals and thus for the purpose of time-series inversion are considered as signal.

In a redundant network of interferograms, for each pixel, considering all triplets of unwrapped interferograms, the following linear system of equations can be used to express the temporal consistency of interferometric phases as

\[
C(\delta\varphi + 2\pi U) = 0 \quad (4.3)
\]
wherein $\delta \varphi$ is a vector of all unwrapped interferometric phases, C is a matrix expressing all possible triplets and U is an Mx1 vector of required integer numbers of phase jumps to meet the consistency of the interferometric phases. The amount of phase jumps required to correct the vector of interferometric phases for a given pixel equals $2\pi U$. We label an interferometric phase as correctly unwrapped if there is no non-zero phase closure associated with that interferometric phase. Assuming $m$ correctly unwrapped interferometric phases, we add a constraint, $DU = 0$, to the system of equation of (3), where $D$ is an $m \times M$ matrix. Elements of each row of D are zero except for the column corresponding to the correctly unwrapped interferometric phase. Considering (3) and the constraint $DU=0$, the following system of equations should be solved for each pixel to estimate $U$:

$$
\begin{bmatrix}
-2\pi C \\
D
\end{bmatrix} U = 
\begin{bmatrix}
C\delta \varphi \\
0
\end{bmatrix}
$$

In equation (4.4) the matrix C is the same for all pixels but D may be different for various pixels. Figure 4.1 shows an example of the design matrices for a given pixel with a network of eight interferograms with unwrapping errors on the sixth interferogram.
Figure 4.1 A network of eight interferograms with unwrapping error in sixth interferogram for a given pixel. C and D are the required matrices to form the system of equations in (4) to detect and correct the unwrapping error in sixth interferogram.

4.5 The algorithm for unwrapping error correction

Given a set of interferograms, unwrapped with any two dimensional unwrapping algorithm, we propose the following algorithm to correct the possible unwrapping error based on the phase closures:

a) Calculate the phase closure of all possible triplets of unwrapped interferograms using equation (2) and generate the matrix C in equation (3).

b) For a given pixel analyze the phase closures to identify the interferograms without unwrapping errors. An interferogram is considered correctly unwrapped if there is no triplet in the network with phase closure more than a threshold (i.e. $\text{abs}(C_{mnk}) < \text{thr}$). In this paper we considered a threshold of 0.1 radians.

c) Generate D matrix using correctly unwrapped interferograms from step a.

d) Obtain the solution of equation (4.4) using L2-norm minimization or any other Lp-norm minimization. In this paper we use Least-squares solution: $\hat{U} = (A^T A)^{-1} A^T L$, where

$$A = \begin{bmatrix} -2\pi C \\ D \end{bmatrix} \quad \text{and} \quad L = \begin{bmatrix} C\delta\phi \\ 0 \end{bmatrix}.$$ 

$$€A = \begin{bmatrix} \delta\phi -2\pi C \\ D \end{bmatrix} \quad \text{and} \quad €L = \begin{bmatrix} 0 \\ C\delta\phi \end{bmatrix}.$$ 

e) Round values of $\hat{U}$ to the nearest integer numbers ($\hat{U} = \text{round}(\hat{U})$) and correct the interferometric phases as $\delta\phi^c = \delta\phi + 2\pi \hat{U}$,

f) Repeat steps (b) to (e) for every other pixel.
Figure 4.2 A simulation to test the unwrapping error correction approach. a) A network of simulated interferograms, b) a simulated velocity field, c) a simulated unwrapped interferometric phase and d) same as c but after adding unwrapping error (2pi phase jump) to the top half of the interferogram.

4.6 Simulated data

To test the proposed algorithm, we simulate a set of interferograms considering the network of interferograms shown in Figure 4.2(a), based on the simulated ground displacement (Figure 4.2(b)) due to a vertical dislocation at the middle of the scene locked at 15 km depth with a long term slip rate of 15 mm/yr. Figure 4.2(c) shows a simulated unwrapped interferometric phase and Figure 4.2(d) shows the same interferogram after adding 2pi phase jump as unwrapping error to the top half of the interferogram. Figure 4.3(a) to 4.3(c) show three cases where we add unwrapping errors to the 10%, 20% and 50% of the interferograms respectively. Interferograms with unwrapping errors have been specified with red lines in the network of interferograms. We correct the unwrapping errors for all the three simulations and evaluate the efficiency of the correction with different approaches. First we invert the interferometric phases before and after the unwrapping error correction and for each pixel we calculate a parameter called temporal coherence, $\gamma$, as
\[ \gamma = \left| \frac{1}{M} \sum_{i=1}^{M} \exp\left[ j(\delta \varphi_i - \delta \hat{\varphi}_i) \right] \right|, \quad 0 \leq \gamma \leq 1 \] 

(4.5)

Since the network of interferograms are fully connected and unwrapping errors are the only source of noise in our simulations, \( \gamma = 1 \) for a given pixel indicates no unwrapping errors in any of the interferograms while \( \gamma < 1 \) implies the existence of unwrapping errors.

Figures 4.3(d) to 4.3(f) show the temporal coherence before unwrapping error correction for the three simulations given 10%, 20% and 50% of interferograms with unwrapping errors. Figures 4.3(g) to 4.3(i) demonstrate the temporal coherence after unwrapping error correction for the three simulations. All the maps of temporal coherences in Figure 3 show maximum coherence of 1 in the bottom parts of the maps where no unwrapping errors were added to the simulated interferograms. The upper parts of the maps of temporal coherence are affected by the unwrapping errors. The unwrapping error correction improves the temporal coherence of the upper parts of the maps from 0.6, 0.46 and 0.07 to 1, 0.91 and 0.27 for the three simulations with 10%, 20% and 50% of the unwrapping errors respectively.

We also evaluate the effect of the unwrapping error correction on the interferometric phases. Figures 4.4(a) to 4.4(c) shows the unwrapping errors (2\pi phase jumps) in the simulated interferograms before and after the correction for three simulations with 10%, 20% and 50% of interferograms with simulated unwrapping errors. The figures show that the correction algorithm detects and corrects 16, 25 and 54 interferograms out of total 16, 32 and 80 interferograms with unwrapping errors for the three simulations with 10%, 20% and 50% of the unwrapping errors respectively.
Figure 4.3 Simulated network of interferograms with unwrapping errors added to a) 10% b) 20% and c) 30% of the interferograms. Red lines represent the interferograms with the unwrapping errors. The map of temporal coherence before the unwrapping error correction for the three simulations with d) 10% e) 20% and f) 50% of unwrapping errors added to the simulated interferograms. (g) to (i) are the same as (d) to (f) but after unwrapping error correction.

Figures 4.4(d) to 4.4(f) show the effect of the unwrapping error correction on the estimated displacement histories for the three simulations. The correction reduces the scatter of the estimated displacement with RMSE of 4, 6 and 17 mm to 0, 5 and 12 mm for the three simulations with 10%, 20% and 50% of the unwrapping errors, respectively.
Figure 4.4 Effect of the unwrapping error correction on the interferometric phases and on the estimated displacement history. (a-c): The unwrapping errors in the simulated interferograms before (black bars) and after (red bars) unwrapping error correction for the three simulations with a) 10%, b) 20% and c) 50% of the simulated unwrapping errors. (d-f): The estimated displacement history before (red circles) and after (black squares) unwrapping error correction compared with the simulated displacement (green dashed line) for the three simulations with d) 10%, e) 20% and f) 50% of the unwrapping errors.

4.7 Real data

We apply the proposed algorithm to a set of ENVISAT SAR data acquired over Baja California from ~2003 to 2011 in descending orbit (Track = 499). We generate small baseline interferograms and multilook each differential interferogram by a factor of 8 by 40 in range and azimuth directions respectively. We use Snaphu for unwrapping each interferogram separately and reference all of them to the same coherent pixel. Without applying any correction to the unwrapped interferograms we invert the connected network of interferograms to obtain the phase history at each epoch. Figure 4.5 demonstrates the temporal coherence of the inversion, computed using equation (4.5).
Figure 4.5 (a) a highly coherent area with red color can be distinguished from an area with moderate coherence, with green color. Third area is Isla Ángel de la Guarda island which shows the lowest temporal coherence due to its spatial separation from the rest of the scene on the peninsula. Choosing a threshold over 0.7 removes all the pixels in green area with moderate coherence values. Figure 4.5(b) shows the temporal coherence after unwrapping error correction. Figure 4.5(c) demonstrates the difference of temporal coherence before and after the unwrapping error correction, and shows significant improvement after the correction.

Figure 4.5(d) shows an interferogram before unwrapping error correction and Figure 4.5(e) shows the same interferogram after the unwrapping error correction. Figure 4.5(f) is the difference of interferometric phase before and after the unwrapping error correction, which is the amount of phase jump added to the interferogram during the unwrapping error correction. Before unwrapping error correction, a phase jump can be seen in Figure 4.5(e) in the area with moderate coherence. This phase jump has been corrected in Figure 4.5(e) as a result of unwrapping error correction.

4.8 Conclusion

We developed an automatic algorithm in time domain to detect and correct the unwrapping errors of Interferometric phases and evaluated the efficiency of the algorithm using simulated and real data. For a connected network of 160 interferograms, the algorithm successfully detected and corrected 16, 24 and 56 interferograms for three simulations with unwrapping errors added to 16, 32 and 80 interferograms, respectively. Applying the algorithm to a network of real interferograms showed improvement of the temporal coherence from ~0.5 to ~0.9.
A redundant network of interferograms is crucial for the success of the algorithm. There is a higher chance to successfully detect and correct the unwrapping errors in the interferograms located in the denser parts of the network. Possible unwrapping errors in isolated interferograms and disconnected from other interferograms in the network can not be detected using this algorithm.

**Figure 4.5** Application of the proposed algorithm for unwrapping error correction on a stack of interferograms formed from ENVISAT SAR images (Track=499): a) temporal coherence before unwrapping error correction, b) temporal coherence after unwrapping error correction, c) the difference between a and b, d) an interferogram formed from two SAR images, acquired on 2004/07/26 and 2006/06/26, before unwrapping error correction e) the same interferogram after unwrapping error correction, and f) the difference of e and d. All figures are overlaid on an SRTM DEM of the area. Incoherent pixels in d, e and f with temporal coherence less than 0.8 are masked out.
Chapter 5. Strain accumulation along the Chaman Fault system observed with InSAR

5.1 Summary
Chaman fault system forms the western boundary of the India plate with Eurasia and is expected to accommodate ~3 cm/yr relative motion between the two plates. Due to a lack of permanent geodetic observations of ground deformation in the region, it’s not known how the fault system accommodates the relative motion between the two plates.

We use an advanced InSAR time-series approach to obtain a large ground displacement velocity field along and across the plate boundary zone to evaluate the contemporary rate of strain accumulation along the Chaman fault system. We develop a new approach, which uses MODIS observations of precipitable water vapor to evaluate the InSAR uncertainty due to the tropospheric delay, called pseudo-uncertainty. We also evaluate the uncertainty of the InSAR velocity field, InSAR uncertainty, using the scatter of the InSAR time-series. High correlation of 86% between the InSAR pseudo-uncertainty and InSAR uncertainty indicates that the scatter of our InSAR time-series is dominated by the random tropospheric delay.

In the central Chaman fault system, latitudes of 28N to 30.5N, the InSAR velocity field demonstrates strain localization on the Ghazaband fault and with a smaller amplitude on the Chaman fault. We estimate slip rates of 16 and 8 mm/yr for Chaman and Ghazaband faults and a locking depth of 10 km for the Ghazaband fault. Further north where the topographic expression of Ghazaband fault vanishes the slip does not transfer to the Chaman fault but rather distributes among several faults in the Kirthar
range and Sulaiman lobe. At latitude 32N we estimate a slip rate of 6-7 mm/yr for the Chaman fault. We detect 340 km creeping segment of the Chaman fault with variable creep rates of 0-8 mm/yr along the strike.

The observed surface creep on the Chaman fault indicates that the fault is partially locked accumulating strain which most likely will be released with M<7 earthquakes as has been observed also in last 100 years. Our InSAR velocity does not show strain localization neither surface creep on the first 100 km of the 300 km segment of the Chaman fault north of 32N. The high rate of strain accumulation on the Ghazaband Fault and lack of evidence for the rupture of the fault in the 1935 Quetta earthquake, implies a significant earthquake hazard to the Balochistan and the populated areas such as the city of Quetta.

5.2 Background

Space geodesy has dramatically improved our understanding of the ground deformation at the three phases of the earthquake cycle including the coseismic ground deformation within few to tens of seconds during the earthquake followed by post-seismic deformation, which within days to decades relaxes to the long-term secular rate of the ground deformation during the interseismic period (tens to thousands of years) [eg: Thatcher and Rundle, 1979; Arnadóttir and Segall, 1994; Segall and Davis, 1997; Pollitz et al., 1998; Burgmann et al., 2002; Fialko et al., 2005; Hsu et al., 2006; Ozawa et al., 2011; Johnson et al., 2012]. Ground deformation observations have been used to constrain the depth to which the faults are locked during the interseismic period (locking depth) [Savage and Burford, 1973; Bilham et al., 1997; Smith-Konter et al., 2011], the rate of aseismic slip below the locking depth (slip rate) [Elliott et al., 2008; Lindsey and
Fialko, 2013; Walters et al., 2013], the rate at which faults may aseismically slip at shallow depths (shallow creep rate) [Bürgmann et al., 1998; Cakir et al., 2005; Wei et al., 2011; Jolivet et al., 2012; Lindsey et al., 2014a; Chaussard et al., 2015], the depth to which the shallow fault creep extends (depth of creep extend) [Kaneko et al., 2013; Cetin et al., 2014] or to estimate the location of the locked (coupled) and creeping regions on a fault plane and their extent of coupling [Jolivet et al., 2014a]. Such studies have been widely conducted over major transform plate boundaries such as the San Andreas Fault and the North Anatolian Fault systems. However little is known about some other major plate boundaries such as the Chaman fault system.

