Advance Supply Contracts for Contingency Inventory

Sercan Demir
University of Miami, sercanxdemir@gmail.com

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UNIVERSITY OF MIAMI

ADVANCE SUPPLY CONTRACTS FOR CONTINGENCY INVENTORY

By

Sercan Demir

A DISSERTATION

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ADVANCE SUPPLY CONTRACTS FOR CONTINGENCY INVENTORY

Sercan Demir

Approved:

Murat Erkoc, Ph.D.
Associate Professor of Industrial Engineering

Shihab S. Asfour, Ph.D.
Professor of Industrial Engineering

Nurcin Celik, Ph.D.
Associate Professor of Industrial Engineering

Raphael Boleslavsky, Ph.D.
Associate Professor of Economics

Guillermo J. Prado, Ph.D.
Dean of the Graduate School
Contingency inventory planning plays a significant role in many businesses. Firms must take low-probability big-impact events such as natural disasters and supply disruptions into consideration in their supply planning since they might cause crippling and irreversible effects on businesses. Companies that have operations in disaster-prone regions often face significant opportunity loss from shortages due to demand surge caused by rare events. With a robust contingency inventory planning policy, a firm can turn the negative effects of a low-probability event into an opportunity. Inventory pooling is one approach to keep contingency inventory for possible future use in case a demand surge caused by a disruptive low-probability event occurs. By pooling the inventory, the participating companies mitigate the risk of running out of stock when the demand surge occurs.

In the first part of this study, we present a game theoretic analysis of a decision problem of two buyers and a supplier who keeps contingency stock for the buyers for the future use in case of a low-probability high-impact event such as a hurricane or an epidemic. The buyers operate their businesses in independent markets. However, they reserve contingency inventory from a single supplier. The proposed sequential game starts with the supplier’s unit reservation fee offer to the buyers. In the second stage, the buyers decide on their reservation quantities. In the last stage, the supplier
acquires contingency inventory based on the reservation amounts. The reservation fees are kept and the reserved inventories are salvaged by the supplier if there is no low-probability event occurs.

In the second part of this study, we present a game theoretic analysis of a decision problem of two buyers and a supplier where the buyers are the first movers (leaders) of the game. This time, the sequential game starts with each buyer’s individual reservation fee decision. At the beginning, buyers simultaneously move and offer deductible and non-refundable reservation fees to a single supplier for a unit product to be held as backup inventory over a single period. In the second stage, the supplier decides on her reservation quantities for each buyer based on their reservation fee offers. If a buyer is inflicted by the low-probability event, her reserved inventory is supplied by the supplier. If her reservation amount is not enough, the supplier can supply additional quantities from the reservation of the other buyer unless she exercises her reservation.

Another approach to tackle shortages involves expedited replenishment. Expedited replenishment opportunities after obtaining updated demand information may exist to cover the potential shortages. However, usually such options impose high procurement costs. In the third part of this study, we study a finite horizon multi-period procurement and inventory control problem of a buyer where she has the option of ordering seasonal products with an advance contract before the beginning of the selling season. Purchasing large quantities with an advance contract creates cost saving opportunity to the buyer since the unit product cost is lower than the subsequent stages. However, at the time of the advance contract, the buyer is less informed about the market demand. The buyer can make additional expedited replenishments
later during the season at predefined modes. During this time interval, the market
signal for the buyer is updated. Our goal in this chapter is to determine the most
efficient way to make procurement decisions for the buyer. The framework developed
in this part of the study provides efficient policies to the buyer who has to deal with
multiple sources of uncertainty such as market signal variability and demand across
multiple periods during the selling season.
to my family
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"I am not leaving a spiritual legacy of dogmas, unchangeable petrified directives. My spiritual legacy is science and reason. What I wanted to do and what I tried to achieve for the Turkish Nation is quite evident. If those people who wish to follow me after I am gone take reason and science as their guides they will be my true spiritual heirs." M.K. Ataturk (1881-1938)

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CHAPTER 1

Introduction

This study investigates two contracting approaches in procurement for availability of stocks: contingency inventory reservation and advanced procurement. The first one relates to designing contracts to enable contingency inventory pools for the buyers who operate in markets that are subject to low probability high impact disruptive events. Contingency inventory planning plays a significant role in many businesses. Rare events such as natural disasters and health epidemics bring businesses face to face with arduous task of reacting to supply disruptions and/or demand surges in timely fashion. If unprepared, the businesses either suffer under crippling financial and operational effects or experience significant opportunity costs due to unfilled surge in demand.

Toyota suffered from severe parts shortages when the magnitude-9.0 earthquake and tsunami destroyed many factories in Northeastern Japan on March 11, 2011. The company’s global production in March 2011 dropped by 29.9% from a year ago, while its sales in Japan tumbled 45% for the month (usatoday.com 2011). In October 2012, Hurricane Sandy ravaged the Atlantic coast with record flooding and winds. As a result of the disaster, 8 million people left without power, thousands of flights canceled, businesses shutdown for days and billions of dollars lost. 1.5 million businesses...
located across 19 counties in the tri-state region potentially affected by Hurricane Sandy (dnb.com 2012). Recently, several countries in Latin America experienced a significant shortage of contraceptives and bug sprays with the emergence of the ZICA outbreak, which made it increasingly difficult to combat the epidemic (foxnews.com 2016). The list in regard to such recent events and their adverse consequences can be expanded. Most businesses are aware of the potential benefits of effective contingency plans that usually involve acquiring back up capacity and/or inventory. However, this is rather a challenging undertaking in that such disruptive events are uncertain and typically low probability. Provisioning for such events typically necessitates taking risks of experiencing large overage costs. On the other hand, for a prepared company, reward is also significant as such events may not only bring a short term spike in demand volume but also strengthen its market presence and share in long term.

Consumable products such as flashlights, batteries, portable generators, and other emergency supplies are the most common necessities during and after natural disasters such as hurricanes. In 2004, when the Hurricane Frances was on its way, the Wal-Mart Stores executives turned that situation into a great opportunity with the help of one of their newest data-driven weapons which they call it predictive technology. Based on the historical shoppers data stored during past-hurricane seasons in their computer network, the company could predict the top-selling items and their expected demand surge. Just before the Hurricane Frances hit Florida’s Atlantic coast, the executives decided to stock Walmart stores in the region based on the data from Hurricane Charley that had struck several weeks earlier. By predicting the demand increase, the company was able to replenish the shelves for certain products in its stores in the path of a hurricane [1]. When the banana plantations in Central America were
destroyed by the Hurricane Mitch, Dole lost 4% of its market share to its much smaller rival Chiquita. While Dole had no alternative supply source, Chiquita had contracts in place with suppliers from Australia, which helped the company continue supplying their products in the market [2].

As mentioned earlier, the second part of our study focuses on advance procurement contracts with market signal updates and multiple replenishment modes. Most companies that are selling seasonal products update their sales forecasts through demand (market) signal. Some of the important market signals used are customer surveys, market trends, market research and advance booking information. In deciding when and how much to order, firms must consider the lead time risk and the demand risk, i.e., the accuracy of their demand forecast [3]. To minimize risk, oftentimes firms prefer to postpone their inventory buildup and quantity commitments until after market signals are obtained and right before the start of the selling season. Usually market signals are obtained shortly before the selling season starts. Early commitment has its own advantages to the firms and under certain situations companies might want to make early commitments to avoid long capacity/production lead times and save cost. Oftentimes, in order to mitigate buyers’ holding risk, suppliers offer their products with discounted prices for advance orders and high quantity purchases. Therefore, replenishing some of inventory through advance ordering provides price incentive to a firm when the price discounts are high enough to offset the firm’s early inventory commitment risk.

A major downside of the advance contracts is the lack of more reliable signals about the market demand for the selling season. Generally, companies sign advance supply contracts with their suppliers committing to a certain quantity of seasonal
product before their demand forecast is finalized. As the selling season approaches, customer demand information is continuously updated through market signals and hence, becomes more accurate. The season demand uncertainty usually cannot be resolved thoroughly until the beginning of the selling season. The demand forecast updates might necessitate making additional product purchasing through expedited orders from local spot markets on the eve of the selling season, and/or during the selling season. In this context, optimal advance supply contract design must incorporate the trade-offs between advance procurement and expedited replenishment costs, and the value of information update.

1.1 Literature Review

Supply chain risks and risk mitigation strategies can be categorized under variety of groups. These risks and risk mitigation strategies are categorized in [4, 5]. For instance, disruptions due to the natural disasters, labor disputes, supplier breakdown, wars, and terrorism as one of the top risk categories for an organization [4]. The researchers in [4] discuss the impact of major mitigation strategies that can be used to reduce supply chain disruption risks. They report the two of them: keeping additional inventory or having redundant suppliers greatly decrease such risks.

Tomlin [6] classifies tactics for managing risks under three major categories: financial mitigation (i.e. business interruption insurance), operational mitigation (i.e. inventory, and sourcing), an operational contingency (i.e. rerouting, and demand management). The difference between mitigation and contingency tactics are that the former includes the tactics that are taken by a firm in advance of a disruption, while the latter has actions taken by a firm in the case of an undesired event (dis-
ruption) occurs. A firm can adapt one or more of these strategies for managing the supply chain disruption risk.

The issue of how and why one supply chain disruption would be more severe than another is discussed in [7]. With the understanding that disruptions are inherent to supply chains, they present risk factors in supply chain, how vulnerable and resilient a supply chain to these risks, and prevention tactics of supply chain disruptions. In recent years, quite a few researchers have focused on studies that aim to develop and recommend robust supply chain strategies that not only enable a firm to manage regular fluctuations efficiently under normal circumstances but also help sustain its operations during under major disruptions [8–10].

In the context of decentralized supply chains, research reported in the literature on risk mitigation typically consider the problem from the perspective of a single buyer. As such, most models consider single-buyer-single-supplier or single-buyer-multiple-supplier settings. In the latter case, usually the models involve a regular supplier and a backup supplier.

A buyback contract between a buyer and a backup supplier to be exercised in case the buyer’s regular supplier experiences disruptions investigated in [11]. In a similar setting, the problem of how a firm should setup a contract with a backup supplier while ensuring the backup supplier to install responsive capacity is analyzed in [12]. Under a multi-period setting, a periodic review inventory system of a buyer that has two suppliers: a regular supplier and a backup supplier that can be used during disruption is studied in [13]. A model to analyze an interaction between a firm and her backup supplier through a revenue-sharing contract by using the combination of decision tree approach and Nash game may be found in [14]. A manufacturer’s
contracting decision problem with her back-up supplier where only the supplier has a concession of having information about supply disruptions is investigated in [15].

In the Chapter 2 and Chapter 3, we consider reservation contracts with deductible reservation fees. In the literature, reservation contracts have been mostly studied in the context of capacity management [16–21]. While the contracts are designed to ensure availability of capacity to meet nominal demand, our context deals with low probability high impact events. A reservation framework in this context is proposed in a recent study by [22]. The authors study a capacity reservation contract between a buyer and her backup supplier when the major supplier’s disruption risk is uncertain. They propose a backup sourcing strategy through advanced reservation contracts in the context of disruption uncertainty and minimum order quantity constraints. In contrast to their work, we consider capacity reservation for back up supplies between multiple buyers and a single supplier. Capacity reservation with multiple buyers is considered in [20, 21] for supplying nominal demand. In our context, where the reservation is made to be prepared for rare events, the implied risk of reservation is higher and hence, the incentives are somewhat different. Typically reserved inventory is either exercised or scraped in large quantities. As such, the reservation fees oftentimes are not the only enabler of reservations under such settings. When the likelihood for exercising reserved inventory is relatively low, working with multiple buyers provides reservation pools, hence, risk pooling opportunities, that incentivize suppliers to offer practicable contracts.

In the literature, pooling inventory to mitigate high risk of overage under rare events is typically tackled by models based on either cooperative games or horizontal competition. In the former group, the authors analyzes the inventory pooling strate-
gies of multiple firms in the presence of uncertain demand [23]. Their motivation derived from two key aspects: How much should each firm contribute to the pool and how should the inventory be allocated in case of shortages occurs to a single or multiple firms? The authors introduced a new concept of an endogenously determined "entitlements", and analyze the shortage allocation problem of the firms under the Nash bargaining solution framework. In their study, the researchers analyze the production capacity pooling for a set of independent firms where any firms can operate in its own facility or invest in a shared facility under the framework of cooperative game theory [24]. A study by [25] proposes a coalition model that consists of a single firm and multiple suppliers where the suppliers face the risk of completely defaulting in fulfilling their orders and can form coalition to mitigate their risk.

Pooling studied under horizontal competition considers a single echelon. Humanitarian organizations’ inventory management system in the context of non-cooperative game theory is discussed in [26]. The authors propose an analytical framework to investigate horizontal cooperation between humanitarian organizations. The study introduced in [27] investigates inventory planning problem in an environment where the companies have to deal with volatile commodity prices, e.g. crude oil. The framework of this study is based on multi-period setting where the firms make inventory sharing decision under a Nash game for a common commodity to meet their stochastic demands in spot and forward markets. The study in [28] examines the joint inventory stockpiling of medical supplies within a community of hospitals in a game-theoretic framework. Hospitals decide on the amount of stockpile they want to keep prior to a disaster, and unlike our paper the players may be located close enough to serve overlapping groups, and a patient may choose any hospital. In our setting, the players
are located in different regions where market overlapping is not possible. The authors in [29] conduct their research on how some countries possessing medicine stockpiles would react if an epidemic started in a country possessing little or no medication. Under a game theoretic framework, their model captures the epidemic dynamics and critical resources of the uncertainty that results in a Nash equilibrium where countries might collaborate in order to contain the epidemic. In contrast to above papers that consider competitive settings, we study pooling via reservations in a two echelon supply chain composed of multiple buyers and a single supplier.

The study in [30] is among the earliest to consider a supply chain that comprises multiple buyers and a single supplier in the context of inventory pooling. They study the impact of carrying shared inventory on the supply chain partners. The quantity decisions are made by the buyers who shoulder all the risk. The supplier participates as a strategic partner by means of setting the wholesale price. This study is extended in [31] by incorporating the transshipment option across buyers in a later paper. Again, the buyers are responsible for all the risks associated with carrying inventory. The paper [32] focuses on the Shapley value allocation of excess profit in the context of inventory pooling where a single supplier and multiple buyers join a coalition based on a cooperative game theory framework. In their model, the quantity decisions are made by the supplier based on service level constraints imposed by the buyers. In their model, supplier bears the inventory costs. Unlike these papers, in our model the risk is shared between the buyers and the supplier, and inventory pooling is realized by means of deductible reservation fees under a competitive setting.

In Chapter 4, we introduced an advance supply contract that allows the buyer to order inventory in high quantity with a lower price. She updates her demand forecast
information as the selling period approaches, and she has an opportunity to replenish
her inventory through spot market. Most papers in this area study the efficient use
of advance demand information and forecast updates in the context of supply chain
coordination.

The authors in [33] study textile/apparel industry where demand uncertainty can
be reduced by correct responses to early sales data, and formulate the production
planning decisions as a two-stage stochastic program while using various relaxations
to obtain feasible solutions and lower bounds on the minimum cost. The study in [34]
investigates an advance contract setting that includes two-stage ordering policy for
seasonal products where the market information collected at the first period is used
to update the demand forecast at the second period. Bayesian update method is used
to update the initial forecast of the product’s demand.

The authors in [17] examine the flexibility a buyer gains through options and how
does this flexibility help her to respond the market changes in a two-period model
where the second period demand correlates with the first period demand.

The model studied in [35] explores backup agreements between a buyer and a
supplier in fashion industry where the buyer commits a certain number of units to
buy when the selling season starts and pays a penalty fee for each unit she refuses to
buy. The proposed contract allows the buyer to update demand forecast by observing
the early season demand and to employ expedited shipments if necessary.

A Bayesian model of demand is considered in [36] where uncertain demand rate has
a prior that is a combination of two normal distributions. The value of information,
production flexibility, and supplier flexibility for a good for which an initial and a
subsequent order may be placed are analyzed in their paper.
The researchers in [37] study a problem of matching supply with demand for products with short life cycles and highly unpredictable demands where the retailer is unable to restock during the selling season and respond to market demand due to the long replenishment lead time and the short sales season. The authors model a case where advance booking discount is used by the retailer in order to convince customers to commit to their orders at a discount price prior to the selling season. This time interval enables the retailer to update the demand forecast before the selling season.

All the papers mentioned above are similar to our study in Chapter 4 in the sense that they all utilize advance demand information to update forecast in revising their forecast. However, in these papers the demand for the next period is updated only based on the demand data observed during the previous periods or pre-selling season. In our model discussed in Chapter 4, the advance demand information is not limited to the previous periods or pre-selling seasons demand information, other indicators such as advance booking information, customer surveys, market trends can be used to update the demand.

The authors in [38] investigate the initial stocking quantity of a retailer who sells a product with stochastic demand over a finite selling season. The authors introduce three types of consumers based on their purchasing strategy. In another study presented in [39], the authors investigate how sharing advance demand information can improve supply chain performance by comparing the two types of advance demand information: aggregated advance demand information and detailed advance demand information. The researcher in [40] analyze the optimal supply chain performance under forecast information sharing, which is between a manufacturer with uncertain demand for a single product and her supplier as the only source for the product.
The manufacturer provides initial forecast to the supplier which the supplier uses to build capacity under two contract regimes: forced compliance and voluntary compliance. In the subsequent stages, the manufacturer adjusts her final order quantity based on her updated forecast. The authors in [41] examine a risk sharing contract in a two-stage supply chain in which the manufacturer decides the initial production quantity in the first stage, and the retailer specifies her order quantity in the second stage after the demand forecast is improved. The contract requires that both parties partially compensate their loss in case of overproduction or overstocking. A dynamic pricing procurement model in which a single supplier and a single buyer interact multiple times during the selling season is analyzed in [42]. In this model, the supplier determines the unit capacity price dynamically during the season. In their research presented in [43], the authors focus on a component-purchasing problem of a retailer and two suppliers with different lead times. The retailer can update her demand forecast after and before ordering from the suppliers, and can partially cancel her order from the supplier whose lead time is longer.

The Stackelberg game is used in [44] to model three selling strategies (advance selling, regular selling and dynamic selling) of a manufacturer, who sells a seasonal product to a retailer under uncertain supply and demand. The authors analyze and compare all three selling strategies from the manufacturer’s and retailer’s perspective. The contracting problem of a supplier who sells perishable goods with a two-period shelf life is studied in [45]. The authors introduce a two-level wholesale price contract, a two-level buy-back contract and a buy-back contract with channel rebates. The researchers in [46] study the problem of assuring credible forecast information sharing between a supplier and a manufacturer. They build a capacity reservation
and advance purchase contract to analyze the role of information sharing on supply chain efficiency. The authors in [47] propose a dual purchase contract to mitigate the negative effects of a wholesale price contract on the manufacturers ability in a supply chain that consists of a manufacturer and a retailer who is facing a newsvendor problem with a forecast update. In the proposed contract, the manufacturer provides a discount for the orders placed before the forecast update, and the new contract form creates a strict Pareto improvement over the wholesale price contract. The author in [48] studies the problem of choosing a product price and commitment time frame in a network consisting of a single supplier and a buyer. He investigates the effect of early and delayed commitment strategies on the supply chain profit distribution between the supplier and the buyer. The results of this paper are extended in [49], including the manufacturer’s demand information updating option. The authors investigate the impact of partial demand information on early commitment and postponement policies of a firm. The demand is partially updated after the capacity is committed yet before production starts. Once the market signal is received, the firm updates its forecast and decides on the production amount which is constrained by the capacity committed earlier.

Advance capacity reservation contracts and partial commitment policies under partial information update in a single supplier-single buyer supply chain are analyzed in [19]. Two-instant replenishment policies of a retailer where the procurements can be made in two stages is considered in [50]. The authors study on the optimal ordering decision of a retailer under uncertain cost and stochastic demand setting for a product. The optimal time for placing an order and market conditions that impact the retailers postponement strategy analyzed in their study. This study is similar to
ours in terms of the multiple stages opportunity of procurement which allows taking advantage of the price discounts and forecast updates.

A model that has two-mode production environment and consists of a manufacturer and a distributor operating in a fashion industry is analyzed in [51]. The authors considers a setting in which the first production mode is cheaper but it requires a long lead time while the second production mode is expensive but it offers shorter lead time. They offer a contract to coordinate the manufacturer and distributor to act in the best interest of the channel.

A single and a multi-period quantity flexibility contracts are introduced in [52]. After the initial order at the beginning of the period, the contract allows demand forecast update with a market signal information in the middle of the period, and additional purchases on contract and from spot market before the demand realized at the end of the period. However, the additional purchase quantity on the contract at a contractual price is limited by the specified flexibility limit while spot market purchase at the market price is not limited.

Another study in [53] introduces a purchase contract with a demand forecast update in which the contract provides a buyer an opportunity to adjust her initial commitment based on the updated demand forecast obtained at a later stage. The problem is formulated as a two-stage dynamic programming model and it is assumed that the buyer incurs a fixed and variable cost for the adjustment of her initial commitment quantity after the market signal realization.

The paper [54] studies a periodic review inventory model with a fixed ordering costs, dual supply modes, and demand forecast updates. The authors in [55] study how does Quick Response Approach affect a manufacturer-retailer inventory decision,
and their service level, wholesale price and volume commitments to improve quick response strategy. The study in [56] develops optimal centralized control policies for the two market stochastic inventory systems. The author in [57] studies the cruise line industry in an oligopolistic competition model where finite number of cruise line companies compete to maximize their profits over a fixed planning horizon. The authors model the problem as a N-person nonzero-sum non-cooperative dynamic game and obtain the Nash Equilibrium strategies of the players.

The most relevant study to the last chapter of this dissertation is the multi-stage replenishment problem of a cruise liner studied in [58]. In our work, we extend their results under more realistic assumptions. First, we assume a more general continuous distribution function for the market signal (e.g., bookings) in contrast to a discrete probability distribution with only two possible outcomes (low vs. high demand). Second, we relax some of their assumptions on exogenous parameters such as purchasing costs, period lengths, and market signal information variance, and conduct a sensitivity analysis to investigate the effects of these parameters on the optimal replenishment policies.

1.2 Dissertation Overview

In the first part of this study, presented in Chapter 2, we propose an advance supply contract covering multiple independent buyers and a supplier. The structure of the contract is based on a game theoretical model in which the players (e.g., buyers and the supplier) move sequentially. In this setting, the supplier is the leader such that the game starts with her unit reservation fee decision. Afterwards, the buyers set
their contingency reservation quantities independently and declare it to the supplier. Hence they are the followers.

Buyers operate in independent markets and procure inventory for their nominal demand from their own suppliers, which are out of the paper’s scope. We rather focus on the contingency inventory reservation from a common supplier. In this context, the supplier in question can be named as a backup supplier since the buyers only need her in case of a contingency. We consider a game theoretical model and investigate conditions that elicit incentives for multiple independent buyers to reserve contingency inventory from a common backup supplier.

Our study is motivated by our involvement with a major distributor of consumable products that serves Trinidad & Tobago, Grenada, and Barbados. The distributor faces severe shortages due to demand surges in the event of a major hurricane, which is common in the region. Since overage costs associated with backup provisioning is usually prohibitive for such companies due to low margins and relatively small volumes, they typically underinvest in or completely forego contingency policies. One conceivable way for such a firm to enable provisioning of contingency inventory is to share the risk with other firms operating in the Caribbean Basin, who faces similar back up needs. The risk can be shared via reserving from a common supplier, who can take advantage of risk pooling and offer economically justified terms for contingency inventory reservation.

The event probabilities for the buyer markets are not identical but assumed to be known. Based on these probabilities, the supplier offers a reservation fee to the buyers who respond with their decisions regarding their respective reservation amounts. If a buyer is inflicted by a low-probability event, additional inventory beyond her
reservation may also be available for her if the other buyers do not need to exercise their reservations. This creates a risk pooling opportunity for the supplier as inventory reserved by a buyer can also potentially be utilized by another. As such, viable conditions for carrying contingency inventory may become possible. In this context, we consider deductible reservation fees, where any payment made by the buyer for reserving inventory is deducted from the wholesale price when the buyer exercises her reservation.

We apply our framework to three settings that differ in terms of stipulations on a buyer’s procurement of inventory from the supplier in excess of her reservation and the deemed ownership of the reserved inventory. In the basic case, a buyer is not required to pay any fees or penalties other than the wholesale price if she requests additional inventory items beyond her reservation quantity. The supplier can meet such requests based on availability, which occurs if another buyer’s reservation is not exercised. In this case, the buyer not only relies on her own reservation but also has the option to dip into others’ reserved inventories. The second setting adds a no-reservation surcharge (penalty) fee that must be paid in addition to the wholesale price for each product requested by a buyer beyond her reserved stock. As such, a buyer takes a risk of not only facing shortages but also paying a penalty fee if she reserves less than her actual need. In this case, the supplier collects and keeps the surcharge fees as it is assumed that she has the ownership of the inventory. In the last setting, the ownership assumption is modified where the reserving buyer is deemed to have partial ownership on her own reservation. In this case, a portion of the surcharge fee collected by the supplier for any inventory item that is reserved by a buyer but
sold to another is transferred to the reserving buyer. Clearly this is possible if the reserving buyer does not choose to exercise her reservation herself.

Our goal in Chapter 2 is to address the following research questions:

- Under what conditions does the risk pooling through a common backup supplier lead to contingency inventory reservation?

- How does the contract structure and the ownership of reserved inventory affect the incentives of all parties in the channel and the contingency inventory outcome?

