Essays on Supplier’s Market Encroachment Strategy: Asymmetric Information, Strategic Inventory, and Sharing Economy

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ESSAYS ON SUPPLIER’S MARKET ENCROACHMENT STRATEGY: ASYMMETRIC INFORMATION, STRATEGIC INVENTORY, AND SHARING ECONOMY

By

Huiqi Guan

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ESSAYS ON SUPPLIER’S MARKET ENCROACHMENT STRATEGY:
ASYMMETRIC INFORMATION, STRATEGIC INVENTORY, AND SHARING ECONOMY

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Essays on Supplier’s Market Encroachment Strategy: Asymmetric Information, Strategic Inventory, and Sharing Economy

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The development of technology provides an opportunity for the upstream manufacturer (supplier) to adapt its traditional business model. For example, the manufacturer may use an online store as a direct selling channel to enter the downstream market, which originally belongs to the retailer (buyer). The change in consumers behavior by the technology may also influence the manufacturer’s business model redesign. For instance, consumers’ peer-to-peer product sharing may lead the manufacturer to build a product sharing platform. Nevertheless, change in business model presents new challenges to traditional manufacturers. Motivated by the need to make decisions in a new business environment, my dissertation addresses different issues for the manufacturer in each essay. The first essay studies the players’ incentives in learning the manufacturer’s direct selling cost. Our analysis shows that the manufacturer may prefer no information advantage over the retailer on its direct selling cost and the retailer may also prefer to have less information. Furthermore, even if the manufacturer and the retailer agree on the information sharing contract, the manufacturer may have no incentive to resolve the uncertainty on its direct selling cost, which indicates the bright side of uncertainty in the supply chain when the manufacturer has a direct selling channel. The second essay analyzes the interaction between a buyer and a supplier where the buyer may hold strategic inventory in expectation
of the supplier’s future direct selling. Our results show that the supplier and the buyer can be better off at the same time when considering the players’ strategies together rather than only considering one player’s strategy. The third essay examines the viability of the platform-building strategy for a premium manufacturer who tries to incorporate the new peer-to-peer product sharing paradigm in its business model. We design the new platform and find the manufacturer can always be better off by launching a platform exclusively for its premium product.
to my family
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HUIQI GUAN

University of Miami
August 2018
Preface

Chapter 2 is co-authored by my supervisor Professor Haresh Gurnani and Professor Zhibin (Ben) Yang. Chapter 3 is co-authored by Professor Yadong Luo, and my supervisors Professor Haresh Gurnani and Professor Xin Geng. Chapter 4 is an ongoing project, which is co-authored by my supervisors Professor Haresh Gurnani and Professor Xin Geng. In all three chapters, I made the main contribution, which includes identifying the topics, developing the models, carrying out the analysis and presenting the results. A version of Chapter 3 has been accepted for publication at *Manufacturing & Service Operations Management* (Guan, Gurnani, Geng, and Luo, 2017). Chapter 2 and Chapter 3 will be modified and submitted for publication in academic peer reviewed journals.
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CHAPTER 1

Introduction

Traditionally, the upstream supplier (manufacturer) only reaches the downstream market through the retailer (buyer). The barrier in the manufacturer’s direct selling can be a lack of experience in sales or the huge fixed cost incurred by the bricks-and-mortar stores. As the prosperity of e-commerce (U.S. Department of Commerce, 2016), shopping online is very popular among consumers, which stimulates a convenient channel, opening online stores, for the sellers to obtain customers. As we witness, more and more traditional manufacturers begin to sell directly through online stores, e.g. Nike, Hewlett Packard, IBM, and Apple (Tsay and Agrawal, 2004). Not as the bricks-and-mortar stores, the online stores do not incur huge fixed cost and can help the manufacturers improve sales ability by massive collection of customer data. Therefore, the manufacturer may adjust its product distribution strategy by launching these direct selling channels as the development of technology.

The manufacturer’s business model change captures new opportunity in the market, but it also presents new challenges for the manufacturer’s decision making in the new business environment. For example, due to the lack of experience in the downstream market, the manufacturer may have little information on its selling cost when it launches the direct selling channel. Whether to acquire more information on
its direct selling cost or not can be an issue for the manufacturer. After having the
cost information, whether to share that with the retailer or not can be another issue.
Besides the difficulties in making its own decision, the change in the manufacturer’s
business model can also raise strategic reaction from its supply chain partner. The di-
rect selling channel is built by the supplier to encroach the buyer’s market. With the
anticipation of the supplier’s direct selling, the buyer may also adopt some strategic
change. For example, the buyer may hold inventory to obstacle the supplier’s direct
selling. The buyer’s strategic reaction can vary the benefit of direct selling for the
manufacturer, which complicates the manufacturer’s decision.

As shown above, the manufacturer’s business model redesign can stem from self-
adjustment in distribution strategy with the technology changing. In addition, it may
also be driven by consumers behavior change. The recent advances in technology, es-
pecially internet-based platforms and algorithms, have significantly fueled the growth
of the peer-to-peer product sharing market across a wide range of sectors, such as hospi-
tality (Airbnb, CouchSurfing), retailing (SnapGoods, Tradesy) and transportation
(Uber, Lyft). In what is called sharing economy or collaborative consumption, indi-
viduals can monetize their under-utilized resources of basically all kinds. Similarly,
people can derive value from products that they do not necessarily own. As such, on
the one hand, collaborative consumption has the advantage of helping the consumers
to get access to the product without ownership concerns. Hence, consumers’ incentive
in purchasing can be diminished, which is an alarm for the manufacturer in the sharing
economy era. On the other hand, the new value of sharing the product can increase
the attractiveness of the product. Therefore, if the manufacturer makes it convenient
for sharing its product, the manufacturer’s sales may grow. For instance, some manu-
facturers have modified their product quality to facilitate peer-to-peer sharing. More active manufacturers even have considered launching the peer-to-peer product sharing platform. For example, Tesla’s CEO Elon Musk announced Tesla will launch its car sharing platform, Tesla Network. Recently, BMW also added peer-to-peer car sharing service on its car sharing platform, ReachNow. However, the manufacturer-built platform will face the competition from an existing platform. For the aforementioned examples of car sharing platform, Uber is a very strong competitor. Considering the competition from the existing platform, the validity of the platform-building strategy for the manufacturer to incorporate consumers sharing behavior is not obvious.

This dissertation is designed to examine the manufacturer’s market entry strategies via revamping its business model. It consists of three essays that support the manufacturer’s managerial decision making in different contexts by building analytical models. They include contexts where the manufacturer and the retailer may have different levels of information on the manufacturer’s direct selling ability, and others where the retailer can adopt strategic reaction in anticipation of the manufacturer’s market entry. In another case, we examine a context where a firm may enter the peer-to-peer product sharing market by building a product sharing platform. As appropriate for the context, the dissertation shows the viability of the manufacturer’s market entry strategy. It also presents the effect of the manufacturer’s business model change on the retailer, consumers and other competitors. In the following, I briefly describe the three essays.

The first essay “The Value of Information with Supplier Encroachment,” examines the players’ incentives to acquire information on the manufacturer’s direct selling cost. We consider one manufacturer and one retailer in the market, where the manufacturer
can directly sell to consumers after supplying the retailer. For this two-tier supply chain structure, we propose three models with different information structures on the manufacturer's direct selling cost. Specifically, in the *no* information model, neither the manufacturer nor the retailer has information on the manufacturer's direct selling cost. In the *asymmetric* information model, only the manufacturer but not the retailer knows the manufacturer's cost information. In the *full* information model, both the manufacturer and the retailer have information on the manufacturer's direct selling cost. Then from the no information model to the asymmetric information model, we know the manufacturer learns the cost information and can derive the manufacturer's incentive in acquiring more information on its cost by comparing these two models. From the asymmetric information model to the full information model, we know the retailer also learns the cost information. Hence, comparing these two models, we can derive the retailer's incentive in learning the manufacturer's cost information, or the manufacturer's incentive to share its private information with the retailer. Our analysis shows the manufacturer can be better off after having more information on the direct selling cost. However, it may also be worse off. Meanwhile, the retailer can be better off. When the channel profit increases, the retailer may have incentive to pay side payments to induce the manufacturer's exploration on its direct selling cost. By sharing the cost information with the retailer, the manufacturer can always be weakly better off while the retailer will be weakly worse off. So the retailer may have no incentive to learn the manufacturer's direct selling cost. When the total channel profit increases, the manufacturer can pay side payments to the retailer for sharing its cost information. Combining the above two comparisons, we can derive the players' profit changes from the no information model to the full information model. We find
the manufacturer and the retailer may both be worse off after the uncertainty of the manufacturer’s direct selling cost is resolved. In other words, if the retailer agrees to share the cost information in advance, the manufacturer and the retailer can both be worse off after the manufacturer uncovers its direct selling cost. Hence, some level of uncertainty on the manufacturer’s direct selling cost in the supply chain can benefit the supply chain members.

The second essay “Strategic inventory and Supplier Encroachment,” focuses on the interaction between the supplier’s and the buyer’s strategies. Specifically, in a two-period model of a dyadic supply chain, we study the interaction between the use of strategic inventory withholding by the buyer and the use of a direct selling channel (encroachment) by the supplier in the second period. While the extant OM literature has individually examined the two strategies, the system-wide combined effect of these strategies has not been studied. We fill the gap by building, and analyzing, two decentralized models of vertical competition. The main model focuses on sequential quantity decisions where the buyer decides on its order quantity and sets its selling quantity, with a first-mover advantage, before the supplier decides on its direct selling quantity. In an extension, we also consider an alternative timing structure of simultaneous quantity competition. For the sequential model, we find that for any finite holding cost, the buyer may withhold strategic inventory in certain cases. This makes the supplier less aggressive in using its direct selling strategy compared to the case when strategic inventory is not an option. Moreover, there exist situations where both the supplier and the buyer make higher profits from the combined use of their respective strategies at the same time. For the simultaneous model, the buyer can achieve higher profit than in the sequential model in certain
scenarios. However, the supplier’s profit may drop below the benchmark case (when neither strategic inventory nor direct selling is present), which is not seen in the sequential model. As such, the buyer can benefit from losing its first-mover advantage whereas the supplier may be worse off. Our study provides useful managerial insights into the strategic (joint) moves of the players in a supply chain. We show that both the supplier and the buyer can benefit from vertical competition. Moreover, the first-mover advantage may not always increase profits for the players.

The third dissertation essay “Peer-to-Peer Sharing Platforms with Quality Differentiation: Manufacturer’s Strategic Decision under Sharing Economy,” studies the manufacturer’s strategies in the new sharing economy paradigm. By comparing the manufacturer’s profits without and with platform-building strategy when facing an existing product sharing platform, we derive the validity of the manufacturer’s platform-building strategy. In other words, our results indicate that the manufacturer can always get larger profit after having an exclusive platform for its premium product. However, if considering the platform building cost, the platform-building strategy can only be profitable for the manufacturer if the quality differentiation between the premium and non-premium product is in the medium range. We also find the manufacturer-built platform may help the manufacturer to encroach the non-premium manufacturers’, i.e. competitors’ market, which indicates the competition among firms can be intensified after the emergence of peer-to-peer product sharing.
CHAPTER 2

The Value of Information with Supplier Encroachment

2.1 Overview

An upstream supplier with a direct selling channel can sell directly to consumers (in addition to selling via a buyer) thereby encroaching on the buyer’s market. For instance, more and more traditional manufacturers begin to sell directly through online stores, e.g. Nike, Hewlett Packard, IBM, and Apple (Tsay and Agrawal, 2004). For this phenomenon, we refer to “supplier encroachment.”

After adding a direct selling channel, the manufacturer can earn profit not only from wholesaling to the retailer, but also from direct selling to consumers. Hence, the direct channel diversifies the manufacturer’s profit source, which adds more power for the manufacturer to control the supply chain. The success of direct selling has been shown by Apple’s sales in smart phones (Bajarin, 2012). Besides extra profit source, direct selling channel may also add potential risk in the supply chain. Nevertheless, the direct selling channel is used by the manufacturer to encroach its retailer’s market, which may increase competition in the downstream market and call resistance from the manufacturer’s retailer. For example, when Nike opened a Niketown store in
downtown Chicago, the retailers carrying Nike products treated it as a serious threat (Collinger, 1998). A more extreme example is for Home Depot, who sent letters to its suppliers to announce its reaction to supplier encroachment before any direct channel initiatives (Brooker, 1999). As indicated by these examples, the manufacturer should be cautious when launching a direct selling channel. Otherwise, the fate of the direct channel can be shut down. Sony announced to close its online store in America in 2015 (Sony, 2016). Now the consumers need to turn to Sony’s participating retailers when they want to purchase Sony’s products online in USA. Two articles from Wall Street Journal, which show the difficulty in operating an online store, may explain the failure of direct selling. Kapner (2014) finds the operational cost for the online stores is higher than bricks-and-mortar retailers and this cost can run as high as 25% of sales. Recently, Whelan (2016) reports that shipping expenses for some online orders can far exceed price of the product. Motivated by Kapner (2014) and Whelan (2016), we conceive the direct selling cost may be a changer for manufacturer’s direct selling strategy. Hence, the first thing we examine is to find the manufacturer’s optimal direct selling strategy based on the manufacturer’s direct selling cost.

As a new entrant to the downstream market, due to the lack of sales experience, the manufacturer may have little information on its direct selling cost. If more information is available, the manufacturer can tailor its direct selling strategy appropriately. Hence, one may think the manufacturer should benefit from uncovering its direct selling cost. However, the manufacturer’s information advantage may alter the retailer’s order incentive and then change the profit from wholesaling. After the manufacturer learns its direct selling cost, on one hand, it can keep it privately and obtain information advantage over the retailer. With information disadvantage, however, the
retailer may make rational inference from the manufacturer’s wholesale price and then a signaling game between the manufacturer and the retailer will arise. In order to signal its high cost information to the retailer, the manufacturer may need to distort its wholesale price downward. Considering this, how will the manufacturer’s incentive in information acquisition be changed? On the other hand, the manufacturer may share the cost information with the retailer, which eliminates the information asymmetry between the two players. It seems the retailer can benefit since it can adjust its order quantity based on the manufacturer’s direct channel strategy. However, the increased information for the retailer also removes the restriction on the wholesale price for the privately informed manufacturer. In other words, the manufacturer can choose its wholesale price more flexibly if the retailer has the same amount of information as it on its direct selling cost. Therefore, the retailer may have no incentive to share the manufacturer’s information. In this project, we will examine the players’ incentives in learning the manufacturer’s direct selling cost. Specifically, we want to answer the following research questions:

1. Does the manufacturer have incentive to learn its direct selling cost?
   
   (1a) Can the manufacturer be better off with more information?
   
   (1b) If the manufacturer cannot be better off, will the retailer be better off? How about the channel profit?

2. Should the manufacturer invest in alternative mechanism (if feasible) for credible information sharing with the retailer by contract?
(2a) Can the manufacturer be better off with information sharing?

(2b) If the manufacturer could be better off, will the retailer be better off? How about the channel profit?

In order to answer these research questions, we consider three information structures on the manufacturer’s direct selling cost, i.e. the full information model, the asymmetric information model, and the no information model. In the full information model, both the manufacturer and the retailer know the manufacturer’s direct selling cost, while in the asymmetric information model, only the manufacturer but not the retailer knows the manufacturer’s direct selling cost. In the no information model, neither the manufacturer nor the retailer knows the manufacturer’s direct selling cost. By comparing the players’ profits and channel profit in different models, we can answer our research questions. Specifically, from the no information model to the asymmetric information model, we know the manufacturer learns its direct selling cost and then we can show the manufacturer’s incentive to acquire information on its direct selling cost by comparing the results in these two models. From the asymmetric information model to the full information model, we know the retailer also learns the manufacturer’s direct selling cost and then we can derive the manufacturer’s incentive to share information with the retailer by showing its profit change. Hence, our paper will shed light on the value of information on the manufacturer’s direct selling cost under the supplier encroachment setting. Our main findings are as follows:

1. Information advantage can benefit the manufacturer.

2. The manufacturer’s gain needs not erode retailer’s gain from the manufacturer’s learning. We find that it can be “win-win” for the manufacturer and the retailer.
3. Interestingly, information advantage may also backfire on the manufacturer, but benefit the retailer and the total supply chain, i.e. we find “lose-win-win”. Thus, the retailer may have incentive to pay side payments to the manufacturer for investigating the manufacturer’s direct selling cost.

4. However, the manufacturer’s information acquisition can add no benefit for the supply chain, i.e., we find “lose-lose-lose” for the manufacturer, the retailer and the total supply chain.

5. The manufacturer with extra direct selling cost always benefits from sharing information with the retailer. This implies that having information advantage is a burden for the manufacturer and then the manufacturer has incentive to share information.

6. However, sharing information does not benefit the retailer. Therefore, information sharing may not be feasible, if it involves voluntary effort from both the manufacturer and the retailer. But the channel profit may increase and then the manufacturer can pay the retailer side payments to learn its cost information.

In the no information model, the manufacturer is uninformative about its cost information, while it knows its cost information perfectly in the asymmetric information model. With more information, the manufacturer can implement more appropriate direct selling strategy to maximize its profit. Hence, the manufacturer can get better off with more information. However, we also find the manufacturer can be worse off after knowing more on its direct selling cost, which indicates the manufacturer may not have incentive to conduct information acquisition. The information disadvantage can render the retailer more concern on the direct channel threat, which diminishes
the retailer’s order incentive. Hence, the manufacturer may need to downward distort its wholesale price. If the price distortions is so severe, the manufacturer can be worse off. Thanks to the manufacturer’s price concession, the retailer can be better off and sometimes the total supply chain profit will also increase. Therefore, the retailer may have incentive to pay side payments to the manufacturer to learn the direct selling cost. It should be cautious that the manufacturer’s learning may hurt both players, which indicates the bright side of uncertainty in the supply chain.

If the manufacturer shares its private information with the retailer, then both the manufacturer and the retailer know the manufacturer’s direct selling cost and we get the full information model. In the full information model, the manufacturer will not suffer from price distortion due to the retailer’s inference, but can choose wholesale price flexibly. Therefore, the manufacturer can benefit from downward information transmission, but the retailer may be worse off. When the channel profit is higher, the manufacturer can pay the retailer side payments to achieve information sharing contract.

As we shown above, information acquisition may result the manufacturer having lower profit, but information sharing can help the manufacturer to obtain higher profit. So if the information sharing with the retailer is always possible, will the supplier have incentive to learn its cost information? We find the supplier may still be reluctant to gather more information. Furthermore, the manufacturer and the retailer can all be worse off. Thus, our results indicate some level of uncertainty remaining in the supply chain may benefit everyone (Christen, 2005).

The reminder of our paper is organized as follows. In § 2.2, we review the literature and describe the model setting in § 2.3. We extend the supplier encroachment
model without restriction on the manufacturer’s direct channel strategy under full information in § 2.4. In § 2.5, we analyze the asymmetric information model. In § 2.6, we add results for the no information model and examine the role of information structure on the supply chain performance. We’ll conclude in § 2.7.

2.2 Literature Review

The literature on supplier encroachment mainly focuses on the situation when the manufacturer actually sells through the direct channel, e.g. Arya et al. (2007), Li et al. (2013) and Cao et al. (2013). One exception is Chiang et al. (2003), in which the manufacturer will sell nothing through the direct channel but can achieve higher profit by the direct selling threat imposed on the retailer. However, in their paper, the manufacturer can never sell positive quantity through the direct channel in the equilibrium. We extend this stream of literature on supplier encroachment by considering a Cournot competition model as in Arya et al. (2007) but without restricting the manufacturer’s direct channel strategy. Under full information setting, we find the manufacturer will never be worse off by launching a direct channel in the extended model, while the retailer and consumers can be better off in a wider range.

The supplier encroachment model with asymmetric information has been considered in Cao et al. (2013), Li et al. (2013, 2015) and Liao (2014). Li et al. (2013) consider the situation when the retailer will use the order quantity to signal its private demand information. Cao et al. (2013) use nonlinear price contracts to screen the retailer’s private selling cost information. Li et al. (2015) also consider the nonlinear price contracts, but to screen the retailer’s private demand information. The closest work to us is Liao (2014). But Liao (2014) restricts the manufacturer’s direct
channel strategy to when the manufacturer actually sells through the direct channel. In our asymmetric information model, we have no restriction on the manufacturer’s direct channel strategy.

In the literature, researchers have also considered the upstream firm having private information, especially on demand (Chu, 1992; Desai, 2000; Guo and Iyer, 2010; Jiang et al., 2016; Lariviere and Padmanabhan, 1997). In Guo and Iyer (2010), the manufacturer can acquire the information on the consumers’ preference. They find the manufacturer may not prefer perfectly accurate information even if information acquisition is costless. For the manufacturer’s incentive to share information, it depends on the market parameters. In their paper, however, the retailer cannot rationally infer information from the manufacturer’s wholesale price. When wholesale price is treated as a signal of the manufacturer’s demand information, Jiang et al. (2016) show that the manufacturer may downward distort the wholesale price to convey a low forecast to the retailer. This wholesale price distortion may result in the manufacturer having lower profit but the retailer having higher profit, compared to the full information case when the manufacturer shares the information with the retailer. They also show more accurate information may hurt both firms if information sharing is impossible. Some other researchers (Dukes et al., 2011, 2017; He et al., 2008) also consider the situation when both the manufacturer and the retailer have private demand information. In He et al. (2008), they show information sharing may not always increase the channel profit. As in Guo and Iyer (2010), they assume the retailer cannot derive any information from the manufacturer’s wholesale price. When the wholesale price is a signal, as in Jiang et al. (2016), Dukes et al. (2017) also find the manufacturer may downward distort the wholesale price to transmit a low demand forecast to the
retailer, which causes information sharing may only benefit the manufacturer at the cost of the retailer and consumers. Gal-Or et al. (2008) consider two competing retailers and show that the more informed manufacturer may only share information with the less informed retailer.

Our paper differs from the aforementioned literature in the following ways. First, our model focuses on the manufacturer’s direct selling cost. Second, in our model, the manufacturer is also the retailer’s potential competitor in the downstream market. This dual channel structure makes it possible for the manufacturer to signal its cost information to the retailer through the wholesale price. Third, we study not only the manufacturer’s incentive to acquire the cost information, but also the effect of the information sharing on the supply chain.

2.3 Model Setting

We consider a supply chain with a manufacturer (supplier, she) and a retailer (buyer, he). The manufacturer distributes a product to consumers in the end market through the retailer. In addition, the manufacturer has a direct channel for direct access to consumers. We assume that the products distributed via these two channels are perfect substitutes from the consumer’s perspective.

The manufacturer and retailer’s actions are conceptualized into three stages. In stage 1, the manufacturer quotes a take-it-or-leave-it wholesale price, \( w \), to the retailer. In stage 2, the retailer orders the quantity of \( q_r \) from the manufacturer. In stage 3, the manufacturer and the retailer simultaneously decide their respective selling quantities, \( q_m \) and \( \tilde{q}_r \), and sell them to the downstream consumer market. We assume that the market-clearing price, denoted as \( p \), is linearly decreasing in the total
quantity sold in the market: \( p = \alpha - q_m - \bar{q}_r \), where \( \alpha \) the highest possible market price. We normalize the retailer’s per-unit selling cost to be zero. The retailer’s total profit is:

\[
\Pi_r(w, q_r, \bar{q}_r, q_m) = (\alpha - q_m - \bar{q}_r)q_r - wq_r. \tag{2.1}
\]

The manufacturer incurs a selling cost of \( s > 0 \) per unit. This reflects the situation where the manufacturer is more costly than the retailer in selling, for example, because the manufacturer is a new entrant to the downstream market and is inexperienced in dealing with consumers or investing in e-commerce (e.g., see The Economist 2013 and Kapner 2014). To further simplify the model, without loss of generality, we normalize the manufacturer’s production cost to be zero per unit. The manufacturer’s total profit is:

\[
\Pi_m(w, q_r, \bar{q}_r, q_m) = (\alpha - q_m - \bar{q}_r - s)q_m + wq_r. \tag{2.2}
\]

Depending on whether the manufacturer and the retailer know the manufacturer’s selling cost, \( s \), we will consider three scenarios of information structure: full information, asymmetric information, and no information. We first explore the full information model in § 2.4 and then analyze the asymmetric information model in § 2.5 and no information model in § 2.6.

### 2.4 Full Information Model

In this section, we analyze the model in which all information is public, including the manufacturer’s direct selling cost per unit, \( s \). Our model setting is similar to that of Arya et al. (2007) after replacing \( a, c \) and \( b \) in Arya et al. (2007) with \( \alpha, s \) and 1 in our model. There are two key differences, however. First, Arya et al. (2007) conduct
analysis with the manufacturer’s selling cost $s < \frac{3\alpha}{5}$. We consider all $s \geq 0$. Second, Arya et al. (2007) assume that the retailer will sell all supply to the market, which in our notation is $\tilde{q}_r \equiv q_r$. Our model allows the retailer to withhold some supply from selling to the market, that is, $\tilde{q}_r \leq q_r$. Therefore, our model is more general and allows us to derive new insights for $s \geq \frac{3\alpha}{5}$.

We first solve this game of perfect information backwards for sub-game perfect equilibrium, starting from stage 3. We then analyze the effect of having a direct channel on the manufacturer’s product distribution, the profits of the supply chain firms and the consumer surplus. The results of this section will also be used as a benchmark to the models with asymmetric information and no information in § 2.5 and 2.6.

### 2.4.1 Model Analysis

In stage 3, the manufacturer and the retailer simultaneously choose their selling quantities, $q_m$ and $\tilde{q}_r$, given that the retailer has received the supply of $q_r$ units at the price of $w$. We solve for Nash equilibrium of the game. The manufacturer and the retailer’s decisions are represented by the following joint optimization programs:

$$
\begin{align*}
\max_{q_m \geq 0} & \quad \Pi_m(\tilde{q}_r, q_m | w, q_r) \\
\max_{0 \leq \tilde{q}_r \leq q_r} & \quad \Pi_r(\tilde{q}_r, q_m | w, q_r).
\end{align*}
$$

Note from equations (2.1) and (2.2) that the wholesale price $w$ is irrelevant in the retailer’s and the manufacturer’s decision making with respect to $\tilde{q}_r$ and $q_m$. We let $\tilde{q}^*_r(q_r)$ and $q^*_m(q_r)$ be the retailer’s and manufacturer’s equilibrium strategies at

---

1We prove that in equilibrium the retailer does sell $\tilde{q}_r = q_r$. However, buyer withholding does happen in real life and we refer to Yang et al. (2017) for more practical examples.
given \( q_r \) and denote the manufacturer’s and the retailer’s profits at the equilibrium as 
\[
\Pi_m(w, q_r) = \Pi_m \left( w, q_r, \bar{q}_r^*(q_r), q_m^*(q_r) \right) \quad \text{and} \quad \Pi_r(w, q_r) = \Pi_r \left( w, q_r, \bar{q}_r^*(q_r), q_m^*(q_r) \right). 
\]

We relegate the analysis and result for stage 3 to Lemma A.1 in the appendix.

In stage 2, having received the wholesale price \( w \) and anticipating the selling quantities in stage 3 to be \( \bar{q}_r^*(q_r) \) and \( q_m^*(q_r) \), the retailer decides his order quantity, \( q_r \), to maximize his profit: 
\[
\max_{q_r \geq 0} \Pi_r(w, q_r).
\]
We let \( q_r^*(w) \) denote the retailer’s optimal order quantity at given \( w \), and relegate the analysis and result of stage 2 to Lemma A.2 in the appendix. A key observation, from Lemmas A.1 and A.2 combined, is that at the optimal order quantity \( q_r = q_r^*(w) \), the retailer’s equilibrium selling quantity in stage 3, \( \bar{q}_r^*(q_r^*(w)) \), equals \( q_r^*(w) \). In words, the retailer always sell off all supply to the consumer market. So, in the rest of analysis we will refer to \( q_r \) as both the order and selling quantity of the retailer and discard the notation of \( \bar{q}_r \).

In stage 1, the manufacturer chooses the wholesale price \( w \) to maximize her profit, in anticipation of the retailer’s optimal order quantity \( q_r^*(w) \) in stage 2 and the equilibrium selling quantities \( \bar{q}_r^*(q_r^*(w)) \) and \( q_m^*(q_r^*(w)) \) in stage 3. The manufacturer’s price decision is represented by the following program:

\[
\max_{w \geq 0} \Pi_m(w, q_r^*(w)).
\]

We solve this program for the optimal \( w \) and apply it to the equilibrium outcomes in stages 2 and 3, deriving the retailer’s order quantity and the manufacturer’s selling quantity in equilibrium. In this process, we also derive the profit of the manufacturer, the retailer and the channel and the consumer surplus, which has the expression of 
\[
\Gamma = \frac{(q_m + q_r)^2}{2}. 
\]
The results are presented in Proposition 1.

**Proposition 1** Under perfect information, in equilibrium, the wholesale price, \( w^* \), the retailer’s order quantity, \( q_r^* \), the manufacturer’s direct-selling quantity, \( q_m^* \), the
manufacturer’s, retailer’s and channel’s profits, $\Pi^*_m$, $\Pi^*_r$ and $\Pi^*_c$, and the consumer surplus, $\Gamma^*$, are as follows:

<table>
<thead>
<tr>
<th>$s$ satisfies</th>
<th>$w^*$</th>
<th>$q^*_r$</th>
<th>$q^*_m$</th>
<th>$\Pi^*_m$</th>
<th>$\Pi^*_r$</th>
<th>$\Gamma^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s &lt; \frac{3\alpha}{5}$</td>
<td>$\frac{3\alpha-s}{6}$</td>
<td>$\frac{3\alpha-s}{2}$</td>
<td>$\frac{s}{2}$</td>
<td>$\frac{-7s+7s^2-6\alpha s}{6}$</td>
<td>$\frac{2s}{2}$</td>
<td>$\frac{\alpha^2}{8}$</td>
</tr>
<tr>
<td>$\frac{3\alpha}{5} \leq s &lt; \frac{5\alpha}{6}$</td>
<td>$\frac{3\alpha-5s}{6}$</td>
<td>$\alpha-s$</td>
<td>$\frac{\alpha}{4}$</td>
<td>$\frac{-3s+\alpha}{2}$</td>
<td>$\frac{(\alpha-s)^2}{2}$</td>
<td>$\frac{\alpha^2}{4}$</td>
</tr>
<tr>
<td>$\frac{5\alpha}{6} \leq s$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

where Channel Profit $\Pi_c = \Pi_m + \Pi_r$ and Consumer Surplus $\Gamma = \frac{(q_m+q_r)^2}{2}$.

We relegate all proofs to the appendix.

### 2.4.2 Effect of Direct Channel

We now examine how the direct channel affects the manufacturer’s and retailer’s decisions. Proposition 1 shows that as the manufacturer’s selling cost $s$ increases, not surprisingly, the manufacturer decreases her direct selling quantity in the market.

When the selling cost is low with $s < \frac{3\alpha}{5}$, as Arya et al. (2007) has found, the manufacture sells strict positive quantity $q^*_m = \frac{3\alpha-5s}{6}$ to the market, encroaching the retailer’s market share. In this paper, we will refer to this role of the direct channel as “encroachment”. When the selling cost is high with $s \geq \frac{3\alpha}{5}$, from Proposition 1 we find that the manufacturer sells $q^*_m = 0$, completely relying on the retailer for selling to the market.

This observation may suggest that the direct channel is irrelevant when the manufacturer’s selling cost is high with $s \geq \frac{3\alpha}{5}$. To verify this notion, we analyze effect of direct channel under $s \geq \frac{3\alpha}{5}$. We compare the equilibrium result under $s \geq \frac{3\alpha}{5}$ in Proposition 1 to the benchmark model without direct channel. This benchmark
model is attainable by applying $s = \infty$ to Proposition 1. In fact, from Proposition 1 for all $s \geq \frac{5\alpha}{6}$ the direct channel is so costly that the model degenerates to be the benchmark model without direct channel. The direct channel truly has no effect under such high selling cost. Thus, it suffices to focus on effect of the direct channel for $\frac{3\alpha}{5} \leq s < \frac{5\alpha}{6}$.

Under $\frac{3\alpha}{5} \leq s < \frac{5\alpha}{6}$, the retailer orders $q_r^* = \alpha - s$ with the direct channel, but orders $\frac{\alpha}{4}$ without direct channel. While the manufacturer does not actually exercise the direct channel, its mere presence imposes a threat of encroachment on the retailer. We refer to this role of the direct channel under $\frac{3\alpha}{5} \leq s < \frac{5\alpha}{6}$ as “threat”, which is absent from Arya et al. (2007). In the following, we define two regions of threat, one strong and one weak. Under $\frac{3\alpha}{5} \leq s < \frac{3\alpha}{4}$, we have $q_r^* > \frac{\alpha}{4}$, that is, the retailer orders more due to the “threat” imposed by the direct channel. Thus, we say the threat in the region with $\frac{3\alpha}{5} \leq s < \frac{3\alpha}{4}$ is strong. To dismiss this “threat”, the retailer increases its order quantity to be such that the manufacturer is assured that she would not benefit from entering a heated market competition. Associated with strong direct channel threat, the manufacturer will set the wholesale price $w^*$ as an increasing function of $s$, but it can be less than the benchmark price $\frac{\alpha}{2}$ in the model without the direct channel. Interestingly, the threat does not always lead to larger order quantity of the retailer. When the selling cost is sufficiently high with $\frac{3\alpha}{4} \leq s < \frac{5\alpha}{6}$, the retailer orders $q_r^* \leq \frac{\alpha}{4}$ and then we say the threat in this region is weak. As the selling cost increases, the strength of the threat peters out. So does the manufacturer’s ability to extract a rent from the retailer. The manufacturer pushes up the wholesale price, causing the retailer’s order quantity to dip drastically. This also indicates in the weak
threat region, the manufacturer needs to rely on the high wholesale price to sustain the direct channel threat on the retailer.

As such, our model captures two roles of the direct channel—encroachment and threat—with one unified framework. Using this framework, we proceed to explore the effect of direct channel on the supply chain firms’ performance.

Arya et al. (2007) make an exciting finding for $\frac{3\alpha}{4\sqrt{2}} \leq s < \frac{3\alpha}{5}$, under which the direct channel is used for “encroachment”, that the retailer, the manufacturer, and consumers are better off at the same time than the case without direct channel. That is, “encroachment” is “win-win-win.” We confirm this result with our model, but also extend the analysis to high range of selling cost $s$, as presented in Corollary 1:

**Corollary 1** Under $\frac{3\alpha}{4\sqrt{2}} \leq s < (1 - \frac{1}{2\sqrt{2}})\alpha$, the manufacturer’s and the retailer’s profit and the consumer surplus, respectively, are greater than those in the model without direct channel.

Let’s focus on the interval of $\frac{3\alpha}{5} \leq s < (1 - \frac{1}{2\sqrt{2}})\alpha$, in which the direct channel presents as a “threat” to the retailer and pushes him to increase the order size. It follows from Corollary 1 that for such $s$ having the direct channel is also “win-win-win” for the manufacturer, the retailer and consumers. To explain the “win-win-win” outcome for $\frac{3\alpha}{4\sqrt{2}} \leq s < \frac{3\alpha}{5}$, Arya et al. (2007) develop a key insight: the encroaching manufacturer has an incentive to lower the wholesale price, which pushes up the retailer’s order; as a result, both the manufacturer and the retail enjoy a benefit. With our model, we show that under $\frac{3\alpha}{5} \leq s < (1 - \frac{1}{2\sqrt{2}})\alpha$, the manufacturer also charges a lower wholesale price, and the retailer orders more than without direct channel. With that,

---

$^2$Using a Bertrand competition model, Chiang et al. (2003) also show that the manufacturer may use the direct channel as a threat of encroachment for strategic channel control. However, in their model, the manufacturer never sells directly.
we find that Arya et al. (2007)’s insight holds even if the direct channel is too costly to be useful for encroachment, but only for “threat.” Thus, our result expands the “win-win-win” finding of Arya et al. (2007) to where the direct channel is used as a threat.

In § 2.5, we will compare the results in this model to the model with asymmetric information. To prepare for discussion there, we summarize the effect of the direct channel in Figure 2.1.

![Figure 2.1: The effect of the direct channel in the full information model.](image)

Note: (1) The direct channel effect on the manufacturer’s profit, the retailer’s profit, and the consumer surplus is denoted by the three letters in the triplet in order. (2) Letter “W” denotes an increase, letter “L” denotes a decrease, and “U” denotes no change. (3) In this illustration, we set \( \alpha = 1 \).

### 2.5 Asymmetric Information Model

In the full information model, we assume the manufacturer will incur extra selling cost for every unit sold to the consumers directly. It’s possible that the manufacturer may not have such disadvantage in her direct channel. For example, after analyzing the big data collected from the online sales, the manufacturer may understand the consumers behaviors better than the retailer. The extra information from the direct channel can help the manufacturer to recommend much more suitable product to the consumers and then reduce the return rate, which may drastically reduce the direct selling cost. Hence, the manufacturer’s direct selling can also be as efficient
as the retailer. Nevertheless, whether the manufacturer is an efficient or inefficient seller in the downstream market may be the manufacturer’s private information. In other words, the manufacturer’s actual direct selling cost can be known only by the manufacturer herself, but not the retailer. In this section, we will examine how the information asymmetry on the manufacturer’s direct selling cost can affect the direct channel effect derived under full information setting. Specifically, we first present the model setting and the equilibrium results for the asymmetric information model. Then, we discuss how the information asymmetry can change the manufacturer’s direct selling strategy. Finally, we will show the change of the effect of direct channel on the manufacturer’s and retailer’s profits, and consumer surplus due to information asymmetry.

2.5.1 Model Formulation and Equilibrium Outcome

As we mentioned before, the manufacturer can be less efficient than, or as efficient as the retailer in the downstream selling. Thus, we assume the manufacturer’s direct selling cost \( s \) is random, which can be \( s_m (>0) \) or \( 0 \), with probability \( \lambda \) and \( 1 - \lambda \), respectively. To facilitate our analysis, we denote the manufacturer with \( s = s_m \) as \( H \)-type and \( s = 0 \) as \( L \)-type. The retailer only knows the manufacturer’s ex-ante type information, while the manufacturer knows the true value of \( s \). After the nature reveals \( s \) to the manufacturer, the following stages of this asymmetric information model are the same as those in the direct channel model in § 2.3. Because the manufacturer chooses her wholesale price \( w \) after knowing her direct selling cost, \( w \) can reflect her information on \( s \). Thus, the retailer can make rational inference about
the manufacturer’s true direct selling cost based on the announced wholesale price. Then, a signaling game between the manufacturer and the retailer arises.

With the direct selling channel, the manufacturer can leverage her profit from two sources: One is wholesaling to the retailer and the other is direct selling to the consumers. However, the profitability of each source is oppositely affected by the retailer’s order quantity. On the one hand, if the retailer’s ordering quantity is large, it can benefit the manufacturer’s wholesale channel but impede the direct selling. On the other hand, if the retailer’s ordering quantity is small, it can harm the manufacturer’s wholesale channel but promote the direct selling. Hence, the manufacturer’s preference over the retailer’s order quantity depends on the comparison between the profit margins in these two channels. If the marginal in the wholesale channel, i.e. the wholesale price $w$, is relatively high, the manufacturer desires more order from the retailer. If the wholesale price $w$ is relatively low, the manufacturer may want to obstacle the retailer’s order but rely on her direct selling channel. Note that the manufacturer is the monopoly supplier in the market and high wholesale price is a natural tool to limit the retailer’s order incentive. Therefore, the manufacturer will never offer such low wholesale price in our model. Hereinafter, we only consider these high $w$ such that the manufacturer will always prefer more order from the retailer.

For any such $w$, the retailer’s order quantity is lower with the $L$-type manufacturer. Hence, the $H$-type manufacturer will have no incentive to mimic as the $L$-type, but not vice versa. By contrast, the $H$-type manufacturer will have an incentive to separate herself so that the retailer can order a correspondingly large quantity. To separate herself from the $L$-type, the $H$-type manufacturer must downward distort her wholesale price until the $L$-type manufacturer finds it more profitable to charge
a high wholesale price rather than mimicking the $H$-type manufacturer. The $H$-type manufacturer weights the trade-off between the signaling cost, i.e. downward distortion in her wholesale price, and the benefit of separating, i.e. larger order quantity from the retailer, by using an appropriately low wholesale price that is in accord with the high expected direct selling cost. When the market conditions (parameters) are such that the required signaling cost to separate is relatively low, the $H$-type manufacturer will separate from the $L$-type. Otherwise, it will give up signaling her high cost and charge a higher, pooling wholesale price, which gives the manufacturer a higher per-unit profit margin at the expense of lowered unit sales due to the retailer’s lower direct selling cost expectation than in the separating case (where the retailer can infer the manufacturer’s high cost). From the $L$-type manufacturer’s perspective, the benefit of pooling (mimicking the $H$-type) is to induce the retailer to set a higher order quantity, and the cost of mimicking is a lower profit margin. If the $H$-type manufacturer’s wholesale price is too low, the $L$-type manufacturer will decide to charge its high, first-best wholesale price (in essence confirming her type) at the expense of reduced unit sales due to the retailer’s lower ordering quantity.

There are two kinds of equilibria in this kind of signaling game. One is separating equilibrium and the other is pooling equilibrium. Under our setting, in the separating equilibrium, the $H$-type manufacturer will signal its high cost by significantly reducing her wholesale price (from its first-best level) to prevent profitable mimicking by the $L$-type manufacturer, which will choose the first-best wholesale price. Under pooling equilibrium, both types of manufacturers will choose the same wholesale price. Thus, the retailer’s belief on the manufacturer’s type information will be unchanged in the pooling equilibrium.
A proper equilibrium concept for our setting is the perfect Bayesian equilibrium (PBE), for example, Fudenberg and Tirole (1991). Since the concept of PBE does not place many restrictions on the off-equilibrium beliefs of the players, it is typical to find multiple equilibria in the analysis of signaling games. In order to eliminate “unreasonable” beliefs, a fairly substantial literature has been on the refinement of PBE. The equilibrium concept that we work with is PBE along with “undefeated equilibrium” proposed by Mailath et al. (1993), in which the equilibrium with the highest profit for the player with the incentive to signal its type information will be selected. In our setting, only the \(H\)-type manufacturer has incentive to signal its private information. Thus, in the possible separating (pooling) equilibrium, only the one with highest profit for the \(H\)-type manufacturer will be selected. Next, we will compare the separating equilibrium and the pooling equilibrium when they both are possible. If the \(H\)-type manufacturer has higher profit in the separating (pooling) equilibrium than in the pooling (separating) equilibrium, then the separating (pooling) equilibrium will be selected.\(^3\) The equilibrium outcomes are formally shown in the following lemma. An illustration with \(\alpha = 1\) is in Figure 2.2.

