The Florida Current: Mean Jet Structure, Meandering, and Velocity Fluctuations Observed with HF Radar

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THE FLORIDA CURRENT: MEAN JET STRUCTURE, MEANDERING, AND VELOCITY FLUCTUATIONS OBSERVED WITH HF RADAR

By

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THE FLORIDA CURRENT: MEAN JET STRUCTURE, MEANDERING, AND VELOCITY FLUCTUATIONS OBSERVED WITH HF RADAR

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High-resolution ocean surface current velocity measurements from high frequency (HF) radar are used to map the Florida Current jet structure, and quantify its fluctuations, in more detail than has previously been possible. Whereas earlier HF radar studies in the Straits of Florida focused on individual events, this research takes the next step by using a 2 year timeseries to quantify and characterize the mean horizontal jet profile, its meandering and structural variability, and the space-time structure of the velocity fluctuations. This dissertation is organized into three research chapters:

In the first chapter, the 2 year mean horizontal profile of the Florida Current is constructed at high spatial (1 km) and temporal (20 min) resolution. To improve the mean calculation, each 2-D map in time is converted from geographical to stream coordinates, where grid points are shifted relative to the jet core (cross-stream/downstream). The core time-mean velocity is 162 cm s\(^{-1}\), compared to 136 cm s\(^{-1}\) in the geographical frame. This difference is due to meandering, which smears energy across grid points in the geographical frame, producing a diffuse jet profile with weaker cross-stream gradients. At 25.4°N, the mean position of the jet is 44 km offshore, over the 650 m isobath. Lateral meandering has a standard deviation of 8 km and a range of 60 km, accounting for 45% of mean eddy kinetic energy. Jet width exhibits an annual cycle in the Straits of Florida, with
a boreal summer maximum and late winter minimum. The summer peak is accompanied by a maximum in volume transport and local meridional wind stress. The winter minimum precedes a peak in core intensity, lateral shear and surface transport. Sub-seasonal fluctuations of these variables peak at 7-10 days and 3 weeks, but exhibit large inter-annual variability.

The second chapter presents two case studies that demonstrate the power of HF radar to: (1) reveal new information regarding flow field kinematics of previously studied features; and (2) measure transient phenomena that have been historically difficult to capture with ship and moored point measurements, or to resolve with satellite imagery. In the first case study, the kinematic properties of a cyclonic vortex are investigated. In contrast to conditions recorded in a period of no eddy activity, the vorticity field revealed a complex structure, with significant contributions from strain, and a large Rossby number indicative of submesoscale dynamics. Strong horizontal current divergence near the core of the eddy was associated with anomalously cold water brought to the surface by upwelling, observed in satellite SST imagery. The particle dispersion metric IROS peaked during the event, indicating cross-shelf exchange of water properties between offshore and coastal regions. In the second case study, a near-inertial signal on the jet’s anticyclonic flank was investigated for the first time. The strongly sheared Florida Current partially masked the structure of the signal, which manifested as a succession of clockwise-rotating eddies in the observed surface currents. The wave trough was not evident when embedded in a laterally sheared northward background flow. The dominant frequency was shifted by ~13% below $f$ in the average, which is consistent with a near-inertial wave propagating in
a background regime with negative vorticity. Near-inertial energy peaked in the negative vorticity trough along the jet’s eastern flank, indicative of wave trapping in the horizontal.

In the final chapter, the characteristic temporal and spatial scales of the fluctuations are calculated based on the flow field’s correlation properties. The Florida Current dominates the ocean circulation in this region, and strongly determines the character of the fluctuations; the strongly sheared northward flow meridionally extends decorrelation length scales and polarizes fluctuating motions in the along-stream direction. The dominant periods of variability are quantified, along with their time dependency that reveals a seasonal variation in periodicity. The slope of the mean kinetic energy wavenumber spectrum is $k^{-3}$, which is consistent with interior quasi-geostrophy theory. This result implies that nonlocal dynamics are dominant in driving local transport and dispersion. Eddy-mean flow interaction is investigated through the conservation of eddy kinetic energy equation, variance ellipses and the Reynolds stress terms. The map of the barotropic energy exchange term reveals a 2-D pattern, where south of 25.5°N there is an upgradient (downgradient) flux in the cyclonic (anticyclonic) shear zone, and vice versa north of 25.5°N. The magnitude of the divergence of energy flux is significant, however, suggesting there is not an equal exchange of energy between the eddy and mean, but rather an export out of the open domain.
“Man stands with bowed head in the presence of nature’s visible grandeurs, such as towering mountains, precipices, or icebergs, forests of immense trees, grand rivers, or waterfalls. He realizes the force of waves that can sweep away light-houses or toss an ocean steamer about like a cork. In a vessel floating on the Gulf Stream one sees nothing of the current and knows nothing but what experience tells him; but to be anchored in its depths far out of the sight of land, and to see the mighty torrent rushing past at a speed of miles per hour, day after day and day after day, one begins to think that all the wonders of the earth combined can not equal this one river in the ocean.”

- John Elliott Pillsbury
“The Gulf Stream” (1890)

“The sea gets deeper as you go further into it.”

- Venetian proverb
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I spent two years of my Ph.D. living on a sailboat, crossing the Florida Current on numerous occasions and seeing with my own eyes the immense power of this ‘river in the ocean’. This was thanks to Conor Smith, the Captain of the S/V Tardis, a great boat mate and sailing teacher, and one of the other most generous people I know!

I was involved in two side projects during my Ph.D. that contributed profoundly to my scientific education and experience. In 2012 Pierre Flament contracted me to work on a rapid HF radar deployment in the Zambales province of Luzon, the Philippines, to develop algorithms for real-time long-range ship tracking. Working in a team of very intelligent people on a fast paced project such as this was a baptism of fire that was exhausting but very enjoyable. In 2013, I was awarded the NortekUSA Student Equipment Grant, and planned and conducted the WHARF experiment (Wave Heights and Currents in the Florida Straits; Archer et al., 2015b). This would not have been possible without the financial support of SECOORA for the offshore mooring deployment. The RSMAS Ocean Technology Group, Mark Graham and Adam Houk, were indispensable – from designing the mooring to instrument preparation and finally the deployment/recovery. They were a hell of a lot of fun to work with too! Professors Bill Johns and Lisa Beal lent us important (and expensive) mooring materials, so we are grateful for their trust and generosity. Thank you to Captain Lake and the crew of the R/V Walton Smith, and Richard Behn and Miguel McKinney of Marine Operations for making the cruises run so smoothly.

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Chapter 1

Introduction

An introduction to the motivation, goal and objectives of this dissertation research.

1.1 Motivation

The ocean and South Florida

South Florida is a region comprising the densely inhabited Miami metropolitan area, the Florida Keys, and several National Parks. The urban center holds over five and a half million people and is constrained to a narrow strip along the coastline, between the Everglades and the Straits of Florida. Just offshore, the third largest barrier reef system in the world nurtures rich ecosystems, multi-species fisheries and several endangered species including sea turtles and manatees. The beaches and coastal waters of the Florida Straits support a multi-billion dollar tourist industry; they are used for swimming, scuba-diving, recreational boating and fishing, and include the world’s largest cruise ship port. Tourism in Florida continues to grow, with 2015 receiving 105 million tourists – the largest number in the state’s history (Visit Florida Website, 2016). With a widening of the Panama Canal due to be completed April 2016, Miami is one of few coast ports south of Virginia deep
enough to welcome the larger “Post-Panamax” vessels, signaling a substantial expected increase in shipping traffic. The coastal ocean is an integral part of South Florida and must be properly managed and protected.

*Application of oceanographic research*

Research efforts aimed at understanding the coastal ocean circulation in this region are extremely valuable to policymakers who must balance societal and environmental concerns. An increased knowledge would benefit a variety of applications, including; search and rescue, maritime security, navigation, fisheries management, commercial shipping, and oil spill mitigation. In addition, sea level rise is a major issue that is relevant to Miami now, and a component of the system that must be considered when predicting flooding events is the contribution of ocean dynamics – including the Florida Current, whose meanders and intensity changes are accompanied by a varying sea surface height gradient (*Ezer et al.*, 2013).

*Real-world example*

A prime example of the relevance of this research is the Deepwater Horizon oil spill in 2010. An explosion on an offshore drilling platform in the Gulf of Mexico resulted in the release of ~780,000 m$^3$ of crude oil into the environment (*National Commission on the BP Deepwater Horizon Oil Spill*, 2011). This was an environmental and economic disaster that caused widespread damage to marine habitats and the Gulf’s fishing and tourism industries. Capabilities for real-time monitoring and prediction of the surface and subsurface flow fields that can forecast the transport of hydrocarbons are vital in such a situation. This is only made possible by the groundwork from extensive scientific research. The state oil company of Cuba plans to drill two offshore exploratory wells to the northwest
of the island in late 2016 or early 2017 (*News Maritime*, 2015), which has led to concern about oil spills that would impact the Straits of Florida. An improved understanding of the ocean current processes that can transport and mix tracers such as oil in the Straits is the goal of this study.

1.2 Observing Ocean Currents with HF Radar

The Florida Current

The Florida Current flows through the Straits of Florida, following the continental shelf as it turns from an eastward course along the Florida Keys to northward offshore of Miami. Within this channel, it dominates the ocean circulation, and its frontal instabilities drive upwelling of nutrient rich water that penetrate the euphotic zone, stimulating primary productivity. Its intense velocities and large lateral shears interact with winds and surface waves to create a complex environment that can be challenging to forecast, and hazardous to navigate. At the larger scale, the Florida Current transports heat northward from the tropics, influencing the global climate. For these reasons, as well as its proximity to the United States coastline and the experimentally convenient channel-like bathymetry, the Florida Current is one of the most studied ocean currents in the world. Extensive research has been conducted regarding its volume transport and variability. Several focused research programs have detailed the vertical structure and low frequency variability of the Florida Current, including: SYNOPS 71 (*Synoptic Observations of current Profiles in the Straits*, 1971), STACS (*Subtropical Atlantic Climate Studies*, 1983), FACTS (*Florida Atlantic Coast Transport Study*, 1985), WBTS (*Western Boundary Time Series*, 1985), POFS (*Physical Oceanography of the Florida Straits*, 1992), and RAPID (*Rapid Climate Change*, 2001). There is now a general agreement on the mean structure, volume transport,
and low frequency variability of the Florida Current. Frontal instabilities along the Florida Current have been observed and modeled. However, there are still many gaps in our understanding. The questions are now directed at the dynamics of the higher frequency instabilities, especially on the eastern front, which has not yet been addressed due to lack of observations.

**High frequency radar - a unique dataset of high horizontal resolution**

Flow features that have smaller horizontal scales and evolve more quickly in time are not as easily observed by traditional in situ instruments or satellites, but can be resolved by high frequency (HF) radar. HF radar is a land-based radar deployed at the coast that operates in the radio high frequency part of the electromagnetic spectrum. It can provide two-dimensional (2-D) maps of coastal ocean surface currents in near-real time, with the ability to sample at intervals as little as a few minutes, and a spatial resolution of less than a kilometer.

**Contributing new insights to a well-studied phenomenon**

This dissertation research uses measurements from HF radar as the primary tool to study the ocean current variability. Where this research can contribute to the existing (and comprehensive) knowledge base in the Straits of Florida is with a study of the 2-D spatial variability of the surface currents over an extended time period. Previous research has been largely lead by in situ measurements, such as hydrographic sections or moorings, which have provided excellent coverage in space, or time, but not both.

**Exploiting the strengths of HF radar**

The focus for this research is to utilize the specific strengths of the HF radar dataset to obtain new information about the ocean currents in this region. The strengths are: (1)
high resolution 2-D spatial and temporal coverage; (2) Long-term timeseries covering the same region; and (3) high resolution measurements in regions that have had little to no in situ observations, and only low resolution satellite coverage. Conversely, the drawback is that it focuses attention on the surface layer, which constitutes only a fraction of the flow through the Straits. For this reason, the HF radar data where possible will be augmented by measurements from in situ instrumentation and remote satellite imagery to obtain subsurface and synoptic observations, respectively.

1.3 Goal and Objectives

The goal of this dissertation research is to quantify the mean jet structure and variability of the Florida Current between 25°N to 26°N, to characterize the temporal and spatial scales of the surface current fluctuations, and determine the spatial pattern of eddy-mean flow energy conversion.

The dissertation is organized into three major research chapters:


To achieve the stated goal, answers will be sought to the following scientific questions, organized by chapter:
1 THE FLORIDA CURRENT IN A STREAM COORDINATE FRAME:
MEAN JET STRUCTURE AND VARIABILITY

*What is the mean horizontal structure of the Florida Current, and how does its time-mean structure change when lateral meandering motion is removed?*

Since the Florida Current flows northward in the center of the HF radar footprint, it provides the opportunity to accurately determine its mean structure over a two year period. By converting to a natural stream coordinate system, in which the origin is fixed to the core of the jet rather than a geographical location, a more realistic time average can be achieved because the smearing due to lateral meandering is removed. What is the contribution from meandering to the mean eddy kinetic energy?

*What are the dominant periods, wavelengths and phase speeds of meandering offshore of Miami?*

The stream coordinate conversion method identifies the offshore position of the jet core at each time step, from which a timeseries of meandering can be retrieved. Whereas previous studies in the Florida Straits relied on inference of meandering from point measurements, HF radar can directly observe the core of maximum velocity and its lateral shifting in time. The characteristic period, wavelength and phase speed of meanders can be determined based on a two year HF radar dataset.

*How does the jet structure vary in time and space?*

Whilst the large-scale mean horizontal structure of the Florida Current has been well documented, its variability in time and space is not well known. Jet variability can be quantified based on several variables, including: width, intensity, lateral shear and surface transport. These variables can be related to local wind forcing, water level fluctuations and the full volume transport measured by a NOAA-operated submarine cable.
2 CYCLONIC AND ANTICYCLONIC FRONTAL INSTABILITY OF THE FLORIDA CURRENT: TWO CASE STUDIES

Using two case studies chosen for exceptional data coverage and signal clarity, the kinematic properties of (1) cyclonic and (2) anticyclonic instability in the Florida Current will be investigated.

What are the kinematic properties of a cyclonic frontal eddy?

A coherent frontal eddy translated through the HF radar domain January 18 to 21, 2005. It was recorded in the surface current velocity field by HF radar and sea surface temperature field by satellite imagery. The kinematic properties of the eddy, including spatial scale and translation speed will be documented. Does the eddy follow mesoscale or submesoscale dynamics? The components of the velocity gradient tensor (vorticity, divergence and strain) will be calculated and contrasted with a period of no eddy activity. Particle dispersion will be quantified using an Eulerian diagnostic (instantaneous rate of separation).

What are the kinematic properties of an anticyclonic shear-zone instability?

A transient, coherent near-inertial signal was recorded by HF radar from October 16 to 21, 2006. The signal will be separated from the sheared background velocity field using digital filtering, and its spatial structure, phase speed and frequency of oscillation will be analyzed. Does the signal conform to near-inertial wave dynamics, or to vortex dynamics? How does the background flow alter the signal in the total velocity vector field?
3 CHARACTERIZING SPACE-TIME STRUCTURE OF FLUCTUATIONS AND EDDY-MEAN FLOW INTERACTION IN THE STRAITS OF FLORIDA

What are the characteristic spatial and temporal scales of the fluctuations?

The decorrelation properties of the flow can be obtained by calculating its autocorrelation in time and space. A function can be applied to the dataset to define subgrid scale processes, and turbulent and integral timescales (Mariano and Chin, 1996). Scales are expected to grow with distance offshore into the Florida Current, so these calculations will be conducted over the HF radar domain to inspect the spatial variability.

What are the dominant periods of variability and do they exhibit seasonality?

Frequency power spectra can be calculated to determine which time scales contain the most energy. The slope will indicate the cascade of energy from longer to shorter periods of motion. Intermittency of a signal will reduce its signature in power spectra, while the periodic energy (e.g. tides) will be emphasized. To study signals with an unsteady and episodic nature, the wavelet transform will be computed, which provides frequency energy content as a function of time.

What is the slope of the kinetic energy wavenumber spectrum, and what dynamical theory do the results support?

The slope of the wavenumber spectrum describes the partition of energy between scales of motion. A steeper slope implies that it is non-local motions that dominate transport and dispersion in the ocean, while a shallower slope indicates relatively more energy at the smaller scales and the importance of the submesoscale. The HF radar dataset offers a unique opportunity to calculate the wavenumber spectrum in the Straits of Florida at high resolution (1 km).
What is the spatial pattern of barotropic energy conversion, and what are the most important terms in the equation?

An assessment of the kinetic energy exchange between the fluctuations and the mean flow in the surface layer will be conducted. Whereas previous energetics studies in the Straits of Florida were based on observations that spanned longitude and depth, this study can contribute new insight by considering both along-stream and cross-stream terms to determine their relative importance to the whole. In this region are eddies unstable (extracting kinetic energy from the mean flow), or do they act upgradient – supplying energy to drive the mean flow?

How do these scales relate to previously studied scales in the Florida Current, shown in Figure 1.1?

Figure 1.1 The scales of time, space, and phase speed of the fluctuations that have been observed in the Straits of Florida. *Phase speed for the 27 hr signal observed by Peters et al. (2002) is 170 cm s⁻¹. [Data source: Lee & Mayer, 1977; Brooks & Mooers, 1977; Johns & Schott, 1987; Boudra et al., 1988; Chew, 1974; Brooks & Niiler, 1975; Lee et al., 1992; Lee et al., 1995; Frantantoni et al., 1998; Lee, 1975; Lee & Mayer, 1977; Haus et al., 2000; Davis et al., 2008; Parks et al., 2009; Shay et al., 2000; Shay et al., 1998; Peters et al., 2002; Soloviev et al., 2003]
It is hoped that the major contribution of this doctoral work is to take the big step forward from individual case studies to long-term time series analysis, in which we can determine the quantitative and statistical details of the time and space scales of these instabilities, and whether they exhibit change over time.

1.4 Outline

The dissertation is divided into 7 chapters, which are briefly summarized below:

CHAPTER 1 “Introduction”. An introduction to the motivation, goal and objectives;

CHAPTER 2 “Ocean Circulation in the Straits of Florida”. A review of the current state of knowledge regarding ocean circulation in the Straits of Florida. Beginning with a historical account of the discovery and exploration of this western boundary current, topics covered include: geography of the region, the Florida Current structure, volume transport, meandering, continental shelf waves, eddies, tides, internal waves and the local wind forcing;

CHAPTER 3 “Instrumentation and experimental design”. An overview of HF radar principles of operation, accuracy, limitations and the derivation of vector velocity. An outline of data quality control and interpolation procedures applied to the dataset used in this research;

The effects of lateral meandering on an Eulerian time-mean field are quantified in relation to eddy kinetic energy. Meandering periodicity, wavelength and phase speed, and jet variables including intensity, width, surface transport and shear are computed and analyzed.

CHAPTER 5 “Cyclonic and Anticyclonic Frontal Instability of the Florida Current: Two Case Studies”. In the first case study, the kinematic properties of a cyclonic vortex are investigated, and the terms in the velocity gradient tensor calculated and compared to a period of no eddy activity. In the second case study, a near-inertial signal on the anticyclonic flank is investigated, including its dominant frequency, wavelength and phase speed. Does the signal conform to near-inertial wave dynamics, or to vortex dynamics? How does the background flow alter the signal in the total velocity vector field?

CHAPTER 6 “Characterizing the Space-Time Structure of Fluctuations and Eddy-Mean Flow Interaction in the Straits of Florida”. The characteristic temporal and spatial scales of the fluctuations are calculated based on the correlation properties of the flow field. The dominant periods of variability are examined, along with their time dependency. The time-mean slope of the kinetic energy wavenumber spectrum is found. Eddy-mean flow interaction is investigated through the conservation of eddy kinetic energy equation, variance ellipses and the Reynolds terms.

CHAPTER 7 “Concluding Remarks”. Principle findings, new contributions and open questions.
Chapter 2

Ocean Circulation in the Straits of Florida

A review of the current state of knowledge regarding ocean circulation in the Straits of Florida. Beginning with a historical account of the discovery and exploration of this western boundary current, topics covered include: geography of the region, the Florida Current structure, volume transport, meandering, continental shelf waves, eddies, tides, internal waves and local wind forcing.

2.1 Historical Perspective

The Florida Current has a rich history. While early explorers of the ocean, including Norse, Arabian and Portuguese, may have encountered the Gulf Stream, it was not explicitly documented until the Spaniard Ponce de León, in 1513 on his quest to discover the fountain of youth, attempted to sail south from what is now St. Augustine. Though they had clear skies and fair winds, they found that they could not make headway against a strong northward flowing current. Two of the vessels closer to the coast managed to anchor, but the third, a brig in deeper water, was “carried away by the current and lost from sight although it was a clear day” (Pillsbury, 1890). The knowledge of this fast current was quickly made use of, and by 1519 was routinely used by Spanish ships bound
for home who came to America by means of the Equatorial Current, but returned through the Florida Straits, riding the Gulf Stream northeast to cape Hatteras and then eastward to Spain.

Figure 2.1 (top) Athanasius Kircher’s “Mundus Subterraneus”, the world’s first published chart showing the ocean currents on it, from 1678. (bottom) A 1786 copy of Benjamin Franklin’s chart of the Gulf Stream originally printed in 1770 [Source: http://www.oldfloridamaps.com]. See Richardson (1980) for a discussion of Franklin’s chart and subsequent copies.

In 1590 John White, the Governor of the colony of Roanoke (the ‘lost colony’), reported that during a trip northward from the Florida Keys, that their ship by necessity “stood to sea for to gaine the helpe of the current, which runneth much swifter farre off
than in sight of the coast… (for) all along the shore are none but eddie currents setting to the south and southwest.” This is the first time that the approximate location of the Florida Current axis, as well as its frontal instabilities, were documented (Pillsbury, 1890). In the seventeenth century, North America was colonized and the Gulf Stream was traversed frequently. At this time, a number of studies of the ocean currents were published (Varenius, 1671; Vossius, 1663). The first chart showing the Gulf Stream was printed by Athanasius Kircher in 1665; it shows the correct general clockwise circulation in the North Atlantic, in addition to some fanciful features, including two currents crossing over one another, and the Loop Current in the Gulf of Mexico flowing linearly northwest (Figure 2.1). While the explanation for the mechanisms driving these currents were often based on fantastic theories and misconceived ideas, the knowledge of what was happening was frequently correct. For instance, it was recognized that the Florida Current velocity was influenced by meridional wind variability, and that this large scale current system was the cause of the strange fruits and woods that washed up on the coastlines of Ireland.

In the mid-1700s, the polymath Benjamin Franklin turned his attention to the Gulf Stream. Franklin was Postmaster General at the time and was brought a complaint that the British mail packets usually took a fortnight longer to cross the Atlantic that merchant ships. In discussion with his cousin and Nantucket ship captain Timothy Folger, he learned why: the packet captains were unaware of this current (or “too wise to be counselled by simple American fishermen”) and so frequently sailed against it. Folger drew out the Gulf Stream for Franklin, based on the Nantucket whaler’s firsthand knowledge of its speed, course and breadth, and Franklin had it printed in 1770 (Figure 2.1). During several voyages across the Atlantic, Franklin took measurements and based on the temperature
could determine whether the vessel was in or out of the Stream, and recorded that the current was significantly cooler on its western flank.

The United States Coast and Geodetic Survey began the age of modern surveying in 1844, under the direction Franklin’s great-grandson Alexander Bache, who measured 14 temperature section between Tortugas and Nantucket (Stommel, 1966). These cruises confirmed the presence of cold ‘veins’ along the western edge of the Stream, although they were misconstrued to be invariable and attributed to bottom topography, which diverted the Gulf Stream into separate bands. In 1887, John Elliot Pillsbury conducted a remarkable study of the Florida Current offshore of Miami, by anchoring his steamer Blake at several stations across the Straits, and recording temperature and velocity at various depths on daily to monthly timescales (Pillsbury, 1980). The anchoring gear and current meter were designed by Pillsbury, and the data that was so meticulously collected of such high caliber that they were used well into the 20th century by scientists.

In the last century, there have been many advances in our understanding of the large-scale ocean dynamics; particularly the recognition of the importance of earth’s rotation and horizontal pressure gradients in driving the time-mean circulation. The reader is referred to Henry Stommel’s 1966 book titled “The Gulf Stream”, which provides an overview of the 20th century work to describe the physical mechanism driving the large-scale ocean in the North Atlantic. A far more detailed historical perspective on the Florida Current and Gulf Stream can be attained from Pillsbury (1890) and Stommel (1966). This chapter now proceeds to an overview of the most recent knowledge concerning the Florida Current structure and it fluctuations.
2.2 Geography

The Straits of Florida is a narrow channel that connects the Gulf of Mexico to the South Atlantic Bight (Figure 2.2). It is bounded by the Florida peninsula to the north and Cuba and the Bahamas to the south and east, respectively. The southern Straits is orientated east-west, with a width of 150 km and depth of 2000 m at the western entrance. Eastward the channel becomes progressively narrower and shallower down to a width of 85 km and depth of 800 m. At approximately 80°W 25°N the channel turns cyclonically to a north-south orientation, generally maintaining its width and depth. A shallow coral bank (Cay Sal) is located at this bend in the channel and acts to partly separate the southern Straits from the Santaren Channel. The Northwest Providence Channel connects between Little Bahama Bank and Grand Bahama.

Figure 2.2 (a) Schematic representation of the Straits of Florida, showing the bathymetry of the channel, the flow pattern of the Florida Current and associated inshore cyclonic frontal eddies. Inset: 3-D structure of a cyclonic frontal eddy [based on Figure 18 of Lee et al. (1981)]. Visualization created in collaboration with artist Yiran Zhu.
For this study, the area of observation covers 79° to 80.5°W and 25° to 26°N. At this location the channel has just made the turn to a meridional orientation, and is contained between South Florida to the west and Bimini to the east. The continental shelf is narrow, the shelf break is located 3.5 km offshore of Miami, increasing to a depth of 800 m in the channel, over a distance of 50 km.

2.3 Structure of the Florida Current

The Florida Straits acts as a conduit for the fast flowing Florida Current – a western boundary current and part of the major current system that connects the Loop Current in the Gulf of Mexico to the Gulf Stream in the North Atlantic (Figure 2.3). The Florida Current is composed of nearly equal parts of waters originating from the South and North Atlantic, but the warm upper layer waters derive mainly from the tropical South Atlantic (Schmitz and Richardson, 1991). This makes it an important link in both the wind driven gyre and the thermohaline circulation. For this reason, and its proximity to the eastern seaboard of the US, it has been the focus of scientific research as early as the 1880s (Pillsbury, 1890). Several focused research programs have aimed to detail the structure and variability of the Florida Current (see Chapter 1 for a list of field campaigns). These programs have produced surges in published research, particularly in the 80s.

Figure 2.4 shows a typical vertical cross-channel section of stratification and meridional (along-shelf) velocity at two latitudes in the center of our research area (Miami at 25.76°N and Fowey Rocks at 25.59°N). These sections are constructed from average values of dropsonde data in a study by Brooks (1975). The two sections indicate the variability between latitudes. For this period of observation (and confirmed by other studies, e.g. Beal et al., 2008) the position of the core is around 25 to 30 km offshore with
velocities exceeding 175 cm s\(^{-1}\) in the mean, and strong shear zones either side. The core moves offshore with depth as is typical of western boundary currents \((Halkin and Rossby, 1985)\). Cross-shelf velocity is more variable at 25°N (south of Brooks (1975) transects), where the current is still turning from a zonal orientation, resulting in a persistent displacement and strong variability in the cross-shelf component \((Fiechter, 2009)\). Between 26°N and 27°N (north of Brooks (1975) transects), the current is much less variable due to strong topographical constraints. Peters et al. (2002) measured the water column at 26°N and noted the strong stratification with significant contributions from both temperature and salinity. They recorded an overall depth-time average squared buoyancy frequency of \(\bar{N}^2 = 1.7 \times 10^{-3} \text{s}^{-2} (\bar{N} = 23.6 \text{cph})\) at the shelf in 60 m water depth.