In order to facilitate our analysis and gradually build our results, we first consider the setting with two buyers who are identical except for their rare event probabilities. Later, we generalize our model to settings with more than two buyers and buyers with asymmetric demand volumes. We are able to demonstrate under what conditions the reservation contracts result in positive contingency inventory reservation. We observe that mere existence of a buyer in the pool without reserving any inventory can actually help the supplier offer terms that lead other buyers to reserve contingency inventory. For such situations, we propose a win-win contract that subsidizes the supplier’s risk through a payment from the reserving buyer so as to incentivize the former one to decrease her reservation fee so that the non-reserving buyer switches her strategy to reservation.

In the second part of the study, presented in Chapter 3, we propose an advance supply contract covering two independent buyers and a single backup supplier. The difference between this contract and the one proposed in Chapter 2 is that, the leader-follower structure of the game. In this setting, buyers start the game as leaders such that they independently offer a unit reservation fee that they are willing to pay for a
unit product to be held by the supplier. As the follower, the supplier respond with the reservation quantities for each buyers depending on the unit reservation fees she is offered. Like the former case, the buyers operates in different markets and subject to the different rare event probabilities. We apply our framework to the setting in which no-reservation surcharge (penalty) fee is added to the wholesale price for each extra unit requested beyond any buyer’s original reserved quantity.

The contract enables the supplier to allocate the contingency stock to any buyer who is in need of inventory more than her reserved quantity unless the other buyer experiences the rare event. Throughout the paper, we show that pooling their contingency inventory improves each buyers ability to efficiently match their unanticipated demand while allowing them to reserve less than their contingent demand.

Our goal in Chapter 3 is to address the following research questions:

- Is it always beneficial for a supplier and multiple buyers to invest in a contingency reservation contract? What are the conditions that makes this investment for appealing for both parties?

- Can independent buyers benefit reserving contingency inventory from a shared supplier?

- Under what conditions pooled reservations lead to sustainable win-win outcomes?

- What are the differences in terms of game outcomes when the supplier is the leader vs buyers are the leaders?
In the **third part** of the study, presented in **Chapter 4**, our research is motivated by the application of advance supply contracts in the cruise line industry where cruise line companies make contracts for the seasonal products before the final number of customer bookings are realized. Advance contracting enables the cruise line companies to purchase products in a cost efficient way. The company that uses advance contract to make large quantity purchases in order to benefit from early commitment and price discount can also make additional purchases just before the departure from the home port or at the intermediate stops. The extra replenishment might be necessary based on the final number of customers on board and consumption rate of the seasonal product. Note that, replenishing inventory through spot market hedges the company against stock-outs, however the company should incur higher unit prices.

After the cruise liner departs from its home port, it anchors at intermediate stops called "ports of call". Each trip between the home port and a port of call or between two ports of call are referred to as a "leg" or "period". While the cruise liner is at any port, it has an opportunity to replenish its inventory before departure. Each time span the cruise liner spend at a port referred to as a "stage". The problem under consideration is a finite horizon multi-period inventory control problem of a cruise line company, and the demand for the seasonal product is stochastic and non-stationary across periods. The demand distribution for each period is a function of a demand (market) signal which is random at the beginning of the planning horizon.

We apply a periodic inventory review policy to our model and assume that replenishments are possible only at the beginning of each period and these periods are not necessarily equal in length. The cruise line company’s unit replenishment costs vary during the selling season. It is assumed that the unit inventory cost paid during the
time of the advance contract is less than the spot market costs at home port and the ports of call. Also, the spot market cost vary at the home port and ports of call. It is assumed that the cruise line company knows both unit replenishment costs at each stage and time intervals between possible replenishments (length of each leg or period) before the demand (market) signal realized. We analyzed the cruise line company’s optimal decision of advance contracting and spot market replenishment quantities under the periodic inventory review policy and assumptions mentioned above.

Our goal in Chapter 4 is to address the following research questions:

- How does the optimal combination of advance contracting and expedited replenishment policy improves the firm’s profit in multi-period inventory control problem?

- How do the exogenous parameters affect the optimal initial replenishment (advance contracting) policy of the firm?

- Does the length of each period (leg) affect the total cost of the firm?

- What is the relation between base-stock level and exogenous cost parameters?

We adopt the stochastic dynamic programming approach for our solution. In addition, line search algorithm method is used to compute the optimal contracting amount in our numerical analysis.

1.2.1 Summary of Contributions

- The analysis of the advance supply contract in Chapter 2 shows that reserving through a common backup supplier can be an effective strategy if the buyers’ margins are sufficiently high. A buyer with comparably lower event probability
may participate in this channel without reserving any inventory if the supplier’s wholesale price and salvage value are low or the unit cost of the product is high. Because of relatively small margins, the supplier offers a reservation fee that is appealing only for the buyer with higher event probability. However, we observe that the mere existence of the nonreserving buyer benefits the channel and results in inventory reservation by the other buyer. The possibility that a nonreserving buyer may end up utilizing the inventory reserved by the others provides incentives to the supplier to offer reservation fees that are attractive for the other buyers.

Our analysis shows that when surcharge fees are included in the reservation contract, the buyer incentives for reservation increases and as such, reservation is more likely. As expected, the contractor’s expected payoff improves in this case. Interestingly, we observe that a buyer, who would reserve regardless of the surcharge fees, may in fact be better off when they are introduced as they may drive the reservation fee cost down. Our conclusions for the case in which a portion of surcharge fees are transferred to the reserving buyer are context specific. Ownership of the reserved inventory and hence the surcharge fee transfer has no affect on equilibrium player payoffs and reservation amounts except when a buyer dominates another with her event probability and demand volume. We also observe in that case that higher demand volume does not necessarily lead to higher reservation amounts. This is an interesting result that does not arise under conventional reservation settings.
• The buyer-leading advance supply contract investigated in Chapter 3 shows that the equilibrium outcomes (reservation quantities of the buyers) depend on the relation among the event probabilities, buyer margins, and supplier’s overage cost. By reserving contingency inventory via a shared supplier, a buyer has the option of not only exercising her own reservation but also the other buyer’s reservation in case the latter one does not need to exercise it. Also, even when a buyer participates pool without any reservation, her mere potential demand provides incentives for the supplier to carry backup inventory at a lower reservation fee for the other buyer. We also contrasted our model with the supplier-leading case in the previous section. Our analysis shows that under similar conditions the equilibrium reservation amounts are no less under a buyer-leading channel compared to the supplier-leading channel.

• The advance supply contract framework presented in Chapter 4 deals with a firm’s optimal advance contracting decision and expedited replenishment policies for a seasonal product under the finite horizon multi-period inventory setting. According to our results, optimal replenishment policies follow base-stock policies for all selling season periods following the realization of the demand signal and we show that there is a unique advance contract amount that minimizes the expected cost of a finite period problem. We also provide numerical examples to analyze the effects of certain exogenous parameters on the optimal replenishment policies for all periods, the initial contract amount, service levels of the periods, and the overall expected cost.
CHAPTER 2

Contingency Inventory Reservation across Independent Retailers under a Supplier Leading Contract

This chapter analyzes reservation contracts for contingency demand that emerges as an immediate result of a rare event such as natural disaster or epidemic. Typically, building inventory for demand surges caused by such low probability high impact events is not economically justified due to high overage risks. We propose a game theoretic framework that mitigates these risks by contingency inventory pooling across independent retailers through a common supplier under a competitive setting. In the proposed game, the supplier offers a deductible reservation fee and the buyers, who face different event probabilities, respond with their reservation amounts. Any reserved item, if not exercised by the reserving buyer, can be shipped to another buyer who may need it. The latter one may be required to pay a surcharge fee for any inventory beyond her original reservation. We consider the case where a portion of this fee is transferred to the reservee. We show that although a buyer with comparably lower event probability may participate in this channel without reserving any inventory, her mere participation in the game benefits the other parties in the supply chain. Interestingly, our analysis reveals that surcharge fees may benefit the buyer
with the higher event probability. Transferring these fees to reserving buyers, however, does not affect the equilibrium reservation amounts and player profits unless the buyer with the higher event probability faces a higher contingency demand. Contrary to the reservation contracts employed to meet nominal demand, we show that larger contingency demand does not always lead to increase in reservation amounts. This is mainly due to fact that exercise of the contingency inventory is typically full-or-none.

2.1 Problem Statement

As a basic building block for our analysis, we consider a single-period setting where a third party contractor carries ”contingency inventory” reserved by multiple buyers operating in independent markets so as to hedge against low-probability high-impact rare events such as natural disasters or epidemics. In this context, an event is deemed as rare if its probability of occurrence is below 0.5. As such, it is assumed that the event probabilities are known for each market and strictly below 0.5. We specifically consider events that create spikes in demand for essential consumables or pharmaceuticals. The analysis is based on a Stackelberg setting where the contractor is the leader and the buyers are the followers in the supply channel. For better tractability, we first assume a two-buyer setting, namely Buyer $A$ and Buyer $B$, where the buyers are identical in terms of their market sizes and resale prices. They are basically distinguished by their event probabilities denoted by $P_A$ and $P_B$ respectively. Without loss of generality, it is assumed that $P_A \leq P_B$. In subsequent sections, we will extend our analysis to multiple buyers and nonidentical markets.

During the regular seasons (i.e., when there is no event) each buyer procures the same product from their own sources in order to satisfy their nominal demand. The
rare event creates additional demand beyond nominal quantities. We let $H$ denote this excess demand, which is assumed to be known. As mentioned above, the retail price for the product is identical in both markets and denoted by $p$. The additional demand due to the rare event offers significant profit potentials for the buyers. On one hand, due to long lead times, it may not be possible for the buyers to make additional orders from their regular suppliers in a timely fashion. On the other hand, carrying local inventory for such events may not be economically justified due to the associated low probabilities. A viable option for the buyers may involve filling their excess demand caused by the rare event via a common contractor who carries contingency inventory for them. We investigate the conditions and strategies under which all parties have incentives to adopt such an option.

In this setting, the buyers reserve contingency inventory from a single contractor. They pay a wholesale price of $w$ for each unit of product that they procure from this contractor. The wholesale price is assumed to be exogenous in our analysis. It is either negotiated beforehand or, in the case of consumable products, determined by the market. The contractor sets a nonrefundable deductible unit reservation fee of $r$ to hold a single unit of product that costs her $c$ to procure. It is assumed that $p > w > c$. Given the contractor’s reservation fee, the buyers move simultaneously and decide on their respective reservation quantities, $Q_i$ $(i = A, B)$. The contractor is required to have her inventory match the total reservation amount. If the demand surge occurs, the buyer must pay the exercise fee of $w - r$ to acquire the product. Variations of reservation contracts in this form are studied in the context of capacity provisioning in the literature. Some examples included in [19, 59, 60]. Initially, we assume forced compliance, which ensures that the contractor makes the contingency
inventory reserved by a buyer available when needed. In case the reservation quantity for a buyer is below her demand, that is the buyer under reserves, she has the option of receiving additional quantities from the contractor subject to availability. This is typically the case when the contractor carries additional inventory reserved by the other buyer who does not need to exercise her inventory option. In some cases, the supplier may charge additional fees for any unreserved inventory demanded by a particular buyer. All or a proportion of the collected fees could be transferred to the other buyer who originally reserved that inventory. We refer to this fee as the no-reservation penalty and denote it by \( u \) and let \( a \) denote the proportion of the no-reservation penalty transferred to the “other” buyer. Any unused inventory at the end of the selling period can be salvaged by the contractor at \( s \) per unit, where \( s < c \).

The low-probability events across two buyers lead to four possible scenario outcomes: (1) no rare event occurs, (2) only buyer \( i \) is inflicted, (3) only the other buyer (buyer \( -i \)) is inflicted, and (4) both buyers are inflicted by the rare events. We let \( \Psi_o \), \( \Psi_i \), \( \Psi_{-i} \), and \( \Psi_{i,-i} \) represent the probability for scenario outcomes 1-4 respectively. The notation is summarized in Table 2.1.
The sequence of the events in the general setting can be summarized as follows:

1) The contractor announces a reservation fee $r$ under given exogenous wholesale price $w$ and no-reservation penalty $u$.

2) Buyers decide on their reservation quantities $Q_i$, where $i = \{A, B\}$, and pay $r$ for each reserved unit.

3) If any buyer is inflicted by the low-probability event, her reserved inventory is supplied by the contractor and the buyer pays $w - r$ for each unit delivered. In case, the reservation amount is below the inflicted buyer’s demand and the other buyer has reserved inventory that is not needed, additional inventory can also be procured at $w + u$ per unit and the contractor transfers $au$ to the buyer who initially reserved the inventory.
Profit of the contractor depends on the revenues from reservation, exercise, and no-reservation penalty fees, unit cost, and the salvage value of the reserved inventory. As such, the constructor's expected profit function can be written as follows:

\[
\Pi_S = (r - c)(Q_A + Q_B) + \Psi_o s(Q_A + Q_B)
\]

\[
+ \Psi_A[(w - r) \min(Q_A, H) + (w + (1 - a)u) \min(H - Q_A, Q_B) + s \max(0, Q_A + Q_B - H)]
\]

\[
+ \Psi_B[(w - r) \min(Q_B, H) + (w + (1 - a)u) \min(H - Q_B, Q_A) + s \max(0, Q_A + Q_B - H)]
\]

\[
+ \Psi_{A,B}(w - r)(Q_A + Q_B)
\]

(2.1)

Each buyer decides on her reservation amount at the beginning of the period before the nature is observed. The reserved inventory is exercised by the buyer only if the rare event occurs. In that case, as mentioned earlier, a buyer has the option of procuring quantities beyond her reservation if additional inventory reserved by the other buyer is available at the contractor. This can happen only if the other buyer does not exercise her reservation. Given the wholesale price \(w\), reservation fee \(r\), no-reservation surcharge \(u\), transfer rate of the surcharge \(a\) and event probabilities \(P_i\), the expected rare-event profit for Buyer \(i\) is then

\[
\Pi_i = -rQ_i + (p - w + r)(\Psi_i + \Psi_{i,-i})Q_i + (p - w - u)\Psi_i \min(Q_{-i}, H - Q_i) + au \min(Q_i, H - Q_{-i})\Psi_{-i}.
\]

(2.2)

We investigate the problem under three settings. As the base case, we first assume that there is no no-reservation surcharge, that is, \(u = 0\). In this case, the buyer can procure additional quantities from the contingency inventory based on availability with no additional cost. Later, we incorporate the surcharge penalty to examine its impact on the player incentives and equilibrium outcomes when the contractor has the
sole ownership of the inventory, that is, no fees are transferred to the reserving buyer. In the third model, we consider the case, where the buyer entitles all or some of the ownership for the reserved inventory. As such, certain portion of the no-reservation surcharge - when applied - is transferred to the buyer, who originally reserved the inventory.

2.2 Base Case: No Surcharge for Unreserved Inventory

We begin our analysis with the base case where no surcharge is applied by the contractor for unreserved inventory. In this case, a buyer has the option of using the inventory reserved by the other buyer without any surcharge if the latter one does not need it. We derive equilibrium conditions using backward induction. As such, we first analyze the second stage where buyers simultaneously decide on their reservation quantities for any given reservation fee. Later, we solve the contractor’s problem based on the second stage equilibrium outcome.

2.2.1 Buyers’ Reservations

The expected profit function for Buyer $i$ given in (2.2) can be rewritten as follows when $u = 0$:

$$\Pi_i = -rQ_i + (p - w + r)P_iQ_i + (p - w)P_i(1 - P_i) \min(Q_{-i}, H - Q_i).$$  \hspace{1cm} (2.3)

We note that the above profit function is piece-wise conditioned on the other buyer’s reservation amount. As such, the first order derivative for Buyer $i$ with respect to her reservation quantity is
\[
\frac{\partial \Pi_i}{\partial Q_i} = \begin{cases} 
-(1-P_i)r + P_i(p-w), & \text{if } Q_i \leq H - Q_{-i} \\
-(1-P_i)r + P_iP_{-i}(p-w), & \text{o/w} 
\end{cases}
\]

(2.4)

The first order derivatives above indicate that the slope of the profit function with respect to the reservation amount depends on the interval defined by the reservation amounts. The following reservation fee thresholds for Buyer \( i \) (\( i = A, B \)) help us identify the trade-off between the buyer’s reservation amount and her expected profits:

\[
\begin{align*}
    r_{i,1} &= \frac{P_i}{1 - P_i}(p-w) \\
r_{i,2} &= \frac{P_iP_{-i}}{1 - P_i}(p-w)
\end{align*}
\]

(2.5) (2.6)

Basically, when the reservation fee is sufficiently low, \( i.e., r < r_{i,2} \), the profit function is strictly increasing with the buyer’s reservation amount. On the other hand, for sufficiently high values, \( i.e., r > r_{i,1} \), it is strictly decreasing. When the reservation fee is neither low nor high, \( i.e., r_{i,2} \leq r \leq r_{i,1} \), the profit is first increasing for \( Q_i \leq H - Q_{-i} \) and then decreasing for higher values of \( Q_i \). Knowing that rare event probabilities are less then 0.5, we can make the following observation regarding the ordering of the reservation fee threshold values:

**Lemma 1** Assuming, \( P_A < P_B \) and \( P_B < 0.5 \), it is always true that \( r_{B,1} > r_{A,1} > r_{B,2} > r_{A,2} \).

**Proof:** First, it is straightforward to observe from (2.5) and (2.6), \( r_{i,1} > r_{i,2} \) for both \( A \) and \( B \). We can also easily observe that \( r_{B,1} > r_{A,1} \) and \( r_{B,2} > r_{A,2} \) since \( P_B > P_A \). To complete the rest of the proof, we \( r_{A,1} > r_{B,2} \) must hold, which reduces to \( P_B(2 - P_A) > 1 \). Clearly, this inequality always holds when \( P_A < P_B < 1/2 \). ■
This ordering helps us identify the equilibrium reservation amounts across two buyers for a given reservation fee. For ease of notation we let \((Q_A, Q_B)\) denote the joint buyer strategies representing the reservation amounts for buyers \(A\) and \(B\) respectively. For example, a strategy outcome \((0, H)\) indicates that the reservation quantities for players \(A\) and \(B\) are 0 and \(H\) respectively. In what follows, we present the equilibrium reservation strategies for value ranges of the reservation fees:

**Proposition 1** The equilibrium buyer reservation quantities in response to the reservation fee ranges are as given in the below table:

<table>
<thead>
<tr>
<th>Reservation Fee Range</th>
<th>Buyers’ Eq. Reservation</th>
</tr>
</thead>
<tbody>
<tr>
<td>([0, r_{A,2}])</td>
<td>((H, H))</td>
</tr>
<tr>
<td>((r_{A,2}, r_{B,2}])</td>
<td>((0, H))</td>
</tr>
<tr>
<td>((r_{B,2}, r_{A,1}])</td>
<td>((x, H - x))</td>
</tr>
<tr>
<td>((r_{A,1}, r_{B,1}])</td>
<td>((0, H))</td>
</tr>
<tr>
<td>((r_{B,1}, \infty))</td>
<td>((0, 0))</td>
</tr>
</tbody>
</table>

Table 2.2: Reservation Fee Ranges and Buyers’ Equilibrium Reservations

**Proof:** Let \(G_i\) and \(\hat{G}_i\) denote the first order derivatives given in (2.4) for Buyer \(i\). Namely, \(G_i = -r + (p - w + r)P_i\) and \(\hat{G}_i = -r + (p - w + r)P_i - (p - w)P_i(1 - P_{-i})\). Clearly a positive (negative) derivative indicates that the buyer profit is increasing (decreasing) with the reservation quantity in the respective range. First, we observe that \(G_i\) and \(\hat{G}_i\) are both nonnegative when \(r \in [0, r_{A,2}]\) for both buyers. As such, both buyers reserve their full demand when the reservation fee is in this range resulting with \((H,H)\). If \(r\) is in \((r_{A,2}, r_{B,2}]\), then \(\hat{G}_A < 0\), while \(G_A, G_B,\) and \(\hat{G}_B\) are all nonnegative. In this case, Buyer \(A\) has no incentive to reserve more than \(H - Q_B\). Since Buyer \(B\), has incentive to reserve the full quantity, \(H\), at equilibrium, Buyer \(A\) does not reserve. This results with the \((0,H)\) strategy outcome. If \(r\) is in \((r_{B,2}, r_{A,1}]\), then \(\hat{G}_i < 0\) and \(G_i \geq 0\) for both buyers. As such, Buyer \(i\) has no incentive to reserve
more than $H - Q_{-i}$. We can easily deduce that none of the buyers reserves more than the full amount, $H$. If any buyer reserves $x$ units ($x \leq H$), the other one, then, reserves $H - x$. The buyers have no incentives to move from this point and as such, equilibrium is reached. Clearly, in this case, there are multiple equilibria. If $r$ is in $(r_{A,1}, r_{B,1}]$, then $G_B > 0$, while $G_A$, $\hat{G}_A$, and $\hat{G}_B$ are all nonpositive. In this case, only Buyer B has incentive to reserve up to $H$. This results with the $(0, H)$ strategy outcome. Finally, when $r > r_{B,1}$, all derivatives are negative implying that buyers have no incentive to make any reservations resulting with the $(0, 0)$ strategy outcome.

The above result shows that, as expected, neither of the buyers will reserve, if the contractor’s reservation fee is too high, i.e., $r > r_{B,1}$. When $r_{A,1} < r \leq r_{B,1}$, while Buyer A has no incentive to reserve any amount at all, Buyer B profits increases in the reservation amount up to $H$ as indicated in (2.4). This results with a unique equilibrium of $(0, H)$. Since Buyer A has a lower expectation for the rare event, she is not inclined to take the risk of investing in reserved inventory. When the reservation fee is lowered to the next range, $r_{B,2} < r \leq r_{A,1}$, both buyers’ profit functions increase with the reservation amount up to $H - Q_{-i}$ leading to multiple equilibria where the total reservation quantities are always $H$. When $r_{A,2} < r \leq r_{B,2}$, Buyer B profits strictly increases in her reservation amount and her best response is to reserve the full amount. Inferring on this, Buyer A does not take the risk of investment in reservation but bets on potential use of Buyer B’s reservation instead. Lastly, when the reservation fee is too low, that is, $r < r_{A,2}$, both buyers end up reserving their full amounts at equilibrium.
2.2.2 Contractor’s Reservation Fee

Inferring on the buyers’ responses, in the first stage, the contractor decides on the reservation fee, $r^*$, that maximizes her expected profit. We know from Proposition 1 that the equilibrium reservation quantities depend on the reservation fee ranges, where, within a given range, the equilibrium outcome stays unchanged. As such, it is straightforward to conclude that the contractor is always better off with choosing the highest reservation fee in a given range. Consequently, the contractor evaluates her profits under each range and chooses the one with the highest pay off. The ordering in Lemma 1 provides the contractor with three options:

**Lemma 2** At equilibrium, the contractor sets her reservation fee to either 1) $r^* > r_{B,1}$ and there will be no reservation; or 2) $r^* \approx r_{B,1}$ leading to $(0, H)$; or 3) $r^* \approx r_{A,2}$ leading to $(H, H)$.

*Proof:* To complete the proof, first observe from (2.1) that the contractor’s payoff function is piece-wise linear in reservation fee, $r$. Under the $(0, H)$ reservation outcome, the first derivative is $1 - P_B$, which is clearly positive since $P_B \leq 1/2$. With the $(H, H)$ outcome, the derivative is $2 - P_A - P_B$, which is also positive. Consequently, we conclude that the contractor’s payoff function is increasing in $r$ and she will always prefer the highest reservation fee for any range given in Table 2. Next, we note that, for $r^*$ in $(r_{B,2}, r_{A,1}]$, there are multiple equilibria and the outcome $(H, 0)$ incurs the highest payoff for the contractor at $r^* = r_{A,1}$ in this range. However, even with this outcome at $r^* = r_{A,1}$, it is easy to observe that the contractor payoff is higher at $r^* = r_{B,1}$ with the the same outcome since $P_B \geq P_A$. As such, we conclude that the contractor will choose either $r_{A,2}$ or $r_{B,1}$, based on the comparison of her profits evaluated under $(0, 0)$, $(0, H)$, and $(H, H)$ strategy outcomes. □
We note that the contractor selects the first option only to avoid negative profits. In this case, there will be no reservation and hence, no profit for any parties. In order to identify the contractor’s best strategy, we need to examine the expected payoffs for the other two options. Under the second option given in the above lemma and (2.1), the contractor’s expected profit becomes:

$$\Pi_S(r_{B,1}) = H[P_B(p - w) + (P_A + P_B - P_A P_B)(w - s) - (c - s)] \quad (2.7)$$

Whereas, the third option results with the following expected payoff:

$$\Pi_S(r_{A,2}) = H\left[\frac{P_A P_B (2 - P_A - P_B)}{1 - P_A} (p - w) + (P_A + P_B)(w - s) - 2(c - s)\right] \quad (2.8)$$

Clearly, if both profit functions return negative values, employing a reservation fee that is low enough to generate any reservation is not justified for the contractor. Otherwise, the contractor’s reservation fee choice is based on the comparison between (2.7) and (2.8). Consequently, the optimal strategy for the contractor can be summarized as follows:

**Corollary 1** The contractor opts out of any reservation policy if and only if

$$\max(\Pi_S(r_{B,1}), \Pi_S(r_{A,2})) < 0.$$  

Otherwise,

i) If $\Pi_S(r_{B,1}) \geq \Pi_S(r_{A,2})$ then $r^* \approx r_{B,1}$ is the best reservation fee choice for the contractor resulting with the $(0, H)$ reservation amounts.

ii) If $\Pi_S(r_{B,1}) < \Pi_S(r_{A,2})$ then $r^* \approx r_{A,2}$ is the best reservation fee choice for the contractor resulting with the $(H, H)$ reservation amounts.