**Lemma 1** In the asymmetric information model, a unique “undefeated equilibrium” outcome exists, and

1. if \((\lambda, s_m) \in \{((\lambda, s_m))|\lambda < \frac{3}{5}\} \cup \{((\lambda, s_m))|\frac{3}{5} \leq \lambda, s_m < s_1\} \cup \{((\lambda, s_m))|\frac{3}{5} \leq \lambda < \frac{47}{50}, s_m > s_2\}\), we have separating equilibrium. The \(L\)-type manufacturer will

\(^3\)If the \(H\)-type manufacturer’s profit is the same in two different equilibria, the equilibrium with higher \(L\)-type manufacturer’s profit will be selected.
Figure 2.2: Equilibrium outcome in \((\lambda, s_m)\) space with \(\alpha = 1\).

choose \(w(L) = \frac{\alpha}{2}\) and the retailer will choose \(q_r(L) = 0\). The \(H\)-type will choose

\[
w(H) = \begin{cases} 
\frac{3\alpha}{\lambda} & \text{if } \frac{5\alpha}{7} < s_m, \\
\frac{\alpha + s_m}{4} & \text{if } \frac{3\alpha}{5} \leq s_m \leq \frac{5\alpha}{7}, \\
\frac{3\alpha - s_m}{6} & \text{if } s_m < \frac{3\alpha}{5},
\end{cases}
\]

and the retailer will order

\[
q_r(H) = \begin{cases} 
\frac{2\alpha}{\lambda} & \text{if } \frac{5\alpha}{7} < s_m, \\
\alpha - s_m & \text{if } \frac{3\alpha}{5} \leq s_m \leq \frac{5\alpha}{7}, \\
\frac{2s_m}{3} & \text{if } s_m < \frac{3\alpha}{5},
\end{cases}
\]

2. if \((\lambda, s_m) \in \{((\lambda, s_m)) \mid \frac{3}{5} \leq \lambda < \frac{47}{49}, s_1 \leq s_m \leq s_2\} \cup \{((\lambda, s_m)) \mid \frac{47}{49} \leq \lambda, s_m \leq s_3\} \cup \{((\lambda, s_m)) \mid \frac{47}{49} \leq \lambda, s_m > s_3\}\right\}, we have pooling equilibrium. The manufacturer will pool at \(\bar{w} = \frac{(1+\lambda)\alpha}{4}\) and the retailer will order \(\bar{q}_r = \frac{\alpha}{4}\) if \((\lambda, s_m) \in \{((\lambda, s_m)) \mid \frac{47}{49} \leq \lambda, s_m > s_3\}\}. Otherwise, the manufacturer will pool at \(\bar{w} = \frac{(2+\lambda)s_m - \alpha}{2}\) and the retailer will order \(\bar{q}_r = \alpha - s_m\). Here, \(s_1 = \frac{3\alpha}{3+2\lambda}, s_2 = \frac{\alpha(\sqrt{49\lambda^2+50\lambda-47+7\lambda+21})}{14(\lambda+2)}, s_3 = \frac{\alpha(\sqrt{2}\sqrt{\lambda(\lambda+1)+2\lambda+6})}{4\lambda+8}\).
The selected equilibrium is very intuitive. When $\lambda$ is small, i.e. $\lambda < \frac{3}{5}$, the expected manufacturer’s direct selling cost, $\lambda s_m$, is also small, which diminishes the retailer’s order incentive. However, the manufacturer needs to be very efficient in direct selling to take over the market left by the retailer. Hence, the $H$-type manufacturer will not pool at any wholesale price, but may downward distort her wholesale price to signal her type information and rely more on the wholesaling to the retailer.

When $\lambda$ is large but $s_m$ is small, $\lambda s_m$ can also be small. Using the same argument as $\lambda$ is small, we know the separating equilibrium is selected. When $s_m$ becomes large, we observe the pooling equilibrium is possible in Figure 2.2. However, the selected equilibrium is changed back to separating when $s_m$ is very large but $\lambda$ is not that large. The reasons are as follows. If $s_m$ is medium, i.e. $\frac{3\alpha}{5} \leq s_m \leq \frac{5\alpha}{7}$, in the separating equilibrium, the $H$-type manufacturer will downward distort the wholesale price to make the retailer treat the direct channel as threat of encroachment, i.e. $q_r(H) = \alpha - s_m$, while the retailer will also order the same amount just to obstacle the $H$-type manufacturer’s direct selling in the pooling equilibrium. Due to the possibility of being $L$-type, the expected direct channel threat in the pooling equilibrium is stronger than in the separating equilibrium for the $H$-type manufacturer, which can result in higher wholesale price. Therefore, the $H$-type manufacturer will pool at some wholesale price, rather than signaling its type information by downward distorting its wholesale price to a very low level. If $s_m$ becomes larger, the $H$-type manufacturer will offer a low enough price to make the retailer feel no threat from the direct channel. However, in the pooling equilibrium, the retailer will still order $\alpha - s_m$ if $s_m$ is not that large but treat the direct channel as no threat if $s_m$ is very large. Hence, when $s_m$ is not that large, the retailer’s order quantity is still large, which makes
pooling profitable for the $H$-type manufacturer. But when $s_m$ becomes very large, the retailer’s order quantity becomes small in the pooling equilibrium. When $\lambda$ is medium, due to the strong direct selling threat from the $L$-type manufacturer, the pooling wholesale price cannot be set very large, which makes separating profitable for the $H$-type manufacturer. When $\lambda$ becomes large, the probability of $L$-type manufacturer goes down and then the pooling wholesale price can be set close to the optimal price for the $H$-type manufacturer, which makes pooling profitable for the $H$-type manufacturer. Thus, the pooling equilibrium is selected when $\lambda$ is very large.

### 2.5.2 Effect of Information Asymmetry on the Direct Channel Strategy

Under full information setting, the $L$-type manufacturer will be a solo seller in the downstream market. Under asymmetric information setting, we see the $L$-type manufacturer’s channel strategy is unchanged in the separating equilibrium. However, in the pooling equilibrium, the $L$-type manufacturer may change to strategy “En- croachment” (E) to share the downstream market with the retailer and become less aggressive. In the pooling equilibrium, the possibility to be $H$-type manufacturer for the $L$-type manufacturer makes the retailer feel the manufacturer’s direct channel threat weak, which amplifies the retailer’s order incentive. It is possible for the retailer to earn negative profit when he encounters the $L$-type manufacturer, but in expectation the retailer will earn positive profit due to the lower wholesale price. This is also intuition for the pooling equilibrium to exist only when the possibility of the $H$-type manufacturer is large enough.
We illustrate the \( H \)-type manufacturer’s strategy in the full information model and in the asymmetric information model in Figure 2.3 when \( \lambda \leq \frac{3}{5} \). When \( \frac{5\alpha}{7} < s_m < \frac{5\alpha}{6} \), the manufacturer will adopt strategy “Threat” (T) when the retailer knows her direct selling cost, while with private information, her strategy is changed to “No Threat of Encroachment” (N), which means the direct channel has no threat on the retailer. Then we know the \( H \)-type manufacturer becomes less aggressive under asymmetric information setting. When \( \lambda > \frac{3}{5} \), we can show the same conclusion. However, due to the possibility of pooling equilibrium, in which the \( H \)-type manufacturer’s direct selling ability is perceived higher by the retailer, the \( H \)-type manufacturer may become more aggressive, compared to the case when \( \lambda \leq \frac{3}{5} \). We formally show our finding in the following proposition.

**Proposition 2** Asymmetric information makes the manufacturer less aggressive (from solo seller to “E” for the \( L \)-type manufacturer and from “T” to “N” for the \( H \)-type manufacturer).

In the separating equilibrium, the retailer knows the manufacturer’s direct selling cost. The \( L \)-type manufacturer with \( s = 0 \) is as efficient as the retailer and then she will be the solo seller in the downstream market. Thus, the \( L \)-type manufacturer’s channel strategy is unchanged by the information asymmetry. But in order to...
obstacle the \( L \)-type manufacturer’s incentive to mimic as \( H \)-type, the \( H \)-type manufacturer has to lower the wholesale price to signal her high cost and dependence on the retailer. Hence, the \( H \)-type manufacturer may become less aggressive under asymmetric information setting.

In the pooling equilibrium, the retailer cannot conclude any information from the wholesale price. For the \( H \)-type manufacturer, since the possibility to be \( L \)-type in retailer’s belief, the retailer will think the manufacturer’s encroachment ability is stronger than what she is. And the retailer will only order the same quantity if he gets more wholesale price concession compared with that when he knows manufacturer’s type information. Thus, the \( H \)-type manufacturer will become less aggressive. Since pooling with the \( H \)-type manufacturer, the \( L \)-type manufacturer can also receive order quantity from the retailer. And the \( L \)-type manufacturer will share the downstream market with the retailer and seems “less aggressive.”

2.5.3 Effect of Information Asymmetry on the Direct Channel Influence on Profits and Consumer Surplus

In order to derive the direct channel effect on manufacturer’s and retailer’s profits and consumer surplus under asymmetric information setting, we first need the results without direct channel under asymmetric information setting. Without direct channel, manufacturer’s direct selling cost is irrelevant to the supply chain whether it is public or private information. Thus, the profits for the manufacturer and the retailer and consumer surplus under asymmetric information setting are the same as those with symmetric information. Therefore, the manufacturer’s profit is \( \Pi_m^N = \frac{\alpha^2}{8} \), retailer’s profit is \( \Pi_r^N = \frac{\alpha^2}{16} \), and consumer surplus is \( \Gamma^N = \frac{\alpha^2}{32} \). With direct channel,
manufacturer’s direct selling cost is an important factor to determine the strategy for the players in the supply chain. The equilibrium results are shown in § 2.5.1. By comparing the relevant profits and consumer surplus with and without direct channel, we can get the benefits for the manufacturer, the retailer and consumers due to direct channel under asymmetric information setting.

Recall from Arya et al. (2007) that the potential mutual benefit for the manufacturer and retailer from supplier encroachment arises because manufacturer’s wholesale price concession stimulates higher retailer’s order incentive, compared to that without the direct channel. In the presence of asymmetric information, the propensity for the manufacturer to downward distort the wholesale price when the direct selling cost is high can expand retailer’s willingness to order more. Consequently, information asymmetry can improve the potential for mutual benefit due to direct channel. However, the wholesale price reduction sometimes needs to be so severe that the H-type manufacturer turns to be worse off. More details are as follows.

In the separating equilibrium, the L-type manufacturer will still be a solo seller in the downstream market. Thus, the benefits for the manufacturer, the retailer and consumers due to direct channel are “W-L-W”, which is unchanged compared with that when the retailer also knows $s$. We present the benefits of direct selling for the manufacturer, the retailer and consumers in Figure 2.4 when the manufacturer is $H$-type and $\lambda \leq \frac{3}{5}$. The benefits of direct channel when the retailer knows the manufacturer’s high cost information ex ante are above the axis and those in the separating equilibrium are below the axis. Then we see, with $H$-type manufacturer, the mutual benefits for the manufacturer and the retailer will be enlarged. But we also see an alarm for the manufacturer. With full information, the manufacturer will
Figure 2.4: Benefits of direct channel with the $H$-type manufacturer.

Note: (1) The direct channel effect on the manufacturer’s profit, the retailer’s profit, and the consumer surplus is denoted by the three letters in the triplet in order. (2) Letter “W” denotes an increase, letter “L” denotes a decrease, and “U” denotes no change. (3) In this illustration, we set $\alpha = 1$.

never be worse off due to direct channel, while with asymmetric information, it is possible for the $H$-type manufacturer to be worse off.

Since the retailer feels a potential stronger threat from the direct channel due to information disadvantage, he will not order the same quantity unless the manufacturer gives larger wholesale price concession. So if the manufacturer finds his direct selling cost is high, she needs to offer deeply wholesale price reduction to help the retailer build the belief that she is $H$-type. If $s_m$ is small, the $H$—type manufacturer will not have large incentive to reduce her wholesale price since she can rely more on her own direct channel. Thus, the direct channel effect will not be distorted. However, when $s_m$ is large, the retailer’s distribution ability is important to the manufacturer. In order to credibly signaling her type information to the retailer, the $H$-type manufacturer will reduce the wholesale price to such a level that the $L$-type has no incentive to mimic. Hence, we see the asymmetric information can enlarge retailer’s profit and consumer surplus. But sometimes, the $H$-type manufacturer has to sacrifice so much to convey her true type information to the retailer that she will be worse off compared with no direct channel. The $H$-type manufacturer’s high direct selling cost limits her chance to earn profit from direct channel, which makes her more dependent
on the retailer. If the $H$-type manufacturer does not offer a lower wholesale price, the retailer will think she is $L$-type and has stronger retail ability and then retailer’s order quantity will shrink severely. Constrained by the large direct selling cost, the $H$-type manufacturer cannot fully take retailer’s market share when retailer’s order quantity becomes low. Thus, the manufacturer with a high direct selling cost has to offer a low enough wholesale price to signal her type information, which can benefit the retailer and the consumers.

Under symmetric information setting, the wholesale price is used for a strategic channel control purpose by the manufacturer to tune the retailing price in the downstream market, while under asymmetric information setting, the wholesale price is also used for information leakage purpose by the manufacturer to signal her true type information to the retailer. Since the retailer will infer manufacturer’s type information from the wholesale price, then retailer’s belief posts a constraint for the range of the wholesale price available for the manufacturer to control the downstream market.

When the $L$-type manufacturer has incentive to mimic as $H$-type, in order to let the retailer believe the manufacturer is $H$-type, the $H$-type manufacturer may need to give more wholesale price concession to the retailer. Thus, the information role of the wholesale price can impede the effectiveness of its channel control role. The interaction between the channel control role and information role of the wholesale price under asymmetric information setting may let the manufacturer earn less but the retailer earn more. Since the retailer is more efficient than the manufacturer selling in the downstream market, with a lower wholesale price, there are more products deployed in the market and then the retailing price in the market is lower and thus, consumers also benefit more in the asymmetric information setting. But sometimes it
is caution for the manufacturer since the wholesale price concession should be lowered to such an extent to signal her type information, which may harm herself.

Following, we examine the benefits for the manufacturer, the retailer and consumers due to direct channel in the pooling equilibrium. We summarize results in the following proposition.

**Proposition 3** In the pooling equilibrium, we have:

1. The L-type manufacturer is always “Win”;

2. The H-type manufacturer is “Win” with a relative small selling cost, i.e. $s_m < \frac{\sqrt{(\lambda^2+\lambda-1)+\lambda+3}}{2\lambda+4} \alpha$ and “Lose”, otherwise.

3. The benefits outcome is “W-W-W” for the H-type manufacturer, the retailer and consumers when $s_m < \max\{\frac{\sqrt{(\lambda^2+\lambda-1)+\lambda+3}}{2\lambda+4} \alpha, \frac{2(\lambda+1)-\sqrt{2}\sqrt{(2\lambda^2-3\lambda+2)}}{4\lambda}\alpha\}$.

From Proposition 3 (1.), we know the L-type manufacturer will always be better off from the direct channel, which is the same as in the full information setting. Under symmetric information setting, from Figure 2.1, we know the manufacturer will always be better off after adding direct selling. However, Proposition 3 (2.) shows the H-type manufacturer will lose due to direct channel if $s_m$ is large. That’s because pooling with the L-type manufacturer makes the manufacturer cannot charge high wholesale price. We also know it is “W-L-W” for the H-type manufacturer, the retailer and the customers in symmetric information setting when $(1 - \frac{1}{2\sqrt{2}})\alpha < s_H < \frac{3}{4} \alpha$. From Proposition 3 (3.), the benefits outcome can be changed to “ W-W-W.” This is also caused by the manufacturer’s low wholesale price.

Combining the above results for separating equilibrium and pooling equilibrium, we have the following proposition.
Proposition 4  With asymmetric information, the H-type manufacturer can be worse off due to direct channel. And due to asymmetric information, the direct channel effect on the profits and consumer surplus can be strengthened, which means there are non-(W-W-W) regions become (W-W-W) compared with full information setting, when the manufacturer is of H-type.

2.6 The Role of Information Acquisition

In § 2.5, we investigate how asymmetric information changes the effect of direct channel on the supply chain performance. Specifically, for the retailer and consumers, they can benefit more from direct channel thanks to information asymmetry, while for the manufacturer, she may even lose due to direct channel under asymmetric information setting, which never happens under full information setting. All of these indicate the important role played by the information structure when dealing with direct channel impact. In this section, we will explore the effect of manufacturer’s learning and retailer’s learning about manufacturer’s direct selling cost on the supply chain performance. We assume the manufacturer always has a direct channel and focus on the information structure effect on profits. We rely on three models to show the role of information. First, the manufacturer and the retailer both have no information about manufacturer’s direct selling cost, which we denote as “no information model.” Then the manufacturer learns her direct selling cost, which we denote as “asymmetric information model.” Finally, the retailer also learns the direct selling cost information, which we denote as “full information model.” We have already shown the results for full information model in § 2.4 and for asymmetric information model in § 2.5. We will add the results for no information model in the
following part. Based on the sequence we introduce these models, we denote the full information model as Model I and the asymmetric information model as Model II and the no information model as Model III.

2.6.1 Model with No Information about Direct Selling Cost

In the no information model, neither the manufacturer nor the retailer knows the manufacturer’s direct selling cost before they make any decisions. We assume the direct selling cost $s$ can be $s_m$ with probability $\lambda$ and 0 with probability $1 - \lambda$. The timeline of the game is as follows:

Stage 1: The manufacturer quotes the wholesale price $w$.

Stage 2: The retailer places the order quantity $q_r$.

Stage 3: The manufacturer and the retailer simultaneously set selling quantities. Then the direct selling cost is realized and the manufacturer and the retailer get their profits.

Since the manufacturer and the retailer make decisions based on the expectation of the direct selling cost in Stage 3, just replacing the direct selling cost, $s$, in the full information model by the expectation of the direct selling cost, $\lambda s_m$, we can get the equilibrium results for this no information model, which are shown in the appendix.

After fully analyzing the three models, we can derive how the information structure affects the supply channel performance by comparing their equilibrium results. The change from Model III to Model II is that the manufacturer learns the direct selling cost. Then we can get manufacturer’s learning effect on the supply chain performance by comparing the ex ante expectations of the profits, which are the expected value
for the players before the manufacturer gets more information about her direct selling cost. The difference of Model I compared with Model II is that the retailer also learns manufacturer’s direct selling cost. Then we can get retailer’s learning effect on the supply chain performance by comparing Model I and Model II. We first present the effect of manufacturer’s learning.

2.6.2 Effect of Manufacturer’s Learning Direct Selling Cost

In order to derive the effect of manufacturer’s learning direct selling cost, we compare the ex ante profits for the manufacturer, the retailer and the total supply chain. We visualize the results in Figure 2.5. With more information on her direct selling cost,

![Figure 2.5: Benefits due to manufacturer’s learning the direct selling cost.](image)

Note: (1) The learning effect on the manufacturer’s profit, the retailer’s profit, and the channel profit is denoted by the three letters in the triplet in order. (2) Letter “W” denotes an increase, and letter “L” denotes a decrease. (3) In this illustration, we set $\alpha = 1$.

the manufacturer can tailor her wholesale price and direct selling quantity based on her direct selling ability. Intuitively, one may think the manufacturer should
be better off after having information advantage on her direct selling cost over the retailer. Indeed, in Figure 2.5, we show the region where the manufacturer can be better off. The retailer can also get better if the manufacturer has more information. However, we also find the manufacturer can be worse off when $\lambda s_m$ is medium.

In the no information model, both players only have the ex ante information on the manufacturer’s direct selling cost and then they will make decisions based on the expected direct selling cost, i.e. $\lambda s_m$. When $\lambda s_m$ is medium, the manufacturer will implement direct channel strategy “T” and charge high wholesale price. Then the retailer’s order quantity is $\alpha - \lambda s_m$. Without information on $s$, the manufacturer will always sell nothing through the direct selling channel even if the realized direct selling cost is $s = 0$. After the manufacturer learns her direct selling cost information privately, the manufacturer can adjust her direct selling quantity based on her cost, while the retailer will still make decisions based on the expected direct selling cost $\lambda s_m$. Hence, threatened by the informative manufacturer’s direct selling, the retailer’s order incentive is diminished. Actually, the retailer’s order quantity is $\alpha - s_m$ in the pooling equilibrium if $s_m$ is medium, which is lower than that with no information setting. Though the pooling price is set higher than that in the no information model, if the retailer’s order is diminished too much, the manufacturer can get lower profit after having more information. In the separating equilibrium, the $H$-type manufacturer needs to offer lower price to signal her high cost information to the retailer. Therefore, if this price distortion is severe, the manufacturer will also be worse off.

**Proposition 5** *The manufacturer can be worse off from having more information about her direct selling cost.*
As $\lambda s_m$ increases, in the no information model, the manufacturer will charge very high wholesale price to sustain her direct channel threat, which results in low retailer’s profit. However, in the separating equilibrium, the price offered by the $H$-type manufacturer is so low that the retailer feels no threat from the direct channel when $s_m$ is large. Hence, the manufacturer can be worse off but the retailer can be better off. We also find the total supply chain profit may increase. Hence, the retailer can pay some side payments for the manufacturer to learn her direct selling cost information.

**Proposition 6** The manufacturer’s learning can benefit the retailer as well as supply chain, though herself is worse off.

When $s_m$ is not that large, we have pooling equilibrium. Due to information advantage, we find the manufacturer offers higher price but the retailer orders lower quantity, $\alpha - s_m$, compared to the no information model. Since the lowered order quantity from the retailer, the manufacturer can be worse off after knowing more on her cost information. For the retailer, he will depress his order incentive but set a high profit margin in the downstream market, considering the possibility of the manufacturer’s low direct selling cost, which can also result in low profit if the ordering quantity is set too low. Hence, we find manufacturer’s learning can make no more profit in the supply chain and then the manufacturer may have no incentive to acquire more information on her cost.

**Proposition 7** The manufacturer’s learning can hurt everyone, i.e. “L-L-L”. Therefore, no information can benefit everyone.

By reverse thinking, from Model II to Model III, we have the effect of uncertainty on the supply chain performance. Thanks to be uncertain about the direct channel
encroachment ability, the manufacturer cannot tailor her wholesale price based on her ability and the threat from the direct channel to the retailer can be weakened and so can the downstream market competition. Thus, the manufacturer will be less aggressive and depend more on the more efficient retailer to distribute the product. Due to the diminished threat from the direct channel, the retailer will order more to seize the opportunity in the downstream market, which results in more profits for the manufacturer and the total supply chain. The uncertainty of the direct selling cost limits manufacturer’s interest in the downstream market, but helps to recover retailer’s order incentive when facing manufacturer’s direct channel threat, which demonstrates the positive effect of uncertainty on the supply chain performance.

2.6.3 Effect of Retailer’s Learning Direct Selling Cost

After the manufacturer learns her direct selling cost, we find the manufacturer may be worse off due the information advantage over the retailer. Then the manufacturer may try to share the cost information with the retailer and the retailer will also learn the manufacturer’s direct selling cost. We now examine the effect of retailer’s learning on the direct selling cost.

In the separating equilibrium, similar as in the full information model, the manufacturer and the retailer both know the true direct channel selling cost. Then we will compare the supply chain performance based on manufacturer’s type. When the manufacturer is $L$-type, the equilibrium outcomes are the same in Model I and Model II. So we focus on the comparison results when the manufacturer is $H$-type. We illustrate these results in Figure 2.6.
Figure 2.6: The benefits of the retailer’s learning manufacturer’s direct selling cost with the \(H\)-type manufacturer.

Note: (1) The learning effect on the manufacturer’s profit, the retailer’s profit, and the channel profit is denoted by the three letters in the triplet in order. (2) Letter “W” denotes an increase, letter “L” denotes a decrease, and “U” denotes no change. (3) In this illustration, we set \(\alpha = 1\).

In Figure 2.6, we note the \(H\)-type manufacturer will never be worse off because of retailer’s learning, while the retailer himself may be worse off after knowing more. We will see similar results in the pooling equilibrium in below. Then we have the following proposition.

**Proposition 8** If the retailer learns the informed manufacturer’s direct selling cost information, he can be worse off, and at the same time the manufacturer can be better off.

In the pooling equilibrium, we compare the ex ante profits for the manufacturer and the retailer and show results in Figure 2.7. As we see, now the total supply chain will be better off. Hence in these regions, the manufacturer may pay side payments to the retailer to share her private information.

If only the manufacturer knows the direct selling cost, the retailer would infer information from the wholesale price. Threatened by the information disadvantage, the retailer’s belief will generate a constraint for the direct channel strategy chosen by the manufacturer. Therefore, the information leakage role of the wholesale price will impede its channel control role. In order to signal the true type information to the retailer, the manufacturer may need to offer larger wholesale price concession to the retailer and distort her direct channel strategy. The manufacturer is forced to
Figure 2.7: Benefits due to retailer’s learning.

Note: (1) The learning effect on the manufacturer’s profit, the retailer’s profit, and the channel profit is denoted by the three letters in the triplet in order. (2) Letter “W” denotes an increase, letter “L” denotes a decrease, and “U” denotes no change. (3) In this illustration, we set $\alpha = 1$.

play a signaling game due to retailer’s information disadvantage. However, when the retailer also knows the direct selling cost, the manufacturer can tailor her wholesale price based on her direct selling cost and freely choose the direct channel strategy. Thanks to retailer’s learning, the manufacturer can focus on the channel control role of the wholesale price and then the retailer will get higher wholesale price. Therefore, the manufacturer can be better off from retailer’s learning, while the retailer may be worse off after knowing more. In all, we know if the manufacturer gets private information for the direct selling cost, she prefers to share it with the retailer. But for the retailer, he prefers to deal with a super informal supplier.

Combining the results in the above section, we find the manufacturer may prefer not to know the direct selling cost information at all if she cannot share it with the retailer effectively, which means the manufacturer may have low incentive to gain
information advantage for her direct channel, which may explain manufacturer’s high direct selling cost in practice (Chao, 2016). Obtaining private information may help the manufacturer compete with the retailer in the downstream market. But when facing a strategic retailer, knowing more for the manufacturer may enlarge the direct channel threat and impede the channel control role of the wholesale price. The special cooptition relationship between the manufacturer and the retailer should be taken into account by the manufacturer with direct channel before trying to learn the direct selling cost information. For the retailer, knowing less not always means disadvantage. Sometimes the information disadvantage may help the retailer to get more wholesale price concession from the informed manufacturer. To be information deficient may also be a threat to the superior informative player in this coopetition framework. Since knowing more for the retailer can make the manufacturer more aggressive and make himself worse off, the retailer may prefer to be uninformed.

If the manufacturer and the retailer agree on sharing the manufacturer’s direct selling cost, will the manufacturer have incentive to acquire her direct selling cost? We find the answer can still be no, as show in Figure 2.8. With more information, the $L$-type manufacturer will always sell in the downstream market and cut off the retailer, while the manufacturer with high cost can set a higher wholesale price than in the no information setting to sustain its direct selling threat. Both will push up the expected wholesale price for the retailer. Though with higher price, the manufacturer can still be worse off if the retailer’s order decreases too much when $s_m$ and $\lambda$ are large. For the retailer, lower selling quantity can also result in sales loss in the downstream market and then less profit for him.
Figure 2.8: Benefits of information acquisition under information sharing contract.

Note: (1) The learning effect on the manufacturer’s profit, the retailer’s profit, and the channel profit is denoted by the three letters in the triplet in order. (2) Letter “W” denotes an increase, letter “L” denotes a decrease, and “U” denotes no change. (3) In this illustration, we set $\alpha = 1$.

2.7 Summary

Low launching cost of the online stores may induce the manufacturer to open a direct selling channel online thereby encroaching the buyer’s market. Due to the lack of experience in sales, however, the manufacturer may incur extra cost in selling to consumers. Then the manufacturer may have incentive to acquire more information on its direct selling cost and then tailor her direct selling strategy based on her direct selling cost. With information disadvantage, the retailer may infer the manufacturer’s direct selling cost from the manufacturer’s wholesale price. Then a signaling game can happen. In order to signal her high cost information, the manufacturer may need to downward distort her direct selling cost. If the manufacturer gets worse off by having information advantage over the retailer, then she may want to share that information.
with the retailer. Then the question is whether the retailer has incentive to share the manufacturer’s cost information. In order to analyze the players’ incentives in learning the manufacturer’s direct selling cost, we compare the results under different information structures on the direct selling cost. Specifically, we consider three information structures, i.e. the full information model, the asymmetric information model and the no information model. We find the region of “win-win-win” for the manufacturer, the retailer and consumers can expand when the manufacturer is privately informed on its direct selling cost, compared to that when the information is public. Furthermore, we show the manufacturer may prefer no information advantage over the retailer on its direct selling cost. On the one hand, if the manufacturer cannot share the information with the retailer, the manufacturer may not have incentive to uncover the cost information but to stay uninformative as the retailer. On the other hand, if the manufacturer can share the information with the retailer, after the manufacturer gathers the information on its direct selling cost, it will be weakly better off for the manufacturer to share that information with the retailer rather than keeping it private. For the buyer, however, we find it may want to keep information disadvantage on the manufacturer’s direct selling cost under the supplier encroachment setting. In addition, we also examine the benefit of uncertainty in the supply chain. If the manufacturer and retailer agree on the information sharing contract before the manufacturer’s information acquisition, the manufacturer may still have no incentive to resolve the uncertainty on its direct selling cost. Hence, our results shows it may not be better for the players to have more information on the direct selling cost when the manufacturer has a direct selling channel.
CHAPTER 3

Strategic Inventory and Supplier Encroachment

3.1 Motivation and Related Literature

When different players in a decentralized supply chain make pricing or quantity decisions, their interactions are usually associated with conflicting objectives. Indeed, both the supplier and the buyer may insist on high margins for themselves and even resort to specific strategies that either grant them more leverage or put the other player in a disadvantaged position. It is well-documented that supply chain players taking actions to maximize their own profit may result in inefficient channel performance; e.g. double marginalization (Spengler, 1950). However, in some cases, the tension-creating competitive strategies may surprisingly benefit all players in the supply chain. The strategic use of inventory, as identified by Anand et al. (2008), is one such example.

The classical reasons for carrying inventory are mostly from an operational perspective (Zipkin, 2000); for instance, firms would carry pipeline inventory, safety stocks or speculative inventory to overcome supply or demand uncertainty. However, there are also strategic reasons why firms invest in inventory. The excess inventory,
which exists for pure strategic purpose, acts as a tool to induce lower wholesale price from the supplier in the future. Such strategic use of inventory has been observed in practice — an anecdotal evidence is provided by Martínez-de-Albéniz and Simchi-Levi (2013, pp 397-398):

...the steel manufacturer Celsa, located in Spain, that procures scrap metal... does not need to carry a high level of scrap metal inventory at any time, because supply is sufficiently diversified (hence no shortage is likely). However, the company stores large piles of it outside its factory. In fact, displaying such high levels of raw materials to the local dealers is an effective negotiation tool that forces them to quote lower prices. This constitutes a credible threat from the buyer: it will only buy more raw materials if the price is low enough.

The effectiveness of strategic inventory as a competitive tool has been shown by Anand et al. (2008). They study a two-period model and find that the buyer’s strategic inventory can force the supplier to lower the second-period wholesale price. Moreover, under certain conditions, the supplier can also benefit from the buyer’s move. This interesting result has been confirmed in other settings too. For example, when the supplier has no visibility on the buyer’s inventory, Roy et al. (2016) illustrate the benefit of strategic inventory. In fact, they find that the supplier may even actively lower the wholesale price to sustain the buyer’s incentive to hold inventory. In addition, when the buyer faces horizontal competition, the strategic use of inventory, although being discouraged, still exists and benefits both the buyer and the supplier (Desai et al., 2010; Saloner, 1986). In a recent paper, Hartwig et al. (2015) provide empirical support for the presence of strategic inventory via laboratory experiments. According to their results, strategic inventory can also increase buyer’s perception of equitable payoff distribution in the supply chain and therefore its positive effects can be even more pronounced than theoretically predicted. Finally, Yang et al. (2017)
show that when the supplier has limited capacity, strategic use of inventory by the buyer can benefit both players.

Despite the fact that the supplier may benefit from the buyer’s use of strategic inventory, this strategy arises from channel conflict and the supplier may not favor it at all times. Instead, the supplier may utilize its own available strategies to limit the buyer’s competitive pressure. For instance, Keskinocak et al. (2008) show that, by constraining its initial capacity, the supplier can mitigate the buyer’s use of strategic inventory; likewise, Arya and Mittendorf (2013) illustrate that the supplier can employ direct-to-consumer rebates to alleviate the buyer’s strategic inventory influence. Interestingly, both of those papers have noted that, with the supplier’s confronting reactions, strategic inventory can still be beneficial to both players. In a similar vein, the supplier can use another (perhaps more commonly seen) strategy to interplay with the buyer’s inventory withholding strategy — direct selling to the end customers.

Traditionally, suppliers used to seldom sell directly to the end market either because of lack of distribution capability or because of high costs. However, the advent of e-commerce and sophisticated technology has greatly facilitated the use of the direct channel and has made suppliers selling to the end customers possible (Tedeschi, 2005). Examples of direct selling exist widely in practice: firms such as Apple, Microsoft, Nike, and Ralph Lauren sell their products through department stores as well as their own online/physical stores. This phenomenon is also referred to as “supplier encroachment.” In this paper, we do not distinguish between “direct selling” and “supplier encroachment,” and will use them interchangeably hereafter.

While supplier encroachment may threaten downstream buyers by reducing their share of market demand, researchers have shown that, in some cases, it can increase
buyer’s profit under either price competition (Chiang et al., 2003) or quantity competition (Arya et al., 2007). In addition, Yoon (2016) shows that the direct channel may result in a spillover effect of the supplier’s cost-reducing investment, and therefore benefit the buyer. However, supplier encroachment does add channel conflict and can be detrimental to the buyer; for example, when demand information is asymmetric (Li et al., 2013) or when product quality is endogenous (Ha et al., 2015), the buyer hardly benefits from the existence of supplier’s direct selling channel.

Although supplier encroachment is commonly seen in practice and has been intensively studied in the literature, its system-wide effect when the buyer can withhold inventory has not been analyzed. Indeed, the above papers all focus on a single period model and ignore the strategic supplier-buyer interaction across periods. Intuitively, when the buyer carries strategic inventory in order to gain competitive power, the supplier can use the direct selling option as an effective strategy to counter the buyer’s threat. Similarly, the buyer’s withholding of strategic inventory can also be seen as a strategic response to potential use of direct selling by the supplier. While the extant literature has individually examined the two strategies and established their respective “bright side” to the channel, their joint interactions have not been analyzed.

This paper fills the gap in the literature by studying a deterministic two-period model wherein the buyer may carry excess inventory across periods and the supplier may sell to the end customers through a direct channel in the second period. As our model incorporates the consideration of both strategic inventory and supplier encroachment, it is closely related to the models in Anand et al. (2008) and Arya

\footnote{We also analyzed the model with direct channel in both periods. As the main results are not affected, the analysis is cleaner, and in order to make suitable comparisons with Anand et al. (2008) and Arya et al. (2007) papers, we consider direct channel in the second period only.}
et al. (2007); see Figure 3.1 for an illustration. Similar to their models, we assume a deterministic system and complete information on each player’s cost. In Anand et al. (2008), the buyer may carry strategic inventory but the supplier does not engage in direct selling; in Arya et al. (2007), the supplier can sell directly but there is no scope for strategic inventory withholding due to the single-period setting. Hence, we are able to analyze a synthesis of both models and thus extend their works. The focus in this paper is to understand the combined effect of strategic inventory and supplier encroachment on channel performance. Specifically, we attempt to answer the following research questions:
How does the interplay between the supplier’s potential use of the direct selling channel and the buyer’s use of strategic inventory affect:

(a) the supplier’s and the buyer’s strategies in equilibrium?

(b) the equilibrium wholesale prices and order quantity?

(c) the supplier’s and the buyer’s optimal profits?

To study the combined effect of the strategies on the equilibrium outcome, we directly compare our results to a benchmark model, which is a simple two-period unrelated dyadic supply chain model with no strategic inventory or direct selling option. The two papers (Anand et al., 2008; Arya et al., 2007) closest to ours study the respective effect of the strategies based on essentially the same benchmark; so noting the differences between our results and theirs also helps answer the research questions. Our main findings are as follows.

First, we give a full characterization of the equilibrium strategies employed by the two players. Depending on the buyer’s unit inventory holding cost and supplier’s unit direct selling cost, the buyer could either hold inventory (H) or not hold inventory (NH), and the supplier could directly sell (D) some positive quantity or sell nothing through the direct channel. When the direct channel is not utilized to sell any quantity, we identify two relevant strategies for the supplier: (1) When the market price equals to the direct selling cost, the supplier is using the option of encroachment as a threat against the buyer. We call this strategy “no direct selling with threat of encroachment” (ND\(T\)). (2) When the buyer’s order behavior is the same as if there were no direct selling channel, we say that the supplier’s strategy is “no direct selling (without threat of encroachment)” (ND). When the “ND\(T\)” strategy
is used, the supplier can avail of selling through the direct channel, and even though in equilibrium it does not, it gives the supplier a competitive lever to use against the buyer. This strategy is sustainable under equilibrium but has not received much attention in the extant literature.

Second, we also draw insights into how price and quantity decisions are affected by the players' use of their respective strategies. We find that when the direct selling cost is low, the supplier will sell directly to the end customers, leading to more competition in the downstream market. In order to soften the competition, the supplier will set a low wholesale price to induce the buyer to purchase more in the first period. Thus, there exist scenarios where the buyer carries inventory with the wholesale price lower than under the benchmark case. When the direct selling cost is high, the strategic use of inventory becomes so prominent that in certain cases it is capable of changing the supplier's strategy from “NDT” to “ND”. In fact, it is possible that the buyer will purchase excess inventory with any finite holding cost, just to make the supplier choose a less aggressive direct selling strategy. These results are in contrast to Anand et al. (2008). In their paper, the buyer will carry inventory only when the holding cost is below a threshold and the supplier will charge higher-than-benchmark wholesale price to limit the use of strategic inventory. Therefore, our result provides with another strategic purpose of the buyer’s inventory withholding decision.

We also show that the supplier’s profit is always higher than that in the benchmark model, whereas the buyer’s profit may be so in some cases, especially when the direct selling cost is large. This result confirms the positive respective effect of both strategies identified in the previous works, and also establishes the positive combined effect of both strategies. Furthermore, we note that a player’s profit is sometimes
negatively correlated with its opponent’s cost to employ the competitive strategy, which implies that having a less cost-efficient rival in vertical competition may not always benefit the other player.

In an extension to our main model, we consider a different timing structure in the quantity competition in the second period. Instead of sequentially (buyer first) deciding the selling quantities, the two players are assumed to make simultaneous quantity decisions. As a result, the buyer does not have the first-mover advantage anymore. This will happen when no player has a leadership advantage over the other. Indeed, the simultaneous game assumption has precedence in extant literature; both Arya et al. (2007) and Ha et al. (2015) have conducted studies under such assumption. By studying the same research questions with a different timing structure, we are able to determine the effect of first-mover advantage for the buyer. Our results indicate that, losing the first-mover position may result in higher profit for the buyer, although it may rely more on using strategic inventory. Moreover, the supplier will enjoy higher profit if the quantity competition is sequential rather than simultaneous. In other words, being a first-mover (second-mover) may not always be an advantage (disadvantage) for the players. This counter-intuitive effect has been shown by Gal-Or (1985) in a different context.

The remainder of this paper is organized as follows. § 3.2 introduces our main model and analyzes the benchmark model. In § 3.3, we summarize the individual effect of use of strategic inventory by the buyer and the individual effect of supplier encroachment. We also extend the model setting in Arya et al. (2007) to consider the case of threat of supplier encroachment. In § 3.4, we analyze the combined effect of strategic inventory and direct selling and determine the equilibrium decisions for
both players. In § 3.5, we study the simultaneous model and discuss the effect of
timing of quantity competition on the equilibrium outcome. Finally, we summarize
and conclude in § 3.6. The summary of all equilibrium results for the sequential
model is in Appendix B.1 and that for the simultaneous model is in Appendix B.2.
All key proofs are provided in Appendix B.3.

3.2 Model and Benchmark Analysis

In this section, we introduce our main model, which is based on sequential quantity
decisions and also outline the results for the benchmark model, where neither strategic
inventory nor direct channel is an option.

3.2.1 Sequential Model

We consider a dyadic supply chain with one supplier and one buyer. There are two
periods in which the buyer purchases from the supplier and sells to a downstream
market. See Figure 3.1(c) for the model framework. Consumer demand is assumed to
be deterministic and identical in each period, and is represented by a linear downward
sloping inverse demand function, \( p = \alpha - q \), where \( p \) and \( q \) are price and quantity of
the product, respectively\(^5\). The buyer may order extra in the first period and carry
excess inventory to the second period. The risk, however, is in increased end-market
competition if the supplier uses a direct selling channel and encroaches into the buyer’s
market in the second period. Both strategies are costly for the two players. We assume
that the buyer incurs per-unit inventory holding cost \( h \geq 0 \) and the supplier incurs a

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\(^5\)The model can be extended to the case when demand in the two periods is not identical. To
draw a meaningful comparison with Anand et al. (2008) and Arya et al. (2007), we use the same
demand model as theirs.
per-unit cost, \( s \in [0, \alpha) \), to sell directly in the end-market. The cost associated with selling through the direct channel may arise from the supplier’s disadvantage due to lack of experience in selling direct to consumers (Arya et al., 2007; Ha et al., 2015; Kapner, 2014; The Economist, 2013). Without loss of generality, we normalize the production cost, the buyer’s selling cost, the salvage value of the good and other fixed costs to be zero. The sequence of events in the main model is as follows. In period one:

Stage 1: The supplier quotes wholesale price \( w_1 \).

Stage 2: The buyer chooses the order quantity \( Q_1 \) and sells \( q_1 \leq Q_1 \) in the end-market. It carries inventory \( I = Q_1 - q_1 \) to the second period. The market clearing price in period one, \( p_1 = \alpha - q_1 \).

In period two:

Stage 1: The supplier quotes wholesale price \( w_2 \).

Stage 2: The buyer chooses the order quantity \( Q_2 \) and sells \( q_2 \leq I + Q_2 \) to the market.

Stage 3: The supplier chooses its selling quantity \( q_s \) through the direct channel.

The market clearing price in period two, \( p_2 = \alpha - (q_2 + q_s) \).

Note that we focus on a sequential move from the buyer to the supplier in the main model. Specifically, at Stages 2 and 3 in the second period, the supplier chooses its selling quantity \( \text{after} \) knowing the buyer’s selling quantity, that is, the buyer has a first-mover advantage in selling to the market. However, alternative timing structures in period two may exist. In § 3.5, we will study the case where the buyer and the supplier make \( \text{simultaneous} \) quantity decisions.
3.2.2 Benchmark Model: Two-Period Model without Strategic Inventory and Direct Channel

The two-period model without strategic inventory and direct channel consists of just two unrelated basic dyadic supply chains, which is illustrated by Figure 3.1(c) when the inventory withholding option and the direct channel are both removed. Thus, it is straightforward to compute the supplier’s wholesale price and the buyer’s order quantity as follows:

\[ \hat{w} = \frac{\alpha}{2} \quad \text{and} \quad \hat{q}_b = \frac{\alpha}{4}. \]

Moreover, the supplier’s and the buyer’s profits over the two periods are given by

\[ \hat{\Pi}_s = 2\hat{w}\hat{q}_b = \frac{\alpha^2}{4}, \]

and

\[ \hat{\Pi}_b = 2[(\alpha - \hat{q}_b)\hat{q}_b - \hat{w}\hat{q}_b] = \frac{\alpha^2}{8}. \]

3.3 Effect of Strategic Inventory and Effect of Supplier Encroachment

We use this section to summarize the results of two previous papers (Anand et al., 2008; Arya et al., 2007) concerning the effect of strategic inventory and effect of supplier’s direct selling channel, respectively. These two papers provide important results on how each strategy individually affects the supply chain in the absence of the other. By comparing with them, we are able to draw meaningful insights into the interplay between the supplier’s and the buyer’s strategies. Subsequently, we extend the model setting in Arya et al. (2007) to include a new strategy of “no direct selling
with threat of encroachment” for the supplier. Finally, to facilitate our study in the following section, we illustrate the results in a two-period model with direct channel option only.