**Figure 2.3** Surface trajectories of the entire NOAA/AOML Drifting Buoy Data Assembly from 1978 to 2013. The figure highlights the dominant current system, including the Loop Current in the Gulf of Mexico, the Florida Current that wraps around the Florida Peninsula, and the Gulf Stream, which separates from the coast off Cape Hatteras. The black box denotes the region of interest. [Figure courtesy R. Lumpkin. Data source: www.aoml.noaa.gov/phod/dac/index.php]
The pycnocline depth is approximately 150 m. The density distribution has isopycnals sloping upward toward the west, which is consistent with thermal wind balance

\[
\frac{\partial V}{\partial z} = \frac{g}{\rho_0 f} \frac{\partial \rho}{\partial x},
\]

where \( V \) is downstream velocity, \( z \) and \( x \) are the position in east-west and up-down coordinates, \( g \) is gravitational acceleration, \( \rho_0 \) is reference density, and \( f \) the Coriolis
frequency (Cushman-Roisin, 1994). The steeper the isopycnals, the greater the vertical shear.

The Florida Current exhibits fluctuations that cover a broad range of time and space scales. A review of the fluctuations that have been observed in the Straits of Florida and published in the literature is presented in Figure 1.1. It reveals a broad spectrum of scales, ranging from slower, larger meanders and Tortugas eddies to smaller, rapidly evolving features such as submesoscale vortices and a super-tidal oscillation. These features are not independent, but strongly influenced by one another and the dynamics of the Florida Current. The following subsections will attempt to address the accumulated research knowledge regarding these separate scales.

2.4 Volume Transport

The mean volume transport of the Florida Current in the northern Straits (26°N to 27°N) has been measured frequently over the last century with a variety of instruments to be approximately 30 Sv (1 ‘Sverdrup’ = 10^6 m^3 s^-1), with a standard deviation of ±3 Sv (e.g. Brooks and Niiler, 1977; Leaman et al., 1987; Larson, 1992; Beal et al., 2008; Meinen et al., 2010). The northern Straits is the latitude of largest volume transport of the Current, composed of water from the main Yucatan Channel (24 to 28 Sv), fed by additional waters as it moves downstream: 1.2 to 2.5 Sv from the Old Bahamas Channel and 2 Sv from the Northwest Providence Channel (Leaman et al., 1987; Leaman et al., 1995; Hamilton et al., 2005).

Volume transport variability

Tidal transport variations are ~ ± 1.5 Sv. Transport variability is dominated by sub-annual fluctuations, which contribute roughly 70% of the total variance (Meinen et al.,
2010), making the quantification of longer-term variability challenging. Significant energy is contained within periods of 3 days to 3 weeks, correlated with changes in meridional wind stress, continental shelf waves, meandering and frontal eddies (Brooks, 1975; Johns and Schott, 1987; Schott et al., 1988; Lee and Williams, 1988; Mooers et al., 2005). Many studies have shown that energy peaks within this period band, but rarely do they align precisely with previous results. The partition of energy within the spectrum is controlled by event-type forcing of randomly timed transient phenomena (wind events, or frontal instabilities) and is therefore somewhat dependent on the time period of study (Lee and Mayer, 1977). There is, however, a seasonality of the transport variance; winter exhibits increased energy in the 2 to 10 day period band (forced by cold front passages) than summer (Lee and Mayer, 1977; Schott et al., 1988). Johns and Schott (1987) note significant coherence between transport and meridional wind stress in several narrow period bands, noting that there is a nearly constant time lag of one day between the wind stress and transport variation.

The annual cycle of volume transport has an amplitude between 2 to 4 Sv, exhibiting a maximum in summer, a minimum in fall, and a secondary minimum in spring (e.g. Niiler and Richardson, 1973; Leaman et al., 1987; Schott et al., 1988; Rousset and Beal, 2011). However, the robustness of this signal over time is questionable, since phase shifting and adjustment from a dominant annual to semi-annual component was found after 1991 (Baringer and Larsen, 2001; Meinen et al., 2010). The mechanism controlling this cycle is still not clear, with a number of processes possibly contributing. It is evidently not due to the seasonal wind stress curl over the North Atlantic (Sverdrup dynamics), since this would generate a maximum transport in the winter. This can partially be explained by the
blocking effect of the Bahamas island chain, which separates the Florida Straits from the interior ocean, preventing the westward-propagating Rossby wave energy to deliver the wind forced signal (Schott and Zantopp, 1985). Neither is the cycle caused solely by the local meridional wind stress fields, although it clearly has a strong influence; it has been shown to account for approximately 25% of the seasonal observed amplitude (Schott et al., 1988). The mechanism by which this occurs is the summer peak in northward wind stress that drives an eastward Ekman transport, steepening the east-west tilt of the pycnocline and accelerating the thermal wind balance. Czeschel et al. (2012) employed an adjoint model approach to test the sensitivities of the transport to wind stress forcing. They identified the primary source of seasonal variability to be caused by wind stress forcing of upwelling/downwelling north of the Straits of Florida along the North American coast, generating fast barotropic waves that propagated southward within a month, driving an annual cycle with an amplitude of ~1 Sv. Such barotropic shelf waves have been observed in the Florida Straits (Schott and Duing, 1976; Brooks and Mooers, 1977). Another potential contribution is from upstream forcing in the Caribbean Sea (Johns et al., 2002). The variability of transport at the annual period is therefore more complicated than a simple cause and effect of one variable. This makes sense, since it is an integrated value that encompasses a wide range of dynamical forcing.

**Interannual and decadal variability**

Interannual and decadal variations have been explored and compared with the North Atlantic Oscillation (NAO) index. It was hypothesized that the NAO and Florida Current transports are negatively correlated, with NAO leading by 1 to 25 months
(Baringer and Larsen, 2001). However, data after 1998 indicates this relationship does not appear to hold (Beal et al., 2008; Meinen et al., 2010).

2.5 Meanders

The nomenclature ‘meander’ was introduced during the Multiple Ship Survey of 1950 (Fuglister and Worthington, 1951) for observed undulations in the Gulf Stream location and structure. In the Florida Straits, a large portion of current variability can be attributed to cross-channel meandering of the Florida Current jet (Lee et al., 1995). Bane and Brooks (1979) describe meanders as northward travelling asymmetric waves with crests the shoreward displacement of the jet axis. Several studies have shown that upwelling occurs on the inshore side of the wave trough, and cold cyclonic eddies are formed that travel with the parent wave (Lee et al., 1981; Lee and Atkinson, 1983; McClain et al., 1984).

Meander characteristics

Length scales of the meanders range from 70 km up to several hundred kilometers, and their periods from several days to weeks, with propagation speeds around 40 cm s⁻¹ (Schmitz and Richardson, 1968; Duing, 1975; Brooks and Niiler, 1977; Johns and Schott, 1987; Zantopp et al., 1987; Lee et al., 1995; Parks et al. 2009). Studies have highlighted that meander amplitudes generally decrease downstream of the western entrance due to the narrowing channel and shoaling topography (Schmitz and Richardson, 1968; Johns and Schott, 1987).

Johns and Schott (1987) used 18 months of current meter data from the STACS program to investigate meanders. Their results show that approximately 25% of the total subinertial (> 2 days) velocity and temperature variance was explained by meandering. The most coherent and energetic meandering signals were centered at 5 and 12 days, with
phase speeds (wavelengths) of 20 km day\(^{-1}\) (340 km) and 36 km day\(^{-1}\) (170 km). They could not determine the source of energy. \textit{Boudra et al.} (1988) ran an isopycnic coordinate numerical model with the STACS observations as initial conditions. They found that a perturbation wavelength of 180 km results in the greatest amplitude, which is similar to the 170 km/5 day period meander observed by \textit{Johns and Schott} (1987).

\textbf{Energetics}

The \textit{Boudra et al.} (1988) model results suggest that baroclinic conversion (potential to eddy kinetic energy) is the primary physical mechanism of meander growth, but the role of barotropic conversion (mean kinetic energy to eddy kinetic energy) is not negligible. It was clear that for the flow to be marginally unstable the surface cross-stream density variation was a necessity; that is, the mass/flow structure is sufficiently baroclinic. Earlier work by \textit{Schmitz and Niiler} (1968) on the energetics of the Florida Current found substantial energy transfer from fluctuations to the mean flow in a narrow zone of about 20 km in the region of cyclonic lateral shear. \textit{Johns and Schott} (1987) also note a significant energy transfer to the mean flow through upgradient eddy momentum and buoyancy fluxes in the cyclonic shear zone, dominated by barotropic (eddy momentum flux) processes. They discuss the data is consistent with a weakly finite-amplitude baroclinic instability process, in which energy is first released from the mean flow by downgradient buoyancy fluxes (at some point farther upstream in the Keys), then later converted back to mean kinetic energy via these up-gradient momentum fluxes. See section 2.7 for further discussion.
**Relationship to wind and volume transport**

A topic of interest in these previous studies was the relationship between meanders, volume transport and wind forcing. *Duining* (1975) and *Brooks* (1979) suggested larger transports occurred when the axis was located offshore of its mean position, although *Brooks* (1979) noted that meanders and transport fluctuations often appeared to occur independently. *Johns and Schott* (1987) found no evidence of strong correlation between these two processes, and little evidence to suggest that wind stress was a primary energy source for the meanders. *Webster* (1961) showed that the energy provided by the wind stress was two orders of magnitude smaller than the transfer of kinetic energy from the meander to the mean flow, and was therefore not the significant source of energy for meandering.

### 2.6 Continental Shelf Waves

The continental shelf is an efficient wave guide for the propagation of subinertial sea level and current fluctuations over large distances (*Mysak*, 1980). Fluid columns displaced across the slope acquire relative vorticity in order to conserve potential vorticity. This forces the displacement of adjacent columns resulting in along-shelf propagation of the disturbance in a cyclonic oscillation about the deeper water (with coast to right in northern hemisphere; *Robinson*, 1964). These features are named ‘continental shelf waves’ (CSWs). They are predominantly forced by wind stress along the shelf associated with synoptic scale weather systems. They have typical amplitudes of a few centimeters, periods of several days and in the Florida Straits, wavelengths of around 200 km (*Duining et al.*, 1977). *Brooks and Mooers* (1977) studied the complicated interaction between CSWs and the sheared Florida Current. The waves can be advected northward by the mean
current and have their propagation characteristics modified. Southward propagating CSWs along the Florida coast can occur due to cyclonic shear approaching the magnitude of $f$, which creates sufficient southward tendency to overcome northward advection by the mean flow. Distinction between meanders and continental shelf waves can be problematic in the presence of the strongly sheared Florida Current: it is often unclear due to hybrid shelf-wave modes with dynamics that are influenced by the velocity and potential vorticity distribution of the flow and local topography (Brooks and Mooers, 1977).

2.7 Eddies

A growing amount of research in the Florida Straits concerns mesoscale and submesoscale eddy events. Since Lee (1975) observed a number of cyclonic current reversals offshore of Boca Raton, oceanographers have measured and modeled eddies from the western entrance to north of the Straits. Nomenclature for these events includes: spin-off eddy (Lee, 1975; Lee and Mayer, 1977), frontal eddy (Fiechter and Mooers, 2003), Pourtales gyre (Lee et al., 1992) and Tortugas gyre/eddy (Lee et al., 1995; Frantantoni et al., 1998) and submesoscale vortex (Shay et al., 2002). It has been shown that these events are an important link between littoral and offshore waters, transporting salt, heat and momentum. They also play a vital role in the transport of nutrients and larvae from the Florida Current to the fringing reefs along the Keys (Lee et al., 1992; Hitchcock et al., 2005; Richardson et al., 2009). Based on the literature, two classes of eddy can be identified in the Straits by their kinematic properties and the forcing mechanisms that generated them:
2.7.1 **Tortugas gyres (or eddies)**

In the Southern Straits of Florida, cyclonic mesoscale eddies are generated near the Dry Tortugas (24°N, 83°W), and are associated with large southward displacements or meanders of the Florida Current axis (*Lee et al.*, 1992; 1995). *Frantantoni et al.* (1998) analyzed three years of advanced very high resolution radiometer (AVHRR) data to study these Tortugas gyres ‘TG’ (*Lee et al.*, 1992). They name them eddies to reflect their transient nature. Typical dimensions change as these eddies move downstream: cross-channel length from 200 km (83°W) to 70 km (81°W) and along-channel length from 115 km (83°W) to 165 km (81°W). Typical translation speeds increased from 5 – 16 km day⁻¹ accompanied by a ~55% reduction in size (fluid loss of 0.7 Sv). Residence times vary between 50 – 140 days, depending on conditions (see below).

*Lee et al.* (1995) found that TG formation is based on the orientation of the Loop Current as it enters the southern Straits. In a well-developed Loop Current (greater northern penetration) the flow overshoots its entry to the southern Straits, causing the formation of a cold recirculation off the Dry Tortugas. ‘Pourtales’ Gyres previously documented between Key West and Isla Morada (*Lee et al.*, 1991) are now identified as the downstream latter stage evolution of TGs. When the Loop Current is not developed (occurs following the shedding of an anticyclonic ring), flow from the Yucatan Channel turns anticyclonically into the Straits and gyre formation does not occur. *Frantantoni et al.* (1998) propose that these TGs are the downstream expression of Loop Current frontal eddies (LCFE) in the Gulf of Mexico. A TG will remain stationary near the Dry Tortugas until it is impacted upon by an LCFE, which may produce residence times of 50-60 days. The Loop Current ring-shedding however may inhibit the southward propagation of
LCFEs, producing a two-fold increase in TG residence times (105-140 days) whilst the TG awaits impact by another LCFE.

Simulated results (Fiechter, 2007) suggest that the TGs do not translate into the meridional part of the Straits, which is consistent with observations that they decrease in size and are sheared apart and dissipated along the upper Florida Keys. However, there is a possibility for these eddies to move close to 25°N, our region of interest. By this latitude the TG has mostly dissipated, and is characterized by a large offshore meander of the Florida Current that has traveled downstream in phase with the eddies. Parks et al. (2009) observed such an event with HF radar and Moderate Resolution Imaging Spectroradiometer (MODIS) imagery in 2005.

It should be noted that Frantantoni et al. (1998) found inconsistencies between the number of eddies they recorded (11) compared with the number predicted by the velocity timeseries of ADCP moorings (17) that were located shoreward of the 30 m isobath. The TGs were identified in the ADCP velocity record as reversals from eastward to westward throughout the water column. This inconsistency may be explained by the results of Shay (1997), who observed submesoscale near-inertial motions (see below), which would have also forced current reversals at the ADCP moorings.

2.7.2 Frontal eddies (spin-off eddies or edge eddies)

Cyclonic, cold-core frontal eddies occur all along the Straits of Florida inshore of the Florida Current (Figure 2.2 and 2.5). They were first described by Lee (1975) and Lee and Mayer (1977) as small diameter (elongated 10 km east-west and 20-30 km north-south) ‘spin-off’ eddies that propagate northward at speeds of 20 to 50 cms⁻¹, observed off Miami and Boca Raton. While the larger TGs have their forcing mechanism in the orientation of
the Loop Current as it enters the Florida Straits, frontal eddies are formed by barotropic instabilities due to small perturbations in regions of high horizontal shear. The lifespan of frontal eddies is estimated to be between one to three weeks, with an average one week occurrence (Lee and Mayer, 1977). They have been consistently observed over a wide range of bottom topographies throughout the year, which suggests their formation is not directly due to topographic constraints or direct wind forcing, although the random passage of wind events can perturb the high shear region of the Florida Current creating these meanders that may lead to eddy formation.

**Figure 2.5** (left) 1 day mean SST from MODIS at 1 km resolution for May 7, 2006 [Data source: http://mur.jpl.nasa.gov/]. Depth contours in meters. Black box indicates HF radar domain. ‘LCFE’= Loop Current Frontal Eddy; ‘TE’= Tortugas Eddy and ‘AE’= Anticyclonic Eddy. (right) Ocean color image reveals two cyclonic frontal eddies on the inshore edge of the Florida Current on December 10, 2013 [Source: http://optics.marine.usf.edu/]. White box denotes same geographical region in both maps.

**Thermal signature**

The passage of these eddies distorts the thermal structure over the shelf break, which is visible in satellite sea surface temperature imagery (**Figure 2.5**). Uplifted
Isotherms associated with the cyclonic circulation create a cool surface band of water near the center of the vortex. This produces a strong SST signature characterized by a warm tongue-like extrusion (termed ‘shingle’) forced by the southward-orientated currents, with a cold upwelling region between the Florida Current and the extruding filament that has an isopycnal uplift of approximately 10 m day\(^{-1}\) in the upper 200 m (Lee and Mayer, 1977). In this sense, they are more akin to roll-vortices produced by wavelike rolling up of the shear zone than isolated rings such as observed in the Gulf Stream after it detaches from the coastline (Lee et al., 1981). An alternative description is that the warm filament contains an anticyclonically rotating current, which explains the ‘gap paradox’ that the filament never closes (Chew, 1981). As can be seen in ocean color imagery (Figure 2.5), this cold nutrient rich water that is upwelled and penetrates the euphotic zone stimulates primary productivity and plays an important role in local fisheries (Lee et al., 1992).

**HF radar observations**

Two cyclonic frontal eddies were mapped by Haus et al. (2000), seaward of Hawk Channel off Key Largo. They occurred when the jet axis was further offshore, with large frontal shears due to wind-forced southwestward inner-shelf currents. Both eddies were elongated in the along-shore direction (19 by 15 km and 47 by 25 km) with fast downstream translation speeds (53 cm s\(^{-1}\) and 80 cm s\(^{-1}\)). Although both had a strong shelf break signature, there was no observable influence on flows over the inner-shelf. Parks et al. (2009) described a cyclonic submesoscale eddy offshore of Miami with a diameter approximately 15 km and moving at 45 cm s\(^{-1}\). A moored acoustic Doppler current profiler (ADCP) revealed it had a barotropic structure, characterized by a westward \(u\)-component through the entire water column over a 48 hour period.
**Numerical simulations**

_Fiechter and Mooers_ (2003) and _Mooers and Fiechter_ (2005) ran a high resolution (2-10 km) curvilinear coastal ocean model (EFS-POM) with realistic topography, and more recently _Bulhões de Morais_ (2010) and _Kourafalou and Kang_ (2012) used a regional Florida Keys version of the Hybrid Coordinate Ocean Model (FKEYS-HYCOM) with high horizontal resolution of 900m. These simulations can generate frontal eddies with translation speed, recurrence period, strength and characteristic length scale that agree well with observations along the length of the Florida Straits. _Gula et al._ (2015), based on a high resolution (750 m) Regional Ocean Modeling System (ROMS) model, analyzed the instability mechanism for cyclonic frontal eddies along the shear zone of the Florida Current as it exits the Straits. They emphasize the role of topographic drag due to the continental slope, which amplifies the cyclonic shear and potential vorticity. The flow becomes barotropically unstable due to horizontal shear instability, rolls up and forms a street of submesoscale vortices. _Gula et al._ (2014) find that all along the Gulf Stream, the interaction between flow and topography acts as an external forcing mechanism to localize the oceanic ‘storm tracks’ of eddies, with eddy fluxes that are downgradient (upgradient) associated with eddy growth (decay). Offshore of Miami, barotropic conversion is weakly negative on the cyclonic inshore edge of the Florida Current, signifying eddy decay.

### 2.8 Submesoscale Processes

**A note on the submesoscale within the Florida Straits**

The submesoscale can be examined based on the dynamics of the flow field (_Thomas et al._, 2008). In a localized upper ocean flow regime, where the relative vorticity \( \zeta \) equals or exceeds the planetary vorticity \( f \), the dynamics will differ from the mesoscale
as the Rossby number \((Ro = \left| \zeta \right| / f)\) tends to \(O(1)\). The bulk Rossby \((Ro_b = U/fL)\) and Richardson \((Ri_b = N^2H^2/U^2)\) numbers are defined by the characteristic speed \((U)\), horizontal \((L)\) and vertical \((H)\) length scales of the flow field. Flows with \(Ro_b \sim Ri_b \sim O(1)\) are characterized by a Burger number \((Bu = N^2H^2/f^2L^2)\) of unity, which indicates the importance of stratification \((N\) is the buoyancy frequency) within a fluid \((Thomas et al., 2008)\). A Burger number near unity implies that \(L\) (horizontal length) scales with the Rossby radius of deformation \((L = NH/f)\).

A recent cruise made two CTD (conductivity-temperature-depth) casts on April 22, 2014. The location of cast #1 relative to the Florida Current, and the vertical profiles of temperature, salinity, density and buoyancy frequency are displayed in Figure 2.6 (cast #1 is representative of both casts). Using the values from these measurements, the mixed layer and first baroclinic mode Rossby radius of deformation can be calculated. The mixed layer length scale, using the bottom of the mixed layer buoyancy frequency \((N_{mil}^2 = 1.1 \times 10^{-3}\) s\(^{-2}\)), the mixed layer depth (45 m) and the local Coriolis parameter (25° 54’ N) is 23 km. The first baroclinic mode radius is defined as \(R_d = \sqrt{g'\bar{H}/f}\), where \(H\) is the mixed layer depth (45 m) and \(g' = \rho_2 - \rho_1/\rho\) is reduced gravity, where \(\rho\) is the vertical average of density, and the density subscripts represent a top (mixed) layer (1) from 0-45m, and a bottom layer (2) below the mixed layer. Using cast #1 data, the Rossby radius of deformation of the first baroclinic mode is calculated to be 17 km.

\(Peters et al.\) (2002) calculated two first baroclinic mode phase speeds of 0.85 and 1.7 ms\(^{-1}\) out past the 200 m isobath. This equates to a Rossby radius of deformation of 14 to 27 km. \(Shay et al.\) (2000) note that the deformation radius on the shelf break is approximately 10km increasing to ~ 30km further offshore. Note the local inertial period
$(2\pi/f)$ in our study region is approximately 28 hours. The Rossby radius of deformation is therefore assumed to fall within the limits stated above (15-30 km).

**Observations of submesoscale processes in the Florida Straits**

A near-inertial oscillation translating eastward along the 150 m isobath off Key West was observed by Shay et al. (1998). The signal moved along the inshore edge of the jet axis at 30 cm $s^{-1}$, exhibiting a dipole-like structure of the current vectors in space. The signal was embedded in the near-inertial passband, and absent in the subinertial band. They suggest it was forced by an abrupt change in wind stress, consistent with analytical model results of Kundu (1984). The Florida Current jet trapped the higher frequency near-inertial motions due to negative vorticity, and amplified these motions in the positive vorticity (cross-shelf gradient $\pm 2f$) regime.

Shay et al. (2000) utilized very high frequency (VHF) radar with a high horizontal resolution of 250 m. Several submesoscale vortices were observed with diameters of 2 to 3 km, over the shelf break at Ft. Lauderdale. The translation speed of these features was approximately 30 cm $s^{-1}$, consistent with that of frontal eddies and near-inertial motions, although these vortices are an order of magnitude smaller. They concluded that the vortex they investigated was linked to Florida Current intrusions over the shelf break because it occurred during a period of weak wind conditions.

Two dominant modes of narrowband frequency embedded in the subinertial Florida Current flow were investigated by Peters et al. (2002); a 10 hour (super-tidal) signal with amplitude near 50 cm $s^{-1}$ and an equally strong 27 hour signal (close to the local inertial period). Both signals appeared barotropic in shallow water (50 m depth), but farther offshore (160 m depth) the 10 hr signal exhibited baroclinicity with a phase reversal at
depth. The 10 hr signal ‘leaned against the shear’ (phase trend in the east-west direction), which is consistent with unstable, growing waves that draw energy from the mean flow. Neither signal could be associated with mesoscale meandering, or the near-inertial broadband oscillation observed by Shay et al. (1998). They solve for a linear barotropic instability process using the observed mean conditions, and find that the scale of the instability does not match, but is mostly likely the result of a baroclinic instability further offshore.

![Figure 2.6](image)

**Figure 2.6** (a) Vertical profile of salinity and temperature from cast #1, inset with the location of measurement relative to the Florida Current (b) Vertical profile of density and the buoyancy frequency. Mixed layer depth is approximately 45 m. Density has been smoothed in the vertical and interpolated into 1 m bins. Even after smoothing, step-like features are evident in the profile; that is, alternating regions of high gradient and low gradient layers. These are presumably due to internal wave displacements of the isopycnals.

### 2.9 Internal Waves

A complex internal wave (IW) field is created by the strongly sheared current velocity regime and narrow channel with steep topographic gradients (Winkel et al., 2002).
IWs contribute to the generation and distribution of turbulent mixing and mass transfer to coral reef communities along the Keys (Leichter et al., 2005). Sources of IW energy are the interaction of the barotropic (surface) tide with the along-shelf topography and Florida Current fluctuations (Boudra et al., 1988; Davis et al., 2008). The IW field is particularly energetic in the spring and summer months (Mar-Oct), due to the uplift of the pycnoclines on the western side of the Straits (Leichter et al., 1996). Propagation of frontal eddies along the Florida Keys reef tract has been associated with enhanced high frequency IW energy, and a peak of diurnal-band spectral power in near-shore barotropic currents (Sponaugle et al., 2005; Davis et al., 2008).

2.10 Tidal and Super-Tidal Motions

The M$_2$ tidal amplitude increases with the northward increase of the shelf width from Florida to Georgia (Redfield, 1958). At the latitude of Miami, barotropic (surface) tidal currents are mixed-semidiurnal with small amplitudes of approximately 3-5 cm s$^{-1}$ (Kielmann and Duing, 1974). Mayer et al. (1984) found a linear relationship between sea level and the northward component of the depth-averaged tidal velocity. They develop a 1-D model to predict barotropic tidal currents near 27°N. The predicted diurnal tidal current is dominant and they are able to account for 70% of the variance of the diurnal and semi-diurnal tidal bands in comparisons with depth-averaged current meter observations. Beal et al. (2008) corrected their volume transport calculations for tides. They calculated a maximum tidal correction of 7 cm s$^{-1}$, with a mean of only 0.16 cm s$^{-1}$. The mean of the five year transport timeseries was unaffected by de-tiding.

Soloviev et al. (2003) did a spectral analysis of sea level and found a semi-diurnal peak close to the 12 hour time period. The corresponding peak is not seen in the velocity
spectrum; instead a 10 hour peak is prominent. This 10 hr signal is at the same frequency as observed by Peters et al. (2002) in June-August 1999. Soloviev et al. (2003) show that this signal is seasonally modulated, being observed primarily in the summer months. They also note that this 10 hour signal is modulated over timescales of about 10 days or less. They argue that the proximity of the 10 hour signal to the semidiurnal constituents M2 and S2 suggest that it is a motion excited by tidal rather than inertial forces. Luther et al. (2001) observed this signal with ADCPs. They suggested it could be the semidiurnal internal tide interacting with the continental slope and Doppler-shifted by the mean current. Peters et al. (2002) argue that it is not possible due to linear wave kinematics; in a time-invariant, spatially variable mean current only the intrinsic frequency measured by an observer moving with the current is changed by Doppler shifting. Relative to earth, any wave generated by M2 or S2 tidal frequency keeps that frequency. Non-linear processes could still shift the frequency, but that is not Doppler shifting.

Niiler (1968) showed with an analytical model that a cross-channel baroclinic tidal oscillation could be excited by a barotropic (surface) wave travelling along the non-uniform channel (requires an abrupt change in depth, which the Florida Straits has). Soloviev et al. (2003) used the dimensions of the Straits to calculate the natural period of an internal seiche to be 10 hours. They postulate that the observed 10 hour oscillation may be a near-resonant baroclinic seiche generated by the barotropic tidal wave in the channel between Florida and the Bahamas. This is consistent with the variability of the signal, by the changing of the seasons, as well as the passage of frontal eddies, which affect the upper ocean stratification and thus the internal seiche from summer to winter, and a period of ~10 days, respectively.
2.11 Atmospheric Climatology

Peng et al. (1999) performed a statistical analysis of surface meteorological data from eight C-MAN (Coastal Marine Automated Network) stations in South Florida. They demonstrated that mean wind speed and direction are statistically homogenous and coherent along the Straits of Florida. Figure 2.7 displays the monthly mean wind vectors from the 8 C-MAN stations from April 1994 to April 1995. It shows the dominant easterly wind regime, with a northeasterly component during the wintertime. This suggests onshore Ekman transport and thus downwelling most of the year along the upper Florida Keys, which highlights the importance of eddy events for cross-frontal transport and upwelling of nutrient rich waters for primary production.