The Chaman fault system is one of the longest (~1000 km) continental transform faults on Earth forming the western boundary of the India plate with Eurasia [Lawrence et al., 1981]. The fault system connects the Makran subduction zone in southern Balochistan, Pakistan, to the Himalayan convergence zone to the north [Lawrence et al., 1981]. Geological studies have estimated long term slip rates of 19-24 mm/yr for the past 20–25 Myr [Lawrence et al., 1992] and 25–35 mm/yr for the past 2 Myr [Beun et al., 1979] without taking in to account the distributed shear [Ambraseys and Bilham, 2003]. Field investigations show an offset of ~1150 m of alluvial fans over last ~34.8 kyr along the Chaman Fault which is equivalent to a slip rate of ~33 mm/yr [Ul-Hadi et al., 2013].

During the last century the plate boundary has been ruptured in four M>7 earthquakes including M7.1 1909 Kachhi [Ambraseys and Bilham, 2003], M7.3 1931 Mach [Szeliga et al., 2009], M7.7 1935 Quetta [Armbruster et al., 1980] and 2013 M7.7 Balochistan earthquakes [Avouac et al., 2014; Barnhart et al., 2014; Jolivet et al., 2014c]. Vicinity of the Chaman fault system to the major cities in Afghanistan and Pakistan including
Karachi, Quetta, Kabul and Kandahar with 11, 0.5, 2 and 0.5 millions population together with the relatively poor construction in this region faces millions of inhabitants of the region to a great hazard. Geodetic measurements of ground deformation over the Chaman fault system help to improve our understanding of the earthquake cycle deformation and to better understand the earthquake hazard in Afghanistan and Pakistan.

The modern geodetic observations of the ground deformation across the Chaman fault system are limited to a few campaign GPS measurements [Szeliga et al., 2012]. More recently, several studies have evaluated the coseismic displacement due to the M7.7 2013 Balochistan earthquake [Avouac et al., 2014; Barnhart et al., 2014; Jolivet et al., 2014c]. Limited Interferometric Synthetic Aperture Radar (InSAR) studies have been also conducted on the Chaman fault near the town of Chaman, Pakistan [Szeliga et al., 2012] and to study the coseismic and postseismic deformation due to the moderate (M<6) earthquakes on the fault system [Furuya and Satyabala, 2008; Fattahi et al., 2015]. Others have used InSAR to evaluate the interseismic coupling in the eastern Makran subduction zone. In absence of permanent geodetic GPS networks, we use InSAR to study the tectonic deformation and to evaluate the strain accumulation along and across the Chaman fault system.

InSAR has been widely used to measure the short-wavelength (eg: <50 km) ground deformation [eg: Pritchard and Simons, 2004; Fielding et al., 2009; López-Quíroz et al., 2009; Weston et al., 2011; Chaussard and Amelung, 2012; Bagnardi et al., 2013; Chaussard et al., 2013], but rarely has been applied to measure longer wavelength deformation across the plate boundaries. Given the long-wavelength ground displacement as the signal of interest, its similar spatial pattern with long-wavelength noise, originated
from atmosphere, processing artifacts, hardware issues and orbital errors, makes it difficult to separate the signal from the noise [Lohman and Simons, 2005a]. Several studies have overcome this limitation by using a combination of GPS and InSAR observations to constrain the long-wavelength and short-wavelength ground displacements from GPS and InSAR respectively [Lundgren et al., 2009; Gourmelen et al., 2010; Wei et al., 2010; Manzo et al., 2012; Wang and Wright, 2012; Kaneko et al., 2013; Tong et al., 2013] or by assumptions about the ground deformation models [Wang et al., 2009a]. These approaches cannot be used over the Chaman fault system due to the lack of auxiliary data and lack of knowledge on the ground deformation across the fault system.

Fattahi & Amelung [2014] showed that removing systematic effects from the displacement histories estimated from the InSAR time-series analysis, enabled them to precisely measure the spatially long-wavelength and temporally secular ground displacements across the southern San Andreas Fault system comparable with independent GPS observations. They also showed that the uncertainty of InSAR data are dominated by tropospheric delay. Here, we use the similar methodology to study the tectonic deformation across and along the Chaman fault system and evaluate the uncertainty of the measurements with two approaches using InSAR data and using the independent multispectral imagery.
Figure 5.1 Tectonic setting of the Chaman fault system in the western India plate boundary zone. Red boxes show approximate regions of the major ruptures in last century. Yellow boxes show the footprint of the Envisat ascending tracks across the fault system.

5.3 Tectonic settings

The Chaman fault system consists of three major predominantly sinistral strike-slip faults including Ornach Nal fault, Ghazaband fault and Chaman fault. To the south the
~250 km long Ornach Nal fault runs from the coast of the Makran subduction zone northward to the latitude of ~28N. Campaign GPS observations suggest a slip rate of 15 mm/yr and a locking depth of 4 km for the Ornach Nal fault [Szeliga et al., 2012]. The rest of the plate motion, at the latitudes south of ~28N, is most likely accommodated by the thrust faults of the Makran ranges including the Hoshab fault, Panjur fault and Siahan fault. This is confirmed by the campaign GPS observations [Szeliga et al., 2012] and by the nearly pure strike-slip faulting of the Hoshab fault during the 2013 Balochistan earthquake. Around latitude of ~28N deformation steps from the Ornach Nal fault westward to the Ghazaband fault.

The Ghazaband fault runs northward and around latitude of 28.5N, where several moderate earthquakes have been recorded in last four decades [Fattahi et al., 2015], the fault bends toward NNE direction fairly parallel to the Chaman fault and runs over 250 km up to the town of Pishin. The Ghazaband fault is expected to accommodate a significant part of the plate motion, although no geodetic observations have confirmed that yet. Around latitude of 30.7 N, north of Quetta, the topographic expression of the Ghazaband fault stops, which means that either slip transfers to the Chaman Fault to the north-west or distributes among different faults to the right on the Sulaiman lobe.

5.4 InSAR velocity field

We use ~6 years (~2003 to ~2010) ascending ENVISAT ASAR data from tracks 70, 299, 27, 256, 485, 213 and 442 (beam IS6) and ~7 years descending ASAR data from track 134 (beam IS2) to investigate the interseismic tectonic deformation across the Chaman fault system from the Makran subduction zone (around latitude 25N) to latitude
of ~33N. We generate zero Doppler single look complex (SLC) images using the Modular SAR Processor software (MSP) from Gamma Remote Sensing, except for autofocus and azimuth compression, for which we use the pseudo inverse Fourier transform instead of regular range-Doppler focusing algorithm (Hyung-Sup Jung, personal communication, 2012). For each track, we obtain small spatial baseline interferograms with perpendicular baselines less than 200 m using the JPL/Caltech ROI_PAC software [Rosen et al., 2004] (See Figure S1 for the networks of interferograms). We use the DORIS orbits and the 3 arc-second digital elevation model from the Shuttle Radar Topography Mission (SRTM) interpolated to 1 arc-second spacing, to simulate and remove the phase due to the topography and earth curvature from each interferogram. We take 8 looks in range and 40 looks in azimuth direction. Using the DEM and orbit state vectors we find the offsets between the SLCs and a master SLC [Sansosti et al., 2006] to coregister the multi-looked and filtered interferograms to a master SAR image. We unwrap the phase of the coregistered interferograms using the statistical-cost network-flow algorithm for phase unwrapping (SNAPHU) [Chen and Zebker, 2001]. We evaluate the phase consistency for triplets of interferograms and correct for phase unwrapping errors using an automated algorithm in the time domain as explained in chapter 4.

We then invert the network of interferograms for the phase velocities between subsequent epochs, which is used to obtain the phase history at each epoch relative to the first acquisition [Berardino et al., 2002]. The networks of interferograms are connected (Figure S5.1 in the supporting information), which results in full rank designed matrices for the inversion of interferograms and ensures unbiased estimation of phase histories.
The connected networks are specifically important because our interferometric phases contain components from non-linear contributions in time such as from tropospheric delay and topographic residuals. Any discontinuity co-occurred with the non-linear phase components, biases the estimated phase histories.

The connected networks of interferograms indicate that possible bias in the inversion is only due to the phase decorrelation, phase inconsistency for example due to independent filtering of interferograms [Agram & Simons, 2015] and remaining phase unwrapping errors. We therefore mask out the incoherent pixels using a temporal coherence threshold [Pepe et al., 2006] of 0.7 to ensure unbiased estimation of the phase history at coherent pixels. In the time domain we then correct for the local oscillator drift of the ASAR instrument [Marinkovic and Larsen, 2013; Fattahi and Amelung, 2014] and for topographic residuals [Fattahi and Amelung, 2013]. We use the ERA-Interim global atmospheric reanalysis model of the European Center for Medium-Range Weather Forecasts [Dee et al., 2011] to correct the stratified tropospheric delay using the approach of Jolivet et al. [2014b].

After converting the corrected phase history to range-change history, we estimate for each pixel the LOS velocity, which is the slope of the linear fit to the range-change history. The range change histories for the coherent pixels of each track are relative measurements with respect to a coherent reference pixel. Accordingly the LOS velocity field shows relative velocity between any pair of coherent pixels. We concatenate the velocity fields of adjacent tracks using the median of the differences in the overlapping areas. The median is less sensitive to localized residual tropospheric delays (not
represented by the ERA-I model) than the mean. After adjusting the adjacent tracks, we use for the overlapping areas the average velocity of the two tracks.

**Figure 5.2** Relative InSAR velocity field obtained from seven ascending tracks of Envisat ASAR data. Dashed lines show the location of the transects across the plate boundary shown in Figure 5.9.
Figure 5.2 shows the InSAR LOS velocity field from the seven ascending tracks. Blue and red colors show ground movements away and towards the satellite, respectively. Overall the velocity field indicates a relative lengthening of the distance between the ground and the satellite of the eastern part of the study area with respect to the western part by 8-15 mm/yr, consistent with the left-lateral motion of the Indian tectonic plate relative to Eurasia towards the Himalaya. The velocity field also shows several areas of subsidence of agricultural areas due to groundwater pumping such as a region of subsidence close to Qalat at latitude of 32N and large subsidence region in the Quetta valley.

5.5 Uncertainty of the InSAR velocity field

For each coherent pixel we estimate the standard deviation of the estimated velocity, \( \sigma_v \), as:

\[
\sigma_v = \sqrt{\frac{\sum_{i=1}^{N}(d_i - \hat{d}_i)^2}{(N-2)\sum_{i=1}^{N}(t_i - \bar{t})^2}}
\]  

(5.1)

where \( d_i \) is the displacement at epoch \( i \), \( \hat{d}_i \) the predicted linear displacement, \( \bar{t} \) is the average of acquisition times and \( N \) the number of epochs (\( t_1 \) to \( t_N \)). \( \sigma_v \) evaluates the uncertainty due to the temporally random residual noise in InSAR time-series.
Since the InSAR observations of range-change history for each track are relative to a coherent reference point, the estimated uncertainties are also relative to that pixel. Figure 5.3(a) shows the estimated velocity uncertainties for all tracks relative to their reference pixels. Given the corrections described in section 3, the estimated velocity uncertainties are mainly caused by residual tropospheric delay and by orbital uncertainties (See Figures S5.2 and S5.3 for velocities and uncertainties from raw time-series and after each correction). The uncertainty due to the orbital errors is expected to be small because the precise orbits of Envisat lead to velocity uncertainties less than 0.5 mm/yr/100 km in both range and azimuth directions [Fattahi and Amelung, 2014], which are significantly smaller than the estimated uncertainties. In the following section we use multispectral satellite imagery to confirm that the velocity uncertainty is dominated by the tropospheric delay.

Figure 5.3 Standard deviation of the InSAR velocity fields relative to the reference pixels (white boxes) representing the uncertainty: a) estimated from InSAR time-series data, b) pseudo-uncertainty from MODIS.
5.6 Pseudo-uncertainties from MODIS

The tropospheric delay affecting the InSAR observations can be subdivided into systematic and stochastic components. The systematic component includes the stratified delay in space and seasonal variations in time and the stochastic component includes the turbulent delay in space and non-seasonal variations in time. The stratified and seasonal delays bias the InSAR velocity depending on topography and SAR acquisition times, respectively. These systematic effects can be modeled and mitigated [eg: Jolivet et al, 2014; Samsonov et al, 2014]. Several studies have evaluated the stochastic component of the delay in space [Hanssen 2001; Emardson et al, 2003; Liu, 2012]. In the following we evaluate the stochastic component of the tropospheric delay in time and develop an approach to obtain the pseudo-uncertainty for InSAR displacement and InSAR velocity fields using independent observations of tropospheric delay.

We use 2002-2011 precipitable water vapor (PWV) products obtained from daily MODIS observations acquired at ~10:00 am local time. More than 75% of all the pixels have at least 2500 cloud free acquisitions (See Figure S5.4 in the supplementary information for a map of the number of cloud free acquisitions). We convert the MODIS PWV products to zenith wet delay (ZWD, one way in centimeter) as \( ZWD = \Pi \times PWV \), where \( \Pi \) is the conversion factor [Bevis et al., 1994]. We assume \( \Pi = 6.2 \) [Bevis et al., 1994; Li et al., 2006].

We model the seasonal component of the ZWD by a summation of sine and cosine functions with annual and semi-annual periodicities as

\[
ZWD(t) = \sum_{k=1}^{2} S_k \sin(2\pi kt) + C_k \cos(2\pi kt)
\]  

(5.2)
with $S_1$, $C_1$ the coefficients of the annual and $S_2$, $C_2$ the coefficients of the semi-annual components, from which their amplitudes and phases can be calculated. We remove the seasonal components and convert the residual zenith wet delay to residual slant wet delay, $\text{RSWD}$, as 

$$\text{RSWD}(t) = \left[ \text{ZWD}(t) - \sum_{k=1}^{2} \left( S_k \sin(2\pi kt) + C_k \cos(2\pi kt) \right) \right] / \cos(\theta),$$

where $\theta$ is the average incidence angle (41 degrees for the Envisat IS6 beam).