### 3.3.1 Review of Previous Papers and Extension

Anand et al. (2008) consider a two-period model with strategic inventory only, as shown in Figure 3.1(a). One of their major results is that the buyer will choose to withhold inventory (H) if \( h < \frac{a}{4} \), and to not withhold (NH) otherwise. The underlying logic is that, by using strategic inventory efficiently, the buyer can force the supplier to reduce the second-period wholesale price, yielding a margin that overcomes the inventory holding cost. Another major contribution of this paper is that two thresholds on the unit holding cost are established concerning the effect of strategic inventory on players’ profits. Both the supplier and the buyer will benefit from the use of strategic inventory (win-win situation) if \( h < \frac{21a}{152} \) (low inventory holding cost). The supplier will benefit from, but the buyer is hurt by the use of strategic inventory if \( \frac{21a}{152} < h < \frac{a}{4} \) (medium inventory holding cost). When \( h \geq \frac{a}{4} \) (high inventory holding cost), there is no inventory withholding, and thus both players’ profits are unchanged.

Arya et al. (2007) study a single-period dual channel model, as illustrated in Figure 3.1(b). In their paper, the discussions focus on the situation where the supplier sells a positive quantity through the direct channel in equilibrium. They find that the supplier will employ strategy “D” if and only if \( s < \frac{3a}{5} \) and charge a wholesale price \( \frac{3a-s}{6} \), which is less than the benchmark wholesale price \( \tilde{w} = \frac{a}{2} \). Partly because of the wholesale price decrease, the buyer can sometimes benefit from the supplier’s
selling through direct channel; specifically, this happens when the unit direct selling cost is not too small, i.e. \( \frac{3\alpha}{4\sqrt{2}} < s < \frac{3\alpha}{5} \). The supplier, on the other hand, will always benefit from selling direct to the customers. This *win-win* situation is the bright side of the supplier encroachment that the authors refer to.

**No Direct Selling with Threat of Encroachment**

As mentioned above, the case when the supplier does not sell through direct channel is not fully analyzed in Arya et al. (2007). We now complement it by studying an extension.

Given a wholesale price \( w \), the buyer’s selling quantity is given by:

\[
q_b(w) = \begin{cases} 
\frac{\alpha - w}{2} & \text{if } 0 \leq w < (2s - \alpha)^+, \\
\alpha - s & \text{if } (2s - \alpha)^+ \leq w \leq \frac{(3s - \alpha)^+}{2}, \\
\frac{\alpha + s - 2w}{2} & \text{if } \frac{(3s - \alpha)^+}{2} < w < \frac{\alpha + s}{2}, \\
0 & \text{if } \frac{\alpha + s}{2} \leq w, 
\end{cases}
\]

which can be derived by letting \( I = 0 \) and replacing \( w_2 \) by \( w \) in equation (A1) in Appendix B.3. On the other hand, the supplier’s direct selling quantity, \( q_s = \frac{(\alpha - q_b(w) - s)^+}{2} \). Then, we substitute \( q_b(w) \) from Equation (3.1) to \( q_s \) and deduce that \( q_s = 0 \), i.e. there is no direct selling, if and only if \( q_b(w) = \frac{\alpha - w}{2} \) or \( q_b(w) = \alpha - s \).

Although in both cases the direct selling quantity \( q_s = 0 \), the actual strategy used by the supplier is subtly different depending on the actual selling quantity of the buyer. In the first case, \( q_b(w) = \frac{\alpha - w}{2} \), which is exactly the selling quantity for the same wholesale price in the basic dyadic supply chain model. Hence, the buyer reacts as if the direct channel does not exist, and we define the supplier’s direct channel strategy as “no direct selling (without threat of encroachment)” (ND). In the second
case, $q_b(w) = \alpha - s$, which implies that the market clearing price equals to the direct selling cost. We define the supplier’s direct channel strategy in this case as “no direct selling with threat of encroachment” (NDT). Indeed, under supplier’s strategy “NDT”, the buyer sells a large enough quantity and leaves no profit margin for the supplier in the downstream market. Hence, the strategy “NDT” emphasizes the case when the supplier can influence the buyer’s decision by simply having the option (but not necessarily exercised) of direct selling.

As a direct extension to Arya et al. (2007), the above definition for the supplier’s “NDT” strategy is based only on the buyer’s equilibrium selling quantity. We differentiate the supplier’s strategy according to its impact on the buyer’s selling behavior, that is, we will look at the buyer’s equilibrium selling quantity in the second period in order to determine whether the direct channel poses a threat or not when $q_s = 0$. However, since our model also considers inventory decision for the buyer, it is possible that the buyer’s inventory withholding behavior may display different patterns for the same strategy used by the supplier.

### 3.3.2 Two-Period Model with Direct Channel Only

To facilitate our discussion in the following section, we modify Arya et al. (2007)’s model to be comparable to ours. Consider a two-period model: the first period is a basic dyadic supply chain model and the second period is the same model as in Arya et al. (2007). Hence, the two periods are unrelated and this model is in effect our main model without the inventory withholding option; model framework is shown in Figure 3.1(c) with the inventory arrow removed.
Since both our model and the benchmark model are two-period, this makes it a fair comparison in terms of players’ profits. Moreover, results from Arya et al. (2007) and its extension can be easily compared with this model. For better illustration, we visualize the equilibrium results for this model. Specifically, we plot the wholesale prices in both periods, the buyer’s first-period order quantity, and the players’ profits in Figure 3.2. Based on the extension in the previous section, we are able to characterize the equilibrium using a wider range of parameter for the supplier’s selling cost. As shown in Figure 3.2, the supplier’s direct channel strategy is “D” if \( s < \frac{3\alpha}{5} \), “ND\(^T\)” if \( \frac{3\alpha}{5} \leq s < \frac{5\alpha}{6} \), and “ND” if \( s \geq \frac{5\alpha}{6} \).

(a) The wholesale price in each period (\( w_1 \) and \( w_2 \)), in comparison with the benchmark wholesale price \( \hat{w} \), and the buyer’s first-period order quantity \( Q_1 \).

(b) Players’ profits, in comparison with the benchmark profits, \( \hat{\Pi}_s \) and \( \hat{\Pi}_b \).

Figure 3.2: Illustration of the equilibrium wholesale prices, order quantity, and players’ profits in the two-period model with direct channel only.

Note: (1) In both graphs, the \( x \)-axis represents the unit direct selling cost, and is divided into three intervals in which the supplier employs different strategies. (2) \( \alpha \) is normalized to 1. (3) The previous two notes will be applied to all the following figures.

In Figure 3.2(a), we observe that \( w_2 = \frac{3s-\alpha}{2} \) when the strategy is “ND\(^T\)”. This is in contrast to the case when the strategy is “D”, as shown in Arya et al. (2007). Here, \( w_2 \) is increasing in the supplier’s direct selling cost \( s \) and can be even greater than \( \hat{w} \). This means that the supplier may exploit the direct channel threat by charging a higher
wholesale price. In Figure 3.2(b), the supplier’s profit is always weakly higher than \(\hat{\Pi}_s\), for all values of \(s\). Besides, the buyer’s profit is above \(\hat{\Pi}_b\) if \(\frac{3\alpha}{4\sqrt{2}} < s < (1 - \frac{\sqrt{2}}{4})\alpha\).

This region not only contains the region identified by Arya et al. (2007), but also includes an extra region under strategy “ND\(^T\)”. Therefore, we extend the bright side of encroachment to the case where the direct channel is a threat.

### 3.4 Combined Effect of Strategic Inventory and Supplier Encroachment

In this section, we study the combined effect of strategic inventory and direct selling channel on the channel performance, including the supplier’s and the buyer’s strategies, the equilibrium prices and order/selling quantities, and their optimal profits. In the following, we first analyze our model to characterize the players’ strategies in equilibrium. Then, we describe the combined effect on wholesale prices, quantity decisions and resulting profits in three scenarios based on the buyer’s inventory holding cost (low, medium, high). In each scenario, we illustrate and discuss the comparison of the equilibrium results to the benchmark model. In addition, we note the results that are in contrast to those when only strategic inventory or direct selling option is available in order to derive insights into the interplay between the two strategies.

Since the buyer’s holding cost \(h\) and the supplier’s direct selling cost \(s\) affect the buyer’s ability to carry strategic inventory and the supplier’s ability to sell directly, in the following proposition, we establish the existence and uniqueness of the sub-game perfect equilibrium for any \((h, s)\) combination.
Proposition 9 For any \( h \geq 0 \) and \( s \geq 0 \), there exists a unique sub-game perfect equilibrium for the sequential model.

Given \( h \) and \( s \), we find the equilibrium paths, including the wholesale prices, order quantity, level of strategic inventory, and selling quantities. As a result, we characterize all possible cases in equilibrium based on the partition of the \( h - s \) plane. In Appendix B.1, we define the regions in Table B.1 and list all equilibrium results in Tables B.2-B.4. In addition, we identify the strategy choices for both players — for the buyer, we look at strategies “withholding inventory” (H) and “no withholding inventory” (NH), and for the supplier, the strategies are “direct selling” (D), “no direct selling with threat of encroachment” (ND\(^T\)) and “no direct selling (without threat of encroachment)” (ND).

Although the equilibrium contracts in different regions may differ in value, the strategy adopted by the two players may coincide in some regions. For example, although the selling quantities may differ, the supplier will sell through the direct channel in both regions 1 and 9; the buyer will withhold inventory in both regions 1 and 8 even though the quantity may differ. At the bottom of Tables B.2-B.4, we list the strategy pairs for the supplier and the buyer.

We look at three scenarios where the buyer’s inventory cost \( h \) is low, medium and high, respectively, and plot the prices, quantities and profits as function of the supplier’s direct selling cost \( s \). Without loss of generality, we assume \( \alpha =1 \) in the rest of the paper. Our approach is motivated by the thresholds on \( h \) found in Anand et al. (2008); we choose three values of \( h \) (0, 0.15 and 0.26) that are in the low, medium and high range, and close to the identified thresholds (0, \( \frac{21}{152} \) and \( \frac{1}{4} \)) in Anand et al.
Note that Figure 3.2 in § 3.3.2 can be seen as a special case where the inventory holding cost \( h \to \infty \).

### 3.4.1 Low Inventory Holding Cost \((h = 0)\)

We start our discussion with the case when \( h = 0 \). The equilibrium wholesale prices, the buyer’s order quantity and inventory, and the players’ profits are shown in Figure 3.3. The first observation is that all outcomes become constant when \( s \) is larger than a constant \((\approx 0.77)\), as shown by the shaded region. This implies that the direct channel has no effect on the supply chain and our model converges to that in Anand et al. (2008) when \( s \) exceeds 0.77. Therefore, in the following discussion, we will only focus on the range, \( s \leq 0.77 \). In addition, the \( s \)-axis is divided into three parts, corresponding to the supplier’s three strategy options (D, ND\(^T\), and ND). Our discussion will follow the order of these strategies.

**Supplier’s direct channel strategy is “D”:** In Figure 3.3(a), we see that \( w_1 < \hat{w} \) (wholesale price in the benchmark model). This is in contrast to the result in Anand et al. (2008) (shaded region), where the supplier will set \( w_1 > \hat{w} \) with the purpose of reducing the buyer’s incentive to order extra and hold excess inventory. Since the supplier can sell directly to the end-market, it does not have to exclusively rely on the buyer’s order to generate profits. In fact, in the extreme case (when \( s = 0 \)), the supplier can completely cut off the buyer in the second period. However, increasing \( w_1 \) together with its own ability of direct selling would lower the buyer’s incentive to order, which hurts the supplier’s profit in the first period. As a result, the supplier would offer a lower-than-benchmark wholesale price in the first period to encourage the
(a) The wholesale price in each period ($w_1$ and $w_2$), in comparison with the benchmark wholesale price $\hat{w}$.

(b) The buyer’s first-period order quantity $Q_1$ and inventory $I$.

(c) Players’ profits, in comparison with the benchmark profits, $\hat{\Pi}_x$ and $\hat{\Pi}_b$.

Figure 3.3: Illustration of the equilibrium wholesale prices, order quantity, inventory, and players’ profits in our model with $h = 0$. Note that Anand et al. (2008) is in the range when $s \geq 0.77$.

*buyer to order more.* The buyer, on the other hand, has an opportunity to withhold inventory.

As $s$ increases, the supplier’s direct selling ability becomes less effective and it relies more on selling to the buyer. Consequently, $w_1$ will be reduced further and $I$ will increase, which leads to the supplier’s convex profit curve in Figure 3.3(c). The increased order amount from the buyer guarantees that the supplier is still better off compared to the benchmark case. On the other hand, comparing the level of strategic inventory in the shaded area in Figure 3.3(b), the buyer’s inventory $I$ can be even higher, leading to a lower $w_2$ (compared to that in Anand et al. (2008)).
Consequently, the buyer can also be better off with large inventory and low wholesale price. Therefore, the bright side of supplier encroachment studied in Arya et al. (2007) can exist even when considering the buyer’s strategic use of inventory. Furthermore, in the presence of the direct channel, the wholesale price is lowered in both periods, and thus the double marginalization effect is alleviated. Hence, the supplier and the buyer may both benefit more than shown in Anand et al. (2008), as their respective profit curves in Figure 3.3(c) may be higher than in the shaded region.

Supplier’s direct channel strategy is “NDT”: Here, the supplier does not actually sell through the direct channel, i.e. \( q_s = 0 \). However, \( w_1, w_2 \) and \( I \) are not constant as in Anand et al. (2008). This shows that the threat of using the direct channel has an impact on the channel performance.

First, a direct comparison with Figure 3.2 shows that when strategic inventory is not an option, the supplier uses the direct channel for \( s \leq 0.6 \). However, from Figure 3.3(a), we see that the supplier’s strategy is “NDT” when \( s \) is slightly less than 0.6. As such, the buyer can strategically withhold inventory to change the supplier’s strategy and become more competitive. Indeed, the buyer can rely on using inventory to set a natural price cap, \( \alpha - I \), in the second period. Hence, if this price cap equals to the direct selling cost \( s \), the supplier will not sell a single unit in the direct channel, and the direct channel just becomes a threat (recall definition in § 3.3.1).

Second, since there is no profit from the direct channel anymore, the supplier utilizes the threat of encroachment to charge a higher wholesale price in both periods. From Figure 3.3(a), we observe that both \( w_1 \) and \( w_2 \) are initially increasing in \( s \), and \( w_1 \) can be even higher than that in Anand et al. (2008). At the same time, the buyer’s inventory holding incentive is reduced, and the strategic inventory also decreases. In
order to derive optimal profit, the supplier will lower $w_1$ again when the order quantity from the buyer becomes too low. As a result, the supplier’s profit is always above the benchmark whereas the buyer’s profit fluctuates around the benchmark profit. This, again, indicates that the bright side of supplier encroachment can be extended to the case when the direct channel is just a threat.

**Supplier’s direct channel strategy is “ND”:** Here, the supplier does not sell in the direct channel and also does not use it as a threat. According to our definition in § 3.3.1, this simply means that the buyer’s second-period selling quantity has the same functional form as if the direct channel does not exist. However, as noted earlier, the buyer’s inventory quantity may display two different patterns in this “ND” region: it first decreases in the supplier’s direct selling ability $s$, and then becomes a constant which is the same as shown in Anand et al. (2008); see Figure 3.3(b).

From Figure 3.2, we note that when strategic inventory is not an option, the supplier uses the “ND” strategy for $s \geq 5/6$, but with strategic inventory, the “ND” strategy is used for $s \geq 3/5$ (see Figure 3.3). As the supplier changes its strategy from “ND$^T$” to “ND”, since the direct channel is no longer a threat, the wholesale price $w_1$ is reduced to induce a higher order from the buyer. Then, the inventory withheld by the buyer increases to the highest level, leading the buyer’s profit to exceed the benchmark profit. Hence, the use of strategic inventory benefits the buyer the most when the supplier just changes its strategy from “ND$^T$” to “ND”. In addition, since the supplier is still better off due to a larger buyer’s order quantity, both players can be better off. Finally, we observe a discontinuity at $s \approx 0.77$, where the buyer’s profit jumps from below the benchmark profit to above and becomes constant. This
observation shows that the existence of direct channel, although not even a threat, can significantly affect the buyer’s profit compared to the benchmark case.

### 3.4.2 Medium Inventory Holding Cost \((h = 0.15)\)

For the medium inventory holding cost case, we illustrate the equilibrium outcomes in Figure 3.4. Comparing with Figure 3.3, we see a similar trend but in the following, we discuss three main differences.

First, we note that the shaded region (based on Anand et al. (2008) with no direct channel) is shifted to the right — it now ranges from \(s \approx 0.81\) instead of \(s \approx 0.77\). Essentially, when the buyer’s holding cost increases, the use of strategic inventory becomes less efficient compared to when \(h = 0\), and therefore, it would be easier for the supplier to utilize the direct channel. This rationale also applies to other regions of \(s\) with different supplier’s strategies.

Second, in Figure 3.4(a), we see that when the supplier uses strategy “D”, it starts to set \(w_1 < \hat{w}\) only when \(s\) is large enough. The reason is that with higher holding cost, the buyer will withhold inventory only when the wholesale price is low enough to make profit. However, the supplier will offer a lower wholesale price only if its direct selling cost is sufficiently large; otherwise selling through the direct channel can be more profitable for the supplier.

Therefore, the buyer’s inventory strategy becomes cost-inefficient and cannot provide enough incentive for the supplier to offer price discount when direct channel is still an efficient option. As a result, starting at some \(s > 0\), the supplier discontinuously reduces \(w_1\) and then the buyer holds positive inventory; see Figure 3.4(b). In addition, in the medium range of \(s\), the supplier may set \(w_2 > w_1\), but the buyer still
(a) The wholesale price in each period ($w_1$ and $w_2$), in comparison with the benchmark wholesale price $\hat{w}$.

(b) The buyer’s first-period order quantity $Q_1$ and inventory $I$.

(c) Players’ profits, in comparison with the benchmark profits, $\hat{\Pi}_s$ and $\hat{\Pi}_b$.

Figure 3.4: Illustration of the equilibrium wholesale prices, order quantity, inventory, and players’ profits in our model with $h = 0.15$. Note that Anand et al. (2008) is in the range when $s \geq 0.81$.

...does not withhold any inventory. These observations indicate that higher inventory holding cost makes the use of strategic inventory a less effective option in competing with the supplier’s direct channel.

Third, the impact of higher inventory holding cost on profits has interesting implications. Let $\Pi_b(s, h)$ be the buyer’s profit for any given $s$ and $h$, and $\Pi_s(s, h)$ be similarly defined for the supplier. Then, for all $s$ in the shaded regions in Figures 3.4(c) and 3.3(c), $\Pi_b(s, 0) > \hat{\Pi}_b > \Pi_b(s, 0.15)$; indeed this observation confirms the findings in Anand et al. (2008) that the buyer’s profit decreases in its holding cost. However, when the supplier’s direct channel comes into play, the negative cor-
relation between the buyer’s profit and the holding cost may be reversed. In fact, there exist some \( s \) values, e.g. \( s \) near \( 3/5 \), such that \( \Pi_b(s, 0) < \hat{\Pi}_b < \Pi_b(s, 0.15) \); see Figures 3.3(c) and 3.4(c). Therefore, in the presence of the direct channel, higher inventory holding cost may benefit the buyer in some cases. On the other hand, the supplier’s profit, although still above the benchmark, becomes smaller as \( h \) increases. Moreover, for any \( s \), we can show that \( \Pi_s(s, 0) \geq \Pi_s(s, 0.15) \) (detailed proof available upon request), which can be easily verified by noting the change in the supplier’s profit in the shaded regions from Figures 3.3(c) to 3.4(c). Hence, when the buyer becomes less cost-efficient, the supplier may not necessarily benefit from it. In our case, higher holding cost for the buyer also limits the supplier from charging a high wholesale price (\( w_1 \) decreases near \( s = 3/5 \)), which is the main driver of the changes in players’ profits as mentioned above.

### 3.4.3 High Inventory Holding Cost (\( h = 0.26 \))

We illustrate the equilibrium outcomes in Figure 3.5. The shaded region now starts at \( s = \frac{5}{6} > 0.81 \) (which is the threshold in Figure 3.4) and the supplier does not offer a price discount in the first period when using strategy “D”. Therefore, the effect of higher cost in making the use of inventory as a less effective strategy still applies in this case. Moreover, similar to the observation in § 3.4.2, when the buyer (supplier) becomes less cost-efficient, the supplier (buyer) may not necessarily benefit from it.

An interesting finding emerges when we compare the result in Anand et al. (2008) concerning the threshold at which the buyer would withhold inventory. As reported in their paper, the buyer will not withhold inventory when \( h \geq 1/4 \). However, even with \( h = 0.26 \), our findings indicate that positive strategic inventory is possible in
certain cases. To explain this, note that when inventory $I$ is positive, the supplier sets a lower wholesale price $w_2$ in the second period compared to the case when strategic inventory option is not available (see Figures 3.2(a) and 3.5(a)). As such, even though holding inventory is costly and profits are lower than that in the benchmark case, using strategic inventory allows the buyer to limit the loss from competing with the direct channel (see 3.2(b) and 3.5(b)).

![Diagram showing equilibrium wholesale prices, order quantity, inventory, and players' profits](image)

(a) The wholesale price in each period ($w_1$ and $w_2$), in comparison with the benchmark wholesale price $\hat{w}$, and the buyer’s order quantity $Q_1$ and inventory $I$.

(b) Players’ profits, in comparison with the benchmark profits, $\hat{\Pi}_a$ and $\hat{\Pi}_b$.

Figure 3.5: Illustration of the equilibrium wholesale prices, order quantity, inventory, and players’ profits in our model with $h = 0.26$. Note that Anand et al. (2008) is in the range when $s \geq \frac{5}{6} \approx 0.83$.

We also note that the buyer may withhold inventory for any finite $h > 0$ when $s$ approaches $\frac{5}{6}$ from below. This extends the result in Anand et al. (2008), where the threshold $h_0 = \frac{1}{4}$ always exists. We formalize this finding in the following proposition.

**Proposition 10** For any $h > 0$, there exists an $\epsilon > 0$, such that $I > 0$ for all $s \in (\frac{5}{6} - \epsilon, \frac{5}{6})$. Moreover, the buyer’s total inventory cost is less than $\frac{11}{72}$.

Although $h$ is large, total holding cost for the buyer is bounded by a constant, because the withheld inventory is small in quantity, as shown in Figure 3.5(a). Indeed, the
strategic inventory would asymptotically converge to zero and Figure 3.5 would be identical to Figure 3.2 (inventory option completely removed) as \( h \to \infty \).

### 3.4.4 Summary of Main Results

We present below the key takeaways concerning the combined effect of strategic inventory and supplier encroachment and offer related managerial insights.

- The strategic use of inventory by the buyer can make the supplier choose a less aggressive direct channel strategy; e.g. switch from “D” to “ND^T“ and from “ND^T“ to “ND“ (comparing Figures 3.2 and 3.3). Further, when the supplier uses the direct channel as a threat, the buyer may withhold inventory with any finite inventory holding cost (see Proposition 10).

- In the presence of strategic inventory, the supplier will never be worse off from the direct selling channel whereas the buyer can be better off only in certain situations. Thus, the bright side of supplier encroachment can occur even when considering buyer’s strategic use of inventory (comparing Figures 3.3(c), 3.4(c) and 3.5(b)). Although the strategic use of inventory may not guarantee the buyer’s profit to be greater than the benchmark profit, it provides the buyer with a competitive option to counter the direct channel challenge from the supplier. Hence, even when it is costly, using strategic inventory can alleviate the damage caused by the supplier’s direct channel threat (see Figures 3.2(b) and 3.5(b)).

- The buyer’s profit may increase even when its holding cost is higher (comparing Figures 3.3(c) and 3.4(c) around \( s = \frac{3}{5} \) or Figures 3.4(c) and 3.5(b) in the shaded
region); similarly, the supplier’s profit may also increase in its selling cost (see profit curves in Figures 3.3(c), 3.4(c) and 3.5(b)). Therefore, being less cost-efficient in a vertically competitive supply chain may not always reduce the player’s profit.

- When the buyer’s inventory holding cost becomes higher, the supplier’s profit may decrease (comparing Figures 3.3(c) and 3.4(c) or Figures 3.4(c) and 3.5(b)); and, when the supplier’s direct selling cost becomes higher, the buyer’s profit may also decrease (see profit curves in Figures 3.3(c), 3.4(c) and 3.5(b)). As such, having a less cost-efficient competitor in the vertical competition may not always benefit the other player.

### 3.5 Simultaneous Quantity Competition

In the second period of the sequential model, the buyer and the supplier make quantity decisions in a sequential manner. Essentially, the buyer has a first-mover advantage as it chooses the order quantity and selling quantity before the supplier decides on the selling quantity through the direct channel. In this section, we consider an alternative timing structure where in the second period, the buyer’s and the supplier’s quantity decisions are all made simultaneously. Similar timing of the game has been considered by Arya et al. (2007) and Ha et al. (2015).

In the following, we first describe the simultaneous model and derive the equilibrium outcome. Similar to the analysis in § 3.4, we then study the combined effect of the use of strategic inventory and direct channel and identify the differences between
the two models. This allows us to analyze the effect of the buyer losing its first-mover advantage on the profits of both players.

3.5.1 Model Description and Equilibrium Results

The sequence of moves in the simultaneous model is the same as in the sequential model in period one, but differs in period two as follows:

In period two:

Stage 1: The supplier quotes a wholesale price \( w_2 \).

Stage 2: The buyer and the supplier independently and simultaneously choose the order quantity \( Q_2 \) and selling quantity \( q_2 \), and the direct selling quantity \( q_s \), respectively. The market clearing price is \( p = \alpha - (q_s + q_2) \).

Essentially, the two players make selling quantity decisions simultaneously since the buyer’s quantity decisions (order quantity \( Q_2 \) and selling quantity \( q_2 \)) are related via inventory by \( Q_2 = (q_2 - I)^+ \) and \( q_2 \leq Q_2 + I \). Therefore, the major difference between the two models is that the buyer loses the first-mover advantage. This change in the game structure alters the players’ incentives to employ their strategies and thus results in different equilibrium outcomes.

**Proposition 11** For any \( h \geq 0 \) and \( s \geq 0 \), there exists a unique sub-game perfect equilibrium for the simultaneous model.

Similar to the sequential model, we tabulate all possible equilibrium outcomes for the simultaneous model (see Appendix B.2). There are nine regions in the partition of the \( h - s \) plane, each of which sees a unique course of actions from the supplier and
the buyer. The regions are defined in Table B.5 and the equilibrium is detailed in Tables B.6 - B.9. Moreover, based on our previous definition of the players’ strategies, we list the supplier and the buyer’s strategy pairs in different regions at the bottom of Tables B.6 - B.9.

3.5.2 Two-Period Simultaneous Model with Direct Channel Only

Similar to § 3.3.2, we first consider a two-period model with direct channel only under the simultaneous setting: the first period is a basic dyadic supply chain model and the second period is the one described in § 3.5.1. Hence, the two periods are unrelated and this model is in effect our simultaneous model without the inventory withholding option. The second-period model has also been considered in Arya et al. (2007) and they derive the effect of supplier encroachment in the simultaneous setting when the supplier actually sells through the direct channel. As noted earlier, they do not consider the “NDT” strategy in their model and we extend their work to the situation when the supplier sells nothing through the direct channel. We illustrate the wholesale prices, buyer’s first-period order quantity and players’ profits in Figure 3.6.

In Figure 3.6, the supplier’s direct channel strategy is “D” when $s < \frac{5}{7}$, “NDT” if $\frac{5}{7} \leq s < \frac{3}{4}$, and “ND” if $s \geq \frac{3}{4}$. Comparing with Figure 3.2, we note that the supplier is more likely to sell through the direct channel in the simultaneous model. As the buyer loses the first-mover advantage, its order incentive is diminished and the supplier may engage in direct selling even if it is not very cost efficient. Moreover, unlike in Figure 3.2(a), now the supplier cannot set wholesale price greater than $\tilde{w}$ with strategy “NDT”. This shows that the direct channel threat is also diminished. In-
(a) The wholesale price in each period ($w_1$ and $w_2$), in comparison with the benchmark wholesale price $\hat{w}$, and the buyer’s first-period order quantity $Q_1$.

(b) Players’ profits, in comparison with the benchmark profits, $\hat{\Pi}_s$ and $\hat{\Pi}_b$.

Figure 3.6: Illustration of the equilibrium wholesale prices, order quantity, and players’ profits in the two-period simultaneous model with direct channel only.

deed, since the selling quantity decisions are made simultaneously, the direct channel threat, which mainly implies potential moves in the future, simply cannot influence the buyer as much as in the sequential model. As a result, in Figure 3.6(b), we see that the supplier’s profit may drop below the benchmark case in both “D” and “ND$^T$”. This indicates that the supplier cannot use the direct channel as efficiently as in the sequential model. Moreover, the bright side of supplier encroachment disappears in the simultaneous setting as the supplier and the buyer cannot be both better off for any value of $s$.

The above discussion is on how the timing of quantity decision affects the supplier’s direct selling strategy when the buyer does not have the strategic inventory option. Next, we include the impact of the buyer’s strategic inventory on the timing effect by comparing the combined effect of the two strategies in sequential and simultaneous models.
3.5.3 Comparison between Sequential and Simultaneous Models

We first summarize the combined effect of strategic inventory and direct channel in three scenarios (for $h = 0, 0.15$ and $0.26$) for the simultaneous model as we did for the sequential model and then point out the differences between them. We will start the discussion with the case when $h = 0$.

**Low Inventory Holding Cost ($h = 0$)**

The supplier’s wholesale prices, buyer’s order quantity and inventory, and the players’ profits are shown in Figure 3.7. The change in timing of quantity competition leads to several differences which we highlight by comparing Figures 3.3(a) and 3.7(a).

First, the shaded region, where the direct channel has no effect on the supply chain, begins at $s \approx 0.72$ which is lower than the threshold in the sequential model (equal to 0.77). This means that the buyer can now use strategic inventory to eliminate the supplier’s use of the direct channel even when the direct selling cost is not as high. *Thus, the direct channel threat on the buyer is diminished when the two players make selling quantity decisions simultaneously.*

Second, when the supplier sells through the direct channel in the two models (region “D”), for the same $s$, the buyer’s strategic inventory is larger in the simultaneous model even with a lower wholesale price discount. *Essentially without the first-mover advantage in the simultaneous model, the buyer is more likely to rely on the strategic use of inventory even if the wholesale price is higher.*

Third, comparing the supplier’s profit curves in Figures 3.3(c) and 3.7(c) indicates that the supplier makes less profit in the simultaneous model for any direct selling cost
(a) The wholesale price in each period ($w_1$ and $w_2$), in comparison with the benchmark wholesale price $\hat{w}$.

(b) The buyer’s first-period order quantity $Q_1$ and inventory $I$.

(c) Players’ profits, in comparison with the benchmark profits, $\hat{\Pi}_s$ and $\hat{\Pi}_b$.

Figure 3.7: Illustration of the equilibrium wholesale prices, order quantity, inventory, and players’ profits in the simultaneous model with $h = 0$. Note that Anand et al. (2008) is in the range when $s \geq 0.72$.

$s$ (detailed proof available upon request); for example, part of supplier’s profit curve in Figure 3.7(c) are very close to the benchmark case. The underlying reasons are as follows. When the supplier sells through the direct channel, the buyer will withhold more inventory than before to offset the loss of first-mover advantage, leading to more competition. When the supplier does not sell directly, the encroachment threat is weakened due to the simultaneous setting, and so the buyer does not purchase as much inventory as before. In either case, the supplier’s profit is hurt. *Hence, when the buyer loses the first-mover advantage, it may not benefit the supplier.* On the other
hand, when $s$ is large, the buyer’s profit is generally higher in the simultaneous model; see Figures 3.7(c) and 3.3(c). Again, this is because the buyer does not need much inventory to counter the direct channel threat, and thus can save on the purchasing cost. Therefore, under certain conditions, the buyer may benefit from losing the first-mover advantage.

**Medium Inventory Holding Cost ($h = 0.15$)**

In Figure 3.8, we show the results for $h = 0.15$ in the simultaneous model. A direct comparison between Figures 3.7 and 3.8 shows that the shaded region now starts at $s \approx 0.74$ versus $s \approx 0.72$ in Figure 3.7.

This can be explained by the same argument that increased holding cost makes the use of strategic inventory less effective in competing with the direct channel. However, the starting point for the shaded region is less than that in Figure 3.4 ($s = 0.81$). Therefore, on losing the first-mover advantage, the buyer actually uses the strategic inventory more effectively even with higher inventory holding cost.

In Figure 3.8(c), we also note that the supplier can be worse off compared to the benchmark case which is in contrast to the findings in Figure 3.4(c). When the supplier uses the “ND$^T$” strategy, due to higher holding cost, the buyer is not inclined to order excess inventory especially without the first-mover advantage. As a result, the reduction in the buyer’s order size hurts the supplier’s profit. However, the buyer can be better off when the supplier uses strategy “ND$^T$”, as shown in Figure 3.8(c); this is not the case in the sequential model. Hence, even with higher inventory holding
(a) The wholesale price in each period ($w_1$ and $w_2$), in comparison with the benchmark wholesale price $\hat{w}$.

(b) The buyer’s first-period order quantity $Q_1$ and inventory $I$.

(c) Players’ profits, in comparison with the benchmark profits, $\hat{\Pi}_s$ and $\hat{\Pi}_b$.

Figure 3.8: Illustration of the equilibrium wholesale prices, order quantity, inventory, and players’ profits in the simultaneous model with $h = 0.15$. Note that Anand et al. (2008) is in the range when $s \geq 0.74$.

cost, the buyer may benefit from losing the first-mover advantage whereas the supplier may become worse off.

High Inventory Holding Cost ($h = 0.26$)

The equilibrium results with $h = 0.26$ are shown in Figure 3.9. On comparing with Figure 3.5, we find that the buyer withholds inventory in the sequential model to change the supplier’s strategy from “ND$^T$” to “ND”, whereas in the simultaneous model, inventory is used to change the supplier’s strategy from “D” to “ND$^T$”. Fur-
thermore, the buyer may hold inventory even when the supplier sells through the direct channel in Figure 3.9(a), which is not the case in the sequential model (see Figure 3.5(a)). This implies that the buyer depends more on the use of strategic inventory even with high holding cost ($h = 0.26$).

![Diagram](a) The wholesale price in each period ($w_1$ and $w_2$), in comparison with the benchmark wholesale price $\hat{w}$, and the buyer’s order quantity $Q_1$ and inventory $I$.

![Diagram](b) Players’ profits, in comparison with the benchmark profits, $\hat{\Pi}_s$ and $\hat{\Pi}_b$.

Figure 3.9: Illustration of the equilibrium wholesale prices, order quantity, inventory, and players’ profits in the simultaneous model with $h = 0.26$. Note that Anand et al. (2008) is in the range when $s \geq \frac{3}{4} = 0.75$.

However, as $h$ increases, we can show that the buyer will not withhold inventory if the holding cost is larger than 0.3, which is in contrast to Proposition 10 in the sequential model\(^6\). Therefore, in contrast to the sequential model, strategic inventory is not used when the holding cost is sufficiently high ($h > 0.3$).

### 3.5.4 Summary of Main Results

We present below the summary of the key differences between the sequential and simultaneous models.

\(^6\)In fact, Figure 3.6(b) shows that when the supplier does not sell directly, the buyer is better off compared to the benchmark case (that is, when inventory holding cost, $h \rightarrow \infty$).
• When the buyer loses the first-mover advantage in the simultaneous model, the supplier becomes more aggressive and will sell through the direct channel even with higher direct selling cost (comparing Figures 3.5 and 3.9). On the other hand, the buyer is more likely to rely on using strategic inventory when the supplier sells through the direct channel as it will withhold inventory even with higher holding cost (comparing Figures 3.5(a) and 3.9(a)). However, when the holding cost is sufficiently high, the buyer will not withhold inventory which contrasts from the result in Proposition 10 for the sequential model.

• When the direct selling channel poses a threat, that is, strategy “ND_T” is used by the supplier, the buyer’s profit may stay above the benchmark case which is not seen in the sequential model (comparing Figures 3.4(c) and 3.8(c) or Figures 3.5(b) and 3.9(b)). This shows that, under certain conditions, the loss of first-mover advantage may actually benefit the buyer.

• Unlike the sequential model, the supplier’s profit may now be lower than the benchmark case (see Figures 3.8(c) and 3.9(b)); furthermore, the win-win situation for the players disappears when the inventory holding cost is high (see Figure 3.9(b)). Therefore, having a less efficient competitor in the vertical competition may sometimes be undesirable for the players.

3.6 Concluding Remarks

Firms in a decentralized supply chain often adopt conflicting strategies to seek a more competitive edge. For example, the buyer may strategically withhold inventory to compete with the supplier, while the supplier may launch a direct selling channel
to compete with the buyer in the downstream market. We considered a two-period model of a dyadic supply chain where both the buyer and the supplier have the option of adopting their aforementioned strategies. While the strategic use of inventory and direct selling channel have been individually shown to increase supply chain profits, our work is the first to study the combined use of the two strategies. Under the sequential framework, we showed that the supplier may reduce the first-period wholesale price in order to induce the buyer to hold inventory, and the buyer may use the inventory to make the supplier choose a less aggressive direct channel strategy. Furthermore, we found that for any finite holding cost, the buyer may withhold strategic inventory. Essentially, strategic inventory is a tool for the buyer to be more competitive with the supplier even though it may still make the buyer worse off. Although the players’ conflicting strategies add more tension to the vertical competition, the win-win situation for the supplier and the buyer can still occur.

We also studied the simultaneous model, where the buyer no longer has the first-mover advantage in the downstream market quantity competition. We found that the buyer relies more on using strategic inventory and the supplier becomes more aggressive in using the direct selling channel. At the same time, the direct channel threat is diminished and the buyer may not need as much inventory as before to change the supplier’s strategy. As a result, the buyer may be better off when the direct channel poses a threat. On the other hand, the supplier is always better off when the inventory holding cost is small; otherwise its profit can be below the benchmark case. Therefore, having a more competitive supply chain partner may sometimes be desirable under vertical competition.
Our research can be extended in a number of ways. Incorporating demand uncertainty into our model would be an interesting theme for future research. Second, we assume that both supply chain members have perfect information about each other. However, in some cases, the buyer’s inventory is private information (Zhang et al., 2010). Similarly, the supplier may also have private information about its direct selling cost. Third, instead of quantity competition, the supplier and the buyer may engage in price competition in the downstream market if encroachment takes place. While the supplier may have reduced interest in direct selling in this case, analyzing the buyer’s inventory strategy would be an interesting direction for future research.
CHAPTER 4

Peer-to-Peer Sharing Platforms with Quality Differentiation: Manufacturer’s Strategic Decision under Sharing Economy

4.1 Background

The recent advances in technology, especially internet-based platforms and algorithms, have significantly fueled the growth of the peer-to-peer product sharing market across a wide range of sectors, such as hospitality (Airbnb, CouchSurfing), retailing (SnapGoods, Tradesy) and transportation (Uber, Lyft). In what is called sharing economy or collaborative consumption, individuals can monetize their under-utilized resources of basically all kinds. Similarly, people can derive value from products that they do not necessarily own. As such, collaborative consumption has the advantages of enhancing consumers’ utility on the one hand and reducing the overall waste for the society on the other hand. Therefore, such businesses have been very well received, and have the potential of shaking many traditional industries. Indeed, a recent report (PwC, 2015) shows that 44% of US consumers are familiar with sharing economy and 72% of those who have tried it believe that they will be a collaborative consumer
in the near future; moreover, the report estimates the global revenue of the sharing economy to increase over twenty times in less than ten years.

In the presence of sharing economy, new economic implications have emerged for everyone involved in the market, including the consumers and the platform managers (Benjaafar et al., 2018). Take the peer-to-peer ride-hailing businesses (exemplified by Uber and Lyft) for example. The consumers are re-thinking the value of the ownership of a car, and are comparing it with just having the access to mobility. The platforms, on the other hand, are trying to leverage on such a paradigm shift in consumer preference by providing the service of matching those who are willing to share with those who have needs. As a result, the consumer behaviors are reshaped. To wit, the consumers with relatively small surplus who used to buy cars may now be diverted by the peer-to-peer sharing market and become platform users instead of buyers. A direct consequence of these implications is an even more profound ripple effect on the upstream car manufacturers. Indeed, as their traditional buyer markets are restructured by the emerging peer-to-peer ride-hailing platforms, the manufacturers are facing both challenges and opportunities from the sharing economy. Attempts to adapt to and even thrive in the new environment have been seen from several car manufacturers in practice. Basically, these firms are observed to make two kinds of strategic decisions to engage in the downstream collaborative consumption.

First, some manufacturers are establishing partnership with the existing peer-to-peer sharing platforms. They implement investment plans to make their products more attractive among consumers. For example, Toyota has made strategic investment in Uber with undisclosed amount (Bluiyan, 2016), and come up with flexible leasing programs for Uber’s Toyota drivers. For another example, GM and Lyft an-
nounced strategic alliance in 2016, which features GM’s $500 million investment and it becoming the preferred vehicle for Lyft drivers (Shontell, 2016). Through the partnership with the existing platforms, these manufacturers may be able to grasp more market by offering attractive terms to consumers.

In contrast to the above approach, the other kind of strategy is more direct, embracing the peer-to-peer sharing market in a more active way. Specifically, manufacturers are building their own platforms to facilitate peer-to-peer ride-hailing services featuring their own products. An illustrating example comes from Tesla’s strategic plan of building a shared fleet from the owners (Musk, 2016); in addition, BMW has employed a similar strategy by launching its peer-to-peer ride-hailing program, ReachNow, in Seattle (ReachNow, 2016). Being more aggressive, this strategy enables firms to operate their own platforms and compete with the existing ones. In this paper, we will focus on this platform-building strategy, and investigate its viability in depth. Compared to the manufacturers that employ the first approach, those trying to build their own sharing platforms belong to the relatively higher-end in brand. Indeed, the platform-building strategy may be especially effective for luxury car manufacturers, because the high-end brand name helps achieve two objectives that contribute the most to the potential success: Quality differentiation and market segmentation. In the following, we elaborate this point by referencing the motivating example of BMW’s DriveNow.

