Color Changes in Stochastic Light Fields Propagating in Non-Kolmogorov Turbulence

Olga Korotkova
*University of Miami, o.korotkova@miami.edu*

Elena Shchepakina

Follow this and additional works at: [https://scholarlyrepository.miami.edu/physics_articles](https://scholarlyrepository.miami.edu/physics_articles)

Part of the [Atomic, Molecular and Optical Physics Commons](https://scholarlyrepository.miami.edu/atomic_molecular_optical_physics_commons)

**Recommended Citation**
[https://scholarlyrepository.miami.edu/physics_articles/2](https://scholarlyrepository.miami.edu/physics_articles/2)

This Article is brought to you for free and open access by the Physics at Scholarly Repository. It has been accepted for inclusion in Physics Articles and Papers by an authorized administrator of Scholarly Repository. For more information, please contact repository.library@miami.edu.
Color changes in stochastic light fields propagating in non-Kolmogorov turbulence

Olga Korotkova1,* and Elena Shchepakina2

1Department of Physics, University of Miami, 1320 Campo Sano Drive, Coral Gables, Florida 33146, USA
2Department of Technical Cybernetics, Samara State Aerospace University, Molodogvardeiskaya 151, Samara 443001, Russia

Received June 10, 2010; revised September 14, 2010; accepted September 15, 2010; posted October 13, 2010 (Doc. ID 129911); published November 4, 2010

The dependence of spectral shifts and switches in optical stochastic beams propagating through nonclassic turbulent medium on the slope of the power spectrum of fluctuations in the refractive index is revealed. © 2010 Optical Society of America
OCIS codes: 010.1300, 010.1690, 030.1640.

The seminal work on the very possibility of optical fields generated by fluctuating, statistically stationary sources propagating in vacuum to exhibit changes in their spectral composition was written by Wolf in 1986 [1]. Since then, this phenomenon has been widely explored, especially in connection with astronomy [2]. Spectral shifts of light waves have also been demonstrated to occur on scattering from media, confined in some region in free space, whether being of a continuous [3,4] or a particular [5] nature, and also on propagation within media with a variety of refractive properties (see [6]). Some studies discovered the phenomenon that the direction of the spectral shift switched after a light beam passed through an aperture [7], a convergent lens [8], or an interface between media with positive and negative phase materials [9]. The dependence of spectral shifts in beamlike stochastic fields on polarimetric properties of their sources was explored in [10,11]. More importantly, spectral changes in light fields can play an important role for solving inverse problems of determining the properties of scattering media [12]. We stress here that the source correlation-induced changes, being an entirely linear phenomenon, must be distinguished from the Doppler-like spectral shifts.

While an enormous body of literature exists that discusses various aspects of optical beam propagation in atmospheric turbulence (see the well-known books [13–15]), only several studies examine specifically the spectral changes [16–21]. These results explored in some detail modulation in the spectra for beams generated by various sources and used the assumption that turbulence has a classic Kolmogorov structure, being characterized by the fractal spatial power spectrum with slope −11/3. This Letter deals with spectral changes in light beams propagating in the non-Kolmogorov turbulence [22], i.e., with generalized slope \( \alpha \), \( 3 < \alpha < 5 \).

We begin by reviewing basic equations characterizing the changes in the spectral composition of optical beams in random media. Suppose a planar, secondary source [23] located in the plane \( z = 0 \) generates a highly directional beam propagating into the half-space \( z > 0 \), which contains atmosphere governed by non-Kolmogorov statistics. The fluctuations in the beam in the source plane may be characterized by the cross-spectral density function of the form [23] \( W^{(0)}(\mathbf{r}_1^0, \mathbf{r}_2^0; \omega) = \langle U^{(0)*}(\mathbf{r}_1^0; \omega) \times U^{(0)}(\mathbf{r}_2^0, \omega) \rangle \), where \( \mathbf{r}_1^0 = (x_1^0, y_1^0, 0) \), \( \mathbf{r}_2^0 = (x_2^0, y_2^0, 0) \), \* denotes complex conjugate, and \( \langle \cdot \rangle \) is a member of a statistical ensemble, denoted by \( \langle \cdot \rangle \), of monochromatic realizations at angular frequency \( \omega \), or, using the relation \( \omega = c\lambda/2\pi \), \( \lambda \) being the wavelength of the form \( W^{(0)}(\mathbf{r}_1^0, \mathbf{r}_2^0; \lambda) = \langle U^{(0)*}(\mathbf{r}_1^0; \lambda)U^{(0)}(\mathbf{r}_2^0, \lambda) \rangle \). Then the spectral density of the field in the source plane is \( \mathcal{S}^{(0)}(\mathbf{r}^0; \lambda) = \mathcal{W}(\mathbf{r}^0, \mathbf{r}^0; \lambda) \), and its normalized version has the form

\[
\mathcal{S}_N^{(0)}(\mathbf{r}; \lambda) = \mathcal{S}^{(0)}(\mathbf{r}^0; \lambda)/\int_0^\infty \mathcal{S}^{(0)}(\mathbf{r}^0; \lambda) d\lambda.
\]