Figure 5.4 a) Time-series and b) distribution of zenith wet delay from MODIS observations for a pixel at coordinates of , . c, d) same for the residual zenith wet delay after removing the seasonal components. Dividing the standard deviation in d by $\cos$ yields

A time series of the ZWD in Figure 5.4(a) for a selected location is characterized by annual and semi-annual amplitudes of 5 cm and 1.7 cm, respectively (calculated as $A_k = \sqrt{C_k^2 + S_k^2}$, $k = 1,2$). The distribution of the ZWD is skewed (Figure 5.4(b)) because of the seasonal effects. After removal of the seasonal bias, the time series appears to be temporally random (Figure 5.4(c)) with a Gaussian distribution and a standard deviation of 3.8 cm (Figure 5.4(d)). Across the whole area, the annual
amplitudes of ZWD range from ~1 cm in the northwest to ~10 cm in the southeast and the semi-annual amplitude from ~1 cm to ~5 cm, respectively, with spatial patterns correlating with the topography (Figure S5.5). A similar map of annual amplitude of ZWD was generated for United States based on GPS observations and compared with the MODIS and ERA-I (Figure S5.6). The bias due to the seasonal delay on the velocity field is a function of the acquisition times and does not necessarily reduce with more acquisitions (Figure S5.7).

Figure 5.5 a): Standard deviation of RSWD from MODIS (seasonal bias removed), (b): Covariance of residual slant wet delay (RSWD) between all pixels of each track and the reference pixels, c): pseudo-uncertainty for displacement time-series, which is equivalent to (a) but relative to the reference points.
The standard deviation of the time-series of RSWD for a given pixel at location \(x\), \(\sigma_{\text{RSWD}(x)}\), represents the scatter of the random component of the tropospheric delay (see Figure 5.5(a)). To quantify the effect of the random tropospheric delay on relative InSAR time-series measurements, we evaluate the variance of the difference in RSWD between pixel \(x\) and the reference pixel (variance of the relative RSWD), \(\sigma^2_{\text{RSWD}(x)-\text{RSWD(\text{ref})}}\), given as:

\[
\sigma^2_{\text{RSWD}(x)-\text{RSWD(\text{ref})}} = \sigma^2_{\text{RSWD}(x)} + \sigma^2_{\text{RSWD(\text{ref})}} - 2\sigma_{\text{RSWD}(x),\text{RSWD(\text{ref})}}
\]

(5.3)

where \(\sigma^2_{\text{RSWD(\text{ref})}}\) is the variance of the RSWD at the reference point, and \(\sigma_{\text{RSWD}(x),\text{RSWD(\text{ref})}}\) the covariance of the time-series of RSWD between the two pixels. Due to the spatial correlation of tropospheric delay, the covariance is non-zero and not negligible. Figure 5.5(b) shows the map of \(\sigma_{\text{RSWD}(x),\text{RSWD(\text{ref})}}\). We refer to \(\sigma_{\text{RSWD}(x)-\text{RSWD(\text{ref})}}\) as the pseudo-uncertainty for the displacement between a given pixel and the reference, \(\sigma^\text{MODIS}_d\), \(\sigma^\text{MODIS}_d := \sigma_{\text{RSWD}(x)-\text{RSWD(\text{ref})}}\). The pseudo-uncertainty (Figure 5.5(c)) represents the scatter of the InSAR time series due to the temporally stochastic component of the tropospheric delay.

Finally we obtain the pseudo-uncertainty for the InSAR velocity between a given pixel and the reference, \(\sigma^\text{MODIS}_v\), as

\[
\sigma^\text{MODIS}_v = \sigma^\text{MODIS}_d \sqrt{\sum_{i=1}^{N} (t_i - \bar{t})^2}
\]

(5.4)
This equation shows that $\sigma_{v}^{\text{MODIS}}$ depends on the scatter of the random tropospheric delay, and is inversely proportional to the number of acquisitions, and the total time span covered by the SAR acquisitions.

The pseudo-uncertainty for velocity for the 7 tracks is shown in Figure 5.3(b). For each track $\sigma_{v}^{\text{MODIS}}$ increases with distance to the reference point to 4-5 mm/yr for tracks 299 and 256. The highest relative uncertainties of ~8 mm/yr occur on southern parts of tracks 485 and 213 at the eastern margin of the Kirthar ranges.

The pseudo-uncertainty from MODIS (Figure 5.3(b)) is fairly similar to the uncertainty derived from the InSAR time-series (Figure 5.3(a)), with a correlation coefficient of 0.86 between the two (see Figure S5.8 in the supplementary information). This strongly suggests that the scatter of our InSAR time-series primarily represents the random tropospheric delay. The high correlation also suggests that our InSAR time-series is not significantly affected by any unidentified non-linear systematic noise in the data (possible linear systematic noise such as local oscillator drift if not corrected, can bias the InSAR velocity field without affecting the InSAR uncertainty). Since the InSAR uncertainty was derived from InSAR data corrected for stratified tropospheric delay, the high correlation indicates that this correction has significantly reduced the seasonal component of the tropospheric delay, so that the variance of the residual tropospheric delay is similar to the variance of the random tropospheric delay.

Small differences between the InSAR uncertainty and the pseudo-uncertainty can be due to the orbital errors, dry delay and seasonal wet delay not accounted for by ERA-I and by diurnal variations in tropospheric delay (the ascending SAR data were acquired at ~10 PM pm local times in the night whereas MODIS imagery was acquired at 10 am).
The InSAR uncertainty in Figure 5.3(a) is generally smaller than the pseudo-uncertainty in Figure 5.3(b) (except for the northern part of track 485). This indicates that the ERA-I model accounts for part of the random tropospheric delay.

Figure 5.6 Standard deviation of InSAR velocity between all possible pairs of a sample of pixels (~ 500 for each track) as a function of distance for different tracks. The last plot shows for all the tracks.
5.7 Covariance matrix of the InSAR velocity

The uncertainties estimated above are relative uncertainties (relative to the reference pixels). In order to express the uncertainty between any pair of coherent pixels, similar to the variance of relative RSWD in equation (5.3), the covariance of noise between those two pixels is required. Since this calculation is computationally expensive for the entire region (for the $4 \times 10^6$ pixels of track 256 the covariance of noise between $16 \times 10^6$ possible pairs of pixels needs to be calculated), we select a sample of coherent pixels and estimate the relative velocity uncertainty between all possible pairs of pixels within this sample. We use samples of 500 pixels for each track, corresponding to 124,750 pairs. Figure 5.6 shows the estimated uncertainties versus distance for each track. Given the estimated uncertainties for all the tracks (See the last plot of Figure 5.6) we calculate the average and standard deviation of the estimated uncertainties over 15 km distance intervals (Figure 5.7).

This figure is similar to the structure function, which are commonly used to form the variance-covariance matrix of InSAR data [eg: Lohmann and Simons, 2005; Sudhaus and Jónsson, 2009; Jolivet et al., 2012]. The structure functions are usually obtained from a specific region of the study area and are based on the assumption that the region is not affected by ground deformation. Our approach for estimating the velocity uncertainty is obtained from a sample of the pixels from the whole area and is independent of linear ground deformation.
Figure 5.7 Average and standard deviation of the relative LOS velocity uncertainty obtained from corrected InSAR time-series as a function of relative distance.

Assuming isotropic noise, which implies that the covariance of noise between a pair of pixels only depends on the distance between the two pixels and not on the location of the pixels or azimuth between the two [Lohman and Simons, 2005], we can use the estimated uncertainties versus distance (Figure 5.7) to form the full covariance matrix of the InSAR velocity field as

\[ C_{ij} = \begin{cases} 
\sigma_s^2 - \sigma^2(r) & , \ i \neq j \\
\sigma_s^2 & , \ i = j 
\end{cases}, \forall \ i,j \in [1,P] \]  

(5.4)

Where \( C_{ij} \) is the covariance of noise between pixels \( i \) and \( j \), \( \sigma(r) \) is the velocity uncertainty for distance \( r \) between the pixels, \( \sigma_s \) is a sill value and \( P \) is the number of pixels. \( \sigma(r) \) is obtained from Figure 5.5 and \( \sigma_s \) over a distance of \( \sim 360 \) km beyond which noise is uncorrelated.
Figure 5.8 Example for the fault parallel velocity due to a strain accumulation on a) two faults with slip rates of 12 and 8 mm/yr and locking depth of 10 km without fault creep at shallow depth (solid black line) and with shallow creep of 3 mm/yr from surface to 3 km depth of Fault 1 (red dashed line). The total fault parallel velocity across the faults is 20 mm/yr, b) one vertical strike-slip fault with a slip rate of 12 mm/yr and locking depth of 10 km (blue solid line); same but surface fault creep at a rate of 3 mm/yr to 3 km depth (red line); and for fault creep through the entire seismogenic depth (black dashed line).

5.8 Modeling strategy

We assume that the LOS velocity field across the Chaman fault system (North of N28°) is caused by both, the accumulation of deformation along, and by partial motion of, one or more vertical strike-slip faults. We model each fault as a combination of a buried infinite screw dislocations in an elastic half-space representing interseismic strain accumulation along a locked fault [Savage and Burford, 1973], and a dislocation extending from the surface to a given depth, representing surface creep [Segall 2010, eq.
2.30. The fault-parallel surface velocity at location \( x \) along a profile perpendicular to \( \text{N parallel} \) faults is given by

\[
v_\parallel(x) = \sum_{i=1}^{N} \frac{S_i}{\pi} \arctan\left(\frac{x - f_i}{D_i}\right) + \frac{C_i}{\pi} \arctan\left(\frac{E_i}{x - f_i}\right)
\]

with \( S_i \) the slip rate, \( D_i \) the locking depth, \( C_i \) the creep rate \( (0 \leq C \leq S) \) and \( E_i \) the creep extent of the \( i \)th fault at location \( f_i \) with respect to the origin (See an example on Figure 5.8(a)). A fault creeping at the same rate as the slip rate through the entire seismogenic depth is given by \( C=S \) and \( E=D \) and \( v(x) \) approaches a Heaviside function scaled by the slip rate (See an example on Figure 5.8(b)).

### 5.9 Results

We examine strain accumulation along four transects roughly perpendicular to the Chaman fault system (Figure 5.9). Two of them cover the central Chaman fault system (the Chaman Fault transect near the city of Qalat and the Chaman-Ghazaband fault transect south of Quetta and Chaman City) and the other two cover the southern Chaman fault system (the Northern Makran-Ornach Nal transect near Khuzdar and the Central Makran-Ornach Nal transect coinciding with the GPS stations of Szeliga et al. [2012]). For the two central Chaman fault transects, we project the LOS velocity, \( v \), to fault parallel velocities, \( v_\parallel \), as

\[
v_\parallel(x) = v / (\sin(az)\cos(h)\sin(\theta) - \cos(az)\sin(h)\sin(\theta))
\]

where \( az \) is the local fault strike (34.5 for the Chaman Fault transect and 22.75 degrees for the Chaman-Ghazaband fault transect), \( h \) the satellite heading angle (347°) and \( \theta \) is the radar incidence angle (41° for the ASAR IS6 beam).
Figure 5.9 Profiles of the LOS velocity field across the Chaman fault system perpendicular to the local fault strike. a) Chaman Fault transect b) Chaman-Ghazaband Fault transect c) Northern Makran-Ornach Nal transect, d) Central Makran-Ornach Nal transect. The transects a-d are oriented N55.5W, N67W, N69W and N65W. a-c are perpendicular to the local strikes of the Chaman, Ghazaband and Hoshab faults, respectively; transect d follows the GPS profile of Szeliga et al. [2012]. Red circles in (d): GPS observations projected into LOS direction with 2 sigma uncertainty from Szeliga et al [2012]. Red lines in (a,b) show the best-fitting models (see text). CF:
5.9.1 Central Chaman fault system

The Chaman Fault and Chaman-Ghazaband Fault transects show relative LOS velocity decrease of ~7 and ~12 mm/yr over ~140 and ~160 km distance, respectively (Figure 7a,b). The transects show LOS velocity discontinuities of 2.5 mm/yr and 1 mm/yr across the Chaman Fault, indicating fault creep from the surface to a given depth, in the following referred to as shallow fault creep. The Chaman-Ghazaband transect suggests the typical arctangent pattern for interseismic strain accumulation along a vertical strike-slip fault. The LOS increase of up to ~10 mm/yr at 30-40 km distance along the Chaman transect and of up to 1 mm/yr at 110-135 km distance along the Chaman-Ghazaband transect most likely represents land subsidence due to groundwater withdrawal for agricultural use. The ~1 mm/yr relative LOS velocity at 10-40 km and the ~3 mm/yr LOS velocity at ~140-160 km along the Chaman-Ghazaband transect could represent vertical deformation related to thrust faults at the margins of the fault system.

The maximum relative uncertainties at the end of a transect relative to the beginning are 3 mm/yr and 3.2 mm/yr, inferred from Figure 6 for 140 and 160 km distance, respectively. The respective equivalent fault parallel rates are ~14+/-6 and ~30+/-6.4 mm/yr along the two transects, respectively.

5.9.2 Range of possible models

For the Chaman-Ghazaband transect we assume that the deformation is caused by strain accumulation along two vertical strike-slip faults, one of which is creeping at the surface (equation 5.9). The model has 8 parameters: 2 slip rates, 2 locking depths, 1 creep
rate, 1 creep extend, 1 location parameter for the Ghazaband fault along the profile and 1 constant offset. The Chaman fault location is constrained by the velocity discontinuity. We mask out the subsidence signals and the vertical signals at the beginning and end of the transect and subsample the data by averaging the LOS velocities over 1 km intervals along the profile (Figure S8b in the supplementary information). The covariance matrix is constructed using equation 5.4 and the information of Figure 5.7 as explained in section 5.7.

Given the fault parallel surface velocities across the fault system we use a Bayesian approach to obtain the range of possible models of interseismic strain accumulation and release in equation 5.5. The posterior Probability Density Function (PDF) of the model parameters, $p(m|d)$, is defined as:

$$p(m|d) \propto p(m)\exp\left[\left(\frac{d-p}{\sqrt{C^{-1}}(d-p)}\right)^T\right]$$

(5.6)

with $d$ and $p$ vectors containing the observations and model predictions, $C^{-1}$ the inverse of the Covariance matrix of the data and $p(m)$ the prior PDF of the model parameters. We use a Gibbs sampling algorithm with 200 000 sweeps to sample the posterior PDF of the models at critical temperature of -1.