Figure 2.7 Monthly mean wind vectors for 8 C-MAN stations (labeled on the x axis). Stations begin at ‘dry’ = Dry Tortugas and are situated along the Straits moving northward until ‘lkw’ = Lake Worth at ~26.5°N. Note ‘spg’ = Settlement Point, at the Bahamian side of the Straits on Grand Bahama. ‘fwy’ = Fowley Rocks, a station situated in our study region. [Source: Peng et al., 1999].
Using wind data from the Fowey Rocks C-MAN station, wind directions and intensities from January to December 2006 (Figure 2.8) are compared with the results of Peng et al. (1999). It highlights the same dominant westward direction as well as a secondary southeast direction. Magnitudes are generally less than 14 ms\(^{-1}\).

These plots indicate the winter/summer wind regimes. In winter, cold frontal passages produce northerly winds with shorter periodicity between 4 to 12 days. In summer, tropical and subtropical depressions generate a mostly stable westward wind regime, with periods from 15 to 30 days (Schott et al., 1988).

**Figure 2.8** Wind rose for Fowey Rocks, using 16° bins, shows the direction and magnitude of hourly wind velocity from January 1, 2006 to December 31, 2006.
Chapter 3

Instrumentation and Data Processing

An overview of HF radar principles of operation, accuracy, limitations and the derivation of vector velocity. An outline of data quality control and interpolation procedures applied to the dataset used in this research.

3.1 High Frequency Radar

High frequency (HF) radar is a shore based electronic system that transmits electromagnetic (EM) waves to remotely observe surface currents, wave height and wind direction. It operates in the radio band between frequencies of 3-30 MHz, or more commonly between 12-25 MHz, which corresponds to wavelengths of 25 to 12 m, respectively. Transmitted electromagnetic waves are backscattered by the rough sea surface and recorded by the receiving antennae. The conductive sea surface guides the radio wave propagation (i.e. trapped ground wave), which allows measurements beyond the horizon with ranges of up to 200 km (Gurgel et al., 1999a,b). At present there are two leading HF radar systems in the United States; Coastal Ocean Dynamics Application Radar (CODAR) and Wellen (wave) Radar (WERA). This work utilizes the latter, and the proceeding explanation of HF radar measurements will be based on the WERA system.
3.1.1 Principles of Operation

The basic physics of backscattering of EM waves from the sea surface was identified by *Crombie* (1955), who observed that the sea echo spectra showed a slight Doppler shift from the transmitted signal. The Doppler shift is the change in frequency (and wavelength) emitted/reflected by an object due to motion. Surface waves with one half of the incident wavelength produce an enhanced backscatter phenomenon known as Bragg scattering (*Figure 3.1*). The energy reflected along the surface of one wave is precisely in phase with another that travelled half a wavelength down and reflected half a wavelength back from the successive wave; this is resonance and amplifies the signal at the receiver (*Teague et al.*, 1997).

![Figure 3.1](image_url) Bragg scattering is an enhanced backscatter phenomenon, with coherent reflection of electromagnetic (EM) waves (thin line) by ocean waves (thick line) with half the incident EM wavelength (top scenario). Incoherent reflection occurs for arbitrary wavelengths (bottom) [Source: *Hilmer*, 2010].

Bragg scattering results in two distinct peaks in the Doppler spectrum, shifted from the transmit frequency by an amount proportional to the deep water phase speed of the Bragg waves, which are known from linear wave theory (*Figure 3.2*). The Doppler frequency shift can be calculated:
\[ f_B = \pm \sqrt{\frac{g f_R}{\pi c}} \]  

(3.1)

where \( g \) is acceleration due to gravity, \( f_R \) is the radar transmit frequency and \( c \) is the speed of light. The presence of an underlying surface current will further shift the Bragg peaks by an amount \( \Delta f \):

\[ \Delta f = \frac{2 V_r f_R}{c} \]  

(3.2)

where \( V_r \) is the radial current along the look direction of the radar. By measuring \( \Delta f \), \( V_r \) can be calculated. At least two radar sites are required to resolve vector current velocities from radial measurements (see Section 3.3).

The effective depth of the measurement depends on the depth of influence of the Bragg waves (Paduan and Graber, 1997), as shown by Stewart and Joy (1974) to be \( d = \lambda/8\pi \). For example, a 16 MHz system will represent the top 0.75 m of the water column. Stable estimates require scattering from hundreds of wave crests plus ensemble averaging of the spectral returns, which sets the time-space resolution limits of the instruments.

The surrounding 2nd order continuous sidebands (Figure 3.2) are generated by multiple scattering and nonlinear hydrodynamic effects. Ocean wave spectra can be obtained from these sidebands by applying inversion techniques, or through empirical calibration with in situ measurements.

Wind is inferred from these Bragg resonant gravity waves that are assumed to be locally generated by the wind field. If the seas are fully developed, the wave field is in equilibrium with the wind and the wave distribution to the wind direction can be modeled. Wave direction is estimated from the relative amplitudes of the two first-order Bragg peaks, the ‘Bragg line ratio’. Waves travelling towards or away from the radar will produce a
large Bragg line ratio (due to predominantly approaching waves and almost no receding waves or vice versa) and waves travelling across the radar look direction will produce a small ratio. If the form of the waves’ directional spreading function is known, the angle of the wind from a radar radial can be determined (Harlan and Georges, 1994).

![Figure 3.2](image)

**Figure 3.2** HF radar Doppler spectrum normalized by the maximum power, showing prominent Bragg peaks due to waves advancing toward and receding from the radar. The additional frequency shift ($\Delta f$) from the 2 vertical red lines is due to the underlying ocean currents. [Edited from Shay et al., 2007]

### 3.1.2 Range Mapping

Range resolution is restricted due to two requirements: scattering has to extend over a large number of EM wavelengths to allow the resonant backscatter from the Bragg waves, and radio interference must be minimized by transmitting signals with narrow bandwidth (Gurgel *et al.*, 1999a,b). Resolution decreases with decreasing frequency (increasing range); thus a compromise must be made between range resolution and the working range.
The transmit signal is linearly chirped upward. The received signal is a broad spectrum, with echoes at frequencies close to the transmit signal for nearby targets and at increasingly lower frequencies as the target distance increases. This technique is called frequency modulated continuous wave mode (FMCW, see Gurgel et al., 1999a). The spatial resolution of the radar is determined by the bandwidth of the chirp (physical range resolution \( \Delta x = c/2B \), where \( B \) is the modulation bandwidth in Hz). For example, a typical bandwidth of 150 kHz gives a range resolution of 1 km. The sampling rate is determined by the chirp length. The Fourier transform of the chirp echoes performs range mapping. After this, the data files contain the backscattered signals from a certain range cell, which are a superposition of backscattered waves from different azimuthal directions at each single antenna.

Note that due to the continuous transmission of the signal, inevitably the receive antennas will be listening for sea echoes whilst the transmitter is active. In order to mitigate the transmit noise at the receive antennas, physical separation between transmit and receive arrays is required. Furthermore, it is ensured that the transmit antenna configuration is such that the receive array lies in a null of the transmit pattern, while the signal is enhanced forward.

The range depends on attenuation of the EM waves (usually ~ 1 to 3 dB per range cell). An empirical relationship for ground wave radar is range \( D = k / f_R \) where \( k \) is a constant \((1.8\times10^{12} \text{ m s}^{-1})\). For a 16 MHz system the range is ~110 km. However, it is subject to relatively large variations; range is hindered by low sea-water conductivity and high sea states (Gurgel et al., 1999b). Background interference from industrial noise and broadcast radio is affected by the ionosphere, which has a strong diurnal cycle, peaking in
the early afternoon (stronger reflections lead to more interference). Thus the working range rarely approaches its maximum.

3.1.3 Azimuthal Resolution

WERA can be configured in either direction finding (same as CODAR) or beamforming mode. Our radar system is configured in beamforming mode, and it is that which shall be discussed here.

Beamforming is a signal processing technique for directional signal reception. Information from different antennas is combined in a way where the expected pattern of radiation is preferentially observed. The time delay in receiving the signal is equal to a phase shift. The array of antennas, each one shifted a slightly different amount, is called a phased array. The signal from each antenna may be amplified by a different weight (weights are designated during calibration). A main lobe is produced, together with nulls and sidelobes. This method has several key advantages. It allows the beam to be steered to achieve particular area coverage, such as localized around a buoy, or spread over a wide area (as well as forming nulls to block interfering radio systems). It also increases the antenna gain, which in turn increases the signal-to-noise ratio and allows WERA to resolve the ocean wave field from 2\textsuperscript{nd} order returns (Gurgel et al., 1999b).

For WERA, a linear array of receive antennas are typically spaced $0.5\lambda$ (transmit wavelength $\lambda = c/f_R$). Spacing is used to create a synthetic aperture antenna; the longer the array the narrower the beam (higher the azimuthal resolution). Beam width in radians scale as $2/(n - 1)$; thus for 16 antennas about $7.5^\circ$, which for a 16 MHz system will take up 150m length of the coast. Steerage of the beam is done by introducing variable phase delays in individual antenna channels. This can be done in hardware or in software.
WERA uses digital beamforming, since beams can be calculated to any shape, and even formed after the experiment.

3.2 Radial Current Accuracy

After the Fourier transform, the backscattered signals are contained within range cells, but are a superposition of backscattered waves from different azimuthal directions at each antenna. Beamforming is performed when the signal-to-noise ratio (SNR) ≥ 6 decibels (dB). Radial currents are calculated from the frequency shift of the Bragg peaks. For $K$ samples of radial current velocity $V_r(i)$ derived from the area surrounding the Bragg peaks, with a backscatter power $P(i)$ and a signal-to-noise ratio $SNR(i)$. The Doppler spectrum consists of $N_{fft}$ spectral lines. In the case of beamforming, these $K$ samples are taken from the area $\pm N_{fft}/64$ around the two first order peaks, so that $K = N_{fft}/16+2$. The average radial current for this sampling period is weighted by the SNR, which gives preference to strong echoes and provides a more stable estimate of $\bar{V}_r$:

$$\bar{V}_r = \frac{\sum_{i=1}^{K} V_r(i) \cdot SNR(i)}{\sum_{i=1}^{K} SNR(i)}$$

(3.3)

The variance $\sigma_r^2$ is calculated as:

$$\sigma_r^2 = \frac{\sum_{i=1}^{K} V_r^2(i) \cdot SNR(i)}{\sum_{i=1}^{K} SNR(i)} - \bar{V}_r^2$$

(3.4)

The accuracy $V_{acc}$ is calculated as:

$$V_{acc} = \frac{\sigma_r^2}{\sqrt{K}}$$

(3.5)

The backscatter power $V_{pwr}$ is calculated as:

$$V_{pwr} = \sum_{i=1}^{K} P(i)$$

(3.6)
The accuracy of the radial currents is estimated by accounting for the signal strength as well as horizontal variations (i.e. shear) within each grid cell for each sample interval. The magnitude of the radial accuracy for two radar sites is given by the sum of squares \((\sigma^2 + \sigma_{\pi}^2)\) time averaged over the domain. Figure 3.6b displays the radial accuracy form September 2004 to June 2005 within the footprint that contains > 70% good data. Within this area the radial accuracy values ranged from 3 to 5 cm s\(^{-1}\). As expected, accuracy decreases with range due to reduced signal strength.

Shay et al. (1995; 1998) and Parks et al. (2009) note that broad Bragg peaks will exist in the spectra when observing western boundary currents with high horizontal shears (since a grid cell may contain changing radial velocities, and hence frequency shifts). Within a grid cell (i.e. 1.2 km\(^2\)) the horizontal shear will be averaged and small scale variability lost.

### 3.3 Vector Calculation

Radial velocities are arranged on a polar grid (range and azimuth, see Figure 3.3a). The most common coordinate system used with HF radar is a Cartesian grid (x, y) because it is more straightforward for analysis. The WERA software provides two options for radial current output: (1) power spectra are interpolated from the polar grid to a user-defined Cartesian grid (using four spectra from the two closest ranges and angles, weighted by distance to the Cartesian grid point), then radial currents calculated; (2) radial currents are calculated on the polar grid. The regular WERA vector calculation uses option (1), and unweighted least squares, to combine radial currents from multiple sites at each grid point.
3.3.1 Unweighted Least Squares (UWLS)

The UWLS method calculates a vector current that minimizes the error between the radial components of the calculated vector and the measured radial velocities. The approach assumes that within the search radius around each grid point is a uniform vector velocity producing the radial velocities. Figure 3.4 displays the geometry to combine two radial components $V_{r1}$ and $V_{r2}$ into the surface current vector $u$. The algorithm used in the WERA HF radar is outlined below (Gurgel, 1994; Barth et al., 2010). An equation to relate each radial current to the $u$ and $v$-component of $u$ is:

\[
\begin{align*}
u \cdot \cos \theta_1 + v \cdot \sin \theta_1 &= V_{r1} \\
u \cdot \cos \theta_2 + v \cdot \sin \theta_2 &= V_{r2}
\end{align*}
\]  

(3.7) (3.8)

For more than two radial components, the matrix form of the above equations is:

\[
\begin{pmatrix}
\cos \theta_1 & \sin \theta_1 \\
\vdots & \vdots \\
\cos \theta_n & \sin \theta_n
\end{pmatrix}
\begin{pmatrix}
u \\
v
\end{pmatrix} =
\begin{pmatrix}
V_{r1} \\
V_{r2}
\end{pmatrix}
\]

(3.9)

Figure 3.3 (left) Polar grid structure for a radar located at point C, with range resolution ($\Delta s$) and azimuthal resolution ($\Delta \theta$). A radar cell is the intersection of a range bin (annulus with thick curves) with an azimuthal bin (dotted lines) [Source: Kim et al., 2008]. (right) The WERA polar grids for Key Biscayne (red) and north Key Largo (blue). From the spatial overlap of these grids, a Cartesian grid is defined, and the radial velocities within a search radius of each Cartesian grid point are used in the combination.
For $n > 2$ the system is overdetermined and can be solved as a linear least-squares problem. In this method, the total error $\varepsilon^2$ is minimized using the $n$ radial components $V_{rj}$ ($j = 1, \ldots, n$) and their error variance $\sigma_{rj}^2$:

$$\varepsilon^2 = \sum_{j=1}^{n} \frac{1}{\sigma_{rj}^2} \left( \cos \theta_j u + \sin \theta_j v - V_{rj} \right)^2$$  \hspace{1cm} (3.10)$$

The matrix form of this error vector is:

$$\varepsilon^2 = |Au - r|^2 \Rightarrow \text{Minimum}$$  \hspace{1cm} (3.11)$$

The WERA software inversely weights the radial velocities by their error variance. Thus, $A$ and $r$ are:

$$A = \begin{pmatrix} \frac{\cos \theta_1}{\sigma_{r1}} & \frac{\sin \theta_1}{\sigma_{r1}} \\ \vdots & \vdots \\ \frac{\cos \theta_n}{\sigma_{rn}} & \frac{\sin \theta_n}{\sigma_{rn}} \end{pmatrix}$$  \hspace{1cm} (3.12)$$

**Figure 3.4** Geometry of the surface current vector from two radial components $V_{r1}$ and $V_{r2}$ [Edited from Barth et al., 2010]
\[ r = \begin{pmatrix} V_{r1} \\ \sigma_{r1} \\ \vdots \\ V_{r2} \\ \sigma_{rn} \end{pmatrix} \]  

(3.13)

The form of the least-squares solution, which solves for \( u \) is:

\[ (A^T A)u = A^T r \]  

(3.14)

The variance of the \( u \) and \( v \)-components (\( \sigma_u^2, \sigma_v^2 \)) is given by the diagonal elements of the inverse covariance matrix \( C \):

\[ C^{-1} = (A^T A)^{-1} \]  

(3.15)

\[ \sigma_u^2 = c_{11} ; \quad \sigma_v^2 = c_{22} \]  

(3.16)

### 3.3.2 Geometric Dilution of Precision

The variances (Equations 3.15 and 3.16) comprise the influence of geometry, termed the geometric dilution of precision (GDOP), which is based on the angle of intersection between the radials from each site. For these radar deployments the acceptable angles of intersection are between \( 30^\circ < \alpha < 150^\circ \). Chapman et al. (1997) defines the GDOP, using the radars mean look direction (\( \alpha \)) and the half angle (\( \theta \)) between intersecting beams:

\[ \sigma_u = \left[ 2 \left( \frac{\cos^2(\alpha) \sin^2(\theta) + \sin^2(\alpha) \cos^2(\theta)}{\sin^2(2\theta)} \right) \right]^{1/2} \sigma_r, \]

\[ \sigma_v = \left[ 2 \left( \frac{\sin^2(\alpha) \sin^2(\theta) + \cos^2(\alpha) \cos^2(\theta)}{\sin^2(2\theta)} \right) \right]^{1/2} \sigma_r, \]

(3.17)

where \( \sigma_r \) represents the measurement error, and it is assumed the noise in each radial measurement is equal. The GDOP is defined as the ratios \( \sigma_u/\sigma_r \) and \( \sigma_v/\sigma_r \) for the \( u \) and \( v \)-components, respectively. The GDOP can be thought of as a multiplier of the noise that is associated with the geometry of the HF radar measurement. Figure 3.5 displays GDOP...
values for the Key Biscayne and north Key Largo sites. The GDOP range is from 1 to 2.5 in the radar domain. In the center of the domain, where intersecting angles are mostly orthogonal, GDOP values are less than 1.5. Towards the periphery of the domain, where intersecting angles are more ‘in-line’ with one another, the GDOP is larger. Note that the GDOP does not quantify the signal-to-noise ratio that may also degrade the signal in the furthest range bins.

![Figure 3.5 Geometric Dilution of Precision (GDOP) over the WERA domain. (left) Green and blue lines indicate u and v-components of GDOP dictated by the geometric limitations of the radial intersections for the two sites, Key Biscayne and north Key Largo. (right) As before, but displaying the total GDOP at each grid point.](image)

3.4 Experimental Design

3.4.1 WERA Set-Up

Since 2004, as part of the Office of Naval Research (ONR) sponsored SouthEast Atlantic Coastal Ocean Observing System (SEACOOS), four HF WERA radar systems have been deployed and operated by the University of Miami’s Rosenstiel School of Marine and Atmospheric Science, along the East Florida Shelf.

This study utilizes data from two 16.045 MHz WERA radars, deployed in June 2004 and located at Crandon Park on Key Biscayne (25° 42.84’N, 80° 9.06’W) and north
Key Largo (25°14.46′N, 80°18.48′W) in South Florida. Each site comprises 4 transmit antennae (Tx) and 16 receiving antennae (Rx). Figure 3.6 depicts the location and coverage of these sites. Table 3.1 lists the parameters used, and capabilities of these systems when operating at 16 MHz.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Capabilities</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating frequency (MHz)</td>
<td>16.045</td>
<td>Average range (km)</td>
<td>80</td>
</tr>
<tr>
<td>Transmit wavelength (m)</td>
<td>18.7</td>
<td>Range cell resolution (km)</td>
<td>1.2</td>
</tr>
<tr>
<td>Bragg wavelength (m)</td>
<td>9.35</td>
<td>Measurement depth (m)</td>
<td>0.75</td>
</tr>
<tr>
<td>Bragg deep water phase speed (m s⁻¹)</td>
<td>3.8</td>
<td>Sampling interval (min)</td>
<td>4.5</td>
</tr>
<tr>
<td>Bragg frequency shift (Hz)</td>
<td>0.408</td>
<td>Azimuth resolution (°)</td>
<td>7.5</td>
</tr>
<tr>
<td>Chirp length (#)</td>
<td>1024</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chirp duration (s)</td>
<td>0.26</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Modulation bandwidth (KHz)</td>
<td>125</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transmit elements (square array) (#)</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Receive elements (#)</td>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transmitter peak power (W)</td>
<td>30</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 3.4.2 Local Winds

A MARS payload (on-board computer system) fixed to Fowey Rocks C-MAN station (25°35.4′N, 80°6.0′W), just inshore of the radar domain (Figure 3.6), measures wind speed and direction, atmospheric pressure and air and water temperatures every hour (Table 3.2). Following Large and Pond (1981), the wind data is converted to the 10 m reference level. The literature review in Chapter 2 demonstrates the necessity of atmospheric measurements for understanding coastal ocean processes.
**Figure 3.6** (left) Location of WERA deployment in South Florida, with depth in color contours. Black box denotes study area. (right) Mean radial accuracy (cm s\(^{-1}\)) for data coverage > 70% from September 2004 to June 2005. Blue lines represent the 120° swath covered by each site. Yellow star refers to location of Fowey Rocks C-MAN station maintained by National Data Buoy Center that records wind velocity. Yellow square is the location of an ADCP mooring (in 86 m water depth).

### Table 3.2  
Measurement capabilities of the MARS payload.  
[Data source: http://www.ndbc.noaa.gov/station_page.php?station=fwyf1]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
<th>Frequency</th>
<th>Ave. Period</th>
<th>Resolution</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind Dir.</td>
<td>0 to 360</td>
<td>1</td>
<td>2/8 min</td>
<td>1.0 deg</td>
<td>+/- 10 deg</td>
</tr>
<tr>
<td>Wind Speed</td>
<td>0 to 62 m/s</td>
<td>1</td>
<td>2/8 min</td>
<td>0.1 m/s</td>
<td>+/- 1 m/s</td>
</tr>
<tr>
<td>Wind Gust</td>
<td>0 to 82 m/s</td>
<td>1</td>
<td>5 sec</td>
<td>0.1 m/s</td>
<td>+/- 1 m/s</td>
</tr>
<tr>
<td>Air Temp.</td>
<td>-40 to 50 C</td>
<td>1</td>
<td>2/8 min</td>
<td>0.1 C</td>
<td>+/- 1 C</td>
</tr>
<tr>
<td>Pressure</td>
<td>800 to 1100 hPa</td>
<td>1</td>
<td>2/8 min</td>
<td>0.1 hPa</td>
<td>+/- 1 hPa</td>
</tr>
<tr>
<td>SST</td>
<td>-5 to 40 C</td>
<td>1</td>
<td>2/8 min</td>
<td>0.1 C</td>
<td>+/- 1 C</td>
</tr>
<tr>
<td>Wave Height</td>
<td>0 to 35 m</td>
<td>1.71</td>
<td>20 min</td>
<td>0.1 m</td>
<td>+/- 0.2 m</td>
</tr>
<tr>
<td>Wave Period</td>
<td>0 to 30 sec</td>
<td>1.71</td>
<td>20 min</td>
<td>1.0 sec</td>
<td>+/- 1 sec</td>
</tr>
<tr>
<td>Wave Spectra</td>
<td>0 to 99 m(^2)/m/Hz</td>
<td>1.71</td>
<td>20 min</td>
<td>0.01 Hz</td>
<td>N/A</td>
</tr>
<tr>
<td>Wave Dir.</td>
<td>0 to 360</td>
<td>1.71</td>
<td>20 min</td>
<td>0.1 deg</td>
<td>+/- 10 deg</td>
</tr>
</tbody>
</table>

### 3.5  Data Processing

#### 3.5.1  Processing Chain

Radial velocities are calculated from interpolated spectra every 20 min using manufacturer supplied software. Vector velocities are calculated every 20 min, using the
two alternating 20 min radial components, with an unweighted least-squares method (Gurgel et al., 1994). Since the vector calculation introduces an error due to GDOP (Figure 3.5), the quality control and filtering procedures were conducted on the radial velocities to mitigate the propagation of error through the processing chain. This approach yields an improvement over the previous method of quality controlling only at the final vector stage (Figure 3.7).

![Figure 3.7](image.png)

**Figure 3.7** Comparison of a timeseries quality controlled at (i) radial stage (blue), versus (ii) vector stage (red). The same simple quality control procedure was performed on each dataset (remove outliers >3 standard deviations from a running 5 day mean and 1 hour Hanning window the timeseries). Here an example period is presented that highlights the efficacy of quality controlling at the radial stage, which prevents propagation of error through the data processing chain. A number of erroneous spikes in the timeseries are not caught in the vector quality controlled timeseries when compared to the radial quality controlled timeseries.

The full data processing chain – from the raw voltage files recorded at the remote sites to the final quality controlled vectors – is displayed in a flow diagram (Figure 3.8). The final quality control and interpolation of the radial and vector currents is applied separately to the dataset used in each chapter, in line with their objectives, and will be discussed next.
3.5.2 Quality Control and Interpolation

There is a certain amount of subjectivity when it comes to data quality control and interpolation. One must have a philosophy regarding the compromise between interpolating to close gaps and reduce outliers (but smoothing the data and damping the signal), versus applying moderate interpolation to ensure retention of the original signal (but leaving larger gaps and missing some spurious outliers). In this dissertation, the latter approach is adopted with the viewpoint that it is better to retain the original signal, since one of the key advantages of the HF radar dataset is its high spatial and temporal resolution that will allow us to study the ocean currents in closer detail than previously possible. An optimal interpolation scheme to combine radials to vectors, while improving 2-D coverage, was tested (Kim et al., 2007), but did not produce convincing improvement in comparison to the more simplistic unweighted least squares method, which is more efficient to run. However, in some cases, 100% coverage is advantageous to apply certain signal processing
techniques. As such, the dataset used in each chapter undergoes specific processing in line with the objectives set forth for that specific study.

Data coverage for the 2 year period used in this study was excellent, with spatial coverage consistently greater than 70% of the maximum footprint (Figure 3.9). The longest down-time of the radars was a 40 day period in October 2005 after the passage of Hurricane Wilma damaged several of the antennas, otherwise the data gaps were generally less than one or two days. Comparison with an in situ ADCP measuring subsurface currents at a depth of 14 m exhibited root mean square differences ∼20 cm s⁻¹, consistent with previous HF radar comparisons in the Straits of Florida (Parks et al., 2009; Martinez-Pedraja et al., 2013; Archer et al., 2015b), and reasonable given the large current magnitudes (>175 cm s⁻¹) and sampling differences between the two instruments (Graber et al., 1997).

Chapter 4

Uses a 2 year dataset from January 2005 to December 2006. Radial grid points were filtered with a 3 hour Hanning window. Data points that exceeded 3 standard deviations (STD) from a running 5 day mean, and grid points that exceeded a STD of 50 cm s⁻¹, with less than 15% data coverage, or GDOP >2.5 were removed from the analysis.

Chapter 5

Uses two shorter period datasets: January 2005 and October 2006. The computed current radials were filtered with a 2 hour Hanning window at each grid point. Data points that exceeded 3 STD from a running 5 day mean, and grid points that exceeded a threshold STD of 40 cm s⁻¹, with less than 25% data coverage, or GDOP >2.5 were removed from the analysis. To apply a Fourier filter to the dataset (Section 5.4.2), data was reduced to a
spatial region of 90% temporal coverage, and interpolated using a cubic smoothing spline with smoothing parameter set at 0.995. The data are quality controlled again and any spurious interpolated data points are removed and replaced with linear interpolation.

Chapter 6

Uses a 2 year dataset from January 2005 to December 2006. Radials were filtered with a 1 hour Hanning window at each grid point. Data points >4 STD from the mean, and grid points with less than 90% data coverage, or GDOP>2.25 were removed from the analysis. Data was despiked, by defining a ‘threshold difference’ between two consecutive data points in a timeseries. Interpolation to 100% coverage was performed with a piecewise cubic hermite interpolating polynomial, quality controlled again and any spurious interpolated data points removed and replaced with linear interpolation.

Figure 3.9 For the Chapter 4 processed dataset: (a) Histogram of percent spatial coverage (b) Total spatial coverage; (c) Downtimes; (d) Timeseries of percent spatial coverage normalized by maximum footprint, from January 2005 to December 2006.
Chapter 4

The Florida Current in a Stream Coordinate Frame: Mean Jet Structure and Variability

A description of the mean horizontal structure of the Florida Current velocity field, its meandering and structural variability. A technique is outlined for converting 2-D HF radar data from Eulerian to semi-Lagrangian stream coordinates. The effects of lateral meandering on an Eulerian time-mean field are quantified in relation to eddy kinetic energy. Meandering periodicity, wavelength and phase speed, and jet variables including intensity, width, surface transport and shear are computed and analyzed.