Upon propagation from the source plane to any plane \( z > 0 \), the cross-spectral density function takes the form

\[
\mathcal{W}(\mathbf{r}_1^0, \mathbf{r}_2^0; \lambda) = \int \int \int \mathcal{W}^{(0)}(\mathbf{r}_1, \mathbf{r}_2^0; \lambda)
\times \mathcal{K}(\mathbf{r}_1^0, \mathbf{r}_2^0; \mathbf{r}_1, \mathbf{r}_2^0; \lambda) d\mathbf{r}_1 d\mathbf{r}_2^0.
\]

where \( \mathcal{K}(\mathbf{r}_1^0, \mathbf{r}_2^0; \mathbf{r}_1, \mathbf{r}_2^0; \lambda) \) is the propagator, depending on the Green’s function of the random medium, of the form [14,24]

\[
\mathcal{K}(\mathbf{r}_1^0, \mathbf{r}_2^0; \mathbf{r}_1, \mathbf{r}_2^0; \lambda) = \left( \frac{1}{2\pi \lambda} \right)^2 \exp \left\{ -\frac{1}{\lambda} \frac{(\mathbf{r}_1^0 - \mathbf{r}_1)^2 - (\mathbf{r}_2 - \mathbf{r}_2^0)^2}{\lambda^2} \right\}
\times \exp \left\{ -\frac{4\pi^2 z}{3\lambda^2} \left( \mathbf{r}_1 - \mathbf{r}_2 \right)^2 + (\mathbf{r}_1 - \mathbf{r}_2)(\mathbf{r}_1^0 - \mathbf{r}_2^0) + (\mathbf{r}_1^0 - \mathbf{r}_2^0)^2 \right\}
\times \int_{0}^{\infty} \kappa^2 \Phi_n(\kappa) d\kappa,
\]

where \( \Phi_n(\kappa) \) is the one-dimensional power spectrum of fluctuations in the refractive index of the turbulent medium, having, for the non-Kolmogorov case, the form [22]

\[
\Phi_n(\kappa) = A(\alpha)C_p^2 \exp \left[ -\kappa^2/\kappa_n^2 \right] \left( \kappa^2 + \kappa_n^2/\lambda^2 \right),
\]

\( 0 \leq \kappa < \infty \), \( 3 < \alpha < 5 \).
where the term \( C^2 \) is a generalized refractive-index structure parameter with units \( m^3 - a \),

\[
\kappa_0 = \frac{2\pi}{L_0}, \quad \kappa_m = c(a) \frac{l_0}{l_0},
\]

\[
c(a) = \left[ \frac{2\pi}{3} \Gamma \left( 5 - \frac{a}{2} \right) A(\alpha) \right]^{\pm},
\]

\[
A(\alpha) = \frac{1}{4\pi^2} \Gamma(\alpha - 1) \cos \left( \frac{\alpha\pi}{2} \right),
\]

where \( L_0 \) and \( l_0 \) are the outer and the inner scales of turbulence, respectively, and \( \Gamma(\cdot) \) is the Gamma function. For the power spectrum \( (3) \) the integral in expression \( (2) \) becomes

\[
I = \int_0^\infty \kappa^3 \Phi_n(\kappa) d\kappa
\]

\[
= \frac{A(\alpha)}{2(a - 2)} C^2 \kappa_0^2 \kappa_m^2 \beta \exp \left( \frac{k_0^2}{k_m^2} \right)
\times \Gamma \left( 2 - \alpha \frac{k_0^2}{k_m^2} \right) - 2k_0^{a - 2}, \tag{4}
\]

where \( \beta = 2k_0^2 - 2k_0^2 + ak_0^2 \) and \( \Gamma(\cdot, \cdot) \) denotes the incomplete Gamma function.

In what follows we are interested in the evaluation of the normalized spectral density of the beam at distance \( z \geq 0 \) from the source plane and at any transverse location \( (x, y) \), given by the expression [23]

\[
S_N(\mathbf{r}, \lambda) = S(\mathbf{r}, \lambda) / \int_0^\infty S(\mathbf{r}, \lambda) d\lambda, \tag{5}
\]

where \( S(\mathbf{r}, \lambda) = \mathbb{W}(\mathbf{r}, \mathbf{r}, \lambda) \) is the spectral density of the field at position \( \mathbf{r} = (x, y, z) \). Substituting Eq. (1) into Eq. (5) and using the result in Eq. (4), one can trace the evolution of the spectral density in the medium of interest. Further, the shifted central wavelength of the beam can be found from the expression [21]

\[
\lambda_1(\mathbf{r}) = \int_0^\infty \lambda S(\mathbf{r}, \lambda) d\lambda / \int_0^\infty S(\mathbf{r}, \lambda) d\lambda. \tag{6}
\]

The normalized spectral shift at position \( \mathbf{r} \) may be quantified by \( Q(\mathbf{r}) = \frac{\lambda_1(\mathbf{r}) - \lambda_0}{\lambda_0} \) being blue if its value is positive and red if its value is negative. Here \( \lambda_0 \) is the central wavelength of the source, which we assume to be position independent.