The modelling suggests slip rates of $8.1 \pm 3.2$ and $16.3 \pm 2.3$ mm/yr for the Chaman Fault and Ghazaband Fault, respectively, a locking depth of $10.6 \pm 2.3$ km for the Ghazaband Fault, and a creep rate of $2.8 \pm 0.4$ mm/yr for the Chaman Fault (table 1). The locking depth and creep extend of the Chaman Fault can’t be constrained because of trade-offs (see Figure S5.10 in the supplementary information).
Table 5.1 The best fitting model parameters for profiles AA’ and BB’ in figure 5.9. S: Slip rate, D: locking depth, C: Creep rate, E: Creep Extend and f: fault location from beginning of the profile. The parameters without uncertainty were fixed for the models and those with 1-sigma uncertainties were estimated using a Gibbs sampling MCMC approach (See supplementary material Figures S5.10 and S5.11 for the plots of marginal posterior density distribution of estimated parameters).

<table>
<thead>
<tr>
<th>Profile</th>
<th>Fault</th>
<th>S [mm/r]</th>
<th>D [km]</th>
<th>C [mm/yr]</th>
<th>E [km]</th>
<th>f [km]</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA’</td>
<td>Chaman</td>
<td>5.7 ± 1.3</td>
<td>15</td>
<td>5</td>
<td>8.5 ± 2.5</td>
<td>60</td>
</tr>
<tr>
<td></td>
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<td>0</td>
<td>0</td>
<td>105</td>
</tr>
<tr>
<td></td>
<td>Fault2</td>
<td>5.7 ± 1.8</td>
<td></td>
<td>0</td>
<td>0</td>
<td>125</td>
</tr>
<tr>
<td>AA’</td>
<td>Chaman</td>
<td>5.4 ± 1.2</td>
<td>10</td>
<td>5</td>
<td>6.9 ± 2.3</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>Fault1</td>
<td>3.1 ± 1.3</td>
<td></td>
<td></td>
<td></td>
<td>105</td>
</tr>
<tr>
<td></td>
<td>Fault2</td>
<td>4.2 ± 1.4</td>
<td></td>
<td></td>
<td></td>
<td>125</td>
</tr>
<tr>
<td>AA’</td>
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<td>6.6 ± 1.2</td>
<td>15</td>
<td>5</td>
<td>7.8 ± 2.4</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>Fault1</td>
<td>7.7 ± 1.3</td>
<td></td>
<td>0</td>
<td>0</td>
<td>125</td>
</tr>
<tr>
<td>AA’</td>
<td>Chaman</td>
<td>6.5 ± 1.1</td>
<td>10</td>
<td>5</td>
<td>6.0 ± 2.0</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>Fault1</td>
<td>6.2 ± 1.1</td>
<td></td>
<td>0</td>
<td>0</td>
<td>125</td>
</tr>
<tr>
<td>BB’</td>
<td>Chaman</td>
<td>8.1 ± 3.2</td>
<td>27.2 ± 8.7</td>
<td>2.8 ± 0.4</td>
<td>6.1 ± 2.2</td>
<td>65</td>
</tr>
<tr>
<td>Ghazaband</td>
<td>16.3 ± 2.3</td>
<td>10.6 ± 2.3</td>
<td>0</td>
<td>0</td>
<td></td>
<td>41.0 ± 0.5</td>
</tr>
</tbody>
</table>

The Chaman Fault transect does not show an arctangent velocity pattern but a roughly linear LOS velocity decrease over 80-140 km, suggesting distributed deformation in this area (there are no surface expressions of major faults). We model this transect using a
vertical fault representing the Chaman Fault and two arbitrarily added faults (at distances of 105 and 125 km), all with a locking depth of 15 km. We find slip rates of $5.7 \pm 1.3$ mm/yr for the Chaman Fault and $2.7 \pm 1.7$ and $5.7 \pm 1.8$ mm/yr for the two arbitrary faults (Table 5.1), corresponding to a total of $\sim 14$ mm/yr slip rate across the transect. The inferred creep rate for the Chaman Fault is 5 mm/yr. A similar fit with locking depth of 10 km, similar to the locking depth of Ghazaband fault, results in similar slip rates and only slightly changes the creep extend from 8.5 km to 6.9 km. Moreover a model with only one arbitrary fault to the east of Chaman fault, slightly increases the slip rate of the Chaman fault to 6.6 and 6.5 mm/yr assuming locking depths of 15 and 10 km respectively (Table 5.1).

5.9.3 Southern Chaman fault system

The Northern Makran-Ornach Nal transect shows 8 mm/yr relative LOS velocity up to 150 km distance (Figure 5.9(c)), consisting of $\sim 7$ mm/yr deformation across the Panjur Fault (at 25-100 km distance) and $\sim 1$ mm/yr discontinuity across the Hoshab Fault. The LOS velocity discontinuities at 60 and 68 km are not due to fault creep but denote the edges of the swath overlap area between the velocity fields from tracks 299 and 27. The 6 mm/yr offset due to inconsistency between tracks 27 and 256 at km 150 has been fixed manually for display (light grey region at km 150). This inconsistency prevents us to accurately quantify the rate of strain accumulation across the fault system at this latitude.

The transect shows 2-3 mm/yr nearly linear LOS velocity increase at distances over 170 km. Given a local fault strike of $21^\circ$, the equivalent shallow creep rate of the
Hoshab Fault inferred is $3.4 \pm 0.2 \text{ mm/yr}$. The fault creep occurs north of the rupture zone of the 2013 M$_{w}$7.7 Balochistan earthquake (ref).

The Central Makran-Ornach Nal transect further south shows 15 mm/yr relative LOS velocity. About 10 mm/yr relative LOS velocity occurs over the western half of the transect (0-150 km distance), which includes several Makran faults. The InSAR velocity is overall consistent with the campaign GPS velocities of Széliga et al. [2012], except for the station PANG which shows 4 mm/yr less velocity relative to station LAKC than InSAR.

The transects include the Ornach Nal fault but we can’t constrain its slip rate because the InSAR observations (acquired from near-polar orbits) are not sensitive to fault-parallel motion along this north-south trending fault. The uncertainties for the relative velocities along the transects are 3 mm/yr for the Northern Makran-Ornach-Nal transect (up to 150 km distance) and 4 mm/yr for the Central Makran-Ornach Nal transect (up to 350 km distance), respectively.

### 5.9.4 Shallow fault creep along the Chaman Fault

Figure 8 shows the InSAR velocity field for the Chaman Fault proper from four ascending tracks (tracks 256, 485, 213 and 442) and one descending track (track 134). The ascending data are nearly identical to Figure 2, except for track 213 for which we have used only the frame with most acquisitions (See Figure S5.12 in the supplementary information for the network). The ascending LOS velocity field shows a discontinuity across the fault due to shallow creep. The descending velocity field does not show this signal because the radar looks nearly perpendicular to the fault and is insensitive to fault parallel displacements.
Figure 5.10 LOS velocity field for the Central Chaman fault system from a) ascending orbits (same as Figure 2 except for track 213, see text) and b) descending orbits (track 134).

We consider seven fault segments with different strikes and 1*1 km² square sections on both sides of the fault at distances of 0.3 km. We estimate the average and standard deviation of the LOS velocity (see table S1 in the supporting information for the coordinates). The difference in the LOS velocity averages gives the LOS creep rate [Chaussard et al., 2015]; the sum of the variances gives the variance of the LOS creep.
rate. We assume horizontal deformation and project the LOS rates to fault-parallel creep rates to obtain the along-strike variation in surface creep rate (Figure 8).

Figure 5.11 Surface creep rate along the Chaman Fault for 10 km intervals along the fault. Distance is from the city of Nushki towards NNW.

The surface creep increases northward from Nushki to \( \approx 8 \text{ mm/yr} \) at 42-45 km distance, decreases to \( \approx 4 \text{ mm/yr} \) near a fault step over (at 65-95 km distance at latitude of \( \approx 30.25 \text{ N} \)) after which it increases. There is no information about the creep rate between km 125 and 230 because of a lack of interferometric coherence. Further north, surface creep occurs at rates of up to 7 mm/yr to a distance of 310 km (north of the epicenter of the 2005 M_w 5.0 Chaman Fault earthquake), where the creep rate starts to decline to reach 0 mm/yr near a distance of \( \approx 340 \text{ km} \).
5.10 Discussion

5.10.1 Distributed Deformation across the Chaman fault system

InSAR observations of ground deformation over the western India plate boundary reveals distributed deformation across different faults of the Chaman fault system. In the southern Chaman fault system, at latitudes south of 28°N, Makran faults including the Hoshab, Panjur and Siahan faults accommodate most part of the plate motion between the India and Eurasia. These thrust faults are also expected to accommodate the deformation due to the subduction of the Arabia beneath the Eurasia, and therefore part of the observed displacement is due to the interseismic subsidence above the Makran megathrust.

In the central Chaman Fault system, at latitudes north of 28N to 30.5 N, the relative plate motion is mainly accommodated by the Chaman Fault and Ghazaband Fault with estimated slip rates of 9 and 15 mm/yr respectively. The plate motion models suggest a relative displacement between India and Eurasia of 30 mm yr at the latitudes of central Chaman Fault system. Therefore, Chaman and Ghazaband faults together accommodate up to ~80% of the total relative motion. The rest of the slip is most likely accommodated by the thrust faults on the eastern side of Ghazaband fault in the Kirthar range and partly by the faults to the west of Chaman fault.

In the northern Chaman fault system, at latitudes north of 30.5N, the deformation is distributed among Chaman fault and most likely on different faults across the Sulaiman lobe. Our InSAR observations do not show an increase in the strain localization and slip rate of the Chaman fault over the northern Chaman fault system compare to the southern Chaman fault system. This indicates that at latitudes north of 30.5 N where the
topographic expression of the Ghazaband fault vanishes, it’s slip does not transfer to the Chaman fault but rather distributes among different faults in the Kirthar range and Sulaiman lobe.

5.10.2 Geodetic versus geologic slip rates for the Chaman Fault

Ul-Hadi et al, [2013] have estimated an average slip rate of $33.3 \pm 3$ mm/yr over last ~34.8 kyr for Chaman Fault. The estimated geologic slip rate is significantly larger than our geodetic estimation of less than 10 mm/yr for Chaman Fault. The difference between the two slip rates is beyond the uncertainty of our geodetic measurements and potentially could represent a temporal variation in the slip rate of the Chaman Fault. However, generally speaking the estimated geological slip rates of strike slip faults could be potentially over-estimated due to the erosion of alluvial fan deposits that leads to anomalously young surface-exposure ages (e.g. Behr et al., 2010) or due to the nature of one-sided piercing lines from terrace risers (Cowgill, 2007).

5.10.3 Strain accumulation along the Chaman fault system

To evaluate the strain accumulation along the Chaman fault, we divide the fault in two segments; the seismically active segment from 29.5N (around the town of Nushki) to 32.2 N and the seismically quiet segment at north of 32.2N. The active segment has experienced several M<7 earthquakes in last 100 years, while the quiet segment lacks seismicity with no report on historical earthquakes except for 1505 earthquake close to Kabul.
5.10.4 Strain accumulation along the seismically active segment of Chaman fault

Our best fitting models suggest slip rates of 1 cm/yr for Chaman fault indicating that the active segment accommodates only ~30% of the total plate motion of ~3 cm/yr. The InSAR velocity field shows shallow fault creep with along strike variation of creep rate from 0 to 8 mm/yr over ~340 km seismically active segment of the Chaman fault.

Several 6<M<7 earthquakes have ruptured the active segment of the Chaman fault in last century. On December 20th, 1892 a M 6.5 earthquake ruptured the Chaman fault and totally destroyed the old town of Chaman without loss of life. The new town of Chaman was later relocated to 10 km west of the fault. The earthquake ruptured to the surface and is associated with possible left lateral slip of 65-80 cm. On October 3rd, 1975 a M 6.8 earthquake ruptured the active segment around Spin Tezha. The earthquake was followed 12 hours later by a M6.5 aftershock. Further south close to Nushki, a M 5.9 earthquake ruptured the fault on March, 16th 1978 with damages to several villages. The last earthquake on the Chaman fault is the M5 Oct 2005 earthquake.

The estimated slip rate of less than 1 cm/yr and the Observed surface creep with maximum rate of 8 mm/yr on the active segment of the Chaman fault indicates that this segment of the fault is not fully coupled (locked). This means that the strain (~30% of the plate motion) is not fully accumulated on the fault but is partially released with aseismic slip on the fault. Aseismic release of the strain generally reduces the magnitude of earthquakes, consistent with the recorded moderate earthquakes on the active segment in the last century.
5.10.5 Strain accumulation along the seismically quiet segment of the Chaman fault

The InSAR velocity field does not show strain localization on the first 100 km, at latitudes of 32.25 N to 33 N, of the 300 km quiet segment of the Chaman fault. Also InSAR observations do not show surface creep on the first 40 km of the quiet segment. Two different scenarios can be considered for strain accumulation on the quiet segment of the Chaman fault. One possibility is that InSAR velocity of the last track is largely biased and therefore the fault accumulates strain along 300 km segment with the same slip rate of the active segment. The other possibility is that at latitudes north of M5 2005 earthquake, the slip distributes on other faults including Gardez fault and therefore the slip rate on the Chaman fault significantly reduces northward.