The comparison between the above two profit functions depends on the cost/price parameters and the event probabilities. It is straightforward to observe that both
profits given in (2.7) and (2.8) are increasing functions of \( p, w \) and \( s \), and decreasing in \( c \). They both are also increasing with \( P_A \) and \( P_B \) since \( P_A \leq P_B \leq 1/2 \). In order to further examine the contractor’s reservation fee choice we let \( \Delta \) denote the difference between the profit functions given in (2.7) and (2.8), that is, \( \Delta = \Pi_S(r_{A,2}) - \Pi_S(r_{B,1}) \). Clearly, if \( \Delta > 0 \), the contractor prefers \( r_{A,2} \) over \( r_{B,1} \) as the reservation price. Opposite is true otherwise. We make the following observation regarding how the cost parameters influence the contractor’s reservation fee choice:

**Lemma 3** Assuming \( 0 < P_A \leq P_B \leq 1/2 \), \( \Delta \) strictly increases in \( w \) and \( s \), and strictly decreases in \( p \) and \( c \).

The proof directly follows from the first order derivatives. The above result indicates that a reservation fee leading to the (H,H) strategy becomes more appealing under higher wholesale price and salvage values. This is expected since higher profit margins and salvage values alleviate the risk of carrying contingency inventory resulting with lower reservation fees and as such, with higher reservation amounts. On the other hand, the diminished margins due to increase in unit cost, \( c \), do not justify lower reservation fee. As such, as the unit cost increases the equilibrium is more likely to lean towards the (0,H) strategy. It is interesting to observe that higher resale price makes the (0,H) strategy more appealing to the contractor compared to the (H,H) strategy. Higher resale prices, and hence, higher buyer margins enable the contractor to charge higher reservation fees. From (2.5) and (2.6), we can easily see that the marginal increase in reservation fee threshold with respect to the resale price is higher in the case of \( r_{B,1} \). As the gap increases between the two threshold reservation fees, the contractor is better off with choosing the higher reservation fee option and col-
lecting a higher upfront payment. This, consequently, leads to the (0,H) equilibrium outcome.

The above result helps us with the following observation:

**Lemma 4** The contractor never chooses a reservation fee that leads to the (H,H) outcome if \( P_A P_B < \frac{(c-s)}{(w-s)} \).

**Proof:** We first observe the fact that (H,H) strategy pair will never be the equilibrium outcome if \( \Delta(r_{A,2},r_{B,1}) < 0 \). At \( p = w \), we get

\[
\Delta(r_{A,2},r_{B,1},p = w) = H(P_A P_B(w-s)-(c-s)) \tag{2.9}
\]

From Lemma 3, we know that \( \Delta(r_{A,2},r_{B,1}) \) decreases in \( p \) and as such, since \( p \geq w \), \( \Delta(r_{A,2},r_{B,1},p = w) \) given in the above equation represents the maximum possible difference between \( \Pi_S(r_{A,2}) \) and \( \Pi_S(r_{B,1}) \). Clearly, its value is strictly negative when \( P_A P_B < \frac{(c-s)}{(w-s)} \), implying that the contractor’s reservation fee choice will never lead to the (H,H) strategy outcome under this condition.

The above result indicates that (H,H) strategy is not appealing to the contractor when the event probabilities are too low compared to the ration between the supplier’s ”overage” and ”underage” costs. We note that the above ratio corresponds to the critical ratio for the contractor in the context of the classical Newsvendor’s problem. The following result provides us more detailed insights regarding the marginal and joint influences of the event probabilities:

**Lemma 5** \( \Delta \) strictly increases in \( P_A \) for \( P_A \in (0,P[B]) \). On the other hand, \( \Delta \) is strictly concave with a unique maximizer at

\[
P_B^* = \frac{1}{2P_A(p-w)}(P_A(1-P_A)(p-s) - (1-2P_A)(p-w)) \tag{2.10}
\]
Proof: The first part of the proof follows from the first derivative with respect to \( P_A \), where

\[
\frac{\partial \Delta(r_{A,2},r_{B,1})}{P_A} = \frac{HP_B}{(1-P_A)^2}((p-w)(1-P_B) + (p-s)(1-P_A)^2) \tag{2.11}
\]

It is straightforward to see that the above function always returns a positive value. Next, to prove that \( \Delta(r_{A,2},r_{B,1}) \) is concave in \( P_B \), we look at the second derivative:

\[
\frac{\partial^2 \Delta(r_{A,2},r_{B,1})}{P_B^2} = \frac{-2HP_A}{(1-P_A)}(p-w) \tag{2.12}
\]

Clearly, the above function is strictly negative for \( 0 < P_A \leq 1 \) establishing the concavity. The unique maximizer given in (2.10) is found by solving the first derivative given in (2.11) for \( P_B \).

We first observe that (H,H) becomes more appealing for the contractor as the rare event probability for Buyer A increases. This is intuitive since as \( P_A \) increases, risk associated with carrying contingency inventory for Buyer A diminishes. As such, when \( P_A \) is sufficiently high, the contractor is justified to select a fee that encourages this buyer to reserve. On the other hand, the impact of \( P_B \) is ambiguous and context specific. Up to a certain level, namely \( P_B^* \), higher \( P_B \) reduces the contractor’s risk for (H,H). However, as the gap between \( P_B \) and \( P_A \) grows, the contractor is better off focusing on Buyer B only, which provides higher upfront payment for the contingency inventory, eventually leading to an equilibrium strategy of (0,H). We note from (2.10) that the difference between the contractor profit functions always decreases in \( P_B \) if

\[
\frac{p-w}{p-s} > \frac{P_A(1-P_A)}{1-2P_A} \tag{2.13}
\]
The inequality in (2.13) holds when the buyer margin is sufficiently large. As such, we expect that the difference in contractor profits increases (resp., decreases) in $P_B$ when the resale price is low (resp., high). This is illustrated in Figure 2.1. The figure depicts the indifference curves (i.e., $\Delta = 0$) for a given resale price between possible equilibrium reservation strategy outcomes. When the resale price is low (Figure 1.a), the indifference curve between (0,H) and (H,H) strategies is decreasing in $P_B$ implying that $\Delta$ increases in $P_B$. On the other hand, under higher resale price (Figure 1.b), the indifference curve is increasing in $P_B$ implying that $\Delta$ decreases in $P_B$. We note that consistent with Lemma 3 and illustrated in Figure 2.1, higher resale price results with a larger region for the (0,H) strategy outcome.

![Figure 1a: Indifference curves for resale price (p) - Low](image1a.png)
![Figure 1b: Indifference curves for resale price (p) - High](image1b.png)

Figure 2.1: Indifference Curves for the Buyers’ Resale Price

We observe that the indifference curve decreases (resp., increases) in $P_B$ when the salvage value, $s$, is sufficiently small (resp., large) as illustrated in Figure 2.2. This implies that $\Delta$ increases in $P_B$ when $s$ is small and decreases in $P_B$ when $s$ is large. This is somewhat a counter-intuitive result at first look. This result can be explained by two observations. First, the risk for the (H,H) strategy under small $s$ and $P_B$
is not justified, and as \( P_B \) increases the the risk diminishes increasing the appeal of the \((H,H)\) strategy. On the other hand, when \( s \) is sufficiently large, the contingency inventory risk is lower and hence, the salvage value does not have as much of an influence on the contractor’s choice between the two strategies. As \( P_B \) increases, the growth in gap between the threshold reservation fees as explained earlier makes the \((0,H)\) strategy more attractive for the contractor. Consistent with Lemma 3 and contrary to the case with the resale price, Figure 2.2 shows that higher salvage value results with a smaller region for the \((0,H)\) strategy.

As illustrated in Figure 2.3, the influence of the wholesale price is completely opposite to the influence of the resale price. The indifference curves indicate that \( \Delta \) decreases in \( P_B \) when \( w \) is small and increases in \( P_B \) when \( s \) is large as suggested by the inequality given in (2.13). Consistent with Lemma 3, higher wholesale price leads to a smaller region for the \((0,H)\) strategy.
2.2.3 Equilibrium Profits

The contractor’s equilibrium profits are given by (2.7) and (2.8) for the (0,H) and (H,H) strategies respectively. The buyer profits at equilibrium can be derived from (2.3) for the (0,H) and (H,H) strategies. If (0,H) is the equilibrium outcome then

\[ \Pi_A(0,H) = (p - w) P_A(1 - P_B) H \] (2.14)
\[ \Pi_B(0,H) = 0 \] (2.15)

For the (H,H) strategy, we get

\[ \Pi_A(H,H) = (p - w) P_A(1 - P_B) H \] (2.16)
\[ \Pi_B(H,H) = H \frac{P_B}{1 - P_A} (p - w)(1 - 2P_A + P_A P_B) \] (2.17)

Under the (0,H) strategy outcome, the contractor’s reservation fee, \( r_{B,1} \) results in market clearing for Buyer B and as such, this buyer makes no positive gain. She is always better off with the (H,H) strategy outcome due to decreased reservation fees.
In this case, although Buyer A’s reservation does not directly contribute to Buyer B’s expected payoff - since buyers do not take advantage of each other’s reserved inventory when they both reserve, - the existence of Buyer A leads to a lower reservation fee eventually benefiting Buyer B. We note that the existence of Buyer A also benefits the contractor even if she does not reserve. Because Buyer A is a potential customer for the inventory reserved by the other buyer, her mere existence in the pool increases the expected payoff for the contractor and makes reservation a viable option for the supply chain.

From (2.15) and (2.17), we observe that Buyer A is indifferent between the two strategy outcomes since \( r_{A,2} \) is the market clearing fee for this buyer. While Buyer A’s payoff solely depends on the other buyer’s reservation under the (0,H) strategy outcome, her payoff under the (H,H) strategy is independent of the Buyer B’s reservation. However, in the latter case, Buyer B’s existence indirectly benefits Buyer A due to reduced reservation fees.

The gap in Buyer B profits between both strategy outcomes may incentivize this buyer to enter into a special contract with the contractor without affecting Buyer A. Consider an addendum to the reservation contract between the contractor and Buyer B, which stipulates that the latter one makes a transfer payment to the former one for each unit of capacity reserved by Buyer A. This is a payment to the contractor in addition to what she collects from Buyer A. We let \( \beta(c - s)Q_A \) denote this payment. In this way, Buyer B subsidizes the contractor’s risk on Buyer A’s reservation, which encourages her to lower the reservation fee sufficiently enough to prompt reservations from both buyers. Such a contract can be designed to achieve a win-win outcome as specified in the following proposition:
Proposition 2 When $\Delta < 0$, with transfer payment $\beta(c - s)Q_A$ from Buyer B, the contractor prefers $r_{A,2}$ over $r_{B,1}$ at equilibrium leading to the (H,H) strategy outcome for any $\beta$ such that

$$\frac{\Pi_B(H,H)}{c - s} \geq \beta \geq \frac{-\Delta}{H(c - s)}$$  \hspace{1cm} (2.18)

Buyer B is better off with this transfer payment if and only if

$$P_A P_B > \frac{c - s}{p - s} \hspace{1cm} (2.19)$$

Buyer A profits are not affected by this contract.

Proof: The lower bound for $\beta$ is derived by solving $\Delta + \beta(c - s)H \geq 0$, at which point the contractor is better off choosing $r_{A,2}$ as the reservation fee leading to the (H, H) strategy outcome. However, this transfer payment is acceptable to Buyer B only if it satisfies her rationality constraint, which in this case is $\frac{P_B}{1 - P_A} (p - w)(1 - 2P_A + P_A P_B) - \beta(c - s) \geq 0$. This constraint leads to the upper bound for $\beta$ given in (2.18). Clearly, a feasible value for $\beta$ is possible only when the upper bound is greater than or equal to the lower bound. This inequality between the bounds given in (2.18) can be easily reduced to the inequality given in (2.19). Since the given transfer payment does not involve Buyer A and her expected profits are same under both strategy outcomes, the proposed addendum does not affect this buyer.

The value range given in (2.18) is possible only if the inequality (2.19) holds. We note that the ratio given in (2.19) is equivalent to the critical ratio for the integrated channel. The above result reveals that when $\Delta < 0$, a contractual setting with a win-win outcome is possible if the event probabilities or the buyer margins are sufficiently large. Otherwise, this arrangement is not justified for both Buyer B and the contractor. This observation is consistent with our earlier discussion regarding
the impact of $p$ on equilibrium outcome. Buyer B finds this contract addendum more useful under higher buyer margins, as the contractor has less incentive for setting a reservation fee leading to the (H, H) under high resale prices. In general, the contractor payoff is higher with larger $\beta$ values and the opposite is true for Buyer B.

At this point, our analysis provides us with two major insights. First, the economical justification of the contingency inventory reservation does not necessarily require that all participating buyers make positive reservations. The possibility of utilization of reserved inventory by a non-reserving buyer, in fact, may benefit both the contractor and the reserving buyer. Second, the contractor may benefit from offering a special transfer-payment contract to the buyer who has the higher event probability. The special handling of this buyer results with a win-win outcome under conditions specified by Proposition 2. In the next section, we extend our analysis to the case where a penalty is charged for the non-reserved items and investigate its impact on the equilibrium reservation strategies and player payoffs.

### 2.3 Surcharge Fee for Unreserved Inventory

In this setting, the buyers are subject to pay a penalty per unit inventory item that they request beyond their reservation. We let $u$ denote this penalty. All penalty payments are kept by the contractor. Such penalty is expected to induce incentives for the buyers to reserve more at higher reservation fees compared to the previous case. In this section, we study the impact of such penalty on player incentives and the equilibrium outcome. We aim to investigate whether a surcharge fee always adversely affects the buyers’ equilibrium profits compared to the no-surcharge setting. We note
that the no-surcharge setting discussed in the previous section is a special case, where $u = 0$. In order to avoid pathological cases we assume that $w + u \leq p$.

2.3.1 Buyers’ Reservations

Under this setting, the expected profit function for Buyer $i$ given in (2.2) can be rewritten as follows:

$$
\Pi_i^{(u)} = -rQ_i + (p - w + r)P_iQ_i + (p - w - u)P_i(1 - P_{-i}) \min(Q_i, H - Q_i). \quad (2.20)
$$

From the first derivatives, we get

$$
\frac{\partial \Pi_i^{(u)}}{\partial Q_i} = \begin{cases} 
-(1 - P_i)r + (p - w)P_i, & \text{if } Q_i \leq H - Q_{-i} \\
-(1 - P_i)r + P_iP_{-i}(p - w) + P_i(1 - P_{-i})u, & \text{o/w} 
\end{cases} \quad (2.21)
$$

The first order derivatives above lead to the following reservation fee thresholds:

$$
r_{i,1}^{(u)} = \frac{P_i}{1 - P_i}(p - w) \quad (2.22)
$$

$$
r_{i,2}^{(u)} = \frac{P_i}{1 - P_i}((1 - P_{-i})u + P_{-i}(p - w)) \quad (2.23)
$$

We observe that while the first threshold is identical to the no-surcharge setting discussed in the previous section (i.e., $r_{i,1}^{(u)} = r_{i,1}$), the second threshold takes a higher value with any positive surcharge (i.e., $r_{i,2}^{(u)} = r_{i,2} + uP_i(1 - P_{-i})/(1 - P_i)$). This implies that a buyer is willing to reserve the full amount regardless of the other buyer’s action at relatively higher reservation fees so as to avoid the surcharge for unreserved inventories.
In this case, the ordering among the threshold reservation fees depends on the value of the surcharge $u$. Let $\hat{u}$ be defined as:

$$\hat{u} = \frac{P_A(1 - 2P_B + P_A P_B)}{P_B(1 - P_A)^2} (p - w)$$ (2.24)

The following result lays out the ordering among the reservation fee thresholds:

**Lemma 6** Assuming, $P_A < P_B$ and $P_B < 1/2$, it is always true that $r_{B,1}^{(u)} > r_{A,1}^{(u)} > r_{B,2}^{(u)} > r_{A,2}^{(u)}$ if and only if $u \leq \hat{u}$. Otherwise, $r_{B,1}^{(u)} > r_{B,2}^{(u)} > r_{A,1}^{(u)} > r_{A,2}^{(u)}$.

**Proof:** It is straightforward to observe from (2.22) and (2.23), $r_{i,1}^{(u)} > r_{i,2}^{(u)}$ for both A and B. Since the inequality $\frac{P_i}{1 - P_i} (p - w) > \frac{P_i}{1 - P_i} ((1 - P_{-i})u + P_{-i}(p - w))$ reduces to $p \geq w + u$ and, it is our rationality assumption for buyers to be in the pool, we deduce that $r_{B,1}^{(u)} > r_{B,2}^{(u)}$ and $r_{A,1}^{(u)} > r_{A,2}^{(u)}$. Also, we can easily observe that $r_{B,1}^{(u)} > r_{A,1}^{(u)}$ since $\frac{P_B}{1 - P_B} > \frac{P_A}{1 - P_A}$ for $P_B > P_A$ and $r_{B,2}^{(u)} > r_{A,2}^{(u)}$ since the inequality reduces to $u(1 - P_A P_B) \geq -(p - w)P_A P_B$ and it holds for any values of the parameters.

To complete the rest of the proof, observe that $r_{A,1}^{(u)} \geq r_{B,2}^{(u)}$ reduces to $u \leq \hat{u}$. As such, in this case, the ordering is $r_{B,1}^{(u)} > r_{A,1}^{(u)} > r_{B,2}^{(u)} > r_{A,2}^{(u)}$. Consequently, when $u > \hat{u}$ we get $r_{B,1}^{(u)} > r_{B,2}^{(u)} > r_{A,1}^{(u)} > r_{A,2}^{(u)}$.

The above observation reveals that the ordering is similar to the no-penalty case when the penalty is below the threshold given in (2.24). With high penalties, since $r_{i,2}$ increase in $u$, the ordering between $r_{A,1}^{(u)}$ and $r_{B,2}^{(u)}$ changes. The orderings are significant in that they lead us to the equilibrium strategies for given reservation fee value ranges. The following proposition presents the equilibrium reservation strategies across both buyers under the presence of $u$:
Proposition 3 Under no-reservation penalty fees, the equilibrium buyer reservation quantities in response to the contractor’s reservation value ranges are as given in the below table:

<table>
<thead>
<tr>
<th>Range ((u \leq \bar{u}))</th>
<th>Buyers’ Eq. Reservation</th>
<th>Range ((u &gt; \bar{u}))</th>
<th>Buyers’ Eq. Reservation</th>
</tr>
</thead>
<tbody>
<tr>
<td>([0, r^{(u)}_{A,2}])</td>
<td>((H, H))</td>
<td>([0, r^{(u)}_{A,2}])</td>
<td>((H, H))</td>
</tr>
<tr>
<td>([r^{(u)}<em>{A,2}, r^{(u)}</em>{B,2}])</td>
<td>((0, H))</td>
<td>([r^{(u)}<em>{A,2}, r^{(u)}</em>{A,1}])</td>
<td>((0, H))</td>
</tr>
<tr>
<td>([r^{(u)}<em>{B,2}, r^{(u)}</em>{A,1}])</td>
<td>((x, H - x))</td>
<td>([r^{(u)}<em>{A,1}, r^{(u)}</em>{B,2}])</td>
<td>((0, H))</td>
</tr>
<tr>
<td>([r^{(u)}<em>{A,1}, r^{(u)}</em>{B,1}])</td>
<td>((0, H))</td>
<td>([r^{(u)}<em>{B,1}, r^{(u)}</em>{B,1}])</td>
<td>((0, H))</td>
</tr>
<tr>
<td>([r^{(u)}_{B,1}, \infty))</td>
<td>((0, 0))</td>
<td>([r^{(u)}_{B,1}, \infty))</td>
<td>((0, 0))</td>
</tr>
</tbody>
</table>

Table 2.3: Reservation Fee Ranges and Buyers’ Equilibrium Reservations with \(u > 0\)

The proof is similar to Proposition 1. We observe that while the equilibrium outcomes are identical with the previous case when \((u \leq \bar{u})\), for \((u > \bar{u})\) all ranges lead to unique equilibrium strategies.

2.3.2 Contractor’s Reservation Fee

The contractor’s expected payoff under this setting can be obtained from evaluating (2.1) at \(a = 0\). As shown in the following result, although there are two separate equilibrium scenario sets depending on the value of \(u\), the contractor’s equilibrium decision results with the same pattern derived in the base case.

Lemma 7 At equilibrium, the contractor sets her reservation fee to either 1) \(r^* > r^{(u)}_{B,1}\) and there will be no reservation; or 2) \(r^{uu} \approx r^{(u)}_{B,1}\) leading to \((0, H)\); or 3) \(r^{uu} \approx r^{(u)}_{A,2}\) leading to \((H, H)\).
The contractor’s payoffs in the second and third scenarios given in the above lemma can be written as follows:

\[ \Pi_S^{(u)}(r_{B,1}^{(u)}) = \Pi_S(r_{B,1}) + HuP_A(1 - P_B) \]  
(2.25)

\[ \Pi_S^{(u)}(r_{A,2}^{(u)}) = \Pi_S(r_{A,2}) + (2 - P_A - P_B)HuP_A \frac{(1 - P_B)}{1 - P_A} \]  
(2.26)

Observe in both cases that the contractor enjoys higher expected payoffs with positive penalty fees. It should be noted that in the second scenario, that is with (H,H) outcome, although the contractor never collects any penalty fees, she realizes higher payoff due to higher reservation fees. Similar to the base case, the contractor’s reservation fee choice depends on the comparison between these two payoff values.

**Corollary 2** The contractor opts out of any reservation policy if and only if

\[ \max(\Pi_S^{(u)}(r_{B,1}^{(u)}), \Pi_S^{(u)}(r_{A,2}^{(u)})) < 0. \]

Otherwise,

i) If \( \Pi_S^{(u)}(r_{B,1}^{(u)}) \geq \Pi_S^{(u)}(r_{A,2}^{(u)}) \) then \( r^{u*} \approx r_{B,1}^{(u)} \) is the best reservation fee choice for the contractor resulting with the (0, H) reservation amounts.

ii) If \( \Pi_S^{(u)}(r_{B,1}^{(u)}) < \Pi_S^{(u)}(r_{A,2}^{(u)}) \) then \( r^{u*} \approx r_{A,2}^{(u)} \) is the best reservation fee choice for the contractor resulting with the (H, H) reservation amounts.

We let \( \Delta^{(u)} \) denote the difference between the profit functions given in (2.25) and (2.26), that is

\[ \Delta^{(u)} = \Pi_S^{(u)}(r_{A,2}^{(u)}) - \Pi_S^{(u)}(r_{B,1}^{(u)}). \]

Basically, if \( \Delta^{(u)} > 0 \), the contractor prefers \( r^{u*} \approx r_{A,2}^{(u)} \) over \( r \approx r_{B,1}^{(u)} \). Opposite is true otherwise. The influence of penalty fee on this difference and hence, on the contractor’s reservation fee choice is summarized as follows:
Lemma 8 Assuming $0 < P_A \leq P_B \leq 1/2$, $\Delta^{(u)}$ strictly increases in $u$.

The proof directly follows from the first derivative with respect to $u$. The above result indicates that a reservation fee leading to $(H,H)$ strategy becomes more appealing with larger penalty fees. This is expected as higher penalties increase the risk of fulfilling the demand surge with higher cost for the buyers. As such, they are willing to pay more for reserving inventory. At the same time, they diminish the risk for the contractor and hence, offer room for charging higher reservation fees. The impact of the other parameters are similar to Lemmas 3 and 5, and the impact of $P_B$ is specified in what follows:

Lemma 9 $\Delta^{(u)}$ is strictly concave in $P_B$ for $1 \geq P_A \geq 0$, with a unique $P_B$ value that maximizes $\Delta^{(u)}$ when the penalty value is positive

$$P_B^* = P_B^* - \frac{(p-w) - P_A(1-P_A)(p-s)}{2P_A(p-w)(p-w-u)}u$$  \hspace{1cm} (2.27)

Proof: Following the proof of Lemma (5), in order to prove that we look at the second derivative of $\Delta^{(u)}(r_{A,2},r_{B,1})$:

$$\frac{\partial^2 \Delta^{(u)}}{P_B^2} = \frac{2HP_A}{1-P_A}(u-(p-w))$$  \hspace{1cm} (2.28)

The above function is strictly negative for $0 < P_A \leq 1$ and $u < p - w$ establishing concavity. The unique maximizer given in (2.27) is found solving the first derivative for $P_B$.

The unique maximizer given in (2.27) is found by solving the first order derivative of $\Delta^{(u)}$ with respect to $P_B$. The above result reveals that impact of $P_B$ on $\Delta^{(u)}$ depends on the surcharge $u$. Specifically, if $(p-w) > P_A(1-P_A)(p-s)$ then for any positive
value of $u$, $P_B^{u*} < P_B^*$. In such case, $\Delta^{(u)}$ decreases in $P_B$ when the surcharge is high. In other words, when the buyer margins are large, high surcharge fee coupled with high event probability for Buyer B leads to the (0,H) outcome at equilibrium. We know from the analysis of the previous section that the (0,H) outcome becomes preferable as $P_B$ increases under higher resale prices. Higher surcharge enforces this leaning as higher $u$ does not only enable higher reservation fee to charge to Buyer B but also higher potential gains for unreserved inventory from Buyer A. Figure 2.4 illustrates this relationship, where the indifference curve is increasing under large values of $u$. As such, the (0,H) region grows as $P_B$ increases. The opposite is observed with small values of $u$. When the buyer margin is small so that $(p - w) < P_A(1 - P_A)(p - s)$, $P_B^{u*} > P_B^*$ holds for any positive values of $u$. We note that in this case the inequality in (2.13) cannot hold and as such, $\Delta^{(u)}$ increases in $P_B$ for any values of $u$, where $u < p - w$.