4.1 Overview

In this chapter we utilize a high resolution, long-term HF radar dataset to study the mean Florida Current and its structural variability over a 2 year period, between January 2005 and December 2006. A geographical temporal average that includes the lateral movement of the Florida Current jet is contaminated; it is broader and weaker than the true horizontal structure of the jet at any snapshot in time. This leads to a bias in the calculated mean cross-stream gradients, which will be underestimated. To remove this contamination, we use a coordinate system based on the physical properties of the stream, rather than a fixed geographical frame. In this semi-Lagrangian ‘stream coordinate’ system, the origin
of reference is defined as the core of the jet, and the observed vector velocities are binned by their cross-stream distance and rotated into local downstream and cross-stream components. Using this method to average the data, the effects of meandering are removed, and the remaining variations represent changes in the current structure. The method also provides a direct timeseries of the core shape and position, from which we obtain statistics on the meandering, core intensity, width, surface transport and shear of the jet.

We quantify how the Florida Current’s horizontal structure changes at annual and sub-seasonal timescales. We show for the first time that there is a clear seasonal change in the jet width, with a maximum occurring in boreal summer and a minimum in winter. This is contrary to results downstream that find the Gulf Stream width to be seasonally invariant (Halkin and Rossby, 1985; Rossby and Zhang, 2001). We relate these changes to the submarine cable volume transport measured at 27°N, mean sea level at a nearby tidal station, and the local wind stress forcing.

In the next section we discuss the precedent for using a stream coordinate method, and the method we developed to convert a 2-D dataset to stream coordinates. In section 4.3 we show the results of this conversion method and compare the mean velocity field in the two coordinate frames. The jet core shape and meandering characteristics are detailed in section 4.4. Section 4.5 presents the dominant timescales of variability in the structure of the jet, both seasonally and at higher frequencies, and in section 4.6 we discuss how these variables may be related. The chapter is summarized in section 4.7.

4.2 Stream Coordinate Method

Converting spatial data that encompasses a meandering jet to stream coordinates has been shown to improve the time-mean representation of the jet. One of the first
applications of the stream coordinate method (also called the jet coordinate system) was in
the atmosphere by Krishnamurti (1959), who binned 4 days of rawinsonde observations of
wind velocity into cross-stream distances to the core of the ‘polar-night’ jet stream, to get
an optimal determination of the jet structure. In the ocean, Halkin and Rossby (1985) were
the first to apply the method to in situ observations, using Pegasus profilers at hydrographic
stations to measure velocity in the Gulf Stream. Subsequent studies have applied the
stream coordinate method to various current systems, using mostly moorings and
hydrographic or acoustic Doppler current profiler (ADCP) sections: in the Gulf Stream
(Hogg, 1992; Johns et al., 1995; Rossby and Zhang, 2001), the North Atlantic Current
(Meinen, 2001), the Kuroshio jet (Hall, 1989; Howe, 2009; Waterman et al., 2011), and
the Subantarctic Front (Phillips and Rintoul, 2002; Meinen and Luther, 2003).

In the Gulf Stream at 73°W, Halkin and Rossby (1985) showed that the temperature
and velocity fields were notably ‘stiff’ in stream coordinates; the jet width and magnitude
remained comparatively invariant regardless of the position of the current. Furthermore,
the eddy kinetic energy (EKE) was reduced by two-thirds when reordered into stream
coordinates. This reveals that meandering of the jet produces significant EKE, which
accounts for a large portion of the mesoscale eddy activity. Rossby and Zhang (2001)
discuss the Gulf Stream structure near 70°W from repeat measurements of shipboard
ADCP and XBTs (expendable bathythermograph). They find that 80% of the EKE can be
accounted for by the shift and rotation of the almost-invariant dynamical structure of the
jet. In the Kuroshio jet, Hall (1989) found similar results, with reductions of EKE up to a
50%. Johns et al. (1995) used a 13 mooring array of current meter observations at 68°W
and noted stream coordinate downstream speeds were roughly twice the size of their
geographical counterparts, and had volume transports approximately 30% larger due to the
removal of regional recirculation effects.

These studies applied the stream coordinate method to derive a horizontal 1-D jet
profile in the cross-stream direction. Our HF radar delivers a horizontal 2-D map of current
velocities that cover the jet’s cross-stream and along-stream structure. By developing an
algorithm to work with this 2-D dataset, we are able to better quantify the horizontal
structural variability of the Florida Current offshore of Miami. The only other studies in
the literature using a similar approach are those by Bingham (1992) and Delman et al.
(2015). Bingham (1992) applied the stream coordinate method in two dimensions using
XBT data of Kuroshio temperature between 130°E to 175°E. Working with a very gappy
dataset in space and time, he was able to reconstruct in the mean the formation and
spreading of North Pacific subtropical mode water more clearly in stream coordinates.
More recently (published after our own algorithm development), Delman et al. (2015)
applied a 2-D stream conversion method to an ocean model simulation of the Kuroshio
Extension. Their objective was to remove the contamination of meandering in the jet
average to better quantify eddy-mean flow interactions. They discovered that in the stream
frame the EKE is substantially lower and identified several patterns of eddy forcing that
are attributable to varicose modes of variability.

4.2.1 Conversion from the Geographical to Stream Coordinate Frame

To convert from geographical to stream coordinates, one must define: (1) the new
origin – taken here as the jet core; (2) the jet core’s downstream direction; and (3) the cross-
stream distance of each measurement from the defined origin. Once these three variables
have been determined, the data can be shifted and rotated to the stream coordinate system.
The Florida Current has a nearly meridional orientation, and its meanders in the HF radar domain are restricted by the channel width. Therefore it can be reasonably assumed that the core of the Florida Current can be identified at one grid point in longitude \( x \) for each monotonic step in latitude \( y \), from the south \( (y = 1) \) to the north \( (y = N) \). This method then works iteratively; for each row from \( y = 1 \) to \( y = N \), the core is defined at a point along \( x \). The conversion steps are outlined for a map of velocity vectors at one time step (Figure 4.1):

1) **Identify the jet core** \( x_0(y) \)

   The jet core is defined as the ridge of maximum velocity (white circles in Figure 4.1). For each latitude \( (y) \), the velocity profile \( V(x) = \sqrt{u(x)^2 + v(x)^2} \) is smoothed in longitude by a running 5-point boxcar filter, and the maximum velocity \( \tilde{V}(x_0) \) is identified as the core \( x_0 \). This produces a set of core locations \( x_0(y) \). This jet core profile \( x_0(y) \) is smoothed using a 1-D spline fit to reduce discontinuities.

2) **Determine the core’s downstream direction** \( \theta(y) \)

   The core’s downstream direction is computed from the average direction of the velocity vector at the core \( \pm 3 \) grid points in \( x \). This produces a set of downstream angles \( \theta(y) \) for the set of core locations \( x_0(y) \).

3) **At each core location** \( x_0(y) \), **identify the grid points on the map that lay along a line perpendicular to the core downstream direction** \( \theta(y) \)

   For each core location, identify the set of grid points \( (x, y) \) that lay along a line nearly perpendicular to the core’s downstream orientation \( \theta(y) \) (not perfectly perpendicular since we are restricted to equally spaced points on a Cartesian grid). Figure 4.1a displays the core locations and their associated grid points that lay along the perpendicular line.

4) **For each perpendicular grid point, calculate its distance from the core, and rotate its u-component (v-component) into a cross-stream (downstream) orientation**
Compute the cross-stream distance $r$ (in km) from each perpendicular grid point to its respective core location $x_0(y)$. The new ‘stream’ grid point is a function of cross-stream distance and assigned the core location’s latitude ($y_c$). Rotate the vectors based on the downstream orientation $\theta(y)$:

\[
\begin{align*}
    u_s(r, y_c) &= u(x, y) \cdot \sin(\theta(y)) - v(x, y) \cdot \cos(\theta(y)) \\
    v_s(r, y_c) &= u(x, y) \cdot \cos(\theta(y)) + v(x, y) \cdot \sin(\theta(y))
\end{align*}
\] (4.1)

5) **Re-grid data from the geographical grid (x, y) to the stream coordinate grid (r, $y_c$)**

Bin the grid points according to their cross-stream distance $r$ from the core, and their assigned along-stream location, which is the latitude of their assigned core location ($y_c$). Thus, the latitude is retained for the core grid points, while a grid point perpendicular to the core grid point may have any original latitude or longitude (*Figure 4.1*). Finally, the data is interpolated onto a uniform grid of 1 km spacing.

Note this method does not assign every grid point on the map to a core location, as can be seen in *Figure 4.1b*. This is because not every grid point lays along a line perpendicular to a core location. After converting to the new coordinate system, the data is quality controlled; again, any data points that exceed 3 standard deviations (STD) from a running 5 day mean are removed, and grid points with less than 30% coverage are thrown away.
4.3 Velocity Structure in Geographical and Stream Coordinates

4.3.1 Mean Horizontal Structure

The 2 year mean jet profile in both geographical and stream coordinate systems is shown together with all observations as a 2-D histogram (Figure 4.2). For the geographical mean, the $u$ and $v$-components of velocity are plotted as a function of longitude (513,957 individual profiles over all latitudes/time). In stream coordinates, the cross-stream and downstream velocities are plotted as a function of distance from the core (223,153 individual jet cross-sections). Confidence intervals are based on the student-t distribution, using the effective degrees of freedom $N^* = N\Delta t/2T^*$, where $N$ is the length of the timeseries, $\Delta t$ is the sampling interval, and $T^*$ is the integral timescale (see next section for $T^*$ calculation).
In the stream coordinate frame, the jet profile of the downstream component is more intense (with a peak velocity 26 cm s\(^{-1}\) higher at 162 cm s\(^{-1}\)) and narrower, with more tightly distributed velocities than its geographical counterpart (Figure 4.2a,b). This difference is due to the meandering motion of the jet, which spreads energy in space, resulting in a weaker, broader mean current, with larger variance. Because the geographical average is taken in both time and latitude, the curved shape of the mean jet through the Straits also contributes to this smearing effect – therefore we converted the time-mean geographical field into stream coordinates using the mean path of the jet to define the stream coordinate axis (red profiles in Figure 4.2b,d). The difference between these two profiles isolates the effect of time-dependent meandering on the mean current structure. The time-mean geographical field in stream coordinates is weaker at the core, as expected, but also at the edges – this is counter-intuitive as one would expect the meandering to spread energy from the core to the flanks, thereby producing larger magnitudes along the shear zones. This does not happen because energy is not redistributed between the downstream/cross-stream components for the time-mean geographical field calculation so the variance is lost in the final mean value. That data coverage differs for two profiles could also affect the comparison.

The 2-D means reveal a clear along-stream variability in the jet (Figure 4.3). In geographical coordinates the Florida Current is still turning from a zonal to meridional orientation as it enters the northern Florida Straits. In stream coordinates, the downstream average reveals an acceleration from south to north of approximately 20 cm s\(^{-1}\). This seems to be consistent with mass conservation: from south to north the cross-sectional area of
the channel decreases almost linearly, and is matched by an increase in the core velocity (Figure 4.4).

![Figure 4.2](image)

**Figure 4.2** Longitudinal (for geographical) and cross-jet (for stream) profiles of (a) Geographical component; (b) Stream downstream component; (c) Geographical component; (d) Stream cross-stream component. Error bars denote the 95% confidence interval based on the student-t distribution. Dashed black line denotes one standard deviation from the mean. All cross-stream profiles are included in this average, regardless of latitude within the Straits. In (b) and (d), the red x’s denote the geographical mean converted to a stream coordinate frame for direct comparison with the stream profile.

The cross-stream velocity is zero at the core of the jet (Figure 4.2d), since by definition the jet core is purely downstream. In this frame, the variability that was associated with the component is significantly reduced, which is expected because the coordinate conversion redistributes the velocity into downstream and cross-stream components, where the cross-
stream component is an order of magnitude weaker. This is seen more clearly in the 2-D means (*Figure 4.3*). The southwest region of elevated \( u \)-component in the geographical mean is due to the curvature of the jet, associated with the downstream (rather than the cross-stream) direction. If we again convert the time-mean geographical field to a stream frame (*Figure 4.3b,e*), the redistribution of geographical mean energy is evident, with the downstream component an order of magnitude larger than the cross-stream. The 2-D stream average of cross-stream velocity reveals an interesting spatial pattern, with positive/negative regions upstream/downstream (*Figure 4.3f*). These positive and negative regions imply convergence and divergence, depending on the sign and relative location to the core: in the southern part of the Straits the flow is divergent on the cyclonic (onshore) side of the current and (weakly) convergent on the anticyclonic (offshore side), whereas in the northern part there is convergence on the onshore side and divergence on the offshore side. The same pattern is also shown in the geographical frame, albeit more weakly because it has been masked in the mean due to meandering. This pattern is robust throughout the 2 year period, and appears in any length of time average. *Bower* (1989) investigated patterns of convergence and divergence in Gulf Stream meanders using RAFOS floats in the thermocline and showed the same patterns of divergence in cyclonic meanders as exhibited in our radar data (with the exception of the southern anticyclonic node – in our data it is near zero rather than convergent). This might be explained by the consistent cyclonic curving of the jet at this latitude; as the jet turns from the zonal to the meridional part of the Straits, it nearly always exhibits a trough-like curvature. The magnitude of this cross-stream velocity is an order of magnitude smaller than the downstream velocity.
Figure 4.3  Two-dimensional time averages for: (a) Geographical $v$-component; (b) Geographical $v$-component mean in the stream frame (downstream velocity); (c) Stream downstream component; dashed black line denotes latitude used for meander, intensity, shear and width timeseries; (d) Geographical $u$-component; (e) Geographical $u$-component mean in the stream frame (cross-stream velocity); and (f) Stream cross-stream component.
In the stream coordinate system, more details regarding the cyclonic shear region have been retained, since beforehand the smearing by meanders contaminated the mean shear zone profile. The mean cyclonic shear zone (defined as $dv/dx$ since this is the dominant term) is twice the magnitude of the anticyclonic zone (Figure 4.5), in agreement with previous studies (Webster, 1961; Brooks and Niiler, 1977). Cyclonic shear peaks 10 km from the core, while anticyclonic shear peaks at 20 km. The spatial pattern of shear displays no significant along-stream variability.

Figure 4.4 (left) Bathymetry (meters) of the Straits of Florida. White dashed lines indicate area of interest, used to calculate channel cross-stream area. (right) Latitudinal increase in downstream velocity corresponds to a reduction in the channel cross-sectional area.
4.3.2 Decorrelation Timescales

At any fixed geographic location in the channel, the measured velocity variability will reflect the time scales of the meandering as well as other possible modes of variability of the jet. One might expect there to be a longer decorrelation timescale in the stream coordinate frame because the meandering has been removed, leaving only the structural changes of the jet. The normalized autocovariance function $\rho_{vv}$ is defined:
\[ \rho_{vv}(\tau) = \frac{C_{vv}(\tau)}{\sigma^2} = \frac{1}{\sigma^2(N-k)} \sum_{i=1}^{N-k} [v_i - \bar{v}][v_{i+k} - \bar{v}], \]  

(4.3)

where \( C_{vv} \) is the autocovariance function, normalized by the variance \( \sigma^2 \), \( v \) is the velocity timeseries, \( N \) is the number of data points, \( \tau = \tau_k = k\Delta t \) \((k = 0, \ldots, M)\) is the lag time for \( k \) sampling time increments \( \Delta t \) (Emery and Thomson, 2001). The integral timescale \( T^* \) is defined:

\[ T^* = \frac{\Delta \tau}{2\sigma^2} \sum_{i=0}^{N'} [C_{vv}(\tau_i) + C_{vv}(\tau_{i+1})]. \]  

(4.4)

where \( N' \leq N - 1 \). \( T^* \) provides the dominant correlation timescale of a timeseries. For times longer than \( 2T^* \) the data is decorrelated.

Figure 4.6 Normalized autocovariance function and integral timescales for (a) and (c) Stream coordinate core versus geographical core; and (b) and (d) Cross-stream distances in stream coordinates.

The decorrelation timescale is very similar in both coordinate frames, at approximately 10 days (Figure 4.6c,d). The same result was found previously by Johns et al. (1995), who proposed that the deformation of the frontal structure occurred in association with the meandering, resulting in similar decorrelation timescales for the two
coordinate frames. The decorrelation timescale is also the same at points across the stream
(Figure 4.6b,d), indicating insignificant differences between the core and the two shear
zones.

4.3.3 Eddy Kinetic Energy

The mean eddy kinetic energy is defined \( EKE = 0.5\left( \langle U'^2 \rangle + \langle V'^2 \rangle \right) \), where \( U \) and \( V \) represent either the \( u- \) and \( v- \)components in geographical coordinates, or cross-stream and
downstream velocities in stream coordinates, brackets denote a time average and the primes
denote a deviation from the average. A comparison between the two coordinate frames for
the period January to June 2005 reveals the spatial pattern of EKE is strongly reduced in
the stream coordinate frame (Figure 4.7). In this frame, the largest values of EKE are now
found exclusively along the cyclonic shear zone, with some weaker amplitude increase to
the far west. In the geographical frame of reference there is an obvious smearing effect on
the EKE values, with elevated values over a wider area across the stream, associated with
lateral meandering of the jet.

We calculate the area integral of the mean EKE field, derived in each coordinate
system, and normalized by the number of grid points – i.e. the mean - since the conversion
method does not retain all the data for the stream coordinate frame, a direct comparison of
mean EKE is not possible. Using this estimate, approximately 45% of the mean EKE field
in geographical coordinates can be explained by the lateral meandering of the jet. This
compares well with previous findings in the other major current systems (Johns et al.,
1995; Halkin and Rossby, 1985; Hall, 1989), when considering the meandering amplitude
of the Florida Current is less than the Gulf Stream and Kuroshio.
4.4 Core Position

4.4.1 Axis Shape

The first step of the stream coordinate method as defined in Section 4.2.1 is to identify the jet core – therefore we can easily extract a direct timeseries of the axis shape and offshore position. The shape of the jet core over the 2 year period is presented in Figure 4.8 as a density contour plot (using 4072 individual jet axis profiles), and reveals the mean location of the core lies over the 650 m isobath, which is 44 km offshore of the Florida coastline at this latitude. The distribution of the jet axis over these two years exhibits a longer tail to the east; this is expected since the bathymetry and coastline restricts westward movement of the core. As shown in the 2-D geographical mean (Figure 4.3a), the jet is still turning cyclonically as it rounds the Florida peninsula heading north. Path
curvature ($\kappa = d^2x/dy^2 / (1+(dx/dy)^2)^{3/2}$; not shown) is always positive (0.01 ± 0.001 km$^{-1}$) and has relatively small variance (standard deviation of 0.006 km$^{-2}$).

**4.4.2 Meandering**

Timeseries of the offshore position of the jet axis at 25.42°N (latitude cross-section denoted in Figure 4.3c) as a function of longitude and displacement from a 2 year mean position reveals strong variability at both short and longer periods (Figure 4.9). The maximum offshore meander observed is 40 km east of the mean position, and the westward maximum is 20 km, creating an overall range of 60 km for the core position over the 2 year period. This is significantly larger than previous estimates in this region of 5 km (Schmitz and Richardson, 1968; Leaman et al., 1986). However, the standard deviation from the mean position is 7.9 km, which agrees more closely with the previous findings. Note that our method provides a direct measure of the meandering, whereas previous studies relied on inference of meandering from point measurements.

**Figure 4.8** Density contour plot of the jet axis profile for every individual time that the jet core was identified. The jet profile is defined by points in longitude and latitude.
Figure 4.9 In the geographical coordinate frame the jet core offshore position is plotted as the thick black line. The dashed white line indicates its mean position. The longitudes of isobaths (150, 350, 600, 800 m) are plotted as vertical lines throughout the timeseries. The colored contour denotes the $v$-component velocity at latitude 25.42°N, at each time step from January 2005 to December 2006.

The dominant meander wavelengths and phase speeds can be obtained from calculating the coherence and phase shift between meander timeseries at two separate latitudes, 25 km apart (Figure 4.10). For a negative phase shift the northern timeseries lags the southern, so meanders at all periods are propagating northward. There is a trend toward larger phase shifts with increasing frequency, indicating higher frequencies have smaller wavelengths. Coherence is high for periods above 5 days, and has two peaks at approximately 3 days and 2.5 days. Power spectra (discussed in next section) reveal dominant meander periods near 3 days, 8 days and 30 days. For a 3 day period meander,
the phase shift is 35°, which gives a wavelength (\( \lambda \)) of 250 km and phase speed (\( c \)) of 83 km day\(^{-1} \). For an 8 day meander, the phase shift is 10° (\( \lambda = 900 \) km, \( c = 112 \) km day\(^{-1} \)). A 30 day period meander has a phase shift of 6° (\( \lambda = 1500 \) km, \( c = 50 \) km day\(^{-1} \)). At these larger wavelengths of \( O(10^3) \) km, the core path variability is more akin to a lateral axis shift than a wave-like meandering motion, as described by Lee and Cornillon (1995).

Figure 4.10 Coherence and phase (in degrees) between two timeseries of meandering taken at 25.42°N and 25.64°N. An increasingly negative phase shift with higher frequency implies shortening wavelengths.

4.5 Jet Structure Variability

4.5.1 Definitions

To quantify how the jet is changing structurally, we define and study several jet variables: intensity, width, surface transport and cyclonic/anticyclonic shear indices. Each variable has been filtered with a 40 hour Hanning window in time. Basic statistics of each variable are given in Table 4.1, and their timeseries are shown in Figure 4.11.
Jet intensity is calculated as the core velocity (at 25.42°N, for consistency with meandering) minus the cross-stream average of the downstream velocity. This variable is a crude measure of the larger scale fluctuations in the North Atlantic western boundary current system, with the local channel-wide fluctuations mostly removed. We define the jet ‘edges’ as 50% of the core velocity, which gives us a measure of jet width. The definition is arbitrary, but consistent for the 2 year timeseries (we originally attempted to use the peak values of lateral shear on each side of the jet but this gave estimates that were too noisy). This width criterion differs from some previous studies that used a threshold velocity value (e.g. Ichikawa et al. (2001) used 20 cm s⁻¹ in the Kuroshio), but we believe it provides a more accurate description of the internal jet width variability, by more clearly distinguishing it from the channel-wide fluctuations associated with local wind forcing. This definition results in the core axis being closer to the western edge of the jet than the eastern edge, since the cyclonic shear on the shoreward side of the jet is typically double that on the anticyclonic side (Figure 4.5). On average, the western edge is approximately 20 km from the core, while the eastern edge is approximately 40 km from the core. The fluctuations in these edges contribute equally to width changes, with amplitudes of approximately 5 km. This is the first time width changes have been documented in the Florida Current. An index of cyclonic lateral shear is defined as the median value of $\frac{dv}{dx}$ from the core to 20 km west, and anticyclonic lateral shear is the median value from the core to 40 km east. Surface transport is calculated as the cross-stream area integral of the downstream velocity, using the HF radar effective depth of 0.75 m in the vertical.
Table 4.1 Basic statistics. Confidence levels are based on a student-t distribution and an effective degrees of freedom using the integral timescale in days.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Integral Timescale</th>
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<tr>
<td>Meandering</td>
<td>0 ± 1.7</td>
<td>7.9</td>
<td>-19</td>
<td>42</td>
<td>4</td>
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<tr>
<td>(km)</td>
<td></td>
<td></td>
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<tr>
<td>Intensity</td>
<td>36.6 ± 3.2</td>
<td>10.4</td>
<td>5</td>
<td>96.7</td>
<td>9</td>
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<tr>
<td>(cm s⁻¹)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Width</td>
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<td>6</td>
<td>41</td>
<td>76</td>
<td>16</td>
</tr>
<tr>
<td>(km)</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Surface Transport</td>
<td>6320 ± 356</td>
<td>918</td>
<td>3082</td>
<td>8962</td>
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<td>(m³ s⁻¹)</td>
<td></td>
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<tr>
<td>Cyclonic Shear</td>
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<td>0.07</td>
<td>1.3</td>
<td>6</td>
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<tr>
<td>Anticyclonic Shear</td>
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<td>0.09</td>
<td>-0.7</td>
<td>0</td>
<td>6</td>
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<tr>
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<td></td>
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<tr>
<td>Volume Transport</td>
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<td>20.9</td>
<td>38.7</td>
<td>35</td>
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<tr>
<td>(Sv)</td>
<td></td>
<td></td>
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<td>Meridional Wind</td>
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<tr>
<td>Stress</td>
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<tr>
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<td>0.3</td>
<td>3</td>
</tr>
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<td>(N m⁻²)</td>
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<td>0.4</td>
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</tr>
<tr>
<td>(m)</td>
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In addition to these HF radar-derived jet variables, we include independent measurements of the Florida Current volume transport, local wind stress and mean sea level. The volume transport timeseries is obtained from the submarine cable at 27°N, maintained by NOAA’s Atlantic Oceanographic and Meteorological Laboratory (www.aoml.noaa.gov/phod/floridacurrent/). The wind stress is calculated as \(\tau = \rho_a C_D |U| U\), where \(\rho_a\) is air density, \(C_D\) is the drag coefficient, and \(U\) is the eastward
(northward) component of wind for \( \tau_x \) (\( \tau_y \)), using data from the Fowey Rocks meteorological station available from NOAA’s National Buoy Data Center (http://www.ndbc.noaa.gov/). The mean sea level is obtained from the Virginia Key tide station provided by NOAA’s Currents and Tides (http://tidesandcurrents.noaa.gov/). Information regarding the data collection and processing is available at the websites provided.

Figure 4.11 Timeseries for each variable defined in the text, at 3hr temporal resolution (except volume transport at 1 day), filtered with a 40hr Hanning window. Black dashed line demarcates 2005 and 2006. Grey lines denote occurrence of tropical storms, in order of appearance: Arlene, Dennis, Katrina, Ophelia, Rita, Tammy, Wilma, and Ernesto (2006).
4.5.2 Annual Cycle

To investigate the Florida Current’s structural variability at the annual period, the timeseries of each variable is averaged over individual months for each year and in total (Figure 4.12). Width shows a clear annual cycle with a minimum in January-February and a maximum in July-August-September, and a secondary minima and maxima in November and December, respectively. Surface transport also has a strong annual cycle, with both years exhibiting a maximum in June-July, and minimum in the fall, with a secondary minima and maxima in April-May and March, respectively. Meandering does not exhibit a coherent annual cycle over the 2 year period. Intensity and the cyclonic shear index exhibit very similar annual cycles, with a peak in February-March and a trough in August-September, which are out of phase with the anticyclonic shear index (in that anticyclonic shear becomes more negative as it gets stronger). A relationship between intensity and the shear indices is expected, since an increase in the core intensity with respect to the channel wide flow would by definition generate more shear.

Mean sea level has a maximum in September-October and a minimum in February-March, with secondary minima and maxima in June-July-August and April-May, respectively. This is out of phase with the surface transport annual cycle. The annual cycle of volume transport differs between the two years. The 2005 signal has been highlighted in previous studies – a maximum during summer time and a minimum in October, with secondary minima and maxima in winter. In 2006, however, the signal has a weak annual pattern, as it just fluctuates about 31.5 Sv. If we consider only 2005, the volume transport matches well with surface transport, jet width, and meridional wind stress, and is out of phase with the mean sea level. For 2006 there is no clear relationship between volume transport and these variables.
What drives these annual signals, and the differences between variables? Local meridional wind stress exhibits a consistent annual cycle, with peak positive values in summer (June-July-August) and a minimum negative value in November, weakening in
December. Surface transport and width both comprise the same phase of maximum and minimum in summer and fall, suggesting a local wind forced response in their cycles. Schott et al. (1988) showed that volume transport is influenced by local meridional wind stress; using data from six mooring arrays between 1982 and 1984 and a simple frictional model they found that wind stress accounts for approximately 22% of the seasonal observed amplitude. Surface transport, core intensity and the shear indices strengthen between February and March. It may be due to the seasonal wind stress curl over the North Atlantic (Sverdrup dynamics) that peaks in February at 26°N (Figure 16a of Rousset and Beal, 2011). Indeed, the intensity is defined so as to reduce the local effect of wind forcing, and the resultant annual cycle more closely follows the basin-wide Sverdrup cycle, while surface transport retains the local summer peak in magnitude that is shown in the local wind stress. Another hypothesis is that the inhibitive southward wind stress during January relaxes to near-zero in February–March, allowing a resurgence in the western boundary current flow, and therefore a secondary peak in surface transport supported by this higher core velocity.