The latest known historical earthquake on the northern parts of the Chaman fault is the 1505 Kabul earthquake. Given a higher bound of slip rate of 1 cm/yr for the quiet segment of the Chaman fault, similar to the active segment, the 300 km segment of the fault during last 500 years has accumulated enough strain for a Mw 7.56 or Mw 7.45 earthquakes assuming locking depths of 15 and 10 km respectively. The magnitudes reduce to 7.36 and 7.25 for 150 km rupture of the fault. Possible distribution of 1 cm/yr slip on other faults to the east of Chaman fault, reduces the rate of strain accumulation on the Chaman fault and accordingly the magnitude of future possible earthquakes on the quiet segment of the fault. However, a small slip rate of 2 mm/yr corresponds to a single Mw 7 event given the rupture of the entire 300 km quiet segment locked at 15 km depth.
5.10.6 Strain accumulation along the Ghazaband fault

The InSAR velocity shows strain localization on the Ghazaband fault without any indication of surface creep. Modeling of the InSAR observations suggests that the Ghazaband fault accumulates strain with a slip rate of ~15 mm/yr beneath a locking-depth of ~10 km. Given these parameters a 250 km segment of the fault can rupture in Mw7 earthquakes every 100 years or in Mw 7.5 earthquakes every 500 years.

The latest large earthquake around the Ghazaband fault was the 1935 ~M7.7 Quetta earthquake, which devastated the city of Quetta and caused up to 35,000 fatalities [Ambraseys and Bilham, 2003]. Several studies attributed the rupture to the Ghazaband fault [Lawrence et al., 1981; Yeats et al., 1997; Széliga et al., 2012]. However, other studies suggested rupture of a subparallel fault tens of kilometers eastward [Armbruster et al., 1980; Ambraseys and Bilham, 2003], which is consistent with the reverse component of the focal mechanism [Singh and Gupta, 1980] and with the lack of reported damage to the Quetta-Spezand-Nushki railroad crossing the Ghazaband fault. The high rate of strain accumulation and lack of evidence for the rupture of the fault in the 1935 Quetta earthquake, present a growing earthquake hazard to the Balochistan and the populated areas such as the city of Quetta.

5.10.7 Uncertainty of the InSAR velocity field (signal, systematic and random noise)

Given linear ground displacement as our signal of interest we subdivided the components of InSAR range-change time-series to signal, systematic and random noise. Systematic noises include topographic residuals, hardware issues such as the local
oscillator drift, seasonal tropospheric delay, and non-linear components of ground deformation such as those caused by coseismic and postseismic transient deformation.

The systematic noises can bias the estimated velocities depending on the acquisition strategy and are not necessarily reduced with more SAR acquisitions over a longer time period. In contrast to the systematic noise, the contribution from the random noise to the InSAR velocity, such as the random tropospheric delay (tropospheric delay after removing the seasonal delay) and the orbital errors, can be reduced with larger number of acquisitions and longer time periods. The random noise of InSAR range change time-series should be evaluated with the variance-covariance matrices.

Similar to systematic biases, most components of the random noise of the InSAR range-change time-series are also spatially correlated. This means that non-diagonal components of variance-covariance matrix of InSAR velocity fields are non-zero. We showed the importance of the covariance of the random tropospheric delay to obtain the relative uncertainty for the InSAR velocities.

5.10.8 Variance Covariance matrix of the InSAR observations

We developed an approach to obtain the relative uncertainty due to the random tropospheric wet delay using the precipitable water vapour products from MODIS observations. Fattahi and Amelung, [2014] conducted a similar study to obtain the velocity uncertainty due to the orbital errors. Agram and Simons, [2015] developed an approach to obtain the covariance matrix due to the phase decorrelation. Since the processes involved in different components of the random noise in InSAR range-change time-series are independent, one can obtain a full variance-covariance matrix of the InSAR range-change time-series by summation of the variance-covariance matrices of
different components, $C_n$, as $C_n = C_{\text{decor}} + C_{\text{orbit}} + C_{\text{trop}}$, where $C_{\text{decor}}$, $C_{\text{orbit}}$ and $C_{\text{trop}}$ are the variance-covariance matrices from phase decorrelation, orbital errors and random tropospheric delay. $C_n$ lacks components from unwrapping errors and ionospheric delay. Both of these two components, similar to the tropospheric delay, are expected to have both systematic and stochastic components. However comparison between the maps of relative InSAR uncertainty and the pseudo-uncertainty due to the random tropospheric delay obtained from MODIS observations, indicates that uncertainty of our InSAR displacement time-series is dominated by the random tropospheric delay.

5.10.9 Possible residual biases in the InSAR velocity field.

Given the corrections for the topographic residuals, local oscillator drift and tropospheric delay, the only source of the systematic bias remaining in our InSAR velocity field is the possible non-linear ground deformation. The velocity field is not affected by the sequence of three $M>6$ earthquakes close to Pishin in Oct 2008, because we exclude all SAR acquisitions after the earthquakes for the tracks 213 and 485. However, the velocity field shows small localized ground displacement due to the 2005 $M_w5$ Chaman [Furuya & Satyabal, 2008] and 2007, $M_w5.5$ Ghazaband [Fattahi et al, 2015] earthquakes. These small earthquakes with ground displacement over $\sim10$-$20$ km do not bias the long-wavelength ground deformation signal. The velocity field could be potentially biased by the viscous relaxation due to large earthquakes in the past 100 years in the region including the $M7.1$ 1909 Kachhi, $M7.3$ 1931 Mach and $M7.7$ 1935 Quetta earthquakes.

The inconsistencies between adjacent tracks (e.g. between tracks 299, 27 and 256 from latitude $\sim26$N to $\sim27$N) are the result of a bias in one of the LOS velocity fields due
to the residual tropospheric delay caused by inaccuracy of the ERA-I model. Another source for inconsistencies is the difference in radar incidence angle $38.6^\circ$ to $43^\circ$ from near range to far range (1 cm of east displacement projects to 6.1 mm LOS displacement in near range and 6.6 mm in the far range). This bias is within the uncertainty of our InSAR velocity field described in section 5.5.

5.11 Conclusions

InSAR time-series analysis demonstrated the distribution of deformation across the Chaman fault system with along strike variation of the deformation. In the central Chaman fault system, the InSAR velocity shows clear strain localization on the Chaman and Ghazaband faults and modeling suggests slip rates of 8 and 16 mm/yr for the two faults respectively, corresponding to the 80% of the total ~3 cm/yr plate motion between India and Eurasia at these latitudes. The slip of the Ghazaband fault further north does not transfer to the Chaman fault but most likely distributes on faults in the Kirthar range and Sulaiman lobe. The velocity field shows 340 km creeping segment of the Chaman fault with along strike variation of creep rate from 0-8 mm/yr. The observed creep indicates that Chaman fault at latitudes of 29.5 to 32.5 is only partially locked accumulating strain which are most likely released with M<7 earthquakes consistent with recorded earthquakes on this segment in last ~100 years.

Our InSAR velocity does not show strain localization, neither fault creep on the first 100 km segment of the 300 km northern segment of the Chaman fault which has been seismically quiet in last 100 years and lacks historical large earthquake except for the 1505 earthquake near Kabul.
Chapter 6. Coseismic and postseismic deformation due to the 2007 M5.5 Ghazaband fault earthquake, Balochistan, Pakistan

6.1 Summary
Time-series analysis of InSAR data reveal coseismic and postseismic surface displacements associated with the 2007 M5.5 earthquake along the southern Ghazaband fault, a major but little studied fault in Pakistan. Modeling indicates that the coseismic surface deformation was caused by ~9 cm of strike-slip displacement along a shallow subvertical fault. The earthquake was followed by at least one year of afterslip, releasing ~70% of the moment of the main event, equivalent to a ~M5.4 event. This high aseismic relative to the seismic moment release is consistent with previous observations for moderate earthquakes (M<6) and suggests that smaller earthquakes are associated with a higher aseismic relative to seismic moment release than larger earthquakes.

6.2 Background
Since 1892 four M>7 earthquakes ruptured the western India plate boundary [Ambraseys and Bilham, 2003], the latest one being the 2013 M7.7 earthquake [Avouac et al., 2014; Barnhart et al., 2014; Jolivet et al., 2014c]. The Ghazaband fault, one of the main structures of the western India plate boundary zone, is a possible source of the M7.7 1935 earthquake which devastated the city of Quetta causing up to 35,000 fatalities [Lawrence et al., 1981; Ambraseys & Bilham, 2003; Szeliga et al., 2012].

Geodetic observations of strain accumulation and release can help to better understand the nature of the Ghazaband fault and the earthquake hazard of Balochistan. Here we report InSAR observations of the coseismic and postseismic deformation
associated with the M5.5 earthquake of 19\textsuperscript{th} October 2007. We estimate the ratio of aseismic moment release by afterslip relative to the coseismic moment release and compare the ratio with 22 other earthquakes with reported geodetic observations of afterslip.

Figure 6.1 a) Location map of the Ghazaband fault. Solid white rectangles: footprint of ascending and descending frames. Black circles show the 1982-1976, M>5 earthquakes from Ambraseys and Bilham, [2003]. Focal mechanisms are from the Global CMT catalogue. Locations of ~M7.7 1935 Quetta earthquake and M6.1, 1978 Nushki earthquake on Chaman fault are from Engdahl and Villaseñor, [2002] and location of M5, 2005 earthquake on Chaman fault north of Chaman city is adjusted from Furuya and Satyabala [2008]. Faults at latitude <29°N are from Lawrence et al. [1981] and for >29°N derived from Google Earth imagery. Orange-shaded rectangle: possible location of 1935 rupture. HF and ONF stand for Hoshab Fault and Ornach Nal Fault respectively. b) Zoom
into epicentral area of the M5.5 Oct 2007 earthquake. Red lines are strike-slip
dislocations, yellow line is reverse-slip dislocation, and blue line is the Gwandan River.c) Optical imagery from Google Earth of the mountain range overlying the reverse-slip
dislocation (see text).

6.3 Tectonic setting

The Chaman fault system, forming the transform to transpressive boundary between
the India and Eurasia tectonic plates, consists of a group of sinistral strike-slip faults
including the Ornach Nal, Ghazaband and Chaman faults (Figure 1a). To the south, the
Chaman fault connects with the faults of the Makran ranges (Siahan, Panjgur, and
Hoshab faults), which accommodate both the shortening due to the convergence of
Arabia and Eurasia and the shear between India and Eurasia. The Ghazaband fault runs
for about 300 km roughly parallel to the Chaman fault.

Szeliga et al. [2012] report velocities from a sparse campaign GPS network at
latitudes 30-32N. Their stations KACH and CHMC suggest \( \sim 12 \) mm/yr relative velocity
across the combined Chaman and Ghazaband fault systems. However, plate motion
models suggest 26 mm/yr fault-parallel and 5-8 mm/yr fault-normal motion between the
Indian and the Eurasian plates [Szeliga et al., 2012].

Four moderate earthquakes (5<M<6) with predominantly strike-slip focal
near latitude 28.5°N (Figure 6.1(a)). In 1993 a M5.5 earthquake ruptured the northern
Ghazaband fault near the city of Pishin [Szeliga, 2010]. These earthquakes are evidence
that the fault accommodates a portion of the deformation.

The Global CMT and USGS solutions place the 19 Oct 2007 event 15 km northeast
of the observed ground deformation likely due to location error given the poor resolution
of global seismic networks [Weston et al., 2011]. The earthquake was followed by a M3.9 aftershock one day later. The topography of the epicentral region and Google-earth imagery suggests for this latitude a system of two subparallel faults (Figure 1b).

6.4 Data analysis and results

We use 2004-2010 ascending (beam IS6) and descending (beam IS2) ENVISAT ASAR data from tracks 27 and 134 to investigate the coseismic and postseismic ground deformation due to the 2007 earthquake using InSAR time-series analysis. We use the JPL/Caltech ROI_PAC software [Rosen et al., 2004] for processing interferograms with perpendicular baselines less than 200 m. We invert the network of interferograms (Supplementary material, Figure S6.1) for the phase history [Berardino et al., 2002] and then correct for the local oscillator drift of the ASAR instrument [Marinkovic and Larsen, 2013; Fattahi and Amelung, 2014], for topographic residuals [Fattahi and Amelung, 2013] and for the stratified tropospheric delay [Jolivet et al., 2014b] using the ERA-Interim global atmospheric reanalysis model [Dee et al., 2011]. From the corrected InSAR time-series we reconstruct the coseismic and postseismic displacements without any assumption about the spatial or temporal deformation model.

Coseismic ground displacement maps in radar line-of-sight direction (LOS) between 2004 and 12 and 20 days after the earthquake obtained from the ascending and descending displacement histories show the ground deformation during and in the first days after the earthquake (Figure 6.2(a) and (b)). We present displacements since 2004 because these early acquisitions are characterized by smaller lateral tropospheric delay variations. The ascending LOS displacement map shows the typical quadruple displacement pattern of strike-slip earthquakes. The displacement towards the satellite in
the southwestern lobe (positive LOS, red colors, Figure 6.2(a)) and away from the satellite in the northeastern lobe (negative LOS, blue colors) is consistent with left-lateral displacement along a NNE trending fault; similar for the descending data but the displacements lobes have opposite signs.

Postseismic displacement maps for the subsequent 3-years period show similar deformation patterns, suggesting afterslip with sign and magnitude comparable to the coseismic slip. This is further supported by the similarity of the east-west and vertical displacement patterns inferred from decomposing the ascending and descending data (Figure S6.2). However, the postseismic vertical displacement field shows a notable difference in the elongated uplift in the northeast (Figure S6.2(d)), which is referred to in the following as the northeastern uplift. This uplift is likely due to afterslip rather than poroelastic deformation, which would result in ground subsidence [Peltzer et al., 1998; Fielding et al., 2009].

6.5 Modeling

We use uniform dislocations in homogeneous elastic half-space to infer the fault slip responsible for the surface displacements. Our data set consists of 2500 pixels of the LOS displacements sampled from a uniform grid for each viewing geometry. We use a Gibbs sampling inversion technique to determine the parameters and uncertainties of the dislocations [Brooks and Frazer, 2005].

For the coseismic observations, we consider a single vertical strike-slip dislocation. We fix the location and strike of the fault and invert for the best-fitting fault length, width, depth and displacement of the dislocation. The coseismic observations are best
explained by ~9 cm of strike-slip displacement along a ~7.7 km long fault at depth (top of the fault) of ~1.7 km (Table1).