![Figure 2.4: Indifference Curves of Surcharge Fees (u) Between (0,H) and (H,H)](image-url)
2.3.3 Equilibrium Profits

The buyer profits at both equilibrium is derived from (2.20) in a way similar to the base case. For the (0,H) equilibrium, the buyers profit functions are:

\[
\Pi_A^{(u)}(0, H) = \Pi_A(0, H) - uP_A(1 - P_B)H
\]

(2.29)

\[
\Pi_B^{(u)}(0, H) = 0
\]

(2.30)

For the (H,H) strategy, we get

\[
\Pi_A^{(u)}(H, H) = \Pi_A(H, H) - uP_A(1 - P_B)H
\]

(2.31)

\[
\Pi_B^{(u)}(H, H) = \Pi_B(H, H) - u\frac{P_A(1 - P_B)^2}{1 - P_A}H
\]

(2.32)

Similar to the no-surcharge setting, Buyer A is again indifferent between the two strategies, however her expected gain is lower than the former scenario. Her margin diminishes because of expected loss from surcharge fee under the (0,H) strategy, while her lower margin only stems from paying higher reservation fee under the (H,H) strategy.

As is the case in the former scenario, Buyer B realizes no gain under (0,H) strategy, while her gains diminishes under the (H,H) strategy outcome compared to the same outcome of the no-surcharge setting due to the higher reservation fee that she needs to pay. (\(r_{A,2}^{(u)} > r_{A,1}\)). However, interestingly, Buyer B may in fact benefit from the inset of the surcharge if the equilibrium outcome in the no-surcharge setting discussed in the previous section is (0,H). The inset of the surcharge may shift the equilibrium to (H,H), under which the Buyer B enjoys a strictly positive profit. This situation is presented by the following:
Lemma 10 When $\Delta < 0$, Buyer B is better off with a positive surcharge, $u$, if

$$p - w \geq u > \frac{-\Delta(1 - P_A)}{HP_A(1 - P_B)^2}$$ (2.33)

It is straightforward to see that when the above inequality holds, $\Delta(u) > 0$ must also hold, implying that the (H,H) strategy is the equilibrium outcome with the inset of the surcharge. As such, the Buyer B realizes higher profits compared to the (0,H) strategy pair which is the resulting equilibrium otherwise.

In case $\Delta(u) < 0$, similar to the previous case, the contractor and Buyer B may engage in an addendum that can be design to achieve a win-win outcome for both parties.

Proposition 4 When $\Delta(u) < 0$, with transfer payment $\beta(c - s)Q_A$ from Buyer B, the contractor prefers $r_{A,2}^{(u)}$ over $r_{B,1}^{(u)}$ at equilibrium leading to the (H,H) strategy outcome for any $\beta$ such that

$$\frac{\Pi_B(H,H)^{(u)}}{c - s} \geq \beta \geq \frac{-\Delta(u)}{H(c - s)}$$ (2.34)

Buyer B is better off with this transfer payment if and only if

$$P_A P_B > \frac{c - s}{p - s}$$ (2.35)

Buyer A profits are not affected by this contract.

The proof is similar to that of Proposition 2.
2.4 Surcharge Fee for Unreserved Inventory with Transfer Payment

In this section we consider the case where a portion of the surcharge fee is transferred to the reserving buyer in the event her reserved inventory is delivered to the other buyer. This setting applies when the reservation is regarded as partial ownership. The share of the reserving buyer in the surcharge fee $u$ is denoted by $a$. As such, for each unreserved inventory delivered to the other buyer, the buyer receives a payoff of $au$. In this case, it is conceivable that a buyer may reserve more than her contingency need if she believes the excess quantity might bring about additional payoffs by means of surcharge transfers. We investigate if this may indeed be the case and analyze the impact of partial ownership entailed by reservation on the equilibrium strategies in general.

2.4.1 Buyers’ Reservations

The expected profit function for Buyer $i$ given in (2.2) can be rewritten as follows:

$$
\Pi_i = -rQ_i + P_i(p - w + r)\min(H, Q_i) + P_i(1 - P_{-i})(p - w - u)\min(Q_{-i}, (H - Q_i)^+) \\
+ (1 - P_i)P_{-i}au\min(Q_i, (H - Q_{-i})^+) + P_iP_{-i}au\min((H - Q_{-i})^+, (Q_i - H)^+). 
$$

(2.36)

The above profit function differs from the previous case with the last term that represents the income due to the surcharge transfers. Similar to the previous cases, we obtain the first order derivative for Buyer $i$ with respect to her reservation quantity as follows:
\[
\frac{\partial \Pi_{i}}{\partial Q_{i}} = \begin{cases} 
-r(1-P_{i}) + P_{i}(p-w) + (1-P_{i})P_{-i}au, & \text{if } Q_{i} \leq H - Q_{-i} \\
-(1-P_{i})r + P_{i}P_{-i}(p-w) + P_{i}(1-P_{-i})u, & \text{if } H \geq Q_{i} > H - Q_{-i} \\
-r + P_{i}P_{-i}au, & \text{if } 2H - Q_{-i} > Q_{i} > H \\
-r, & \text{if } Q_{i} > 2H - Q_{-i} 
\end{cases}
\] (2.37)

As mentioned above, since a reserving buyer can potentially make additional income from the surcharge transfer, she may choose to reserve more than her contingency need. This results in additional ranges in her piece-wise profit function and hence, an additional reservation fee threshold. We denote that threshold by \(r_{3}\). Consequently, we get the following threshold values for Buyer \(i\) (\(i = A, B\)):

\[
r_{i,1}^{(au)} = \frac{P_{i}(p-w) + (1-P_{i})P_{-i}au}{1-P_{i}} \quad (2.38)
\]

\[
r_{i,2}^{(au)} = r_{i,2}^{(u)} \quad (2.39)
\]

\[
r_{3}^{(au)} = P_{i}P_{-i}au \quad (2.40)
\]

In this case, we observe that the first threshold takes a higher value than the thresholds obtained in the previous two scenarios with any positive surcharge and transfer rate (\(i.e., r_{i,1}^{(au)} = r_{i,1}^{(u)} + auP_{-i}\)). The second threshold is unchanged (\(i.e., r_{i,2}^{(au)} = r_{i,2}^{(u)}\)) compared to the no-transfer setting. We note that the new threshold \(r_{3}^{(au)}\) is identical across both buyers.

As in the previous case, the surcharge fee affects the ordering among the threshold reservation fees. Let \(\hat{u}^{(a)}\) be defined as follows:

\[
\hat{u}^{(a)} = \frac{(p-w)P_{A}(1-2P_{B}+P_{A}P_{B})}{(1-P_{A})P_{B}(1-P_{A}+a(1-P_{B}))} \quad (2.41)
\]
It is easy to verify that \( r_{A1}^{(au)} \leq r_{B2}^{(au)} \) if and only if \( u > \hat{u}'(a) \). Consequently, we deduce the following conclusion in regards to the ordering of the thresholds:

**Lemma 11** Assuming, \( P_A < P_B \) and \( P_B < 1/2 \), it is always true that \( r_{B1}^{(au)} > r_{A1}^{(au)} > r_{B2}^{(au)} > r_{A2}^{(au)} > r_3^{(au)} \) if and only if \( u \leq \hat{u}' \). Otherwise, \( r_{B1}^{(au)} > r_{B2}^{(au)} > r_{A1}^{(au)} > r_{A2}^{(au)} > r_3^{(au)} \).

The proof is straightforward and similar to that of Lemma 6. The above result leads to the following equilibrium reservation strategies across both buyers:

**Proposition 5** Under no-reservation penalty fees and surcharge transfer policy, the equilibrium buyer reservation quantities in response to the contractor’s reservation value ranges are as given in the below table:

<table>
<thead>
<tr>
<th>Range ((u \leq \hat{u}))</th>
<th>Buyers Eq. Reservation</th>
<th>Range ((u &gt; \hat{u}))</th>
<th>Buyers’ Eq. Reservation</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0, r_{A1}^{(au)}))</td>
<td>((H, H))</td>
<td>((0, r_{A1}^{(au)}))</td>
<td>((H, H))</td>
</tr>
<tr>
<td>((r_{A1}^{(au)}, r_{A2}^{(au)}))</td>
<td>((H, H))</td>
<td>((r_{A2}^{(au)}, r_{A1}^{(au)}))</td>
<td>((0, H))</td>
</tr>
<tr>
<td>((r_{A2}^{(au)}, r_{B2}^{(au)}))</td>
<td>((0, H))</td>
<td>((r_{B2}^{(au)}, r_{A2}^{(au)}))</td>
<td>((0, H))</td>
</tr>
<tr>
<td>((r_{A1}^{(au)}, r_{B2}^{(au)}))</td>
<td>((x, H - x))</td>
<td>((r_{B2}^{(au)}, r_{B1}^{(au)}))</td>
<td>((0, H))</td>
</tr>
<tr>
<td>((r_{B1}^{(au)}, \infty))</td>
<td>((0, 0))</td>
<td>((r_{B1}^{(au)}, \infty))</td>
<td>((0, 0))</td>
</tr>
</tbody>
</table>

Table 2.4: Reservation Fee Ranges and Buyers’ Equilibrium Reservations with \( u > 0 \) and \( 0 < a < 1 \)

The proof is similar to Proposition 1. We notice that the equilibrium outcomes are similar to the no-transfer setting. The result reveals that at equilibrium none of the players will have incentives to reserve more than their contingency need, namely, \( H \). The intuition is that a buyer would choose to reserve more than \( H \) only when the reservation fee and hence, the risk is sufficiently low. However, in this case, the other buyer would also take advantage of the low reservation fee and reserve. Consequently, none ends up reserving more than \( H \).
**Remark 1** *The surcharge transfer does not give any incentives to the buyers to reserve more than their own need.*

### 2.4.2 Contractor’s Reservation Fee and Equilibrium Profits

The contractor’s expected payoff under this setting can be obtained from evaluating (2.1) at \( a > 0 \).

**Lemma 12** *At equilibrium, the contractor sets her reservation fee to either 1) \( r^* > r_{B,1}^{(au)} \) and there will be no reservation; or 2) \( r_{aus} \approx r_{B,1}^{(au)} \) leading to \((0,H)\); or 3) \( r_{aus} \approx r_{A,2}^{(au)} \) leading to \((H,H)\).*

The contractor’s payoffs at the above thresholds are

\[
\Pi_S^{(au)}(r_{B,1}^{(au)}) = \Pi_S^{(u)}(r_{B,1}^{(u)}) \quad (2.42)
\]

\[
\Pi_S^{(au)}(r_{A,2}^{(au)}) = \Pi_S^{(u)}(r_{A,2}^{(u)}) \quad (2.43)
\]

We observe that the contractor’s expected profit is independent of the transfer rate \( a \) and equal to the previous case, where \( a = 0 \). This result reveals that the surcharge transfer rate has no affect on her expected profits in both scenarios. Although this is expected for the \((H,H)\) case where no transfer takes place, the result in regards to the \((0,H)\) outcome is not straightforward. In the latter case, the contractor has to transfer a portion of the surcharge fees to the reserving buyer. As such, it is expected that her profit under \((H,H)\) strategy pair is lower with transfer. However, the contractor offsets the revenue loss due to the surcharge transfer by setting a higher reservation fee \( r_{B,1}^{(au)} \). We note from 2.38 that the higher reservation fee is accepted by Buyer B.
since her risk is mitigated by the prospect of the surcharge transfer. Consequently, we make the following observation:

**Remark 2** The partial transfer of the no-reservation surcharge has no effect on the contractor’s equilibrium strategy choice when the buyers’ contingency demands are equal.

Based on this result, we conclude that the contractor’s equilibrium strategy is as defined in Corollary 2 and thus, depends on $\Delta^{(u)}$ regardless of the transfer rate $a$. Moreover, using (2.36) we can easily derive the equilibrium profits for the buyers, and observe that they are also independent of $a$ and as such, equal to $\pi^{(u)}_i(0, H)$ and $\pi^{(u)}_i(H, H)$ for $i = A, B$. Consistent with the contractor’s case, the expected surcharge transfer received by any buyer is negated by the increased payments on inventory reservation. At the end, the transfer of the surcharge does not impact the equilibrium outcome and the expected profits except for the value of the reservation fee $t^{(au)}_{B,1}$, which increases with $a$. Figure 2.5 illustrates this relationship, where the indifference curve is not affected by the low or large values of $a$.

![Figure 2.5: Indifference Curves of Transfer Rates (a) Between (0,H) and (H,H)](image-url)
2.5 Voluntary Compliance

So far in our analysis we assume forced compliance where the contractor is obligated to actually procure and carry an inventory amount, $I$, that matches the amount of inventory reserved by the buyers, i.e., $I = Q_A + Q_B$. Under voluntary compliance, the contractor may choose to acquire inventory below the reservation amounts, i.e., $I < Q_A + Q_B$, and hence take the risk of not fully satisfying the reserved amounts. Typically, in such cases the contractor pays a "shortage penalty" for each unit of inventory reserved but not delivered. Depending on the penalty level, she may have incentives for "double booking", which impacts the reservation amount decisions for the buyers. Unless the contractor sends a credible signal to the buyers regarding her compliance, the buyers will be reluctant to make any reservations. Such signal can be ensured if the shortage penalty is large enough.

We let $t$ denote the unit shortage penalty paid to the buyers in the event the reserved inventory cannot be delivered. Then the contractor's profit function for given $r, Q_A$, and $Q_B$ becomes

$$
\Pi_S = r(Q_A + Q_B) - cI + \Psi_s sI + \Psi_A[(w - r) \min(Q_A, I)]
+ (w + (1 - a)u) \min(H - Q_A, \max(Q_C - Q_A, 0)) + s \max(0, Q_C - H) - t \max(Q_A - Q_C, 0)]
+ \Psi_B[(w - r) \min(Q_B, Q_C) + (w + (1 - a)u) \min(H - Q_B, \max(Q_C - Q_B, 0))]
+ s \max(0, Q_C - H) - t \max(Q_B - Q_C, 0)] + \Psi_{A,B}[(w - r)Q_C - t \max(Q_A + Q_B - Q_C, 0)]
$$

(2.44)

In this scenario, the contractor receives a reservation fee for the total reservation quantity. However she only pays for the quantity she holds in her inventory. The
following proposition presents the sufficient condition for the shortage penalty that ensures the contractor’s full compliance with the reservation contract:

**Proposition 6** The contractor voluntarily complies to buyers’ reservation amounts for any given \( u \) and \( a \) if

\[
t \geq r + \frac{(c - s) - (w - s)P_AP_B}{P_AP_B} \tag{2.45}
\]

**Proof:** We show that the contractor’s inventory decision is strictly increasing when the inequality in the proposition holds resulting in full match of the total reservations. First we consider the case \( I < \min(Q_A, Q_B) \). In this case, the derivative of the contractor’s expected profit function with respect to \( I \) is

\[
\frac{\partial \Pi_S}{\partial I} = -(c - s) + (w - r + t - s)(P_A + P_B - P_A P_B) \tag{2.46}
\]

It is straightforward to see that the above function returns a positive value if (2.45) holds, since it is assumed that \( P_A \leq P_B \leq 0.5 \).

When \( Q_i > I \geq Q_{-i} \), the derivative is

\[
\frac{\partial \Pi_S}{\partial I} = -(c - s) + (w - r + t - s)P_i \tag{2.47}
\]

which is strictly positive for any values of \( t \) defined by (2.45). Finally, when \( Q_A + Q_B > I \geq \max(Q_A, Q_B) \) we get

\[
\frac{\partial \Pi_S}{\partial I} = -(c - s) + (w - r + t - s)P_A P_B \tag{2.48}
\]

which is nonnegative if only if (2.45) holds.

The condition given in the above proposition is a sufficient condition. Lower penalties can be adopted for \( \Delta^{(u)} < 0 \), where only one of the buyers reserves. Intuitively, higher reservation fees imply higher shortage penalties. We note from
(2.45) that the shortage penalty is greater than or equal to the reservation fee when 
\[(c - s) - (w - s)P_A P_B \geq 0.\] That is, in case of shortage, not only the reservation fee is returned to the buyer but also an additional amount is paid (i.e., \(t - r\)). Recall from Lemma 4 that in this case, the contractor never offers a reservation fee leading to the (H,H) strategy outcome. When the inequality is reversed, the penalty is below the reservation fee. That is, only a portion of the reservation fee is returned to the buyer in case of shortage. Nevertheless, the contractor voluntarily complies with the reservation amounts as long as inequality (2.45) holds.

2.6 Beyond Two Buyers

In order to facilitate our analysis and mathematical tractability we consider a two-
buyer setting in the previous sections. As the number of buyers increases, the possible equilibrium outcomes also increases. The increase is not linear due to combination of reservation strategy options. The complexity is exacerbated by the fact that there may be instances where multiple buyers may demand the same inventory originally reserved by another buyer. In this case, possible strategy outcomes are influenced by how the unused inventory of a reserving buyer is shared among the other buyers. In this section, we discuss a three-buyer setting so as to illustrate this effect before presenting our results pertaining to more general settings.

Suppose we now have three buyers, namely, Buyers A, B, and C. Without loss of generality, we assume that \(P_A < P_B < P_C\). In order to facilitate our analysis, we assume that any inventory unused by the reserving buyer is shared uniformly across others who demand it. As such, each buyer’s reservation decision is not only influenced by the reservation fee and the other buyers’ reservation amounts but also
by her share on excess inventory if she does not reserve the full amount. Consequently, we can write the expected profit function for Buyer C as follows:

\[
\Pi_C = -rQ_C + (p - w + r)P_CQ_C + (p - w - u)P_C[(1 - P_A)(1 - P_B) \min(H - Q_C, Q_A + Q_B) \\
+ P_A(1 - P_B) \min(H - Q_C, (Q_B - \min(H - Q_A, Q_B/2))^+) \\
+ (1 - P_A)P_B \min(H - Q_C, (Q_A - \min(H - Q_B, Q_A/2))^+)
\]

(2.49)

The profit functions for Buyers A and B are symmetric and can be written in similar fashion. Recall from the two-buyer case that each buyer reserves either nothing (i.e., 0) or the full amount (i.e., \(H\)) at equilibrium. Since the buyer profits are piece-wise linear in reservation amounts, a buyer either ensures her contingency supply by full reservation or bets on the inventory reserved by the other buyer. On the other hand, in the three-buyer case, the number of break-points increases as observed in (2.49) leading to reservation amounts that may differ from 0 or \(H\) at equilibrium. Let the strategy outcome for reservation amounts are represented by \((Q_A, Q_B, Q_C)\) for Buyers A, B, and C respectively. The possible equilibrium outcomes are specified by the following proposition:

**Proposition 7** In a three buyer setting, where \(P_A < P_B < P_C\), exactly one of the following reservation strategies can be the equilibrium outcome: \((0, 0, 0)\), \((0, 0, H)\), \\
\((0, H/2, H)\), \((H/2, H/2, H)\), \((0, H, H)\), and \((H, H, H)\).

**Proof:** As in the two buyer-case, the profits functions of the buyers are piece-wise linear in their reservation amounts. Different from the previous case, there are more breakpoints since there are more event scenarios. This implies that there will be more reservation fee thresholds for each buyer. However, we note that the highest
threshold is independent of the number of players and determined by only the buyer with the highest event probability as defined in (2.5) and (2.22). Clearly, at that level the equilibrium is \((0, 0, H)\). After this point, the reservation outcomes depend on the breakpoints in the buyer profits. Given \((0, 0, H)\) and provided that Buyer C does not face any event, available inventory for an other buyer is either \(H\) if the third buyer is also event-free or \(H/2\) if the third buyer is also struck by the rare event. As such, the next breakpoint for Buyer B is \(H/2\). Consequently, the next reservation fee option for the supplier leads to \((0, H/2, H)\). Depending on the order of threshold values, as the supplier decreases her reservation fee, the reservation equilibrium switches to either \((0, H, H)\) or \((H/2, H/2, H)\). When the reservation fee is sufficiently low, all buyers reserve their full contingency volumes, i.e., \((H, H, H)\).

As in the two-buyer case, there is a threshold reservation fee for each strategy outcome. Clearly, lower reservation prices lead to outcomes with larger total reservations. The supplier’s reservation fee choice is determined by the comparison of her expected profits evaluated at the reservation fee threshold levels that correspond to strategy outcomes listed in the above proposition. The equilibrium outcome depends on the cost parameters, the values of even probabilities, and the gap among these probabilities. Similar to the two-buyer analysis, we expect that as the market price \(p\) increases, the supplier selects a higher reservation fee leading to smaller total amounts. The opposite is expected as the wholesale price \(w\) increases. Consistent to the two-buyer case, the impact of other parameters are context dependent and varies based on interaction among each other.

Based on our analysis up to this point, we can extend some of our observations to the general case with \(N\) buyers. Let \(Q_i^e\) denote the equilibrium reservation amount
for Buyer $i$. We recall that in all of the scenarios we analyze so far, the buyer with the low adverse event probability always reserves less than or equal to the buyer with higher adverse event probability in any equilibrium outcome. This is expected since the overage risk for the low probability buyer is always higher. We can easily extend this observation to the general case:

**Corollary 3** In a multiple buyer setting, for any given buyer pair, say, Buyer $i$ and Buyer $j$, if $P_i \geq P_j$ then $Q^e_i \geq Q^e_j$ always hold at equilibrium.

As mentioned earlier, the equilibrium reservation amounts are realized at the break points of the buyers’ piece-wise profit functions. As the number of buyers increases so do the break points since the inventory sharing scenarios grow. However, in this case, it is expected that the reservation fee ranges that lead to the same total reservation amount decreases since the highest reservation fee threshold is determined only by the buyer with the highest event probability. As such, the supplier profits do not vary significantly across reservation strategies that give the same total amount. Consequently, the supplier’s best response, hence the equilibrium outcome, can be approximated by focusing only the reservation strategies that involve full (i.e., $H$) or no reservation (i.e., 0) for each buyer.

### 2.7 Asymmetric Buyers

Our analysis up to this point assumes that the buyers are identical in terms of their contingency needs. Clearly, in a more general setting, buyers may represent markets that are different in size. In this section, we relax our assumption and consider the setting where buyer contingency demands are nonidentical. As such, we let $H_A$
and $H_B$ denote the contingency demands for Buyer A and Buyer B, respectively. Without loss generality, we still assume that $P_A \leq P_B$. In this case, the profit function for Buyer $i$ can be rewritten as follows:

$$\Pi_i = -rQ_i + P_i(p - w + r)\min(H_i, Q_i) + P_i(1 - P_i)(p - w - u)\min(Q_{-i}, (H_i - Q_i)^+)$$

$$+ (1 - P_i)P_Bau\min(Q_i, (H_{-i} - Q_{-i})^+) + P_iP_{-i}au\min((H_{-i} - Q_{-i})^+, (Q_i - H_i)^+).$$

(2.50)

This setting leads to two separate scenarios: In the first one, demand for the buyer with the lower event probability is smaller than the other buyer. That is, $H_A \leq H_B$. In the second scenario, the opposite is true, i.e., $H_A > H_B$. In the first case, the expected sales volume is smaller for Buyer A not only because of the lower probability event but also because of lower contingency demand. For ease of representation, we first consider the case $a = 0$, that is all surcharge payments are collected by the contractor. As discussed next, we observe that the outcome for this case is similar in most part to the symmetric demand case discussed in the previous sections:

**Proposition 8** Assuming $H_A \leq H_B$ and $P_A \leq P_B$, reservation fee thresholds are identical to those given in (2.22) and (2.23). Consequently, at equilibrium the contractor sets her reservation fee to either 1) $r^* > r_{B,1}$ and there will be no reservation; or 2) $r^* \approx r_{B,1}$ leading to $0, H_B)$; or 3) $r^* \approx r_{A,2}^{(u)}$ leading to $(H_A, H_B)$.

**Proof:** Following the proof of Proposition 1, let $G_i$ and $\hat{G}_i$ denote the first order derivatives given in (2.50) for Buyer i. Namely, $G_i = -r + (p - w + r)P_i$ and $\hat{G}_i = -r + (p - w + r)P_i - (p - w)P_i(1 - P_{-i})$. Under the assumption of $p \geq w + u > w \geq c$ and $P_A < P_B \leq 0.5$, the first order derivatives rank as $\hat{G}_A < \hat{G}_B < G_A < G_B$. Clearly,
$G_i$ and $\hat{G}_i$ are both nonnegative when $r \in [0, r_{A,2}]$ for both buyers. As such, both buyers reserve their full demand when the reservation fee is in this range resulting with $(H_A, H_B)$. If $r$ is in $(r_{A,2}, r_{B,2}]$, then $\hat{G}_A < 0$, while $G_A$, $G_B$, and $\hat{G}_B$ are all nonnegative. In this case, Buyer A has no incentive to reserve more than $H_A - Q_B$ while Buyer B reserves her full need resulting with $(0, H_B)$. If $r$ is in $(r_{B,2}, r_{A,1}]$, then $\hat{G}_i < 0$ and $G_i \geq 0$ for both buyers. As such, Buyer $i$ has no incentive to reserve more than $H_i - Q_{-i}$. Since the contingency demand is asymmetric, unlike the symmetric case, there will be no multiple equilibria in this reservation fee range. The buyer whose contingency inventory is higher reserves her full demand while the other does not reserve anything, i.e., $(0, H_B)$ is still the equilibrium outcome. If $r$ is in $(r_{A,1}, r_{B,1}]$, then $G_B > 0$, while $G_A$, $\hat{G}_A$, and $\hat{G}_B$ are all nonpositive. In this case, only Buyer B has incentive to reserve up keeping the outcome at $(0, H_B)$. Finally, when $r > r_{B,1}$, all derivatives are negative implying that buyers have no incentive to make any reservations resulting with the $(0,0)$ strategy outcome.