Why does the 2006 volume transport cycle differ so much from 2005, unlike surface transport, width and mean sea level? The annual signal of local wind stress forcing is not appreciably different between these two years. Since the data is not sufficiently long enough to study inter-annual fluctuations, we can only speculate that the cause may be due to baroclinicity of the current. Rousset and Beal (2011) showed that the annual cycle of the Florida Current velocity structure is dominated by a first baroclinic mode-type response across the Straits, largely in the top 150 m. The volume transport is an integral value, which encompasses a wide range of processes that may not be exhibited in the surface
transport and width measured by HF radar. For example, Czeschel et al. (2012) found that barotropic shelf waves were the primary source of seasonal volume transport variability. A barotropic response may not be so well exhibited at the surface layer if superimposed on a baroclinic current. In addition, there is a one degree separation in latitude between the radar and submarine cable, which can contribute to the differences because of the additional input of water mass from the Northwest Providence Channel, just north of the radar domain (see Figure 2.1). This channel has recently been shown to act a conduit for the transmission of eddy variability from the interior Atlantic basin (Frajka-Williams et al., 2013; Domingues, Baringer and Goni, “Remote forcing of the Florida Current on Seasonal Time-scales”, in preparation). The variability of transport at the annual period is therefore more complicated than a simple cause and effect of one variable, as it is an integrated value that encompasses a wide range of dynamical forcing. To elucidate the driving mechanisms for these variables at the annual timescale requires a longer timeseries.

4.5.3 Sub-Seasonal Periodicity

Higher frequency fluctuations of these variables are investigated to assess how they vary on timescales of days to weeks, and whether the amplitude and frequency of their variability exhibits a seasonality. Variance-preserving power spectra are calculated using Welch’s averaged periodogram method, with five 58 day half-overlapping segments windowed with Hanning weights in the time domain, Fourier transformed, and ensemble averaged. Spectra for summer and winter seasons from both years are calculated, and a seasonal mean is obtained by averaging the two years over bandwidth intervals of 0.025 day$^{-1}$. Summer is defined as May to October and winter is November to April, defined using the wind climatology by Peng et al. (1999).
**Figure 4.13** Variance-preserving spectra for the different variables under study, for summer (May-October) and winter (November-April) in 2005 and 2006, and the seasonal average — individual spectra are averaged over bandwidth to obtain seasonal average. Wind stress observations (meridional $y$ and zonal $x$) for summer 2005 were obtained from Sombrero Key C-MAN station (http://www.ndbc.noaa.gov/station_page.php?station=smkf1) nearby, because Fowey Rocks had a large gap in the timeseries.

All HF radar-derived variables exhibit sub-seasonal variance dominated by the 3 day to 3 week period band (**Figure 4.13**), with variability tailing off at longer periods (even considering the inherent bias of variance-preserving spectra). There is a clear seasonality in distribution of variance across frequency. The winter season has significantly elevated
variance at higher frequencies compared to the summertime (except for width), with a 7-10 day peak that dominates the spectral curve. Meandering and intensity also have a ~3 day spectral peak, which is again more dominant during the winter. During the summer, variance is spread more evenly across frequencies, and with greater variance at the lower frequencies than winter (>20 days), except for surface transport. This seasonality is presumably a function of local wind forcing. During summertime, wind is predominantly easterly, due to subtropical depressions with periods of 15 to 30 days. During wintertime, cold front passages bring northerly winds with periods of 4 to 12 days, which provides an energy input to the ocean with this shorter periodicity. The 3 day peaks seen in the meandering and intensity may also be due to the passage of frontal eddies that are associated with the troughs of meanders and have been shown to pass by the radar domain at periods of 1 to 3 days (*Archer et al.*, 2015a)

The mean seasonal wind stress spectra do not show strong seasonality between the winter and summer, as would be expected, because 2005 was one of the most active hurricane seasons in history, so these high speed wind events have biased the mean summer spectral shape. Seven named storms passed by south Florida, including Hurricanes Katrina, Rita and Wilma (*Figure 4.11*). This is seen in the 2005 summer spectra of meridional and zonal wind stress, both of which have the highest variance over all seasons studied. The 2006 hurricane season was relatively docile, with only one tropical storm (Ernesto) passing over south Florida, and this is displayed in the 2006 summer wind stress spectra, especially the meridional component that is an order of magnitude smaller in variance, and more characteristic of the summer reduction in wind energy compared to winter.
Volume transport variability is most energetic between 10 to 40 days. There is a seasonal shift toward lower frequency variability in summertime than winter – but as we discuss below – whether this can be attributed to seasonal or inter-annual variability is not clear. The HF radar-derived surface transport reveals similar peaks to the volume transport at these lower frequencies, although at the surface the dominant variability is in the high frequency 5 to 10 day weather band. Mean sea level also shows lower frequency variability with the largest peak in the 10 to 20 (20 to 30) day range for winter (summer) and a smaller peak at 5 (8) days. The seasonal difference is more robust for mean sea level than volume transport, exhibited consistently in both years.

All variables exhibit strong inter-annual variability. For wind stress, as discussed above, the summer 2005 season had the most energetic fluctuations, while winter 2006 slightly exceeded 2005. For width and volume transport, the winter and summer of 2005 showed the most variance. We speculate this could be connected to the larger 2005 variance in sea surface height anomaly east of the Bahamas as shown by Frajka-Williams et al. (2013). For meandering, intensity and surface transport, the winter 2005 dominated the spectra, so much so that the large seasonal differences in these variables (winter amplification) can be mostly attributed to this one season. While this energetic winter 2005 was seen in both the HF radar-derived variables and the volume transport, the wind stress was no greater than the winter 2006 season, suggesting other mechanisms involved for the increased variance. It is therefore difficult to ascribe particular characteristics to the seasons with confidence, because we only have a 2 year dataset that exhibits strong inter-annual variability. Seasonality aside, the dominant periods of variability we find here for meandering and volume transport corroborate previous studies in the Straits of Florida (Lee
and Mayer, 1977; Johns and Schott, 1987; Zantopp et al., 1987; Schott et al., 1988), which is encouraging given the different observational methods employed. Previous studies worked with point measurements from current meter moorings while we use the HF radar-derived timeseries of the axis position. This implies that the meandering motion dominates the velocity records at the current meter moorings, allowing previous researchers to extract meandering motion without a direct timeseries of core location.

4.6 Discussion

4.6.1 Investigating Relationships between Variables

All significant correlations at the 99% confidence level (Emery and Thomson, 2001) are presented in Table 4.2. We calculated each correlation with and without (shown here) the seasonal cycles removed and found only small magnitude changes (some increased, some decreased). With a few exceptions, there are no strong correlations between any of the variables; this is presumably due to the complex nature of the interactions. We performed a coherency analysis in the frequency domain between every variable, and while we found several significant frequencies of correlation for many of these comparisons in each year (we had to split the timeseries up due to the data gap caused by Hurricane Wilma), there was no convincing consistency in the frequencies between the two years. We suspect this is in part due to the random nature of the forcing at periods between 3 days to 3 weeks (wind events and frontal instabilities), as has been found by previous studies, and means the analysis of different time periods leads to slightly different results. Wavelet coherence between the variables confirmed this non-stationarity in time, both in magnitude and phase. But otherwise, no extra insight was gained, so in this paper
we restrict our presentation of the relationships between variables to simple correlation values.

Table 4.2 Correlations between the different variables at the 99% confidence level, based on a normal distribution and using effective degrees of freedom (Equation 3.15.11a from Emery and Thomson, 2001). $r$ is the correlation coefficient, $r_0$ is the null hypothesis and $N_f$ is the effective degrees of freedom. The bold line divides the HF radar-derived variables from the ancillary observations.

<table>
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<tr>
<th>$r$</th>
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4.6.2 Correlation between HF Radar-derived Variables

The strongest correlations (0.5 to 0.8) are found between intensity, width and shear indices. This is an expected result, explained both by ocean dynamics and how we defined the variables (definitions in Section 4.5.1). For example, jet width is intrinsically linked to lateral shear – it must be anti-correlated (note that anticyclonic shear is a negative value so being out of phase implies a positive correlation, found here). This is due to the definition of width using a ratio (0.5) of velocity between the core and ‘edge’; thus, for an increase in width, the shear must decrease. Meandering only correlates significantly, albeit
weakly, with cyclonic shear (-0.3), indicating that as the jet axis moves shoreward, shear increases. The absence of correlation between these variables also provides insight – for example, surface transport is not significantly correlated with the core intensity, indicating that the channel wide flow is more important for transport changes than solely the high intensity core variability. Likewise, that meandering does not correlate with most variables implies that the position of the jet in the channel does not have a large effect on its surface structure, contrary to the suggestion by Johns et al. (1995) that jet structure co-varies with meandering.

4.6.3 Correlation between HF Radar and Ancillary Observations

The most interesting correlations are between the HF radar-derived variables and independent observations of volume transport, wind stress and mean sea level. A correlation of $r = 0.48$ between surface and cable transport indicates that 23\% ($r^2$) of the volume transport variability at 27°N can be explained by surface fluctuations at 25.42°N. Schott et al. (1986) compared HF radar northward velocities averaged zonally at 27°N to cable transport and found a correlation of 0.85, with significant coherence above 5 days. Their higher values are most likely explained by their radar domain being located at the same latitude as the cable measurements, therefore containing the same signal input from the Northwest Providence Channel. Surface transport (volume transport) is correlated at -0.46 (-0.31) with mean sea level at Virginia Key FL, suggestive of a steeper gradient across the channel during times of stronger velocities (sea level lower on western side of channel), confirming geostrophic balance.

Surface transport also correlates well with local meridional wind stress (0.46). A northward wind drives an eastward Ekman transport, leading to divergence (convergence)
and upwelling (downwelling) at the western (eastern) boundary, therefore steepening the east-west tilt of the pycnocline and enhancing northward geostrophic flow (Rousset and Beal, 2011). However, meridional wind stress does not correlate significantly with mean sea level at the western station, which would be anticipated for such an Ekman-produced sea level response. This may be due to the strong relative vorticity of the Florida Current that interacts with the wind stress and affects Ekman divergence, complicating the scenario (Niiler, 1969). Besides a wind-induced geostrophic response, the other mechanism to explain this surface transport correlation is a directly-forced surface Ekman flow, in which wind-forced currents theoretically move 45° to the right of the wind (in the N. hemisphere). A coherence plot showing meridional wind stress versus both HF radar-derived surface transport and cable volume transport reveals that both mechanisms may be occurring at the surface (Figure 4.14). Johns and Schott (1987) found that volume transport changes lagged wind stress by approximately 1 day, with three significantly coherent period bands (phase shifts) at 20 days (20°), 8 days (45°) and 3.5 days (80-90°). For 2005 we find similar results for cable volume transport within reasonable error bounds – 3 significant periods of coherence at 40 days (5° phase lag), 6.5 days (35°) and 4 days (80°). These phase shifts imply a lag period between half to two thirds of a day. The surface transport is also significantly coherent with wind stress at these same periods, and for the two peaks at 40 days and 6.5 days has the same phase shift as shown by volume transport, suggestive of wind-induced geostrophic response (Figure 4.14). However, at the shorter periods (<4 days), the phase shifting is reduced, indicating a much shorter lag between the wind and surface currents, which could support the second mechanism for correlation (that is,
directly-forced surface Ekman flow). Refer to the appendix for a comparison between our observations and the theoretical Ekman response.

The width exhibits a weak but significant positive correlation with meridional wind stress, which may be partially attributed to our definition; for example, a uniform increase in northward surface flow across the channel will by definition slightly increase the width (other factors assumed constant), because the jet edges are defined as half the maximum velocity – thus, a uniform increase velocity moves the edges slightly further away from one another (by half the uniform increase).

As was found for sub-seasonal periodicity, there is a strong inter-annual variability between the two years seen in the correlation values, with elevated (reduced) correlations in 2005 (2006). For example, surface transport correlation with meridional wind stress is 0.55 (0.43) for 2005 (2006). Width and volume transport become significantly correlated (0.47) if only calculated in 2005.

4.6.4 Conditional Averaging

By averaging the data based on threshold conditions, we can more intuitively display the effect of these conditions on the mean jet profile. In the following results (Figure 4.15) we have divided the timeseries into three parts and averaged, based on small, intermediate, and large values of surface transport, intensity, width, volume transport and meridional wind stress (wind stress is separated into negative, near zero, and positive). Surface transport equates to a channel-wide increase in downstream velocity, with no change in lateral shear (Figure 4.15). Intensity describes the core velocity fluctuations with the channel wide flow signal removed, and by definition affects the shear (maximum values in intensity also have maximum values in shear). The width of the jet is clearly not
strongly related to the core velocity, but does correlate (as shown above) with shear. Both volume transport and meridional wind stress are associated with a uniform increase in velocity across the channel, but no change in shear. This indicates that core intensity variations do not play a dominant role in the volume transport compared to the full channel-wide velocity, which may explain why the annual cycle of volume transport does not contain the February-March secondary peak that is exhibited by intensity (Figure 4.12).

**Figure 4.14** Coherence between meridional wind stress and (blue) HF radar-derived surface transport; or (red) Cable volume transport, for 2005. Grey bands denote significant coherence peaks shared by both HF radar and cable timeseries with the wind stress. Black circles highlight periods when phase shifting was the same, and the black arrow highlights the phase shift difference at the 4 day period band.
Figure 4.15 In stream coordinates, the mean cross-section of (top) downstream velocity and (bottom) lateral shear conditionally averaged for time periods isolated using small, intermediate and large values of intensity, width and volume transport. For meridional wind stress $\tau_y$, the timeseries is separated into three groups: negative, weakly negative/positive, and positive.
4.7 Summary

The mean horizontal profile of the Florida Current has been mapped in 2-D using HF radar data between January 2005 and December 2006. By converting from the traditional Eulerian geographical coordinate frame to a semi-Lagrangian stream frame, an improved average has been obtained, since we removed the jet path variability from biasing the mean jet profile. In the stream frame, the time-mean is more representative of the instantaneous jet, which is stronger and narrower than in the geographical field. By improving the time-mean calculation of the Florida Current, eddy-mean flow interaction terms in the energy and vorticity equations can be more accurately investigated.

The stream conversion method provides a direct timeseries of the core’s offshore position. This has allowed us to investigate the period, wavelength, phase speed and amplitude of lateral meandering, which is responsible for approximately 45% of the mean eddy kinetic energy. While meandering exhibits strong inter-annual variability, it is dominated by high frequency fluctuations in the 3-10 day period band, and to a lesser extent, at the 30 day period. The meander range over the 2 year period was 60 km, with a standard deviation of 8 km. Phase speeds ranged between 50 to 112 km day$^{-1}$, and wavelengths from 250 to 1500 km. These longer wavelengths exceed the length of the Straits by an order of magnitude, and imply a shifting of the core axis rather than a wave-like meandering motion (Lee and Cornillon, 1995).

Jet structure variability has been quantified using the following metrics: jet intensity, width, surface transport and cyclonic/anticyclonic shear indices. These variables exhibited consistent annual cycles for 2005 and 2006. Width is shown for the first time to have a strong annual cycle with a maximum in summer and a minimum in winter. The
summer maximum exhibited by width and surface transport corresponds to a peak in the local meridional wind stress. The surface transport timeseries is also coherent with meridional wind stress at shorter periods of 40, 6.5 and 4 days, with phase lags indicative of two mechanisms linking these variables: (1) an Ekman-driven eastward transport that tilts the pycnocline and increases the geostrophic current; and (2) wind stress forcing at the surface generating flow with near-zero lag at 45° to the wind direction. The data also reveals a late winter maximum in core intensity and shear, with a secondary peak in surface transport, following the width minimum. This is an interesting discovery, which requires more data to be explained. For now, we present two hypotheses: (1) it is driven by the wind stress curl in the North Atlantic (Sverdrup dynamics), which also peaks around this time (Rousset and Beal, 2011) – a strengthened western boundary current system leads to increased core velocity but not necessarily width, since a width increase requires a channel-wide flow increase that is associated more with local wind forcing; and (2) the local meridional wind stress relaxes to near-zero at this time, after a period of strong southward wind stress that is inhibitive to northward ocean currents – we speculate that such a relaxation in wind stress could allow a ‘resurgence’ in the western boundary current flow, which would correspond to an increased core velocity and therefore shear across the channel.

At the sub-seasonal timescales, the most energetic periods for all HF radar-derived variables are 3 days, 7 to 10 days and 3 weeks, while volume transport and mean sea level exhibit lower frequency peaks on the order of 10 to 40 days. These fluctuations are non-stationary in time and frequency, because they are forced by randomly timed transient phenomena: local wind stress forcing (3 to 10 days), local frontal instabilities of the Florida
Current (3 days), and internal ocean variability (30 days). While we found there is a seasonality in the signals, there is also a strong inter-annual variability between 2005 and 2006, making seasonal patterns difficult to distinguish without a longer timeseries to analyze.

These high resolution results provide a useful benchmark for the evaluation of numerical models, as well as the National Weather Service estimates of the Florida Current daily frontal position based on the Naval Oceanographic Office Gulf Stream analysis. Knowledge of how this jet changes seasonally is of particular interest to ocean users, including the U.S. Coast Guard and local fishermen. By developing the stream coordinate method to work with 2-D HF radar maps, we have improved the time averaged representation of the Florida Current. This method can be applied to the continuing HF radar dataset to begin assessing whether the Florida Current is changing at a longer timescale relevant to the climate.
Chapter 5

Cyclonic and Anticyclonic Frontal Instability of the Florida Current: Two Case Studies

In the first case study, the kinematic properties of a cyclonic vortex are investigated, and the terms in the velocity gradient tensor calculated and compared to a period of no eddy activity. In the second case study, a near-inertial signal on the anticyclonic flank is investigated, including its dominant frequency, wavelength and phase speed. Does the signal conform to near-inertial wave dynamics, or to vortex dynamics? How does the background flow alter the signal in the total velocity vector field?

5.1 Overview

This chapter documents new observations of frontal instabilities of the Florida Current using HF radar. These two case studies demonstrate the power of HF radar for coastal ocean observing. In the first case, a study of a submesoscale frontal eddy in the cyclonic shear-zone is presented. The emphasis is on the ability of HF radar to provide new insight into spatial variability of these features, using the 2-D velocity field and its derivatives to investigate their kinematics. Understanding the flow field provides insight into particle dispersion, which if known could help in search and rescue (SAR) operations
and pollution mitigation. These eddies also contribute to cross-shelf exchange of mass and nutrients, which has implications for biological productivity along the Florida Keys and South Florida coastlines (e.g. Lee et al., 1992).

### 5.1.1 Open Scientific Questions

In a numerical study of the Gulf Stream along the South Atlantic Bight, Xue and Bane (1997) investigated frontal instabilities either side of the jet core. They note clockwise rotation on the offshore side of the meander crests. Fiechter and Mooers (2003) modeled Florida Current instabilities, and whilst their focus was on cyclonic eddies (like all studies in the Straits of Florida), they noted the presence of frontal instabilities in the eastern shear zone.

Observations from moored current meters through the Straits have hinted at anticyclonic shear zone instabilities (Lee et al., 1995; Leaman et al., 1995). When the Florida Current is in an offshore meander over the Pourtales Terrace, it interacts with the Cay Sal Bank (Leaman et al., 1995). Using SST imagery, Lee et al. (1995) observed an offshore meander that was partially diverted clockwise around the Cay Sal Bank and into the Santaren Channel, which set up a cyclonic rotation. These results suggest that eddies could be formed upstream because of instability of the meandering jet impinging on the steep shelf break of the Cay Sal Bank. This interaction may form either anticyclonic (directly) or cyclonic (indirectly through the Santaren Channel) circulations. Another mechanism could be wind stress perturbations on the laterally sheared jet, which because of its unstable nature can encourage fast-growing modes (Lee and Mayer, 1977).

Anticyclonic shear-zone instabilities in the Straits of Florida have not received attention in the research literature, thus very little is known about their kinematics or
dynamics. In this study, HF radar will be used to investigate the spatial and temporal characteristics of an anticyclonic instability, to begin to elucidate the kinematics of these features. This is an example of HF radar’s unique ability to measure transient events that are difficult to capture with ship and in situ point measurements, or to resolve using satellite imagery. These features could have implications for mixing and cross-shelf exchange on the eastern side of the channel.

5.2 Cyclonic Shear Zone Instability

5.2.1 Observed Surface Current Field

A cyclonic frontal eddy was observed translating downstream, inshore of the Florida Current, from January 18 to 21, 2005 (Figure 5.1). The eddy was almost stationary in the southernmost part of the domain for ~48 hrs. Then on January 20 at approximately 12:00 (all time in GMT) it began propagating northward along the 200 m isobath, over a 36 hr period. During the passage, downstream current velocities in the jet approached 200 cm s⁻¹, and the southward tangential flow of the eddy reached 80 cm s⁻¹. The eddy was nestled in the trough of a meander, which translated with the feature.

The length scale of the eddy was approximately 20 km. This value is estimated based on the tangential velocities not contaminated by the strong mean flow. As the eddy translated downstream it moved inshore, and thus its shoreward side gradually exited the HF radar footprint. The length scale is near the first baroclinic mode Rossby radius of deformation, which has been measured to be between 15 to 30 km in the Florida Straits (Shay et al., 2000; Peters et al., 2002). The eddy is defined as submesoscale, based on its $O(1)$ Rossby number (see later discussion). The signal propagated north at ~46 km day⁻¹ (53 cm s⁻¹), measured using the slope ($\Delta$ latitude / $\Delta$ time) of the $u$-component (Figure 5.2).
Figure 5.1 Four snapshots of the surface current vectors (hourly averaged) depict a submesoscale frontal eddy, which propagates northward along the inshore edge of the Florida Current. Dashed line in (b) depicts transect used for Hovmöller (Figure 5.2), and the black velocity vectors in (c) highlight the radial horizontal profile of the eddy and the Florida Current.

Associated with the passage of this frontal eddy was a strong SST front along the western wall of the Florida Current (Figure 5.3). The 1 day mean cross-frontal SST gradient was 0.9°C km⁻¹ on 20 January, compared to 0.04°C km⁻¹ along the anticyclonic front east of the core, and 0.02°C km⁻¹ at the cyclonic front at a latitude outside the feature (Figure 5.3d). A map of the 1 day mean surface current velocity field superimposed on the SST image reveals the warm meander, and within its trough the cold frontal eddy. The translating feature has been smeared by the 1 day average, but there is a correlation between
the vectors and SST gradients. Warmer water surrounds a core of cooler water, presumably upwelled because of divergence at the surface, since horizontal temperature advection is unlikely considering the temperature of the ambient surface water. Cross-frontal structure at the latitude of the eddy’s core is shown in Figure 5.3b-d. The flow is cyclonic and divergent to the west of the jet core, and maximum values correspond to the Florida Current front where the gradient in the velocity is greatest.

![Hovmöller diagram](image)

**Figure 5.2** Hovmöller diagram of the u-component of velocity (cm s\(^{-1}\)) as a function of latitude (y-axis) and time day/month (x-axis). Black dashed line indicates the slope (= speed) of the signal propagation: 66 km/34 hours = 46 km day\(^{-1}\), or 53 cm s\(^{-1}\). The transect follows the path of the eddy, shown in Figure 5.1b.

Pressure charts (not shown) indicate the progression of a cold front that passed the Straits on 15 January. A southward wind approached 10 m s\(^{-1}\) on January 17, with wind stress (surface frictional velocity) over 50 cm s\(^{-1}\) (Parks et al., 2009), which forced a southward countercurrent. As the cold front moved through, the wind weakened and shifted to the southwest on January 20. At this time the eddy began propagating north.
Figure 5.3  (a) 1 day mean SST (°C) from MODIS at 1 km resolution [Data source: http://mur.jpl.nasa.gov/]. Superimposed on this is the 1 day mean HF radar derived surface current field, which reveals upwelling in the core of the cyclonic submesoscale eddy. White dotted lines 1 and 2 denote the latitude of the cross-sections plotted below. (b) Cross-section of $u$- and $v$-component of velocity along line 1, (c) vorticity and divergence normalized by local Coriolis frequency along line 1 and (d) SST along line 1 (solid line) and line 2 (dot-dashed line). Thin dotted lines indicate the standard deviation over 25.34°-25.47°N (line 1) and 25.09° - 25.20° N (line 2).
5.3 Flow Field Kinematics

The dispersion of passive tracers, such as phytoplankton or oil, is primarily controlled by ocean currents and wind. Understanding transport of passive tracers on the ocean surface has practical application, most notably for SAR operations and oil spill mitigation. Disregarding the effects of wind and unresolved small-scale processes, passive tracer (or particle) dispersion is a function of the velocity gradient tensor, the components of which can be calculated with the HF radar dataset. It is of interest to compare how the flow field kinematics change between an eddy event (e.g. frontal eddy) and normal background conditions.

5.3.1 Velocity Gradient Tensor

A 2-D surface velocity field, \( \mathbf{u}(\mathbf{x}, t) = (u(x, y, t), v(x, y, t)) \), can be expanded into a Taylor’s series near a reference point \( x_0 \) (Flament and Armi, 2000). Discarding higher order terms gives:

\[
\mathbf{u}(\mathbf{x}, t) = \mathbf{u}_0 + \mathbf{a} (x(t) - x_0(t))
\]

where \( \mathbf{u}_0 = \mathbf{u}(x_0) \) is the mean velocity and \( \mathbf{a} \) is the second-order velocity gradient tensor:

\[
\mathbf{a}_{ij} \equiv \nabla \mathbf{u} = \begin{bmatrix}
\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\
\frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}
\end{bmatrix}
\]

The following elemental components may be defined (Saucier, 1953):

\[
\text{vorticity} \quad \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \equiv \zeta
\]
A purely rotational flow (vorticity) does not separate particles. Particle separation is controlled by the combined effect of divergence and non-divergent strain (Futch, 2009). An eddy core is an ‘elliptic’ regime where vorticity dominates over strain and particle trapping and transport occurs. The core is surrounded by a hyperbolic regime where strain dominates over vorticity (McWilliams, 1984), and filamentation and mixing occurs leading to dispersion.

5.3.2 Lagrangian and Eulerian Diagnostics of the Flow Field

There are numerous techniques to quantify dispersion and resolve coherent flow features in a horizontal velocity field (e.g. d’Ovidio et al., 2009; Beron-Vera et al., 2008; Poje et al., 2010). Some methods are based on Lagrangian, time-dependent information (e.g. Lyapunov exponents), while others require only an Eulerian snapshot of the velocity field (e.g. Okubo-Weiss). In general, Lagrangian techniques are the preferred approach because they integrate in time, allowing resolution of coherent structures in the flow field, whereas Eulerian methods will resolve an instantaneous flow field that cannot distinguish between coherent and transient features. Advantages to the Eulerian approach when using real data are the ease of calculation and retained spatial coverage. Several studies have shown the utility of the Eulerian strain and divergence field for measuring particle dispersion (Futch, 2009; Poje et al., 2010; Haza et al., 2010).
Here we apply the Eulerian method, based on a consideration of the oceanography in the Straits; the strong Florida Current advects flow patterns quickly through the domain. Techniques based on integration time that attempt to capture coherent features of the flow would suffer from either lack of data (because seeded particles quickly exit the region), or would have to drastically reduce the length of integration in order to retain spatial information, which would converge to near-instantaneous values.