To illustrate the dependence of the spectral changes on parameter \( a \) numerically, we employ the isotropic Gaussian–Schell-model beams [23]. The crosspectral density matrix of such a beam in the source plane \( z = 0 \) has the form

\[
\mathbb{W}^{(0)}(\mathbf{r}_1^0, \mathbf{r}_2^0; \lambda) = I_0(\lambda) \exp \left[ \frac{-(\mathbf{r}_1^0 - \mathbf{r}_2^0)^2}{4\sigma^2} \right]
\times \exp \left[ \frac{-(\mathbf{r}_1^0)^2}{2\delta^2} \right],
\]

where the values of the parameters must obey the beam conditions [23]. Without loss of generality we assume that the initial spectral composition consists of a single Gaussian spectral line, i.e., \( I_0(\lambda) = \exp[-(\lambda - \lambda_0)^2/(2\Lambda^2)] \), with a peak value of 1, being centered at wavelength \( \lambda_0 \), and having rms width \( \Lambda \). It was found in [25] that, for an arbitrary power spectrum of fluctuations in the refractive index, the spectral density of a Gaussian–Schell-model beam has the form

\[
S(\mathbf{r}, \lambda) = \frac{I_0(\lambda)}{\Delta^2(z)} \exp \left[ -\frac{\mathbf{r}^2}{2\sigma^2 \Delta^2(z)} \right],
\]

\[
\Delta^2(z) = 1 + \left( \frac{\lambda z}{2\pi\sigma} \right)^2 \frac{1}{\pi^2} \frac{1}{\delta^2} + \frac{2\pi^2 z^2 I}{3\delta^2}, \tag{7}
\]

with \( I \) being defined in Eq. (4).

On substituting Eq. (7) into Eqs. (5) and (6) it is possible to determine spectral density and spectral shift, respectively, in Gaussian–Schell-model beams propagating in the non-Kolmogorov turbulence. Because of the inverse dependence of \( S(\mathbf{r}, \lambda) \) on \( \lambda \) [see Eq. (7)], it is clear that the spectrum is broadened as the beam propagates. Moreover, since the central wavelength \( \lambda_1(\mathbf{r}) \) and, hence, the shift \( Q(\mathbf{r}) \) also generally vary with \( z \) and \( \lambda \), the spectrum is expected to be shifted, with the shift being difficult to assert qualitatively. Hence, in what follows, we illustrate the spectral shifts by numerical examples.

We use the following parameters for the source and the atmosphere, unless other parameters are specified in the figure captions: \( \lambda_0 = 0.5435 \times 10^{-6} \) m, \( \Lambda = \lambda_0/3 \), \( \sigma = 10^{-2} \) m, \( \delta = 10^{-3} \) m, \( C_2^N = 10^{-13} m^{-3/2} \), \( L_0 = 1 \) m, and \( l_0 = 10^{-3} \) m.

Figure 1 shows the changes in the on-axis behavior of the normalized spectral density \( S_N \) in the source plane and in the field, calculated at three different distances from the source. We note that, for very steep power spectra \( (a = 3.01) \) and \( a = 3.1 \), the spectral density gradually recovers with propagation distance from the source. This effect is delayed for a Kolmogorov power spectrum \( (a = 3.67) \) and completely disappears for flatter power spectra \( (a = 4.99) \).

In Fig. 2 we show the density plot of the actual central wavelength \( \lambda_1 \) and the contour plot of its normalized shift \( Q \) as a function of \( z \) and \( r = |\mathbf{r}| \) for several values of slope \( a \) of the atmospheric power spectrum. In the region close to the \( z \) axis, the blueshift of the spectrum is well pronounced for \( a = 4.9 \), but is somewhat compensated by

![Image](image-url)
positions initial redshift is a possibility, which turns to blueshift after propagation, resulting in a red–blue switch. Thus, both types of spectral switches have been found.

The ability of non-Kolmogorov turbulence to strongly mitigate spectral changes originally caused by source correlations may have crucial consequences for spectroscopic measurements relating to astrophysics and atmospheric remote sensing.

O. Korotkova’s research was funded by the U.S. Air Force Office of Scientific Research (USAFOSR) through grant FA 95500810102 and the U.S. Office of Naval Research (ONR) through grant N0018909P1903. E. Shchepakina’s research was funded by the Russian Foundation of Fundamental Investigations through grant 10-08-00154a.

References


---

The ability of non-Kolmogorov turbulence to strongly mitigate spectral changes originally caused by source correlations may have crucial consequences for spectroscopic measurements relating to astrophysics and atmospheric remote sensing.

O. Korotkova’s research was funded by the U.S. Air Force Office of Scientific Research (USAFOSR) through grant FA 95500810102 and the U.S. Office of Naval Research (ONR) through grant N0018909P1903. E. Shchepakina’s research was funded by the Russian Foundation of Fundamental Investigations through grant 10-08-00154a.

References