The proof of the above result follows from the fact that the values of reservation fee thresholds are independent of the contingency demand volumes. It can be easily shown that the results given by Lemma 3 also apply in this case. Suppose now, $\hat{\Delta}^u$ represents the difference in contractor profits between the $(H,H)$ and $(0,H)$ strategy outcomes. That is, $\hat{\Delta}^u = \Pi_S(H_B, H_A) - \Pi_S(0,H_B)$. The influence of the gap between the demand volumes is then captured by the following Lemma:

**Lemma 13** $\hat{\Delta}^u$ increases with $H_A$ if and only if inequality given in (2.19) holds. Otherwise, $\hat{\Delta}^u$ is nonincreasing in $H_A$. On the other hand, assuming $P_A \leq P_B$ and $p - w > u$, $\hat{\Delta}^u$ strictly decreases in $H_B$. 


The proof is straightforward from the first derivatives with respect to \( H_A \) and \( H_B \). The above results implies that the increase in the contingency demand volume of the lower probability buyer does not necessarily provides the contractor with incentives to select a fee that leads to reservation by both buyers. Recall that inequality (2.19) holds when the cost of the product \( (c) \) is sufficiently low and/or the scrap value \( (s) \) and the resale price \( (p) \) are sufficiently high. In this case, additional volume is regarded as higher sales and revenue for the contractor. Consequently, \((H_A, H_B)\) becomes more appealing for her. Whereas, when the inverse is true, the risk that the contractor faces due to high overage costs or lower upfront payments (due to smaller resale prices) does not justify higher reservation amounts, making \((0, H_B)\) more appealing for the contractor. Likewise, as \( H_B \) increases, the contractor prefers increasing the reservation fee and collecting higher payments upfront instead of facing higher risks by lowering the fees and increasing the reservation amounts. Consequently, \((0, H_B)\) becomes more appealing to her. Interestingly, this result reveals that in the context of contingency inventory reservation, higher demand does not necessarily leads to larger reserved inventory at equilibrium.

Proposition 8 indicates that the equilibrium structure in terms of reservation amounts are similar to the symmetric buyers case when \( a = 0 \) in that when there is reservation either only the buyer with higher event probability reserves her full contingency volume or both do the same. However, our next result shows that Contrary to the symmetric case, the equilibrium outcome may be affected when \( a > 0 \), that is when the reserving buyer receives a part of the surcharge paid by the other buyer. Recall that in this case, buyers’ risks can be mitigated by the probability that some of their reservation costs can be recuperated. In the symmetric case, the outcome
does not change since the demand volumes are identical and the contractor employs higher reservation fees that offsets the transfer amounts. When the demand volumes are not identical, we have a somewhat different outcome:

**Lemma 14** Assuming $P_A \leq P_B \leq 0.5$, $H_A < H_B$, and $a > 0$, if $\max(r_{A,1}^{(au)}, r_{B,1}) < r \leq r_{B,1}^{(au)}$ then the unique equilibrium strategy for the buyers is the $(0, H_A)$ outcome.

*Proof:* For the proof of the first part, first we know that, since the buyer profits are piece-wise linear in their reservation amounts, when the reservation fee is too high, there will be no reservation. At $(0,0)$, the next break point for buyers is $H_A$ since $H_A \leq H_B$. For $Q_i \leq H_A$, the first derivative of the buyer profit with respect to the reservation amount is:

$$-r + P_i(p - w + r) + (1 - P_i)P_{-i}au.$$  \hfill (2.51)

Clearly, the above function returns a nonnegative value only if $r \leq r_{i,1}^{au}$. Since $P_b > P_A$, $r_{B,1}^{au} > r_{A,1}^{au}$. This implies that when the reservation fee falls below $r_{B,1}^{au}$ the equilibrium reservation outcome switches from $(0,0)$ to $(0, H_A)$, in which case the first derivative for Buyer B becomes:

$$-r + P_i(p - w + r),$$  \hfill (2.52)

which is negative as long as $r < r_{B,1}$. Hence we can conclude that $(0, H_A)$ is the equilibrium outcome for the buyers if and only if $\max(r_{A,1}^{(au)}, r_{B,1}) < r \leq r_{B,1}^{(au)}$. □

When $H_A < H_B$, at a sufficiently high reservation fee Buyer B may not have sufficient margin to order her full amount $(H_B)$, however, the possibility that her reservation can be compensated partially by the surcharge transfer may provide her with incentives to reserve upto $H_A$. This way, her reservation covers a portion of her
contingency demand. For $r \leq r_{B,1}$, the equilibrium structures will be similar to the case with no transfer payments as presented in Proposition 8.

A more complicated case arises when $H_A > H_B$. Under this condition, although Buyer A has a lower rare-event probability, her larger market may lead to additional strategy outcomes in comparison to the previous case. We note that her expected volume may be higher than Buyer B. As in the previous case, the reservation fee thresholds do not change since they are independent of the market volumes. However, same thresholds bring about new strategy outcomes as presented by the following proposition:

**Proposition 9** Let $\hat{u}$ be defined by (2.24). Assuming $H_A > H_B$ and $P_A \leq P_B$, 

i) If $u \leq \hat{u}$, the contractor sets her reservation fee to either 1) $r > r_{B,1}$ and there will be no reservation; or 2) $r \approx r_{B,1}$ leading to $(0, H_B)$; or 3) $r \approx r_{A,1}$ leading to $(H_A, 0)$; or 4) $r \approx r_{B,2}^{(u)}$ leading to $(H_A - H_B, H_B)$; or 5) $r \approx r_{A,2}$ leading to $(H_A, H_B)$.

ii) If $u > \hat{u}$, the contractor sets her reservation fee to either 1) $r > r_{B,1}$ and there will be no reservation; or 2) $r \approx r_{B,1}$ leading to $(0, H_B)$; or 3) $r \approx r_{A,1}$ leading to $(H_A - H_B, H_B)$; or 4) $r \approx r_{A,2}^{(u)}$ leading to $(H_A, H_B)$.

**Proof:** Since the reservation fee thresholds are independent of buyers’ contingency needs, both buyers reserve full demand when the reservation fee fall in the range of $(0, r_{A,2}]$ regardless of the $u$ value. It is straightforward to observe that $u > \hat{u}$ implies that $G_A < \hat{G}_B$, resulting in $\hat{G}_A < G_A < \hat{G}_B < G_B$. On the other hand, $u < \hat{u}$ leads to $\hat{G}_A < G_A < \hat{G}_B < G_B$.

The above orderings imply the following: If the reservation fee ranges in $(r_{A,2}, r_{A,1}]$ with $u > \hat{u}$ or in $(r_{A,2}, r_{B,2}]$ with $u \leq \hat{u}$, Buyer B has incentive to reserve her full amount while Buyer A has incentive to bring the total reservation amount up to
$H_A$. Consequently, the equilibrium outcome will be $(H_A - H_B, H_B)$. When $u > \hat{u}$, only Buyer B has incentive to reserve under higher reservation fees. As such, the equilibrium outcome is $(0, H_B)$ for $r$ in $(r_{A,2}, r_{B,1}]$. However, for $u \leq \hat{u}$, the equilibrium outcome is $(H_A, 0)$ for $r$ in $(r_{B,2}, r_{A,1}]$ and $(0, H_B)$ when $r$ is in $(r_{A,1}, r_{B,1}]$. If the reservation fee is greater than $r_{B,1}$, none of the buyers has incentive to reserve and the equilibrium outcome will be $(0, 0)$.

Clearly, the highest reservation threshold is always tailored towards the buyer with the higher probability event. When the demand for the buyer with the lower probability event is higher, her amount becomes an option for the total reservation amount, which is reflected by the outcomes $(H_A, 0)$ and $(H_A - H_B, H_B)$. In general, the impact of parameters are similar on the contractor’s reservation fee choice moving from a reservation outcome with smaller amount to larger amounts.

Contrary to the previous case, $(0, H_A)$ is not a possible outcome for $a > 0$ when $H_A > H_B$. In this case, as the reservation fee is reduced the reservation outcome switches from $(0, 0)$ to $(0, H_B)$ first. Since $p - w > u$ and $a < 1$, it is straightforward to conclude that the reservation outcome switches from this point to either $(H_A, 0)$ or to $(H_A - H_B, H_B)$ as given in Proposition 9. Consequently, the equilibrium structure is similar to Proposition 9 except $r_{i,1}$ replaced by $r_{i,1}^{(au)}$. We observe in all cases that no buyer ever reserves more than her own contingency volume.
2.8 Concluding Discussion

An effective contingency-management strategy is one of the key component of a firm’s supply chain strategy, especially for a firm that operates in disaster prone regions. Seller of consumable goods often face a significant opportunity loss from shortages due to the demand surge caused by rare natural disasters and low probability epidemics, e.g., extreme weather conditions, geophysical events, and outbreaks. In these volatile environments, the physical centralization of contingency stocks with multiple retailers decreases the risk of losing extra sale opportunity rising from unexpected events. This centralization also should benefit the mediator, e.g. the contractor, who takes the risk of inventory ownership. Otherwise, she will not be part of the centralization agreement.

In this paper, we introduce a reservation contract for contingency inventory management in which the buyers pool their contingency needs by means of reserving inventory from a shared contractor. The need for contingency inventory emerges as a result of a rare event, which has a probability of occurrence less than 0.5 and creates a surge in demand. The contractor announces a reservation fee for a unit product to be reserved for a single period. This reservation fee is deducted from the exogenous wholesale price if a buyer purchase her own reservation by exercising the contract. However, this fee is non-refundable. If the reserved amount is not exercised by the reserving buyer, the contractor keeps the reservation fee. In that sense, the contractor’s investment risk is partially mitigated by the reservation fee paid by the buyers.

Since we assume that the buyers operate in separate and independent locations, they do not engage in mutual aid agreements, such as sharing supplies with those who experiences a shortage. However, the contractor is able to supply any buyers who
are in need based on availability. Buyer contingency need decision is independent; however this decision affects the other players in the pool. A buyer decision regarding her reservation quantity depends not only on her contingency need but also the other buyer’s decision. Both buyers compete with each other in order to maximize their profits. We model the buyers’ decision problem as a Nash Game in which all the buyers announce their decisions simultaneously. The contractor decides on her reservation fee before the uncertainty regarding the rare-events are resolved. As such, her reservation fee decision is based on the occurrence probabilities of the rare-events.

In our study, we consider three main settings with two competing buyers. In the first and basic case, a buyer with rare-event can procure inventory beyond her reservation amount with no additional cost based on availability. Typically, this is possible if another buyer does not exercise her own reservation. Second setting introduces a surcharge for each additional item procured by the buyer beyond her reservation. In the third and last setting, a portion of the surcharge fee collected by the contractor from a buyer, who receives an inventory item beyond her reservation, is transferred to the buyer who has originally reserved that item.

The analysis of the first setting reveals that reserving through a common contractor can be an effective strategy if the buyer margins are sufficiently high. A buyer with comparably lower event probability may participate in this channel without reserving any inventory if the contractor’s wholesale price and salvage value are low or the unit cost of the product is high. Because of relatively small margins, the supplier offers a reservation fee that is appealing only for the buyer with higher event probability. However, we observe that the mere existence of the nonreserving buyer benefits the channel and results in inventory reservation by the other buyer. The possibility that
a nonreserving buyer may end up utilizing the inventory reserved by others provides incentives for the supplier to offer reservation fees that are attractive for the other buyers. We observe that while the low-probability buyer typically enjoys similar expected payoffs across two possible equilibria; one resulting with no reservation from her and the other under which she reserves. On the other hand, the high-probability buyer is always better off in an outcome where all buyers reserve. Therefore, we propose a coordinating contract between the high-probability buyer and the contractor that ensures that the latter one chooses a reservation fee enabling the low-probability buyer to reserve. This contract results with a win-win outcome for all parties. We identify the conditions under which such contracting approach can be successfully employed.

The inclusion of surcharge fees lowers the risk of the supplier and hence, increases the number of settings under which reservation contracts produce positive reservation amounts. Interestingly, we observe that surcharge fees can benefit the high-probability buyer. The intuition is that the surcharge gives incentives to the supplier to lower her reservation fee to a threshold that enables lower-probability buyer to reserve. Consequently, the high-probability buyer benefits from the reduction in reservation fee, which would not be possible without the surcharge fees. We identify the conditions under which the surcharge fee makes this effect. In the third setting, we observe that the equilibrium reservation amounts and profits are not affected by the transfer of any portion of the transfer fee to the reserving buyer when the contingency demand volume of the low-probability buyer is less than or equal to that of the high-probability buyer. The supplier offsets the transfer amount by increasing her reservation fee. Such increase takes place only for the outcome where one of the buyers opts out by not
reserving any inventory. In this case, since the reserving buyer expects additional income from surcharge transfers, she is willing to accept higher a reservation fee. Transfer fees impact the outcome only when the high-probability buyer has a larger contingency demand volume. In this case, when the reservation fee is relatively high, this buyer is better off reserving the contingency volume of the other buyer with the expectation that her reservation can be used by the other buyer while also covering a portion of her own contingency needs.

In our analysis, we also consider that case of voluntary compliance. In this case, the supplier may choose to take the risk of not fully complying with procuring the full reservation amount. This way, she offsets her overage risk. We determine the minimum shortage penalty, above which the supplier has no incentives to under procure. With the inclusion of this penalty term in the reservation contract, the buyers can rely on their reservation amounts.

Another interesting result, that contradicts the literature on conventional reservation contracts, is the conclusion that the total reservation amount does not necessarily increase with the total contingency demand volumes. As the gap between demand volumes increases, the supplier may be better off switching from a lower reservation fee resulting in full reservations from all buyers to a higher reservation fee resulting in reservation only from the high-probability buyer. In other words, the supplier may benefit from collecting higher payments upfront and taking smaller overage risk rather than facing higher risks by lowering the fees and increasing the reservation amounts.

Our model can be extended in multiple ways. First possible extension is to consider case where buyers are the channel leaders. In this setting, buyers may move first and offer their reservation fees to the supplier, who responds with the quantity
that she is willing to reserve for the buyers. Second, we assumed simultaneous moves for the buyers. In an alternative setting, the supplier may negotiate reservations with the buyers sequentially rather than simultaneously. Finally, another possible extension includes a bargaining framework for determining the wholesale price and the surcharge fee.
CHAPTER 3

Buyer Driven Reservation Contracts for Contingency Inventory

This chapter proposes a reservation contract for contingency inventory management between two independent distributors (buyers) and a single supplier using a game theoretic framework. In the game setting, the buyers are leaders and they simultaneously move to offer nonrefundable-deductible reservation fees to the supplier to reserve backup inventory for low-probability big-impact events. Once the reservation fee offers are revealed, the supplier decides the inventory amount she wants to keep for each buyer. By reserving through a shared supplier, the buyers enable a contingency inventory pool which not only alleviates overage risk for the supplier but also enables a buyer receive inventory more then her individual reservation in case of contingency from unused reservation of the other buyer. We investigate the equilibrium reservation amounts for this setting and identify conditions under which reservation contracts can be successfully employed for contingencies. We also contrast our results with the supplier-leading channel. We show that in a market where the buyers have more negotiation power, reservation contracts for contingency inventory are more likely to achieve inventory buildup under relatively lower event probabilities resulting with higher expected payoffs for the buyers.
3.1 Problem Setting

We consider a supply chain with a single supplier and two independent buyers, namely Buyer A and Buyer B. Under a single-period setting, the supplier carries backup inventory for the buyers. The backup inventory is kept to respond to contingencies that may arise due to low-probability high-impact events such as natural disasters or epidemics. Buyers procure their products for their nominal demands from their own sources which are separate independent entities. Since our analysis only focuses on the backup supply reservation, buyers’ procurement for their nominal demands is out of this study’s scope. In this context, the supplier serves the buyers as a third party in case of rare events, which creates additional demand for products such as consumables and pharmaceuticals. Typically, the buyers need a common supplier for the contingency demand mainly for two reasons. First, due to long lead times, it may not be possible for the buyers to make additional orders from their regular suppliers in a timely fashion in the event of the demand surge. Second, carrying local inventory for such events may not be economically justified due to the associated low probabilities. In this study, we investigate the conditions and strategies under which the backup supplier and the buyers enter into a reservation contract for contingencies such as the rare events explained above. It is assumed that the probabilities for these rare events are known for each market and strictly below 0.5.

Under a leader-follower game, the buyers simultaneously move and offer their unit reservation fees to the supplier. The reservation fee is denoted by \( r_i \) for buyer \( i \), where \( i = \{A, B\} \). The supplier responds by deciding the amount of inventory that she will carry for each buyer, which is represented by \( Q_i \) for buyer \( i \). To facilitate our analysis and derive managerial insights on key aspects, we assume that the buyers operate in
similar markets in terms of size and resale prices. We let \( H \) denote the excess demand created by the rare event for each buyer and the retail price for the product is denoted by \( p \). The buyers are different in terms of their event probabilities denoted by \( P_A \) and \( P_B \) respectively for Buyer A and B. Without loss of generality, it is assumed that \( P_A \leq P_B \).

The reservation fees paid for each unit by the buyers are nonrefundable and thus, can be regarded as a nonrefundable deductible down payments for the contingency inventory. This is analogous to fees paid for options. If the rare event occurs, buyer \( i \) has the option of completing her procurement of the reserved item (that is, exercising her option) by paying \( w - r_i \), where \( w \) is the wholesale price for each unit of product charged to both buyers. In our analysis, the wholesale price is assumed to be either negotiated beforehand or, in the case of consumable products, determined by the market. As such, it is exogenous. Similar models have been employed in the literature for different settings such as the ones studied by [19], [59], and [60]. We assume forced compliance for the supplier. That is, the supplier has to procure the reserved quantities that she announces to the buyers. Each unit costs \( c \) to the supplier. To avoid pathological cases, it is assumed that \( p > w > c \) so that reservation can be a viable option for all parties. Unused inventory reserved by buyers can be salvaged by the supplier at the end of the planning period with an income of \( s \) per unit, where \( s < c \).

Although, the contingency demand is known for a buyer, she may not choose to reserve the full amount due to the risk that the reserved inventory will not be needed. Then, the buyer can procure additional quantities from the other buyer’s reserved inventory through the supplier provided that it is not used by the reserving buyer. In
this case, the supplier charges an additional per unit fee for the amount in excess of the buyer’s reserved quantity. This fee is referred to as the no-reservation surcharge and denoted by $u$.

The rare events lead to four realization scenarios: (1) no contingency event occurs, (2) only Buyer $A$ is inflicted, (3) only buyer $B$ inflicted, and (4) both buyers are inflicted by the rare events. We let $\Psi_o$, $\Psi_A$, $\Psi_B$, and $\Psi_{AB}$ represent the probability for scenario outcomes one to four respectively. Here, it is straightforward to deduce that

$$\Psi_o = (1 - P_A)(1 - P_B); \quad (3.1)$$
$$\Psi_i = P_i(1 - P_{-i}); \quad (3.2)$$
$$\Psi_{AB} = P_AP_B. \quad (3.3)$$
The nomenclature used in the analysis is listed in Table 3.1 and the sequence of the events are summarized in the following:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>Buyer’s unit market price</td>
</tr>
<tr>
<td>$w$</td>
<td>Wholesale price of a unit product</td>
</tr>
<tr>
<td>$u$</td>
<td>No-reservation surcharge</td>
</tr>
<tr>
<td>$c$</td>
<td>Cost of a unit product for the contractor</td>
</tr>
<tr>
<td>$H$</td>
<td>Player $i$’s contingency inventory need (demand surge) if inflicted by the rare event</td>
</tr>
<tr>
<td>$P_i$</td>
<td>Player $i$’s rare event probability ($i = A, B$)</td>
</tr>
<tr>
<td>$s$</td>
<td>The salvage value of each unit of unsold product at the end of the period</td>
</tr>
</tbody>
</table>

**Decision Variables**

- $r_i$: unit reservation fee offered to the supplier by each buyer ($i = \{A, B\}$, and deductible if the contract exercised, non-refundable if the contract not exercised)
- $Q_i$: Player $i$’s reservation quantity ($i = A, B$)

Table 3.1: Nomenclature

1) Each buyer announces an individual reservation fee $r_i$ without knowing each others decision under a given exogenous wholesale price $w$ and no-reservation penalty $u$.

2) The backup supplier decides on her reservation quantities $Q_i$ for each buyer, where $i = \{A, B\}$, and receives $r_i$ for each unit.

3) If buyer $i$ is inflicted by the rare event, her reserved inventory is supplied by the backup supplier and the buyer pays $w - r_i$ for each unit delivered. In case the reservation amount is not enough for the inflicted buyer, the supplier can supply additional quantities from the reservation of the other buyer unless she
exercises her reservation. The supplier charges $w + u$ for each unit supplied to the buyer in excess of her reservation.

We first write down the buyer profits. Each buyer offers her individual reservation fee at the beginning of the period before the nature is observed. The contingency inventory contract is exercised by a buyer if and only if she experiences rare event that cause demand surge in her market. In that case, a buyer has the option of procuring quantities beyond her reserved amount subject to availability at the supplier’s side. Given this, we can write the expected profit for buyer $i$ as the following:

$$\Pi_i = -r_i Q_i + (p - w + r_i)(\Psi_i + \Psi_{AB}) Q_i + (p - w - u)\Psi_i \min(Q_i, H - Q_i)$$ (3.4)

As discussed above, the profit of the backup supplier depends on the revenues from reservation, exercise, and no-reservation penalty fees, unit cost, and the salvage value of the reserved inventory. As such, we get the following expected profit function for the supplier:

$$\Pi_S = (r_A - c)Q_A + (r_B - c)Q_B + \Psi_o s(Q_A + Q_B)$$
$$+ \Psi_A [(w - r_A)Q_A + (w + u) \min(H - Q_A, Q_B) + s \max(0, Q_A + Q_B - H)]$$
$$+ \Psi_B [(w - r_B)Q_B + (w + u) \min(H - Q_B, Q_A) + s \max(0, Q_A + Q_B - H)]$$
$$+ \Psi_{A,B} [(w - r_A)Q_A + (w - r_B)Q_B]$$ (3.5)

Given the above payoff functions and sequence of events, we analyze the equilibrium outcome in the next section. We aim to generate equilibrium reservation fee pairs $(r_A, r_B)$ that result in equilibrium reservation amounts $(Q_A, Q_B)$. 


3.2 Equilibrium Analysis

We derive equilibrium conditions using backward induction. As such, we first analyze the second stage where the supplier sets her reservation quantities for each buyer given buyers’ reservation fee pair \((r_A, r_B)\) determined in the first decision stage. We note that the supplier profit function is piece-wise conditioned on the trade-off between the reservation amounts assigned to individual buyers. Consequently, from the first order derivative of the supplier’s profit function given in (3.5) with respect to the supplier’s reserved quantity for buyer \(i\), we get

\[
\frac{\partial \Pi_S}{\partial Q_i} = \begin{cases} 
  r_i - c + P_i(w - r_i) + P_{-i}(1 - P_i)(w + u) + (1 - P_i)(1 - P_{-i})s, & \text{if } Q_i \leq H - Q_{-i} \\
  r_i - c - P_i(r_i + u) + P_iP_{-i}(w + u - s) + s, & \text{o/w}
\end{cases}
\]

(3.6)

The first order derivatives above indicate that the slopes of the profit function with respect to the reservation amount \(Q_i\) in both regions depend on the reservation fee \(r_i\) offered by buyer \(i\). This results in two reservation fee thresholds for each buyer. For \(Q_i \leq H - Q_{-i}\), we get

\[
r_{i,1} = \frac{c - s - P_i(w - s) - P_{-i}(1 - P_i)(w + u - s)}{1 - P_i}
\]

(3.7)

and for \(Q_i > H - Q_{-i}\), we get

\[
r_{i,2} = \frac{c - s - P_iP_{-i}(w + u - s) + uP_i}{1 - P_i}
\]

(3.8)

When the reservation fee is sufficiently high, \(i.e., r > r_{i,2}\), the profit function is strictly increasing with the reservation amount given to buyer \(i\). On the other hand, for sufficiently low values, \(i.e., r < r_{i,1}\), it is strictly decreasing. When the reservation fee is neither low nor high, \(i.e., r_{i,1} \leq r \leq r_{i,2}\), the profit is first increasing for \(Q_i \leq H - Q_{-i}\) and then decreasing for higher values of \(Q_i\). Based on this observation
we can obtain the optimal response of the supplier to a given reservation fee pair $(r_A, r_B)$ as summed by the following proposition:

**Proposition 10** At the second decision stage, the supplier’s best response mapping is given by Table 3.2.

<table>
<thead>
<tr>
<th>Buyer A</th>
<th>[0, $r_{A1}$]</th>
<th>[$r_{A1}$, $r_{A2}$]</th>
<th>$r_{A2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0, $r_{B1}$]</td>
<td>(0, 0)</td>
<td>(0, $H$)</td>
<td>(0, $H$)</td>
</tr>
<tr>
<td>[$r_{B1}$, $r_{B2}$]</td>
<td>(0, $H$)</td>
<td>($H$, $x - H$)</td>
<td>(0, $H$)</td>
</tr>
<tr>
<td>$r_{B2}$</td>
<td>($H$, 0)</td>
<td>($H$, 0)</td>
<td>($H$, $H$)</td>
</tr>
</tbody>
</table>

Table 3.2: Buyers’ Reservation Fee Strategies and Supplier’s Equilibrium Reservations

**Proof:** Let $G_i$ and $\hat{G}_i$ denote the first order derivatives given in (3.6) for the supplier. Namely, $G_i = r_i - c + P_i(w - r_i) + P_{-i}(1 - P_i)(w + u) + (1 - P_i)(1 - P_{-i})s$, and $\hat{G}_i = r_i - c - P_i(r_i + u) + P_iP_{-i}(w + u - s) + s$. Clearly a positive (negative) derivative indicates that the supplier’s profit is increasing (decreasing) with the reservation quantity. Note that, $G_i > \hat{G}_i$ always holds. First, we note that $G_i$ and $\hat{G}_i$ are both nonnegative when $r_i \geq r_{i,2}$ for $i = A, B$. As such, the supplier reserves the buyers’ full contingency demands ($H, H$). On the other hand, when $r_i < r_{i,2}$, the supplier has no incentives to carry any reservation inventory for either of the buyers. If the buyers choose their reservation fees at different regions, that is, $r_i \geq r_{i,2}$ while $r_{-i} < r_{-i,2}$, then from (3.6), $G_i \geq 0$ and $G_i < 0$ and $G_{-i}$. Consequently, the supplier reserves the full amount $H$ for buyer $i$ and none for buyer $-i$. When the offered reservation fees are such that $r_{i,1} \leq r_i < r_{i,2}$ for both buyers, the outcome of the game will be $(x, H - x)$, since the supplier’s profits are nondecreasing with $Q_i$ only in $[0, Q_{-i}]$. In
this case, the supplier reserves a total amount of $H$ and she is indifferent to how this amount is allocated to the individual buyers.