First, the quality of the car, as perceived via the general notion of the brand name, plays a vital role in the platform-building strategy. As Richard Steinberg, CEO of DriveNow, points out (PwC, 2015):

\[\text{We used to be the provider of premium cars and now we’re the provider of premium mobility services as well as premium cars... [Millenials] are not}\]
interested in parking, insurance, vehicle acquisition. But they still have mobility needs. Public transit, Uber, all the various sharing tools are at their disposal – but there’s not personal mobility. So that’s where we fit in.

Indeed, compared to using the service from the existing platform, which offers rides on random, and mostly non-luxurious, cars, the consumers who use the manufacturer-built, exclusive platform will be able to ensure the quality of their rides. Therefore, the manufacturer’s platform is likely to be more competitive with respect to the consumers who are brand-sensitive but would not choose to purchase the higher-end cars. Put differently, the platform-building strategy enables BMW to compete with the existing platforms through quality differentiation.

Second, due to the quality differentiation, the manufacturer may utilize the self-built platform to segment its potential market, and even to reach out to the market segment that used to be untouchable. One common doubt on the platform-building strategy stems from the risk of cannibalizing the sales of the manufacturer’s main product; indeed, as mentioned before, the previous buyers may be converted to non-buyers who use the platform service. However, when taking market positioning and targeting into account, the strategy may be more promising than thought. On sharing platforms and cannibalization, Richard Steinberg comments:

*The market BMW Group competes in is a premium market for our new car sales. And the younger generation that’s using car share and using our service is not necessarily in the market for a premium automobile. They might be interested in a base or a non-premium car or a used car, but not so much in the premium category. So, are we cannibalizing ourselves? No. Are we eating into some other automakers’ businesses? Perhaps.*

Based on the CEO’s argument, by targeting on the proper market segment, the manufacturer-built platform may not only preserve the sales of the manufacturer’s
main product, but also have the potential threat to encroach the market of other non-luxury car manufacturers.

The above justification of the DriveNow program is plausible, but lacks rigorousness. For example, since the matching service from the sharing platform is somewhat dynamic, which depends on the availability of cars to be shared and the amount of needs for rides, it is not obvious that BMW’s premium market will be absolutely free of cannibalization. Besides, how exactly the quality level influences the effectiveness of the platform-building strategy is unclear. The primary objective of this paper is, therefore, to build a formal model to analyze the platform-building strategy for a premium product manufacturer facing an existing peer-to-peer sharing platform in the downstream market. In doing so, we manage to study beyond the motivating example of BMW, and provide insights for firms in similar settings. Our research questions are as follows.

(1) With the manufacturer building a premium (higher quality) peer-to-peer sharing platform, how does it compete with the existing platform and what would the equilibrium market outcome, if exists, be like?

(2) Given a certain quality level of the premium product, when adopting the platform-building strategy, would the manufacturer

- cannibalize its own sales of the premium product?
- encroach other non-premium manufacturers’ markets?
- achieve higher profit than the status quo?

Our main result indicates that the premium manufacturer can always achieve higher profit by implementing a platform-building strategy. By comparing with a
model without sharing platforms, we also find the manufacturer-built platform may change the manufacturer from losing to winning after the emergence of peer-to-peer product sharing. Other research questions on the manufacturer’s and non-premium manufacturers’ sales will be answered through numerical study in the future study.

After a sharing platform $P_1$ appears, the consumers in the premium market can fill their needs on demand through the rental service from $P_1$ rather than purchasing the premium product. It seems that consumers sharing behavior creates a competitor for the manufacturer, which challenges the manufacturer’s monopoly power and can cannibalize its sales. Though $P_1$ adds extra value of sharing to the product owners, due to the inconvenience in operations (i.e. the majority of shared product on $P_1$ is low quality), the existing platform may not provide the service to differentiate the premium and non-premium products shared on the platform. Without the high quality demonstration, the value of sharing the premium product can be very low. Under some conditions, the added value of sharing the premium product may not make up the loss in sales for the premium manufacturer, which deteriorates the manufacturer’s profit. By launching an exclusive product sharing platform $P_2$, the manufacturer can help the owners to signal the high quality of their shared product to renters and drastically increase the value of sharing its product. Moreover, $P_2$ may help the manufacturer to encroach the non-premium market. Therefore, the value of sharing the premium product can be so high that it can make the ownership more attractive and outweigh the benefit of using a shared product. Then the manufacturer can get more sales and achieve higher profit even than that without peer-to-peer product sharing.
The rest of this paper is organized as follows. § 4.2 reviews the related works in the extant literature. § 4.3 carefully sets up the model and describes the model formulation in details. In § 4.4, we analytically solve the sub-game to establish the existence and uniqueness of the equilibrium for the platform pricing, obtain the manufacturer’s optimal decisions and examine the viability of the platform-building strategy. Finally, § 4.5 concludes with a discussion on the managerial relevance of our results and the future study. All proofs are relegated to the appendix.

4.2 Literature Review

Sharing economy, a relatively new concept, and its derivative business models in various industry sectors have been receiving considerable attentions. Since the core of the sharing economy is the peer-to-peer sharing platform along with the two-sided market it services, much research has been devoted to understanding the economic incentives of the platform managers and the consumers who use the service, as well as to addressing the marketing and operations issues they face. First, (Benjaafar et al., 2018) propose an innovative model formulation to understand how a peer-to-peer sharing platform matches supply with demand from the two-sided market. Fraiberger and Sundararajan (2017) analyze the segmentation of the sharing market and identify the target customers who tend to use the platform. These papers provide some fundamental economic implications of the peer-to-peer sharing market. Then, several other papers focus on the platform’s pricing issues, which include pricing for the demand (renters) and setting wages for the supply (owners). Banerjee et al. (2016) show that static pricing outperforms dynamic pricing, but is less robust to the system parameters. Taylor (2017) and Bai et al. (2016) consider the price and
wage decisions based on several operational factors in the on-demand service setting. Moreover, Cachon (2003) and Hu and Zhou (2017) also study the pricing problem of the platform in order to understand the matching mechanism in more specific settings. Finally, Gurvich et al. (2016) tackle the similar problem but focus on the self-scheduling issue that may arise in the car-sharing context.

Our paper is rooted in the nascent and growing literature on sharing economy, and follows the basic analytical approaches and modeling techniques laid out by the above works. However, our focus is not on the sharing platform or the consumers, but on the upstream manufacturer who supplies the physical products that are shared in the downstream market. Therefore, this paper is more related to one interesting branch of the sharing economy literature, which studies its impact on other channel players. Using different model formulations, Jiang and Tian (2016) and Weber (2016b) both show that, although peer-to-peer sharing intuitively reduces the incentive for the consumers to own a product, it can benefit the retailer who sells to the sharing market if the production cost is relatively high. Moreover, Razeghian and Weber (2017) focuses on the product durability and show that the presence of a peer-to-peer sharing platform never decreases the incentive for the retailer to provide durability and the optimal durability increases in production cost. In a two-tier supply chain setting, Tian and Jiang (2018) explore the upstream manufacturer’s capacity and wholesale price decisions when its buyer is selling to a sharing market.

All the above papers assume that the sharing platform belongs to a third party. To the best of our knowledge, there are only a couple of papers that look at manufacturer-built peer-to-peer sharing platforms. One example is Weber (2016a), who studies how to set retailing price and the sharing tariff for firm’s “smart” product. The platform is
firm-run in that peer-to-peer sharing may be done under the supervision of the firm. The other paper, which is more closely related to ours, is Abhishek et al. (2018). They study and compare three business models for the manufacturer in the presence of a downstream sharing market. In one of the business models, the manufacturer sells and also runs a peer-to-peer sharing platform for its own product. They find that it is optimal for the firm to run a frictionless platform in this case.

Similar to Abhishek et al. (2018), we also focus on the emerging business model with manufacturer-built sharing platform. However, our paper is different from theirs in two important ways, which are our main contributions to this stream of literature. First, in our model, the manufacturer also needs to consider an existing platform when adopting the platform-building business model. Hence, our paper is among the few first ones to analyze the case where multiple sharing platforms co-exist and compete with each other. Second, we explicitly model the quality level as perceived via the brand name of the manufacturer that considers to build a sharing platform, and base our main results on this important marketing factor. Indeed, the product quality without doubt plays a vital role when the manufacturer is selecting a proper business model.

4.3 Model Description

In this section, we outline the model setup. We start from the supply side of the market by introducing the upstream manufacturers as the product providers and the peer-to-peer sharing platforms as the service provider. Then, we characterize the market segments that we are interested and the consumer behaviors. These basic model ingredients and their relations are illustrated by Figure 4.1 below. To fur-
ther detail our model formulation, we characterize the consumers utility, the market clearing mechanism, and the sequence of events.

4.3.1 Manufacturers and Products

Consider a premium manufacturer and a group of non-premium manufacturers (see Figure 4.1). The premium manufacturer is our focal manufacturer. In other words, we focus on maximizing its profit and provide insights for its strategic decision makings. This manufacturer’s product is perceived as high quality (hence premium) product; e.g., it may have a higher end brand name. On the other hand, the group of non-premium manufacturers are the non-focal counterpart, which supply similar products with relatively lower quality. We treat this group of manufacturers as a whole, and therefore only look at the average quality level. One example of such setting is mentioned in the comment made by the CEO of DriveNow. There, BMW is the premium manufacturer with a more luxurious brand, and the “other automakers” (such as Toyota, Ford, Hyundai, and so forth) that produce base or non-premium cars belong to the group of the non-focal manufacturers. Similarly, such model formulation also fits when Daimler or Tesla is in consideration. We remark that our model does not attempt to explicitly characterize the competition between the premium and non-premium manufacturers. Rather, we mainly examine the impact of the focal manufacturer’s strategy on its own market shares, as well as that of the non-premium manufacturers.

For the non-premium manufacturers, we assume that on average the perceived quality level is $q_1 > 0$, which is exogenously given; and for the premium manufacturer, 

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7Hereafter, when the context is clear, we refer to the premium manufacturer simply as “the manufacturer” for short.
its product’s quality level is \( q_2 > q_1 \). One of the main objectives of our paper is to understand the role of the quality level \( q_2 \) in the focal manufacturer’s strategy decision process. Therefore, we do not consider \( q_2 \) as a decision variable, but only observe its impact on the focal manufacturer’s equilibrium outcomes such as market share and profit. Besides, since the quality level we study here mostly relates to the brand perception/awareness of a firm, it is not easy to change in the short run.

4.3.2 Peer-to-Peer Sharing Platforms

A peer-to-peer sharing platform provides service to match consumers who own the product (owners) with those who need to use it (renters). Most likely the owner consumer will not fully utilize the product. Therefore, the platform helps make use of the otherwise wasted resource by assigning a renter to purchase a temporary use of the product from the owner. As a service provider, the platform often extracts a proportion of the rental price as its commission. Such peer-to-peer sharing platforms have been sprouting in many aspects of people’s daily life with numerous sorts of products to shared among consumers, and are embracing increasingly more popularity.

In our model, we look at two peer-to-peer sharing platforms (see Figure 4.1). First, there is an existing platform \( (P_1) \), which has been established for some time by some firm that is outside the industry of manufacturing the shared product. In other words, the platform simply provides the resource-matching service, but does not participate in producing the resource. Therefore, we assume that the existing platform is not able to distinguish the quality of the shared product. Indeed, since products of any brand may be shared by the owners on this platform, the renters cannot form precise expectation on the quality of the to-be-matched product (the matching process is
usually non-transparent). For example, the ride-sharing apps such as Uber and Lyft are perhaps among the earliest platforms. When using these platforms, renters may be matched to a car of many possible makers; moreover, in reality, the shared products are mostly relatively low-quality (non-premium). Therefore, we assume that the perceived quality of the shared products on $P_1$ by the renters is $q_1$. Note that this is a potential reason why the owners of the premium products are reluctant to share on this platform – the high quality will not be recognized by the renters. In addition, let $\alpha_1 \in (0, 1)$ be the commission collected by $P_1$, which is an exogenous parameter. The rental price, $p_1$, however, is endogenously determined by the matching process (to be explained later).

Second, we consider a peer-to-peer sharing platform built by the manufacturer, which we call $P_2$. This platform, in contrast to $P_1$, is exclusive to the premium product. Put differently, the manufacturer only allows its own product to be shared on the platform $P_2$. As a result, the quality of the shared products on the manufacturer-built platform is perfectly predictable, namely $q_2$. Hence, platform $P_2$ is partly targeting at those renters who are quality-sensitive and are willing to pay more to rent on such a platform that guarantees premium shared products. Back to the motivating example, the platform DriveNow is a BMW-built platform that exclusively allows BMW’s consumers to share their newly released models with peers. In this case, the renters can be sure of the product quality level. As the builder of the platform $P_2$, the manufacturer needs to determine the commission $\alpha_2$, as well as the rental price $p_2$ (again, by the matching process to be explained later). Finally, note that while $P_1$ already exists and in operation, the launch of the manufacturer-built platform $P_2$ is a strategic decision to be made.
4.3.3 Consumer Markets

Facing manufacturers as the product ownership sellers and the platforms as the product usage providers, consumers may take various actions based on their preferences. To capture essences of the implications of the manufacturer’s strategy decision, we focus on the following segments of the consumer market (see Figure 4.1).

First, there is a non-premium market, in which consumers never consider to own a premium product. Apparently, this is not the target market of the premium manufacturer with respect to the ownership transfer. Within this market, we further divide the consumers into two parts: (1) Non-owner market $M_0$. Consumers in this market never consider to own any product. They are purely renters and rely on the sharing platform to have access to the usage of a product. Take car-sharing platform for example, a non-owner market can be observed in NYC (due to inconvenient traffic conditions) and Silicon Valley (due to tech-oriented/high environmental-awareness population). Hence, if there are multiple platforms available, they will consider and compare the options of being renters on each platform. Assume that the non-owner market has size $m_0$. (2) Market of potential owners of non-premium products $M_1$. In this market, consumers will consider and compare the options of buying a non-premium product (and thus becoming owners) or becoming renters on some sharing platform. Note that, since the manufacturer-built platform $P_2$ does not admit non-premium products, these consumers, if become owners, can only use the existing platform $P_1$. Assume the size of this market to be $m_1$.

Second, there is a premium market, $M_2$. Here, the consumers, if decide to be an owner, will only consider to own a premium product. These consumers can be considered as, for example, loyal to the manufacturer, and are the target of the
manufacturer’s sales force. Moreover, as owners of a premium product, the consumers can share their products on either $P_1$ or $P_2$. On the other hand, instead of being owners, the consumers in this market can also be renters; again, they can use either platform at their choice. Let the size of $M_2$ be $m_2$. Due to the exclusive nature of the premium market, the potential owners of the premium product is likely to be dominated by that of the non-premium products. Hence, in this paper, we will make the following assumption to reflect this intuition.

**Assumption 1** *The size of the potential owners of the the non-premium market is larger than the size of the premium market. That is, $m_1 > m_2$.***

Note this assumption is practically plausible, but technically not necessary. Our analysis carries through without it.

Finally, note that all consumers in the above three markets actually have an outside option (such as taking the public transportation in the car-sharing example). Hence, unless one of the available options outlined above is preferred by the consumers, they will simply walk away and take the outside option.

### 4.3.4 Consumer Utility

The consumer utility may consist of several different parts. The main part is the value derived from the product usage. Suppose that consumers in all markets have the usage level $x \in [0, 1]$, which represent the fraction of their times in which they are in the need of consumption.\(^8\) The derived value from a unit-time usage of the product, however, may be different for each consumer. In other words, consumers heterogeneity

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\(^8\)Our assumption of single usage level for all consumers is an approximation of reality. We treat multiple usage level case later as an extension.
arises from the unit-time usage valuation, which is denoted by a random variable $v$. We assume that $v \sim U[0, 1]$ is uniformly distributed for all consumers. For other parts of the consumer utility, it depends on whether the consumer is an owner of a product or a renter on the platform. An owner consumer will share the product whenever not in use by himself and thus derive utility from the compensation; however, he derives disutility from the cost of the product ownership. A renter consumer, on the other hand, will derive disutility from the cost of renting on the platform. Finally, a consumer can always resort to an outside option, which we assume to have utility zero. The above framework, which is also used by notable previous works such as Benjaafar et al. (2018) and Abhishek et al. (2018), serves as the foundation of our
consumer utility model. In the following, we give the full description of a consumer’s
utility function, depending on which market segment he is from, which platform he
participates in, and what role he takes in the peer-to-peer sharing.

Owners Utility.

Let $u_{o_i}^{(j)}$ be the utility of a consumer in market $M_j$ as an owner on the platform $P_i$
($i = 1, 2; j = 1, 2$). Hence, for the premium market $M_2$, we have

$$u_{o_1}^{(2)}(v) = xq_2 v + (1 - x)(1 - \alpha_1)p_1 - w,$$
$$u_{o_2}^{(2)}(v) = xq_2 v + (1 - x)(1 - \alpha_2)p_2 - w. \quad (4.1)$$

The first part in the above utility expression is the consumer’s valuation on the
product usage. Different from previous works, the innovative aspect of our model
is the introduction of quality level. To investigate how quality plays a role in the
manufacturer’s strategy decision in presence of sharing platforms, we assume that
consumer’s usage valuation is proportionate to the product quality. The second part
of the utility function is the compensation for sharing the product. On platform $P_i$,
the unit-time rental price is $p_i$ (recall that $\alpha_i$ of it is extracted by the platform) and
we assume that the owner consumer’s product is available for sharing whenever it is
not used by himself, i.e., $(1 - x)$ of his time. Lastly, the consumer’s utility has a term,
$w > 0$, that accounts for the cost of ownership per unit-time (to keep the prices on the
same scale). Normally, based on the average life expectancy of the product, $w$ can be
related to the manufacturer’s product selling price; so we let $w$ be the manufacturer’s
decision variable.

For the market segment $M_1$, we can similarly formulate the consumer utility func-
tions:

$$u_{o_1}^{(1)}(v) = xq_1 v + (1 - x)(1 - \alpha_1)p_1 - w_1,$$
$$u_{o_2}^{(1)}(v) = 0. \quad (4.2)$$

Since $P_2$ is an exclusive platform, consumers who own non-premium products cannot derive any utility, and therefore $u_{o2}^{(1)} = 0$. To ease exposition and simplify analysis, we further assume that $w_1 = 0$. For one thing, the non-premium manufacturers are not our focus and their product selling prices are considered exogenous. For another, the question we are interested in is how the manufacturer’s strategy could affect the non-premium manufacturers’ market share. Hence, assuming $w_1 = 0$ is in fact to look at the case where the premium manufacturer faces very cost-efficient (consistent with the low-quality assumption) non-premium manufacturers. Therefore, if the premium manufacturer can eat some market share from this non-premium group, it can certainly do so when $w_1 > 0$.

**Renters Utility.**

Let $u_{ri}^{(j)}$ be the utility of a consumer in market $M_j$ as a *renter* on the platform $P_i$ ($i = 1, 2; j = 0, 1, 2$). First of all, note that the renter consumer’s utility is market independent. That is, a renter’s utility only depends on which platform he decides to use. For the existing platform $P_1$, due to its lack of quality recognizability and the popularity of the non-premium products, renters’ usage valuation is associated with the quality level $q_1$. In addition, they need to pay the rental price for the amount of their usage time $x$. On the manufacturer-built platform $P_2$, the renters pay a different rental price $p_2$ per unit-time, but will know for sure that the quality of the product is $q_2$. However, here we make an important assumption concerning renters utility on $P_2$: The renters will not derive the full value of product usage on $P_2$. Indeed, for a premium product, it is plausible that just using it without ownership transfer may not possess the same value as using it as an owner. Usually, the notion of
“social comparison” can generate some social status utility to the owners of premium (luxurious) products, whereas such utility is absent for the renters. However, despite of the partial value derivation, we make the following assumption.

**Assumption 2** The renters on $P_2$ can only derive a fraction, $\mu \in (0, 1)$, of the utility of the premium product owners. Moreover, $\mu q_2 > q_1$.

Based on the above discussions, we write the consumer utility functions as

$$u^{(j)}_{r1}(v) = x(q_1v - p_1), \quad u^{(j)}_{r2}(v) = x(\mu q_2v - p_2), \quad j = 0, 1, 2. \quad (4.3)$$

It is noteworthy that the consumers utility that involves platform $P_2$ will all be zero if the manufacturer chooses to not build the platform in the first place. Hence, if $P_2$ does not exist, then $u^{(j)}_{o2} = u^{(k)}_{r2} = 0$ for $j = 1, 2$ and $k = 0, 1, 2$.

Finally, we remark that the above utility functions are based on simple logics that does not involve uncommon or even eccentric consumer behaviors. For example, we do not consider the case where a consumer uses the platform as a renter while he owns a product at the same time. In such case, due to the relatively large sunk cost of owning a product, we assume that he will always use if when needed rather than resort to shared product on the platform. Another example of irregular behaviors is that a consumer values the income from the platform higher than his self-use of the product. Although we acknowledge the possibility of purchasing the product to purely provide sharing service without self-use, we do not consider such case in our paper. For one thing, this is not commonly observed in practice; for another, we focus on the case where the self-use is critical or inevitable in one’s life, and therefore the consumers must use the products for themselves. It is noteworthy that we can modify our model to justify the exclusion of the above consumer behaviors. However, we keep our model for simplicity.
4.3.5 Platform Matching Mechanism

A peer-to-peer sharing platform will set a rental price that clears the market of products sharing. In our model, the supply and the demand on a platform are the total available usage time that can be shared by all owners and the total usage time needed by all renters, respectively. Hence, the supply and the demand on platform $P_i$ are given by

\[
S_i = \sum_{j=1,2} (1-x)O_i^{(j)}, \quad D_i = \sum_{j=0,1,2} xR_i^{(j)}, \quad i = 1, 2, \tag{4.4}
\]

where $O_i^{(j)}$ ($R_i^{(j)}$) is the size of owner (renter) consumers on $P_i$ from market $M_j$ ($j = 0, 1, 2$). The owners and renters sizes are determined after the consumers consider and compare the utility derived from each feasible option. In particular, for $j = 1, 2$, $i, k = 1, 2$ and $k \neq i$, we have the owner size

\[
O_i^{(j)} = m_j \text{Prob} \left( u_{oi}^{(j)}(v) \geq \max\{u_{ok}^{(j)}(v), u_{o1}^{(j)}(v), u_{o2}^{(j)}(v), 0\} \right); \tag{4.5}
\]

and the renter size

\[
R_i^{(j)} = m_j \text{Prob} \left( u_{ri}^{(j)}(v) \geq \max\{u_{rk}^{(j)}(v), u_{r1}^{(j)}(v), u_{r2}^{(j)}(v), 0\} \right), \tag{4.6}
\]

\[
R_i^{(0)} = m_0 \text{Prob} \left( u_{ri}^{(0)}(v) \geq \max\{u_{rk}^{(0)}(v), 0\} \right).
\]

Therefore, given all other parameters, the supply and the demand on platform $P_i$ are governed by the rental price $p_i$ ($i = 1, 2$). That is, they are both functions of the price. To clear the market, the price is determined so that the amount of the available resources provided by the owners (supply) is at least equal to the amount of the resources needed by the renters (demand). More precisely, we define the platform matching mechanism by the search of the following equilibrium rental price:

\[
p_i^* = \inf \{p_i > 0 \mid S_i(p_i) \geq D_i(p_i)\}. \tag{4.7}
\]
It is intuitive (and proved in appendix) that the supply increases, and the demand decreases, in the rental price. Therefore, the above matching mechanism implies the following two properties of the equilibrium platform price $p^*_i$. First, if $p^*_i > 0$, then $S_i(p^*_i) = D_i(p^*_i)$, under which case the supply and demand are tuned by the price to equal each other. Second, if $S_i(p^*_i) > D_i(p^*_i)$, then $p^*_i = 0$. In such case, we find that no positive price could exactly match the supply with the demand. This means that the platform has excessive supply, and therefore it provides free service to clear the relatively small amount of demand. Note that, in spite of these two properties, it is possible that $p^*_i = 0$ and $S_i(p^*_i) = D_i(p^*_i)$.

Based on the second property, we deduce the following statement: If for any $p_i > 0$, $S_i(p_i) > D_i(p_i)$ holds regardless of other parameters, especially the manufacturer’s decision such as the selling price, then $p^*_i = 0$. Therefore, to exclude this trivial case, we make the following important assumption.

**Assumption 3** We only focus on the parameters such that

$$(1 - x)m_1 < x(m_0 + m_2).$$  \hspace{1cm} (4.8)

The rationale of this assumption is exactly to prevent the case where the supply on the platform always outnumbers the demand. Indeed, if the inequality (4.8) does not hold, then the demand on any platform will be too small regardless of other variables such as selling price, and commission rates.

Despite of Assumption 3, we remark that, in rare cases, the equilibrium rental price may still be zero. In such cases, it is the manufacturer’s decision on selling price $w$ that determines the platform equilibrium. Put differently, the zero rental price is a result of the manufacturer’s search for the optimal selling price. To be specific, in these cases, the manufacturer will set so small a $w$ that too many consumers become
owners and therefore are able to provide services on the platform, which brings the rental price down to zero.

4.3.6 Sequence of Events

We formulate a multi-stage game among the manufacturer, the existing platform and the consumers, as illustrated by Figure 4.2 below. The sequence of events (hence our analysis) is largely determined by the manufacturer’s strategic decision on the launch of its own platform $P_2$. Therefore, in the very beginning, i.e. Stage 0, the manufacturer decides whether to build the platform $P_2$ or not, based on its product
quality $q_2$. There are two alternatives. Strategy 1 refers to the status quo, and therefore the manufacturer does not build $P_2$ when choosing Strategy 1. On the other hand, Strategy 2 is to build $P_2$ as an exclusive platform. Then, in Stage 1, the manufacturer continues to determine the selling price $w$ for its premium product. If Strategy 2 is chosen, then the manufacturer also needs to decide the platform commission $\alpha_2$ in this stage. Entering Stage 2, we have a sub-game between the platform(s) and the consumers with respect to the matching process of the peer-to-peer sharing. Specifically, given the rental price set by the platform\(^9\), consumers consider and compare feasible options (to be an owner or a renter on which platform) and find the one yielding the highest utility. Of course, if the manufacturer chooses Strategy 1, then the consumers can only consider platform $P_1$. The Stage 2 sub-game is solved by finding the equilibrium rental price(s) for the platform(s) based on the matching mechanism explained in the previous subsection.

We utilize backward induction to solve this game. First, given all other parameters, we find the equilibrium outcomes for the Stage 2 sub-game as functions of $q_2$, $w$ and $\alpha_2$ (only under Strategy 2). Then, we solve for the final equilibrium outcomes as functions of $q_2$. Finally, for each $q_2$, we compare the two strategies based on the manufacturer’s optimal profit and the equilibrium ownership shares.

### 4.4 Equilibrium Analysis

In this section, we analyze the equilibrium outcomes of the two strategies, respectively, and compare the two to determine the viability of the platform-building strategy.

\(^9\)The commission $\alpha_1 \in (0, 1)$ is assumed to be fixed so that platform $P_1$ only have one decision variable, i.e., $p_1$.  

Recall that we have made Assumptions 1 to 3 regarding the non-decision parameters such as the markets sizes, usage level, and etc. All our analysis will be under these assumption. Let $z = (q_2, q_1, \alpha_1, \mu, x, m_0, m_1, m_2)$ be a vector of these parameter. Hence, in addition to the natural conditions that $z > 0$ and $0 < \alpha_1, \mu, x < 1$, hereafter we will focus on the parameters such that

$$z \in \{m_2 < m_1, (1 - x)m_1 < x(m_0 + m_1), \mu q_2 > q_1\}.$$  \hspace{1cm} (4.9)

\subsection*{4.4.1 Strategy 1: No Sharing Platform P_2}

Suppose that the premium manufacturer chooses to stay status quo and does not build platform $P_2$. We start our analysis with the second stage. In this stage, the sharing platform $P_1$ will select the rental price and the consumers will form their behaviors based on utility maximization. Then we examine the premium manufacturer’s optimal selling price in stage 1 after knowing the rental price and consumers behaviors.

\subsubsection*{4.4.1.1 Platform Equilibrium.}

Assume that the premium manufacturer’s selling price $w$ is given. Then, for any rental price $p_1 > 0$ on the existing platform, we can derive the consumers utility and analyze how they react to each feasible option. To facilitate the analysis, we visualize the consumers choices by plotting the utility curves for consumers in different markets, which is based on the discussion in § 4.3.4, in particular equations (4.1), (4.2) and (4.3). See Figure 4.3 below for an illustration. In the following, we apply equations (4.5) and (4.6) in § 4.3.5 to characterize consumers behavior in each market.
Market $M_0$: A non-owner consumer with usage value $v$ will choose to be a renter on $P_1$ if and only if $u^{(0)}_{r1} \geq 0$. That is, on the non-owner market,

$$R^{(0)}_{1} = (1 - \hat{v}_1)m_0,$$

where $u^{(0)}_{r1}(\hat{v}_1) = 0$ (upper left in Figure 4.3).

Market $M_1$: A potential owner of non-premium products will actually purchase and own a product if the utility of being a renter on $P_1$ is lower. Comparing (4.2) and (4.3) and noting that $w_1 = 0$ by assumption, we find that the later is dominated by the utility of being an owner. As a result, all of the consumers become owners on $P_1$ (upper right in Figure 4.3):

$$O^{(1)}_{1} = m_1, \quad R^{(1)}_{1} = 0.$$
Market $M_2$: A consumer on the premium market $M_2$ may consider to be an owner of a premium product or a renter on $P_1$, depending on the utility comparison; see (4.5) and (4.6). As shown in Figure 4.3 (lower plots), we have two cases as a result of the comparison. Define $\tilde{v}_0$ and $\tilde{v}_1$ by $u^{(2)}_{o1}(\tilde{v}_0) = 0$ and $u^{(2)}_{o1}(\tilde{v}_1) = u^{(2)}_{r1}(\tilde{v}_1)$, respectively. Then, depending on the selling price $w$ and the rental price $p_1$, these values have different orders, which determines the specific case:

(A) When $0 \leq \tilde{v}_0 \leq \hat{v}_1$, consumers may become an owner of a premium product:

$$O^{(2)}_1 = (1 - \tilde{v}_0)m_2, \quad R^{(2)}_1 = 0.$$  

(B) When $\hat{v}_1 < \tilde{v}_1 < 1$, consumers may become an owner of a premium product or a renter on $P_1$:

$$O^{(2)}_1 = (1 - \tilde{v}_1)m_2, \quad R^{(2)}_1 = (\tilde{v}_1 - \hat{v}_1)m_2.$$  

Note that we require $\tilde{v}_0 \geq 0$ and $\tilde{v}_1 < 1$ because otherwise the outcomes are dominated and will not appear in equilibrium.

Based on the above discussion, we have two different combinations of the consumer behaviors in all markets. For each of the two cases, we apply equation (4.4) and solve the platform’s matching problem (4.7) to obtain the rental price $\bar{p}_1$. If this rental price satisfies the conditions that define the corresponding case, then it is an equilibrium rental price. Note that $\bar{p}_1$ is a function of the selling price $w$ and the parameter vector $z$. Therefore, in the following proposition, we characterize the platform equilibrium for the Stage 2 sub-game.

**Proposition 12** Let the manufacturer adopt strategy 1. For any parameter vector $z$ satisfying (4.9), there exist $\bar{W}_{max}(z)$ and $\bar{W}_{min}(z)$, functions of $z$, such that $\bar{W}_{max} > \bar{W}_{min} \geq 0$ and the following hold:
(i) If $w \notin [\bar{W}_{\min}, \bar{W}_{\max}]$, then choosing the selling price $w$ is sub-optimal for the manufacturer.

(ii) If $w \in (\bar{W}_{\min}, \bar{W}_{\max})$, then there exists a unique equilibrium rental price.

(iii) If $z \in \{xm_0 \leq (1-x)(m_1 + m_2)\}$, then only Case (B) will occur in equilibrium for any $w \in (\bar{W}_{\min}, \bar{W}_{\max})$; moreover, $\bar{p}_1(w) \geq 0$ is linear and increasing in $w$.

(iv) If $z \in \{xm_0 > (1-x)(m_1 + m_2)\}$, then there exists a $\bar{w} \in (\bar{W}_{\min}, \bar{W}_{\max})$ such that Case (A) occurs when $w \in (\bar{W}_{\min}, \bar{W}_{\max})$; moreover, $\bar{p}_1(w) > 0$ is continuous, piecewise linear, and increasing in $w$.

Proposition 12 shows the importance of the parameters in our model. The interval of candidates for optimal selling price is parameter-dependent. Therefore, the analysis are divided into two scenarios according to the range of $z$ shown in Proposition 12(iii) and (iv). Since the above proposition plays a crucial role in the further analysis of the full game, let us understand this result in more depth. First, the intuition behind Proposition 12(i) is as follows. For $w$ either too high or too low, the outcome will either be strictly dominated or simply does not permit an equilibrium. Hence, the manufacturer must choose a selling price in between in the equilibrium of the full game. Second, Proposition 12 not only enables us to find out the unique platform equilibrium, but also reveals the functional structure of the equilibrium price. In particular, $\bar{p}_1(w)$ is a linear function if $xm_0 \leq (1-x)(m_1 + m_2)$, and has two linear pieces otherwise. Finally, Proposition 12(iii) implies that Case (A) will occur only when $m_0$ is relatively large compared to $m_1 + m_2$ and $w$ is not too high. Indeed, low selling price is a natural driver for more ownership of the premium product. Moreover, when there are many non-owners (large $m_0$), the demand on the platform (for sharing
service) increases. As a result, the supply will increase too, which is the second driver for more ownership in the premium market (ownership in $M_1$ has already reached its maximum).

4.4.1.2 Optimal Selling Price and Equilibrium Profit.

Now that we have characterized the platform equilibrium, we move back to Stage 1 to solve the manufacturer’s profit maximization problem. To determine the optimal selling price $w$, we first write out the profit function, which is the product of $w$ and the sales volume. Since the manufacturer will only sell to the premium market $M_2$, the sales volume is exactly the ownership $O_1^{(2)}$. Using Proposition 12, we obtain $\bar{p}_1$ for any given $w$ and $z$. Then, we can substitute it to $O_1^{(2)}$ according to specific cases; thus the equilibrium ownership $\bar{O}_1^{(2)}(w, z) = O_1^{(2)}(w, \bar{p}_1(w), z)$. Similar to before, the parameter vector $z$ is used to determine whether Case (A) or (B) will occur. Therefore, the manufacturer’s profit, when expecting the ensuing platform equilibrium, is given by

$$\hat{\pi}_1(w, z) = w\bar{O}_1^{(2)}(w, z).$$

We identify some structural properties of the profit function, which will facilitate the search for the optimal selling price.

Proposition 13 Let $z$ be given and satisfy (4.9).

(i) If $z \in \{xm_0 \leq (1-x)(m_1 + m_2)\}$, then $\bar{O}_1^{(2)}(w, z)$ is linear and decreasing in $w$. The profit function $\hat{\pi}_1(w, z)$ is a concave quadratic function of $w$.

(ii) If $z \in \{xm_0 > (1-x)(m_1 + m_2)\}$, then $\bar{O}_1^{(2)}(w, z)$ is continuous, piecewise linear, and decreasing in $w$. The profit function $\hat{\pi}_1(w, z)$ is a continuous and piecewise concave quadratic function of $w$. 
As can be observed, the division of cases is the same as given in Proposition 12(iii), and the resulting profit function is defined on the same intervals of $w$ that are given in Proposition 12.

Based on Proposition 13, the concavity of the profit function allows fast search of optimality; and the comparison between the optimality on different pieces can be done easily. Therefore, the maximized profit can be found in an efficient way:

$$
\pi_1^*(z) = \max_{w \in [\bar{W}_{min}, \bar{W}_{max}]} \hat{\pi}_1(w, z).
$$

(4.10)

The optimal selling price $w$, along with other equilibrium quantities such as the sales volume, can be obtained once (4.10) is solved.

### 4.4.2 Strategy 2: Build Sharing Platform $P_2$

When the manufacturer builds its own sharing platform, we follow the similar steps in the previous subsection to derive the equilibrium results. However, there are two more decisions for the manufacturer to make. First, at Stage 2, the manufacturer needs to set a rental price for its own platform. This decision is made at the same time when the existing platform is also deciding a rental price. Therefore, we have a competition between the two platform at the end stage, and the platform equilibrium involves a pair of rental prices that match the supply with the demand on the respective platform. Second, at Stage 1, in addition to the selling price $w$, the manufacturer needs to announce the commission rate $\alpha_2$ for the to-be-built platform $P_2$.

While the rental price as a new decision variable may significantly complicate our analysis, the commission rate is not so much a concern. In fact, as an interesting result, we show that the decision on the commission rate can be trivial to the manufacturer.
Lemma 2 Suppose that the platform-building strategy is used. Given any selling price \( w \) and commission rate \( \alpha_2 > 0 \), the manufacturer can weakly improve its total profit by setting \( \alpha_2 = 0 \) and charge a higher selling price \( w' > w \).

Therefore, Lemma 2 helps our analysis via dimension collapse on the decision variables. Instead of jointly optimizing over \((w, \alpha_2)\), we now can set \( \alpha_2 = 0 \) and concentrate on the single variable \( w \). The underlying reason for this result is as follows. Being a manufacturer-built platform, the management of \( P_2 \) can be more flexible. Particularly, the manufacturer may give up the profit from the platform but instead charge a higher \( w \) to gain extra profit from the traditional product-selling business. Indeed, operating a frictionless platform increases the value of the shared product, making ownership more attractive, which justifies the higher selling price. Thus, Lemma 2 can be understood as a means for the manufacturer to re-distribute the profit from its two sources of business. In the following analysis, we only look at the equilibrium results with \( \alpha_2 = 0 \).

4.4.2.1 Platform Equilibrium.

To solve the platform equilibrium, we again start from the study on the consumer behaviors in different markets. First of all, we provide some necessary conditions for the equilibrium pair of rental prices. To simplify our exposition, define \( \hat{v}_1, \hat{v}_2 \) and \( \hat{v}_{12} \) by \( u_{r1}^{(i)}(\hat{v}_1) = 0, u_{r2}^{(i)}(\hat{v}_2) = 0 \) and \( u_{r1}^{(i)}(\hat{v}_{12}) = u_{r2}^{(i)}(\hat{v}_{12}) \).

Lemma 3 In any equilibrium with strategy 2, we find \( p_1 < p_2 \). Furthermore, we know the renters’ utility on either platform will not dominate that on the other platform, i.e., \( \hat{v}_1 < \hat{v}_2 \) and \( \hat{v}_{12} < 1 \).
Lemma 3 confirms the intuition that the renters need to pay a higher price on the premium sharing platform. Moreover, since $p_2 > p_1 > (1 - \alpha_1)p_1$, the lemma also implies that owners of the premium product obtain higher compensation from the platform $P_2$ than from $P_1$. Therefore, from (4.1) and (4.2), we have $u_{o_2}^{(2)} > u_{o_2}^{(1)}$, i.e., the owners from the premium market will not participate on $P_1$. The last two necessary conditions for platform equilibrium are from the fact that both platforms will manage to survive in equilibrium. To be specific, in equilibrium, the renters’ utility on either platform will not dominate that on the other platform; otherwise the latter will not have any users, which induces a deviation of that platform to set a lower rental price.

Now, based on Lemma 3, we visually illustrate the utility curves of consumers with different options; see Figure 4.4. Then, we characterize the possible consumer behaviors for the three markets. Let the selling price $w$ and the pair of rental prices $(p_1, p_2)$ be given.

**Market $M_0$:** Consumers in this market only consider the options of being a renter on either $P_1$ or $P_2$. From Lemma 3, the utility curves will intersect as depicted in Figure 4.4 (upper leftmost). Hence, consumers could become a renter on $P_2$, a renter on $P_1$, or take the outside option. Since the market size is $m_0$, we obtain that

$$R_1^{(0)} = (\hat{v}_{12} - \hat{v}_1)m_0, \quad R_2^{(0)} = (1 - \hat{v}_{12})m_0.$$  

**Market $M_1$:** Consumers in this market, when $P_2$ is not built, used to be all owners of the non-premium products. However, with $P_2$ built, there is possibility that the manufacturer may use $P_2$ to encroach market $M_1$. Specifically, when some of the consumers find that the utility of being a renter on $P_2$ is higher than owning a non-premium product, they will switch from the purchase behavior to the participation
on $P_2$ as a renter. Define the threshold usage value by $\tilde{v}_2$ where $u_{o1}^{(1)}(\tilde{v}_2) = u_{r2}^{(1)}(\tilde{v}_2)$ (see the upper middle and upper rightmost in Figure 4.4). Then, depending on the value of $\tilde{v}_2$, there are two cases:

(I) Encroachment occurs when $\tilde{v}_2 < 1$. In this case, we have

$$O_1^{(1)} = \tilde{v}_2 m_1, \quad R_2^{(1)} = (1 - \tilde{v}_2) m_1.$$

(II) All consumers in market $M_1$ are owners of non-premium products when $\tilde{v}_2 \geq 1$. In this case, we have

$$O_1^{(1)} = m_1, \quad R_2^{(1)} = 0.$$

**Market $M_2$:** Consumers in this market have three sets of possible behaviors depending on how their utility curves intersect with each other. To have a clear discus-
sion on the possible cases, we first define the following values: Let \( v_0, v_1 \) and \( v_2 \) be the intersection of \( u_{o2}^{(2)} \) and the \( v \)-axis, \( u_{r1}^{(2)} \) and \( u_{r2}^{(2)} \), respectively, i.e., \( u_{o2}^{(2)}(v_0) = 0 \), \( u_{o2}^{(2)}(v_1) = u_{r1}^{(2)}(v_1) \) and \( u_{o2}^{(2)}(v_2) = u_{r2}^{(2)}(v_2) \). Based on the specific value of the above three critical points, we have three cases regarding the consumers behaviors on the premium market \( M_2 \) (see the lower half in Figure 4.4). We classify these cases below.

(a) When \( 0 \leq v_0 \leq \hat{v}_1 \), consumers may be owners of the premium product and will not consider to be renters on either \( P_1 \) or \( P_2 \). In particular,

\[
O_2^{(2)} = (1 - v_0)m_2, \quad R_2^{(2)} = 0, \quad R_1^{(2)} = 0.
\]

(b) When \( \hat{v}_1 < v_1 \leq \hat{v}_{12} \), consumers may be owners of the premium product or renters on \( P_1 \). In particular,

\[
O_2^{(2)} = (1 - v_1)m_2, \quad R_2^{(2)} = 0, \quad R_1^{(2)} = (v_1 - \hat{v}_1)m_2.
\]

(c) When \( \hat{v}_{12} < v_2 < 1 \), consumers may be owners of the premium product, renters on \( P_2 \), or renters on \( P_1 \). In particular,

\[
O_2^{(2)} = (1 - v_2)m_2, \quad R_2^{(2)} = (v_2 - \hat{v}_{12})m_2, \quad R_1^{(2)} = (\hat{v}_{12} - \hat{v}_1)m_2.
\]

Hence, we exclude the apparent non-equilibrium results and require that \( 0 \leq v_0, v_2 < 1 \); as a result, the premium product will always have ownership market.