Figure 6.2 (a, b) Coseismic and (c, d) postseismic LOS displacement maps obtained from the time-series for the Ghazaband fault earthquake from ascending and descending Envisat satellite tracks (IS6 and IS2 beams, respectively). (e,f,g,h) Predicted LOS displacements from the best fitting models. Black and pink lines are modeled strike-slip and dip-slip dislocations, respectively. (i,j,k,l) Differences between the best-fitting models and the observations (residuals). Arrows show satellite flight directions and black bars show the look directions. The location is given by Figure 1b.

For the postseismic observations the model consists of a strike-slip dislocation with the location of the coseismic model (fault 1) and a dip-slip dislocation (fault 2). We invert
for the displacement, depth, length and width of fault 1 and for the dip and dip-slip displacement of fault 2. The residual atmospheric noise in the InSAR data and the small displacement signal does not allow to constrain the length, width and depth of the second dislocation. The moment release from fault 2 is only 7% of the total postseismic moment release (Table 6.1). The postseismic observations are best explained by ~7 cm left-lateral displacement for fault 1 and by ~3 cm reverse slip along fault 2 at depths of ~1.3 and ~2 km respectively (Table 6.1, Figures 6.2(g) and h). The Marginal Posterior Density Distributions (PDD) for the estimated fault parameters (Supplementary materials, Figures S3 and S4) show that the fault source parameters have roughly Gaussian distributions.

For the coseismic data, the ascending residual map shows a narrow stripe of LOS displacement very close to the fault (Figure 6.2(i)), which occurs probably because local fault slip variations are not represented by the uniform dislocation. The residual between two faults can be due to the non-linear processes such as damages at fault zone. The descending data, which are not sensitive to along-strike fault displacements (the radar looks nearly perpendicular to the fault) do not have such residuals. The residuals in the NE lobe of the descending data (Figure 6.2(j)) may be due to early afterslip as it spans 8 more days than the ascending data. The residuals in Figure 6.2(k) (the blue color at the bottom) are most likely due to the residual tropospheric delay in the data.

6.6 Discussion

The InSAR data provide insights into a tectonically complex section of the Ghazaband fault system. Coseismic ground deformation was caused by shallow slip along a subvertical, left-lateral, strike-slip fault and postseismic deformation by slip along
the same fault as well as by dip-slip on a neighboring thrust fault. The geodetic moment magnitude of ~M5.5 is consistent with the seismic moment magnitude of M5.5 from the Global CMT catalog.

Table 6.1 Best-fitting model parameters for the coseismic and postseismic displacements. The uncertainties were derived from the Posterior Density Distribution of the fault parameters obtained from the Gibbs sampling algorithm (See Supplementary materials, Figures S6.3 and S6.4 for the plots of the Posterior Density Distribution). To calculate the moment magnitude we assume rigidity equal to 3e10 Pa.

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6.6.1 Shallow slip

The earthquake was associated with coseismic and postseismic fault slip at shallow depth (~1 to 2 km depth of the model dislocation upper edges). Shallow slip was also observed in the 1993 M5.5 earthquake along the northern strand of the Ghazaband fault (focal depth of 2.25 km, [Szeliga, 2010]) and in the 2005 M5.0 Chaman fault earthquake (maximum slip depth of 2 km, [Furuya and Satyabala, 2008]). The 2013 M7.7 Balochistan earthquake also largely ruptured near the surface [Avouac et al., 2014], as did several historic earthquakes along the Chaman fault [Szeliga et al., 2012]. This could suggest that shallow fault slip is widespread in this plate boundary zone, in contrast to other tectonic settings where some earthquakes may be associated with slip deficits at shallow depth [Fialko et al., 2005; Dolan and Haravitch, 2014]. Shallow slip deficit can be partly explained by shallow creep distributed across the fault during the interseismic period [Kaneko and Fialko, 2011] or by distributed yielding [Lindsey et al., 2014b]. Xu et al., [2015] suggest that the shallow slip deficit is apparent and an artifact due to interferometric decorrelation near earthquake ruptures. This interpretation is consistent with the presence of shallow slip and high interferometric coherence near the Ghazaband fault trace.
Figure 6.3 Relative ascending LOS displacement histories across the fault for (a) the southwestern deformation lobe between points A and A’, (b) the northeastern uplifting area between points B and B’ (see Figure 2a for locations), together with the linear fits for the pre-seismic and the logarithmic and exponential fits to the postseismic data.

6.6.2 Afterslip

The InSAR data and modeling show that the earthquake was followed by 7 cm of slip along the strike-slip fault and by 3 cm of slip along the shallow thrust fault (Table 1). The LOS displacement histories for the southwestern deformation lobe and for the northeastern uplift area, relative to points on the opposite sides of the fault, show the displacements before, during and after the earthquake (Figure 6.3(a),(b)). The AA’ displacement history is noisier than the BB’ displacement history because the distance between the points is larger and therefore the difference is more affected by the residual atmospheric delay. The slight slope of less than 0.5 mm/yr of the AA’ displacement history is most likely due to the noise of the InSAR data and not due to the interseismic shallow fault creep, as confirmed by the absence of a trend in the less noisy BB’ displacement. The afterslip lasted over 2 years in the southeastern area (Figure 6.3(a))
and at least one year in the northeastern area (Figure 6.3(b)). The postseismic displacement histories are well fitted by logarithmic or exponential functions. This 2007 Ghazaband fault earthquake has a ratio of 70±10% between postseismic and coseismic moment release, which is a lower bound estimate given that the first 12 days and 20 days of afterslip are included in the coseismic displacement from ascending and descending data, respectively.

Space-geodetic observations of afterslip over the past two decades provide an opportunity to better understand the nature of afterslip. Afterslip generally surrounds the high-slip patches of the coseismic rupture [Marone et al., 1991; Perfettini and Avouac, 2007] and has been observed following strike-slip [Freed, 2007; Furuya and Satyabala, 2008; Hearn et al., 2009], thrust [Podgorski et al., 2007] and normal crustal earthquakes [Hamiel et al., 2012], following subduction earthquakes [Miyazaki, 2004; Hsu et al., 2006; Sun et al., 2014] and following a detachment fault earthquake [Owen and Bürgmann, 2006].
Figure 6.4 Ratio of aseismic moment release by afterslip relative to the coseismic moment release for earthquakes with reported afterslip (See Supplementary Material Table S1). The red bar for the Ghazaband earthquake shows the 2σ uncertainty. The red arrow for the 2011, Tohoku-Oki earthquake indicates that afterslip still continues and the ratio will increase with future observations.

Figure 6.4 shows the ratio of the aseismic moment release by afterslip relative to the coseismic moment release for 22 earthquakes in different tectonic settings (references are in Table S6.1). The ~70% ratio for the M5.5 on the Ghazaband fault falls below the ratios of 500%, 280% and 300% for the M5 2005 Chaman fault, the M6 2004 Parkfield, California, and the M4.7 2008 Mogul, Nevada earthquakes, respectively [Freed, 2007; Furuya and Satyabala, 2008; Bell et al., 2012], and above the ratios of 15%, 2%, 29-32% for the M7.3 1992 Landers, M7.1 1999 Hector Mine, and M7.5 1999 Izmit, Turkey earthquakes, respectively [Shen et al., 1994; Jacobs et al., 2002a; Wang et al., 2009b]. The distribution shows that moderate earthquakes (M<6) are followed by proportionally more aseismic moment release than large earthquakes (M>6). This difference could reflect that large earthquakes produce a more complete stress drop than smaller ones and
thus proportionally less elastic strain energy remains stored in the crust. We note that these ratios have to be interpreted with caution because many studies lack continuous geodetic observations immediately after the earthquake and because afterslip may continue for many years at a decaying rate such as for the Izmit earthquake [Çakir et al., 2012].

![Marginal Posterior Density Distribution for the depth of coseismic (red) and postseismic (blue) dislocations.](image)

**Figure 6.5** Marginal Posterior Density Distribution for the depth of coseismic (red) and postseismic (blue) dislocations.

### 6.6.3 Depths of coseismic slip and afterslip

The PDDs of the depths of the upper edge of the dislocations estimated using the Gibbs sampling approach show that the postseismic dislocation is slightly shallower than the coseismic dislocation (means of 1.3 and 1.7 km, respectively, Figure 6.5). This, together with the smaller width of the postseismic dislocation (6 km for the postseismic versus 8 km for the coseismic dislocation, Table 6.1) suggests that the afterslip was generally shallower than the coseismic slip, consistent with the narrower spatial pattern
of the postseismic (Figure S6.2(b)) compared to the coseismic surface displacements (Figure S6.2(a)). The shallower depth of afterslip is consistent with inferred velocity-strengthening behavior along the shallow parts of faults related to a lower degree of sediment consolidation [Scholz, 1998; Wei et al., 2013]. However, the small difference between the depths of the coseismic and postseismic dislocations (Figure 6.5) suggests possible overlap between the coseismic and postseismic patches. The overlap of the coseismic and postseismic regions can represent conditionally stable frictional behavior of the shallower parts of the fault with coseismic slip nucleated on velocity-weakening asperities at depth and propagated to the conditionally stable regions, which can also slip aseismically [Scholz, 1998; Noda and Lapusta, 2013]. However, the small surface displacement signal does not allow to infer the spatial variation of slip at depth and thus we can not rule out an alternative scenario of lateral variations in frictional behavior with seismic movements of velocity-weakening patches and aseismic movements of velocity-strengthening patches [eg: Hsu et al., 2006].

**Postseismic push-up**

The reverse-slip dislocation explaining the postseismic uplift locates under a ~7 km long unnamed mountainous ridge subparallel to the Ghazaband system (Figure 6.1b). The ridge is the highest topographic expression in the area (with elevation of ~300 m above the surroundings). The spatial match with the detected uplift suggests that the ridge may have been created by repeated push-ups similar to the 2007 event. Inspection of optical remote sensing imagery (Google Earth imagery) shows that the ridge is the only location in the region where inclined sedimentary strata are exposed at the surface (Figure 6.1c). This suggests erosional unroofing of recent uplift and that the ridge is the geomorphic
expression of the contractional deformation associated with a restraining stepover of the Ghazaband fault system and/or transpressional tectonics.

6.7 Conclusions
Modeling the InSAR observations for the M5.5 Oct 2007 earthquake on the southern end of the Ghazaband fault suggests that the coseismic slip was produced by \(~9\) cm of left-lateral displacement on a shallow subvertical fault. The earthquake was followed by \(~7\) cm slip above and along the main rupture and by a few cm of triggered slip along a nearby thrust fault. The InSAR time-series shows that afterslip lasted for at least one year and released \(~70\%\) of the moment of the main event. This ratio between aseismic and seismic moment release is consistent with previous observations of high aseismic moment release for moderate earthquakes (M<6) and higher than the ratio for larger earthquakes.
Chapter 7. Conclusions

In this dissertation I developed an InSAR time-series analysis approach to measure the long-wavelength tectonic deformation and to evaluate the uncertainty of the measurements. I applied the developed InSAR method to Envisat ASAR acquisitions across and along the western boundary of the India and Eurasia tectonic plates to study the tectonic deformation and rate of strain accumulation across and along the Chaman fault system a transcurrent fault system accommodating shear between the two plates. In the following I discuss in more details the main conclusions of this dissertation on the technical aspects of InSAR time-series analysis as well as the tectonic deformation along the Chaman fault system.

7.1 Signal, systematic noise and random noise in InSAR time-series analysis

Measuring long-wavelength tectonic displacement with InSAR requires proper definition of signal, systematic and random noise in different steps of the InSAR time-series analysis. Signal and systematic noise should be modeled with functional model of the Gauss-Markov model and the random noise should be evaluated with the stochastic model using variance-covariance matrices.

Table 7.1 summarizes different components of signal, systematic and random noise for different steps of the InSAR time-series analysis and table 7.2 shows examples of the different components in table 7.1. For the inversion of a connected network of interferograms to estimate the InSAR range-change time-series, which is equivalent to GPS coordinate time-series, contributions from ground displacement, topographic
residuals, tropospheric delay, orbital errors and systematic effects such as local oscillator
drift should be considered as signal. All these components are spatially correlated and
ideally a time-series of any of them can be estimated with InSAR. Phase-unwrapping
errors can bias the InSAR range-change time-series and thus should be considered as
systematic noise for the inversion. Phase decorrelation and inconsistency due to imperfect
processing should be considered as random noise and should be evaluated with a
stochastic model.

Given ground displacement as the signal of interest, topographic residuals, local
oscillator drift and seasonal components of the tropospheric delay should be considered
as systematic noise of the InSAR range-change time-series. They can be modeled and
mitigated from the range-change time-series to obtain the displacement time-series. If
only tectonic deformation is of interest, which was the case in this dissertation, all non-
tectonic components of ground displacement should be considered as systematic noise.
Random orbital errors and stochastic components of tropospheric delay (tropospheric
delay after removing seasonal components) should be considered as random noise to the
displacement time-series.

Given secular rate of tectonic ground displacement as our signal of interest, the noise
components are same as what I characterized for the displacement time-series, except that
all non-linear components of ground displacement should be also considered as
systematic noise.

Systematic noises bias the signal as a function of acquisition strategy including both
acquisition time and imaging geometry. For example the bias caused by the seasonal
delay to the displacement time-series, is a function of acquisition times and the bias due
to the topographic residual is a function of the imaging geometry. In this dissertation I developed two methods to model the biases due to the topographic residuals and unwrapping errors. I also removed the bias due to the local oscillator drift of ASAR instrument and corrected the tropospheric delay using ERA-I model. The tropospheric delay correction using the ERA-I model significantly removes the bias due to the seasonal delay and also improves the uncertainty due to the random tropospheric delay. I developed two methods to evaluate the contribution of the stochastic components of noise caused by orbital errors and random tropospheric delay to the InSAR uncertainty.

**Table 7.1** Attributing different components of interferometric phase observations to signal, random noise and Systematic noise for estimating InSAR range-change time-series, displacement time-series and for estimating InSAR velocity field.