Since the supplier’s profit function is piece-wise linear in reservation quantities and the buyers’ profits are also piece-wise in reservation fees, we can easily conclude that the buyer reservation fees would be only slightly higher than the threshold values given in (3.7) and (3.8). That is, the reservation fee offer from buyer $i$ will be $r_i = 0$, $r_i = r_i,1 + \epsilon$, or $r_i = r_i,2 + \epsilon$, where $\epsilon \to 0$. Consequently, we obtain the following result:

**Proposition 11** At equilibrium;

1. None of the buyers offer a positive reservation fee and there will be no reservations at equilibrium if and only if
   \[ c - s > P_B(p - s) + P_A(1 - P_B)(w + u - s); \]  
   \[ (3.9) \]

2. There is a unique equilibrium resulting with the reservation fee pair $(0, r_{B1})$ and thus, reservation amounts of $(0, H)$ if
   \[ P_B(p - s) + P_A(1 - P_B)(w + u - s) \geq c - s > P_A(p - s) + P_B(1 - P_A)(w + u - s) \]
   \[ (3.10) \]

3. There are two equilibrium outcomes if
   \[ P_A(p - s) + P_B(1 - P_A)(w + u - s) \geq c - s > P_A P_B(p - s) \]  
   \[ (3.11) \]

   In the first equilibrium, the buyer reservation fees are $(r_{A1}, 0)$ leading to the reservation amounts $(H, 0)$. In the second equilibrium, the buyer reservation fees are $(0, r_{B1})$ leading to the reservation amounts $(0, H)$. The total reservation amount is always $H$ under this condition.
iv) The unique equilibrium reservation fee pair is \((r_A, r_B)\) resulting in reservation amounts of \((H, H)\) if and only if

\[ P_A P_B (p - s) \geq c - s. \] (3.12)

**Proof:** It can be easily verified that the following ordering of payoff functions hold if inequality (3.9) holds:

\[
\Pi_A(0, 0) > \Pi_A(r_A, 0) > \Pi_A(r_A, r_B) = \Pi_A(r_A, r_B) = \Pi_A(r_A, r_B)
\]

\[
\Pi_B(0, 0) > \Pi_B(0, r_B) > \Pi_B(0, r_B) = \Pi_B(r_A, r_B) = \Pi_B(r_A, r_B)
\]

The inequality clearly implies that both buyers are better off with opting out of offering any reservation fees and as such, from Proposition 1, the supplier does not reserve any amount for either of the buyers.

First we note that the following inequality always hold for \(P_B \geq P_A\):

\[ P_B(p - s) + (w + u - s)(P_A - P_A P_B) \geq P_A(p - s) + (w + u - s)(P_B - P_A P_B) \] (3.13)

When (3.10) holds, \(r_A = 0\) becomes a dominating strategy for Buyer A since this buyer’s profit function is strictly decreasing with the reservation fee regardless of the other buyer’s action. On the other hand, the profit function of Buyer B is increases with \(r_B\) in \((0, r_B]\) and decreases with \(r_B\) for \(r_B > r_B\) implying that the Buyer B has incentives to offer a reservation fee of \(r_B\) leading to \((0, H)\) reservation amounts.

In the case (3.11) holds, the game between the buyers transforms into a matrix game. If buyer \(i\) offers a reservation fee of 0, then the best response of buyer \(-i\) is to offer \(r_{-i,1}\) to which the buyer \(i\)’s best response is 0. As such both \((r_A, 0)\) and \((0, r_B)\) are pure Nash equilibria for the buyers. From Proposition 1, this results in reservation outcomes \((H, 0)\) and \((0, H)\) respectively.
Finally, when (3.12) holds, offering the reservation fee $r_{i,2}$ is the dominant strategy for each buyer regardless of the action of the other. As such, $(r_{A2}, r_{B2})$ is the unique Nash equilibrium for the buyers resulting in the reservation amount pair $(H, H)$.

The inequality (3.9) implies that the procurement cost for the product is too high or equivalently, rare event probabilities are too low for the supplier, so that she requires substantial amounts of down payments from the buyers for alleviating her risk for carrying contingency inventory. However, the buyers will be reluctant to invest on reservation because their profit margins do not justify such investment under very small probabilities. As the rare event probabilities and/or revenue margins increase, the buyers will be more willing to pay higher reservation fees, whereas as the unit overage cost (i.e., $c - s$) decreases the supplier will be willing to receive smaller reservation fees. Consequently, when (3.10) or (3.11) is satisfied, the equilibrium will result with a positive reservation quantity amounting to $H$ in total. When (3.12) holds the opportunity cost for the buyers are sufficiently high and thus, both offer reservation fees leading to full reservations.

The above results help us investigate the influence of model parameters on the equilibrium outcomes and obtain several managerial insights. Our first observation is due to the supplier’s wholesale price. As clear from the inequalities given in Proposition 11, higher wholesale price makes reservation a viable outcome for all parties. As $w$ increases the inequality (3.9) transform to (3.10) and then to (3.11) leading to positive reservation amounts. Higher wholesale price improves the supplier profit margins which reduce the risk for the supplier. Consequently, she will be willing to procure reservation inventory for the buyers at lower fees. This is also evident from the threshold values given in (3.7) and (3.8) both of which are decreasing in $w$. Inter-
Estingly, increasing $w$ not lead to condition given in (3.12). As illustrated in Figure 3.1, the equilibrium outcomes obtained in Proposition 11 lead to three regions. The first region corresponds to the $(0, 0)$ reservation outcome resulting with 0 reservation. The second region represents the equilibrium outcomes with total reservation amounts of $H$, namely, $(0, H)$ and $(H, 0)$. The total reservation amount of $2H$, corresponding to the $(H, H)$ outcome, is captured by the third region. As illustrated in the figure, moving form a low wholesale price to a larger one does not effect the $2H$ region whereas the 0 region shrinks while the $H$ region expands. In other words, multiple equilibrium region grows towards the unique equilibrium region of 0 as the wholesale price increases. The full reservation equilibrium $2H$ mainly depends on the buyers’ market price not the supplier’s wholesale price.

We can also observe from Figure 3.1 that as the individual rare event probabilities increase, the equilibrium moves towards an outcome with larger reservation amounts. This is expected in that as the probabilities increase the no-exercise risks diminish for the buyers and the supplier.

The impact of surcharge, $u$, is very similar to the wholesale price as illustrated in Figure 3.2. The surcharge penalty encourages the buyers to reserve more aggressively by urging them to offer higher reservation fees. As the surcharge reservation penalty $u$ increases, the buyers become inclined to increase their reservation fee offers that leading to an equilibrium with non-zero inventories. As is the case for the wholesale price, higher $u$ does not lead to the $2H$ reservation outcome. Clearly, under $2H$ the surcharge is irrelevant since none of the buyers ever needs to procure more than their reservations.
The unit product cost ($c$) and the unit salvage value ($s$) are the key factors for the equilibrium outcome as apparent from the inequalities given in Proposition 11. Large unit product costs diminish the supplier’s margin implying risk of holding contingency inventory for the supplier. In order to balance the holding risk, the supplier expects higher reservation fee offers from the buyers. As a result, buyers should offer higher reservation fees in order to incentivize the supplier to keep non-zero contingency
inventory. As illustrated in Figure 3.3, both $H$ and $2H$ regions diminish as the supplier’s unit cost increases. Hence the supplier reacts to the buyers’ reservation fee offers with less flexibility while the buyers need to raise their offers in order to reach the same equilibrium under higher unit product cost.

![Figure 3.3: The Effect of the Supplier’s Unit Cost on the Buyers’ Equilibrium](image)

The salvage value hedges the supplier against excessive stock by mitigating her holding risk. As a result, high salvage value decreases the buyers’ reservation fee thresholds given in (3.7) and (3.8). As the salvage value $s$ increases, the unique equilibrium of $(r_{A2}, r_{B2})$ becomes more appealing for the buyers since large salvage value decrease the threshold values for reservation fees and make the reservation more affordable for the buyers. The non-zero equilibrium regions $H$ and $2H$ expand with higher salvage values as illustrated by Figure 3.4. The supplier will be more willing to respond with higher inventory reservation quantities to the buyers’ reservation fee offers when the salvage value is high.
The buyers’ market price $p$ is another important parameter that affects all the ranges given in Proposition 11. Clearly buyers’ marginal profits increase with the market price of the product. As their profit margins grow, the buyers are willing to pay higher reservation fees and hence entice the supplier keep more inventories for possible rare events (Figure 3.5).

We can use the equilibrium reservation fees and amounts obtained in the above discussion to derive expected equilibrium profits for all parties. Clearly, the $(0, 0)$
reservation fee equilibrium does not lead to non-zero profits for any of the players. For other cases, the following lemma lays out the player profits:

**Lemma 15** The \((0, r_{B1})\) strategy outcome leads to:

\[
\begin{align*}
\Pi_A(0, r_{B1}) &= P_A(1 - P_B)(p - w - u)H \\
\Pi_B(0, r_{B1}) &= [P_B(p - s) + P_A(1 - P_B)(w + u - s) - c + s]H \\
\Pi_S(0, r_{B1}) &= 0; \tag{3.14}
\end{align*}
\]

The \((r_{A1}, 0)\) strategy outcome leads to:

\[
\begin{align*}
\Pi_A(r_{A1}, 0) &= [P_A(p - s) + P_B(1 - P_A)(w + u - s) - c + s]H \\
\Pi_B(r_{A1}, 0) &= P_B(1 - P_A)(p - w - u)H \\
\Pi_S(r_{A1}, 0) &= 0; \tag{3.15}
\end{align*}
\]

The \((r_{A2}, r_{B2})\) strategy outcome leads to:

\[
\begin{align*}
\Pi_A(r_{A2}, r_{B2}) &= [P_A(p - w - u) + P_A P_B(w + u - s) - c + s]H \\
\Pi_B(r_{A2}, r_{B2}) &= [P_B(p - w - u) + P_A P_B(w + u - s) - c + s]H \\
\Pi_S(r_{A2}, r_{B2}) &= (P_A + P_B - 2P_A P_B)(w + u - s)H. \tag{3.16}
\end{align*}
\]

The buyers’ equilibrium profits are directly obtained from (3.4). We note that under all equilibrium outcomes resulting with a total reservation amount of \(H\), the supplier’s payoff is zero. This is due to the fact that the buyers have the first mover advantage and the reserving buyer’s reservation fee will be on the threshold of non-negative payoff for the supplier. Clearly, if the supplier has strictly positive outside option, the buyers will be required to increase their reservation fees strictly above these thresholds. The supplier enjoys strictly positive payoffs only under the \((H, H)\)
outcome. In this case, reserving capacity is a dominating strategy for each buyer, where each buyer offers a higher fee to guarantee reservation knowing that the other one will do the same. Eventually, this benefits the supplier. In general, it can be shown from the profit functions given in the above Lemma that the total buyer profits are also better under this equilibrium if the buyer prices are sufficiently high or equivalently if the supplier’s wholesale price and surcharge fee are sufficiently low.
3.3 Comparison with the Supplier Leading Game

In this section, we compare the results of the buyer leading reservation game to the setting where the supplier is the leader. In this setting, the supplier moves first and offers the reservation fees for the buyers and the latter follow by deciding on their reservation amounts. This model has been investigated by [61] and in the previous chapter. The authors consider a case where price discrimination is not allowed and hence, the supplier’s reservation fee is equal for both buyers. The following proposition, which is due to their work, lays out the equilibrium outcome for this case:

**Proposition 12** In the supplier leading game, at equilibrium, the supplier sets her reservation fee, \( \hat{r}^* \), to either 1) \( \hat{r}^* > \hat{r}_{B1} \) and there will be no reservation; or 2) \( \hat{r}^* \approx \hat{r}_{B1} \) leading to \((0, H)\); or 3) \( \hat{r}^* \approx \hat{r}_{A2} \) leading to \((H, H)\), where

\[
\hat{r}_{B1} = \frac{P_B}{1 - P_B}(p - w) \tag{3.17}
\]

and

\[
\hat{r}_{A2} = \frac{P_A}{1 - P_A}((1 - P_B)u + P_B(p - w)) \tag{3.18}
\]

The proof is provided in Lemma 2. In contrast to the buyer leading setting, the \((H, 0)\) reservation outcome is never an equilibrium in the supplier leading channel. Moreover, the equilibrium is always unique in this case since there is no Nash competition at the first decision stage and the buyer profits are either strictly increasing or strictly decreasing with the reservation fee values.

Let \( G(P_A, P_B) \) be defined as follows:

\[
G(P_A, P_B) = P_A P_B (p - s) - \frac{P_B(1 - 2P_A + P_A P_B)}{1 - P_A}(p - w) + \frac{P_A(1 - P_B)^2}{1 - P_A}u \tag{3.19}
\]

The following observation is made for the supplier-leading channel:
Proposition 13 At equilibrium in the supplier leading game:

i) There will be no reservation at equilibrium if and only if

\[ c - s > (p - s)P_B + (w - s)P_A(1 - P_B) + uP_A(1 - P_B) \]  \hspace{1cm} (3.20)

ii) There is a unique equilibrium resulting with the reservation quantity pair \((0, H)\)

if

\[ G(P_A, P_B) < c - s \leq (p - s)P_B + (w - s)P_A(1 - P_B) + uP_A(1 - P_B) \]  \hspace{1cm} (3.21)

iii) There is a unique equilibrium resulting with the reservation quantity pair \((H, H)\),

and thus reservation fee of \(r_{A2}\) if

\[ c - s \leq G(P_A, P_B) \]  \hspace{1cm} (3.22)

The proof is similar to that of Proposition 11 based on supplier equilibrium profits derived in [61]. Interestingly, we observe that the first boundary is identical in buyer-leading and supplier-leading channels. This implies that minimum conditions for any reservation to take place are the same in both cases. However, the boundary between \(H\) and \(2H\) regions are different. Specifically, it can be easily verified that the boundary given in (3.11) is strictly greater than that of (3.22) implying that \(H\) region is larger in the supplier-leading game as illustrated in Figure 3.6. The dash line in the figure represents the boundary between the \(H\) and \(2H\) regions for the supplier-leading channel. This result leads to the following conclusion:
Corollary 4  Total equilibrium reservation amount in the buyer-leading channel is no less than the total equilibrium reservation amount in the supplier-leading channel.

It is obvious that being the leader at the game gives a competitive edge to a player, who starts the game by offering a unit reservation fee. When the buyers are the channel leaders, they can drive the game in a direction such that the backup supplier makes little or no profit; yet she still may build backup inventory for the buyers. The buyers have the advantage to offer reservation fees that are no more than the thresholds values for either ranges since in each range the quantity reserved by the supplier remains the same. On the other hand, in the supplier leading setting, the supplier can mitigate her risk by offering reservation fees enabling positive amounts of reservations under relatively higher rare-event probabilities and always enjoys a payoff that is no less than the buyer-leading channel. The following result generalize the comparison of reservation fees between two channel structures:

![Figure 3.6: Supplier Leading vs. Buyer Leading Setting Comparison](image)
Proposition 14 In any equilibrium with the same total reservation amounts, the reservation fees in the buyer-leading channel are always smaller than or equal to that of the supplier-leading channel. Moreover, while the equilibrium reservation fees decrease in event probabilities in the buyer-leading channel, opposite is true for the supplier-leading channel.

Proof: To complete the first part of the proof, it is sufficient to compare the reservation thresholds for equilibrium outcomes resulting with \( H \) and \( 2H \). We note from Proposition 12 that in the first case, only Buyer \( B \) reserves inventory and her reservation fee, \( \hat{r}_{B1} \), is given by (3.17). Also, from (3.7) and (3.8), we can easily verify that \( r_{B1} \leq r_{A1} \) since \( P_B \geq P_A \). Taking the differences of the reservation fees \( \hat{r}_{B1} \) and \( r_{B1} \), we get

\[
\hat{r}_{B1} - r_{B1} = \frac{P_B(p - s) + P_A(1 - P_B)(w + u - s) - (c - s)}{1 - P_B}
\]

(3.23)

We note from Proposition 11 that in order for Buyer \( B \) to reserve inventory in the buyer-leading channel, the above function must return a nonnegative value implying that \( \hat{r}_{B1} \geq r_{B1} \) must hold. When this condition does not hold but inequality in (3.10) holds then only Buyer \( A \) is willing to reserve in the buyer-leading channel. In this case, using the similar approach we can show that \( \hat{r}_{A1} \geq r_{A1} \) and since \( \hat{r}_{B1} \geq \hat{r}_{A1} \), \( \hat{r}_{B1} \geq r_{A1} \) must also hold. When the total reservation amounts are \( 2H \) in both channels, in a similar way to (3.23), we can calculate the differences in equilibrium reservation fees, which result in \( P_A P_B(p - s) - (c - s) \). From Proposition 11, we know that this difference must be nonnegative due to inequality (3.12). As such, the reservation fees in the supplier-leading channel must be no less than those in buyer-leading channel. The impact of event probabilities are straightforward from the first order derivatives of reservation fees given in (3.7), (3.8), (3.17), and (3.18).
As the event probability of a buyer increases, the supplier’s risk in that buyer’s reservation decreases. As such, in the buyer-leading channel this provides an opportunity for the buyer to reduce her reservation fee. On the other hand, the supplier can charge a higher fee in the supplier-leading channel because the buyer’s possibility for exercising the reserved inventory increases.

One interesting result that can be observed in Figure 3.6 is that as the rare event probabilities increase, the total equilibrium reservation amount is always non-decreasing in event probabilities in a buyer-leading channel. As either of $P_A$ and $P_B$ increases, the equilibrium regions transforms from a smaller reservation amount to a larger one. However, this is not always the case in a supplier-leading channel. The boundary $G(P_A, P_B)$ in (3.22) decreases in $P_B$ when the retailers’ revenue margins are sufficiently high. This results with an increase in the $H$ region over the $2H$ region under the supplier-leading channel. As illustrated in Figure 3.6, when the rare even probability for Buyer B increases, the equilibrium outcome may switch from $(H, H)$ to $(0, H)$. The intuition is that the supplier offers a uniform price for both buyers and as the probability of the high-probability buyer increases the supplier may be better off by charging a high reservation fee and only reserve for this buyer rather than charging a lower reservation fee and reserving for both.
3.4 Concluding Discussion

In this section, we introduce a buyer-leading reservation contract for contingency inventory management where the buyers pool their contingency needs for possible future use in case a demand surge caused by a disruptive rare event occurs. At the beginning, buyers simultaneously move and offer deductible and non-refundable reservation fees to a single supplier for a unit product to be held by the supplier as backup inventory over a single period. The supplier moves next and decides on her inventory to be reserved for the buyers. This reservation fee is deducted from the supplier’s wholesale price if a buyer exercises her own reservation. If the contract is not exercised, the supplier keeps the reservation fee. In that sense, the supplier’s investment risk is partially mitigated by the reservation fee paid by the buyers. Another factor that mitigates the supplier’s risk is the possibility that any reservation that has not been exercised by the reserving buyer can be demanded by the other buyer. In the model, we also consider a surcharge fee applied for any inventory procurement that has not been originally reserved by the buyer.

We investigated the equilibrium reservation amounts under this setting in this paper. Basically, the equilibrium outcomes depend on the relation among the event probabilities, buyer margins, and supplier’s overage cost. We analyzed how these factors impact the total reservation amounts. Reserving through a shared supplier benefits the buyers in two ways. First, by reserving backup inventory via a shared supplier, buyers build an inventory pool that can be tapped into in case of rare events. Hence, in the case of rare event a buyer has the option of not only exercising her own reservation but also the other buyer’s reservation in case the latter one does not need to exercise it. Second, even when a buyer participates in this pool without
any reservation, her mere potential demand provides incentives for the supplier to carry backup inventory at a lower reservation fee for the other buyer. Eventually, this enables a backup inventory pool which would not be possible otherwise.

We also contrasted our model with the supplier-leading case. Our analysis shows that under similar conditions the equilibrium reservation amounts are no less under a buyer-leading channel compared to the supplier-leading channel. Reservation for contingency inventory is viable under a supplier-leading channel if the probabilities for rare events are sufficiently high. Our results indicate that in a market where the buyers have more negotiation power, reservation contracts for contingency inventory are more effective achieving inventory buildup under relatively lower probabilities and higher risks with higher expected payoffs for the buyers.
CHAPTER 4

Advance Supply Contracts with Multi-Period Expedited Replenishment Modes

In this chapter, we investigate a finite horizon multi-period inventory control problem of a buyer where she might purchase large quantity of seasonal product with an advance contract at the beginning of the selling season. The demand for the mentioned product is assumed to be stochastic and non-stationary across periods. Besides the proposed contract that allows the buyer to purchase the seasonal product in large quantities, she can make additional purchases from spot market with higher unit cost right after the selling season starts. The demand distribution for each period is a function of a market (demand) signal which is random at the beginning of the planning horizon. The firm has options of contracting its supplies before the realization of the market signal to benefit from price discounts or replenishing its inventories during the selling season after the market signal is observed, this time by paying higher spot market prices.

Under our model, periodic inventory review policy is adopted and it is assumed that the replenishments are possible only at the beginning of each period which are not necessarily equal in length. Unit replenishment costs vary during the selling season.
Due to the early commitment to the high quantities, the firm might take advantage of the price discount. While the unit replenishment cost paid for the advance contract is less than the spot market cost, it is assumed that the spot market costs vary at the home port and the ports of call. Both unit replenishment costs and time intervals between possible replenishments (length of each leg) are known before the realization of the market signal.

Our research is motivated by applications in cruise line industry where the companies purchase large quantities of seasonal products with the advance contracts before they realize the final number of bookings. Based on the finalized bookings before the departure and the consumption rates during the trip, additional purchases can be made at intermediate stops with higher prices. The firm should consider two sources of uncertainty: demand variability (standard deviation of market signal) and consumption variability (standard deviation of daily demand). The firm’s problem is to decide how much inventory to order with the advance contract while considering these sources of variability. The optimal inventory quantity purchased through the advance contracting and expedited replenishment via spot market is investigated in this chapter.

We employ the stochastic dynamic programming method to analyze the firm’s optimal inventory purchase decision that is the total quantity purchased through the advance contracting and spot market (expedited replenishment). Optimal replenishment policy in each selling season period (i.e. periods after the realization of demand signal) follows base-stock policy. Our results show that a unique advance contracting amount exists that minimizes the total expected inventory cost the firm incurs throughout the finite selling periods. Exogenous parameters (e.g. unit cost with the
advance contract, unit cost in spot market, landing cost, etc.) affect the optimal replenishment quantity in each period. In the last section of this chapter, we show numerical examples to analyze the effects of these exogenous parameters on the firm’s optimal advance contract quantity, spot market quantity in each period, and service level in each periods, and the total expected inventory cost.

4.1 Problem Setting

4.1.1 A Three-Stage Optimal Inventory Policy

In the multi-period inventory model studied in this chapter, advance contracting period (Stage 0) is the first stage in the planning horizon. At this stage, although the firm has not received any signal on the selling season demand yet, it has the option of contracting an initial purchase amount, $y_0$, under the unit price $c_0$. At this stage, since the firm might order large quantities of inventory, the unit price is lower than the next stages. Right before the start of the selling season (Stage 1), the firm updates its demand information based on the realized market demand and if needed, increases its current inventory through expedited replenishments by paying the spot market price, $c_1$ ($c_1 > c_0$).