We take the same approach as in analysis for strategy 1. Here, considering two cases for \( M_1 \) (I, II) and three cases for \( M_2 \) ((a), (b), (c)), we have six combinations of the possible consumer behaviors in all markets. For each case, we use equation (4.4) to solve the matching problem (4.7) jointly for both platforms. The solution is a pair of rental price \((p_1^*, p_2^*)\); and it represents an equilibrium if the prices satisfy
the defining conditions of the corresponding case. Like before, these equilibrium prices are functions of \( w \) and parameter vector \( z \). Furthermore, the characterization of the platform equilibrium also depends on the parameters, as shown in the next proposition.

**Proposition 14** Let the manufacturer adopt strategy 2. For any parameter vector \( z \) satisfying (4.9), there exist \( W_{\text{max}}(z) \) and \( W_{\text{min}}(z) \), functions of \( z \), such that \( W_{\text{max}} > W_{\text{min}} \geq 0 \) and the following hold:

(i) If \( w \notin [W_{\text{min}}, W_{\text{max}}] \), then choosing the selling price \( w \) is sub-optimal for the manufacturer.

(ii) If \( w \in (W_{\text{min}}, W_{\text{max}}) \), then there exists a unique pair of nonnegative equilibrium rental prices \( (p^*_1(w), p^*_2(w)) \).

(iii) There exists a partition of the interval \( (W_{\text{min}}, W_{\text{max}}) \) that consists of a set of sub-intervals such that one distinct case occurs in equilibrium when \( w \) is on each sub-interval; moreover, both \( p^*_1(w) \) and \( p^*_2(w) \) are linear in \( w \) on each sub-interval.

(iv) There exists a \( \bar{W} \in [W_{\text{min}}, W_{\text{max}}) \) such that the manufacturer manages to encroach to the market \( M_1 \) only if \( W_{\text{min}} < w < \bar{W}(z) \).

The idea of Proposition 14 is similar to Proposition 12, which is to establish the existence and uniqueness of the platform equilibrium as a sub-game, and to characterize the equilibrium based on the parameter values. However, under the platform-building strategy, the parameter vector \( z \) is more intricately involved. As shown in the appendix, depending on the specific parameters, the interval \( (W_{\text{min}}, W_{\text{max}}) \), which
includes the candidates for the final optimal selling price, is different. Moreover, the partition described in Proposition 14(iii), along with the possible cases that occur in equilibrium, depend on the parameters. For example, for a certain \( z \), we may have a partition
\[
(W_{\text{min}}, W_{\text{max}}) = (W_1, W_2] \cup (W_2, W_3] \cup \cdots \cup (W_k, W_{k+1}),
\]
where \( W_1 = W_{\text{min}} \) and \( W_{k+1} = W_{\text{max}} \). In addition, the number of sub-intervals in this partition also depends on \( z \); i.e. \( k = k(z) \). The details of the partition for every \( z \) will be explicitly given in the proof.

Given the partition, we derive quite a few useful properties from Proposition 14. The foremost one is the functional structure of the equilibrium rental prices: Both \( p_1^* \) and \( p_2^* \) are linear in \( w \) on each sub-interval of the partition. In addition, they are continuous at all kinks. The piecewise linearity has laid the foundation of our further analysis. Moreover, Proposition 14(iii) also indicates that one and only one of the six cases (I(a), I(b), I(c), II(a), II(b), II(c)) will occur in equilibrium when \( w \) is on each of those sub-intervals, and they are different to each other. This means that each of the possible cases corresponds to a cluster of selling prices (i.e., if two selling prices induce the same case in equilibrium, then so does every selling price in between them). In a sense, the possible cases are monotonic with respect to \( w \).

To wit, Proposition 14(iv) provides a demonstration of such monotonicity. There is a threshold of \( w \) such that Case (I) will occur only if the selling price is smaller than the threshold. Intuitively, when \( w \) is relatively small, the manufacturer may enlarge its ownership base, which leads to a reduction of its platform’s rental price (because of more supply) and possible conversion of consumers in \( M_1 \) from owners of non-premium product to renters on platform \( P_2 \). When \( w \) is large, however, this
will not be the case, and we will only observe Case (II). Note that, as shown in the appendix, it is possible that $\tilde{W} = W_{\text{min}}$ and encroachment never happens.

4.4.2.2 Optimal Selling Price and Equilibrium Profit.

We now examine the manufacturer’s problem in Stage 1 and solve for the optimal selling price. Since we have made the assumption that $\alpha_2 = 0$, the manufacturer does not profit from its platform. Rather, its only profit is from selling the premium product. Hence, similar to strategy 1, the manufacturer maximizes the product of $w$ and the sales volume. Unlike strategy 1, the sales volume is given by $O_2^{(2)}$. Therefore, when expecting the ensuing platform equilibrium, the equilibrium ownership is given by $O_2^{(2)*}(w, z)$, and the profit function is

$$\hat{\pi}_2(w, z) = wO_1^{(2)*}(w, z).$$

Similar to other equilibrium quantities, both the ownership and the profit are functions of $w$ that are defined over $w \in (W_{\text{min}}, W_{\text{max}})$. Furthermore, they are piecewise functions according to the partition of sub-intervals. We show that they possess desirable properties that facilitate the computation to a large extent.

**Proposition 15** Let $z$ be given and satisfy (4.9).

(i) $O_1^{(2)*}(w, z)$ is piecewise linear in $w$; moreover, on each piece, it is decreasing in $w$.

(ii) The profit function $\hat{\pi}_2(w, z)$ is continuous and piecewise in $w$. Moreover, on each piece, it is a concave quadratic function.

Based on Proposition 15, we can efficiently solve for the optimality on each piece (sub-interval) of the profit function, and obtain the global optimal profit by direct
comparison. Hence,

\[ \pi^*_2(z) = \max_{w \in [W_{min}, W_{max}]} \hat{\pi}_2(w, z). \] (4.11)

Once (4.11) is solved, the optimal selling price \( w \) and the optimal premium product ownership can be easily obtained.

### 4.4.3 Viability of the Platform-Building Strategy

In the previous two subsections, we respectively solved the manufacturer’s problem up to Stage 1. That is, for a given parameter vector \( z \), we can find the optimal profit under strategy 1 (status quo), \( \pi^*_1(z) \), and the optimal profit under strategy 2 (platform-building), \( \pi^*_2(z) \). Now, we proceed to compare the two strategies and thus solve Stage 0 of the game. As our main result, we establish the dominance of the platform-building strategy.

**Proposition 16** Let \( z \) satisfy (4.9) and be fixed. The manufacturer can always achieve higher profit by building its own peer-to-peer sharing platform. That is,

\[ \pi^*_2(z) > \pi^*_1(z). \]

The intuition behind the above result is as follows. Although the selling price of the premium product is set higher than the non-premium products, i.e., \( w > w_1 = 0 \), there are two advantages that platform-building strategy may render to the manufacturer. First, the self-built platform \( P_2 \) is quality differentiated from the existing platform. Even the renter’s utility on \( P_2 \) cannot be dominated by that on \( P_1 \) (\( \mu q_2 > q_1 \)). Hence, the existence of \( P_2 \) poses a competition against \( P_1 \), especially over the non-owner market \( M_0 \). Further, since \( P_1 \) is not able to distinguish the quality of the shared products, building \( P_2 \) helps the premium manufacturer enhance the value of
the ownership of its product. Second, as implied by Lemma 2, the manufacturer, when building its own platform, may take advantage of the flexibility of leveraging profits from the two sources of business. When taking out the commission rate, the manufacturer makes the sharing process on $P_2$ frictionless and the platform more attractive. The existing platform, however, will always charge some fees to sustain its business. Because of these two advantages, the manufacturer will find it optimal to build a sharing platform with higher quality when facing an existing platform.

While Proposition 16 establishes the absolute dominance of strategy 2 over strategy 1, we may further understand the advantage of the platform-building strategy by referencing to a benchmark case where no sharing platform exists. Before the sharing economy emerged, the manufacturer was a monopoly selling to its premium market $M_2$. Hence, we can straightforwardly derive that the manufacturer’s optimal profit

$$\pi_0^* = \frac{xq_2m_2}{4}.$$  

In this case, due to the lack of sharing platform, consumers in different market segmentations do not have means to interact. With the emergence of peer-to-peer sharing, a platform such as $P_1$ comes into play and interacts with the manufacturer’s traditional product selling business. In our model, we have described such case and derived the manufacturer’s optimal profit $\pi_1^*$. Note that the sharing option may divert some potential owners to become renters on the one hand, but may also increase the utility of the product owners (they can monetize the unused time of the product) on the other hand. Hence, it is unclear how $\pi_1^*$ and $\pi_0^*$ compare to each other. Then, as the manufacturer builds its own peer-to-peer sharing platform, we have shown that the optimal profit $\pi_2^* > \pi_1^*$. However, due to the similar reason, $\pi_2^*$ may or may not exceed $\pi_0^*$. 

From the no-sharing benchmark to the appearance of a third-party platform, and to the adoption of the platform-building strategy, we have witnessed an evolution of the manufacturer’s business ecosystem. Although strategy 2 is strictly preferred by the manufacturer to strategy 1, its attractiveness may be subtly different depending on what the manufacturer has been through as the business ecosystem evolves. To be specific, there are three possible scenarios. (1) $\pi^*_2 > \pi^*_1 > \pi^*_0$. The emergence of the platform $P_1$ has already benefited the manufacturer, and building its own platform simply makes it even better off. (2) $\pi^*_2 > \pi^*_0 > \pi^*_1$. The manufacturer was hurt by the sharing economy at its first appearance; however, actively adopting a platform-building strategy helps the manufacturer turn around the adverse situation. (3) $\pi^*_0 > \pi^*_2 > \pi^*_1$. The sharing economy always results in loss for the manufacturer. Building its own platform only alleviates the harm. Therefore, in view of the changes in the manufacturer’s business ecosystem, the platform-building strategy is the most appealing under the second scenario, where it is the key to turn the sharing economy from against to for the manufacturer.

4.5 Summary and Future Study

In this essay, we consider the manufacturer’s strategy in incorporating the consumers’ new behavior, i.e. peer-to-peer product sharing. The new emergence peer-to-peer product sharing phenomenon can mutually benefit the product owner and the renter, while it may reduce the consumers’ incentive to purchase a new product and then drastically dip the manufacturer’s sales. Moreover, due to the inconvenience in operations, the existing platform, which matches supply and demand between the product owners and riders, may not provide the service to differentiate the premium and
non-premium products shared on the platform. It also drives down the value of sharing of the premium product. Hence, the attractiveness of the premium product can reduce further in the sharing economy which may result in lower profit for the manufacturer. Therefore, providing an exclusive product sharing platform may help the manufacturer gain competitive advantage in the premium market. Indeed, we find the manufacturer can always be better off after launching a peer-to-peer product sharing platform exclusively for its premium product. Our results indicate that the manufacturer can charge no commission fee for the owners who share products on its platform. By implementing a platform-building strategy, the manufacturer can charge higher price due to the frictionless sharing platform. Moreover, by comparing with a model without sharing platforms, we find the platform-building strategy may change the manufacturer from losing due to consumers product sharing to winning.

In the future study, we further compare the two strategies using numerical methods. Our study attempts to address a couple of issues. (1) As a key differentiating factor, how does the quality of the premium product affect the performance of the platform-building strategy? (2) What are the impacts of the market sizes and the usage level on the profit advantage of the platform-building strategy? We also plan to study two extensions to our main model. (1) Multiple usage levels. Instead of single-type consumer in usage level, we assume that there are two groups of consumers (across all markets) with usage levels $0 < x_L < x_H < 1$. (2) Quality-differentiating $P_1$. Suppose that the existing platform offers quality-differentiating service. That is, it offers two options to the renters, premium and non-premium products, and charges two different prices depending on renters’ choice. In both cases, we conduct numer-
ical experiments to confirm our main results, and also derive useful insights for the manufacturer in these particular settings.
CHAPTER 5

Conclusion

The thesis studies several issues in the manufacturer’s market entry strategy: the value of information on the direct selling cost, the impact of the buyer’s strategic inventory and the viability of platform-building strategy. By examining each of the schemes, we provide managerial insights for the manufacturer on how to adapt its traditional business model with the new business environment.

The first essay analyzes the players’ incentives in learning the manufacturer’s direct selling cost. We find the region of “win-win-win” for the manufacturer, the retailer and consumers can expand when the manufacturer is privately informed on its direct selling cost, compared to that when the information is public. Furthermore, we show the manufacturer may prefer no information advantage over the retailer on its direct selling cost. On the one hand, if the manufacturer cannot share the information with the retailer, the manufacturer may not have incentive to uncover the cost information but to stay uninformative as the retailer. On the other hand, if the manufacturer can share the information with the retailer, after the manufacturer gathers the information on its direct selling cost, it will be weakly better off for the manufacturer to share that information with the retailer rather than keeping it private. For the buyer, however, we find it may want to know less information on the manufacturer’s direct
selling cost under the supplier encroachment setting. In addition, we also examine the benefit of uncertainty in the supply chain. If the manufacturer and retailer agree on the information sharing contract before the manufacturer’s information acquisition, the manufacturer may still have no incentive to resolve the uncertainty on its direct selling cost. Hence, our results shows it may not be better for the players to have more information on the direct selling cost when the manufacturer has a direct selling channel.

The second essay studies how a strategic buyer can react to the manufacturer’s direct selling strategy by holding inventory. We show the strategic inventory held by the buyer can change the supplier’s direct channel strategy. Hence, the supplier can become less aggressive. However, the supplier and the buyer can both be better off by considering the players’ strategies together rather than only considering one player’s strategy. The aforementioned results are derived in a sequential model when the supplier decides its selling quantity after the buyer’s decision. We also consider a simultaneous model, in which the supplier and buyer will simultaneously choose the selling quantity in the downstream market. Compared to the sequential model, the buyer loses first-mover advantage in the simultaneous model. Without first-mover advantage, we find the buyer may hold inventory even with higher inventory holding cost but the supplier will directly sell even with higher direct selling cost.

The third essay deals with the manufacturer’s platform building strategy in the new sharing economy paradigm. Though we may have an existing platform to match supply and demand for the peer-to-peer product sharing, due to the inconvenience in operations, the existing platform may not provide the service to differentiate the premium and non-premium products shared on the platform. Thus, providing an ex-
clusive product sharing platform is an opportunity in the market for the manufacturer with premium product. Indeed, we find the premium manufacturer can always be better off after launching a peer-to-peer product sharing platform just for its premium product. We show the manufacturer can charge no commission fee for the owners sharing product on its platform. However, if the platform will incur some cost, our results indicate it can only be profitable for the manufacture to adopt a platform-building strategy if the premium and non-premium products have a medium quality differentiation.


APPENDIX

A Proofs for Chapter 2

Lemma A.1 In stage 3, the selling quantities for the retailer and the manufacturer are as follows:

1: If \( q_r < \min\{\frac{\alpha+s}{3}, \alpha - s\} \), \((\tilde{q}_r, q_m) = (q_r, \frac{\alpha-q_r-s}{2})\);

2: If \( s < \frac{\alpha}{2} \) and \( q_r \geq \frac{\alpha+s}{3} \), \((\tilde{q}_r, q_m) = (\frac{\alpha+s}{3}, \frac{\alpha-2s}{3})\);

3: If \( s \geq \frac{\alpha}{2} \) and \( q_r \geq \alpha - s \), \((\tilde{q}_r, q_m) = \left( \min\{q_r, \frac{\alpha}{2}\}, 0 \right)\).

Proof: Since the manufacturer and the retailer simultaneously choose their selling quantities, then retailer’s selling quantity \( \tilde{q}_r \) is the best response to manufacturer’s direct selling quantity \( q_m \) and manufacturer’s direct selling quantity \( q_m \) is also the best response to retailer’s selling quantity \( \tilde{q}_r \). Hence, retailer’s selling quantity is

\[
\tilde{q}_r = \arg\max_{0 \leq q \leq q_r} \{\alpha - q_m - q\} q,
\]

and manufacturer’s direct selling quantity is

\[
q_m = \arg\max_{0 \leq q} \{\alpha - \tilde{q}_r - q - s\} q.
\]

Then we know \( \tilde{q}_r = \min\{\frac{\alpha-q_m}{2}, q_r\} \) and \( q_m = \frac{(\alpha-\tilde{q}_r-s)^+}{2} \).
First, we consider \( q_r \leq \frac{a}{2} \). Below we divide the proof for the lemma into three cases.

**Case 1: \( s < \frac{a}{2}, q_r < \frac{a+s}{3} \)**

If \( \tilde{q}_r < q_r \), then \( \alpha - \tilde{q}_r - s > \alpha - \frac{a+s}{3} - s = \frac{2a-4s}{3} > 0 \). So \( q_m = \frac{(a-\tilde{q}_r-s)^+}{2} = \frac{a-\tilde{q}_r-s}{2} \).

Since \( \tilde{q}_r < q_r < \frac{a+s}{3} \), we know \( \frac{a-q_m}{2} = \frac{a-\alpha-\tilde{q}_r-s}{2} = \frac{a+\tilde{q}_r+s}{4} > \tilde{q}_r \), which contradicts \( \tilde{q}_r = \min\{\frac{a-q_m}{2}, q_r\} \). So in this case the only possibility in the equilibrium is \( \tilde{q}_r = q_r \).

If \( \tilde{q}_r = q_r \), then \( q_m = \frac{(a-q_r-s)^+}{2} = \frac{a-q_r-s}{2} \) and \( \frac{a-q_m}{2} = \frac{a-\alpha-\tilde{q}_r-s}{2} = \frac{a+q_r+s}{4} > q_r \). Since the retailer can not sell more than \( q_r \), then we know \( (\tilde{q}_R, q_m) = (q_r, \frac{a-q_r-s}{2}) \).

**Case 2: \( s < \frac{a}{2}, q_r \geq \frac{a+s}{3} \)**

As in Case 1, we can show \( \tilde{q}_r < \frac{a+s}{3} \) is not possible in the equilibrium. Since \( q_r \leq \frac{a}{2} \), then \( \tilde{q}_r \leq q_r \leq \frac{a}{2} < \alpha - s \). If \( \tilde{q}_r > \frac{a+s}{3} \), then \( q_m = \frac{(a-\tilde{q}_r-s)^+}{2} = \frac{a-\tilde{q}_r-s}{2} \). But \( \frac{a-q_m}{2} = \frac{a-\alpha-\tilde{q}_r-s}{2} = \frac{a+\tilde{q}_r+s}{4} < \tilde{q}_r \), which is a contradiction. Thus, in this case the only possibility in the equilibrium is \( \tilde{q}_r = \frac{a+s}{3} \). If \( \tilde{q}_r = \frac{a+s}{3} \), then \( q_m = \frac{(a-\tilde{q}_r-s)^+}{2} = \frac{a-2s}{3} \).

And \( \frac{a-q_m}{2} = \frac{a-\alpha-2s}{2} = \frac{a+s}{3} \). So \( (\tilde{q}_r, q_m) = \left( \frac{a+s}{3}, \frac{a-2s}{3} \right) \).

**Case 3: \( s \geq \frac{a}{2} \)**

In this case, we know \( \alpha - s \leq \alpha \).

If \( q_r < \alpha - s \) and \( \tilde{q}_r < q_r \), then we know \( q_m = \frac{(a-\tilde{q}_r-s)^+}{2} = \frac{a-\tilde{q}_r-s}{2} \). Since \( \tilde{q}_r < q_r < \alpha - s \leq \frac{a+s}{3} \), we know \( \frac{a-q_m}{2} = \frac{a-\alpha-\tilde{q}_r-s}{2} = \frac{a+\tilde{q}_r+s}{4} > \tilde{q}_r \), which is a contradiction. So in this case the only possibility in the equilibrium is \( \tilde{q}_r = q_r \). If \( \tilde{q}_r = q_r \), then \( q_m = \frac{(a-q_r-s)^+}{2} = \frac{a-q_r-s}{2} \). And \( \frac{a-q_m}{2} = \frac{a-\alpha-q_r-s}{2} = \frac{a+q_r+s}{4} > q_r \). Since the retailer can not sell more than \( q_r \), then we know \( (\tilde{q}_r, q_m) = (q_r, \frac{a-q_r-s}{2}) \).

If \( q_r \geq \alpha - s \), as before, we can show in the equilibrium \( \tilde{q}_r < \alpha - s \) is not possible. Thus in this case the only possibility in the equilibrium is \( \tilde{q}_r \geq \alpha - s \). Then we
know \( q_m = \frac{(\alpha - q_r - s)^+}{2} = 0 \) and \( \frac{\alpha - q_m}{2} = \frac{\alpha}{2} \geq q_r \). Thus, \( \tilde{q}_r = \min\left\{ \frac{\alpha - q_m}{2}, q_r \right\} = q_r \) and \( q_m = \frac{(\alpha - q_r - s)^+}{2} = 0 \).

Now we consider the situation when \( q_r \geq \frac{\alpha}{2} \). If \( q_r \geq \frac{\alpha}{2} \) and \( s < \frac{\alpha}{2} \), we know \( q_r \geq \frac{\alpha + s}{3} \) and as before we have \((\tilde{q}_r, q_m) = \left( \frac{\alpha + s}{3}, \frac{\alpha - 2s}{3} \right) \). If \( q_r \geq \frac{\alpha}{2} \) and \( s \geq \frac{\alpha}{2} \), we know \( q_r \geq \alpha - s \) and as before we have \((\tilde{q}_r, q_m) = (\frac{\alpha}{2}, 0) \).

Figure A.1: Illustration for Stage 3 equilibrium results

We use Figure A.1 to illustrate the above lemma. When \( q_r \) is small, the retailer will sell all he orders even when the manufacturer’s encroachment threat is high, as we see in region (1). But when \( q_r \) is large, if manufacturer’s direct selling cost is small, the retailer will withhold some quantity since the severe competition with the manufacturer, as shown by region (2); if the manufacturer’s direct selling cost is high, the threat from the direct channel is low and then the retailer can still sell all he
orders, as in region (3), but no larger than the optimal quantity that a monopoly player will sell in the downstream market, as in region (4).

**Lemma A.2** In Stage 2, the retailer chooses order quantity as follows:

1. If \( s < \frac{\alpha}{2} \),
   \[
   q_r(w) = \begin{cases} 
   0 & \text{if } w \geq \frac{\alpha + s}{2}, \\
   \frac{\alpha + s - 2w}{2} & \text{if } \frac{\alpha + s}{2} \geq w \geq \frac{\alpha + s}{6}, \\
   \frac{\alpha + s}{3} & \text{if } \frac{\alpha + s}{6} \geq w \geq 0.
   \end{cases}
   \]

2. If \( s \geq \frac{\alpha}{2} \),
   \[
   q_r(w) = \begin{cases} 
   0 & \text{if } w \geq \frac{\alpha + s}{2}, \\
   \frac{\alpha + s - 2w}{2} & \text{if } \frac{\alpha + s}{2} \geq w \geq \frac{3s - \alpha}{2}, \\
   \alpha - s & \text{if } \frac{3s - \alpha}{2} \geq w \geq 2s - \alpha, \\
   \frac{\alpha - w}{2} & \text{if } 2s - \alpha \geq w \geq 0.
   \end{cases}
   \]

**Proof:** When \( s < \frac{\alpha}{2} \), we need to compare retailer’s profit in two cases: \( q_r \leq \frac{\alpha + s}{3} \) and \( q_r \geq \frac{\alpha + s}{3} \). If \( q_r \geq \frac{\alpha + s}{3} \), then \( \alpha \), and retailer’s profit \( \Pi_r = \max_{q_r} \{-wq_r + (\alpha - \frac{\alpha - 2s}{3} - \frac{\alpha + s}{3})\alpha + s\} \). Then the retailer will choose \( q_r = \frac{\alpha + s}{3} \), which is included in the situation when \( q_r \leq \frac{\alpha + s}{3} \). If \( q_r \leq \frac{\alpha + s}{3} \), then \( \tilde{q}_r = q_r \) and \( \Pi_r = \max_{q_r} \{-wq_r + (\alpha - q_m - q_r)q_r\} \). Thus, the retailer will choose \( q_r = \frac{\alpha + s - 2w}{2} \) if \( \frac{\alpha + s}{2} \geq w \geq \frac{\alpha + s}{6}, q_r = 0 \) if \( w \geq \frac{\alpha + s}{2} \), and \( q_r = \frac{\alpha + s}{3} \) if \( w \leq \frac{\alpha + s}{6} \).

When \( s \geq \frac{\alpha}{2} \), we need to compare retailer’s profit in three cases: \( q_r \leq \alpha - s, \alpha - s \leq q_s \leq \frac{\alpha}{2} \), and \( q_r \geq \frac{\alpha}{2} \). If \( q_r \geq \frac{\alpha}{2} \), then \( \tilde{q}_r = \frac{\alpha}{2}, q_m = 0 \) and retailer’s profit \( \Pi_r = \max_{q_r} \{-wq_r + (\alpha - \frac{\alpha - 2s}{2} - \frac{\alpha + s}{2})\alpha + s\} \). Hence, \( q_r = \frac{\alpha}{2} \), which is included in the situation when \( \alpha - s \leq q_r \leq \frac{\alpha}{2} \). If \( \alpha - s \leq q_r \leq \frac{\alpha}{2} \), then \( \tilde{q}_r = q_r, q_m = 0 \) and \( \Pi_r = \max_{q_r} \{-wq_r + (\alpha - q_r)q_r\} \). Then \( q_r = \frac{\alpha - w}{2} \) if \( w \leq 2s - \alpha \) and \( q_r = \alpha - s \) if \( w \geq 2s - \alpha \). If \( q_r \leq \alpha - s \), then \( \tilde{q}_r = q_r, q_m = \frac{\alpha - q_r - s}{2} \) and \( \Pi_r = \max_{q_r} \{-wq_r + (\alpha - q_m - q_r)q_r\} \). Then
\[ q_r = \frac{\alpha + s - 2w}{2} \text{ if } \frac{\alpha + s}{2} \geq w \geq \frac{3s - \alpha}{2}, \quad q_r = 0 \text{ if } w \geq \frac{\alpha + s}{2}, \text{ and } \quad q_r = \alpha - s \text{ if } w \leq \frac{3s - \alpha}{2}. \]

In all, we know the retailer will choose
\[ q_r = 0 \text{ if } w \geq \frac{\alpha + s}{2}, \quad q_r = \frac{\alpha + s - 2w}{2} \text{ if } \frac{\alpha + s}{2} \geq w \geq \frac{3s - \alpha}{2}, \quad q_r = \alpha - s \text{ if } 2s - \alpha \leq w \leq \frac{3s - \alpha}{2} \text{ and } q_r = \frac{\alpha - w}{2} \text{ if } 0 \leq w \leq 2s - \alpha. \]

**Proof of Proposition 1:** Substituting \( q_r^*(w) \) from Lemma A.2 into \( \Pi_m(w, q_r^*(w)) \), we can easily derive the manufacturer’s optimal wholesale price \( w^* \) and then we can derive the results in the proposition.

**Proof for Lemma 1:** Similar as in the full information model, we first derive the manufacturer’s and retailer’s selling quantities in the downstream market, which is shown in the following lemma, when the manufacturer’s wholesale price and the retailer’s belief are given.

**Lemma A.3**  
If the retailer’s updated belief for the H-type manufacturer is \( \sigma(H|w) := \sigma \), then the selling quantity for the retailer and the manufacturer are as follows:

1. If \( q_r < \min\left\{ \frac{\alpha + s(\sigma)}{3}, \frac{1 + \sigma}{3 + \sigma} \alpha, \alpha - s_m \right\} \), \( \tilde{q}_r = q_r, q_m(H) = \frac{\alpha - \tilde{q}_r - s_m}{2} \) and \( q_m(L) = \frac{\alpha - \tilde{q}_r}{2} \), where \( s(\sigma) = \sigma s_m \).

2. If \( 3s_m + s(\sigma) < 2\alpha \) and \( q_r \geq \frac{\alpha + s(\sigma)}{3}, \tilde{q}_r = \frac{\alpha + s(\sigma)}{3}, q_m(H) = \frac{\alpha - \tilde{q}_r - s_m}{2} \) and \( q_m(L) = \frac{\alpha - \tilde{q}_r}{2} \).

3. If \( 3s_m + s(\sigma) \geq 2\alpha \) and \( \frac{1 + \sigma}{3 + \sigma} \alpha \geq q_r \geq \alpha - s_m, \tilde{q}_r = q_r, q_m(H) = 0 \) and \( q_m(L) = \frac{\alpha - \tilde{q}_r}{2} \).

4. If \( 3s_m + s(\sigma) \geq 2\alpha, \) and \( q_r \geq \frac{1 + \sigma}{3 + \sigma} \alpha, \tilde{q}_r = \frac{1 + \sigma}{3 + \sigma} \alpha, q_m(H) = 0 \) and \( q_m(L) = \frac{\alpha - \tilde{q}_r}{2} \).

**Proof:** Since the manufacturer has private information about direct selling cost, she can tailor the direct selling quantity according to her type. For the H-type manufacturer, the direct selling quantity is \( q_m(H) = \arg\max_{q \geq 0} (\alpha - \tilde{q}_r - q - s_m)q \).
and then \( q_m(H) = \frac{(\alpha - \tilde{q}_r - s_m)^+}{2} \). For the \( L \)-type manufacturer, the direct selling quantity is \( q_m(L) = \text{argmax} (\alpha - \tilde{q}_r - q)q \) and then \( q_m(L) = \frac{(\alpha - \tilde{q}_r)^+}{2} \). The retailer updates his information for the possibility the manufacturer to be \( H \)-type as \( \sigma(H|w) \). Then he believes \( q_m = q_m(H) \) with probability \( \sigma(H|w) \) and \( q_m = q_m(L) \) with probability \( 1 - \sigma(H|w) \). Therefore, the retailer’s selling quantity decision is formulated by \( \tilde{q}_r = \text{argmax}_{0 \leq q \leq \tilde{q}_r} \sigma(H|w)\{\alpha - q_m(H) - q\}q + (1 - \sigma(H|w))\{\alpha - q_m(L) - q\}q \) = \text{argmax}_{0 \leq q \leq \tilde{q}_r} \{\alpha - [\sigma(H|w)q_m(H) + (1 - \sigma(H|w))q_m(L)] - q\}q.

Denote \( \tilde{q}_m = \sigma(H|w)q_m(H) + (1 - \sigma(H|w))q_m(L) \).

Then we know:

1. If \( \frac{\alpha - \tilde{q}_m}{2} \leq q_r \), then \( \tilde{q}_r = \frac{\alpha - \tilde{q}_m}{2} \);
2. If \( \frac{\alpha - \tilde{q}_m}{2} > q_r \), then \( \tilde{q}_r = q_r \);

and \( q_m(H) = \frac{(\alpha - \tilde{q}_r - s_m)^+}{2} \) and \( q_m(L) = \frac{(\alpha - \tilde{q}_r)^+}{2} \).

We will divide the proof to two cases, which are Case 1: \( 3s_m + s(\sigma) \leq 2\alpha \), and Case 2: \( 3s_m + s(\sigma) > 2\alpha \).

**Case 1: \( 3s_m + s(\sigma) \leq 2\alpha \).**

In this case, we know \( \frac{\alpha + s(\sigma)}{3} \leq \frac{1 + \sigma}{3 + \sigma} \alpha \leq s_m \). Based on the retailer’s order quantity \( q_r \), we need to separate to several sub-cases.

**Subcase 1.1: \( q_r \leq \frac{\alpha + s(\sigma)}{3} \).**

Then we know \( \tilde{q}_r \leq q_r \leq \frac{\alpha + s(\sigma)}{3} \leq \alpha - s_m \) and \( \tilde{q}_m = \frac{\alpha - \tilde{q}_r - s_m}{2} \). If \( \tilde{q}_r < q_r \), then we know \( \frac{\alpha - \tilde{q}_m}{2} = \frac{\alpha - \frac{\alpha - \tilde{q}_r - s_m}{2}}{2} = \frac{\alpha + s(\sigma)}{4} > \tilde{q}_r \) since \( \tilde{q}_r < q_r \) \( \leq \frac{\alpha + s(\sigma)}{3} \). Then the retailer has incentive to sell more than \( \tilde{q}_r \). So in this case the only possibility in the equilibrium is \( \tilde{q}_r = q_r \). If \( \tilde{q}_r = q_r \leq \alpha - s_m \), then \( \frac{\alpha - \tilde{q}_m}{2} = \frac{\alpha - \frac{\alpha - \tilde{q}_r - s_m}{2}}{2} = \frac{\alpha + s(\sigma)}{4} \).
In this case, we know 

\[ q_r, q_m(H) = \frac{\alpha - q_r - s_m}{2} \quad \text{and} \quad q_m(L) = \frac{\alpha - q_r}{2}. \]

Subcase 1.2: \( \alpha - s_m \geq q_r \geq \frac{\alpha + s(\sigma)}{3} \).

As in Subcase 1.1, we can show \( \tilde{q}_r < \frac{\alpha + s(\sigma)}{3} \) is not possible in the equilibrium.

If \( \alpha - s_m \geq q_r \geq \frac{\alpha + s(\sigma)}{3} \), then \( \frac{\alpha - \tilde{q}_r - s_m}{2} = \frac{\alpha - \tilde{q}_r}{2} = \frac{\alpha + \tilde{q}_r + s(\sigma)}{4} < \tilde{q}_r. \)

Then the retailer has incentive to sell less than \( \tilde{q}_r \). Thus, in this case the only possibility in the equilibrium is \( \tilde{q}_r = \frac{\alpha + s(\sigma)}{3} \). If \( \tilde{q}_r = \frac{\alpha + s(\sigma)}{3} \), then \( \tilde{q}_m = \frac{\alpha - 2s(\sigma)}{3}. \)

And \( \frac{\alpha - \tilde{q}_m}{2} = \frac{\alpha + s(\sigma)}{3} \). So in the equilibrium \( \tilde{q}_r = \frac{\alpha + s(\sigma)}{3}, q_m(H) = \frac{\alpha - \frac{\alpha + s(\sigma) - s_m}{3}}{2} \)

and \( q_m(L) = \frac{\alpha - \frac{\alpha + s(\sigma)}{3}}{2} \).

Subcase 1.3: \( \alpha \geq q_r \geq \alpha - s_m \).

As in Subcase 1.2, we can show if \( \tilde{q}_r \leq \alpha - s_m \), then the only possible equilibrium is with \( \tilde{q}_r = \frac{\alpha + s(\sigma)}{3} \). If \( \alpha - s_m \leq \tilde{q}_r \leq q_r \leq \alpha \), then we know \( q_m(H) = 0, q_m(L) = \frac{\alpha - \tilde{q}_r}{2} \) and \( \tilde{q}_m = \frac{(1 - \sigma)(\alpha - \tilde{q}_r)}{2} \).

Since \( \frac{\alpha - \tilde{q}_m}{2} = \frac{\alpha - (1 - \sigma)(\alpha - \tilde{q}_r)}{2} = \frac{(1 + \sigma)\alpha + (1 - \sigma)\tilde{q}_r}{4} \leq \tilde{q}_r \), then the retailer has incentive to sell less than \( \tilde{q}_r \). Thus, the retailer will sell no more than \( \alpha - s_m \) and we go to situation when \( \tilde{q}_r \leq \alpha - s_m \). In all, we know in the equilibrium \( \tilde{q}_r = \frac{\alpha + s(\sigma)}{3}, q_m(H) = \frac{\alpha - \frac{\alpha + s(\sigma) - s_m}{3}}{2} \) and \( q_m(L) = \frac{\alpha - \frac{\alpha + s(\sigma)}{3}}{2} \).

Case 2: \( 3s_m + s(\sigma) \geq 2\alpha \).

In this case, we know \( \alpha - s_m \leq \frac{\alpha + s(\sigma)}{3} \) and \( \alpha - s_m \leq \frac{1 + \sigma}{3 + \sigma} \alpha \). Based on the retailer’s order quantity \( q_r \), we need to separate to several sub-cases.

Subcase 2.1: \( q_r \leq \alpha - s_m \).

Then we know \( \tilde{q}_r \leq q_r \leq \alpha - s_m \leq \frac{\alpha + s(\sigma)}{3} \). As in the Subcase 1.2, we know the only possible equilibrium is with \( \tilde{q}_r = q_r, q_m(H) = \frac{\alpha - q_r - s_m}{2} \) and \( q_m(L) = \frac{\alpha - q_r}{2} \).

Subcase 2.2: \( \alpha - s_m \leq q_r \leq \frac{1 + \sigma}{3 + \sigma} \alpha \).
As in Subcase 2.1, we can show if $\tilde{q}_r \leq \alpha - s_m$, then the only possible equilibrium is with $\tilde{q}_r = \alpha - s_m$. Since $q_r \geq \alpha - s_m$, the retailer may want to sell more.

If $\alpha - s_m \leq \tilde{q}_r < q_r \leq \frac{1+\sigma}{3+\sigma} \alpha$, then we know $q_m(H) = 0$, $q_m(L) = \frac{\alpha - \tilde{q}_r}{2}$, and $\tilde{q}_m = \frac{(1-\sigma)(\alpha - \tilde{q}_r)}{2}$. Since $\frac{\alpha - \tilde{q}_m}{2} = \frac{\alpha - (1-\sigma)(\alpha - \tilde{q}_r)}{2} = \frac{1+\sigma}{4} \alpha + (1-\sigma)q_r > \tilde{q}_r$, then the retailer has incentive to sell more. Since the retailer cannot sell more than $q_r$, then we know in the equilibrium $\tilde{q}_r = q_r$, $q_m(H) = 0$ and $q_m(L) = \frac{\alpha - \tilde{q}_r}{2}$.

Subcase 2.3: $\frac{1+\sigma}{3+\sigma} \alpha < q_r \leq \alpha$.

As in Subcase 2.2, we can show if $\tilde{q}_r \leq \frac{1+\sigma}{3+\sigma} \alpha$, then the only possible equilibrium is with $\tilde{q}_r = \frac{1+\sigma}{3+\sigma} \alpha$. If $\frac{1+\sigma}{3+\sigma} \alpha < \tilde{q}_r < q_r \leq \alpha$, then we know $q_m(H) = 0$, $q_m(L) = \frac{\alpha - \tilde{q}_r}{2}$, and $\tilde{q}_m = \frac{(1-\sigma)(\alpha - \tilde{q}_r)}{2}$. Since $\frac{\alpha - \tilde{q}_m}{2} = \frac{\alpha - (1-\sigma)(\alpha - \tilde{q}_r)}{2} = \frac{1+\sigma}{4} \alpha + (1-\sigma)q_r < \tilde{q}_r$, then the retailer has incentive to sell less. Then we know in the equilibrium $\tilde{q}_r = \frac{1+\sigma}{3+\sigma} \alpha$, $q_m(H) = 0$ and $q_m(L) = \frac{\alpha - \tilde{q}_r}{2}$.

Combine the two cases and we can get the lemma.

Next, we will derive the retailer’s order quantity given his updated belief for the $H$–type manufacturer. In order to facilitate the following discussion, we first denote $w_1(\sigma) = \frac{2s_m - \alpha + \sigma s_m}{2}$, $w_2(\sigma) = \frac{2s_m - \alpha + \sigma (2s_m - \alpha)}{2}$, $w_3(\sigma) = \frac{1-\sigma^2}{2(3+\sigma)} \alpha$. If the retailer updates his belief for the manufacturer to be H-type as $\sigma(H|w) = \sigma$, based on Lemma A.3, the retailer will choose $q_r$ to maximize his profit $\Pi_r(q_r|w, \sigma) = -wq_r + \left\{ \alpha - [\sigma q_m(H) + (1-\sigma)q_m(L)] - \tilde{q}_r \right\} q_r$.

**Lemma A.4** If the retailer’s updated belief for the H-type manufacturer is $\sigma(H|w) := \sigma$, then his order quantity is as following:
1. $3s_m + s(\sigma) \leq 2\alpha$.

$$q_r(w, \sigma) = \begin{cases} 
0 & \text{if } \frac{\alpha+s(\sigma)}{2} \leq w, \\
\frac{\alpha+s(\sigma)-2w}{2} & \text{if } w_1(\sigma) \leq w \leq \frac{\alpha+s(\sigma)}{2}, \\
\frac{\alpha+s(\sigma)}{3} & \text{if } 0 \leq w \leq \frac{\alpha+s(\sigma)}{6}, 
\end{cases}$$

where $s(\sigma) = \sigma s_m$.

2. $3s_m + s(\sigma) \geq 2\alpha$.

$$q_r(w, \sigma) = \begin{cases} 
0 & \text{if } \frac{\alpha+s(\sigma)}{2} \leq w, \\
\frac{\alpha+s(\sigma)-2w}{2} & \text{if } w_1(\sigma) \leq w \leq \frac{\alpha+s(\sigma)}{2}, \\
\alpha - s_m & \text{if } w_2(\sigma) \leq w \leq w_1(\sigma), \\
\frac{(1+\alpha-2w)}{2(\alpha+1)} & \text{if } w_3(\sigma) \leq w \leq w_2(\sigma), \\
\frac{1+\alpha}{3+\alpha} & \text{if } 0 \leq w \leq w_3(\sigma). 
\end{cases}$$

**Proof:** We will consider the two cases one by one.

1. $3s_m + s(\sigma) \leq 2\alpha$.

From Lemma A.3, we need to compare the profit for the following two cases: $q_r \leq \frac{\alpha+s(\sigma)}{3}$ and $q_r \geq \frac{\alpha+s(\sigma)}{3}$. If $q_r \geq \frac{\alpha+s(\sigma)}{3}$, then we know $\bar{q}_r = \frac{\alpha+s(\sigma)}{3}, \bar{q}_m = \frac{\alpha-2s(\sigma)}{3}$ and retailer’s profit $\Pi_r(q_r|\sigma) = -wq_r + (\alpha - \frac{\alpha-2s(\sigma)}{3} - \frac{\alpha+s(\sigma)}{3})\frac{\alpha+s(\sigma)}{3}$, which is a decreasing function of $q_r$. Then the retailer will choose $q_r = \frac{\alpha+s(\sigma)}{3}$.

So we only need to consider the case $q_r \leq \frac{\alpha+s(\sigma)}{3}$. If $q_r \leq \frac{\alpha+s(\sigma)}{3}$, then we know $\tilde{q}_r = q_r, \tilde{q}_m = \frac{\alpha-q_r-s(\sigma)}{2}$ and $\Pi_r(q_r) = -wq_r + (\alpha - \tilde{q}_m - q_r)q_r = -wq_r + (\alpha - \frac{\alpha-q_r-s(\sigma)}{2} - q_r)$, which is increasing when $q_r \leq \frac{\alpha+s(\sigma)-2w}{2}$ and decreasing when $q_r \geq \frac{\alpha+s(\sigma)-2w}{2}$. By setting $0 \leq \frac{\alpha+s(\sigma)-2w}{2} \leq \frac{\alpha+s(\sigma)}{3}$, we get the restriction on $w$ as $\frac{\alpha+s(\sigma)}{2} \geq w \geq \frac{\alpha+s(\sigma)}{6}$. So if $w \geq \frac{\alpha+s(\sigma)}{2}$, the retailer’s profit function is a decreasing function of $q_r$ and then the retailer will order nothing. If $\frac{\alpha+s(\sigma)}{6} \leq \frac{\alpha+s(\sigma)}{2}$, then
\(w \leq \frac{\alpha + s(\sigma)}{2}\), the retailer’s profit function is increasing when \(q_r \leq \frac{\alpha + s(\sigma) - 2w}{2}\) and decreasing when \(q_r \geq \frac{\alpha + s(\sigma) - 2w}{2}\) and then the retailer will choose \(q_r = \frac{\alpha + s(\sigma) - 2w}{2}\).