An instantaneous rate of separation (IROS) is the Eulerian metric that determines how an infinitesimally small particle will be moved by an instantaneous velocity field, and is equal to the finite-time Lyapunov exponent (FTLE) at time $t = 0$ (Futch, 2009). It can be calculated from the sum of divergence ($d$) and total strain ($s_n$ and $s_s$). The FTLE picks out features that dominate over longer time periods, whereas IROS acts as a guide to how the particles react in the moment (Futch, 2009). High values of IROS indicate regions of elevated particle dispersion.

### 5.3.3 Eulerian Velocity Field during an Eddy Event

The components of the velocity gradient tensor exhibited large magnitude changes during the eddy passage (Figure 5.4). In the absence of an eddy event, the vorticity structure is dominated by the jet shear; there is uniform positive vorticity to the west of the jet axis, switching to negative vorticity on the eastern side, with magnitudes close to $f$ (Figure 5.4c). However, when the submesoscale frontal eddy moved through the domain, the vorticity showed strong non-uniform fluctuations in time and space (of both positive and negative sign) that approached $11f$. For this reason, the dynamics of the frontal eddy are clearly within the submesoscale, because the Rossby number (vorticity normalized by $f$) is very large (Thomas et al., 2008).
Figure 5.4  A snapshot of current vectors superimposed with fields of vorticity (a, c) and IROS (b, d) during an eddy event (Jan 20, 2005 at 16:00; a, b) and during a time with no eddy activity (Oct 4, 2006 at 00:00; c, d). The ‘no eddy’ time period was identified as an example with relatively uniform downstream velocity, to contrast to the eddy event.

The IROS field during the eddy passage was similarly complex with strong magnitudes that revealed regions with a strong dispersive nature. A comparison of IROS and vorticity reveals co-location of peak values, which indicates that regions of strong vorticity in eddy cores do not necessarily correspond to particle trapping in the Florida Current. This is because the eddy core is not purely rotational, as deformation plays a significant role. During a quiescent period with no eddy activity, the IROS field comprised mostly low values across the domain (Figure 5.4d).
Figure 5.5 Maximum values of the field extracted for the time periods (a, b) 2005: Jan 20, 00:00 to Jan 21, 12:00 (eddy event) and (c, d) 2006: Oct 3, 12:00 to Oct 4, 16:00 (no eddy). Divergence is plotted with a $2f$ solid line contour (a, c) and IROS is plotted with a $4f$ solid line contour (b, d).

The field of maximum value (divergence and IROS) extracted from each period of interest (the eddy and no eddy cases) reveals the nature of the flow field (Figure 5.5). For the eddy passage, there is a clear ‘track’ in which maximum values exceed background levels. There was strong divergence associated with the passage of the eddy, which peaked at $4f$ (Figure 5.5a). This is consistent with the pattern of SST discussed previously (Figure 5.3). IROS exhibits strong values during the event, implying that there is strong particle dispersion because of the presence of the eddy. Regions of strong divergence and IROS translate downstream with the eddy, which produces the track-like pattern in the maximum
value field. When there is no eddy activity, the domain comprises small background levels, except along the periphery of the footprint, where the GDOP is higher.

Velocity gradients during an eddy event exhibit very strong fluctuations in comparison to the Florida Current flow field with no eddy activity. There was a complicated pattern of vorticity and deformation suggestive of strong particle leakage out of the eddy core, associated with high values of IROS. Divergence was strongly positive, and consistent with concurrent MODIS SST imagery of cold water anomaly near the eddy core, associated with upwelling. These results indicate the energetic nature of these frontal eddies. Using the HF radar dataset to study the flux of kinetic energy between the mean and perturbations during both an eddy and no eddy period could shed light on the impact of eddies with respect to the energetics of the Florida Current.

5.4 Anticyclonic Shear Zone Instability

5.4.1 Observed Surface Current Field

Four consecutive eddy-like features were observed translating through the radar domain from October 15 to 21, 2006. However, unlike the near-ubiquitous cyclonic frontal eddies observed between the jet and Florida coastline, these features moved along the outer eastern flank of the Florida Current. All four features exhibited clockwise rotation at the surface, in a water depth of approximately 650 m. The October 19 feature, which was best resolved in the radar footprint, is shown in Figure 5.6. This event produced a strong propagating signal in the time-longitude Hövmöller plot of $u$-component velocity (Figure 5.7a). The phase propagated northward at approximately 80 cm s$^{-1}$. By contrast, observed Florida Straits cyclonic eddy translation speeds range between 6 to 19 cm s$^{-1}$ (Tortugas
eddies), 46 to 93 cm s\(^{-1}\) (frontal eddies) and 17 to 46 cm s\(^{-1}\) (submesoscale features) (Fratantoni et al., 1998; Lee and Mayer, 1977; Shay et al., 1998).

Figure 5.6 Four snapshots of the surface current vectors (hourly averaged) reveal the evolution of a clockwise-rotating eddy observed by WERA HF radar, on October 19, 2006. Dashed line in (a) depicts transect used for Hovmöller (Figure 5.7).

On October 14 the wind speed increased from 5 to 12 m s\(^{-1}\) and shifted from a variable northerly wind to a steady easterly throughout the event. An easterly wind could force the currents shoreward (to the west), which was observed on October 18, and may increase Florida Current magnitude in the surface layer via wind-driven Ekman velocity. There was an observable increase in the jet’s surface velocity at this time, peaking on
October 19 (Figure 5.7a). However, since the disturbance was generated upstream of the observational domain, without additional data the contribution from the wind cannot be determined.

5.4.2 Separating the Signal from the Background Flow

The signal exhibited a periodicity close to the local inertial period ($2\pi f^{-1}$), which for latitudes from 25° to 25.7° ranges from 27.6 to 28.3 hrs. To separate from the background flow (background is herein defined as the current field unassociated with the signal), the timeseries at each grid point was decomposed into subinertial (>48 hrs), near-inertial (20-36 hrs) and high-frequency (< 20 hrs) currents. The near-inertial bandwidth was assigned based upon the analysis of Mooers and Brooks, (1977), who noted that because of the strongly sheared background flow, the inertial frequency can be shifted by up to 30% of $f$ in the Straits of Florida. After conducting sensitivity tests, the Fourier filter proved optimal for the decomposition (Walters and Heston, 1982). The Fourier filter requires a complete timeseries, which imposed restrictions on the spatial coverage of the dataset. The diminished spatial coverage does not fully cover the features that pass through (Figure 5.8b), although it does capture the rotation along the western periphery. This does not significantly affect the outcome; it can be shown that Reynolds decomposition, which uses all the data, gets the same qualitative result. Tidal constituents were not removed, because of the complication of contamination by the episodic Florida Current meandering over daily timescales. Previous studies have shown that tidal velocities in the Straits are <10 cm s$^{-1}$ (Kielmann and Duing, 1974; Mayer et al., 1984; Peters et al., 2002). Tidal forcing is continuous and periodic, while eddy events are transient and highly intermittent.
Figure 5.7 Hövmöller plots ($u$-component contours plotted on time vs. latitude axes) at longitude 79.8°W (transect plotted in Figure 5.6a) for (a) observed and (b) near-inertial currents. Solid contour lines denote negative values. In (b) note the clear propagation of negative-$u$, which is masked out in the observed currents by the strong northward Florida Current.

5.4.3 Near-Inertial Oscillation

The signal was embedded in the near-inertial band, as shown by decomposed surface current maps (Figure 5.8). The subinertial band comprised meandering, while the high frequency band (not shown) exhibited neither coherent structure nor significant amplitude. Once isolated from the background flow field, the signal is oscillatory (Figure 5.7b). In the surface vector maps, the near-inertial currents reveal what can be interpreted as the crest and trough of a wave (Figure 5.9). A clockwise rotation of the vectors in time
produces horizontal convergence (crest) and divergence (trough) of the near-inertial currents.

**Figure 5.8** Frequency decomposition of the surface velocity field at October 19, 02:00 for (a) observed, (b) observed: region of 100% coverage that can be filtered, (c) subinertial and (d) near-inertial components (*colorbar scale for near-inertial currents is from 0 to 35 cm s$^{-1}$).

Near-inertial motions can be generated by fluctuations in local wind stress (D’Asaro, 1985), or ‘loss of balance/spontaneous adjustment’ by western boundary currents, mesoscale eddies, and submesoscale frontogenesis (Ford, 1994; D’Asaro et al., 2011; Alford et al., 2013). After a transient forcing event, and in the absence of all other forces, horizontal currents move under their own inertia, and on a rotating Earth in the Northern (Southern) Hemisphere will complete clockwise (counter-clockwise) oscillations at the inertial frequency $f$ (Cushman-Roisin, 1994). However, in the real ocean these
motions are often shifted off $f$ because of other forces. A horizontal sheared background flow, with relative vorticity $\zeta_g$, can lower the bound of the internal waveband from $f$ to an effective frequency $f_{\text{eff}} = f + \zeta_g/2$ (Mooers, 1975). Kunze (1985) showed that when a near-inertial wave propagates through a horizontal gradient of $f_{\text{eff}}$, its wave vector must evolve to satisfy the dispersion relation, which leads to refraction and partial or total reflection. Horizontal gradients in $f_{\text{eff}}$ result in a non-uniform wave field. Waves generated in regions where $f_{\text{eff}} < f$ are trapped, as they encounter turning points outside of the negative vorticity trough (Lee and Eriksen, 1997). Elevated near-inertial kinetic energy on the anticyclonic (negative vorticity) side of a front has been observed in numerous field studies (e.g. Kunze and Sanford, 1984; Granata et al., 1995; Rainville and Pinkel, 2004; Nagai et al., 2013).

**Figure 5.9** Near-inertial current vector map on Oct 19, 00:20 reveals the crest (convergence) and trough (divergence) of a near-inertial wave.

Within the southern Straits of Florida, Shay et al. (1998) documented near-inertial motions with horizontal wavelengths of 40 km that were trapped and advected by the Florida Current. Vertical current structure measurements from an ADCP revealed vertical
wavelengths between 50-100 m, and phase propagation reversals at a critical layer (the depth where the speed of the wave group equals that of the current; Kunze, 1985). Our case differs in that the near-inertial signal was observed in the anticyclonic shear zone of the Florida Current. The strongly sheared background flow partially masked the near-inertial current field, which is manifested as a succession of clockwise-rotating eddies in the observed surface current maps. The wave trough is not evident in the total surface currents when embedded in a laterally sheared flow regime. Some caution should be invoked, however, since this method filters Eulerian data to look at a translating Lagrangian feature. Further analysis must be conducted to relate this signal to near-inertial wave dynamics.

5.4.4 Idealized Model

To elucidate the geometric effects of a background shear flow on the signal pattern observed in our HF radar domain, a simple analytical model of an asymmetric jet with lateral shear is superimposed with a dipole perturbation. Streamfunctions for the jet ($\psi_J$), perturbation ($\psi_e$) and total flow ($\psi_T$) are:

$$\psi_J = A \cdot e^{-aL_x} \quad (5.7)$$

$$\psi_e = B \cdot \sin(m\pi^{-1}) \cdot \sin(l + \phi) \quad (5.8)$$

$$\psi_T = \psi_J + \psi_e \quad (5.9)$$

where $A$ is the amplitude of the jet core, $a$ is a scaling factor for the lateral shear, $L_x=L_y=100$ are the zonal and meridional extent of the domain (size is arbitrary). $B$ is the wave amplitude, $m = \pi /40$ is defined at $x = 10:50$ (where 40 is the width of the jet), $l = 2\pi/L_y$ is the meridional wavenumber, and $\phi$ the phase. The model has been assigned parameters to resemble the data; specifically the wavelength/domain ratio and phase of the
disturbance. The idealized fields are compared to the observations of the total surface currents, the band-passed near-inertial currents and the low-passed sub-inertial flow (Figure 5.10).

![Model Streamfunction, Model Vector, HF Radar Vector](image)

**Figure 5.10** (left column) Model streamfunction; (middle column) model vector plots; and (right column) observed vector plots, for (top) laterally sheared jet; (middle) dipole perturbation and (bottom) total field.

The model confirms that for a dipole perturbation embedded in a laterally sheared anticyclonic background flow, only closed clockwise rotation is apparent in the total flow field. The counter-clockwise rotating eddy acts to distort the streamfunction contours in the region (see the total fields in Figure 5.10), but there is no closed circulation. Accounting for complications in the observed currents, such as differences in jet and eddy
orientation, the simple model can replicate the basic flow pattern. This model reveals qualitatively how a horizontally sheared mean flow can mask a rotary perturbation signal.

5.4.5 Near-Inertial Wave Kinematics

The hypothesis was that these transient clockwise-rotating features are a succession of stationary eddies advected northward by the Florida Current. However, a systematic frequency analysis of the signal reveals these features to be strongly embedded in the near-inertial passband, and absent from the subinertial. The characteristics of the signal in frequency and space resemble a near-inertial oscillation. In this section, the properties of the signal are examined and compared to near-inertial wave theory.

5.4.6 Subinertial Velocity Field

The jet core was located in the western part of the radar domain, along the continental shelf. There was a thin region of cyclonic shear to the west of the axis, and a much wider anticyclonic shear region to the east (Figure 5.8c). Within the core, subinertial velocities ($v$) reached 200 cm s$^{-1}$, decreasing to less than 40 cm s$^{-1}$ towards the east, over a distance ($x$) of 40 km. This equates to a sheared background flow, with a normalized vorticity of $f^{-1} \partial v / \partial x = -0.6$. Similarly, the cyclonic shear zone exhibited equal magnitudes in vorticity. This suggests that any near-inertial wave propagating in this field will experience strong horizontal gradients of $f_{eff}$, which could lead to frequency shifts and trapping in the negative vorticity trough.

5.4.7 Wavelength

To determine the horizontal wavelength of the near-inertial signal, a series of trial wavenumbers ($2\pi/L$, where $L$ is the wavelength) were fitted to the band-passed data at grid points along lines of constant longitude, using a plane wave model (Shay et al., 1998):
\begin{align*}
u(y) &= A_1 \cos(ly) + B_1 \sin(ly) + u_r(y) \quad (5.10) \\
v(y) &= A_2 \cos(ly) + B_2 \sin(ly) + v_r(y) \quad (5.11)
\end{align*}

where \([u, v]\) are the observed near-inertial data, \(A_1,2\) and \(B_1,2\) are the velocity amplitudes (Fourier coefficients), \(l\) is the meridional wavenumber (trial wavelengths defined between 1 to 300 km) and \([u_r, v_r]\) is the residual current not explained by the model. A ‘carrier’ wavenumber is defined, which maximizes the correlation coefficient \(R\) between the observed and modeled data (Jaimes and Shay, 2010):

\[ R = \sqrt{\frac{(r_u^2 + r_v^2)}{2}} \quad (5.12) \]

where \([r_u, r_v] = ss_{xy}^2 / s_{xx} s_{yy}\), \(r_u\) and \(r_v\) are the correlation coefficients between observed and modeled velocity for the \(u\)- and \(v\)-components of velocity, \(s_{xx}\) and \(s_{yy}\) are the variance matrices of observed and modeled velocities, respectively and \(s_{xy}\) is the covariance matrix.

For each longitude, the latitudinal average was removed at each grid point in latitude, and the least-squares fit was performed over two inertial periods (IP), the time period when the signal was at its strongest. Note this approach assumes there is a dominant single carrier wavenumber for each longitude.

The model reveals an average wavelength of \(~110\) km (Figure 5.11). The wavelength is close to the Eady model most unstable mode of \(3.9R_d\), where \(R_d\) is Rossby radius of deformation, which in the SoF is \(~30\) km (Peters et al., 2002).
Figure 5.11  (a) Histogram of the modeled wavelengths between longitudes 79.5°W to 79.9°W and over a time period of 1.5 IP (43 hrs) and (b) Results of least-squares fit of the near-inertial currents at longitude 79.8°W.

5.4.8 Frequency

The dominant frequency of the oscillation is calculated with Equations 5.10 and 5.11, by substituting in frequency and temporal variations at each grid point in place of wavenumber and spatial variations. The near-inertial components at each grid point were fit to a series of trial frequencies between 0.5 to 1.5 $f$ (intervals of 0.05). The carrier frequency ($2\pi/T$, where $T$ is the wave period) is defined as the value which maximizes the correlation between observed and modeled data, over the two IPs. Over the domain, calculated carrier frequencies range between 0.8 to 1.3 $f$, although a shifting to lower frequencies dominates, and the average is $0.87f$, with correlations between the model and data as high as 0.95 (Figure 5.12). The width of the peaks is because of the broadband character of near-inertial motions.
Figure 5.12 Trial frequency versus correlation between the model and data. Black vertical line denotes the carrier frequency, which has the best fit to the data.

5.4.9 Effects of Subinertial Vorticity

The frequency shift of the signal below $f$ agrees with theoretical results of a near-inertial wave propagating in a region of negative vorticity (Kunze, 1985). The mean near-inertial current amplitude distribution exhibits a peak aligned with the subinertial vorticity trough (Figure 5.13). This suggests trapping of the near-inertial signal generated in the vorticity trough. Trapping occurs because of wave refraction in a spatially non-uniform vorticity field – the wave cannot propagate freely away as it is refracted back and forth between regions of less negative vorticity, and leads to peaks in near-inertial energy within the trough (Kunze, 1985; Lee and Eriksen, 1997).
5.4.10 Open Questions

The observed signal is consistent with near-inertial wave propagation in geostrophic shear (Kunze, 1985). The vector rotation at each grid point over most of the HF radar footprint is clockwise, which is consistent with near-inertial oscillations in the northern hemisphere. However, the rotation of vectors within the jet core and the strong cyclonic shear region is counter-clockwise (not shown). HF radar measures the Eulerian frequency $\omega = \omega_0 + \mathbf{k} \cdot \mathbf{V}$, where $\omega_0$ is the intrinsic frequency, $\mathbf{k}$ is the wavevector and $\mathbf{V}$ is the subinertial velocity. The Eulerian frequency is constant in a steady flow, but in our domain it has a spatial gradient because of non-uniform $f_{eff}$ and $\mathbf{V}$. The term $\mathbf{k} \cdot \mathbf{V}$ represents the Doppler shift (advection) by the background flow. In the North Atlantic subtropical zone, Mied et al. (1987) found evidence of near-inertial waves strongly influenced by Doppler shifting. Preliminary results here indicate this Doppler shift may be significant enough to dominate the near-inertial oscillation frequency within the core. However, without additional observations it is difficult to form a solid conclusion of the
mechanism of the observed signal. One avenue for further insight could be to develop the idealized model by individually applying the dynamics of a vortex (e.g. a Rankine model) and a near-inertial wave. The lateral shear and magnitude of the jet core can also be adjusted to best fit the observations. Since this study is the initial analysis of a new signal observed in the anticyclonic shear zone of the Florida Current, more observations of such events, covered by both remote and in situ instrumentation, as well as numerical modeling efforts are required to fully explain these complex features.

5.5 Summary

The deployment of HF radar along the South Florida coastline has improved our ability to monitor the ocean surface currents within the Straits. This has been shown by two case studies, which have demonstrated HF radar’s ability to (1) examine how the flow field kinematics are significantly altered during the passage of a submesoscale frontal eddy, and (2) document a near-inertial velocity signal along the anticyclonic flank of the Florida Current that has not been previously addressed in the literature.

In the first case study, the passage of a submesoscale cyclonic frontal eddy moving quickly downstream was captured in the HF radar footprint. In contrast to conditions recorded in a period of no eddy activity, during the event the vorticity field revealed a complex structure, with significant contributions from strain, and a Rossby number that greatly exceeded unity, implying the flow field was governed by submesoscale dynamics. Indeed, there was strong horizontal current divergence near the core of the eddy, associated with anomalously cold water brought to the surface by upwelling, observed in MODIS SST satellite imagery. IROS, which is a metric of particle dispersion, exhibited high values that translated with the eddy, indicating the potential for strong dispersion of a passive tracer.
This has important implications for cross-shelf exchange of water properties between offshore and coastal regions, and is important information for SAR operations and pollution mitigation.

In the second case study, a transient, coherent signal in the near-inertial passband was identified. It was found that the strongly sheared Florida Current partially masked the structure of the near-inertial oscillation, which was manifested as a succession of clockwise-rotating eddies in the observed surface currents. The wave trough was not evident when embedded in a laterally sheared northward background flow. The dominant frequency was shifted by ~13% below $f$ in the average, which is consistent with a near-inertial wave propagating in a background regime with negative vorticity. The spatial pattern of frequency was highly anisotropic, because of the variations in the subinertial current velocity and its associated vorticity. Near-inertial energy peaked in the negative vorticity trough along the eastern flank, indicative of wave trapping in the horizontal. These results suggest the observed signal was governed by near-inertial wave dynamics. However, since this is a preliminary study of these features, further work is required to clarify their mechanisms.

These example cases, in addition to previous modeling and observational studies, reveal the highly intermittent nature of the flow within the Straits, which comprises periods of strong fluctuations along both shear zones. Future work needs to take the big step forward from individual case studies to long-term timeseries analysis, which can determine the quantitative and statistical details of the time and space scales of these instabilities, and whether they exhibit change over time. The ultimate goal is to incorporate this information into improving model forecasts of the current and wave field in the Straits of Florida.
Chapter 6

Characterizing the Space-Time Structure of Fluctuations and Eddy-Mean Flow Interaction in the Straits of Florida

The characteristic temporal and spatial scales of the fluctuations are calculated based on the correlation properties of the flow field. The dominant periods of variability are examined, along with their time dependency. The time-mean slope of the kinetic energy wavenumber spectrum is found. Eddy-mean flow interaction is investigated through the conservation of eddy kinetic energy equation, variance ellipses and the Reynolds stress terms.

6.1 Overview

Surface current fluctuations in the Straits of Florida are intrinsically linked to the Florida Current, which may generate, or trap and advect these features leading to mixing and cross-shelf exchange (Peters et al., 2002; Lee et al., 1991). While Chapter 5 focused on two eddy events and their kinematic properties, this chapter takes a broader view, aiming to characterize the temporal and spatial scales of the fluctuations. An open question in the research community concerns the importance of submesoscale fluctuations in driving transport and dispersion. Here we contribute to this discussion by calculating the two year mean kinetic energy wavenumber spectrum to determine
the slope, and hence the partition of energy with spatial scale that indicates the relative importance of the submesoscale flow field. Elucidating the interaction between the mean and fluctuating currents is an important link for understanding and accurately modeling the dynamics of the global ocean circulation. Eddy feedback mechanisms have a profound effect on the time mean flow field, so correctly parameterizing the influence of unresolved eddies in models is critical. In this chapter, eddy-mean flow interaction is investigated through consideration of the energetics, specifically the barotropic eddy momentum fluxes to discover whether eddies act in a time mean sense to accelerate or decelerate the Florida Current offshore of Miami.

In the next section, the characteristic time and space scales are quantified based on their correlation properties. Then the dominant periods of variability are identified in the frequency domain, and the two year mean kinetic wavenumber spectrum is calculated for wavelengths from 5 to 60 km. Section 6.5 investigates the energy conversion terms in two dimensions. Finally, the results are summarized and put into context with previous work in Section 6.6.

6.2 Space-Time Structure of the Surface Velocity Field

6.2.1 Decomposing the Flow into Mean and Fluctuations

To investigate the time and space scales of the fluctuations in the Straits of Florida, the first step is to define ‘mean’ and ‘eddy’. This work applies the Reynold’s approach, where the instantaneous zonal and meridional velocities $u$ and $v$ at each grid point are decomposed into their time-mean ($\bar{u}$, $\bar{v}$) and fluctuating ($u'$, $v'$) components:

$$u(x, y, t) = \bar{u} + u'(x, y, t) \quad v(x, y, t) = \bar{v} + v'(x, y, t)$$  \hspace{1cm} (6.1)

Typical timeseries of $u$ and $v$ are shown in Figure 6.1, and reveal different timescales of motion superimposed on the mean flow, from low frequency meandering of the Florida Current exhibited in $v$, to higher frequency tidal oscillations shown by both velocity components.
Eddy kinetic energy $EKE = 0.5 \left( \bar{u}^2 + \bar{v}^2 \right)$ provides a measure of the intensity of the fluctuations (Figure 6.2). Within the Straits of Florida, peak variance is found along the inshore region between the coastline and the Florida Current jet. EKE, as defined, can be attributed to fluctuations in the flow field ranging from lateral meandering and pulsing of the Florida Current, the passage of submesoscale frontal eddies, wind-forced circulations, and surface manifestations of internal waves and tides. In addition to physical processes, instrument noise can contribute to the EKE calculation, which may explain peak values in the southwest region of the domain during 2006. Winter 2005 is unique for having much higher values of EKE across the radar domain. This energetic time period, between January and April 2005 and observable in the timeseries of Figure 6.1, was associated with elevated variance in meandering, jet intensity, and surface and volume transport (see Figure 4.13).

Figure 6.1 Time series of $u$ and $v$ from January 2005 to December 2006 at an example grid point, 25.28°N, 79.86°W. The large gap in the timeseries (November 2005) was due to radar damage by Hurricane Wilma.
6.2.2 Decorrelation Length Scales

The scales of space and time variability in the surface current field can be quantitatively characterized in terms of its correlation properties. Correlations between a reference point and all other grid points on the map provide a measure of the typical scales of variability in different geographical locations (Figure 6.3). Two main points can be drawn: (1) for all locations, the $v$-component correlations exhibit strong directionality in comparison to the more isotropic patterns of the $u$-component; and (2) the correlations at the mid- and offshore reference points show larger correlated regions compared to the inshore location. The domain averaged spatial lag correlation functions for $v$ and $u$ are shown in Figure 6.4, and also reveal the asymmetry in the north-south currents. For the $v$-component, there is an approximate 2:1 ratio for the $y$ versus $x$ lag, with values
falling to below 0.5 within 15-25km in the zonal direction, while in the meridional the correlation retains high values past 20km. The NNE-SSW inclination in the field can be attributed to the orientation of the Florida Current jet. For the $u$-component, the correlation pattern is more isotropic, with a slight extension of higher values to the west, and values that drop to 0.5 in 20-25km.

![Figure 6.3](image)

**Figure 6.3** Correlation between a reference point (denoted in each map by the diamond ‘◊’) and all other grid points on the map for $v$ (top) and $u$ (bottom) components of velocity. (a) Domain center; (b) offshore; (c) inshore; (d) north; (e) south.

To investigate decorrelation length scales within different frequency bands, coherence was calculated between a central reference point and all other grid points (**Figure 6.5a**). Coherence provides a measure of the amplitude of correlation at specific frequencies, and the phase difference can give information on the propagation of the signal. Coherence was averaged across frequency for the following 3 periods: sub-inertial (>48 hrs), near-inertial and diurnal tides (20 < $f$ < 36 hrs) and high frequency and semi-diurnal tides (<20 hrs). The near-inertial bandwidth was assigned based upon the analysis of Mooers and Brooks (1977), who noted that because of the strongly sheared background flow, the inertial frequency can be shifted by up to 30% off $f$ in the Florida Straits. Another approach is to filter the data within the desired frequency band, and then correlate in time. This was performed to compare with the coherence, and the two methods produced very similar
results. Coherence is more desirable, however, because it provides the phase difference between the timeseries.

The amplitude of spatial coherence shows that sub-inertial motions have the largest decorrelation length scales, exhibiting strongest directionality in the along-stream axis, with a phase difference that indicates northward propagation. The near-inertial currents have a smaller decorrelation length scale in the along-stream direction than the sub-inertial, with a similar scale in the cross-stream. The phase map reveals an eastward to northeastward direction of propagation, with larger phase differences indicating smaller wavelengths. This is most likely attributed to the lateral meandering of the jet in the channel, which would create a similar out-of-phase pattern. The high frequency motions have the smallest decorrelation length scale, as expected. The phase map is more complex, with negative lags to the northwest and southeast of the reference point, and positive lags to the northeast and southwest.
Figure 6.5 (a) Coherence (left) and phase (right) between a central reference point and all other grid points on the map for the $v$-component. (b) Coherence and phase of $v$ as a function of longitude (left) and latitude (right), for a reference grid point (same as above) at all frequencies.
To consider all frequencies, the coherence is plotted at the same reference point but as a one-dimensional function of latitude (or longitude) and frequency (Figure 6.5b). The diurnal and semi-diurnal tide generate high coherence across nearly all latitudes and longitudes. There is a general trend toward decreasing length scale as frequency increases, except at the near-inertial passband. Coherence plotted against latitude shows the clear dominance of the Florida Current, with the highest coherence in the sub-inertial band, and the phase map indicating northward propagation of the signal. For coherence with longitude, the sub-inertial band does not dominate in the same way as with latitude. The phase map shows at the lowest frequencies (<0.4 cpd) the direction of propagation is westward, whereas the higher frequencies are dominantly eastward.