During this and subsequent stages, which are not necessarily equal in length, the firm can continue replenishing its inventory from the spot market as it becomes necessary. Figure 4.1 summarizes the time-line and stages of the N-period model.
The finite horizon multi-period inventory control problem investigated in this chapter incorporates a single seasonal product, single location, and multi-period model. It is assumed that at the beginning of the season, all the costs \( (c_0, c_1, \text{ and } c_2) \) and lengths of all stages \( (t_1, t_2) \) are known and they cannot be changed. At the time of contracting, (i.e., at Stage 0), the market signal and thus the season demand distribution are unknown to the firm. Once the demand signal is received, the demand distribution is established for all season periods. The demand is uncertain and demand distribution is known from past experiences but cannot be certainly determined in advance even after market signal is received. Demand across periods is independent but nonstationary as we assume that the length of the periods are exogenous and non-uniform.

Furthermore, in our consideration, the unit procurement costs are not necessarily identical across periods reflecting the fact that prices do not necessarily stay constant during the selling season. In addition to the purchasing costs, the firm incurs shortage penalty in each stage and landing cost at the end of the final stage. We also assume no backlogging and thus, shortages at any period imply lost sales. The problem
is formulated as a stochastic dynamic program model that aims to minimize the total cost of purchasing, landing at the end of the final stage, and shortages over the planning horizon. The planning horizon starts at the time when initial purchase amount (e.g., $y_0$) is contracted at Stage 0.

Our research is motivated by the applications in cruise line industry where contracts are made before the final number of bookings is realized. We can address the cruise liner example to further describe the model. Demand across all trip legs (periods) are i.i.d (independent and identically distributed) random variables that are conditioned to the number of bookings or number of people on-board as the cruise departs from the home port.

The market signal information (number of bookings) received at Stage 1 is a perfect information for demand distributions of the successive periods. Let the market signal information is denoted by $B$. Suppose the daily demand per passenger follows a Normal distribution with mean $\mu_D$ and variance $\sigma_D^2$. Subsequently, we can say that total demand over a period of $t$ days, is also normally distributed with a mean of $B*t*\mu_D$ and a variance of $B*t*\sigma_D^2$. Given $B$ and $t$, the cruise liner’s optimal inventory policy can be solved through dynamic programming with backward induction.

Figure 4.2 below illustrates the three-stage, two-period setting for the cruise liner. The cruise liner will sail from the home port and travel on sea for $t_1$ days until she arrives at a port of call. This part of the trip constitutes the first leg of the voyage. In the second leg, the cruise liner leaves the port of call and arrives at its final destination (usually back to home port) in $t_2$ days.
Figure 4.2: Timeline and Stages of the Three-Stage Setting

At Stage 0, which is the beginning of the planning horizon, all the purchasing costs, landing cost, and the time for possible replenishments are known and they cannot be changed. However, the bookings information (market signal) and thus the demand distribution of the periods are not completely known yet. Based on its expectations on bookings, the cruise liner makes an initial purchase through an advance contract with a unit price of $c_0$ at this stage. On the next stage, Stage 1, the cruise liner observes the booking information. By observing booking information, the demand forecast for the inventory item can be updated and additional purchase can be made from local suppliers in the home port before the departure. At Stage 1, just before the departure from the home port and after the bookings are finalized, there is also another opportunity for replenishment from the local suppliers at the home port city if necessary.

Due to the necessity of short lead-time and smaller batch size, the cruise liner’s unit cost of the item, $c_1$, is usually higher than the contracted price ($c_1 > c_0$). The cruise departs the home port and first period demand is realized during the first leg of the voyage based on the passengers’ consumption rate. Stage 2 begins when the
cruise arrives at the intermediate location (port of call), where there exists a local market making it possible for the cruise liner to make additional replenishments if needed. It is also the case that the unit cost of procurement at this stage, $c_2$, is higher than the unit cost with the advance contract. Hence, in general $c_0 < c_i$ for $i = 1, 2$.

Moreover, since the cruise lines are service oriented, shortage cost, $p$, is substantially high compared to all other costs implying $p > c_i > c_0$ where $i = 1, 2$. The cruise liner incurs a unit landing cost, $h$, to dispose the inventory leftover at the end of the final stage.

We will use the following notation for the Three-Stage problem starting with Stage 0:

- $B$: Market (demand) signal
- $\xi_n$: Random variable representing the total demand for period $n$ ($n > 0$)
- $t_n$: Total time for the $n$th period of the selling season ($n > 0$)
- $\phi^{t_n}(\xi_n|B)$: Probabilistic density function of demand over $t_n$ given $B$ ($n > 0$)
- $\Phi^{t_n}(\xi_n|B)$: Cumulative probability function for demand over $t_n$ given $B$ ($n > 0$)
- $x_n$: Inventory level at the end of stage $n$ ($n \geq 0$)
- $\mu(B)$: Expected demand rate per unit time in any period given $B$
- $y_n$: Inventory level at the beginning of stage $n$ ($n \geq 0$) after orders have been received
- $c_n$: Unit purchase price at stage $n$ ($n \geq 0$)
- $p$: Unit shortage penalty
- $h$: Unit landing cost at the end of the final stage
- $f_{N-n}(x_n|B)$: Expected cost over the remaining $N - n$ stages starting with an initial inventory of $x_n$ given $B$
We write the recursive function starting from the last stage and continue same manner until Stage 0. At the end, starting from the last stage, optimal policy for each stage can be determined one-by-one by using backward induction method.

4.1.2 Stage 2 Problem

Stage 2 is the last stage of the voyage, and at this stage, there is only $t_2$ periods of time left and given the inventory remained at the end of first period ($t_1$) and the demand signal, optimal expected cost for the remaining period of time can be written as:

$$f_1(x_1|B) = \min_{y_2 \geq x_1} Z_1(y_2; x_1|B)$$ (4.1)

where

$$Z_1(y_2; x_1|B) = c_2(y_2 - x_1) + p \int_{y_2}^{\infty} (\xi_2 - y_2)\phi^{t_2}(\xi_2|B)d\xi_2 + h \int_{0}^{y_2} (y_2 - \xi_2)\phi^{t_2}(\xi_2|B)d\xi_2$$ (4.2)

This function is the total cost function for any continuous demand distribution at the last stage of the voyage. The first part of the equation represents the purchasing cost the cruise liner incurs at Stage 2. If the total demand during the last period ($t_2$) is greater than the inventory level at the beginning of the same period ($y_2$), the cruise liner incurs a shortage penalty cost proportional to the number of stock-outs. This cost is represented by the first integral equation. In addition, if the total demand during $t_2$ is less than $y_2$, the cruise liner incurs landing cost to dispose the leftovers at the end of the second period. This cost is represented by the second integral equation, where $h$ is the unit disposal cost of the product.

To exclude the uninteresting cases, we assume that $p > c_2 + h$ throughout the paper. Now we can present the following observation:
Lemma 16 The optimal order quantity at Stage 2 where expected cost for the remaining period of time is represented by convex function $f_1(x_1|B)$ can be found according to a base-stock policy as follows:

$$y_2 = \begin{cases} K_2(B) & \text{if } x_1 < K_2(B) \\ x_1 & \text{otherwise} \end{cases} \quad \text{(4.3)}$$

The order-up-to level is

$$K_2(B) = \Phi_t^{-1}\left[\frac{p - c_2}{p + h}\right]|B|. \quad \text{(4.5)}$$

Proof: First we assume that $f_1(x_1|B)$ is a convex function and compute the optimal inventory level for Stage 2 using first order optimality condition. Later, we complete our proof showing that $f_1(x_1|B)$ is indeed convex. First order optimality condition leads to $\Phi_t^{-1}\left[\frac{p - c_2}{p + h}\right]|B|$ where $p \geq c_2 + h$ guarantees this critical ratio is a probability. Let $K_2(B) = \Phi_t^{-1}\left[\frac{p - c_2}{p + h}\right]|B|$ and note that $\Phi_t^{-1}\left[\frac{p - c_2}{p + h}\right]|B|$ is an increasing function of $y_2$ therefore if $x_1$ is greater than $K_2(B)$, then $Z_1(y_2;x_1|B)$ increases in $y_2$ everywhere, which states that there is no need for any more inventory. Otherwise, it is better to increase the inventory level up to $K_2(B)$. This leads to the piece-wise recourse function given in 4.6 for the last stage.

If the incoming inventory from the previous stage is sufficiently large to cover the order up-to-level $K_2(B)$ in this stage, there is no need to order any inventory. This means the inventory level at the beginning of the Stage 2 is equal to the inventory level at the end of the Stage 1. If the incoming inventory is not equal or greater than the order-up-to level, the cruise liner should order the difference between the
order-up-to level and on-hand inventory, e.g. $K_2(B) - x_1$. From stochastic ordering that is $\Phi^{t_2}(x) > \Phi^{t_2+\epsilon}(x)$, we can easily deduce that $K_2(B)$ increases with $t_2$.

Based on the above result, the Stage 2 cost can be written as follows:

$$f_1(x_1|B) = \begin{cases} 
  c_2(K_2(B) - x_1) + \mathcal{L}_2(K_2(B)|B) & \text{if } x_1 < K_2(B) \text{ (i.e., } y_2 = K_2(B)) \\
  \mathcal{L}_2(x_1|B) & \text{if } x_1 \geq K_2(B) \text{ (i.e., } y_2 = x_1). 
\end{cases} \quad (4.6)$$

where $\mathcal{L}_2(\theta|B) = p \int_{\theta}^{\infty} (\xi - \theta) \phi^{t_2}(\xi|B)d\xi + h \int_{0}^{\theta} (\theta - \xi) \phi^{t_2}(\xi|B)d\xi$.

Next, we show the convexity of $f_1(x_1|B)$.

**Lemma 17** $f_1(x_1|B)$ is quasi-convex in $x_1$.

**Proof:** The first and the second order condition of the cost Stage 2 function given in (4.6) is

$$\frac{\partial f_1(x_1|B)}{\partial x_1} = \begin{cases} 
  -c_2 & \text{if } x_1 < K_2(B) \\
  -p(1 - \Phi^{t_2}(x_1|B)) + h(\Phi^{t_2}(x_1|B)) & \text{if } x_1 \geq K_2(B) 
\end{cases} \quad (4.7)$$

$$\frac{\partial^2 f_1(x_1|B)}{\partial x_1^2} = \begin{cases} 
  0 & \text{if } x_1 < K_2(B) \\
  \phi^{t_2}(x_1|B)(p + h) & \text{if } x_1 \geq K_2(B) 
\end{cases} \quad (4.8)$$

It is easy to conclude that $\frac{\partial^2 f_1(x_1|B)}{\partial x_1^2} \geq 0 \ \forall \ x_1 \in \mathbb{R}$. Therefore, $f_1(x_1|B)$ is a convex function.

Above result implies the Stage 2 cost strictly decreases in inventory level if its lower than the optimal order-up-to level in this stage. Because as a result of having incoming inventory from the previous period, cruise liner needs to purchase less inventory which decreases the stage 2 cost.
4.1.3 Stage 1 Problem

The selling season for the cruise liner begins with the Stage 1 since the demand signal is observed and the cruise liner updates its demand information at this stage. The objective of this stage is to determine whether additional purchasing is necessary given the realized value of market signal $B$ in addition to the contracted amount at Stage 0. Based on the additional purchasing decision, the cruise liner needs to determine the optimal amount to order at this stage, $y_1$, from the spot market that minimizes the expected total cost through the end of Stage 2. The expected cost for the remaining period of time can be written as:

$$f_2(x_0|B) = \min_{y_1 \geq x_0} Z_2(y_1; x_0|B)$$ where

$$Z_2(y_1; x_0|B) = c_1(y_1 - x_0) + p \int_{y_1}^{\infty} (\xi_1 - y_1) \phi_{\xi_1}(\xi_1|B)d\xi_1$$

$$+ f_1(0|B) \int_{y_1}^{\infty} \phi_{\xi_1}(\xi_1|B)d\xi_1 + \int_{0}^{y_1} f_1(y_1 - \xi_1|B) \phi_{\xi_1}(\xi_1|B)d\xi_1$$

We note that $x_0 = y_0$. Since there is no consumption at Stage 0, the inventory level at the end of Stage 0 is equal to the inventory level at the beginning of Stage 1. It is straightforward to see that in addition to the purchase and shortage costs, Stage 2’s expected cost functions are also included in the expected cost function of Stage 1. The second integral equation represents the stage 2’s expected cost when there is no leftover remains from Stage 1. The third integral equation represents the stage 2’s expected cost, when some leftovers remain from the Stage 1. The above function leads us to the following result:
Proposition 15  The optimal order quantity at Stage 1 follows a base-stock policy and given as

\[ y_1 = \begin{cases} 
K_1(B), & \text{if } y_0 < K_1(B) \\
y_0, & \text{otherwise.}
\end{cases} \quad (4.11) \]

where \( K_1(B) \) is the unique solution to:

\[ c_1 - p\Phi^t_2(y_1|B) + \int_0^{y_1} \frac{\partial f_1(y_1 - \xi|B)}{\partial y_1} \phi^t_1(\xi|B) d\xi = 0 \quad (4.12) \]

Proof: Similar to the Stage 2 problem, the objective is to determine the critical inventory level for the first period, \( y^*_1 \), that minimizes \( Z_2(y_1;x_0|B) \). The critical inventory level for the first period, \( y^*_1 \), is the solution of the following first order condition:

\[ \frac{\partial Z_2(y_1; x_0|B)}{\partial y_1} = c_1 - p(1 - \Phi^t_1(y_1|B)) + \int_0^{y_1} \frac{\partial f_1(y_1 - \xi|B)}{\partial y_1} \phi^t_1(\xi|B) d\xi \quad (4.13) \]

The last part of 4.13 can be extended as:

\[ \frac{\partial f_1(y_1 - \xi|B)}{\partial y_1} = \begin{cases} 
-c_2 & \text{if } y_1 - \xi < K_2(B) \\
\frac{\partial \mathcal{L}_2(y_1 - \xi|B)}{\partial y_1} & \text{if } y_1 - \xi \geq K_2(B)
\end{cases} \quad (4.14) \]

Note that

\[ \frac{\partial \mathcal{L}_2(y_1 - \xi|B)}{\partial y_1} = -p \int_{y_1 - \xi}^{\infty} \phi^t_2(\xi|B) d\xi + h \int_0^{y_1 - \xi} \phi^t_2(\xi|B) d\xi 
= -p \left[ 1 - \Phi^t_2(y_1 - \xi|B) \right] + h \Phi^t_2(y_1 - \xi|B) 
= (p + h) \Phi^t_2(y_1 - \xi|B) - p \quad (4.15) \]

Therefore,

\[ \frac{\partial f_1(y_1 - \xi|B)}{\partial y_1} = \begin{cases} 
-c_2 & \text{if } y_1 - \xi < K_2(B) \\
(p + h) \Phi^t_2(y_1 - \xi|B) - p & \text{if } y_1 - \xi \geq K_2(B)
\end{cases} \quad (4.16) \]
and the second order condition can be written as:

\[
\frac{\partial^2 f_1(x_1|B)}{\partial x_1^2} \begin{cases} 
0 & \text{if } y_1 - \xi_1 < K_2(B) \\
(p + h)\phi^t_2(y_1 - \xi_1|B) & \text{if } y_1 - \xi_1 \geq K_2(B)
\end{cases} \tag{4.17}
\]

Taking the second order condition of \( Z_2(y_1; x_0|B) \) with respect to \( y_1^* \)

\[
\frac{\partial^2 Z_2(y_1; x_0|B)}{\partial y_1^2} = p \phi^t_1(y_1|B) + \int_0^{y_1} \frac{\partial^2 f_1(y_1 - \xi_1|B)}{\partial y_1^2} \phi^t_1(\xi_1|B) d\xi_1 + \frac{\partial f_1(0|B)}{\partial y_1} \phi^t_1(y_1|B)
\]

\[
\tag{4.18}
\]

Since \( y_1 = x_1 + \xi_1, \ dy_1 = dx_1 \). Therefore,

\[
\frac{\partial f_1(0|B)}{\partial y_1} = \frac{\partial f_1(0|B)}{\partial x_1} = -c_2 \quad \text{since} \quad 0 < K_2(B) \tag{4.19}
\]

As a result,

\[
\frac{\partial^2 Z_2(y_1; x_0|B)}{\partial y_1^2} = (p + \frac{\partial f_1(0|B)}{\partial y_1})\phi^t_1(y_1|B) + \int_0^{y_1} \frac{\partial^2 f_1(y_1 - \xi_1|B)}{\partial y_1^2} \phi^t_1(\xi_1|B) d\xi_1
\]

\[
= (p - c_2)\phi^t_1(y_1|B) + \int_0^{y_1-K_2(B)} (p + h)\phi^t_2(y_1 - \xi_1|B)\phi^t_1(\xi_1|B) d\xi_1
\]

\[
+ \int_{y_1-K_2(B)}^{\infty} 0 \phi^t_1(\xi_1|B) d\xi_1 \tag{4.20}
\]

Since \( p > c_2 \) and \( \int_0^{y_1-K_2(B)} (p + h)\phi^t_2(y_1 - \xi_1|B)\phi^t_1(\xi_1|B) d\xi_1 > 0 \),

\[
\frac{\partial^2 Z_2(y_1; x_0|B)}{\partial y_1^2} > 0, \text{ therefore } Z_2(y_1; x_0|B) \text{ is strictly convex in } y_1. \]

Hence the root of the equation in 4.13, which we denote by \( K_1(B) \), minimizes \( Z_2(y_1; x_0|B) \). Note from 4.13 that \( K_1(B) \) is stationary point. Hence, from convexity it must minimize \( Z_2(y_1; x_0|B) \).

Basically, \( K_1(B) \) is the base-stock level for Stage 1. Although there is no closed-form solution for \( K_1(B) \), its value can be found by using a simple line search. We
note that, since random variable $\xi_1$ is stochastically increasing in $B$, there exists a threshold value, say $B_0 = K_{1}^{-1}(y_0)$ for which $y_1^* = y_0$. Clearly, for any $B$ below this threshold, the firm does not need to make any replenishments from the spot market at the beginning of the selling season. Now we have to justify using first order optimality conditions by showing that $f_2(x_0|B)$ is a convex function.

Optimal cost denoted by $f_2(x_0|B)$ is a piece-wise function of the advanced contract amount, $x_0$, as shown below.

$$f_2(x_0|B) = \begin{cases} 
  c_1(K_1(B) - x_0) + L_1(K_1(B)|B) + f_1(0|B) \int_{K_1(B)}^{\infty} \phi^{t_1}(\xi|B)d\xi \\
  + \int_{0}^{K_1(B)} f_1(K_1(B) - \xi|B)\phi^{t_1}(\xi|B)d\xi \\
  L_1(x_0|B) + f_1(0|B) \int_{x_0}^{\infty} \phi^{t_1}(\xi|B)d\xi + \int_{0}^{x_0} f_1(x_0 - \xi|B)\phi^{t_1}(\xi|B)d\xi \\
  \text{if } x_0 < K_1(B) \text{ (i.e., } y_1 = K_1(B)), \\
  \text{if } x_0 \geq K_1(B) \text{ (i.e., } y_1 = x_0). 
\end{cases}$$

(4.21)

where $L_1(\theta|B) = p \int_{0}^{\infty}(\xi - \theta)\phi^{t_1}(\xi|B)d\xi$. It is straightforward to conclude that $\partial^2 f_2(x_0|B)/\partial x_0^2 \geq 0 \ \forall \ x_0 \in \mathbb{R}$. Therefore, $f_2(x_0|B)$ is a convex function.

The foregoing proposition implies that it is optimal for the firm to increase its inventory level beyond the advance contract amount only if the realized market signal is above a certain threshold. Although there is no closed form solution of $K_1(B)$, it is still possible to analyze the effects of certain parameters on the value of $K_1(B)$. The following lemma states the relationship between replenishment costs and $K_1(B)$. 
Lemma 18  Order-up-to level of Stage 1, $K_1(B)$, decreases in $c_1$ and increases in $c_2$.

Proof: We know that $K_1(B)$ is the solution of equation (4.13). First we analyze, the case when $K_1(B) < K_2(B)$ for $c_1$ and $c_2$.

If $K_1(B) < K_2(B)$, then we can derive the following equation, $p - c_1 = (p - c_2)\Phi^t_1(K_1(B)|B)$.

When we take the derivatives of the both sides with respect to $c_2$, we have

$$(p - c_2)\phi^t_1(K_1(B)|B)\partial K_1(B)/\partial c_2 - \Phi^t_1(K_1(B)|B) = 0. \tag{4.22}$$

Note that $p > c_2$ and $\Phi^t_1(K_1(B)|B) > 0$, therefore; we can say that $\partial K_1(B)/\partial c_2 > 0$. That means the order-up-level of Stage 1, $K_1(B)$ increases in $c_2$.

For $c_1$, when taking the derivative of the above equation with respect to $c_1$, we get

$$(p - c_2)\phi^t_1(K_1(B)|B)\partial K_1(B)/\partial c_1 = -1 \tag{4.23}$$

Since $p - c_2 > 0$, it follows that $\partial K_1(B)/\partial c_1 < 0$. This concludes that the order-up-level of Stage 1, $K_1(B)$ decreases in $c_1$.

By doing the similar analysis for the case of $K_1(B) \geq K_2(B)$, the same results can be obtained for $c_1$ ad $c_2$.

Lemma 18 implies that order-up-to level of the first stage decreases as the replenishment cost of this stage increases. This is reasonable since as the cost of replenishment in the first stage increases, it becomes advantageous to buy the items necessary for the second stage at the beginning of the second stage instead of the first stage.

Of course, the effect of the second stage replenishment cost is opposite in this case. Order-up-to level of the first stage increases as the replenishment cost of the following
stage increases. It is beneficial to make procurement at the first stage for the second stage when the second stage cost is very high.

**Lemma 19** $f_2(x_0|B)$ is quasi-convex in $x_0$.

*Proof:* The proof is similar to Lemma 17 and straightforward. □

As we discussed earlier, unfortunately we do not have closed form for $K_1(B)$. However, we can derive analytical bounds on $K_1(B)$ as stated in the next lemma.

**Lemma 20** The base-stock level of Stage 1, $K_1(B)$ lies between $K_1^{Min} \leq K_1(B) \leq K_1^{Max}$, where

\[ K_1^{Min} = \Phi^{-1}_{t_1} \left( \frac{p-c_1}{p+h} | B \right), \text{ and } K_1^{Max} = \Phi^{-1}_{t_1+t_2} \left( \frac{p-c_1}{p+h} | B \right). \]

*Proof:* The above result can be shown to be true with the following explanation. $K_1^{Min}$ is the minimum base stock level that covers the expected total demand for period 1 only, while $K_1^{Max}$ covers the expected demand for the whole selling season, i.e. both period 1 and period 2. Note that $K_1(B)$ can not be less than $K_1^{Min}$ or greater than $K_1^{Max}$. Because the expected demand during the selling season will fall between these boundaries. Also, note that $\Phi(K_1^{Min}) > \Phi(K_1^{Max})$. From stochastic dominance, it is straightforward to see that $K_1^{Min} < K_1^{Max}$. The cruise liner’s optimal purchasing quantity falls between these boundaries depending on the market signal. □

**Lemma 21** Let $SL_1$ be the service level of Stage 1, then the following must be true:

\[ \frac{p-c_1}{p+h} \leq SL_1 \] (4.24)

*Proof:* If $y_0 < K_1(B)$, then the cruise liner will purchase inventory to increase her inventory level to the base-stock level. Assume that the unit purchase price at
stage 1, $c_1 = 0$. In that case the service level will be $SL_1 = \frac{p}{p + h}$. If $\Phi^t(y_0) > \frac{p}{p + h}$ then $SL_1 > SL_2$.

The cruise liner can purchase inventory either for period 1 or both periods (whole selling season) at the Stage 1. The stock-out costs are the same for in both cases, i.e. $p - c_1$. The overage cost is higher if the cruise liners ends up with having inventory at the end of the last period, i.e., $c_1 + h$. Since, the leftovers at the end of period 1 can be used at period 2, the overage cost is lower, i.e., $c_1 - c_2$. 

4.1.4 Stage 0 Problem

This stage marks the beginning of the planning horizon where the market signal is not observed by the firm yet. In other words, market (demand) signal is uncertain at this stage. At the beginning of this stage, we assume that we have an initial inventory level equal to zero (i.e., \( f_3(0) \)). At this stage, there is a cost saving opportunity for the firm that can be executed through advance ordering contract. The unit purchase price, \( c_0 \), at this stage is lower than all the following stages. Let \( k_1(y_0) \) represent \( K_1^{-1}(y_0) \).

\[
f_3(0) = \min_{y_0, s \geq 0} Z_3(y_0, s)
\]  

\[
Z_3(y_0, s) = c_0 y_0 + \int_0^{k_1(y_0)} M_2(y_0|s)h(s)ds + \int_{k_1(y_0)}^{\infty} [c_1(K_1(s) - y_0) + M_2(K_1(s)|s)] h(s)ds
\]

where

\[
M_2(\theta|B) = p \int_{\theta}^{\infty} (\xi_1 - \theta) \phi^{t_1}(\xi_1|B) d\xi_1 + f_1(0|B) \int_\theta^{\infty} \phi^{t_1}(\xi_1|B) d\xi_1 + \int_0^{\theta} f_1(\theta - \xi_1|B) \phi^{t_1}(\xi_1|B) d\xi_1
\]

This function is the cost function for any continuous demand distribution at the Stage 0 where the initial contracted quantity decision has been made. In addition to the initial purchasing cost, Stage 1’s and Stage 2’s expected cost are included in the expected cost function of Stage 0. The first integral in 4.26 represents the portion of the expected cost where the market signal falls between zero and initial contracted quantity, which means the cruise liner decides an initial contract quantity greater than the total realized demand signal. The second integral of the equation 4.26 represents...
the portion of the cost where the market signal is greater than the initial contracted quantity. That means the cruise liner decides an initial contract quantity less than the total realized demand signal. Based on the decision at Stage 0, the cost of the following stages are shown in equation 4.27.