If \(\frac{\alpha + s(\sigma)}{6} \geq w \geq 0\), the retailer’s profit function is increasing when \(q_r \leq \frac{\alpha + s(\sigma)}{3}\) and decreasing when \(q_r \geq \frac{\alpha + s(\sigma)}{3}\) and then the retailer will choose \(q_r = \frac{\alpha + s(\sigma)}{3}\).

2. \(3s_m + s(\sigma) \geq 2\alpha\).

From Lemma A.3, we need to compare the profit for the following three cases:

If \(q_r \leq \alpha - s_m\), \(\alpha - s_m \leq q_r \leq \frac{1+\alpha}{3+\sigma} \alpha\) and \(q_r \geq \frac{1+\alpha}{3+\sigma} \alpha\). If \(q_r \geq \frac{1+\alpha}{3+\sigma} \alpha\), then we know \(\bar{q}_r = \frac{1+\alpha}{3+\sigma} \alpha\), \(\bar{q}_m = \frac{(1-\sigma)(\alpha - \bar{q}_r)}{2} = \frac{(1-\sigma)\alpha}{3+\sigma}\) and the retailer’s profit \(\Pi_r(q_r, \sigma) = -wq_r + (\alpha - \frac{(1-\sigma)\alpha}{3+\sigma})\bar{q}_m\) which is a decreasing function of \(q_r\). Then the retailer will choose \(q_r = \frac{(1-\sigma)\alpha}{3+\sigma}\). So we only need to consider the case \(\alpha - s_m \leq q_r \leq \frac{(1-\sigma)\alpha}{3+\sigma}\).

If \(\alpha - s_m \leq q_r \leq \frac{(1-\sigma)\alpha}{3+\sigma}\), then we know \(\bar{q}_r = q_r\), \(\bar{q}_m = \frac{(1-\sigma)(\alpha - q_r)}{2} = \frac{(1-\sigma)(\alpha - q_r)}{2}\) and \(\Pi_r(q_r) = -wq_r + (\alpha - \bar{q}_m - q_r)q_r = -wq_r + (\alpha - \frac{(1-\sigma)(\alpha - q_r)}{2} - q_r)q_r\), which is increasing when \(q_r \leq \frac{(1+\alpha)\alpha - 2w}{2(1+\sigma)}\) and decreasing when \(q_r \geq \frac{(1+\alpha)\alpha - 2w}{2(1+\sigma)}\). By setting \(\alpha - s_m \leq \frac{(1+\alpha)\alpha - 2w}{2(1+\sigma)} \leq \frac{1+\alpha}{3+\sigma} \alpha\), we get the restriction on \(w\) as \(w_2(\sigma) \geq w \geq w_3(\sigma)\).

If \(q_r \leq \alpha - s_m\), then we know \(\bar{q}_r = q_r\), \(\bar{q}_m = \frac{\alpha - q_r - s(\sigma)}{2}\) and \(\Pi_r(q_r) = -wq_r + (\alpha - \bar{q}_m - q_r)q_r = -wq_r + (\alpha - \frac{\alpha - q_r - s(\sigma)}{2} - q_r)q_r\), which is increasing when \(q_r \leq \frac{\alpha + s(\sigma) - 2w}{2}\) and decreasing when \(q_r \geq \frac{\alpha + s(\sigma) - 2w}{2}\). By setting \(0 \leq \frac{\alpha + s(\sigma) - 2w}{2} \leq \alpha - s_m\), we get the restriction on \(w\) as \(\frac{\alpha + s(\sigma)}{2} \geq w \geq w_1(\sigma)\). In all, we know if \(w \geq \frac{\alpha + s(\sigma)}{2}\), the retailer’s profit is decreasing with respect to \(q_r\) and then the retailer will choose \(q_r = 0\). If \(w_1(\sigma) \leq q_r \leq \frac{\alpha + s(\sigma)}{2}\), the retailer’s profit is increasing when \(q_r \leq \frac{\alpha + s(\sigma) - 2w}{2}\) and decreasing when \(q_r \geq \frac{\alpha + s(\sigma) - 2w}{2}\) and then the retailer will choose \(q_r = \frac{\alpha + s(\sigma) - 2w}{2}\). If \(w_2(\sigma) \leq q_r \leq w_1(\sigma)\), the retailer’s profit is increasing when \(q_r \leq \alpha - s_m\) and decreasing when \(q_r \geq \alpha - s_m\) and then the retailer will choose \(q_r = \alpha - s_m\). If \(w_3(\sigma) \leq q_r \leq w_2(\sigma)\), the retailer’s profit is increasing.
when \( q_r \leq \frac{(1+\sigma)\alpha-2w}{2(\sigma+1)} \) and decreasing when \( q_r \geq \frac{(1+\sigma)\alpha-2w}{2(\sigma+1)} \) and then the retailer will choose \( q_r = \frac{(1+\sigma)\alpha-2w}{2(\sigma+1)} \). If \( 0 \leq w \leq w_3(\sigma) \), the retailer’s profit is increasing when \( q_r \leq \frac{1+\sigma}{3+\sigma}\alpha \) and decreasing when \( q_r \geq \frac{1+\sigma}{3+\sigma}\alpha \) and then the retailer will choose \( q_r = \frac{1+\sigma}{3+\sigma}\alpha \).

From Lemma A.4 and A.3, we know \( \tilde{q}_r = q_r \) in the equilibrium. Since our goal is to find the equilibrium outcome, then we just set \( \tilde{q}_r = q_r \) afterwards. Let \( \Pi_m(w, \sigma|\tau) \) be the manufacturer’s profit if her type is \( \tau \) and the offered wholesale price is \( w \) and the retailer’s belief for the manufacturer to be H-type is \( \sigma \) after observing the offered wholesale price \( w \). Then we know \( \Pi_m(w, \sigma|\tau) = wq_r + (\alpha - \tilde{q}_r - q_m - s_r)q_m = wq_r + \left\lfloor \frac{(\alpha - q_r - s_r)^+}{4} \right\rfloor = wq_r + \left\lfloor \frac{(\alpha - q_r - s_r)^+}{4} \right\rfloor \). Substituting \( q_r \) from Lemma A.4, we can get the formula for \( \Pi_m(w, \sigma|\tau) \).

Specifically, we know the L-type manufacturer’s profit if she offers wholesale price \( w \) is as follows:

\[
\Pi_m(w, \sigma|L) = \begin{cases} 
\frac{\alpha^2}{4} & \text{if } w \geq \frac{\alpha+s(\sigma)}{2}, \\
w\frac{\alpha+s(\sigma)-2w}{2} + \left(\frac{\alpha-s(\sigma)-2w}{4}\right)^2 & \text{if } \frac{\alpha+s(\sigma)}{6} \leq w \leq \frac{\alpha+s(\sigma)}{2}, \\
w\frac{\alpha+s(\sigma)}{3} + \left(\frac{\alpha-s(\sigma)}{4}\right)^2 & \text{if } w \leq \frac{\alpha+s(\sigma)}{6}, 
\end{cases}
\]

if \( 3s_m + s(\sigma) \leq 2\alpha \); and

\[
\Pi_m(w, \sigma|L) = \begin{cases} 
\frac{\alpha^2}{4} & \text{if } w \geq \frac{\alpha+s(\sigma)}{2}, \\
w\frac{(1+\sigma)\alpha-2w}{2(\sigma+1)} + \left(\frac{(1+\sigma)\alpha-2w}{4}\right)^2 & \text{if } w_1(\sigma) \leq w \leq \frac{\alpha+s(\sigma)}{2}, \\
w(\alpha - s_m) + \frac{s_m^2}{4} & \text{if } w_2(\sigma) \leq w \leq w_1(\sigma), \\
w\frac{(1+\sigma)\alpha-2w}{2(\sigma+1)} + \left(\frac{(1+\sigma)\alpha-2w}{4}\right)^2 & \text{if } w_3(\sigma) \leq w \leq w_2(\sigma), \\
w\frac{1+\sigma}{3+\sigma}\alpha + \left(\frac{1+\sigma}{3+\sigma}\alpha\right)^2 & \text{if } w \leq w_3(\sigma), 
\end{cases}
\]

if \( 3s_m + s(\sigma) \geq 2\alpha \).
The H-type manufacturer’s profit if she offers wholesale price $w$ is as follows:

$$\Pi_m(w, \sigma|H) = \begin{cases} 
\frac{(\alpha-s_m)^2}{4} & \text{if } w \geq \frac{\alpha+s(\sigma)}{2}, \\
\alpha+s(\sigma) - 2w + \frac{(\alpha-\alpha-s(\sigma)-2w-s_m)^2}{4} & \text{if } \frac{\alpha+s(\sigma)}{6} \leq w \leq \frac{\alpha+s(\sigma)}{2}, \\
w\frac{\alpha+s(\sigma)}{3} + \frac{(\alpha-\alpha-s(\sigma)-s_m)^2}{4} & \text{if } w \leq \frac{\alpha+s(\sigma)}{6},
\end{cases}$$

if $3s_m + s(\sigma) \leq 2\alpha$; and

$$\Pi_m(w, \sigma|H) = \begin{cases} 
\frac{(\alpha-s_m)^2}{4} & \text{if } w \geq \frac{\alpha+s(\sigma)}{2}, \\
w\frac{\alpha+s(\sigma)-2w}{2} + \frac{(\alpha-\alpha-s(\sigma)-2w-s_m)^2}{4} & \text{if } w_1(\sigma) \leq w \leq \frac{\alpha+s(\sigma)}{2}, \\
w(\alpha-s_m) & \text{if } w_2(\sigma) \leq w \leq w_1(\sigma), \\
w\frac{(1+\sigma)(\alpha-2w)}{2(\sigma+1)} & \text{if } w_3(\sigma) \leq w \leq w_2(\sigma), \\
w^{\frac{1+\sigma}{3+\sigma}} \alpha & \text{if } w \leq w_3(\sigma),
\end{cases}$$

if $3s_m + s(\sigma) \geq 2\alpha$.

We use $w(\tau)$ to denote the wholesale price offered by the manufacturer of type $\tau$. There are two kinds of equilibrium in this kind of signaling game. One is separating equilibrium and the other is pooling equilibrium. Under our setting, in the separating equilibrium, different type manufacturer gives out different signal, i.e. $w(L) \neq w(H)$. In the pooling equilibrium, different type manufacturer gives out the same signal, i.e. $w(L) = w(H)$. Next, we will first derive the most profitable separating equilibrium and then the most profitable pooling equilibrium. Finally, we will compare the $H$-type manufacturer’s profits in the separating equilibrium and pooling equilibrium to get the “undefeated equilibrium”.

**Separating Equilibrium.** Base on our notation, $\Pi_m(w, 0|L)$ is the $L$-type manufacturer’s profit if the retailer knows her type and $\Pi_m(w, 1|L)$ is her profit if the retailer mistook her as $H$-type. From the above formula, we can readily derive the highest value of $\Pi_m(w, 0|L)$ is $\frac{\alpha^2}{4}$. Hence, the $L$-type manufacturer can at most get
profit \( \frac{\alpha^2}{4} \) if the retailer knows her cost information. On the other hand, by setting a large enough wholesale price, the \( L \)-type manufacturer can cut off the retailer and then get profit \( \frac{\alpha^2}{4} \) as a monopoly player in the downstream market. Since \( w(L) \neq w(H) \), then the retailer knows the manufacturer is \( L \)-type when observing \( w(L) \). Hence we have \( \Pi(w(L), 0|L) = \frac{\alpha^2}{4} \). When \( s_m \leq \frac{3\alpha}{4} \), it is also easy to show \( \Pi_m(w, 1|L) \) is increasing when \( w \leq \frac{3\alpha + s_m}{6} \) and decreasing when \( w \geq \frac{3\alpha + s_m}{6} \). When \( \frac{3\alpha}{4} \leq s_m \leq \frac{6\alpha}{7} \), \( \Pi_m(w, 1|L) \) is increasing when \( w \leq \frac{3s_m - \alpha}{2} \) and decreasing when \( w \geq \frac{3s_m - \alpha}{2} \). When \( \frac{6\alpha}{7} \leq s_m < \alpha \), \( \Pi_m(w, 1|L) \) is increasing when \( w \leq \frac{5\alpha}{7} \) or \( 2s_m - \alpha \leq w \leq \frac{3s_m - \alpha}{2} \) and decreasing when \( \frac{5\alpha}{7} \leq w \leq 2s_m - \alpha \) or \( \frac{3s_m - \alpha}{2} \leq w \). And we note \( \Pi_m(2s_m - \alpha, 1|L) > \frac{\alpha^2}{4} \) when \( \frac{6\alpha}{7} < s_m < \alpha \). Hence, by setting \( \Pi_m(\tilde{w}, 1|L) = \frac{\alpha^2}{4} \) and \( \tilde{w} < \frac{\alpha + s_m}{2} \), we have

\[
\tilde{w} = \begin{cases} 
\frac{3\alpha - s_m}{6} & \text{if } s_m < \frac{3\alpha}{5}, \\
\frac{\alpha + s_m}{4} & \text{if } \frac{3\alpha}{5} \leq s_m \leq \frac{5\alpha}{7}, \\
\frac{3\alpha}{7} & \text{if } \frac{5\alpha}{7} < s_m < \alpha.
\end{cases}
\]

From the above discussion, we know \( \Pi(w, 1|L) > \frac{\alpha^2}{4} \) when \( \tilde{w} < w < \frac{\alpha + s_m}{2} \) and then \( w(H) \) cannot be set in the range \( \tilde{w} < w(H) < \frac{\alpha + s_m}{2} \). It is also easy to show \( \Pi_m(w, 1|H) \) is increasing when \( w \leq \tilde{w} \) and \( \Pi_m(\tilde{w}, 1|H) > \Pi_m(\frac{\alpha + s_m}{2}, 1|H) = \frac{(\alpha - s_m)^2}{4} \). Hence, we know \( \Pi_m(w(H), 1|H) \leq \Pi_m(\tilde{w}, 1|H) \). In other words, in any separating equilibrium, the \( H \)-type manufacturer’s profit cannot be larger than \( \Pi_m(\tilde{w}, 1|H) \). Moreover, this is the maximum profit the \( H \)-type manufacturer can get among all the separating equilibria. By setting the retailer’s belief as

\[
\sigma(H|w) = \begin{cases} 
1 & \text{if } w \leq \tilde{w}, \\
0 & \text{if } w > \tilde{w},
\end{cases}
\]

we can have a separating equilibrium with \( w(H) = \tilde{w} \) and \( w(L) = \frac{\alpha}{2} \). By setting \( w(L) = \frac{\alpha}{2} \), the retailer will believe the manufacturer as \( L \)-type and \( \Pi_m(w(L), 0|L) = \frac{\alpha^2}{4} \).
$\frac{a^2}{4}$, which is the highest profit the $L$-type manufacturer can choose. From the above argument, we also know the $L$-type manufacturer has no incentive to mimic as $H$-type, i.e. $\Pi_m(w(L), 0|L) \geq \Pi_m(w, 1|L)$ when $w \leq \bar{w}$. We also know when $w \leq \bar{w}$, then $H$-type manufacturer’s highest profit is at $w = \bar{w}$. Then in order to proof our claim, we just need to show $\Pi_m(w(H), 1|H) \geq \Pi_m(w, 0|H)$ when $w \geq \bar{w}$. From the above formula of $\Pi_m(w, 0|H)$, we know $\Pi_m(w, 0|H)$ is increasing when $w \leq \frac{3a - 2s_m}{6}$ and decreasing when $w \geq \frac{3a - 2s_m}{6}$. By directly comparing the maximum value of $\Pi_m(w, 0|H)$ and $\Pi_m(\bar{w}, 1|H)$, we can show $\Pi_m(w, 0|H) \leq \Pi_m(\bar{w}, 1|H)$. Hence the $H$-type manufacturer has no incentive to mimic as the $L$-type manufacturer.

**Pooling Equilibrium.** We will first find the pooling price related to the most profitable pooling equilibrium for the $H$-type manufacturer. Then we will eliminate the pooling equilibrium that can be defeated by the separating equilibrium. Finally, we will show the retailer’s belief for the pooling equilibrium that can defeat the separating equilibrium. To facilitate our demonstration, we denote $s_1 = \frac{2a}{3 + 2\lambda}$, $s_2 = \frac{a(\sqrt{49\lambda^2 + 50\lambda - 37} + 7\lambda + 21)}{14(\lambda + 2)}$, and $s_3 = \frac{a(\sqrt{2}\sqrt{\lambda \lambda + 1} + 2\lambda + 6)}{4\lambda + 8}$. And it is easy to show $6a > \max\{s_2, s_3\} \geq \min\{s_2, s_3\} > \frac{5a}{7}$ if $\lambda > \frac{3}{5}$.

When $s_m < s_1$, we can check $\Pi_m(w, \lambda|L)$ is increasing when $w \leq \frac{3a + 4s_m}{6}$ and decreasing when $w \geq \frac{3a + 4s_m}{6}$. When $\frac{3a}{3 + \lambda} \leq s_m \leq \frac{3(1 + \lambda)a}{3 + 4\lambda}$, $\Pi_m(w, \lambda|L)$ is increasing when $w \leq w_1(\lambda)$ and decreasing when $w \geq w_1(\lambda)$. When $\frac{3(1 + \lambda)a}{3 + 4\lambda} \leq s_m < \alpha$, $\Pi_m(w, \lambda|L)$ is increasing when $w \leq \frac{a(2\lambda^2 + 5\lambda + 3)}{8\lambda + 6}$ or $w_2(\lambda) \leq w \leq w_1(\lambda)$ and decreasing when $\frac{a(2\lambda^2 + 5\lambda + 3)}{8\lambda + 6} \leq w \leq w_2(\lambda) \text{ or } w_1(\lambda) \leq w$. And we note $\Pi_m(w_2(\lambda), \lambda|L) > \frac{a^2}{4}$ when
\[
\frac{3(1+\lambda)\alpha}{3+4\lambda} \leq s_m < \alpha. \text{ Hence, by setting } \Pi_m(\hat{w}, \lambda|L) = \frac{\alpha^2}{4} \text{ and } \hat{w} < \frac{\alpha + \lambda s_m}{2}, \text{ we have }
\]

\[
\hat{w} = \begin{cases} 
\frac{3\alpha - \lambda s_m}{6} & \text{if } s_m < s_1, \\
\frac{\alpha + \lambda s_m}{4} & \text{if } s_1 \leq s_m \leq \frac{(3+2\lambda)\alpha}{3+4\lambda}, \\
\frac{3(1+\lambda)\alpha}{6+8\lambda} & \text{if } \frac{(3+2\lambda)\alpha}{3+4\lambda} < s_m < \alpha.
\end{cases}
\]

Denote the wholesale price offered by the manufacturer in the pooling equilibrium as \(\bar{w}\). Then we know \(\bar{w}\) cannot be set lower than \(\hat{w}\).

It is easy to check \(\Pi_m(w, \lambda|H)\) is increasing when \(w \leq \frac{3\alpha-(2-\lambda)s_m}{6}\) and decreasing when \(w \geq \frac{3\alpha-(2-\lambda)s_m}{6}\) if \(s_m \leq \frac{3\alpha}{4+\lambda}\); \(\Pi_m(w, \lambda|H)\) is increasing when \(w \leq \frac{(1+\lambda)\alpha}{4}\) or \(w_2(\lambda) \leq w \leq w_1(\lambda)\) and decreasing when \(\frac{(1+\lambda)\alpha}{4} \leq w \leq w_2(\lambda)\) or \(w \geq w_1(\lambda)\) if \(\frac{3\alpha}{4} < s_m < \alpha\). Therefore, we know \(\Pi_m(w, \lambda|H)\) obtains its highest value at \(\frac{3\alpha-(2-\lambda)s_m}{6}\) if \(s_m \leq \frac{3\alpha}{4+\lambda}\) and \(w_1(\lambda)\) if \(\frac{3\alpha}{4+\lambda} \leq s_m \leq s_3\) and \(\frac{(1+\lambda)\alpha}{4}\) if \(s_3 \leq s_m < \alpha\). We also note \(\frac{3(1+\lambda)\alpha}{6+8\lambda} > \frac{(1+\lambda)\alpha}{4}\) if \(\lambda < \frac{3}{4}\) and \(\frac{3(1+\lambda)\alpha}{6+8\lambda} < \frac{(1+\lambda)\alpha}{4}\) if \(\lambda > \frac{3}{4}\). Therefore, in the pooling equilibrium that the \(H\)-type manufacturer has the highest profit, we have

\[
\bar{w} = \begin{cases} 
\frac{3\alpha - \lambda s_m}{6} & \text{if } s_m < s_1, \\
w_1(\lambda) & \text{if } s_1 \leq s_m \leq s_4, \\
\frac{3(1+\lambda)\alpha}{6+8\lambda} & \text{if } s_4 < s_m < \alpha,
\end{cases}
\]

if \(\lambda \leq \frac{3}{4}\) and

\[
\bar{w} = \begin{cases} 
\frac{3\alpha - \lambda s_m}{6} & \text{if } s_m < s_1, \\
w_1(\lambda) & \text{if } s_1 \leq s_m \leq s_3, \\
\frac{(1+\lambda)\alpha}{4} & \text{if } s_3 < s_m < \alpha,
\end{cases}
\]

if \(\lambda > \frac{3}{4}\). Here, \(s_4 = \frac{\alpha(\lambda+3-\sqrt{16\lambda^4+32\lambda^3+\lambda^2-6\lambda+9})}{4\lambda+4}\).

We will now eliminate the cases if the pooling equilibrium can be defeated by the separating equilibrium. First, we find \(\Pi_m(\hat{w}, 1|H) > \Pi_m(\frac{3\alpha - \lambda s_m}{6}, \lambda|H)\) when \(s_m < s_1\).
Second, when \( \lambda < \frac{3}{5} \), we note \( \frac{5\alpha}{7} < \frac{(3+2\lambda)\alpha}{3+4\lambda} < s_4 \) and \( \Pi_m(\tilde{w}, 1|H) > \Pi_m(\frac{3(1+\lambda)\alpha}{6+8\lambda}, \lambda|H) \) if \( s_4 < s_m < \alpha \). When \( \lambda < \frac{3}{5} \), we know \( s_1 > \frac{5\alpha}{7} \) and we can show \( \Pi_m(\tilde{w}, 1|H) > \Pi_m(w_1(\lambda), \lambda|H) \). Next, we only focus on the situation when \( \lambda \geq \frac{3}{5} \). Therefore, we know \( w_1(\lambda) > 2s_m - \alpha \) when \( s_m \geq s_1 \). When \( s_m \leq \frac{5\alpha}{7} \), if \( s_1 < s_m \leq \frac{5\alpha}{7} \), we know \( w_1(\lambda) > \frac{\alpha + s_m}{4} \geq 2s_m - \alpha \) and then \( \Pi(\tilde{w}, 1|H) < \Pi(w_1(\lambda), \lambda|H) \). When \( s_m > \frac{5\alpha}{7} \), we know \( \tilde{w} = \frac{3\alpha}{7} \). And we can show \( \Pi(\tilde{w}, 1|H) < \Pi(w_1(\lambda), \lambda|H) \) if \( s_m < s_2 \) and \( \Pi(\tilde{w}, 1|H) < \Pi(\frac{(1+\lambda)\alpha}{4}, \lambda|H) \) if \( \lambda > \frac{47}{49} \).

In order to show the lemma, we only need to show there exists a pooling equilibrium with pooling price \( \tilde{w} \) when \( s_1 \leq s_m \leq s_2 \) if \( \lambda < \frac{47}{49} \) and when \( s_1 \leq s_m < \alpha \) if \( \lambda \geq \frac{47}{49} \).

Next, we will show a pooling equilibrium with \( \tilde{w} \) is possible with retailer’s belief

\[
\sigma(H|w) = \begin{cases} 
1 & \text{if } w < \tilde{w}, \\
\lambda & \text{if } w = \tilde{w}, \\
0 & \text{if } w > \tilde{w},
\end{cases}
\]

if \( s_m \leq \frac{5\alpha}{7} \) and with retailer’s belief

\[
\sigma(H|w) = \begin{cases} 
0 & \text{if } w \neq \tilde{w}, \\
\lambda & \text{if } w = \tilde{w},
\end{cases}
\]

if \( s_m \geq \frac{5\alpha}{7} \).

Since \( \Pi_m(\tilde{w}, \lambda|L) \geq \frac{\alpha^2}{4} \), we know the \( L \)-type manufacturer has no incentive to choose \( w > \tilde{w} \). We note \( \Pi_m(w, 1|L) \) is an increasing function of \( w \) when \( s_m \leq \frac{6\alpha}{7} \) and \( 2s_m - \alpha \leq w_1(\lambda) < w_1(1) \). Then we know \( L \)-type manufacturer has no incentive to choose \( w < \tilde{w} \) when \( s_1 \leq s_m \leq s_2 \) and \( \lambda < \frac{47}{49} \) and when \( s_1 < s_m < s_3 \) and \( \lambda \geq \frac{47}{49} \).

Since \( \Pi_m(\frac{(1+\alpha)}{4}, \lambda|L) > \frac{\alpha^2}{4} = \max_w \Pi_m(w, 0|L) \), we know the \( L \)-type manufacturer has no incentive to choose \( w \neq \frac{(1+\alpha)}{4} \). Hence, the \( L \)-type manufacturer has no incentive to
choose other wholesale price. When \( s_m < \frac{5\alpha}{7} \), we know \( \Pi_m(w, 1|H) \) is an increasing function of \( w \) if \( w \leq w_1(\lambda) \) and \( \Pi_m(w_1(\lambda), 1|H) > \max_{w \geq w_1(\lambda)} \Pi_m(w, 0|H) \). When \( s_m \geq \frac{5\alpha}{7} \), we can show \( \Pi_m(\bar{w}, 1|H) > \max_{w \geq 0} \Pi_m(w, 0|H) \). Hence, we know the H-type manufacturer will have no incentive to deviate. 

Based on the equilibrium results, we can readily derive all the other propositions and then we omit all the proofs.

## B Proofs for Chapter 3

### B.1 Equilibrium results for the sequential model

To facilitate our demonstration, we denote \( x, y, z, w, s_1, s_2 \) and \( s_3 \) as follows:

\[
x = \sqrt{4\alpha s - \alpha^2 - 3s^2}; \quad y = 40\alpha^6 + 1745s^6 - 5884\alpha s^5 + 7994\alpha^2 s^4 - 5664\alpha^3 s^3 + 2228\alpha^4 s^2 - 464\alpha^5 s + 4(-8\alpha^5 + 79s^5 - 327\alpha s^4 + 444\alpha^2 s^3 - 270\alpha^3 s^2 + 76\alpha^4 s)x; \quad z = \frac{1}{4(x^2 + 4x - 2\alpha x)}(-5\alpha^3 + 37\alpha s^2 - 56\alpha^2 s^2 + 28\alpha^2 s + (19\alpha^2 + 57s^2 - 68\alpha s)x) + \frac{\sqrt{y}}{2(2s + x - \alpha)x}; \quad w = 11\alpha^2 + 24s^2 - 32\alpha s + (8\alpha - 12s)x; \quad s_1 \) is the solution for \( 31\alpha^4 + 410s^4 - 768\alpha s^3 + 565\alpha^2 s^2 - 202\alpha s + 2(-124\alpha^3 + 657s^3 - 1146\alpha s^2 + 657\alpha^2 s)x = 0 \) with respect to \( s \) such that \( \frac{17}{30}\alpha \leq s \leq \frac{3}{5}\alpha \); \( s_2 \) is the solution for \( 12\alpha^2 + 25s^2 - 33\alpha s + (13\alpha - 21s)x = 0 \) with respect to \( s \) such that \( \frac{3}{5}\alpha \leq s \leq \frac{5}{6}\alpha \); \( s_3 = \frac{\sqrt{37 - 3\sqrt{65} + 4}}{6}\alpha \). The \( h - s \) plane is divided into 11 mutually exclusive and collectively exhaustive regions as shown in Table B.1.

In Table B.1, \( h_0 = \frac{a}{4}, \ h_1 = \frac{(42-\sqrt{154})\alpha}{90}, \ h_2 = \frac{7(21s-11\alpha)}{51}, \ h_3 = \frac{30s-17\alpha}{6}, \ h_4 = \frac{30s-17\alpha+\sqrt{135a^2+345s^2-320s}}{6}, \ h_5 = z, \ h_6 = \frac{33s-12\alpha^2-25s^2+(21s-13\alpha)x}{4(2\alpha-3s+x)} + \frac{\sqrt{w}}{2}, \ h_7 = \frac{8(a-s)^2-4a^2+3(a-3s+x)^2}{8(3s-2a-x)} - \frac{a}{2}, \ h_8 = \frac{8(a-s)^2-4a^2+3(a-3s+x)^2}{8(3s-2a-x)} - \frac{a}{2}, \ h_9 = \frac{12a^2+25s^2-33a+(13a-21s)x}{4(3s-2a-x)}; \ h_{10} = h_7 - \frac{\sqrt{-w}}{2}, \) and \( h_{11} = \frac{a-\sqrt{17}\sqrt{3a^2+6s^2-8as+(4\alpha-6s)x}}{4}. \)
Table B.1: Definition for the regions.

<table>
<thead>
<tr>
<th>Region</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>{(h, s)</td>
</tr>
<tr>
<td>2</td>
<td>{(h, s)</td>
</tr>
<tr>
<td>3</td>
<td>{(h, s)</td>
</tr>
<tr>
<td>4</td>
<td>{(h, s)</td>
</tr>
<tr>
<td>5</td>
<td>{(h, s)</td>
</tr>
<tr>
<td>6</td>
<td>{(h, s)</td>
</tr>
<tr>
<td>7</td>
<td>{(h, s)</td>
</tr>
<tr>
<td>8</td>
<td>{(h, s)</td>
</tr>
<tr>
<td>9</td>
<td>{(h, s)</td>
</tr>
<tr>
<td>10</td>
<td>{(h, s)</td>
</tr>
<tr>
<td>11</td>
<td>{(h, s)</td>
</tr>
</tbody>
</table>

Note: \(x^+ := \max\{0, x\}\), \(c_1 = \frac{3(3696-17\sqrt{154})\alpha}{17683}\), \(c_2 = \frac{(\sqrt{13}+26)\alpha}{39}\), \(c_3 = \frac{3(3696-17\sqrt{154})\alpha}{17683}\) and \(MH = \min\{h_9, h_{10}\}\).

For each region, we summarize the equilibrium results of the supplier’s first- and second-period wholesale prices, \(w_1\) and \(w_2\), the buyer’s first- and second-period order quantities, \(Q_1\) and \(Q_2\), the buyer’s withholding inventory, \(I\), the buyer’s first- and second-period selling quantities, \(q_1\) and \(q_2\), the supplier’s direct selling quantity, \(q_s\), the buyer’s and supplier’s total profits, \(\Pi_b\) and \(\Pi_s\), and the supplier and buyer’s strategy pair through Tables B.2-B.4.

Table B.2: Equilibrium results for regions 1 and 2.

<table>
<thead>
<tr>
<th>Region</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>({w_1, w_2})</td>
<td>\{\frac{11\alpha-13h-s}{154}, \frac{11\alpha+10h-6s}{154}}</td>
<td>\{\frac{13\alpha-3h-b_1}{6}, \frac{3\alpha-b_1}{2}}</td>
</tr>
<tr>
<td>({Q_1, Q_2})</td>
<td>\{\frac{77\alpha-288h+119s}{318}, \frac{3(17h+28s)}{154}}</td>
<td>\{\frac{23\alpha+6h-45s}{12}, \frac{3\alpha-3s}{2}}</td>
</tr>
<tr>
<td>(I)</td>
<td>\{\frac{29\alpha-15h}{154}}</td>
<td>\{\frac{3\alpha-3s}{2}}</td>
</tr>
<tr>
<td>({q_1, q_2})</td>
<td>\{\frac{11\alpha+13h+s}{308}, \frac{10h-51h}{77}}</td>
<td>\{\frac{11\alpha+6h-15s}{12}, \alpha-s}</td>
</tr>
<tr>
<td>(q_s)</td>
<td>\{\frac{17\alpha+5h-13h}{154}}</td>
<td>0</td>
</tr>
<tr>
<td>(\Pi_b)</td>
<td>\frac{41940h^2-28h(1079h-99s)+49(12h^2+345s^2+22as)}{49864}</td>
<td>(b_1)</td>
</tr>
<tr>
<td>(\Pi_s)</td>
<td>\frac{270h^2-252h+0(33\alpha^2+69s^2-444s)}{616}</td>
<td>(b_2)</td>
</tr>
<tr>
<td>Strategy Pair</td>
<td>({D, H})</td>
<td>({ND^1, H})</td>
</tr>
</tbody>
</table>

Note: \(b_1 = \frac{265a^2+36h^2+172h-180h+s+657s^2-810as}{72}\) and \(b_2 = \frac{360h^2-127\alpha^2-36h^2-204\alpha-513s^2+552as}{72}\).
B.2 Equilibrium results for the simultaneous model

As before, we first show the mutually exclusive and collectively exhaustive division for the $h - s$ plane in Table B.5.

In Table B.5, $s_0 = \frac{(137\sqrt{7}+12\sqrt{7}\sqrt{5} - 9 - 865)\alpha}{238\sqrt{5} - 1306}$, $h_0 = \frac{9\alpha}{4}$, $\hat{h}_1 = \frac{3(155 - \sqrt{674})s}{950}$.

\[
\hat{h}_2 = \frac{3(546\alpha - 337\alpha)}{520}, \quad \hat{h}_4 = \frac{10(7\sqrt{5} - 11)\alpha + 2\sqrt{(2s - \alpha))((40\sqrt{5} - 11)\alpha + (22 - 56\sqrt{5})s)}{12}, \\
\hat{h}_3 = \frac{157s - 104\alpha}{30}, \quad \hat{h}_5 = \frac{157s - 104\alpha + \sqrt{10}\sqrt{175s^2 + 889s^2 - 1300s^2}}{30}, \\
\hat{h}_6 = \frac{3\alpha - 5\sqrt{45s^2 + 48s^2 - 96s^2 + 2\sqrt{3(57s^2 + 80s^2 - 136s^2)}}}{12}, \quad \text{and} \quad \hat{h}_7 = \frac{15\alpha - 17\sqrt{45s^2 + 48s^2 - 96s^2}}{60}.
\]
Table B.5: Definition for the regions in the simultaneous model.

<table>
<thead>
<tr>
<th>Region</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>${(h, s)</td>
</tr>
<tr>
<td>2</td>
<td>${(h, s)</td>
</tr>
<tr>
<td>3</td>
<td>${(h, s)</td>
</tr>
<tr>
<td>4</td>
<td>${(h, s)</td>
</tr>
<tr>
<td>5</td>
<td>${(h, s)</td>
</tr>
<tr>
<td>6</td>
<td>${(h, s)</td>
</tr>
<tr>
<td>7</td>
<td>${(h, s)</td>
</tr>
<tr>
<td>8</td>
<td>${(h, s)</td>
</tr>
<tr>
<td>9</td>
<td>${(h, s)</td>
</tr>
</tbody>
</table>

Note: $x^+ := \max\{0, x\}$, $c_4 = \frac{5(1685-2\sqrt{674})\alpha}{11518}$, $c_5 = (1 - \frac{\sqrt{91}}{34})\alpha$, and $c_6 = \frac{5(1685-2\sqrt{674})\alpha}{11518}$.

The equilibrium results of the supplier’s first- and second-period wholesale prices, $w_1$ and $w_2$, the buyer’s first- and second-period order quantities, $Q_1$ and $Q_2$, the buyer’s withholding inventory, $I$, the buyer’s first- and second-period selling quantities, $q_1$ and $q_2$, the supplier’s direct selling quantity, $q_s$, the buyer’s and supplier’s total profits, $\Pi_b$ and $\Pi_s$, and the supplier and buyer’s strategy pair for each region are summarized through Tables B.6 - B.9.

Table B.6: Equilibrium results for regions 1 and 2 in the simultaneous model.

<table>
<thead>
<tr>
<th>Region</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>${w_1, w_2}$</td>
<td>${337\alpha-40\alpha-11x, 337\alpha-40\alpha-20y}$</td>
<td>${29x-15\alpha-10y, 2s - \alpha}$</td>
</tr>
<tr>
<td>${Q_1, Q_2}$</td>
<td>${3033\alpha-990\alpha+455x, 10(60\alpha+48y)}$</td>
<td>${60\alpha+30\alpha-227x, 2(2s-\alpha)}$</td>
</tr>
<tr>
<td>$I$</td>
<td>${337\alpha-30\alpha+11x, 627x-529y}$</td>
<td>${25x+10\alpha-20y, \alpha - s}$</td>
</tr>
<tr>
<td>${q_1, q_2}$</td>
<td>${337\alpha-30\alpha+11x, 627x-529y}$</td>
<td>${30x-20y, \alpha - s}$</td>
</tr>
<tr>
<td>$\Pi_b$</td>
<td>$b_3$</td>
<td>$b_5$</td>
</tr>
<tr>
<td>$\Pi_s$</td>
<td>$b_4$</td>
<td>$b_6$</td>
</tr>
<tr>
<td>Strategy Pair</td>
<td>$(D, H)$</td>
<td>$(ND^1, H)$</td>
</tr>
</tbody>
</table>

Note: $b_3 = \frac{340707\alpha^2+217070\alpha x+101100\alpha y-1848732\alpha x+1268979\alpha^2+22242\alpha y}{5441312}$, $b_4 = \frac{9909\alpha^2+950\alpha x-9390\alpha y+13131x^2-12132\alpha x}{24264}$, $b_5 = \frac{1129\alpha^2+100\alpha x+600\alpha y-580\alpha x+2091\alpha x-301\alpha \alpha}{400}$, and $b_6 = \frac{3140\alpha x-1797\alpha^2-300\alpha x+2080\alpha y-4983\alpha^2+625\alpha \alpha}{600}$. 

Table B.7 : Equilibrium results for regions 3 and 4 in the simultaneous model.

<table>
<thead>
<tr>
<th>Region</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>{w_1, w_2}</td>
<td>{\frac{13\alpha-14s}{6}, 2s-\alpha}</td>
<td>{c_{11}, \frac{\alpha-2I}{2}}</td>
</tr>
<tr>
<td>{Q_1, Q_2}</td>
<td>{\frac{13\alpha-14s}{12}, \frac{2(2s-\alpha)}{3}}</td>
<td>{I + \frac{\alpha-w_1}{2}, \frac{\alpha-2I}{4}}</td>
</tr>
<tr>
<td>I</td>
<td>\frac{5\alpha-7s}{3}</td>
<td>\frac{10\alpha+\sqrt{5(2s-\alpha)-14s}}{6}</td>
</tr>
<tr>
<td>{q_1, q_2}</td>
<td>\frac{7(2s-\alpha)}{12}, \alpha-s</td>
<td>(\frac{\alpha-w_1}{2}, \frac{\alpha+2I}{4})</td>
</tr>
<tr>
<td>q_s</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>\Pi_b</td>
<td>\frac{16h(7s-5\alpha)-189\alpha^2-372s^2+540as}{4}</td>
<td>\frac{f_{11}}{4}</td>
</tr>
<tr>
<td>\Pi_s</td>
<td>\frac{217\alpha^2+388s^2-556as}{72}</td>
<td>\frac{(\alpha-2I)^2}{8} + w_1(\frac{\alpha-w_1}{2} + I)</td>
</tr>
<tr>
<td>Strategy Pair</td>
<td>(ND, \ H)</td>
<td>(ND, \ H)</td>
</tr>
</tbody>
</table>

Note: \(c_{11} = \frac{(\alpha-w_1)}{4}\) and \(f_{11} = \frac{(\alpha-w_1)^2}{4} + \frac{4\alpha^2-3(\alpha-2I)^2}{16} - I(h + w_1).

Table B.8 : Equilibrium results for regions 5 and 6 in the simultaneous model.

<table>
<thead>
<tr>
<th>Region</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>{w_1, w_2}</td>
<td>{c_{22}, \frac{\alpha-2I}{2}}</td>
<td>{9\alpha-2h, \frac{2(3\alpha+3h)}{17}}</td>
</tr>
<tr>
<td>{Q_1, Q_2}</td>
<td>{I + \frac{\alpha-w_1}{2}, \frac{\alpha-2I}{4}}</td>
<td>{15\alpha-18h, \frac{3\alpha+8h}{17}}</td>
</tr>
<tr>
<td>I</td>
<td>\frac{\sqrt{5\alpha^2+48s^2-96\alpha s}}{4}</td>
<td>\frac{5(\alpha-4h)}{17}</td>
</tr>
<tr>
<td>{q_1, q_2}</td>
<td>\frac{(\alpha-w_1)}{2}, \frac{\alpha+2I}{4}</td>
<td>\frac{4\alpha+6h}{34}, \frac{10\alpha-10h}{34}</td>
</tr>
<tr>
<td>q_s</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>\Pi_b</td>
<td>\frac{(\alpha-w_1)^2}{4} + \frac{4\alpha^2-3(\alpha-2I)^2}{16} - I(h + w_1)</td>
<td>\frac{155\alpha^2+304\alpha h^2-118ah}{1156}</td>
</tr>
<tr>
<td>\Pi_s</td>
<td>\frac{(\alpha-2I)^2}{8} + w_1(\frac{\alpha-w_1}{2} + I)</td>
<td>\frac{9\alpha^2+8h^2-4ah}{34}</td>
</tr>
<tr>
<td>Strategy Pair</td>
<td>(ND, \ H)</td>
<td></td>
</tr>
</tbody>
</table>

Note: \(c_{22} = \frac{3\alpha-4h-\sqrt{45\alpha^2+48s^2-96\alpha s}}{4}\).

Table B.9 : Equilibrium results for regions 7, 8 and 9 in the simultaneous model.