6.2.3 Decorrelation Time Scales

The autocorrelation of the timeseries at each grid point provides a measure of the decorrelation timescale over the radar domain.

\[
R(\tau) = \frac{C_{vv}(\tau)}{\sigma^2} = \frac{1}{\sigma^2(N-k)} \sum_{i=1}^{N-k} [v_i - \bar{v}] [v_{i+k} - \bar{v}],
\]

where \(C_{vv}\) is the autocovariance function, normalized by the variance \(\sigma^2\), \(v\) is the velocity timeseries, \(N\) is the number of data points, and \(\tau = \tau_k = k\Delta t\) \((k = 0, ..., M)\) is the lag time for \(k\) sampling time increments \(\Delta t\) (Emery and Thomson, 2001). The autocorrelation function can be integrated over a certain number of lags to determine the integral timescale \(T^* = \int R(\tau) d\tau\), the dominant timescale of correlation within a timeseries. For times longer than \(2T^*\) the data are decorrelated. In addition to providing information of the temporal scales, the integral timescale is also valuable for determining the statistical degrees of freedom in the dataset. The initial integral timescale calculation yielded no near-constant value; that is, no plateau was reached. To obtain a value for the integral timescale of the flow in this region, it has been suggested to take the first zero crossing of the autocorrelation function (Poulain and Niiler, 1989). However, this can inaccurately represent the timescale of the
flow, because the negative lobe of the autocorrelation should be considered in the calculation, which can reduce the timescale considerably (Klocker et al., 2012). The optimal method in this case is to fit an autocorrelation function to the observed $R$ (Mariano and Chin, 1996; Garrafo et al., 2001; Peters et al., 2002; Lumpkin et al., 2002). We fit an autocorrelation function of a form that composes both a wave component and a turbulent component:

$$R^*(\tau) = (1 - \epsilon^2) \cos\left(\frac{\pi \tau}{2T_d}\right) e^{-\left(\frac{\tau}{\tau_e}\right)}, \quad (6.3)$$

where $\epsilon^2$ accounts for the noise such as measurement variability and sub-grid scale processes, $T_d$ is the first zero crossing of the observed $R$, and $\tau_e$ is the e-folding (or turbulent) timescale. This function differs slightly from previous studies that used a Gaussian decay; this edit was necessary because the autocorrelation function falls off more steeply than Gaussian, so Equation 6.3 provides a better fit to the observed data (Figure 6.6). The three parameters ($\epsilon^2$, $T_d$, $\tau_e$) are determined using the feature-based technique of Mariano and Chin (1996). Note $\epsilon^2$ is determined from the difference between $R$ at zero lag and after 10 hours. The first zero crossing ($T_d$) is found where $R$ changes from positive to negative. The turbulent timescale $\tau_e$ is determined from the first two parameters, finding the best fit of the function to the observed $R$ through least squares. The values of $\epsilon^2$ and $T_d$ are tweaked within 10% of their initial values to get the final best fit value. The integral timescales at each grid point are calculated using the exact integral of $R^*$ from zero to infinite lag (Gradshteyn and Ryzhik, 1980, pg. 477, eq. 3.893.2):

$$\int_0^\infty R^*(\tau) d\tau = (1 - \epsilon^2) \left[\tau_e^{-1} + \frac{\tau_e \pi^2}{4T_d^2}\right]^{-1} \quad (6.4)$$
Figure 6.6 Observed v-component autocorrelation (black) and fitted function (blue) for one grid point (25.65°N, 79.7° W).

Spatial maps of the integral timescale $T^*$ and the three observed parameters $\epsilon^2$, $T_d$, and $\tau_e$ for the $v$ and $u$-components are shown in Figures 6.7 and 6.8, respectively. The integral timescale ranges from 2 to 10 days for $v$ and 1 to 2 days for $u$. There is a strong spatial inhomogeneity in both maps, due to the dominating effect of the lower frequency Florida Current variability – the $v$-component $T^*$ is largest at the same location as the time mean jet core. Likewise, the $u$-component $T^*$ is largest in the same region as the maximum time mean $u$-component, which comprises a component of the downstream jet. The decorrelation timescale of $v$, or zero crossing lag, exhibits a decay from a western maximum to eastern minimum, the gradient perpendicular to the orientation of the shelf break and along-stream flow. The values are significantly higher, and more variable, than the integral timescale, ranging between 5 to 35 days. This demonstrates the importance of choosing the correct method to calculate the integral timescale. The zero crossing of $u$ ranges between 4 to 11 days, and more closely resembles the integral timescale than $v$. Both the $u$ and $v$ zero crossing maps match the pattern of the standard deviation – the higher the standard deviation, the longer the
autocorrelation takes to drop below zero. This is presumably due to low frequency Florida Current variability along the cyclonic frontal region of the jet that produces both higher variance and correlates longer in time. The e-folding timescales for $v$ reveal much larger time scales within the jet core (up to 14 days), dropping significantly in the shear regions to below 2 days. For $u$, e-folding timescales are below 2 days everywhere. The maps of noise – normalized sub-grid scale and measurement variance – reveal larger values for $u$ than for $v$, due to the lower ratio of signal to noise in comparison with the stronger downstream flow.

**Figure 6.7** For the $v$-component: Spatial maps of the integral timescale ($T^*$), observed decorrelation time scale ($T_d$) where $R$ crosses zero, observed e-folding scale ($\tau_e$) and noise ($\epsilon^2$).
For the $u$-component: Spatial maps of the integral timescale ($T^*$), observed decorrelation time scale ($T_d$) where $R$ crosses zero, observed e-folding scale ($\tau_e$) and noise ($\epsilon^2$).

### 6.3 Frequency Domain

Domain averaged power spectra of the hourly-sampled surface currents are presented in Figure 6.9, based upon 4530 grid points for 2005. The spectra are calculated using Welch’s modified periodogram method, with nine 56 day half-overlapping segments windowed with Hanning weights in the time domain, Fourier transformed, and ensemble averaged. The spectra reveal dominant periods of variability at the sub-inertial frequency ($< 0.5$ cpd, or 48 hrs), the major tidal constituents $O_1$ (25.82 hrs), $K_1$ (23.93 hrs), $N_2$ (12.66 hrs), $M_2$ (12.42 hrs) and $S_2$ (12 hrs), and a steepening of the spectra at frequencies over 1 day, with noise dominating at the highest frequencies. The local inertial
period ranges from 27.6 to 28.3 hrs in the radar domain, which spans the latitude from 25°N to 25.7°N.

Figure 6.9 Domain-averaged power spectra of the (top) v and (bottom) u-components of velocity, with 95% confidence intervals for a chi-squared distribution.

To take a closer look at the sub-inertial fluctuations, variance-preserving spectra (for which the area under the curve is proportional to the variance) were calculated to highlight the lower frequency peaks. The spectra in Figure 6.10 show the domain-averaged low-pass filtered (>48 hrs) surface current components u and v for 2005. The dominant signals in the v-component are at 3-4 days, 6-7 days, 9-10 days and approximately 30 days, with increasing variance at the larger periods.
For the $u$-component, the dominant scales of variability are at 3 days, 5 days, 7 days and approximately 20 days, with the highest variance exhibited at the 5 day period. Motions with periods between 3 to 10 days can be forced by local wind events, or shear-induced instabilities of the Florida Current. Longer period motions of 20 to 30 days can be due to longer wavelength jet meanders or the wind forcing from subtropical depressions. The $v$-component exhibits greater contribution from the lower frequency forcing of the Florida Current, while the $u$-component reveals highest variance between 3 to 7 days, consistent with the periodicity of frontal instabilities (Lee and Mayer, 1977).

**Figure 6.10** Variance-preserving spectra of the (top) $v$ and (bottom) $u$-components of the sub-inertial (>48 hrs) velocity, plotted as a function of period, increasing toward the left.
Unlike tides, these low frequency motions are nonstationary due to their transient forcing mechanisms. Wavelet analysis is a method that can resolve the time-frequency space of a signal, which allows us to see how these motions are modulated throughout the year (Figure 6.11). This method works through the convolution of the time series with an oscillatory wavelet function (the ‘mother wavelet’). By varying the scale of the wavelet – adjusting its width in the frequency domain – and translating it along the timeseries, the local power of each frequency (corresponding to the wavelet scale) is obtained in time. The wavelet function should be chosen based upon the character of the timeseries; for a surface current timeseries, a good choice is the Morelet wavelet, which is a plane wave modulated by a Gaussian:

\[
\psi_0(\eta) = \pi^{-1/4} e^{i\omega_0 \eta} e^{-\eta^2 / 2}
\]  (6.5)

where \(\eta\) is a non-dimensional time parameter and \(\omega_0\) is a non-dimensional frequency. The wavelet is localized in time so that the oscillations decay from the origin, has zero mean, and is normalized at each scale to have unit variance. We run the continuous wavelet transform in Fourier space, based on Torrence and Compo (1998), with a wavenumber \(\omega_0\) of 6 (Farge, 1992). By operating in frequency rather than time, the wavelet transform is calculated via the convolution theorem as the inverse Fourier transform of the product:

\[
W_n(s) = \sum_{k=0}^{N-1} \hat{x}_k \hat{\psi}^*(s \omega_k) e^{i\omega_k n \delta t}
\]  (6.6)

where \(\hat{x}_k\) is the discrete Fourier transform of the timeseries \(x_n\), \(\hat{\psi}^*(s \omega_k)\) is the complex conjugate of the Fourier transform of \(\psi_0(\eta)\), which has been normalized to have unit energy, \(k\) is the frequency index, \(n\) the time index, \(s\) the scale and \(\omega_k\) the angular frequency, defined:

\[
\omega_k = \begin{cases} 
\frac{2\pi k}{N \delta t} : & k \leq \frac{N}{2} \\
\frac{-2\pi k}{N \delta t} : & k > \frac{N}{2}
\end{cases}
\]  (6.7)
Since the scale of a Morelet function is almost equal to the Fourier period (Torrence and Compo, 1998), the terms ‘scale’ and ‘period’ will be used interchangeably here. The scales were selected to resolve the sub-inertial frequencies, from the shortest period of 2 days up to the longest period of 280 days for the 2005 timeseries. However, since this analysis deals with a finite-length timeseries, edge effects can cause errors due to the assumption of a cyclical signal. The timeseries is padded with zeroes so that the total length $N$ is brought up to the closest power of two, which restricts edge effects and speeds up the Fourier transform. The ‘cone of influence’ is the region of the spectrum where edge effects are important, defined as the e-folding time of the autocorrelation of wavelet power for each period (Torrence and Compo, 1998). For periods greater than 42 days, edge effects become large so the spectrum is cut-off at this scale.

The wavelet transform of $v$ reveals the same peaks of variability as the variance-preserving spectra of Figure 6.10. The largest variance is attributed to periods between 25-35 days, consistent throughout the year. At periods of 7-10 days, there is a clear seasonality, with peaks in the winter months, and quiescent periods during the summer. The shortest period motions of 3-4 days appear during the spring time, from March to May. For the $u$-component, dominant periods of variability are at 16-30 days, 5-12 days and 1-3 days. The same seasonality is exhibited, with higher variance in the winter and spring seasons than fall and winter. During the winter and spring, wind forcing has a shorter periodicity due to the passage of cold fronts.

The variance-preserving spectrum of $u$ (Figure 6.10) exhibits maximum variance at a relatively sharp peak of 5 days, while the wavelet transform indicates greater variance at the longer periods. This may be due to the inherent bias of wavelet analysis – at shorter periods (smaller scales), the wavelet is broader in frequency, and so sharper spectral peaks are smoothed out.
To investigate cross-channel changes in the surface current fluctuations, variance-preserving spectra were calculated for each grid point along a line of constant latitude (25.23°N), for the $u$ and $v$-components of velocity (Figure 6.12). For all spectra the tidal constituents exhibit zonal lines in the frequency contours across the channel. The $v$ spectra reveal elevated energy west of 79.9°W, with broad peaks centered on the sub-inertial and near-inertial frequencies, while to the east the energy is reduced in these bands except at the lowest frequencies. This dividing longitude of 79.9°W is also
the approximate mean location of the Florida Current axis, in which the sub-inertial vorticity changes sign from negative to the east, to positive towards the west (Archer et al., 2015a). Additionally, this longitude coincides with the shelf break, where the bathymetry shoals to the west from ~600 m to less than 150 m at 81.1°W, and eastward the deeper water (~800m) extends out to 79.4°W. The \( u \) spectra have low frequency variance nearly uniform across the channel, with the exception of the furthest western region. The inertial energy is strongest in the shallower water to the west, but near-inertial frequencies follow the same pattern as the sub-inertial.

**Figure 6.12** Variance-preserving spectra as a function of longitude for (top) \( v \)-component, (bottom) \( u \)-component.
6.4 Wavenumber Domain

6.4.1 Background

While oceanic kinetic energy is dominated by the mesoscale eddy field with length scale $\mathcal{O}(100 \text{ km})$, it is dissipated at the viscous scale of length $\mathcal{O}(1 \text{ mm})$. As energy is input into the ocean through external forces, such as large-scale winds, energy must be balanced through dissipation at the viscous scale. This implies a cascade of energy from the large to the small scale, where in between lays an ‘inertial’ range where forcing and dissipation are negligible (Kolmogorov, 1941). The energy pathway from basin-wide gyres to mesoscale eddies has been extensively studied, as well as small-scale three-dimensional turbulence. The submesoscale, which lay in between traditional quasi-geostrophic theory and fully three-dimensional motions, is not as well understood. This has been due to limitations in instrument sampling resolution and computational power. The questions that remain are: how is energy transferred to the submesoscale, and how important is the submesoscale in mixing and energy dissipation? One way to investigate these questions is through the calculation of kinetic energy wavenumber spectra, which characterize the partition of energy between the mesoscale and submesoscale. The steeper (flatter) the slope, the less (more) energy is contained in the smaller scale motions relative to the larger scales, and the less (more) important they are for controlling transport and dispersion.

Classical interior quasi-geostrophic (QG) theory is based upon interior potential vorticity anomalies away from boundaries, so that surface buoyancy anomalies can be ignored (Charney, 1971). It predicts a wavenumber spectral slope of $k^{-3}$, where $k$ is wavenumber. In this scenario, particle dynamics are governed by the nonlocal large scale flow field that dominates the energy spectrum and is more important for mixing.
Conversely, other theories predict a shallower slope, in which the submesoscale flow field is more turbulent and plays a prominent role in driving mixing and transport of tracers. One such theory is mesoscale-driven frontogenesis (Lapeyre and Klein, 2006), whereby the strain field surrounding the cores of mesoscale eddies leads to a sharpening of buoyancy fronts, subsequent roll-up instabilities, and a nonlinear cascade of energy into the submesoscale (Held et al., 1995). This is called surface quasi-geostrophy, and is focused entirely on surface buoyancy anomalies, under the assumption of uniform interior potential vorticity (Blumen, 1978). It predicts a surface kinetic and potential energy spectral slope of $k^{-5/3}$. When ageostrophic effects are accounted for (Boyd, 1992), which speed up the frontogenesis, the slope scales as $k^2$. Another theory is mixed layer baroclinic instability, where energy is drawn from lateral buoyancy gradients of the mesoscale eddy field via an Eady-type instability mechanism, which energizes the full depth of the mixed layer, while decaying rapidly below (Callies and Ferrari, 2015).

**Altimetry**

Sea surface height (SSH) derived from satellite altimetry has made it possible to calculate the wavenumber spectrum on a global scale. Under QG turbulence theory, the SSH wavenumber spectrum should follow a $k^5$ power law. An early study (Stammer, 1997) based on TOPEX/Poseidon and Jason-1/2 altimeters obtained a global SSH spectrum of $k^{-4.6}$, which corresponds to a velocity spectrum of $k^{-2.6}$, consistent with interior QG theory. However, the velocity spectrum in the Gulf Stream region (high eddy energy) was closer to $k^{-2}$ (also found by La Traon et al., 1990). The scale range was from 50 km to 100 km, with a peak at twice the wavelength of the Rossby radius of deformation. La Traon et al. (2008), using all available altimeter products over a 4 year period, found that all high eddy activity areas have a $\sim k^{-5/3}$ velocity spectrum, which follows surface QG theory.
Xu and Fu (2011) conducted a global survey of wavenumber spectra from the Jason-1/2 altimeter, between wavelengths 70-250km, to evaluate the geographic pattern of the spectral slope and its consistency with the current theories. They found steepest spectra dominated in the high eddy activity regions (i.e. the major current systems). However, the corresponding velocity wavenumber spectra never reach $k^{-3}$; even in the high eddy energy regions, slopes are significantly flatter than what is predicted by interior QG theory. The pattern of the spectral slope follows that of the global distribution of eddy variability; the higher the eddy variability, the steeper the slope. This indicates that the ratio between the geostrophic currents to the submesoscale fluctuations is higher. Conversely, in the lower energy regions (e.g. central subtropical Atlantic and Pacific), the ratio is smaller; larger-scale currents are weaker and the smaller-scale motions contain more energy relative to the large scale.

**Shipboard ADCP tracks**

Wang et al. (2010) calculated the wavenumber spectrum in the Gulf Stream mixed layer with both altimetry and shipboard ADCP measurements (the Oleander project dataset) between 20-250 km. From altimetry, the velocity spectrum yielded a $k^{-2}$ slope, which supports previous surface QG studies. The ADCP measurements, however, yielded the interior QG power law of $k^{-3}$. They explain this discrepancy is likely due to noise contamination in the SSH measurements. Callies and Ferrari (2013) utilized the same Oleander dataset down to 1 km resolution. They noted a transition shift below 20 km, where the spectral slope flattens out to $k^{-2}$, consistent with the horizontally isotropic model of the internal wave spectrum (the Garrett-Munk spectrum; Munk, 1981). Callies and Ferrari (2013) also analyze a shipboard ADCP dataset in the eastern subtropical North Pacific (Spice dataset; Ferrari and Rudnick, 2000), and find flatter slopes inconsistent with both interior and surface QG theories. Callies et al. (2015) observe seasonality in submesoscale motions, which explains the
shallower spectral slope under the theory of mixed layer baroclinic instability (Boccaletti et al., 2007). Most recently, Rocha et al. (2015), using 13 years of upper ocean (0-200 m) shipboard ADCP measurements in Drake Passage, found between 10 to 200 km that wavenumber spectra exhibit the $k^3$ power law, again confirming interior QG turbulence theory. They separate the spectra into across- and along-track components, and find the ratio of these components departs from isotropic interior QG theory (Charney, 1971). They attribute the departure to ageostrophic motions (in particular inertia-gravity waves) that are especially dominant at their smallest resolvable scale range 10-40 km (local deformation radius is 16 km).

**HF radar**

Altimetry provides observations from 70 to 250 km, which do not capture submesoscale processes. Several HF radar studies have now calculated the wavenumber spectra and provide more detail regarding the energy cascade at this intermediate scale. Lekien and Coulliette (2007) were the first to apply HF radar to calculate kinetic energy wavenumber spectra. Using hourly data, averaged in circles of radii 3 km mapped on a 1 km grid, they found a slope of $k^3$ in Monterey Bay. Haza et al. (2010) used VHF radar in the Gulf of La Spezia, with a range of 7 km and horizontal resolution of 250 m. They computed the average wavenumber spectra over ten meridional sections along a 2.5 km wide strip in the middle of the observation domain. This is done at 4 snapshots during the experiment to investigate how the spectrum changes in time. The broadband slope of the spectrum is generally flatter than $k^3$ (closer to $k^{5/3}$), but since the variability in time is so strong the spectrum is not in equilibrium but dominated by transient events with length scales close to $R_d$. These results highlight that a time average is required to resolve the general pattern of the spectral slope. Kim et al. (2011) performed a comprehensive analysis of the U.S. West Coast HF radar dataset. The wavenumber spectra from three different resolutions (1, 6 and 20 km), with scales from $O(1000)$ km
to $O(1) \text{km}$ show a slope of $k^{-2}$ at high wavenumber. They note that although the spectra vary with location because of spatial differences in forcing mechanisms, there is a robust $k^{-2}$ decay.

The conflicting results of wavenumber spectra indicate the current uncertainty in our understanding of observed wavenumber spectra across the world’s oceans. Research to-date (e.g. Xu and Fu, 2011; Callies and Ferrari, 2013) hints we should expect the energy partition between the mesoscale and submesoscale near strong baroclinic currents, such as the Florida Current, to follow interior QG theory to first order. Here we are able to add to this discussion using our unique HF radar dataset in the Straits of Florida that covers the Florida Current.

### 6.4.2 Kinetic Energy Wavenumber Spectrum in the Straits of Florida

To obtain a kinetic energy wavenumber spectrum from the surface velocity field, the following steps are taken:

1. At every grid point in $x$ and $y$, apply the Reynolds decomposition to the $u$ and $v$ velocity components ($u' = u - \bar{u}$, where $u'$ is the fluctuating field and $\bar{u}$ is the temporal mean field). This decomposition removes the mean Florida Current from the dataset.

2. To simplify the calculation, two rectangular regions are selected to obtain equal length transects in the north-south and east-west planes (Figure 6.13a). Within the north-south ‘box’, there are 38 meridional transects of length 66 km, and within the east-west box there are 30 zonal transects of length 60 km. Over the 2 year period, this equates to 501,518 zonal profiles and 630,942 meridional profiles of $u'$ and $v'$.

3. For each transect of $u'$ and $v'$:
   - Remove the spatial mean and linear trend
- Multiply by a Hanning window (the spectra is later adjusted for the reduced variance created by implementing this step).

- Compute the power spectra:

\[ P_u(k) = \left| \hat{u}'(k) \right|^2, \quad P_v(k) = \left| \hat{v}'(k) \right|^2, \quad (6.8) \]

where \( k \) is the along transect wavenumber and the caret denotes the discrete Fourier transform. The kinetic energy wavenumber spectrum \( K(k) \) is:

\[ K(k) = 0.5 \left[ P_u(k) + P_v(k) \right] \quad (6.9) \]

- The spectral matrix is averaged in space and time to obtain the averaged 1-D spectrum.

Windowing is necessary to taper the data to zero at the ends to reduce edge effects; without this the slope of the spectrum changes significantly. We also calculated the spectrum via the Welch method, using three 50% overlapping segments, and obtained the same overall result.

The 2 year average kinetic energy wavenumber spectrum is plotted in Figure 6.13b. Between 25 and 60 km, the slope decays slowly at \(-k^{5/3}\). Then, at approximately 25 km, which is the Rossby radius of deformation in this region (Archer et al., 2015a), the flow moves from the geostrophic to submesoscale regime, and the slope steepens to \(-k^3\). These slopes are consistent with the theory of interior quasi-geostrophy, in which it is the nonlocal dynamics that govern diffusion and mixing at scales smaller than the Rossby radius. This agrees with Wang et al. (2010) and Callies and Ferrari (2015) who found similar slopes in the Gulf Stream region. Below 10 km, the slope flattens to \(-k^2\). This is consistent with internal wave dynamics and is the slope predicted by the Garrett-Munk spectrum for internal waves with frequencies between \( f \) and \( N \) (Munk, 1981). The spectrum is cut off at a wavelength of 4 km (even though our data are at 1 km resolution) because below this scale noise dominates the calculation, flattening out the slope to near zero. The spectrum of a pseudorandom
normally distributed dataset with equal variance to the observations is calculated and plotted in Figure 6.13a as a comparison. The individual spectra $P_u(k)$ and $P_v(k)$ are displayed in Figure 6.13c and reveal as one might expect, that $P_v(k)$ contains more energy. Under isotropic 2-D flow, the ratio $P_v(k)/P_u(k)$ should be equal to $n$, where $n$ defines the slope of the spectrum $K(k) \sim k^{-n}$. The ratio is indeed very close with a value of $n = 3.02$. To determine the influence of near-inertial motions on the spectrum, the data was low pass filtered at 36 hours, which removes the higher frequency motions including tides. This reduces the energy of the spectrum, with an increasingly larger reduction at the higher wavenumbers. This indicates, as one might expect, that the higher frequency motions have smaller spatial scales. That the spectrum still flattens out at the highest reported wavenumbers suggests that it is noise, rather than the internal wave continuum, that is causing this flattening, since the internal wave frequencies have been removed by the filtering.

6.5 Energetics: Eddy-Mean Flow Interaction

6.5.1 Background

One approach to understanding eddy-mean flow interaction is through an examination of the energy flow; the magnitude and direction of the energy flow helps unravel the dynamics of the system. Barotropic instabilities draw energy from the kinetic energy of the mean flow, while baroclinic instabilities draw their energy from the mean potential energy. In this study, which utilizes surface current velocity measurements, we focus by necessity on the barotropic component. While this is far from satisfactory, we note previous studies of the energy conversions from barotropic and baroclinic instabilities in the Florida Current and Gulf Stream show that the barotropic term dominates (Dewar and Bane, 1985; Gula et al., 2015).
Figure 6.13  (a) Radar domain (black line = 100% coverage region) and area of calculation for zonal (black) and meridional (green) transects.  (b) 2 year spatial average kinetic energy wavenumber spectrum, and an estimate of the noise level of the velocity data.  (c) The individual spectra of $u$ and $v$.  (d) Kinetic energy wavenumber spectrum for observed (blue) and 36 hour low passed data (red).
To study the barotropic energy flow in a system, one can derive the conservation of eddy kinetic energy equation by multiplying the \( u \) and \( v \) momentum equations by their fluctuating components \( u' \) and \( v' \) and time averaging their sum, which yields:

\[
\frac{D}{Dt} \left( \frac{u'^2 + v'^2}{2} \right) = - \left[ \frac{\partial}{\partial x} \left( \frac{u'^2 + v'^2}{2} \right) + \frac{\partial}{\partial y} \left( \frac{u'^2 + v'^2}{2} \right) \right] - \frac{1}{\rho_0} \left[ \frac{\partial}{\partial x} u'p' + \frac{\partial}{\partial y} v'p' + \frac{\partial}{\partial z} w'p' \right] - \frac{\sigma w}{\rho_0} \frac{\partial}{\partial x}(u'v' + v'u') \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right] + g\alpha w'T' - \text{Dissipation} \quad (6.10)
\]

where \( p \) is pressure, \( g \) is gravitational acceleration, \( T \) is temperature, mean kinetic energy is \( \text{MKE} = 1/2 (\bar{u}^2 + \bar{v}^2) \) and EPE is eddy potential energy. The third term on the right hand side represents the conversion of mean kinetic energy into eddy kinetic energy. If the term in brackets is positive (ignoring the minus sign in front of it), it represents an upgradient flux, where energy is transferred from the eddy to the mean flow. Each component of this term can be easily calculated with HF radar over two dimensional maps to examine their relative magnitudes and how they vary across the channel. Whereas previous energetics studies in the Straits of Florida were based on observations that spanned longitude and depth, this study can contribute new insight by considering along-stream variations with a high spatial resolution (1 km) two dimensional gridded data product.