Consequently, we observe the following:

**Lemma 22** There is a unique value of $y_0$ that minimizes the expected cost for the planning horizon.

**Proof:** This is an unconstrained optimization problem, we have to find the first and the second optimality conditions. The first derivative of $Z_3(y_0, s)$ is below:

$$
\frac{\partial Z_3(y_0, s)}{\partial y_0} = c_0 + \int_0^{k_1(y_0)} \frac{\partial}{\partial y_0} M_2(y_0|s)h(s)ds - \int_{k_1(y_0)}^{\infty} c_1 h(s)ds + M_2(y_0|k_1(y_0)) h(k_1(y_0)) \frac{dk_1(y_0)}{dy_0} - M_2(K_1(k_1(y_0)) | k_1(y_0)) h(k_1(y_0)) \frac{dk_1(y_0)}{dy_0} \quad (4.28)
$$

When $B = k_1(y_0)$, we know that $K_1(B) = y_0$ from the Stage 1 base-stock policy, so $M_2(y_0|k_1(y_0)) = M_2(K_1(k_1(y_0)) | k_1(y_0))$. As a result the third and the fourth terms in 4.28 are equal to each other. After cancelling them from 4.28, we can rewrite $Z_3(y_0, s)$ as follows:

$$
\frac{\partial Z_3(y_0, s)}{\partial y_0} = c_0 + \int_0^{k_1(y_0)} \frac{\partial}{\partial y_0} M_2(y_0|s)h(s)ds - \int_{k_1(y_0)}^{\infty} c_1 h(s)ds
$$

Since $M_2(y_0|B)$ and its first order condition with respect to $y_0$ can be written as follows:

$$
M_2(y_0|B) = p \int_{y_0}^{\infty} (\xi_1 - y_0)\phi^{f_1}(\xi_1|B)d\xi_1 + f_1(0|B) \int_{y_0}^{\infty} \phi^{f_1}(\xi_1|B)d\xi_1 + \int_{0}^{y_0} f_1(y_0 - \xi_1|B)\phi^{f_1}(\xi_1|B)d\xi_1 \quad (4.29)
$$
and

$$\frac{\partial M_2(y_0|B)}{\partial y_0} = -p \left(1 - \Phi^{t_1}(y_0|B)\right) + \int_0^{y_0} \frac{\partial f_1(y_0 - \xi_1|B)}{\partial y_0} \phi^{t_1}(\xi_1|B) \, d\xi_1$$  \hspace{1cm} (4.30)

Hence the first derivative of $Z_3(y_0, s)$ can be written as follows:

$$\frac{\partial Z_3(y_0, s)}{\partial y_0} = c_0 - \int_{k_1(y_0)}^{\infty} c_1 h(s) \, ds + \int_0^{k_1(y_0)} -p \left(1 - \Phi^{t_1}(y_0|B)\right)h(s) \, ds$$

$$+ \int_0^{k_1(y_0)} \int_0^{y_0} \frac{\partial f_1(y_0 - \xi_1|B)}{\partial y_0} \phi^{t_1}(\xi_1|s)h(s) \, d\xi_1 \, ds$$  \hspace{1cm} (4.31)

Taking the second derivative of $Z_3(y_0, s)$,

$$\frac{\partial^2 Z_3(y_0, s)}{\partial y_0^2} = c_1 h(k_1(y_0)) \frac{\partial k_1(y_0)}{\partial y_0} + p \int_0^{k_1(y_0)} \phi^{t_1}(y_0|s)h(s) \, ds$$

$$- p(1 - \Phi^{t_1}(y_0|k_1(y_0)))h(k_1(y_0)) \frac{dk_1(y_0)}{dy_0}$$

$$+ \int_0^{k_1(y_0)} \int_0^{y_0} \frac{\partial^2 f_1(y_0 - \xi_1|s)}{\partial y_0^2} \phi^{t_1}(\xi_1|s)h(s) \, d\xi_1 \, ds$$

$$+ h(k_1(y_0)) \frac{dk_1(y_0)}{dy_0} \int_0^{y_0} \frac{\partial f_1(y_0 - \xi_1|s)}{\partial y_0} \phi^{t_1}(\xi_1|k_1(y_0)) \, d\xi_1$$  \hspace{1cm} (4.32)

Now we have to show that the equation 4.32 is non-negative in order to prove the convexity of $Z_3(y_0, s)$.

First, we can state that,

$$p \int_0^{k_1(y_0)} \phi^{t_1}(y_0|s)h(s) \, ds \geq 0$$  \hspace{1cm} (4.33)

and,

$$\int_0^{k_1(y_0)} \int_0^{y_0} \frac{\partial^2 f_1(y_0 - \xi_1|s)}{\partial y_0^2} \phi^{t_1}(\xi_1|s)h(s) \, d\xi_1 \, ds \geq 0$$  \hspace{1cm} (4.34)

Because of the convexity of $f_1(x_1|B)$. Let,

$$\frac{\partial^2 Z_3(y_0, s)}{\partial y_0^2} = P_0(y_0) + F_0(y_0)$$  \hspace{1cm} (4.35)
where,

\[
P_0(y_0) = p \int_0^{k_1(y_0)} \phi^{t_1}(y_0|s) h(s) \, ds + \int_0^{k_1(y_0)} \int_0^{y_0} \frac{\partial^2 f_1(y_0 - \xi_1|s)}{\partial y_0^2} \phi^{t_1}(\xi_1|s) h(s) \, d\xi_1 \, ds
\]

(4.36)

and

\[
F_0(y_0) = h(k_1(y_0)) \frac{dk_1(y_0)}{dy_0} \left[ c_1 - p(1 - \Phi^{t_1}(y_0|k_1(y_0))) + \int_0^{y_0} \frac{\partial f_1(y_0 - \xi_1|s)}{\partial y_0} \phi^{t_1}(\xi_1|k_1(y_0)) \, d\xi_1 \right]
\]

(4.37)

From 4.33 and 4.34, we know that \( P_0(y_0) \geq 0 \). We need to show that \( F_0(y_0) \geq 0 \) to prove convexity. From equation 4.13, the first order optimality conditions of Stage 1,

\[
c_1 = p(1 - \Phi^{t_1}(y_1|B)) - \int_0^{y_1} \frac{\partial f_1(y_1 - \xi_1|B)}{\partial y_1} \phi^{t_1}(\xi_1|B) \, d\xi_1
\]

(4.38)

The sum of the terms in the bracket in 4.37 is equal to zero since \( B = k_1(y_0) \) in 4.37 which means \( y_0 = K_1(B) \), and according to Stage 1 policy, \( y_1 = x_0 = y_0 \) when \( y_0 = K_1(B) \). Therefore, \( F_0(y_0) = 0 \) at optimality. This implies that

\[
\frac{\partial^2 Z_3(y_0, s)}{\partial y_0^2} = P_0(y_0) + F_0(y_0) > 0
\]

(4.39)

Thus there must exist a value, \( y_0^* \) that minimizes the expected cost of the 3-Stage inventory control problem.

Unfortunately, a closed form solution to Stage 0 problem cannot be derived. However, an improved line search and simulation can be used to compute the optimal contract amount and the optimal expected cost. Similar to the analysis done in the previous sections, the effects of purchasing costs of different stages can be seen in the following proposition.
4.2 Numerical Study and Sensitivity Analysis

4.2.1 Solution Procedure

In this section, we propose the line search method as the solution approach for the cruise liner’s multi-period inventory decision problem in order to compute the optimal advance contract and spot market quantities. This method is an iterative process to find a local minimum of an objective function.

The line search method first finds a descent direction along which the objective function will be reduced and then computes a step size that determines how far x should move along that direction [62]. This method is guaranteed to converge the optimal solution as it is proven that expected costs at all stages are strictly convex. This procedure recursively calculates optimal inventory levels for selling season periods (Period 1 and Period 2), given the market signal $B$. Then, by iterating over $B$ which can be converted into $y_0$ using $B = k_1(y_0)$, the algorithm searches for optimal advance contract quantity $y_0$ value.

We run the solution procedure on the Monte Carlo Simulation in VBA and MS Excel. The steps of the algorithm is below:

1- Set an initial random market signal $B_0$ (NormInv function in excel is used with the parameters of the normal distribution). Multiple random demand signals are created for every $y_0$.

2- Base stock level at Stage 2, $K_2(B = B_0)$, is computed using Lemma 16. Since the base stock level at Stage 1 can not be computed at this step, the lower and upper limit for the base stock level of Stage 1, $K_{1min}$ and $K_{1max}$, are computed for each random demand signal. When searching for the optimal base stock level
of Stage 1, $K_1$, the step size of the line search method is calculated based on these upper and lower limits.

3- For each $B$, the optimal $K_1$ is searched by calculating the total cost of the selling season. This cost equals purchasing cost, landing cost, and shortage cost. The macro changes the $K_1$ value cleverly using the properties of convex function and stops at the one $K_1$ value which makes the total cost minimum.

4- Step 3 is repeated for all of the random market signals created.

5- For each $y_0$ value, the average cost of the selling season and other indicators (Fill Rate and Service Level) are calculated.

6- As the advance contract quantity $y_0$ increased by the predetermined heuristic step size, the same computation at Step 5 is applied. The advance contracting value $y_0$ that returns the lowest overall cost is selected for that loop.
4.2.2 Numerical Analysis and Simulation Result

In this section, we provide numerical examples for the three-stage inventory control problem of a cruise liner. Monte Carlo simulations are performed to test the effect of the mean \( \mu_B \) and standard deviation \( \sigma_B \) of the market signal (bookings), the mean \( \mu_D \) and standard deviation \( \sigma_D \) of the daily consumption, and other parameters (i.e., unit purchase price at each stage \( c_i \), total time for each period of the selling season \( t_i \), shortage penalty \( p \), and landing cost \( h \)) on the initial advance contract quantity \( y_0 \). Our goal in this chapter is to find the optimal initial advance contract amount that minimizes the total selling season cost of the cruise liner under the different scenarios with the parameters mentioned above.

We conduct our experiments to explore the effects of some parameters while keeping some parameters constant. For instance, in the first example below, advance contract amount is computed under three different initial contract prices \( c_0 = 0.25, 0.5, 1 \) and five different standard deviations of the demand signal \( \sigma_B = 5, 10, 20, 30, 50 \). Other simulation parameters are assumed to be: \( c_1 = 2, c_2 = 3, p = 8, h = 0.1, t_1 = 3, t_2 = 4, \mu_B = 100, \mu_D = 1, \sigma_D = 0.5 \). Given the market signal information (bookings) and the daily consumption mean and standard deviation, the market signal information in period \( n \) is distributed with mean of \( B \cdot t_n \cdot \mu_D \) and a standard deviation that is equal to \( \sqrt{B \cdot t_n \cdot \sigma_D} \). The result of the simulation run is shown below.
Table 4.1 shows the initial advance contract amounts under the combination of different market signals ($B$) and advance contract unit purchase prices ($c_0$). The initial advance contract amount increases as the variability of the market signal increases. Since advance contract offers a cost saving opportunity, the cruise liner hedges herself against the possible stock-outs by purchasing more aggressively as the uncertainty of the market signal increases.

Lower unit purchase price at Stage 0 (advance contract unit purchase price) encourages the cruise liner to purchase higher quantities since the risk of early commitment diminishes. On the other hand, increasing the purchase price at Stage 0 discourages the firm since the early commitment risk of the firm will be elevated with higher costs. Hence, postponing the purchase until the next stages is a strategy for the firm if the risk is high at Stage 0. Since the holding cost between the stages is not included in the total cost, the cruise liner does not have carrying risk between each stage. This also encourages the firm to hold higher quantities at the very beginning of the selling season. Especially, under the high demand signal variation, the firm commits high quantities even if the advance contract price is getting higher. One of the insights behind this result is that the lower advance contract cost and zero

<table>
<thead>
<tr>
<th>Sigma B</th>
<th>$c_0$ 0.25</th>
<th>$c_0$ 0.5</th>
<th>$c_0$ 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>750</td>
<td>730</td>
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<td>850</td>
</tr>
<tr>
<td>50</td>
<td>1150</td>
<td>1100</td>
<td>1050</td>
</tr>
</tbody>
</table>

Table 4.1: Optimum Advance Contract Amount/Data of Figure 4.3
carrying cost between stages encourages the firm to purchase high percentages of the total inventory needed at Stage 0.

The graphical representation of Table 4.1 is given in Figure 4.3.

Table 4.2 shows the total inventory cost that the cruise liner incurs for the selling season based on the committed advance contract quantities given in Table 4.1. As the variability of the demand signal increases, the total inventory cost of the cruise liner increases since the cruise liner buys more inventory to cover the expected demand during the selling season. The total inventory cost increases as the advance contract unit purchase price increases.

Figure 4.3: Impact of the Market Signal Variation on the Advance Contract Amount
Table 4.2: Total Inventory Cost/Data of Figure 4.4

<table>
<thead>
<tr>
<th>Sigma B</th>
<th>c0 = 0.25</th>
<th>c0 = 0.5</th>
<th>c0 = 1</th>
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<tr>
<td>50</td>
<td>364</td>
<td>675</td>
<td>1244</td>
</tr>
</tbody>
</table>

The graphical representation of Table 4.2 is shown in Figure 4.4. The total inventory cost is always lower when the advance contract unit purchase fee is low. As the purchase price and the demand signal variability increases, the cruise liner incurs a higher total inventory cost. Figure 4.4 shows the total cost under different contract unit purchase prices and market signal deviations.
We analyze the effect of the duration of the first leg \((t_1)\) and the unit purchase cost \((c_1)\) of Stage 1 in the **second example**.

In this example, the advance contract amount is computed under five different Stage 1 unit purchase prices \((c_1 = 1, 1.5, 2, 2.5, 2.75)\) and five different durations for the first period \((t_1 = 1, 2, 3, 4, 5)\). Other simulation parameters are: \(c_0 = 0.5\), \(c_2 = 3\), \(p = 8\), \(h = 0.2\), \(\mu_B = 100\), \(\sigma_B = 25\), \(\mu_D = 1\), \(\sigma_D = 0.5\). Note that the duration of the second leg \((t_2)\) depends on the duration of the first leg \((t_1)\), since we fix the total selling season at a constant, i.e. \(t_1 + t_2 = 7\).

<table>
<thead>
<tr>
<th>(c_1)</th>
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<th>2</th>
<th>2.5</th>
<th>2.75</th>
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<tr>
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<td>301</td>
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<td>760</td>
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<td>301</td>
<td>203</td>
<td>102</td>
</tr>
<tr>
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<td>803</td>
<td>354</td>
<td>204</td>
<td>103</td>
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<td>804</td>
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</tr>
<tr>
<td>2.75</td>
<td>802</td>
<td>803</td>
<td>805</td>
<td>805</td>
<td>152</td>
</tr>
</tbody>
</table>

Table 4.3: Optimum Advance Contract Amount/Data of Figure 4.5

Table 4.3 shows optimum advance contract amounts for the cruise liner under the combination of the different unit purchase costs \((c_1)\) and the durations \((t_1)\) of Stage 1. The cruise liner commits higher advance contract quantities as the Stage 1 purchase cost increase. This is reasonable because increasing cost of the next Stage will increase the postponement risk of the firm.

One of the interesting results is that as the duration of the first leg increases, the initial commitment quantity \((y_0)\) at Stage 0 diminishes. The cruise liner will use the postponement strategy because of the increasing uncertainty of the market signal effect at Stage 1. At Stage 0 the market signal is unknown and the cruise liner
does not have perfect information about the total number of passengers. She has another option to purchase inventory at Stage 1, right after the market signal reveals. Since the longer period length increases the uncertainty, she prefers to commit less quantities at Stage 0. However, if the duration of the first leg is shorter, the uncertainty becomes less detrimental, therefore the cruise liner is likely to commit higher quantities.

Figure 4.5 shows the total contracted quantities based on the purchase cost \( c_1 \) and the duration \( t_1 \) of the Stage 1.

![Figure 4.5: Impact of the Unit Purchase Cost and Duration of Stage 1 on the Initial Contract Amount](image)

Figure 4.5 illustrates the change in the initial inventory commitment of the cruise liner based on different Stage 1 costs and durations. The sharp decreases can be seen as the duration of the first leg increases when the first stage cost is relatively low. However, the cruise liner tends to commit higher initial inventory reservations when the first stage cost is high even when there are long period lengths. The higher unit
cost increases the cruise liner’s postponement risk, hence she wants to hedge herself against possible higher unit cost payment even if the uncertainty is high because of the longer first period length.

Table 4.4 below shows the total inventory cost of the selling season based on the cruise liner’s advance contract quantity decision shown in Table 4.3. As the initial commitment decreases, the total inventory cost of the cruise liner increases. When the cruise liner commits low quantities before the selling season starts, she will need to replenish some of her inventory from the local spot markets at higher prices after the selling period starts and this will increase her total expected inventory cost. This decline in the initial commitment is caused by the increasing effect of the uncertainty of the market signal at Stage 1 as a result of the longer duration of Stage 1. In addition, increasing Stage 1 costs ($c_1$) encourages the firm to purchase more inventory at Stage 0 because of the high replenishment cost in the next stages.

<table>
<thead>
<tr>
<th>c1</th>
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<th>5</th>
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<td>499</td>
<td>498</td>
<td>1074</td>
<td>1241</td>
</tr>
<tr>
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<td>499</td>
<td>499</td>
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<tr>
<td>2.75</td>
<td>500</td>
<td>499</td>
<td>498</td>
<td>497</td>
<td>1633</td>
</tr>
</tbody>
</table>

Table 4.4: Total Inventory Cost/Data of Figure 4.6
Figure 4.6 plots the data given in Table 4.4 on a graph in which the sudden increment in the total inventory cost can be seen when the duration of the first period increases.

In Figure 4.6, we see that if the firm purchases less inventory through the advance contract due to the uncertainty at Period 1 and relies on the spot market option at subsequent stages, the total inventory cost increases dramatically. This cost increase becomes more detrimental especially if the cost at Stage 1 ($c_1$) is high and the duration of the first leg is long. There are some cases where early commitment decreases the inventory cost even if the duration of the first leg is long.

The third example tests the effect of the unit purchase price ($c_2$) and the duration ($t_2$) of Stage 2 on the initial contract quantity. In this example, the advance contract amount is computed under five different Stage 2 unit purchase prices ($c_2 = 1.25, 1.5, 2, 2.5, 3$) and five different durations for the second period ($t_2 = 2, 3, 4, 5, 6$).
Other simulation parameters are: $c_0 = 0.5, c_1 = 1, p = 5, h = 0.1, \mu_B = 100, \sigma_B = 5, \mu_D = 1, \sigma_D = 0.5$. Note that the duration of each leg depends on each other, since we fix the duration of the total selling season at a constant, i.e. $t_1 + t_2 = 7$.

Table 4.5 below shows the effects of the duration of the second leg and Stage 2 unit cost on the initial contract amount.

<table>
<thead>
<tr>
<th>$C_2$</th>
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<th>3</th>
<th>4</th>
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<td>727</td>
<td>730</td>
</tr>
<tr>
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<td>700</td>
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<td>712</td>
<td>720</td>
<td>733</td>
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<tr>
<td>2</td>
<td>163</td>
<td>253</td>
<td>706</td>
<td>713</td>
<td>736</td>
</tr>
<tr>
<td>2.5</td>
<td>153</td>
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<td>350</td>
<td>750</td>
<td>756</td>
</tr>
<tr>
<td>3</td>
<td>140</td>
<td>240</td>
<td>320</td>
<td>770</td>
<td>780</td>
</tr>
</tbody>
</table>

Table 4.5: Impact of Unit Purchase Cost and Duration of Stage 2 on the Initial Contract Amount

We observe that the initial contract amount decreases with high Stage 2 costs if the duration of the second leg is sufficiently short (i.e. $t_2 = 2, 3$). On the other hand, it increases with high Stage 2 cost if the second leg duration is longer (i.e. $t_2 = 4, 5, 6$). This suggests that the risk associated with the change in the Stage 2 cost is directly affecting the initial contract amount for short and long periods lengths. If Period 2 is relatively short, when the cost of Stage 2 is high, purchases are postponed to Stage 1 due to the increasing downside risk. While Period 2 gets shorter, Period 1 becomes longer and this increases the risk of demand uncertainty in that period. For this reason, it is better for the firm to use the postponement strategy and wait until the demand signal is revealed at Stage 1. On the other hand, when Period 2 is longer, the risk associated with demand uncertainty decreases due to the shorter duration of Period 1. As the uncertainty risk on Period 1 decreases and Stage 2 cost
increases, the firm commits higher initial contract quantities at the beginning of the selling season.

Figure 4.7 plots the data given in Figure 4.5 on a graph in which the sudden increment in the total inventory cost can be seen when the duration of the second period increases since this will cause less uncertainty for the first period.

The expected total cost of the firm based on the initial contract amounts in Figure 4.5 is shown below in Table 4.6.

<table>
<thead>
<tr>
<th>C2</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<tr>
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</tr>
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</table>

Table 4.6: Total Inventory Cost/Data of Table 4.5
The total inventory cost of the firm increases when Stage 2 cost is higher, especially when the demand uncertainty risk is high in Period 1 due to the short duration of Period 2. Since the firm postpones the orders until receiving the market signal because of the risk due to the uncertainty, higher Stage 1&2 costs increase her expected inventory cost. On the other hand, when the risk is lower due to the shorter Period 1 (when Period 2 is longer), the firm commits more initial inventory and her expected inventory cost diminishes. This commitment is higher when Stage 1&2 cost are high due to the increasing purchasing risk in the later stages.

The increase in the initial contract amount for different Stage 2 costs can be seen in Figure 4.7. This increase is expected because of the resolved uncertainty in Period 1 due to the shorter leg duration as Period 2 duration increases.

Figure 4.8 plots the data given in Table 4.6. The sudden decrement in the total inventory cost is caused by the cruise liner’s large initial order due to the resolved uncertainty at Period 1. At the high Period 2 costs, the large commitment of the firm reduces her total inventory cost dramatically.

![Figure 4.8: Impact of the Unit Purchase Cost and Duration of Stage 2 on the Total Inventory Cost](image-url)
4.3 Concluding Discussion

In this section, we introduce a finite horizon multi-period inventory control problem for a seasonal product where the demand for the product is stochastic and non-stationary across periods. Advance market information (demand signal) can be gathered prior to the beginning of the selling season. However, before acquiring this information, the firm can make an early commitment through advance contract in order to benefit from price discounts. It is also possible to make replenishments from the local spot markets during the selling season with higher costs compared to the cost offered by the advance contract before the realization of market information.

Neither the costs during the selling season at the beginning of each stage, nor the lengths of these stages are necessarily equal to each other. Our goal in this chapter is to determine the optimal combination of advance contracting and expedited replenishment quantity in order to minimize the total inventory cost of the firm.

We analytically prove that order-up-to (base-stock) level policies are optimal for Stage 1 and Stage 2 for the 3-Stage problem. We also provide the relationship between order-up-to level of Stage 1 and replenishment costs of Stage 1 and Stage 2. We establish an upper and a lower boundary for Stage 1 base-stock level.

Later on, we show additional analytical results that capture the relationship between costs at each stage, period durations, and service and order-up-to levels. We prove the convexity of the expected cost function and show that there is a unique contract amount that minimizes the expected cost of a 3-Stage problem. Proving that there is a unique solution for Stage 0 cost function is important for us to justify the applicability of the method (line-search) we use in numerical results. As an important
result of this, we show that the initial contract amount \( y_0 \) increases in both Stage 1 and Stage 2 purchasing costs \((c_1 \text{ and } c_2)\) and decreases in initial contract cost \((c_0)\).

In the last part of this section, we provide numerical examples to analyze the effects of certain exogenous parameters on order-up-to levels, initial contract amount and expected inventory cost. Numerical examples show that the market signal information variability has considerable effect on initial contract amount decisions. We observe that if the initial contract cost is high, replenishment is postponed to the following stages where demand signal information is known; on the other hand, when initial cost is low, the optimal contract amount goes up because the risk is lower. We investigate the expected cost in a similar manner when the standard deviation of the market signal information varies. As expected, we see an increasing trend of expected cost with the variability of the market signal information also as the initial contract cost increases, expected cost increases, too. The lengths of the periods during the selling season are also investigated. Our numerical example shows that when the length of the first period becomes longer, the initial contract amount decreases because of the increasing uncertainty of the first period. On the other hand, advance contracting becomes more appealing when the length of the first period is shorter, because the initial commitment risk is lower in this case.

The framework developed in this study provides efficient inventory control policies to the decision maker who have to deal with multiple sources of uncertainties mentioned above.
As a direction for future research,

1- It would be interesting to extend the model for multi-period contracting, a case where at the beginning of Stage 0, contracts are allowed not only for Stage 1 but also for the following stages after Stage 1. The value of the advance market information increase enables the firm to benefit from price incentives due to early contracts with suppliers for all stages following Stage 0.

2- Spot market cost uncertainty can also be investigated under stochastic setting since this study considers only deterministic stock market prices.

3- Consideration of multi-product case with space limitation is another interesting extension for future study.
Bibliography