<table>
<thead>
<tr>
<th>Region</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>{w_1, w_2}</td>
<td>{\frac{\alpha}{2}, \frac{3\alpha-s}{10}}</td>
<td>{\frac{\alpha}{2}, 2s-\alpha}</td>
<td>{\frac{\alpha}{2}, \frac{\alpha}{2}}</td>
</tr>
<tr>
<td>{Q_1, Q_2}</td>
<td>{\frac{\alpha}{2}, \frac{\alpha-s}{7}}</td>
<td>{\frac{\alpha}{2}, \alpha-s}</td>
<td>{\frac{\alpha}{2}, \frac{\alpha}{7}}</td>
</tr>
<tr>
<td>I</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>{q_1, q_2}</td>
<td>{\frac{\alpha}{2}, \frac{\alpha-s}{10}}</td>
<td>{\frac{\alpha}{2}, \alpha-s}</td>
<td>{\frac{\alpha}{2}, \frac{\alpha}{7}}</td>
</tr>
<tr>
<td>q_s</td>
<td>\frac{9\alpha-10s}{10}</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>\Pi_b</td>
<td>\frac{\alpha^2}{16} + \frac{3s^2}{25}</td>
<td>\frac{17\alpha^2}{16} + s^2 - 2\alpha s</td>
<td>\frac{\alpha^2}{8}</td>
</tr>
<tr>
<td>\Pi_s</td>
<td>\frac{3\alpha^2}{8} + \frac{3s^2}{20} - \frac{\alpha s}{2}</td>
<td>\frac{3\alpha s - \frac{3\alpha^2}{8} - 2s^2}{4}</td>
<td>\frac{\alpha^2}{4}</td>
</tr>
<tr>
<td>Strategy Pair</td>
<td>(D, \ NH)</td>
<td>(ND, \ NH)</td>
<td>(ND, \ NH)</td>
</tr>
</tbody>
</table>
B.3 Proofs for Other Results

All of our results are based on the equilibrium behavior of the supplier and the buyer. Therefore, we provide the detailed proof for Proposition 9 here and include a brief outline of the proof for Proposition 11 (which shows equilibrium results for the simultaneous model in the same logic).

**Proof for Proposition 9:** We solve the equilibrium by backward induction, starting with the supplier’s decision for its direct selling quantity $q_s$. At Stage 3 in the second period, after knowing the buyer’s selling quantity $q_2$, the supplier will choose $q_s = \frac{(a-q_2-s)^+}{2}$ to maximize its direct selling profit, $(\alpha - q_2 - q_s - s)q_s$.

At Stage 2 in the second period, anticipating $q_s$, the buyer will choose $q_2$ to maximize its second-period profit, $\Pi_{b2}(q_2, Q_2, w_2, I) = (\alpha - q_2 - q_s)q_2 - w_2 Q_2$, with the constraint $q_2 \leq I + Q_2$. Then we know $q_2$ is $\min\{I + Q_2, \frac{\alpha + s}{2}\}$ if $s \leq \frac{a}{3}$ and $\min\{I + Q_2, \alpha - s\}$ if $\frac{a}{3} < s \leq \frac{a}{2}$ and $\min\{I + Q_2, \frac{a}{2}\}$ if $s > \frac{a}{2}$. Then based on the supplier’s wholesale price $w_2$, the buyer will maximize $\Pi_{b2}(q_2, Q_2, w_2, I)$ by choosing its order quantity $Q_2$ as follows:

$$Q_2(w_2, I) = \begin{cases} 
(\alpha - w_2 - I)^+ & \text{if } 0 \leq w_2 < (2s - \alpha)^+, \\
(\alpha - s - I)^+ & \text{if } (2s - \alpha)^+ \leq w_2 \leq (\frac{3s-\alpha}{2})^+, \\
(\frac{a+s-2w_2}{2} - I)^+ & \text{if } (\frac{3s-\alpha}{2})^+ < w_2 < (\frac{a+s}{2} - I)^+, \\
0 & \text{if } (\frac{a+s}{2} - I)^+ \leq w_2. 
\end{cases}$$  

(A1)

When $s \leq \frac{a}{3}$, if $I \geq \frac{a+s}{2}$, then $q_2 = \frac{a+s}{2}$ and $Q_2 = (q_2 - I)^+ = 0$. If $I \leq \frac{a+s}{2}$ and $Q_2 \leq \frac{a+s}{2} - I$, then $q_2 = I + Q_2$ and $q_s = \frac{a-q_2-s}{2} = \frac{a-(I+Q_2)-s}{2}$. Substituting them to $\Pi_{b2}(q_2, Q_2, w_2, I)$, we know the buyer will choose $Q_2$ as $\frac{a+s-2w_2}{2} - I$ if $0 \leq w_2 < \frac{a+s}{2} - I$ and 0 if $w_2 \geq \frac{a+s}{2} - I$. If $I \leq \frac{a+s}{2}$ and $Q_2 \geq \frac{a+s}{2} - I$, then $q_2 = \frac{a+s}{2}$ and $Q_2 = \frac{a+s}{2} - I$, 

which is included in the above case. In a similar way, we can derive $Q_2$ for $s > \frac{\alpha}{3}$ and then we omit the details.

At Stage 1 in the second period, anticipating $Q_2, q_2$ and $q_s$, the supplier will choose $w_2$ to maximize its second-period total profit: $\Pi_s(w_2, I) = w_2Q_2 + (\alpha - q_2 - q_s - s)q_s$. When $s \leq \frac{\alpha}{3}$, if $I \geq \frac{\alpha + s}{2}$, or $I \leq \frac{\alpha + s}{2}$ and $w_2 \geq \frac{\alpha + s}{2} - I$, we know $Q_2 = 0$. Then $\Pi_s(w_2, I)$ is just the direct channel profit and independent of $w_2$. When $I \leq \frac{\alpha + s}{2}$ and $w_2 \leq \frac{\alpha + s}{2} - I$, then $Q_2 = \frac{\alpha + s}{2} - 2w_2 - I$, $q_2 = I + Q_2 = \frac{\alpha + s}{2} - 2w_2$ and $q_s = \frac{\alpha - q_s - s}{2} = \frac{\alpha - 3s + 2w_2}{4}$. Substituting in $\Pi_s(w_2, I)$, we know the supplier will choose $w_2$ as $\frac{3\alpha - s - 4I}{6}$ if $I \leq 2s$ and $\frac{\alpha + s}{2} - I$ if $I \geq 2s$. When $\frac{\alpha}{3} < s \leq \frac{\alpha}{2}$, if $I \geq \alpha - s$, then $Q_2 = 0$ and $\Pi_s(w_2, I)$ is independent of $w_2$. If $I \leq \alpha - s$, we know $w_2$ is $\frac{3\alpha - s - 4I}{6}$ if $I \leq \frac{3\alpha - 5s}{2}$ and $\frac{3s - \alpha}{2}$ if $I \geq \frac{3\alpha - 5s}{2}$. When $\frac{\alpha}{2} < s$, if $I \geq \frac{\alpha}{2}$, or $\alpha - s \leq I \leq \frac{\alpha}{2}$ and $w_2 \geq \alpha - 2I$, then $Q_2 = 0$ and $\Pi_s(w_2, I)$ is independent of $w_2$. If $\alpha - s \leq I \leq \frac{\alpha}{2}$ and $w_2 \leq \alpha - 2I$, we have $w_2 = \frac{\alpha - 2I}{2}$.

If $I \leq \alpha - s$ and $w_2 \geq 2s - \alpha$, we know the supplier will choose $w_2$ as $\frac{3\alpha - s - 4I}{6}$ if $I \leq \frac{3\alpha - 5s}{2}$ and $\frac{3s - \alpha}{2}$ if $I \geq \frac{3\alpha - 5s}{2}$. If $I \leq \alpha - s$ and $w_2 \leq 2s - \alpha$, the supplier will choose $w_2$ as $\frac{\alpha - 2I}{2}$ if $I \geq \frac{3\alpha - 4s}{2}$ and $2s - \alpha$ if $I \leq \frac{3\alpha - 4s}{2}$. Note $\frac{3\alpha - 4s}{2} > \frac{3\alpha - 5s}{2}$. When $s \leq \frac{3\alpha}{5}$, then $w_2$ is $\frac{3\alpha - s - 4I}{6}$ if $I \leq \frac{3\alpha - 5s}{2}$ and $\frac{3s - \alpha}{2}$ if $\frac{3\alpha - 5s}{2} \leq I \leq \frac{3\alpha - 4s}{2}$. If $\frac{3\alpha - 4s}{2} \leq I \leq \alpha - s$, $\Pi_s(w_2, I)$ is bimodal with critical points at $\frac{3s - \alpha}{2}$ and $\frac{\alpha - 2I}{2}$. To facilitate our analysis, we define $x = \sqrt{4\alpha s - \alpha^2 - 3s^2}$ and $I^* = \frac{2\alpha - 3s + \sqrt{4\alpha^2 - \alpha^2 - 3s^2}}{2} = \frac{2\alpha - 3s + x}{2}$. Then $I^* = 0$ if $s = \frac{\alpha}{2}$ or $\frac{5\alpha}{6}$. Since $\Pi_s(\frac{3s - \alpha}{2}, I) - \Pi_s(\frac{\alpha - 2I}{2}, I) = \frac{3s - \alpha}{2}(\alpha - s - I) - \frac{(\alpha - 2I)^2}{8}$, $0 < I^* < \alpha - s$ and $I^* > \frac{3\alpha - 4s}{2}$ when $\frac{\alpha}{2} < s < \frac{5\alpha}{6}$, then $\Pi_s(\frac{3s - \alpha}{2}, I) - \Pi_s(\frac{\alpha - 2I}{2}, I)$ is positive if $I < I^*$ and negative if $I > I^*$. Hence, the supplier will choose $w_2 = \frac{3s - \alpha}{2}$ if $\frac{3\alpha - 4s}{2} \leq I < I^*$ and $w_2 = \frac{\alpha - 2I}{2}$ if $I^* \leq I \leq \alpha - s$ (the reason is in the following paragraph). When $\frac{3\alpha}{5} \leq s < \frac{5\alpha}{6}$, the supplier will choose $w_2 = \frac{3s - \alpha}{2}$ if $0 \leq I < I^*$ and $w_2 = \frac{\alpha - 2I}{2}$ if $I^* \leq I \leq \alpha - s$. When $\frac{5\alpha}{6} \leq s$, we have $w_2 = \frac{\alpha - 2I}{2}$ if $I \leq \alpha - s$. 


When \( I = I^* \), the supplier is indifferent to choose \( \frac{3s-\alpha}{2} \) and \( \frac{\alpha-2I^*}{2} \) and then the buyer has the same sourcing cost for the new ordered quantity accordingly due to \( q_s = 0 \). Moreover, \( q_2 = \frac{\alpha-ws}{2} = \frac{\alpha+2I^*}{4} \) if \( w_2 = \frac{\alpha-2I^*}{2} \) and \( \alpha-s \) if \( w_2 = \frac{3s-\alpha}{2} \). Since \( \alpha-s < \frac{\alpha+2I^*}{4} < \frac{\alpha}{2} \) when \( \frac{\alpha}{2} < s < \frac{5\alpha}{6} \), we know the buyer’s profit will jump upward when the supplier offers \( \frac{\alpha-2I^*}{2} \) other than \( \frac{3s-\alpha}{2} \). If in any equilibrium \( I = I^* \) and \( w_2 = \frac{3s-\alpha}{2} \), the buyer can carry \( \epsilon \) (small enough) more inventory to force the supplier to offer wholesale price \( w = \frac{\alpha-2(I^*+\epsilon)}{2} \) and then enjoy higher profit. This is a contradiction. Then in any possible equilibrium, if \( I = I^* \), the supplier should offer \( w_2 = \frac{\alpha-2I^*}{2} \).

Before we go on, we summarize the supplier’s and buyer’s second-period profits based on inventory \( I \). The supplier’s second-period profit \( \Pi_s(I) \) is \( \frac{(\alpha-3s)^2}{16} \) if \( I \geq \frac{\alpha+s}{2} \) and \( s \leq \frac{\alpha}{3} \), \( \frac{(a-s-I)^2}{4} \) if \( 2s \leq I \leq \frac{\alpha+s}{2} \) and \( s \leq \frac{\alpha}{3} \), \( \frac{3\alpha^2-6\alpha s^2-6\alpha s+4s^2+4I^2}{12} \) if \( I \leq \min\{2s, \frac{3\alpha-5s}{2}\} \) and \( 0 \leq s < \frac{3\alpha}{5} \), \( \frac{(3s-\alpha)(\alpha-s-I)}{2} \) if \( \frac{3\alpha-5s}{2} \leq I \leq \alpha-s \) and \( \frac{\alpha}{3} < s < \frac{\alpha}{2} \), or, \( \frac{(3\alpha-5s)^+}{2} \leq I < I^* \) and \( \frac{\alpha}{2} < s < \frac{5\alpha}{6} \); \( \frac{(a-2I)^2}{8} \) if \( I^* \leq I \leq \frac{\alpha}{2} \) and \( \frac{\alpha}{2} < s < \frac{5\alpha}{6} \), or, \( 0 \leq I \leq \frac{\alpha}{2} \) and \( s \geq \frac{5\alpha}{6} \); \( 0 \) if \( \alpha-s \leq I \) and \( \frac{\alpha}{3} < s \leq \frac{\alpha}{2} \), or, \( I \geq \frac{\alpha}{2} \) and \( s \geq \frac{\alpha}{2} \).

The buyer’s second-period profit \( \Pi_b(I) \) is \( \frac{(a+s)^2}{8} \) if \( I \geq \frac{\alpha+s}{2} \) and \( s \leq \frac{\alpha}{3} \), \( \frac{(a+s-I)^2}{4} \) if \( 2s \leq I \leq \frac{\alpha+s}{2} \) and \( s \leq \frac{\alpha}{3} \), \( \frac{I(9a-8I)+4s^2+5Is}{18} \) if \( I \leq \min\{2s, \frac{3\alpha-5s}{2}\} \) and \( 0 \leq s < \frac{3\alpha}{5} \), \( s(\alpha-s) - \frac{(3s-\alpha)(a-s-I)}{2} \) if \( \frac{3\alpha-5s}{2} \leq I \leq \alpha-s \) and \( \frac{\alpha}{3} < s \leq \frac{\alpha}{2} \), or, \( \frac{(3\alpha-5s)^+}{2} \leq I < I^* \) and \( \frac{\alpha}{2} < s < \frac{5\alpha}{6} \), \( \frac{4a^2-3(a-2I)^2}{16} \) if \( I^* \leq I \leq \frac{\alpha}{2} \) and \( \frac{\alpha}{2} < s < \frac{5\alpha}{6} \), or, \( 0 \leq I \leq \frac{\alpha}{2} \) and \( s \geq \frac{5\alpha}{6} \); \( s(\alpha-s) \) if \( \alpha-s \leq I \) and \( \frac{\alpha}{3} < s \leq \frac{\alpha}{2} \), \( \frac{a^2}{4} \) if \( I \geq \frac{\alpha}{2} \) and \( s \geq \frac{\alpha}{2} \).

At Stage 2 in period one, the buyer chooses its order quantity \( Q_1 \) and selling quantity \( q_1 \) to maximize its two-period profit \( \Pi_b(Q_1) = (\alpha-q_1)q_1 + \Pi_b(I) - hI - w_1Q_1 \), where \( I = Q_1 - q_1 \). Substituting \( Q_1 = I + q_1 \), then \( \Pi_b(Q_1) = [(\alpha-q_1)q_1 - w_1q_1] + [\Pi_b(I) - (w_1+h)I] \). The first part is buyer’s profit from selling \( q_1 \) in period one and
the second part is its profit for carrying inventory $I$ to period two. Then the decision for $Q_1$ breaks down to two parts: how much to sell in period one and how much to hold as inventory. It is easy to get $q_1(w_1) = \frac{\alpha - w_1}{2}$. Substituting $\Pi_{b2}(I)$, we can derive buyer’s inventory decision.

When $s \leq \frac{\alpha}{3}$, if $I \leq 2s$, the global maximal value of $\Pi_{b2}(I) - (h + w_1)I$ occurs at $I = \frac{9\alpha - 18h + 5s - 18w_1}{16}$. Thus, its maximum point $I$ is $\frac{9\alpha - 18h + 5s - 18w_1}{16}$ if $\frac{\alpha}{2} - \frac{3s}{2} - h \leq w_1 \leq \frac{\alpha}{2} + \frac{5s}{18} - h$ and 0 if $w_1 \geq \frac{\alpha}{2} + \frac{5s}{18} - h$ and 2$s$ if $0 \leq w_1 \leq \frac{\alpha}{2} - \frac{3s}{2} - h$ (If $\frac{\alpha}{2} - \frac{3s}{2} - h < 0$, then we will truncate the conditions related to $w_1$ at 0. For the following cases, we use the same treatment.). If $\frac{\alpha + s}{2} \geq I \geq 2s$, the global maximal value of $\Pi_{b2}(I) - (h + w_1)I$ occurs at $I = \frac{\alpha - 2h + s - 2w_1}{2}$. Then its maximum point $I$ is $\frac{\alpha - 2h + s - 2w_1}{2}$ if $0 \leq w_1 \leq \frac{\alpha}{2} - \frac{3s}{2} - h$ and 2$s$ if $w_1 \geq \frac{\alpha}{2} - \frac{3s}{2} - h$. If $\frac{\alpha + s}{2} \leq I$, $\Pi_{b2}(I) - (h + w_1)I$ is a decreasing function of $I$ and its maximum point $I$ is $\frac{\alpha + s}{2}$.

Combining the three ranges for $I$, we know the buyer will choose $I$ as $\frac{\alpha - 2h + s - 2w_1}{2}$ if $0 \leq w_1 \leq \frac{\alpha}{2} - \frac{3s}{2} - h$ and $\frac{9\alpha - 18h + 5s - 18w_1}{16}$ if $\frac{\alpha}{2} - \frac{3s}{2} - h \leq w_1 \leq \frac{\alpha}{2} + \frac{5s}{18} - h$ and 0 if $\frac{\alpha}{2} + \frac{5s}{18} - h \leq w_1$.

When $\frac{\alpha}{3} < s \leq \frac{\alpha}{2}$, if $I \leq \frac{3\alpha - 5s}{2}$, we can derive the maximum point $I$ as before. If $\frac{3\alpha - 5s}{2} \leq I \leq \alpha - s$, $\Pi_{b2}(I) - (h + w_1)I$ is increasing if $w_1 \leq \frac{3s - \alpha}{2} - h$ and decreasing if $w_1 \geq \frac{3s - \alpha}{2} - h$. Then its maximum point $I$ is $\alpha - s$ if $w_1 < \frac{3s - \alpha}{2} - h$ and $\frac{3\alpha - 5s}{2}$ if $w_1 > \frac{3s - \alpha}{2} - h$ and indifferent between these two values when $w_1 = \frac{3s - \alpha}{2} - h$.

However, when $w_1 = \frac{3s - \alpha}{2} - h$, the supplier’s profit is $w_1 \frac{\alpha - w_1}{2} + w_1 I + \frac{(\alpha - s - I)(3s - \alpha)}{2}$, which is decreasing of $I$. Thus, in any possible equilibrium, the buyer needs to choose $I = \frac{3\alpha - 5s}{2}$ when $w_1 = \frac{3s - \alpha}{2} - h$. If $\alpha - s \leq I$, $\Pi_{b2}(I) - (h + w_1)I$ is decreasing in $I$ and then its maximum point $I$ is $\alpha - s$. Combining the three ranges for $I$, we know the
buyer will choose $I$ as $\alpha - s$ if $w_1 < \frac{3s-\alpha}{2} - h$ and $\frac{3\alpha - 5s}{2}$ if $\frac{3s-\alpha}{2} - h \leq w_1 \leq \frac{5s}{2} - \frac{5\alpha}{6} - h$ and $\frac{9\alpha - 18h + 5s - 18w_1}{16}$ if $\frac{5s}{2} - \frac{5\alpha}{6} - h \leq w_1 \leq \frac{\alpha}{2} + \frac{5s}{18} - h$ and $0$ if $\frac{\alpha}{2} + \frac{5s}{18} - h \leq w_1$.

When $\frac{\alpha}{2} < s < \frac{3\alpha}{5}$, if $I \leq \frac{3\alpha - 5s}{2}$, as before, we know the maximum point $I$. If $\frac{3\alpha - 5s}{2} \leq I < I^*$, $\Pi_{b2}(I) - (h + w_1)I$ is increasing if $0 \leq w_1 < \frac{3s-\alpha}{2} - h$ and decreasing if $w_1 > \frac{3s-\alpha}{2} - h$ and constant if $w_1 = \frac{3s-\alpha}{2} - h$. If $I^* \leq I \leq \frac{\alpha}{2}$, $\Pi_{b2}(I) - (h + w_1)I$ attains its global maximal value at $I = \frac{3\alpha - 4h - 4w_1}{6}$. Denote $\hat{w} = \frac{3(3s-\alpha-x)}{4} - h$. Then the maximum point $I$ is $\frac{3\alpha - 4h - 4w_1}{6}$ if $0 \leq w_1 \leq \hat{w}$ and $I^*$ if $w_1 \geq \hat{w}$. If $I \geq \frac{\alpha}{2}$, $\Pi_{b2}(I) - (h + w_1)I$ is a decreasing function of $I$ and then its maximum point $I$ is $\frac{\alpha}{2}$, which is included in the above case $I^* \leq I \leq \frac{\alpha}{2}$.

Note the buyer’s profit $\Pi_{b2}(I)$ is $s(\alpha - s) - \frac{(3s-\alpha)(\alpha - s - I)}{2}$ if $I$ goes up to $I^*$ and $\frac{4a^2 - 3(\alpha - 2I)^2}{16}$ if $I$ goes down to $I^*$. Since $\{s(\alpha - s) - \frac{(3s-\alpha)(\alpha - s - I)}{2} - \frac{4a^2 - 3(\alpha - 2I)^2}{16}\}_{I=I^*} = -2a^2 + 4s^2 - 7\alpha s + (3s-\alpha)x$ $\frac{8}{s} < 0$ when $\frac{\alpha}{2} < s \leq \frac{5\alpha}{6}$, then the buyer’s profit function will jump upward if $I$ increases to $I^*$, which demonstrates the power of buyer’s strategic inventory. We also note $\hat{w} < \frac{3s-\alpha}{2} - h$ when $\frac{\alpha}{2} \leq s \leq \frac{3\alpha}{5}$. Next, we will derive the buyer’s inventory decision based on $w_1$. When $w_1 \leq \hat{w}, \Pi_{b2}(I) - (w_1 + h)I$ is increasing if $I \leq \frac{3\alpha - 4h - 4w_1}{6}$ and decreasing if $I \geq \frac{3\alpha - 4h - 4w_1}{6}$. Hence, the buyer chooses $I = \frac{3\alpha - 4h - 4w_1}{6}$. When $\hat{w} \leq w_1 \leq \frac{3s-\alpha}{2} - h$, similar as before, the buyer chooses $I = I^*$. When $\frac{3s-\alpha}{2} - h < w_1 \leq \frac{5s}{2} - \frac{5\alpha}{6} - h$, $\Pi_{b2}(I) - (w_1 + h)I$ is increasing if $I \leq \frac{3\alpha - 5s}{2}$ and decreasing if $\frac{3\alpha - 5s}{2} \leq I < I^*$ and decreasing if $I \geq I^*$. Then the buyer needs to compare its profit at $I = \frac{3\alpha - 5s}{2}$ and $I = I^*(w_2(I) = \frac{a-2h}{2})$. Since $[\Pi_{b2}(I) - (h + w_1)I]|_{I=\frac{3\alpha - 5s}{2}} - [\Pi_{b2}(I) - (h + w_1)I]|_{I=I^*} = \frac{17\alpha s - 17s^2 - 4a^2 + 4h(2s-\alpha + x) + (3s-9s)x}{8} + \frac{2s-\alpha + x}{2}w_1$, then the buyer chooses $I = \frac{3\alpha - 5s}{2}$ if $w_1 > a := \frac{4a^2 + 17s^2 - 17\alpha s + (9s-3s)x}{4(2s-\alpha + x)} - h$ and $I = I^*$ if $w_1 < a$ and be indifferent to these two values if $w_1 = a$. It can be shown that $\frac{5s}{2} - \frac{5\alpha}{6} - h > a > \frac{3s-\alpha}{2} - h$. However, when $w_1 = a$, the supplier’s
profit is \( w_1(\frac{\alpha-w_1}{2} + I) + \frac{(3s-\alpha)(\alpha-s-I)}{2} \) if \( I = \frac{3\alpha-5s}{2} \) and \( w_1(\frac{\alpha-w_1}{2} + I) + \frac{(\alpha-2I)^2}{8} \) if \( I = I^* \). The difference is \( \frac{(2s-\alpha+x)}{4} (3s - \alpha - 2a) \). Then the buyer chooses \( I = \frac{3\alpha-5s}{2} \) if \( a < \frac{3s-\alpha}{2} \), i.e. \( h > \frac{2a^2 + 5s^2 - 7\alpha s + (3s-\alpha)x}{8s - 4\alpha + 4x} \) and \( I = I^* \) if \( a > \frac{3s-\alpha}{2} \). If \( a = \frac{3s-\alpha}{2} \), the supplier’s profit is the same no matter what the buyer chooses. Since we focus on pure strategy equilibrium, we require the buyer to choose \( I = I^* \). When \( \frac{5s}{2} - \frac{5\alpha}{6} - h \leq w_1 \leq \frac{\alpha}{2} + \frac{5s}{18} - h, \Pi_{b2}(I) - (w_1 + h)I \) is increasing if \( I \leq \frac{9\alpha-18h+5s-18w_1}{16} \) and decreasing if \( \frac{9\alpha-18h+5s-18w_1}{16} \leq I < I^* \) and decreasing if \( I \geq I^* \). Thus, the buyer needs to compare its profit at \( I = \frac{9\alpha-18h+5s-18w_1}{16} \) and \( I = I^*(w_2(I) = \frac{\alpha}{2} - I) \).

Since \( \left[ \Pi_{b2}(I) - (h + w_1)I \right] \bigg|_{I=\frac{3\alpha-5s}{2}} - \left[ \Pi_{b2}(I) - (h + w_1)I \right] \bigg|_{I=I^*} \geq 0 \) if \( w_1 > a \) and \( \frac{9\alpha-18h+5s-18w_1}{16} \leq \frac{9\alpha-18h+5s-18w_1}{16} \bigg|_{w_1=\frac{\alpha}{2} - \frac{5s}{6} - h} = \frac{3\alpha-5s}{2} < I^* \), then the buyer will choose \( I = \frac{9\alpha-18h+5s-18w_1}{16} \). When \( w_1 \geq \frac{\alpha}{2} + \frac{5s}{18} - h \), similar as before, the buyer will choose \( I = 0 \).

When \( \frac{3\alpha}{5} \leq s < \frac{5\alpha}{6} \), if \( I < I^* \), \( \Pi_{b2}(I) - (h + w_1)I \) is increasing if \( w_1 \leq \frac{3s-\alpha}{2} - h \) and decreasing if \( w_1 \geq \frac{3s-\alpha}{2} - h \). If \( I^* \leq I \leq \frac{\alpha}{2} \), the buyer will choose \( I = \frac{3\alpha-4h-4w_1}{6} \) if \( 0 \leq w_1 \leq \hat{w} \) and \( I = I^* \) if \( w_1 \geq \hat{w} \). If \( I \geq \frac{\alpha}{2} \), \( \Pi_{b2}(I) - (h + w_1)I \) is a decreasing function of \( I \) and then the buyer will choose \( I = \frac{\alpha}{2} \), which is included in the above case \( I^* \leq I \leq \frac{\alpha}{2} \).

Note the buyer’s profit function may jump upward at \( I = I^* \). Similar as before, we can derive the result when \( 0 \leq w_1 \leq \frac{3s-\alpha}{2} - h \). When \( \frac{3s-\alpha}{2} - h < w_1 < \Pi_{b2}(I) - (w_1 + h)I \) is decreasing if \( 0 \leq I < I^* \) and decreasing if \( I \geq I^* \). Thus, we need to compare the buyer’s profit at \( I = 0 \) and \( I = I^*(w_2(I) = \frac{\alpha}{2} - I) \). Since \( \left[ \Pi_{b2}(I) - (h + w_1)I \right] |_{I=0} - \left[ \Pi_{b2}(I) - (h + w_1)I \right] |_{I=I^*} = \frac{2a^2 + 13s^2 - 11\alpha s + 8hI^* + (3s-9\alpha)x}{8} + I^*w_1 \), then we know the buyer will choose \( I \) as 0 if \( w_1 > b := \frac{11\alpha s - 2s^2 - 13s^2 + (9s-3\alpha)x}{4(2\alpha - 3s + x)} - h = h_8 + \frac{3s-\alpha}{2} - h(h_8 \) is defined in Appendix B.1.) and \( I^* \) if \( w_1 < b \). It can be shown \( b > \frac{3s-\alpha}{2} - h \). However, when
$w_1 = b$, the supplier’s profit is $w_1 I + \frac{(3s-\alpha)(\alpha-s-I)}{2}$ if $I = 0$ and $w_1 I + \frac{(\alpha-2I)^2}{8}$ if $I = I^*$ and the difference is $\frac{I'^*(3s-\alpha-2b)}{2}$. Thus, the buyer will choose $I = 0$ if $b < \frac{3s-\alpha}{2}$, i.e. $h > h_8$ and $I = I^*$ if $b > \frac{3s-\alpha}{2}$. If $b = \frac{3s-\alpha}{2}$, the supplier’s profit is the same no matter what the buyer chooses. As before, we require the buyer to choose $I = I^*$.

When $\frac{5a}{6} \leq s$, if $0 \leq I \leq \frac{a}{2}$, $\Pi_{b2}(I) - (h + w_1)I$ attains its global maximal value at $I = \frac{3\alpha-4h-4w_1}{6}$. Then the buyer will choose $I = \frac{3\alpha-4h-4w_1}{6}$ if $w_1 \leq \frac{3\alpha}{4} - h$ and $I = 0$ if $w_1 \geq \frac{3\alpha}{4} - h$. If $I \geq \frac{a}{2}$, we know the buyer will choose $I = \frac{a}{2}$, which is included in the above case $0 \leq I \leq \frac{a}{2}$.

An Stage 1 in period one, anticipating the buyer’s reaction, the supplier will choose $w_1$ to maximize its two-period profit $\Pi_s(w_1) = w_1 Q_1 + \Pi_{s2}(I)$, where $Q_1 = q_1 + I = \frac{\alpha-w_1}{2} + I$. Substituting $Q_1 = \frac{\alpha-w_1}{2} + I$, we know the supplier’s profit formula is changed to $w_1 (\frac{\alpha-w_1}{2} + I) + \Pi_{s2}(I)$. Substituting $\Pi_{s2}(I)$ and then substituting $I(w_1)$, we can derive the optimal $w_1$ for the supplier.

**Lemma B.5**  
*Given $s$ and $h$, the supplier will choose first-period wholesale price $w_1$ as follows:*

1. When $0 \leq s < \frac{11a}{21}$,

   $$w_1 = \begin{cases} 
   \frac{a}{2} & \text{if } h_1 < h, \\
   \frac{77a-18h-7s}{154} & \text{if } 0 \leq h \leq h_1.
   \end{cases}$$

2. When $\frac{11a}{21} \leq s \leq \frac{17a}{30}$,

   $$w_1 = \begin{cases} 
   \frac{a}{2} & \text{if } h_1 < h, \\
   \frac{77a-18h-7s}{154} & \text{if } h_2 < h \leq h_1, \\
   \frac{15s-5a-6h}{6} & \text{if } 0 \leq h \leq h_2.
   \end{cases}$$
3. When \( \frac{17\alpha}{30} < s < \frac{3(3696-17\sqrt{154})}{17683} \alpha \),

\[
  w_1 = \begin{cases} 
    \frac{\alpha}{2} & \text{if } h_1 < h, \\
    \frac{77\alpha-18h-7s}{154} & \text{if } h_2 < h \leq h_1, \\
    \frac{15s-5\alpha-6h}{6} & \text{if } h_3 \leq h \leq h_2, \\
    2\alpha - \frac{5s}{2} & \text{if } 0 \leq h < h_3.
  \end{cases}
\]

4. When \( \frac{3(3696-17\sqrt{154})}{17683} \alpha \leq s < s_1 \),

\[
  w_1 = \begin{cases} 
    \frac{\alpha}{2} & \text{if } h_4 < h, \\
    \frac{15s-5\alpha-6h}{6} & \text{if } h_3 \leq h \leq h_4, \\
    2\alpha - \frac{5s}{2} & \text{if } 0 \leq h < h_3.
  \end{cases}
\]

5. When \( s_1 \leq s < \frac{3\alpha}{5} \),

\[
  w_1 = \begin{cases} 
    \frac{\alpha}{2} & \text{if } h_4 < h, \\
    \frac{15s-5\alpha-6h}{6} & \text{if } h_3 \leq h \leq h_4, \\
    2\alpha - \frac{5s}{2} & \text{if } h_5 < h \leq h_3, \\
    a(I(a) = I^*) & \text{if } 0 \leq h \leq h_5.
  \end{cases}
\]

6. When \( \frac{3\alpha}{5} \leq s \leq \frac{2\alpha}{3} \),

\[
  w_1 = \begin{cases} 
    \frac{\alpha}{2} & \text{if } h_6 < h, \\
    b(I(b) = I^*) & \text{if } 0 \leq h \leq h_6.
  \end{cases}
\]

7. When \( \frac{2\alpha}{3} < s \leq s_2 \),

\[
  w_1 = \begin{cases} 
    \frac{\alpha}{2} & \text{if } h_7 < h, \\
    b(I(b) = 0) & \text{if } h_8 < h \leq h_7, \\
    b(I(b) = I^*) & \text{if } 0 \leq h \leq h_8.
  \end{cases}
\]
8. When $s_2 \leq s < \frac{\sqrt{3}+26}{39}\alpha$,

$$w_1 = \begin{cases} 
\frac{\alpha}{2} & \text{if } h_7 < h, \\
b(I(b) = 0) & \text{if } h_8 < h \leq h_7, \\
b(I(b) = I^*) & \text{if } h_9 < h \leq h_8, \\
\frac{\alpha+2I^*}{2} & \text{if } 0 \leq h \leq h_9.
\end{cases}$$

9. When $\frac{\sqrt{3}+26}{39}\alpha \leq s < s_3$,

$$w_1 = \begin{cases} 
\frac{\alpha}{2} & \text{if } h_7 < h, \\
b(I(b) = 0) & \text{if } h_{10} < h \leq h_7, \\
\frac{\alpha+2I^*}{2} & \text{if } 0 \leq h \leq h_{10}.
\end{cases}$$

10. When $s_3 \leq s < \frac{5\alpha}{6}$,

$$w_1 = \begin{cases} 
\frac{\alpha}{2} & \text{if } h_7 < h, \\
b(I(b) = 0) & \text{if } h_{11} < h \leq h_7, \\
\frac{\alpha+2I^*}{2} & \text{if } 0 \leq h \leq h_{11}, \\
\frac{9\alpha-2h}{17} & \text{if } 0 \leq h < h_{11}.
\end{cases}$$

11. When $\frac{5\alpha}{6} \leq s$,

$$w_1 = \begin{cases} 
\frac{\alpha}{2} & \text{if } h_0 \leq h, \\
\frac{9\alpha-2h}{17} & \text{if } 0 \leq h < h_0.
\end{cases}$$

Here $h_0$ to $h_{11}$ are defined in the Appendix B.1 after the main paper.

Proof: 1. $s \leq \frac{\alpha}{3}$. If $w_1 \geq \frac{9\alpha+5s-18h}{18}$, the interior critical point of $\Pi_s(w_1)$ is $w_1 = \frac{\alpha}{2}$ and then its maximum point $w_1$ is $\frac{\alpha}{2}$ if $h \geq \frac{5s}{18}$ and $\frac{9\alpha+5s-18h}{18}$ if $h \leq \frac{5s}{18}$. If $\frac{\alpha-3s-2h}{2} \leq w_1 \leq \frac{9\alpha+5s-18h}{18}$, the interior critical point of $\Pi_s(w_1)$ is $w_1 = \frac{7\alpha-18h-7s}{154}$ and then its maximum point $w_1$ is $\frac{7\alpha-18h-7s}{154}$ if $h \leq \frac{56s}{153}$ and $\frac{9\alpha+5s-18h}{18}$ if $h \geq \frac{56s}{153}$. If
$0 \leq w_1 \leq \frac{a-3s-2h}{2}$, the interior critical point of $\Pi_s(w_1)$ is $w_1 = \frac{5a-2h-s}{10}$ and then its maximum point $w_1$ is $\frac{a-3s-2h}{2}$ since $\frac{5a-2h-s}{10} - \frac{a-3s-2h}{2} = \frac{4h+7s}{5}$. If $w_1 = \frac{a-3s-2h}{2}$, we know $\frac{a-2h+s-2w_1}{2} = 2s = \frac{9a-18h+5s-18w_1}{16}$ and then the supplier’s profit function is continuous at $w_1 = \frac{a-3s-2h}{2}$. So it is suboptimal for the supplier to choose $w_1 \leq \frac{a-3s-2h}{2}$.

Note $\frac{5s}{18} \leq \frac{56s}{153}$. If $h \leq \frac{5s}{18}$, $\Pi_s(w_1)$ is increasing in $[0, \frac{77a-18h-7s}{154}]$ and decreasing in $[\frac{77a-18h-7s}{154}, +\infty)$. So the supplier will choose $w_1 = \frac{77a-18h-7s}{154}$. If $\frac{5s}{18} \leq h \leq \frac{56s}{153}$, $\Pi_s(w_1)$ is increasing in $[0, \frac{77a-18h-7s}{154}]$ and decreasing in $[\frac{77a-18h-7s}{154}, \frac{9a+5s-18h}{18}]$ and increasing in $[\frac{9a+5s-18h}{18}, \frac{a}{2}]$ and decreasing in $[\frac{a}{2}, +\infty)$. Then $\Pi_s(w_1)$ is bimodal and we need to compare its value at $\frac{77a-18h-7s}{154}$ and $\frac{a}{2}$. Since $\Pi_m(\frac{77a-18h-7s}{154}) - \Pi_m(\frac{a}{2}) = \frac{810h^2-750hs+161s^2}{1848} = 0$ at $h = h_1$ and $\frac{5s}{18} \leq h_1 \leq \frac{56s}{153}$ when $s \leq \frac{a}{3}$, the supplier will choose $w_1$ as $\frac{77a-18h-7s}{154}$ if $h < h_1$ and $\frac{a}{2}$ if $h > h_1$ and be indifferent to these two values if $h = h_1$. However, the buyer prefers lower wholesale price. Then we require the supplier to choose $w_1 = \frac{77a-18h-7s}{154}$ if $h = h_1$. If $h \geq \frac{56s}{153}$, $\Pi_s(w_1)$ is increasing in $[0, \frac{a}{2}]$ and decreasing in $[\frac{a}{2}, +\infty)$. Thus, the supplier will choose $w_1 = \frac{a}{2}$.

2. $\frac{a}{3} < s \leq \frac{a}{2}$. If $\frac{15s-5a-6h}{6} \leq w_1 \leq \frac{9a+5s-18h}{18}$, as before, we know the maximum point of $\Pi_s(w_1)$ is $w_1 = \frac{77a-18h-7s}{154}$ if $h_2 \leq h \leq \frac{56s}{153}$ and $w_1 = \frac{9a+5s-18h}{18}$ if $h \geq \frac{56s}{153}$ and $w_1 = \frac{15s-5a-6h}{6}$ if $h \leq h_2$. In this case, $h_2 < 0$. If $\frac{3s-a-2h}{2} \leq w_1 \leq \frac{15s-5a-6h}{6}$, the interior critical point of $\Pi_s(w_1)$ is $w_1 = 2a - \frac{5s}{2}$ and then the maximum point is $w_1 = \frac{15s-5a-6h}{6}$ since $2a - \frac{5s}{2} > \frac{15s-5a-6h}{6}$. If $0 \leq w_1 < \frac{3s-a-2h}{2}$, $\Pi_s(w_1)$ is increasing since $\frac{3a}{2} - s > \frac{3s-a-2h}{2}$. In all, we can derive $w_1$ as in the case when $s \leq \frac{a}{3}$.

3. $\frac{a}{2} < s < \frac{3a}{5}$. Note $2a - \frac{5s}{2} > a$. If $a < w_1 \leq \frac{15s-5a-6h}{6}$, the maximum point of $\Pi_s(w_1)$ is $w_1 = \frac{15s-5a-6h}{6}$ if $h \geq h_3$ and $w_1 = 2a - \frac{5s}{2}$ if $h \leq h_3$. Denote $\tilde{w} = \frac{3a-3s+x}{2} = \frac{a+2h}{2}$.
Under the condition $\hat{w} \leq w_1 < a$, the interior critical point of $\Pi_s(w_1)$ is $w_1 = \hat{w}$ and $\Pi_s(w_1)$ is increasing since $\hat{w} > 2\alpha - \frac{5s}{2} > a$. If $0 \leq w_1 \leq \hat{w}$, the interior critical point of $\Pi_s(w_1)$ is $w_1 = \frac{9\alpha - 2h}{17}$ and its maximum point is $w_1 = \hat{w}$ since $\frac{9\alpha - 2h}{17} > \hat{w}$.

Note $\frac{3\alpha - 4h - 4w_1}{6} = I^*$ when $w_1 = \hat{w}$. Then $\Pi_s(w_1)$ is continuous at $w_1 = \hat{w}$. In all, $\Pi_s(w_1)$ is increasing when $w_1 < a$. When $w_1$ increases to $a$, $\lim_{w_1 \to a} I(a) = I^*$ and then $\Pi_{s2}(I^*) = \frac{(\alpha - 2I)^2}{8}|_{I = I^*}$. When $w_1$ decreases to $a$, $\lim_{w_1 \downarrow a} I(w_1) = \frac{3\alpha - 5s}{2}$ and then $\Pi_{s2}(\frac{3\alpha - 5s}{2}) = \frac{(\alpha - s - I)(3s - \alpha)}{2}|_{I = \frac{3\alpha - 5s}{2}}$. Since $[w_1(\alpha - w_1 + I) + \Pi_{s2}(I)]|_{I = \frac{3\alpha - 5s}{2}} - [w_1(\frac{\alpha - w_1}{2} + I) + \Pi_{s2}(I)]|_{I = I^*} = \frac{(2s - \alpha + x)}{4}(3s - \alpha - 2w_1)$, then the supplier’s profit will be discontinuous at $w_1 = a$ if $a \neq \frac{3\alpha - \alpha}{2}$. We will derive the optimal $w_1$ for the supplier by discussing the following four subcases.

**Subcase 1:** $s < \frac{11\alpha}{21}$. Then $h_2 < 0$ and $h_3 < 0$. So the supplier only needs to compare its profit at $a(I(a) = I^*)$, $\frac{77\alpha - 18h - 7s}{154}$ and $\frac{\alpha}{2}$. It can be shown $\Pi_s(\frac{77\alpha - 18h - 7s}{154}) > \Pi_s(a)$ when $h \leq \frac{5s}{153}$ and $\Pi_s(\frac{\alpha}{2}) > \Pi_s(a)$ when $h \geq \frac{5s}{153}$. Thus, we do not need to consider $w_1 = a$ and can determine $w_1$ as before.