<table>
<thead>
<tr>
<th>Components of the interferometric phase</th>
<th>Estimating range change time-series</th>
<th>Displacement time-series</th>
<th>Estimation velocity field From range-change time-series</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear ground displacement</td>
<td>Signal</td>
<td>Signal</td>
<td>Signal</td>
</tr>
<tr>
<td>Non-linear ground displacement</td>
<td>Signal</td>
<td>Signal</td>
<td>Systematic noise</td>
</tr>
<tr>
<td>Topographic residual</td>
<td>Signal</td>
<td>Systematic noise</td>
<td>Systematic noise</td>
</tr>
<tr>
<td>Local oscillator drift</td>
<td>Signal</td>
<td>Systematic noise</td>
<td>Systematic noise</td>
</tr>
<tr>
<td>Tropospheric delay (Seasonal delay)</td>
<td>Signal</td>
<td>Systematic noise</td>
<td>Systematic noise</td>
</tr>
<tr>
<td>Tropospheric delay (random delay)</td>
<td>Signal</td>
<td>Random noise</td>
<td>Random noise</td>
</tr>
<tr>
<td>Orbital error</td>
<td>Signal</td>
<td>Random noise</td>
<td>Random noise</td>
</tr>
<tr>
<td>Unwrapping error</td>
<td>Systematic noise and Random noise</td>
<td>Systematic noise and Random noise</td>
<td>Systematic noise and Random noise</td>
</tr>
<tr>
<td>Phase decorrelation</td>
<td>Random noise</td>
<td>Random noise</td>
<td>Random noise</td>
</tr>
<tr>
<td>Phase inconsistency</td>
<td>Random noise</td>
<td>Random noise</td>
<td>Random noise</td>
</tr>
</tbody>
</table>
Table 7.2 Examples for different processes that cause different phase components listed in Table 7.1

<table>
<thead>
<tr>
<th>Components of the interferometric phase</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear ground displacement</td>
<td>secular tectonic deformation due to strain accumulation</td>
</tr>
<tr>
<td>Non-linear ground displacement</td>
<td>co-seismic displacement, post-seismic displacement, non-linear surface displacement due to volcanic activity, seasonal displacement due to seasonal loading and unloading, seasonal water withdrawal and recharge, solid earth tides, atmospheric loading, …</td>
</tr>
<tr>
<td>Topographic residual</td>
<td>DEM error, geoid undulation, imperfect consideration of imaging geometry</td>
</tr>
<tr>
<td>Systematic hardware or software issues</td>
<td>Local oscillator drift</td>
</tr>
<tr>
<td>Unwrapping error</td>
<td>Systematic noise and Random noise</td>
</tr>
<tr>
<td>Phase decorrelation</td>
<td>Temporal decorrelation, geometric decorrelation, thermal decorrelation</td>
</tr>
<tr>
<td>Phase inconsistency</td>
<td>inconsistent filtering, multi-looking, …</td>
</tr>
</tbody>
</table>

7.2 Topographic residuals

Inaccuracy of the DEMs used for simulation of topographic phase and also imperfect simulation and simplification of imaging geometry results in topographic residuals which bias the displacement time-series depending on the perpendicular baseline history of the set of SAR acquisitions. This effect at a given epoch is proportional to the perpendicular
baseline between the SAR acquisition at that epoch and the reference acquisition. Therefore, the DEM error can significantly affect the displacement time-series even if small baseline interferograms are used. The proposed method for topographic residual correction, efficiently corrects the displacement time-series.

For simple ground displacement time-series, which are well approximated by the assumed deformation model, the new method for topographic residual correction is equivalent to the original SBAS method. For complex time-variable displacement histories (typical for volcanic unrest or including earthquakes), the proposed method for topographic residual correction yields more accurate estimations of the DEM error and of the displacement time-series. This occurs because the new method is applied in the time domain and therefore independent of the interferogram network. In contrast, in the original SBAS method, the estimated DEM error may depend on the interferogram network.

7.3 Uncertainty due to orbital errors

Considering the orbital error as a stochastic component in the InSAR range-change time-series, I evaluated the contribution of the error to the uncertainty of the InSAR velocity field. I expressed the uncertainty of the InSAR velocity fields in terms of the uncertainties of the velocity gradients in range and azimuth directions (range and azimuth uncertainties). I found that these uncertainties depend on the orbital uncertainties, the number and time span of SAR acquisitions. For modern SAR satellites with precise orbits such as TerraSAR-X and Sentinel-1, the range uncertainty is 0.2 mmyr−1 100 km−1. For older satellites with less accurate orbits such as ERS and Envisat, for the same time span, the uncertainty is about 1.5 and 0.5 mmyr−1 100 km−1, respectively.
However, in practice the InSAR measurements can be biased by sensor hardware and by processing approximations. For Envisat an important effect is the drift of the local oscillator. The accuracies quoted above can only be achieved if systematic errors are identified and corrected for. The InSAR uncertainty is dominated by the tropospheric delay and not by the orbital errors.

7.4 Uncertainty due to the tropospheric delay

The tropospheric delay affecting the InSAR range-change time-series can be subdivided into systematic and stochastic components. The systematic component includes the stratified delay in space and seasonal variations in time and the stochastic component includes the turbulent delay in space and non-seasonal variations in time. The stratified and seasonal delays bias the InSAR velocity depending on topography and SAR acquisition times, respectively. The turbulent tropospheric delay has been previously evaluated with structure functions. The non-seasonal components due to the temporally random tropospheric delay should be evaluated with a stochastic model.

I developed a new approach to evaluate the non-seasonal components of the tropospheric delay in a stochastic model, using precipitable water vapor products from MODIS observations. The non-seasonal component of the tropospheric delay is spatially correlated and therefore the covariance between two given pixel is non-zero and not negligible.

The relative velocity uncertainty due to the random tropospheric delay depends on the scatter of the random tropospheric delay, and is inversely proportional to the number of acquisitions, and the total time span covered by the SAR acquisitions. The relative uncertainty between two pixels generally increases with distance and is spatially variable.
High correlation of 0.86 between velocity uncertainty estimated from InSAR displacement time-series and the uncertainty obtained from MODIS observations, confirms that the InSAR uncertainty is dominated by the tropospheric delay.

7.5 Tectonic deformation across the Chaman Fault system

InSAR time-series analysis of ENVISAT ASAR observations demonstrated the distribution of deformation across the Chaman fault system. In the central Chaman fault system, the InSAR velocity shows clear strain localization on the Chaman and Ghazaband faults and modeling suggests slip rates of 8 and 16 mm/yr for the two faults respectively, corresponding to the 80% of the total ~3 cm/yr plate motion between India and Eurasia at these latitudes.

Further north, where the topographic expression of the Ghazaband Fault vanishes at latitudes north of ~30.5N, the slip of the Ghazaband fault does not transfer to the Chaman fault but most likely distributes on faults in the Kirthar range and Sulaiman lobe. The velocity field shows 340 km creeping segment of the Chaman fault with along strike variation of creep rate from 0-8 mm/yr. The observed creep indicates that Chaman fault at latitudes of 29.5 to 32.5 is only partially locked accumulating strain which are most likely released with M<7 earthquakes consistent with recorded earthquakes on this segment in last century.

The high rate of strain accumulation on the Ghazaband fault and lack of evidence for the rupture of the fault in the 1935 Quetta earthquake, present a significant earthquake hazard to the Balochistan and the populated areas such as the city of Quetta.
7.6 Future directions

Given different stochastic components of noise, a full variance covariance matrix of InSAR products is crucial to express the uncertainty of the InSAR time-series products. The work in this dissertation on evaluating the uncertainty due to the orbital errors and random tropospheric delay can be combined with recent expression of InSAR uncertainty due to the phase decorrelation from Agram & Simons, [2015] to form a full variance covariance matrix of noise. Such variance-covariance matrix still lacks information about the uncertainty due to the residual unwrapping errors and possible stochastic components of ionospheric delay. The latter has also significant systematic components, which should be modeled and mitigated.

The random noise in this dissertation was assumed to be temporally uncorrelated white noise. This was chosen because of the limited number of InSAR time-series epochs. However, with auxiliary data for example from MODIS observations of the precipitable water vapor and numerical weather models, the temporal correlation of the random tropospheric delay can be investigated.

The algorithm for unwrapping errors correction can be improved by constraints from space. The current implementation of the algorithm is computationally expensive. This can be potentially improved by first identifying regions with same unwrapping errors and then estimating the phase jumps for each region instead of each pixel.

More acquisitions from future SAR missions are crucial to better understand the strain accumulation along the most northern segment of Chaman fault, close to Kabul, where 300 km segment of the fault has been seismically quiet in last 500 years. The InSAR velocity field over the western India plate boundary zone should be modeled with taking in to account the deformation due to both subduction of Arabia plate beneath the
Eurasia in the Makran subduction zone and shear due to the northward motion of India plate.
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Appendix 2.1
Baseline of an interferogram, generated from two SAR images acquired at \( t_A \) and \( t_B \), is the spatial separation of two SAR images. Given \( \vec{O}(t_i) \) as the vector of the SAR satellite’s position at time \( t_i \) in an arbitrary coordinate system representing the orbital parameters, the spatial baseline between two SAR acquisitions acquired at times \( t_A \) and \( t_B \), \( \vec{B}(t_A, t_B) \), can be expressed as the difference of orbital parameters at these epochs as follows:

\[
\vec{B}(t_A, t_B) = \vec{O}(t_B) - \vec{O}(t_A).
\]  

(A.1.1)

Let’s rewrite this equation by simply adding and subtracting the orbital parameters of the reference SAR acquisition (at epoch \( t_0 \)):

\[
\vec{B}(t_A, t_B) = \tilde{\vec{O}}(t_B) - \tilde{\vec{O}}(t_A) + \vec{O}(t_0) - \vec{O}(t_0) \\
= [\tilde{\vec{O}}(t_B) - \vec{O}(t_0)] - [\tilde{\vec{O}}(t_A) - \vec{O}(t_0)] \\
= \vec{B}(t_B, t_0) - \vec{B}(t_A, t_0)
\]

(A.1.2)

wherein \( \vec{B}(t_B, t_0) \) and \( \vec{B}(t_A, t_0) \) are the spatial baselines between SAR acquisitions acquired at times \( t_B \) and \( t_0 \) and between \( t_A \) and \( t_0 \) respectively. Considering \( t_0 \) as the reference temporal acquisition, let us simplify this equation as follows:

\[
\vec{B}(t_A, t_B) = \vec{B}(t_B) - \vec{B}(t_A)
\]  

(A.1.3)

where \( \vec{B}(t_B) \) and \( \vec{B}(t_A) \) by definition are equal to \( \vec{B}(t_B, t_0) \) and \( \vec{B}(t_A, t_0) \). Considering parallel and perpendicular components of the spatial baseline as \( \vec{B} = (B_\perp, B_\parallel) \), the following equation for the perpendicular baseline is valid:

\[
B_\perp(t_A, t_B) = B_\perp(t_B) - B_\perp(t_A)
\]  

(A.1.4)

In this paper, \( B_\perp(t_i) \) is named baseline history, which at each epoch, \( t_i \), is the perpendicular baseline between SAR images acquired at that epoch and reference epoch.
Appendix 2.2

Considering (7) and the definition of phase velocity as $v_i = \frac{\phi(t_i) - \phi(t_{i-1})}{t_i - t_{i-1}}$ with $i = 1, ..., N$, we can obtain the following formula for phase velocity based on the DEM error and parameters of the assumed deformation model:

$$v_i = \frac{\phi(t_i) - \phi(t_{i-1})}{t_i - t_{i-1}} = \frac{\varphi(t_i) - \varphi(t_{i-1})}{t_i - t_{i-1}} + \frac{4\pi B_i(t_i) - B_i(t_{i-1})}{(A2.1) r \sin(\theta)} z^e + \psi(t_i) - \psi(t_{i-1})$$

$$+ \frac{1}{2} \left[ \begin{array}{cccc} t_1 - t_0 & 0 & 0 & \vdots & 0 \\ 0 & t_2 - t_1 & 0 & \vdots & 0 \\ 0 & 0 & \ddots & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & t_N - t_{N-1} \end{array} \right] B_i(t_i) - B_i(t_{i-1}) + \frac{1}{2} \left( \begin{array}{c} \frac{(t_1 - t_0)}{6} \frac{(t_1 - t_0)^2}{2} \\ \frac{(t_2 - t_1 - 2t_0)}{2} \frac{(t_2 - t_0)^3 - (t_1 - t_0)^3}{6(t_2 - t_1)} \\ \vdots \vdots \vdots \vdots \vdots \\ \frac{(t_N - t_{N-1} - 2t_0)}{2} \frac{(t_N - t_0)^3 - (t_{N-1} - t_0)^3}{6(t_N - t_{N-1})} \end{array} \right]$$

Appendix 2.3

To show that DEM error estimation in the time domain using phase velocity history is equivalent to the interferogram domain with a sequential network, we obtain the design matrix ([BM,C] of equation (25) in [Berardino et al., 2002]) for DEM error correction of a sequential network of interferograms based on original SBAS. In this appendix we use the same symbols as [Berardino et al., 2002].:
The system of equations to estimate the DEM error (equation (25) in [5]) takes the form below:

\[ [BM, C] \begin{bmatrix} \bar{v} \\ \bar{a} \\ \Delta \bar{a} \\ z^e \end{bmatrix} = \begin{bmatrix} \phi_1 - \phi_0 \\ \phi_2 - \phi_1 \\ \vdots \\ \phi_N - \phi_{N-1} \end{bmatrix}. \] (A.3.3)

The observation vector in the right side of this equation contains interferometric phases identical to the differences in phase history between two consequent epochs. Since we use in the time-domain the phase velocity history, let’s convert the observation vector to phase velocity by dividing both sides of (A.3.3) by the corresponding time differences:

\[ \begin{bmatrix} 1 & \frac{(t_1 - t_0)}{2} & \frac{1}{6} (t_1 - t_0)^2 & \frac{4\pi B_{11}(t_0, t_1)}{\lambda r \sin(\theta)} \\ \frac{(t_2 - t_0)}{2} & \frac{(t_2 - t_0)(t_2 + t_1 - 2t_0)}{2} & \frac{1}{6} (t_2 - t_0)^3 - (t_1 - t_0)^3 & \frac{4\pi B_{12}(t_1, t_2)}{\lambda r \sin(\theta)} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{(t_N - t_{N-1})}{2} & \frac{(t_N - t_{N-1})(t_N + t_{N-1} - 2t_0)}{2} & \frac{1}{6} (t_N - t_{N-1} - t_0)^3 - (t_{N-1} - t_0)^3 & \frac{4\pi B_{1N}(t_{N-1}, t_N)}{\lambda r \sin(\theta)} \end{bmatrix} \begin{bmatrix} \bar{v} \\ \bar{a} \\ \Delta \bar{a} \\ z^e \end{bmatrix} = \begin{bmatrix} \bar{v}_1 \\ \bar{v}_2 \\ \vdots \\ \bar{v}_N \end{bmatrix}. \] (A.3.4)

The design matrix of this equation is identical to (10). This shows that time-domain DEM error estimation from phase velocity history is equivalent to interferogram-domain estimation using a sequential network.
In a same way it can be shown that time-domain DEM error estimation using phase history is equivalent to interferogram-domain estimation using a tree-like network.