### 6.5.2 Reynolds Momentum Fluxes

The meridionally averaged Reynolds stresses \( \overline{u'u'} \), \( \overline{u'v'} \) and \( \overline{v'v'} \) are shown in Figure 6.14, together with the time mean zonal profile of the \( u- \) and \( v- \) components, with standard errors. Standard errors are calculated as \( \sigma/\sqrt{N} \), where \( \sigma \) is the standard deviation from which the mean is calculated and \( N \) is the number of independent observations, based on the decorrelation timescale of the flow. All three terms exhibit positive spikes on the inshore edge of the Florida Current core, where the
cyclonic shear is greatest, and weaker values in the core and anticyclonic shear region. This appears to be a robust pattern, exhibited in all previous studies offshore of Miami, regardless of instrument type or averaging interval (Webster, 1961; Schmitz and Niiler, 1969; Brooks and Niiler, 1977). Two-dimensional maps are presented in Figure 6. 15, and reveal the meridional heterogeneity in $u' u'$, and $u' v'$. Previous studies offshore of Miami were based on cross-sections and so were unable to resolve the two-dimensional pattern of momentum fluxes. The cross-stream term $u' v'$ exhibits a nearshore meridional positive-negative pattern, which is related to the quasi-stationary meander of the jet at this latitude, where it is still turning in the channel: it is positive on the downstream side of the wave crest and negative on the upstream slope. It also coincides with a shoreward movement of the 200 m isobath, creating a narrow shelf region. The same pattern was found by Nishida and White (1982) in the quasi-stationary meanders of the Kuroshio Extension. The positive value of this term implies an eastward transport of northward momentum, accelerating the mean flow.

![Figure 6.14](image.png) (top) Meridionally averaged time mean $v$- and $u$-components; (bottom) Meridional mean of Reynolds stresses $u' u'$, $u' v'$ and $v' v'$ averaged over 2 years. Shaded regions denote standard errors.
6.5.3 Barotropic Kinetic Energy Transfer

The horizontal kinetic energy transfer term between the fluctuations and mean flow can be expanded:

\[
\mathbf{u}' \mathbf{v}' \cdot \nabla \mathbf{u} + \mathbf{u}' \mathbf{v}' \cdot \nabla \mathbf{v} = \mathbf{u}' \mathbf{u}' \frac{\partial \mathbf{u}}{\partial x} + \mathbf{u}' \mathbf{v}' \frac{\partial \mathbf{u}}{\partial y} + \mathbf{u}' \mathbf{v}' \frac{\partial \mathbf{v}}{\partial x} + \mathbf{v}' \mathbf{v}' \frac{\partial \mathbf{v}}{\partial y} \tag{6.11}
\]

Error is calculated for the transfer terms (and the terms which follow) by assuming the Reynolds stress and mean velocity gradients are not independent (Szabo and Weatherly, 1979), for example:

\[
Err \left( \mathbf{u}' \mathbf{v}' \frac{\partial \mathbf{v}}{\partial x} \right) = Err (\mathbf{u}' \mathbf{v}') \cdot \frac{\partial \mathbf{v}}{\partial x} + \mathbf{u}' \mathbf{v}' \cdot Err \left( \frac{\partial \mathbf{v}}{\partial x} \right) + Err (\mathbf{u}' \mathbf{v}') \cdot Err \left( \frac{\partial \mathbf{v}}{\partial x} \right) \tag{6.12}
\]

![Figure 6.15 Spatial coverage of Reynolds stresses](image)

For the full term comprising all four components, the error is taken as the square root of the squared sum of all errors. The meridionally averaged and zonally averaged one-dimensional profiles of the energy terms are presented in Figure 6.16. For the cross-stream profile (meridionally averaged), the net horizontal kinetic energy conversion is positive on the inshore side of the Florida Current, and weakly negative in the offshore side. This means that eddies are releasing energy in the cyclonic shear zone, and gaining energy in the anticyclonic shear zone. This picture is consistent with previous studies of the Florida Current and Gulf Stream energetics (Webster, 1961; Schmitz and Niiler, 1969; Brooks and Niiler, 1977). However, several previous studies also noted that when integrated across the jet, the net conversion from eddies to the mean is negligible (Schmitz and Niiler,
The along-stream pattern of energy conversion is shown in Figure 6.16, which plots the zonally averaged energy terms as a function of latitude. When integrated across the jet, the values of the energy conversion terms are an order of magnitude smaller. A downstream variability in the sign of the energy transfer term emerges in this perspective, the term oscillates from positive to negative. While the error bars are quite large, there is still a significant change of sign with latitude. Previous studies in the Straits of Florida have alluded to this difference in stability; Johns and Schott (1987) and Leaman et al. (1987) found upgradient fluxes from eddies to the mean, while Lee et al., (1986) found a downgradient flux. These differences imply changing stability of the Florida Current, perhaps associated with topographic changes in latitude. Gula et al. (2015) ran a ROMS model to study Gulf Stream dynamics along the US eastern seaboard and showed that the horizontal kinetic energy conversion term exhibits strong along-stream variability, which is a function of topographic features along the shelf.

It is of note that the largest contributor to the eddy energy transfer is the downstream conversion term $\overline{v'u'} \partial \overline{v} / \partial y$, rather than the more commonly considered cross-stream term $\overline{u'u'} \partial \overline{v} / \partial x$. This has been noted before in energetics studies of western boundary currents (Szabo and Weatherly (1979) for the Kuroshio Extension, and Hager (1977) for the Gulf Stream). The reason for this lies in the magnitude of the downstream Reynolds stress term $\overline{v'u'}$ (Figure 6.14), which dominates over the cross-stream shear of the downstream component $\partial \overline{v} / \partial x$.

Velocity variance ellipses characterize the degree of anisotropy of the fluctuations, and the orientation of the ellipse relative to the mean shear indicates the direction of the horizontal eddy momentum flux (Morrow et al., 1994). The principal direction of the velocity variance aligned with the major axis is:

$$\theta = \tan^{-1} \left( \frac{\sigma_{11} - \overline{u'u'}}{\overline{u'u'}} \right)$$

(6.13)
where variance in the major axis is given by:

\[
\sigma_{11} = \frac{1}{2} \left( u'u' + v'v' + \sqrt{(u'u' - v'v')^2 + 4(u'v')^2} \right) 
\]  
(6.14)

and along the minor axis:

\[
\sigma_{22} = (u'u' + v'v') - \sigma_{11}. 
\]  
(6.15)

Figure 6.16 (top) Turbulent kinetic energy transfer term (gray bold line) with standard error, and its two largest component terms meridionally averaged. (bottom) Energy conversion term zonally averaged. Positive values indicate a loss of eddy energy to the mean flow.
An isotropic ellipse indicates random fluctuations in all directions, while a line ellipse at 45° indicates $u'$ and $v'$ are perfectly correlated (Cronin and Watts, 1996). For eddies to extract (supply) energy from the mean, the orientation of the ellipse must lean against (with) the shear (Pedlosky, 1987). The variance ellipses are presented in Figure 6.17, together with the direction and magnitude of the mean flow. Also shown is a schematic representation of how the ellipse orientation interacts with the mean flow. As the eddy energy transfer terms indicate (Figure 6.16), the inshore cyclonic shear zone is a region where eddies are supplying energy to the mean flow (ellipses leaning against shear), while in the offshore anticyclonic shear zone eddies are gaining energy (ellipses leaning with the shear).

![Figure 6.17](image)

Figure 6.17 (left) Schematic showing how the orientation of the ellipse acts to supply or extract momentum from the jet. (right) Eddy variance ellipses plotted over mean current magnitude (colored contours) and direction (arrows).

The eddy kinetic energy conversion term has an analogous term in the mean kinetic energy equation:

\[
\bar{u}\nabla \cdot (\bar{u}'u') + \bar{v}\nabla \cdot (\bar{u}'v') = \bar{u}\frac{\partial}{\partial x} \bar{u}'u' + \bar{u}\frac{\partial}{\partial y} \bar{u}'v' + \bar{v}\frac{\partial}{\partial x} \bar{u}'v' + \bar{v}\frac{\partial}{\partial y} \bar{v}'v' \tag{6.16}
\]

This term describes the conversion of mean kinetic energy into eddy kinetic energy. In a closed system, Equation 6.11 and 6.16 balance each other exactly, so that a loss of mean kinetic
energy equates to a gain in eddy kinetic energy (and vice versa). However, for an open system, such as the Straits of Florida, the two terms differ by a divergence of energy flux across the open boundary (Harrison and Robinson, 1978):

\[
\begin{align*}
\mathbf{u}'\mathbf{u}' \cdot \nabla \mathbf{u} + \mathbf{u}'\mathbf{v}' \cdot \nabla \mathbf{v} + \left[ \mathbf{u} \nabla \cdot (\mathbf{u}'\mathbf{u}') + \mathbf{v} \nabla \cdot (\mathbf{u}'\mathbf{v}') \right]
= \nabla \cdot \mathbf{u}'\mathbf{u}' + \nabla \cdot \mathbf{v}'\mathbf{v}'
\end{align*}
\]

(6.17)

This divergence term, if non-negligible, implies non-local energy exchange, where energy released from the eddy or mean flow is not used to sustain the mean/eddy in the local region, but is exported out of the local domain. The largest component of each of the terms in Equation 6.17 (the downstream component) is plotted in Figure 6.18, as a function of longitude and latitude, and in both dimensions in Figure 6.19. There are two observable features: (1) the mean kinetic and eddy kinetic energy terms are generally anti-correlated; and (2) the terms are not equal – the mean kinetic energy term is larger and therefore the divergence term is significant over most of the region. The general pattern that emerges is that in the cross-stream, eddies are losing energy in their interaction with the mean flow inshore, while the mean flow is gaining energy, and in the offshore side both eddies and the mean are gaining energy. The divergence term on the inshore side is not significantly different from zero; this suggests a direct exchange of energy between eddies and the mean flow, in which eddies are acting to accelerate the mean flow. In the core and anticyclonic zone the divergence is significantly negative, which implies there is an energy flux across the open boundary. In the along-stream direction, the mean kinetic energy term is larger in magnitude at all latitudes, again indicating that a flux of energy across the open boundaries is necessary to supply the mean kinetic energy field. This is apparent in the two-dimensional plots, where it can be seen the mean kinetic energy term dominates the balance and the divergence is large. These results indicate care must be taken when relating the eddy energy transfer term to an exchange of energy between the mean and fluctuating...
flow field. The divergence term needs to be accounted for, and if significant, indicates that energy is converted but not exchanged locally between the eddy and mean flow. While this has been found in other studies (Rhines, 1977; Harrison and Robinson, 1978; Bryden, 1982; Nishida and White, 1982; Dewar and Bane, 1985; Chen et al., 2014), it has not before been explicitly shown in the Straits of Florida.

Figure 6.18  Turbulent kinetic energy transfer term, mean kinetic energy transfer term and divergence term plotted as a function of (top) longitude and (bottom) latitude. Positive values indicate a loss of energy (for the divergence term this implies energy is fluxed out of the domain).
6.5.4 Eddy Viscosity

Eddy viscosity, which is analogous to molecular viscosity but on a much larger scale, quantifies the turbulent transfer of momentum by eddies, and is often used to parameterize mixing in numerical models. For the upgradient momentum flux observed in the inshore cyclonic zone of the Florida Current, this implies a so-called negative viscosity. It can be calculated as:

$$K_H = \frac{-u'v'}{\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)/2}$$  (6.18)

The pattern of eddy viscosity closely follows the lateral shear of the Florida Current; it is negative in the cyclonic zone and positive in the anticyclonic zone, so there is a discontinuity at the axis (Figure 6.20). Values range between $\pm 1 \times 10^4$ m$^2$ s$^{-1}$, depending on the location relative to the core. This is similar in scale to that calculated by Bryden (1982) in the cyclonic shear zone of the Gulf Stream ($-0.9 \times 10^4$ m$^2$ s$^{-1}$).
6.6 Discussion and Summary

The northward velocity component $v$ exhibits a mean decorrelation length scale that is strongly polarized due to the influence of the Florida Current jet, which can trap and advect fluctuations northward. The $v$-component decorrelates to 0.5 in 20 km zonally, and over 30 km meridionally. The eastward component $u$ exhibits a more isotropic decorrelation structure, reaching 0.5 within ~25 km in both planes. The decorrelation length scale varies with the frequency of the fluctuations. The low frequency fluctuations ($> 48$ hrs) exhibit the strongest meridional directionality, and propagate northward. At the near-inertial frequency, length scales are reduced, but still show directionality in the along-stream direction. Fluctuations in this frequency band have a mean eastward
direction of propagation, or out-of-phase relationship. The high frequency (< 20 hrs) motions, including the semi-diurnal tidal constituents, exhibit the smallest length scales, and a more complex phase map. Coherence as a function of latitude reveals the strong sub-inertial correlation from south to north in the domain, consistent with the correlation maps. Other latitude coherent peaks include the diurnal and semi-diurnal tidal frequencies and as well as the broadband near-inertial frequency. In longitude, the low frequency phase map explicitly shows westward propagation, which may be due to westward propagating signals from the interior ocean basin. Indeed, the Northwest Providence Channel has recently been shown to act a conduit for the transmission of eddy variability from the interior Atlantic basin (Frajka-Williams et al., 2013). Decorrelation time scales also display a strong spatial dependence, with the longer timescales associated with the location of the Florida Current jet core. The integral timescale, as calculated from the observed data, does not converge within the length of the timeseries. We follow Mariano and Chin (1996) by fitting an autocorrelation function to the data, based upon a wave and turbulent component, and a term accounting for the noise. We find the integral timescale as the exact integral of the autocorrelation function. Via this method, we obtain $T^*$ values over the domain that range from 2 to 10 days for $v$ and 1 to 2 days for $u$.

The frequency spectra reveal peaks at the major tidal constituents ($O_1$, $K_1$, $N_2$, $M_2$ and $S_2$), the near-inertial band (20 to 36 hrs) and the sub-inertial band (>48 hrs). In the sub-inertial, we find dominant signals in $v$ at 2-4 days, 6-7 days, 9-10 days and 30 days. For the $u$-component, dominant periods are 3 days, 5 days, 7 days and 20 days. These periods are consistent with previous studies in the Florida Straits (e.g. Düing et al., 1977; Schott et al., 1988; Johns and Schott, 1987; Shay et al., 1998). We did not encounter the 10 hr signal that has been investigated in previous studies of HF radar and ADCP measurements (Peters et al., 2002; Soloviev et al., 2003). This baroclinic supertidal oscillation appears during the summer, and exhibits strong inter-annual variability, with summer 1999
having much greater spectral energy than 2001 and 2002, and not prominently observed during summer 2000 (Soloviev et al., 2003). Neither in the power spectra nor through bandpass filtering did we see the 10 hr signal with a summer maximum. Both studies that have investigated this signal were focused on the strongest 1999 season, and were inshore (<150 m water depth) of our radar domain. Either the signal did not exhibit the same magnitude during 2005 and 2006, or our data were too far offshore to observe it.

Seasonality in the low frequency period signals are revealed using a wavelet transform. Peak variability at 7-10 days is observable in the winter months (September to April) and quiescent during the summer, so is most likely attributed to the passage of cold fronts with the same periodicity. At the shorter periods of 3-4 days, there is a peak in variance between March and May. These motions coincide with peaks in jet intensity, meandering and sea level fluctuations, and smaller peaks in surface transport and meridional wind stress (see Figure 4.13 in Chapter 4).

We calculate a time and domain averaged 1-D kinetic energy wavenumber spectral slope of $k^{-3}$, in the scale range between 4 and 60 km, which is consistent with interior QG theory, and agrees with recent research based on observations in the Gulf Stream (Wang et al., 2010; Callies and Ferrari, 2015). This result implies that nonlocal dynamics are dominant in driving local transport and dispersion at submesoscales (Beron-Vera and Olascoaga, 2009). At wavelengths below 10 km the spectrum flattens out to $k^{-2}$, which is the slope predicted by an ocean regime dominated by internal waves (Garrett-Munk spectrum). However, this may be attributed to noise in our data, since low pass filtering to remove the internal wave frequencies did not change this flattening effect. We divided the spectra into winter and summer to investigate seasonality, and found elevated variance during the winter but no significant change in the slope, which indicates there is no change in the mesoscale to submesoscale transition. This is contrary to Callies et al. (2015), who showed wintertime spectra
with shallower $k^2$ slopes that implies a regime shift with relatively higher energy in the submesoscale. We also compared wavenumber spectra averaged in both the cyclonic and anticyclonic regions, and found no change in slope, only a more energetic spectrum in the cyclonic region. Our calculation of the spectrum is based on several limiting assumptions, chiefly that the flow is isotropic and stationary so can be characterized with a one-dimensional spectrum. We have limited our study to 1-D spectra of zonal and meridional transects, a preferable method would be to apply a 2-D Fourier transform and reduce to a 1-D spectrum by summing the densities within discrete annuli in wavenumber space (Errico, 1985).

Eddy-mean flow interaction has been investigated via the energy conversion term in the conservation of eddy kinetic energy equation. The overall picture of eddy-mean flow interaction in the channel between 25°N and 26°N is divided; south of ~25.5°N there is an upgradient flux in the cyclonic inshore edge of the Florida Current. Based on the peak value of the energy conversion (0.023 cm$^2$ s$^{-3}$), if acting in isolation, it could spin-up the Florida Current of speed $U=150$ cm s$^{-1}$ in $T = U^2 / 0.023 \approx 11$ days. Seaward of the Florida Current core, in the anticyclonic shear zone, there is a weakly downgradient flux indicating a transfer of kinetic energy from the mean to fluctuations. North of ~25.5°N the pattern reverses; there is a downgradient transfer on the inshore cyclonic side, and upgradient transfer on the offshore side. We tested varying the time period over which the mean is calculated and found this result to be robust for any period sufficiently long enough (periods too short are noisy because they fail to capture a sufficient number of wavelengths of the fluctuations to obtain an averaged effect). This along-stream reversal appears to be a function of the time mean quasi-stationary meander of the Florida Current as it follows the continental shelf around the peninsula; inshore it is upgradient on the downstream side of the wave crest (< ~25.5°) and downgradient on the upstream slope (> ~25.5°). We note here that we have calculated the energy
transfer terms in the stream coordinate system and find a qualitatively similar result. However, that analysis (not presented here) in stream coordinates is incomplete because it does not re-derive the conservation of eddy kinetic energy equation with each coordinate frame shift (which occurs every time step).

In this chapter, we have described the fluctuating surface velocity field in the Straits of Florida between 25° to 26°, over a range of temporal and spatial scales. The fundamental message is the Florida Current dominates the ocean circulation in this region, and strongly determines the character of the fluctuations. This strongly sheared northward flowing current meridionally extends decorrelation length scales and polarizes fluctuating motions in the along-stream plane. So it is perhaps not surprising that the kinetic energy wavenumber spectral slope is $k^{-3}$, given the energy contained within the mesoscale flow field in this channel. HF radar offers excellent spatial resolution coverage of the surface velocity field, and has allowed us to explore the energy exchange between the mean and fluctuations in two dimensions to show the along-stream and cross-stream pattern within the channel. However, without accompanying subsurface measurements of current velocity and density, it is difficult to construe what the total sum amounts to in terms of exchange of energy between the mean and fluctuations. Examining the total energy budget, both the barotropic and baroclinic transfer terms in the full water column and the along-stream direction is an important step that remains to be undertaken.
Chapter 7

Concluding Remarks

*Principle findings, new contributions and open questions.*

In 1513 Ponce de León first described the Florida Current, and over five hundred years since this first description we are still working to unravel the dynamics of this powerful and important ocean current. Much progress has been made in the interim; we have discovered why it exists (western intensification), mapped its mean location from source to end, measured its hydrographic structure and fluctuations and recreated its leading order dynamics in numerical models. We have come a long way from the early hypotheses of its source, such as the outpouring of rivers in the Gulf of Mexico (*Thevet*, 1575), or the attraction of the sun that forms a ‘long mountain of water’ along the equator that is carried toward the west and breaks on the South American coast, running south and north (*Vossius*, 1663). These advances have been made possible through progress in numerous fields, including fluid dynamics, meteorology, and computer science. In the last century and a half after Pillsbury anchored the steamer *Blake* midstream in the Florida Current, a number of large field campaigns have been conducted to better understand the transport variability and fluctuations of the Florida Current. In this dissertation, the overarching aim has been to contribute to this rich history of
scientific investigation by utilizing a new and powerful instrument that provides us with long-term high resolution surface current measurements over a large surface area of the Florida Straits channel. This dataset has allowed us to examine the Florida Current with new methodologies unavailable to past investigators.

We began by describing the mean horizontal structure of the Florida Current, and its variability via a timeseries analysis of the following metrics: meandering, intensity, width, shear and surface transport. The relationship between these variables, and with independent wind and volume transport data revealed the complex nature of this western boundary current forced both locally and remotely, with high variance at sub-seasonal and inter-annual timescales. Next we focused on two coherent eddy events that had exceptional data coverage and signal clarity, and contrasted to a quiescent period of no eddy activity. These two case studies were examined in terms of their kinematics as they moved through the radar domain; they demonstrated the complex dynamics in this region, where transient instabilities drive surface divergence, particle dispersion and cross-shelf exchange under conditions of significant strain and vorticity. This study also revealed how these instabilities are very quickly advected through the Straits, passing through the radar domain (with a meridional extent of approximately 80 km) in a matter of hours to days. In the final research chapter of this dissertation, we take a broader view of these instabilities by characterizing the time and space scales of the fluctuating velocity field, from hours to days to months. The spatial and temporal dependence of the dominant periods of variability are examined, and the partition of energy across wavenumber is computed to distinguish the dynamical regime in the Florida Straits. Eddy-mean flow interaction is addressed by calculating the magnitude and spatial pattern of the barotropic energy conversion term in the equation for the conservation of eddy kinetic energy.
7.1 New Contributions

This doctoral work offers new contributions to the existing body of knowledge of the Florida Current:

- It is shown for the first time that the width exhibits an annual cycle in the Straits of Florida, with a boreal summer maximum and late winter minimum. This summer maximum is also exhibited by surface transport and corresponds to a peak in the local meridional wind stress.

- A stream coordinate method is developed to work with a 2-D HF radar dataset. In this new coordinate frame, the time-averaged Florida Current velocity field is stronger and narrower – more akin to an instantaneous profile of the jet. The core of the jet accelerates between 25°N and 26°N, presumably due to the narrowing channel profile. Meandering motion, which is removed by this method, is shown to account for approximately 45% of the eddy kinetic energy of the fluctuations.

- An anticyclonic frontal instability was described for the first time. During October 2006, a transient, coherent near-inertial signal was identified. This signal was masked by the strongly sheared Florida Current flow, and once separated by bandpass filtering, revealed counter-rotating horizontal oscillations, with a wavenumber and frequency consistent with near-inertial wave dynamics.

- Examination of the flow field kinematics during the passage of a cyclonic frontal eddy revealed the event was associated with large magnitude fluctuations in the particle separation metric IROS (instantaneous rate of separation), indicative of cross-shelf transport and mixing.

- Time and length scales of fluctuations are approximately 20 to 30 km, increasing offshore. The $v$-component exhibits strong directionality, with a 2:1 ratio between meridional and
zonal scales, while the $u$-component is more isotropic. Length scales decrease with increasing frequency, from sub-inertial (>48 hrs) to near-inertial (20-36 hrs) and high frequency (<20 hrs).

- Integral timescales are a function of offshore distance due to the presence of the Florida Current, ranging from 2 to 10 days for $v$ and 1 to 2 days for $u$.

- The 1-D kinetic energy wavenumber spectrum was calculated for the first time in the Straits of Florida, and found to exhibit a $k^{-3}$ decay slope, which is consistent with interior quasi-geostrophy theory, implying it is the non-local larger-scale dynamics that drive local transport and dispersion in this region.

- The barotropic energy conversion from eddy to mean reveals a 2-D pattern, where south of 25.5°N there is a weakly negative (positive) upgradient (downgradient) flux in the cyclonic (anticyclonic) shear zone, and vice versa north of 25.5°N. This along-stream reversal in pattern appears to be a function of the time-mean quasi-stationary meander of the Florida Current as it follows the continental shelf in its cyclonic turn around the peninsula. The magnitude of the divergence of energy flux is significant, however, suggesting there is not an equal exchange of energy between the eddy and mean, but rather an export out of the open domain.

This work also confirms several previous results using a new dataset:

- Sub-seasonal variance between 3 days to 3 weeks dominates the fluctuating flow field.

- The fluctuations are highly nonstationary, with strong inter-annual variability and seasonality. Higher frequency variance (7 to 10 days) exists in the winter months (September to April), with lower frequency periods (>30 days) present year round.
- Cyclonic frontal eddies are associated with strong surface divergence, leading to upwelling of cold sub-surface water in the core of the eddy, observable in satellite SST imagery.

### 7.2 Open Questions

There are still many questions to be answered regarding ocean dynamics within the Straits of Florida; some confined to the specific nature of the Straits, others global in scope. Several questions emerged during this dissertation research that could not be investigated within the time frame and data used:

- Following the winter minimum in width, there is a sharp peak in jet intensity and shear, and a secondary peak in surface transport. This is an interesting discovery and requires more data to be explained.
- The 10 hour signal previously observed in the Straits (Peters et al., 2002; Soloviev et al., 2003) was not evident in the two year dataset analyzed here. Is this due to different physical conditions between years, or our data not capturing the signal, and if the latter, why?
- What is the subsurface structure of the near-inertial anticyclonic signal that was observed for the first time seaward of the Florida Current? With measurements at depth, the nature of the fluctuations we observed could be better understood. Near-inertial waves propagating downwards in this baroclinic current would experience a critical layer, leading to direction changes and wave amplification and breaking. What is the recurrence frequency of these anticyclonic features? What is their contribution to transport and mixing?
- While it has been found that along the Gulf Stream north of 27°N there are preferred large-scale regions of eddy growth and decay locked by topographic features (and explained by the direction of barotropic energy flux), the results in this dissertation identify a similar pattern
may exist within the Straits. An investigation of the along-stream formation, evolution and
decay of frontal eddies and the associated energetics terms would shed light on this.

As an ocean observing tool, HF radar still has a lot to offer in the way of new insights. In addition to
surface current maps, phased-array radar systems deliver the ability to measure the directional wave
spectrum and surface wind field. These observations, when used collectively, enable a thorough
examination of the coastal ocean surface current and wave field, and can help elucidate mechanisms
still not fully understood, such as wave-current interaction and Ekman divergence in geostrophic
shear.
References


Appendix

Comparison of Observations to Ekman Theory

To investigate further, we can compare the gain between the wind and HF radar measurements to the expected response from Ekman theory. The expected surface current response (Ekman, 1905) is:

\[ V_0 = \frac{0.0127}{\sqrt{\sin|\phi|}} U_{10} \]

where \( \phi \) is latitude and \( U_{10} \) is wind speed at 10m above the sea surface. For a 10 m s\(^{-1}\) wind speed (common in the Straits of Florida), the expected Ekman surface velocity response at 25.42°, in the downstream direction (\( v = V_0 \cdot \sin(\pi/4) \)) is 0.21 m s\(^{-1}\).

To calculate the relationship between wind and currents in the same units, rather than use the HF radar derived surface transport (m\(^3\) s\(^{-1}\)), we take the cross-stream average surface downstream velocity at 25.42° (m s\(^{-1}\)). We calculate the frequency response function \( H_{xy} = G_{xy}/G_{xx} \) (Emery and Thomson, 1998), where \( G_{xy} \) is the cross-spectral density and \( G_{xx} \) is the autospectrum of the wind. Gain is defined as the magnitude \( |H_{xy}| \) and \( \varphi(f) \) is the phase shift \( (H_{xy} = |H_{xy}|e^{-i\varphi(f)}) \). The values of gain at the three periods
(40 days, 6.5 days and 4 days) are: 0.056, 0.019 and 0.019 (Figure A1). Therefore at the 4 and 6.5 day periods, a 10 m s⁻¹ wind speed produces a surface velocity response of \(10 \times 0.019 = 0.19\) m s⁻¹. For the 40 day period, the response is 0.56 m s⁻¹.

The 4 day period response of 0.19 m s⁻¹ is very close in magnitude to the expected Ekman response of 0.21 m s⁻¹, consistent with our hypothesis of a directly-forced surface Ekman flow at this period. However, we note that the 6.5 day period gain also has a similar magnitude as the expected Ekman surface flow, even though the phase shift suggests a wind-forced geostrophic response, as we discussed in Section 4.6.3.

![Figure A1](image.png)

*Figure A1.* (top) Coherence between meridional wind and HF radar mean surface velocity across 25.42°; (middle) phase shift; and (bottom) gain.