**Subcase 2:** $\frac{11\alpha}{21} \leq s < \frac{17\alpha}{30}$. Then $h_2 \geq 0$, $h_3 \leq 0$ and $h_2 < \frac{5s}{18}$. If $h > h_2$, we can derive $w_1$ as in the Subcase 1. If $h \leq h_2$, $\Pi_s(w_1)$ is increasing when $a < w_1 \leq \frac{15s - 5\alpha - 6h}{6}$ and decreasing when $w_1 \geq \frac{15s - 5\alpha - 6h}{6}$. Thus, we need to compare the value of $\Pi_s(w_1)$ at $a(I(a) = I^*)$ and $\frac{15s - 5\alpha - 6h}{6}$. It can be show $\Pi_s(\frac{15s - 5\alpha - 6h}{6}) - \Pi_s(a)$ is increasing with respect to $h$ and positive at $h = 0$ when $\frac{\alpha}{2} < s \leq \frac{17\alpha}{30}$. Then the supplier will choose $w_1 = \frac{15s - 5\alpha - 6h}{6}$.

**Subcase 3:** $\frac{17\alpha}{30} < s \leq \frac{462\alpha}{797}$. Then $0 < h_3 < h_2 \leq \frac{5s}{18}$. If $h < h_3$, $\Pi_s(w_1)$ is increasing if $a < w_1 \leq 2\alpha - \frac{5s}{2}$ and decreasing if $w_1 \geq 2\alpha - \frac{5s}{2}$. If $h_3 \leq h$, we can derive $w_1$ as in the Subcase 2. It can be shown $\Pi_s(2\alpha - \frac{5s}{2}) - \Pi_s(a)$ is increasing with respect to $h$. Then $\Pi_s(2\alpha - \frac{5s}{2}) - \Pi_s(a) \geq \{\Pi_s(2\alpha - \frac{5s}{2}) - \Pi_s(a)\}_{h=0} = \frac{1}{32(2s - \alpha + x)^2}(31\alpha^4 +$
410s^4 - 768\alpha s^3 + 565\alpha^2 s^2 - 202\alpha^3 s + 2(-124\alpha^3 + 657s^3 - 1146\alpha s^2 + 657\alpha^2 s)x), which is positive when \( \frac{a}{2} < s < s_1 \) and negative when \( s_1 < s < \frac{3a}{5} \), where \( s_1 \) is the solution for \( \{\Pi_s(2\alpha - \frac{5s}{2}) - [w_1(\frac{a-w_1}{2}) + I] + \Pi_s(I)\}_{I=I^*, w_1=a} \) with respect to \( s \). We can show \( s_1 > \frac{462a}{797} \). Then the supplier will choose \( w_1 = 2\alpha - \frac{5s}{2} \) if \( h < h_3 \).

**Subcase 4:** \( \frac{462a}{797} < s < \frac{3a}{5} \). Then \( 0 \leq h_3 < \frac{5s}{18} < h_2 \). If \( h < h_3 \), \( \Pi_s(w_1) \) is increasing in \((a, 2\alpha - \frac{5s}{2})\) and decreasing in \([2\alpha - \frac{5s}{2}, +\infty)\). If \( h_3 \leq h < \frac{5s}{18} \), \( \Pi_s(w_1) \) is increasing in \((a, \frac{15s-5\alpha-6h}{6})\) and decreasing in \([\frac{15s-5\alpha-6h}{6}, +\infty)\). If \( \frac{5s}{18} \leq h < h_2 \), \( \Pi_s(w_1) \) is increasing in \((a, \frac{15s-5\alpha-6h}{6})\) and decreasing in \([\frac{15s-5\alpha-6h}{6}, \frac{9a+5s-18h}{18}]\) and increasing in \([\frac{9a+5s-18h}{18}, \frac{a}{2}]\) and decreasing in \([\frac{a}{2}, +\infty)\). If \( h_2 \leq h < \frac{56s}{153} \), \( \Pi_s(w_1) \) is increasing in \((a, \frac{a}{2})\) and decreasing in \([\frac{a}{2}, +\infty)\). Note \( \Pi_s(\frac{15s-5\alpha-6h}{6}) - \Pi_s(\frac{a}{2}) = \frac{(-154a^2-36h^2-204ah+360hs-55s^2+588\alpha s)}{72} \) and its first order derivative with respect to \( h \) is \( 5s - \frac{17a}{6} - h = h_3 - h \). Then \( \Pi_s(\frac{15s-5\alpha-6h}{6}) - \Pi_s(\frac{a}{2}) \) is decreasing in \( h \) when \( \frac{5s}{18} \leq h \leq h_2 \). Note \( \Pi_s(\frac{15s-5\alpha-6h}{6}) - \Pi_s(\frac{a}{2}) \) is positive when \( \frac{462a}{797} \leq h \leq \frac{3(3696-17\sqrt{154})a}{17683} \) and negative when \( \frac{3(3696-17\sqrt{154})a}{17683} < h < \frac{3a}{5} \). We can show \( s_1 > \frac{3(3696-17\sqrt{154})a}{17683} \) and \( h_1 < h_2 \) if \( s > \frac{3(3696-17\sqrt{154})a}{17683} \). Thus, when \( s > \frac{3(3696-17\sqrt{154})a}{17683} \) and \( h_2 \leq h \leq \frac{56s}{153} \), the supplier will choose \( w_1 = \frac{a}{2} \).

When \( \frac{462a}{797} < s < \frac{3(3696-17\sqrt{154})a}{17683} \), as in Subcase 3, the supplier will choose \( w_1 \) as \( 2\alpha - \frac{5s}{2} \) if \( h < h_3 \) and \( \frac{15s-5\alpha-6h}{6} \) if \( h_3 \leq h \leq h_2 \) and \( \frac{7\alpha-18h-7s}{154} \) if \( h_2 < h \leq h_1 \) and \( \frac{a}{2} \) if \( h_1 < h \).

When \( \frac{3(3696-17\sqrt{154})a}{17683} \leq s < s_1 \), there exists \( h_4 \) such that \( \frac{5s}{18} \leq h_4 \leq h_2 \) and \( [\Pi_s(\frac{15s-5\alpha-6h}{6}) - \Pi_s(\frac{a}{2})] \) is zero if \( h = h_4 \) and positive if \( h < h_4 \) and negative if \( h > h_4 \).
As before, we require the supplier to choose \( w_1 = \frac{15s-5\alpha-6h}{6} \) if \( h = h_4 \). Thus, the supplier will choose \( w_1 = 2\alpha - \frac{5s}{2} \) if \( h < h_3 \) and \( \frac{15s-5\alpha-6h}{6} \) if \( h_3 \leq h \leq h_4 \) and \( \frac{\alpha}{2} \) if \( h_4 < h \).

When \( s_1 \leq s < \frac{3\alpha}{5} \), we know \( \{\Pi_s(2\alpha - \frac{5s}{2}) - \Pi_s(a)\}_{h=0} \leq 0 \) but \( \{\Pi_s(2\alpha - \frac{5s}{2}) - \Pi_s(a)\}_{h=h_3} > 0 \). Then there exists a unique \( h_5 \leq h_3 \) such that \( \Pi_s(2\alpha - \frac{5s}{2}) - \Pi_s(a) \) is negative if \( h < h_5 \) and positive if \( h > h_5 \). As before, we require the supplier to choose lower wholesale \( w_1 = a \) if \( h = h_5 \). Then the supplier will choose \( w_1 \) as \( a(I(a) = I^*) \) if \( h \leq h_5 \) and \( 2\alpha - \frac{5s}{2} \) if \( h_5 < h < h_3 \) and \( \frac{15s-5\alpha-6h}{6} \) if \( h_3 \leq h \leq h_4 \) and \( \frac{\alpha}{2} \) if \( h_4 < h \). If \( 0 \leq h \leq h_5 \), we can show \( a > \frac{3s-\alpha}{2} \) and then we know \( I(a) = I^* \).

4. \( \frac{3\alpha}{5} \leq s < \frac{5\alpha}{6} \). If \( w_1 > b \), the interior critical point of \( \Pi_s(w_1) \) is \( w_1 = \frac{\alpha}{2} \). We know \( \frac{\alpha}{2} > b \) if \( h > h_7 \) and \( \frac{\alpha}{2} < b \) if \( h < h_7 \). Denote \( t_1 = \frac{15s-9\alpha-5x}{4} \) and \( t_2 = \frac{31s-29\alpha-17x}{20} \). If \( \hat{w} \leq w_1 < b \), the interior critical point of \( \Pi_s(w_1) \) is \( \hat{w} \), which is less than \( \hat{w} \) if \( h < t_1 \) and greater than \( b \) if \( h \geq h_9 \). If \( 0 \leq w_1 \leq \hat{w} \), the interior critical point of \( \Pi_s(w_1) \) is \( w_1 = \frac{9\alpha-2h}{17} \) and then its maximum point is \( w_1 = \frac{9\alpha-2h}{17} \) if \( h \leq t_2 \) and \( w_1 = \hat{w} \) if \( h > t_2 \). It is easy to check \( t_2 \geq t_1 \). If \( w_1 \) goes up to \( b \), \( \lim_{w_1 \uparrow b} I(w_1) = I^* \) and \( \Pi_{s_2}(I^*) = \frac{(\alpha-2I)^2}{8} | I = I^* \) and if \( w_1 \) goes down to \( b \), we know \( \lim_{w_1 \downarrow b} I(w_1) = 0 \) and \( \Pi_{s_2}(0) = \frac{(3s-\alpha)(\alpha-I-s)}{2} | I = 0 \).

Since \( [w_1(\frac{\alpha-w_1}{2} + I) + \Pi_{s_2}(I)]_{I=0} - [w_1(\frac{\alpha-w_1}{2} + I) + \Pi_{s_2}(I)]_{I=I^*} = \frac{I^*(3s-\alpha-2w_1)}{2} \), then the supplier’s profit will be discontinuous at \( w_1 = b \) if \( b \neq \frac{3s-\alpha}{2} \) and jump up if \( h > h_8 \) and down if \( h < h_8 \). Next, we will divide our discussion to five more subcases to get \( w_1 \).

**Subcase 1:** \( s \leq \frac{2\alpha}{3} \). Then \( t_2 < 0, t_1 < 0, h_9 < 0, h_7 - h_8 = \frac{3s-2\alpha}{2} \leq 0 \) and \( \Pi_s(w_1) \) is increasing when \( w_1 < b \). Thus, the supplier will choose \( w_1 = b \) if \( 0 \leq h \leq h_7 \) and \( w_1 = \frac{\alpha}{2} \) if \( h > h_8 \). When \( \max\{h_7, 0\} \leq h \leq h_8 \), the supplier will compare the value of
\[ \Pi_s(w_1) \text{ at } b(I(b) = I^*) \text{ and } \frac{a}{2}. \] Note \( \Pi_s(\frac{a}{2}) - \Pi_s(b) \) is increasing in \( h \) when \( h_7 < h < h_8 \) and \( \frac{3a}{5} \leq s \leq \frac{2a}{3}. \) Since \( \{\Pi_s(\frac{a}{2}) - \Pi_s(b)\}_{h=0} \leq 0, \{\Pi_s(\frac{a}{2}) - \Pi_s(b)\}_{h=h_7} \leq 0 \) and \( \{\Pi_s(\frac{a}{2}) - \Pi_s(b)\}_{h=h_8} \geq 0, \) then there exists a unique \( h \) such that \( \max\{0, h_7\} \leq h \leq h_8 \) and \( \Pi_s(\frac{a}{2}) - \Pi_s(b) = 0, \) which we denote as \( h_6. \) Therefore, the supplier will choose \( w_1 = b \) when \( h < h_6 \) and \( w_1 = \frac{a}{2} \) when \( h > h_6 \) and be indifferent to these two values when \( h = h_6. \) As before, we require the supplier to choose \( w_1 = b \) if \( h = h_6. \)

**Subcase 2:** \( \frac{2a}{3} < s < s_2. \) Then \( t_1 < 0, t_2 < 0, h_7 > h_8 > 0 > h_9. \) Since \( \left[w_1 \left(\frac{a-w_1}{2} + I\right) + \Pi_{s2}(I)\right]_{t=0} - \left[w_1 \left(\frac{a-w_1}{2} + I\right) + \Pi_{s2}(I)\right]_{t=I^*} \) at \( w_1 = b \) is negative if \( h < h_8 \) and positive if \( h > h_8, \) the supplier will choose \( w_1 = b(I(b) = I^*) \) if \( h < h_8 \) and \( w_1 = b(I(b) = 0) \) if \( h_8 < h \leq h_7 \) and \( w_1 = \frac{a}{2} \) if \( h > h_7. \) When \( w_1 = b \) and \( h = h_8, \) the buyer is also indifferent to choose \( I = 0 \) or \( I = I^*. \) We have required the buyer to choose \( I = I^* \) when \( w_1 = b \) and \( h = h_8. \) Thus, if \( h = h_8, \) the supplier will choose \( w_1 = b \) and then \( I(b) = I^*. \)

**Subcase 3:** \( s_2 \leq s < \frac{26+\sqrt{13}}{39} \alpha. \) Then \( h_7 > h_8 > h_9 \geq \max\{0, t_2\} \geq 0 > t_1. \) When \( h > h_9, \) we can derive \( w_1 \) as in the Subcase 2. When \( \max\{t_2, 0\} \leq h \leq h_9, \Pi_s(w_1) \) is increasing in \( [0, \tilde{w}] \) and decreasing in \( [\tilde{w}, +\infty). \) So the supplier will choose \( w_1 = \tilde{w}. \) When \( h \leq \max\{t_2, 0\}, \Pi_s(w_1) \) is increasing in \( [0, \frac{9\alpha-2h}{17}] \) and decreasing in \( [\frac{9\alpha-2h}{17}, \tilde{w}] \) and increasing in \( [\tilde{w}, \tilde{w}] \) decreasing in \( [\tilde{w}, +\infty). \) Thus, the supplier needs to compare its profit at \( \frac{9\alpha-2h}{17} \) and \( \tilde{w}. \) Note \( \Pi_s(\tilde{w}) - \Pi_s(\frac{9\alpha-2h}{17}) \) is increasing in \( h \) when \( h \leq \frac{a}{4} \) and \( t_2 \leq \frac{a}{4}. \) Then \( \Pi_s(\tilde{w}) - \Pi_s(\frac{9\alpha-2h}{17}) \geq \{\Pi_s(\tilde{w}) - \Pi_s(\frac{9\alpha-2h}{17})\}_{h=0} = \frac{25\alpha^2}{34} + \frac{3s^2}{2} - 2\alpha s + (\alpha - \frac{3}{2} s)x, \) which is positive if \( s < s_3 \) and negative if \( s_3 < s < \frac{5\alpha}{6}. \) Since \( \frac{26+\sqrt{13}}{39} \alpha < s_3, \) the supplier will choose \( w_1 = \tilde{w}. \)

**Subcase 4:** \( \frac{26+\sqrt{13}}{39} \alpha \leq s \leq \frac{37+\sqrt{57}}{60} \alpha. \) Then \( h_7 > h_9 > h_8 > t_2 > 0 \geq t_1. \) Note \( \{\Pi_s(\tilde{w}) - \Pi_s(\frac{9\alpha-2h}{17})\}_{h=t_2} = \frac{2(3\alpha^2+6s^2-8\alpha s+(4\alpha-6s)x)}{25} > 0 \) when \( \frac{3\alpha}{5} \leq s < \frac{5\alpha}{6}. \) Therefore,
when \( s < s_3 \), the supplier will choose \( w_1 = \tilde{w} \) when \( h \leq t_2 \). When \( s \geq s_3 \), there exists a unique \( h \) such that \( \Pi_s(\tilde{w}) - \Pi_s(\frac{9\alpha - 2h}{17}) = 0 \) and \( 0 \leq h \leq t_2 \), which we denote as \( h_{11} \). The supplier will choose \( w_1 \) as \( \frac{9\alpha - 2h}{17} \) if \( h < h_{11} \) and \( \tilde{w} \) if \( h_{11} < h \leq t_2 \) and be indifferent to these two values if \( h = h_{11} \). As before, we require the supplier to choose \( w_1 = \frac{9\alpha - 2h}{17} \) if \( h = h_{11} \). When \( t_2 \leq h \leq h_8 \), the supplier will choose \( w_1 = \tilde{w} \). When \( h_8 < h \leq h_9 \), the supplier will compare the value of \( \Pi_s(w_1) \) at \( \tilde{w} \) and \( b(I(b) = 0) \). Note \( \Pi_s(\tilde{w}) - \Pi_s(b) \) is decreasing in \( h \) when \( h \leq h_9 \). Since \( \{\Pi_s(\tilde{w}) - \Pi_s(b)\}_{h=h_8} > 0 \) and \( \{\Pi_s(\tilde{w}) - \Pi_s(b)\}_{h=h_9} < 0 \), then there exists a unique \( h \) such that \( \Pi_s(\tilde{w}) - \Pi_s(b) = 0 \) and \( h_8 < h < h_9 \), which we denote as \( h_{10} \). Therefore, the supplier will choose \( w_1 = \tilde{w} \) if \( t_2 \leq h < h_{10} \) and \( w_1 = b(I(b) = 0) \) if \( h_{10} < h \leq h_9 \) and be indifferent of these values if \( h = h_{10} \). As before, we require the supplier to choose \( w_1 = \tilde{w} \) if \( h = h_{10} \). If \( h_9 \leq h \leq h_7 \), the supplier will choose \( w_1 = b(I(b) = 0) \). If \( h_7 < h \), the supplier will choose \( w_1 = \frac{a}{2} \).

In all, we know when \( s < s_3 \), the supplier will choose \( w_1 \) as \( \tilde{w} \) if \( h \leq h_{10} \) and \( b(I(b) = 0) \) if \( h_{10} < h \leq h_7 \) and \( \frac{a}{2} \) if \( h > h_7 \). When \( s \geq s_3 \), the supplier will choose \( w_1 \) as \( \frac{9\alpha - 2h}{17} \) if \( h \leq h_{11} \) and \( \tilde{w} \) if \( h_{11} < h \leq h_{10} \) and \( b(I(b) = 0) \) if \( h_{10} < h \leq h_7 \) and \( \frac{a}{2} \) if \( h > h_7 \).

**Subcase 5:** \( \frac{\sqrt{97} + 37}{66} \alpha \leq s < \frac{5\alpha}{6} \). Then \( h_7 > h_9 > h_8 > t_2 \geq t_1 \geq 0 \). When \( h \leq t_1 \), \( \Pi_s(w_1) \) is increasing in \([0, \frac{9\alpha - 2h}{17}]\) and decreasing in \([\frac{9\alpha - 2h}{17}, +\infty)\). So the supplier will choose \( w_1 = \frac{9\alpha - 2h}{17} \) if \( h \leq t_1 \). If \( h \geq t_1 \), the situation is the same as in the Subcase 4.

5. \( \frac{5\alpha}{6} \leq s \). If \( w_1 \geq \frac{3\alpha}{4} - h \), the interior critical point of \( \Pi_s(w_1) \) is \( w_1 = \frac{a}{2} \) and its maximum point is \( w_1 = \frac{a}{2} \) if \( h \geq h_0 = \frac{a}{4} \) and \( w_1 = \frac{3\alpha}{4} - h \) if \( h < h_0 \). If \( 0 \leq w_1 \leq \frac{3\alpha}{4} - h \),
the interior critical point of $\Pi_s(w_1)$ is $w_1 = \frac{9\alpha - 2h}{17}$ and its maximum point is $w_1 = \frac{9\alpha - 2h}{17}$ if $h < h_0$ and $w_1 = \frac{3\alpha}{4} - h$ if $h \geq h_0$. Due to the continuity of $\Pi_s(w_1)$, we know the supplier will choose $w_1 = \frac{9\alpha - 2h}{17}$ if $h < h_0$ and $w_1 = \frac{\alpha}{2}$ if $h \geq h_0$.

Based on the above backward induction results, we can directly obtain the players’ equilibrium behavior, as summarized in Tables B.2-B.4 in the Appendix B.1.

**Proof for Proposition 10:** From the equilibrium results in Table B.3, we know $I > 0$ in region 7. We can show $h_{10}$ is an increasing function of $s$. When $s$ goes to $\frac{5\alpha}{6}$, we know $x$ goes to $\frac{9}{2}$ and then $4\alpha^2 - 8(\alpha - s)^2 - 3(\alpha - 3s + x)^2$ goes to $\frac{7\alpha^2}{9}$, $2\alpha + x - 3s (= 2I^*)$ and $w$ go to 0. Thus, we have $h_{10}$ goes to $+\infty$ when $s$ goes to $\frac{5\alpha}{6}$. Moreover, we know $x \leq s$ and then $-6\alpha^2 - 13s^2 + 17\alpha s + (9s - 5\alpha)x \leq -6\alpha^2 - 13s^2 + 17\alpha s + (9s - 5\alpha)s = -6\alpha^2 - 4s^2 + 12\alpha s$\leq \frac{11\alpha^2}{9}$ if $\frac{3\alpha}{5} \leq s < \frac{5\alpha}{6}$. Therefore, we have $h_{10} * I^* \leq h_7 * I^* = \frac{6\alpha^2 - 13s^2 + 17\alpha s + (9s - 5\alpha)x}{8} \leq \frac{11\alpha^2}{72}$. Hence, for small enough $\epsilon$, we can derive the proposition.

**Proof outline for Proposition 11:** At Stage 2 in the second period, the supplier will choose $q_s$ to maximize its direct selling profit, $(\alpha - q_2 - q_s - s)q_s$, and the buyer will choose $q_2$ to maximize its second-period profit, $\Pi_{b2}(q_2, w_2, I) = (\alpha - q_s - q_2)q_2 - w_2Q_2$ where $Q_2 = (q_2 - I)^+$. Since the two players make decisions *simultaneously*, we know $q_s = \frac{\alpha - q_2 - s}{2}, q_2 = \max\{\frac{\alpha - q_2 - w_2}{2}, I\}$ if $q_2 \geq I$ and $q_2 = \min\{\frac{\alpha - q_s}{2}, I\}$ if $q_2 \leq I$. This is the major difference compared with the proof of Proposition 9.

When $I < \alpha - s$, if $q_2 \leq I$, then $q_s = \frac{\alpha - q_2 - s}{2}$. If $q_2 = \frac{\alpha - q_s}{2}$, we have $(q_2, q_s) = (\frac{\alpha - q_2 - s}{2}, \frac{\alpha - q_2 - s}{2})$ and need $\frac{\alpha + s}{3} \leq I$. If $q_2 = I$, we have $(q_2, q_s) = (I, \frac{\alpha - I - s}{2})$ and need $\frac{\alpha + s}{3} \geq I$. If $I \leq q_2 \leq \alpha - s$, then $q_s = \frac{\alpha - q_2 - s}{2}$. If $q_2 = \frac{\alpha - q_s - w_2}{2}$, we have $(q_2, q_s) = (\frac{\alpha + s - 2w_2}{3}, \frac{\alpha - 2s + w_2}{3})$ and need $2s - \alpha \leq w_2 \leq \frac{\alpha + s - 3I}{2}$. If $q_2 = I$, we have $(q_2, q_s) = (I, \frac{\alpha - I - s}{2})$ and need $w_2 \geq \frac{\alpha + s - 3I}{2}$. If $q_2 \geq \alpha - s > I$, then $q_s = 0$ and we have
\[ q_2 = \frac{\alpha - q_s - w_2}{2}. \] Thus, we have \((q_2, q_s) = \left( \frac{\alpha - w_2}{2}, 0 \right)\) and need \(w_2 \leq 2s - \alpha\). When \(I \geq \alpha - s\), if \(q_2 < \alpha - s \leq I\), we know \(q_2 = \frac{\alpha - q_s}{2}\) and then \((q_2, q_s) = \left( \frac{\alpha + s}{3}, \frac{\alpha - 2q_s}{3} \right)\) and we need \(s < \frac{\alpha}{2}\); if \(\alpha - s \leq q_2 \leq I\), then \(q_s = 0\) and we know \(q_2 = \frac{\alpha}{2}\) if \(s \geq \frac{\alpha}{2}\) and \(I \geq \frac{\alpha}{2}\), and if \(s > \frac{\alpha}{2}\) and \(I \leq \frac{\alpha}{2}\); if \(q_2 \geq I\), then we know \(q_2 = \frac{\alpha - w_2}{2}\) if \(w_2 \leq \alpha - 2I\) and \(I\) if \(w_2 \geq \alpha - 2I\). Based on the above results, we can derive the buyer’s selling quantity \(q_2\).

When \(s < \frac{\alpha}{2}\), if \(I \leq \frac{\alpha + s}{3} \leq \alpha - s\), we know the buyer will choose \(q_2\) as \(\frac{\alpha + s - 2w_2}{3}\) if \(0 \leq w_2 \leq \frac{\alpha + s - 3I}{2}\) and \(I\) if \(w_2 \geq \frac{\alpha + s - 3I}{2}\); if \(\frac{\alpha + s}{3} \leq I\), we know the buyer will choose \(q_2\) as \(\frac{\alpha + s}{3}\). When \(s \geq \frac{\alpha}{2}\), if \(\alpha - s \leq I \leq \frac{\alpha + s}{3}\) and the buyer will choose \(q_2 = \frac{\alpha + s - 2w_2}{3}\) if \(2s - \alpha \leq w_2 \leq \frac{\alpha + s - 3I}{2}\) and \(q_2 = I\) if \(w_2 \geq \frac{\alpha + s - 3I}{2}\) and \(q_2 = \frac{\alpha - w_2}{2}\) if \(0 \leq w_2 \leq 2s - \alpha\); if \(\alpha - s \leq I \leq \frac{\alpha}{2}\), then the buyer will choose \(q_2 = \frac{\alpha - w_2}{2}\) if \(0 \leq w_2 \leq \alpha - 2I\) and \(q_2 = I\) if \(w_2 \geq \alpha - 2I\); if \(I \geq \frac{\alpha}{2}\), then the buyer will choose \(q_2 = \frac{s}{2}\) for any nonnegative wholesale price \(w_2\) offered by the supplier.

Therefore, the supplier’s direct selling quantity is \(q_s = \frac{(\alpha - q_2 - s)^+}{2}\) and the buyer’s second-period order quantity is \(Q_2 = (q_2 - I)^+\). Based on \(Q_2\) and \((q_2, q_s)\), the supplier will choose \(w_2\) to maximize its second-period profit, \(\Pi_s(w_2, I) = w_2Q_2 + (\alpha - q_2 - q_s - s)q_s\). We denote the supplier’s and buyer’s optimal second-period profits as \(\Pi_{s2}(I)\) and \(\Pi_{b2}(I)\), respectively. After knowing the supplier’s reaction to its inventory \(I\), at the Stage 2 in period one, the buyer will choose its order quantity \(Q_1\) and selling quantity \(q_1\) to maximize its two-period profit, \(\Pi_b(Q_1) = (\alpha - q_1)q_1 + \Pi_{b2}(I) - hI - w_1Q_1\), where \(I = Q_1 - q_1\). Anticipating buyer’s reaction to \(w_1\), the supplier will choose \(w_1\) to maximize its two-period profit, \(\Pi_s(w_1) = w_1Q_1 + \Pi_{s2}(I)\), where \(Q_1 = q_1 + I = \frac{\alpha - w_1}{2} + I\). All of these can be done similarly as in the proof of Proposition 9. Thus, we omit the details, which are available upon request.
Based on the backward induction results, we can directly obtain the players’ equilibrium behavior, as summarized in Tables B.6 - B.9 in Appendix B.2.

C Proofs for Chapter 4

Use the same notations as in the main paper and we have

\[
\bar{v}_0 = \frac{w - (1 - x)(1 - \alpha_1)p_1}{xq_2}, \quad \bar{v}_1 = \frac{w - (1 - x)(1 - \alpha_1)p_1 - xp_1}{x(q_2 - q_1)},
\]

\[
\hat{v}_0 = \frac{p_1}{q_1}, \quad \hat{v}_1 = \frac{p_2}{\mu q_2}, \quad \hat{v}_{12} = \frac{p_2 - p_1}{\mu q_2 - q_1},
\]

\[
v_0 = \frac{w - (1 - x)(1 - \alpha_2)p_2}{xq_2}, \quad v_1 = \frac{w - (1 - x)(1 - \alpha_2)p_2 - xp_1}{x(q_2 - q_1)},
\]

\[
v_2 = \frac{w - (1 - x)(1 - \alpha_2)p_2 - xp_2}{x(1 - \mu)q_2}.
\]

Following our analysis in §4.4, we will first examine the Stage 2 sub-game equilibrium with each manufacturer’s strategy. We denote the threshold of usage value to be an owner in Market $M_2$ as $v^*$, i.e. the consumers in Market $M_2$ choose to purchase the premium products if and only if $v \geq v^*$.

**Strategy 1: No Sharing Platform $P_2$**

When $0 \leq \bar{v}_0 \leq \hat{v}_1$, we have Case (A) and $v^* = \bar{v}_0$. Then $S_1(p_1) = (1 - x)((1 - \bar{v}_0)m_2 + m_1)$ and $D_1(p_1) = x(1 - \hat{v}_1)m_0$. Substituting the formulas of $\bar{v}_0$ and $\hat{v}_1$, we know $S_1(p_1)$ is increasing of $p_1$ and $D_1(p_1)$ is decreasing of $p_1$. Since $0 \leq \bar{v}_0 \leq \hat{v}_1 < 1$, we know $S_1(p_1) = (1 - x)((1 - \bar{v}_0)m_2 + m_1) > (1 - x)(1 - \hat{v}_1)(m_2 + m_1)$. If $(1 - x)(m_2 + m_1) \geq xm_0$, then $S_1(p_1) > D_1(p_1)$ regardless of other variables. Hence, the necessary condition for Case (A) is $(1 - x)(m_2 + m_1) < xm_0$, which is stronger than Assumption 3.
Now we will derive the possible equilibrium selling price \( w \) for Case (A) under the condition \((1 - x)(m_2 + m_1) < xm_0\). We find it is better to use \( v^* \) to simplify the demonstration of the analysis. From the formula of \( \bar{v}_0 \), we readily derive \( p_1 = \frac{w - 2q_2v^*}{(1 - \alpha_1)(1 - x)} \). By setting \( S_1(p_1) = D_1(p_1) \), we get \( v^* = ((1 - \alpha_1)(1 - x)^2m_1q_1 + (1 - \alpha_1)(1 - x)^2m_2q_1 + (w - (1 - \alpha_1)(1 - x)xm_0q_1)/((1 - \alpha_1)(1 - x)^2m_2q_1 + x^2m_0q_2) \). It is easy to see \( v^* \) is an increasing function of \( w \) and the condition for \( v^* \) is \( 0 \leq v^* \leq \hat{v}_1 < 1 \). Then we can derive the possible selling price for Case (A). For Case (B), we do the similar thing and then combined the results in Cases (A) and (B), we can derive the possible selling price for strategy 1. The results are in Table C.1.

Table C.1 : Conditions for each case with Strategy 1.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Case (A)</th>
<th>Case (B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( xm_0 \leq (1 - x)(m_1 + m_2) )</td>
<td>( I^{(1)} = \emptyset )</td>
<td>( I^{(2)} = [\bar{w}_0, \bar{w}_3] \cup (\bar{w}_3, \bar{w}_4) )</td>
</tr>
<tr>
<td>( xm_0 &gt; (1 - x)(m_1 + m_2) )</td>
<td>( I^{(1)} = [\bar{w}_1, \bar{w}_2] )</td>
<td>( I^{(2)} = (\bar{w}_2, \bar{w}_4) )</td>
</tr>
</tbody>
</table>

Note: (1) The definition of each interval \( I^{(j)} \) depends on the condition. (2) When \( w \in [\bar{w}_0, \bar{w}_3] \), we have \( \bar{p}_1 = 0 \).

In Table C.1, \( \bar{w}_1 \) to \( \bar{w}_4 \) are defined as follows:

\[
\bar{w}_0 = \frac{1}{2}x(q_2 - q_1),
\]

\[
\bar{w}_1 = \frac{(a_1 - 1)q_1(x - 1)(A(r + x - 1) + x - 1)}{Arx},
\]

\[
\bar{w}_2 = \frac{(A(r + x - 1) + x - 1)((a_1 - 1)q_1(x - 1) + q_2x)}{Arx + x - 1},
\]

\[
\bar{w}_3 = x(q_1 - q_2)(A(r + x - 1) + x - 1),
\]

\[
\bar{w}_4 = \frac{d_1}{Arx + x}.
\]

Here, \( A = m_0 + m_1, r = \frac{m_0}{A}, m_2 = 1 \) and \( d_1 = A(q_1(x - 1)(a_1(r + x - 1) - r(x + 1) + 1) + q_2rx^2) + x((a_1 - 1)q_1(x - 1) + q_2x) \).
Strategy 2: Building Sharing Platform $P_2$

Similar as for without platform $P_2$, we can derive the possible selling prices and summarize the results in Table C.2.

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Case 1a</th>
<th>Case 1b</th>
<th>Case 1c</th>
</tr>
</thead>
<tbody>
<tr>
<td>$xm_0 &lt; (1-x)m_2$</td>
<td>$I_1 = [w_1, w_2]$</td>
<td>$I_2 = (w_2, w_6)$</td>
<td>$I_3 = [w_3, w_9], I_4 = [w_5, w_7], I_5 = (w_6, w_10)$</td>
</tr>
<tr>
<td>$C_1$</td>
<td>$t_0 &lt; 0$</td>
<td>$I_2 = I_0 \cup (w_4, w_6)$</td>
<td>$I_5 = (w_6, w_{10})$</td>
</tr>
<tr>
<td>$t_0 &gt; 0$</td>
<td>$I_1 = [w_1, w_2]$</td>
<td>$I_2 = (w_2, w_6)$</td>
<td>$I_5 = (w_6, w_{10})$</td>
</tr>
<tr>
<td>$C_2$</td>
<td>$t_2 &lt; t \leq t_1$</td>
<td>$I_1 = [w_1, w_3]$</td>
<td>$I_4 = [w_5, w_7], I_6 = [w_{8}, w_9], I_5 = (w_6, w_{10})$</td>
</tr>
<tr>
<td>$0 &lt; t \leq t_2$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: (1) The interval of selling price for $j$-th case is $I_j$, which depends on the conditions and is null set φ in the blank. (2) $C_1 := (1-x)m_2 \leq xm_0 \leq (1-x)(m_1+m_2)$, $C_2 := (1-x)(m_1+m_2) < xm_0, t_0 = xm_0 - (1-x)m_1, t = \mu q_2 - q_1, I_0 = [w_0, w_1], I_3 = [w_5, w_7], I_4 = [w_3, w_9], I_5 = [w_6, w_{10}]$, and $I_6 = [w_7, w_{10}]$. (3) When $w \in I_0$, we have $p_1 = 0$.

Here, $t_1 = \frac{q_1 r (\alpha_1 (x-1) + 1)(A(r+x-1) + x)}{(x-1) r}, t_2 = -\frac{q_1 (\alpha_1 (x-1) + 1)(A(r+x-1) + x)}{(x-1) x}$,

$w_0 = \frac{q_2 (\mu + x - \mu x)}{2} q_1, w_1 = ((Ax + x - 1) r (A q_1 q_2 (\mu (x-1)) (\alpha_1 (r-1) (x-1)^2 - r(x-1)^2 - 2x + 1) + x(-\alpha_1(r-1)(x-1)^2 + r(x^2 - x + 1) + x - 1) + (\alpha_1 - 1) q_1^2 r^2 (r-1)(x-1)^2 + \mu q_2^2 r x^2 (\mu (x-1) - x)) + q_2^2 x^2 (\mu (x-1) - x) (\mu q_2 - q_1))/((2Ax q_1 (\alpha_1 (r-1)(x-1)+r+x^2-1)-\mu q_2 x^2) + A q_1 (\alpha_1 (r-1)(2x^2-3x+1)+r(x^3-x^2+2x-1)+x^3-2x+1)-\mu q_2 x^2 (r(x-1) + x)) + (x-1) x^2 (q_1 - \mu q_2))$,

$w_2 = (A^2 (\mu - 1) q_1 q_2 r x^2 (\alpha_1 (x-1) + 1)(r+x-1) + A q_1 q_2 (\mu (\alpha_1 (x-1)(r(x^3+x-1)+(x+1)(x-1)^2) + r(x^3-x^2+2x-1) + x^3-2x+1) - x^2 (\alpha_1 (x-1)(r(x+x-1) + 2r x + x-1)) - (\alpha_1 - 1) q_1^2 (r-1)(x-1)^2 + \mu q_2^2 r x^2 (\mu (-x) + \mu x)) + q_2^2 x^2 (q_1 (\alpha_1 (x-1)(x-1)^2+2\mu (x-1) - 2x+1)+\mu q_2 (\mu (-x) + \mu + x))/((Ar + 1)((\alpha_1 - 1) q_1 (x-1) + \mu q_2 x)),$

$w_3 = \frac{x (Ax+x-1) q_2 (A(r+x-1)+\mu (x-1)) - A q_1 (A(r+x-1)+x))^2}{A(r(x-1)-2x+1)}, w_4 = -((q_1 - \mu q_2)(-1 + x + Ax)((-1+\alpha_1) q_1 (-1+r)(-1+x)^3 + q_2 (1+r(-1+\mu (-1+x)) - x)x^2))/(x(-\mu q_2((-1+}
\[x^2 + r(-1 + x + Ax^2) + q_1(Ar^2(1 + \alpha_1(-1 + x)) + (-1 + x)^2 + r(-1 + x)(1 + A(1 - \alpha_1 + x)))\]

\[w_5 = (A^2q_1rx^2(\alpha_1(x - 1) + 1)(q_1 - q_2)(r + x - 1) + A(x - 1)(-q_1q_2(\mu(\alpha_1(x - 1) - 1)(r(2x - 1) + (x - 1)^2) + r(x^3 - 2x^2 + 3x - 1) + 2x^2 - 3x + 1) + (\alpha_1 - 1)r(x - 1)x^2) + q_1^2(\alpha_1(x - 1)(r(x^2 + 2x - 1) + (x - 1)^2) + r(3x - 1) + 2x^2 - 3x + 1) + \mu q_2^2rx^2(\mu(x - 1) - x)) + (x - 1)x(q_1 - \mu q_2)(q_1(x - 1)(\alpha_1(x - 1) + 1) + \mu q_2(\mu(-x) + \mu + x)))/(x(Ar(q_1(\alpha_1(x - 1) + x^2 - x + 1) - \mu q_2(x - 1)x) + (x - 1)x(q_1 - \mu q_2))\]

\[w_6 = -\frac{(x-1)(A\alpha_1+q_1-A\alpha_1)(q_1-A\alpha_1)(q_1-A\alpha_1)^2+\mu q_2 r x^2}{Ar(x (\alpha_1)(r - 1)(x - 1) + r + x^2 - 1) - \mu q_2 x^2},
\]

\[w_7 = (q_1q_2r^2(\mu(x - 1) + 1)(r + x - 1) + (\alpha_1 - 1)(x - 1)^2) - \mu(x - 1)^2(\alpha_1(r - 1)(x - 1)^2 + r(-2x^2 + 2x - 1) + (x - 1)^2)) + (\alpha_1 - 1)q_1^2(r - 1)(x - 1)^4 - \mu q_2^2 rx^2(x - 1)(\mu(x - 1) - x)))/\(r(x - 1)x((\alpha_1 - 1)q_1(x - 1) + \mu q_2(x))\),
\]

\[w_8 = \frac{A(qrx(\mu(-x) + \mu x) + q_1r(x-rx+1) + q_2\mu(\mu(-x) + \mu x))}{Ar \times x},
\]

\[w_9 = (q_1(-1 + r)(-1 + x^2(-1 + x + Ar^2)) + q_2r^2(-1 + x + A(-1 + r + x)) - \mu(-1 + x)((-1 + x)^2 + Ar^2 x^2 + r(1 - 3x + 2x^2)))/(x(r(1 + x + Ar^2)),
\]

\[w_{10} = \frac{(x-1)(A(q_1(r - 1)(x-1) - \mu q_2 r x) - \mu q_2(x - 1))}{Arx}.
\]

Here, \(A = m_0 + m_1, r = \frac{m_0}{A}\) and \(m_2 = 1\).

**Proof of Lemma 2:** For any possible equilibrium, assume that the manufacturer chooses \((w, \alpha_2)\), and we have rental prices \((p_1, p_2)\). Then the manufacturer’s profit is \(wO_2^{(2)} + (1 - x)\alpha_2 p_2 O_2^{(2)} = (w + (1 - x)\alpha_2 p_2)O_2^{(2)}\). If the manufacturer charges higher selling price \(w' = w + (1 - x)(1 - \alpha_2)p_2\) but no commission fee \(\alpha_2' = 0\), then we can have the same rental prices \((p_1, p_2)\) since the utility of the owner, \(u_{o2}^{(2)} = xq_2v + (1 - x)(1 - \alpha_2)p_2 - w = xq_2v + (1 - x)p_2 - w',\) is unchanged. Hence, the owner size will be unchanged. Therefore, the manufacturer’s profit is unchanged. So for any manufacturer’s equilibrium profit, we can derive the same profit but without commission fee. Hence, we can always assume \(\alpha_2 = 0\).
Proof of Lemma 3: If $p_2 = 0$, then all the consumers will choose to be a renter on $P_2$ and we will have no owners on $P_2$. This is a contradiction. So $p_2 > 0$. If $p_1 \geq p_2$, the all the premium owners will rent out their products on $P_1$, but all the renters will want to rent a product on $P_2$. This is also a contradiction. Hence, we have $p_1 < p_2$. If $p_2 > 0$, then the consumer with very low usage value will choose platform $P_1$. Hence, $P_2$ cannot dominate $P_1$. ■

Proof of Theorem 16: Let’s assume the manufacturer’s optimal selling price with strategy 1 is $\bar{w}$ and the rental price is $\bar{p}_1$. If the manufacturer with strategy 2 also chooses selling price $\bar{w}$ and we can find positive renting price pair ($p_1 > 0, p_2 > 0$). We claim $p_2 > \bar{p}_1$. Otherwise, $p_1 < p_2 \leq \bar{p}_1$ and then the total demand will surpass the total supply, which is impossible. Due to the higher value of sharing the product, we know $\pi_2(\bar{w}, q_2) > \pi_1(\bar{w}, q_2)$. Since $\pi_2(q_2)$ is derived with the optimal selling price of the manufacturer with strategy 2, then we know $\pi_2(q_2) \geq \pi_2(\bar{w}, q_2) > \pi_1(\bar{w}, q_2) = \pi_1(q_2)$. Hence, the manufacturer will be better off with strategy 2. If we cannot find positive matching price for each platform when the manufacturer chooses $\bar{w}$, since $\bar{w} < \bar{w}_2 < w_9$, we know the lower bound, which is denoted as $\hat{w}$, that the manufacturer with strategy 2 can choose should be greater than $\bar{w}$. However, due to the extra value of sharing on $P_2$, even if the manufacturer charges higher price $\hat{w} > \bar{w}$, the manufacturer’s sales with strategy 2 will be no less than that when the manufacturer with strategy 1 chooses $\bar{w}$. This yields $\pi_2(q_2) \geq \pi_2(\hat{w}, q_2) > \pi_1(\bar{w}, q_2) = \pi_1(q_2)$. In all, by setting a not too high selling price, we know the manufacturer with strategy 2 can always obtain larger sales, compared to with strategy 1. Hence, the manufacturer can be better after launching its own product sharing platform. ■
For other propositions, after we have the possible selling prices for each case, we can readily derive the results and then we omit all the proofs.
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