**Appendix 3.1: Effect of second order terms in evaluating the orbital errors**

In this appendix we assess the magnitude of the second order terms using numerical examples of the baseline error components. Equation (6) including second order terms gives

\[
d\phi(r,s) = \frac{\partial \phi}{\partial \vartheta}_{r_0,s_0} d\vartheta + \frac{\partial \phi}{\partial s}_{r_0,s_0} ds + \frac{\partial^2 \phi}{\partial \vartheta^2}_{r_0,s_0} d\vartheta^2 + \frac{\partial^2 \phi}{\partial s^2}_{r_0,s_0} ds^2 + \frac{\partial^2 \phi}{\partial \vartheta \partial s}_{r_0,s_0} d\vartheta ds + \ldots \quad (A.1)
\]

The second order terms control the curvature of the phase gradient in range and azimuth directions; these terms are functions of \(B_\parallel\), \(B'_\parallel\) and \(B'_\perp\) as,

\[
\frac{\partial^2 \phi}{\partial \vartheta^2} d\vartheta^2 = -\frac{1}{2} B_\parallel d\vartheta^2 \quad (A.2),
\]

\[
\frac{\partial^2 \phi}{\partial s^2} ds^2 = \frac{1}{2} B'_\parallel ds^2 \quad (A.3),
\]

\[
\frac{\partial^2 \phi}{\partial \vartheta \partial s} d\vartheta ds = \frac{1}{2} B'_\perp d\vartheta ds \quad (A.4).
\]

For numerical examples we use \(B_\parallel = B'_\parallel = 10 cm\), \(B'_\parallel = 1 mm / 100 km^2\) and \(B'_\perp = 2.8 cm / 100 km\) and consider the ERS and Envisat satellites (\(d\vartheta = 8^\circ\), \(ds = 100 km\), \(\lambda = 5.6 cm\)). The value for \(B'_\parallel\) is the average of the actual baseline curvatures which vary from 0.1 \(mm / 100 km^2\) to 10 \(mm / 100 km^2\) for the data analyzed in this paper.

The first order terms in range and azimuth directions (equation 9, 10) generates a phase gradient equivalent to \(~1.4 cm\) range change over 100 km in range direction (50% of one
fringe) and 2.8 cm over 100 km in azimuth direction (one fringe). The second order terms add 1 mm in range direction (eq. A2, 7% of the first order term, 3.5% of one fringe) and 0.5 mm in azimuth direction (eq. A3, 2% of the first order term, 2% of one fringe). This assumption that the magnitude of the baseline curvature error is in the order of the average baseline curvature itself, can be a very conservative scenario. In a more realistic scenario with the error of the baseline curvature one order of magnitude smaller than the baseline curvature itself, the curvature contribution becomes negligible. The third term (equation A4) introduces 0.2 cm over 100 km across one frame of Envisat or ERS data (7% of one fringe).

In summary, the second order terms of the Taylor expansion are responsible for curvature in the interferograms. In range direction the phase variation caused by the curvature is less than 10% of the linear phase ramps. In azimuth direction, the phase curvature is likely to be very small or negligible because baseline curvature itself is very small.

Appendix 3.2: Velocity uncertainty as functions of horizontal and vertical baseline uncertainties

Given horizontal and vertical baseline representation in equations (3) and (4), equations (18) and (19) can be rewritten as

$$
\sigma_{\parallel} = \sqrt{\sigma_{B_h}^2 \cos^2(\theta_0) + \sigma_{B_v}^2 \sin^2(\theta_0)} / |\Delta t| \quad (B.1),
$$

$$
\sigma_{\alpha} = \sqrt{\sigma_{B_v}^2 \sin^2(\theta_0) + \sigma_{B_v}^2 \cos^2(\theta_0)} / |\Delta t| \quad (B.2).
$$
Appendix 3.3: Oscillator Drift correction for Envisat data

We correct for the local oscillator drift (OD) of Envisat’s ASAR instrument, using the empirical model of Marinkovic and Larsen [2013], which adjusts the range change history for each pixel. For a given pixel the correction $C$ is

$$C = (3.87 \times 10^{-7})x\delta\rho\delta t \quad (C.1).$$

with $x$ the dimensionless pixel count in range direction, $\delta\rho$ the range pixel size, $\delta t$ the time difference between a given epoch and the reference epoch. This correction for each pixel should be referenced to the same reference pixel as InSAR data and then removed from each epoch.
Figure S3.1 phase components for a time-series epoch: a) from raw time-series, b) after correction for local oscillator drift, c) after topographic residual correction, d) after wet delay correction using MERIS data, e) after dry delay correction using ERA-I model.
Figure S5.1 Network of interferograms for 7 ascending tracks used to produce the velocity field in Figure 2
Figure S5.2  effect of different corrections on the InSAR velocity fields for different tracks: a) obtained from raw time-series, b) after local oscillator drift correction, c) after topographic residual correction, and d) after tropospheric delay correction. The velocities have not been adjusted for constant offset among adjacent tracks.
Figure S5.3 effect of different corrections on the InSAR uncertainty for different velocities obtained from: a) raw time-series, b) after local oscillator drift correction, c) after topographic residual correction, and d) after tropospheric delay correction. The velocities have not been adjusted for constant offset among adjacent tracks.
Figure S5.4 Map showing for each pixel the number of cloud free images out of a total of 3163 MODIS acquisitions for 2002 to 2011 (only ~10:00 am acquisitions). 75% of the study area has at least 2500 cloud free acquisitions.
Figure S5.5 The amplitude of a) annual and b) semi-annual ZWD from MODIS observations. (c): Digital Elevation Model from Shuttle Radar Topography Mission (SRTM) for the study area.
Figure S5.6 Amplitude of the annual zenith wet delay for a) United States estimated from GPS, b) south-west US from GPS c) south-west from MODIS, d) south west US from ERA-I. e) the DEM of the region.

Figure S5.7 Effect of temporal sampling on the bias due to seasonal delay. Grey dots are the time-series of ZWD, solid line is the best fitted annual and semi annual delay, the
green circles and red circles show the acquisition times of tracks 256 and 27 respectively. The bias to the velocity due to the annual wet delay using red acquisitions, green acquisitions and both red and green acquisitions is 5.2, -0.2 and 2.3 mm/yr respectively. The figure shows that the bias due to seasonal delay does not necessarily reduce with more acquisitions.

**Figure S5.8** Comparison between velocity uncertainties estimated from InSAR (Figure 3a) and the pseudo-uncertainty from MODIS (Figure 3b). The pseudo-uncertainty represents the wet random tropospheric delay. The correlation of 0.86 indicates that the InSAR uncertainty is dominated by wet random tropospheric delay.
Figure S5.9 same as Figures 7(a) and (b), but the black dots show the sampled data used for modeling.
Figure S5.10 Marginal posterior density distribution of model parameters and their contours of trade offs obtained from Gibbs sampling with 200000 sweeps at the critical temperature of 1 for profile AA’ in Figure 5.9. E1: creep extend of Chaman fault, S1: slip rate of Chaman fault, S2: slip rate of the first arbitrary fault at km 45, S3: slip rate of the first arbitrary fault at km 65, f2: location of Ghazaband fault, offset: a constant offset.
Figure S5.11 Marginal posterior density distribution and contour of model parameters obtained from Gibbs sampling with 200000 sweeps at the critical Temperature of 1 for profile BB’ in Figure 5.9. C1: creep rate of Chaman fault, E1: creep extend of Chaman fault, S1: slip rate of Chaman fault, S2: slip rate of Ghazaband fault, D1: locking depth of Chaman fault, D2: locking depth of Ghazaband fault, f2: location of Ghazaband fault, offset: a constant offset.
Figure S5.12 Networks of interferograms for tracks 213 and 134 in Figure 5.10. The network of interferograms for other tracks are the same as Figure S5.1.

Figure S6.1 A network of interferograms for (left) ascending and (right) descending tracks. Each red circle shows the SAR acquisition in a perpendicular-baseline versus time plot and each line represents the interferogram formed from the two SAR acquisitions. Both networks are fully connected and thus the design matrix to invert for the phase history is full rank.
Figure S6.2 (a,b) Horizontal and (c,d) vertical components of the coseismic and postseismic ground displacements, respectively, obtained by combining the ascending and descending data following [Wright et al., 2004].
Figure S6.3 Marginal Posterior Density Distribution for Co-seismic fault model parameters derived from Gibbs-sampling.
Figure S6.4 Marginal Posterior Density Distribution for post-seismic faults model parameters derived from Gibbs-sampling assuming one vertical and one dipping dislocation.
Table S6.1 Reported or inferred ratio of the afterslip moment release relative to the coseismic moment release.

<table>
<thead>
<tr>
<th>#</th>
<th>Earthquake</th>
<th>Date</th>
<th>Faulting</th>
<th>Coseismic Mw</th>
<th>Ratio of afterslip to coseismic moment magnitude released</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Manyi (Tibet)</td>
<td>Nov 8, 1997</td>
<td>Strike-slip</td>
<td>7.6</td>
<td>20%</td>
<td>[Ryder et al., 2007]</td>
</tr>
<tr>
<td>2</td>
<td>Hector Mine</td>
<td>Oct, 16 1999</td>
<td>Strike-slip</td>
<td>7.1</td>
<td>2%</td>
<td>[Jacobs et al., 2002b]</td>
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<tr>
<td>3</td>
<td>Izmit</td>
<td>Aug, 17, 1999</td>
<td>Strike-slip</td>
<td>7.5</td>
<td>29%-32%</td>
<td>[Burgmann et al., 2002; Wang et al., 2009b]</td>
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<tr>
<td>4</td>
<td>Parkfield</td>
<td>2004</td>
<td>Strike-slip</td>
<td>6.0</td>
<td>280%</td>
<td>[Freed, 2007]</td>
</tr>
<tr>
<td>5</td>
<td>Chaman</td>
<td>Oct 2005</td>
<td>Strike-slip</td>
<td>5</td>
<td>560%</td>
<td>[Furuya and Satyabala, 2008]</td>
</tr>
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<td>6</td>
<td>Mogul swarm</td>
<td>2008</td>
<td>Strike slip</td>
<td>4.7</td>
<td>280%</td>
<td>[Bell et al., 2012]</td>
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<tr>
<td>7</td>
<td>Landers</td>
<td>1992</td>
<td>Strike-slip</td>
<td>7.3</td>
<td>15%</td>
<td>[Shen et al., 1994]</td>
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<td>8</td>
<td>Boumerdes</td>
<td>May 21, 2003</td>
<td>Thrust</td>
<td>6.9</td>
<td>31%</td>
<td>[Mahsas et al., 2008]</td>
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<td>9</td>
<td>Chi Chi</td>
<td>1999</td>
<td>Thrust</td>
<td>7.6</td>
<td>7%</td>
<td>[Hsu et al., 2002]</td>
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<td>Wenchuan</td>
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<td>[Hao et al., 2013]</td>
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<td>Sefidabeh</td>
<td>1994</td>
<td>Thrust</td>
<td>6.5</td>
<td>9%</td>
<td>[Copley and Reynolds, 2014]</td>
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<td></td>
<td>Region</td>
<td>Year</td>
<td>Mechanism</td>
<td>Magnitude</td>
<td>Slip (%)</td>
<td>Reference</td>
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<td>12</td>
<td>Northridge</td>
<td>1994</td>
<td>Thrust</td>
<td>M6.7</td>
<td>22%</td>
<td>[Donnellan and Lyzenga, 1998]</td>
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<td>13</td>
<td>L’Aquila</td>
<td>2009</td>
<td>Normal</td>
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<td>[D’Agostino et al., 2012]</td>
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<td>Mozambique</td>
<td>Feb, 22, 2006</td>
<td>Normal</td>
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<td>[Copley et al., 2012]</td>
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<td>15</td>
<td>Nima-Gaize</td>
<td>Jan, 9, 2008</td>
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<td>6.4</td>
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<td>[Ryder et al., 2010]</td>
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<td>Oblique-thrust</td>
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<td>7%</td>
<td>[Segall et al., 2000]</td>
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<td>17</td>
<td>San Simeon</td>
<td>2003</td>
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<td>14%</td>
<td>[Johanson and Bürgmann, 2010]</td>
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<tr>
<td>18</td>
<td>Nias Simeulue</td>
<td>2005</td>
<td>Thrust-subduction</td>
<td>8.7</td>
<td>18%</td>
<td>[Hsu et al., 2006]</td>
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<td>2011</td>
<td>Thrust-subduction</td>
<td>9</td>
<td>10%</td>
<td>[Ozawa et al., 2011; Evans and Meade, 2012]</td>
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<td>2010</td>
<td>Thrust-subduction</td>
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<td>[Tanimoto and Ji, 2010]</td>
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<td>Sumatra-Andaman</td>
<td>2004</td>
<td>Thrust-subduction</td>
<td>9.15</td>
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