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Changes in the Statistical Properties of Stochastic Anisotropic Electromagnetic Beams on Propagation in the Turbulent Atmosphere

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Changes in the statistical properties of stochastic anisotropic electromagnetic beams on propagation in the turbulent atmosphere

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Abstract: We report analytic formulas for the elements of the 2×2 cross-spectral density matrix of a stochastic electromagnetic anisotropic beam propagating through the turbulent atmosphere with the help of vector integration. From these formulas the changes in the spectral density (spectrum), in the spectral degree of polarization, and in the spectral degree of coherence of such a beam on propagation are determined. As an example, these quantities are calculated for a so-called anisotropic electromagnetic Gaussian Schell-model beam propagating in the isotropic and homogeneous atmosphere. In particular, it is shown numerically that for a beam of this class, unlike for an isotropic electromagnetic Gaussian Schell-model beam, its spectral degree of polarization does not return to its value in the source plane after propagating at sufficiently large distances in the atmosphere. It is also shown that the spectral degree of coherence of such a beam tends to zero with increasing distance of propagation through the turbulent atmosphere, in agreement with results previously reported for isotropic beams.

OCIS codes: (030.1640) Coherence; (010.1300) Atmospheric propagation; (260.5430) Polarization.

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1. Introduction

Studies of light beams propagating through the turbulent atmosphere are very important for many applications such as tracking, remote sensing, and free-space optical communications. In the past three decades, statistical properties of scalar partially coherent (stochastic) beams propagating in the atmospheric turbulence have been investigated extensively [1-9]. Quite recently a class of electromagnetic stochastic beams became of a great interest. After the unified theory of coherence and polarization had been presented by Wolf [10], it became possible to determine the changes in statistical properties of such beams, in particular, in the spectral density, in the spectral degree of coherence, and in their polarization properties. All aforementioned properties of electromagnetic stochastic beams can be determined from the basic quantity of the unified theory, i.e. from their 2×2 electric correlation matrix, also known as the cross-spectral density matrix [11, 12].

Until now the unified theory has been used only for the analysis of the changes in the statistical properties of *isotropic* electromagnetic beams propagating through the turbulent atmosphere [13-17]. In particular, it has been shown in these studies that after propagating at sufficiently long distances through the homogeneous and isotropic turbulent atmosphere all polarization properties, including the spectral degree of polarization of the beam, return to their values in the source plane. In very recent publications coherence and polarization properties of partially coherent general beams propagating in atmospheric turbulence have been investigated by Eyyuboglu *et al*. [18, 19]. The effects of coherence on anisotropic electromagnetic Gaussian-Schell model beams propagating through free space have also been studied [20].

In this paper, we derive a general analytic formula for the elements of cross-spectral density matrix of an *anisotropic* stochastic electromagnetic beam propagating in the homogeneous and isotropic turbulent atmosphere. The changes in the spectral density, in the spectral degree of coherence, and in the spectral degree of polarization of this class of beams on propagation in the atmosphere are investigated in details.

To illustrate the theory we consider an example illustrating the evolution of the statistical properties of a typical anisotropic electromagnetic Gaussian Schell-model beam propagating in the atmosphere with Kolmogorov power spectrum.

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2. Theory

Let us consider a stochastic, statistically stationary, electromagnetic beam-like field generated by a source located in the plane z = 0 and propagating into the positive half-space z > 0, filled with the turbulent atmosphere (see Fig. 1).

The second-order correlation properties of such a beam at a pair of points $\mathbf{r}_1, \mathbf{r}_2$ can be characterized by the 2×2 electric cross-spectral density matrix [10]

$$\overleftrightarrow{W}(\mathbf{r}_1, \mathbf{r}_2, \omega) \equiv [W_{ij}(\mathbf{r}_1, \mathbf{r}_2, \omega)] = [\langle E_i^*(\mathbf{r}_1, \omega) E_j(\mathbf{r}_2, \omega) \rangle], \quad (i = x, y; \ j = x, y). \tag{1}$$

Here the asterisk denotes the complex conjugate and the angular brackets stand for the ensemble average in the sense of coherence theory in the space-frequency domain [21]. x and y are two mutually orthogonal directions perpendicular to the beam axis. $\boldsymbol{E} = (E_x, E_y)$ is a statistical ensemble of the fluctuating component of the transverse electric field, specified by the position vector $\mathbf{r} \equiv (\boldsymbol{\rho}, z)$ (see Fig.1). The propagation of each of the two transverse components of the electromagnetic beam in the turbulent atmosphere can be treated by the use of the extended Huygens-Fresnel integral formula [22]:

$$E_i(\boldsymbol{\rho}, z, \omega) = -\frac{ik}{2\pi z} \exp(ikz) \int\int E_i^{(0)}(\boldsymbol{\rho}', \omega) \exp\left[\frac{ik}{2z}(\boldsymbol{\rho} - \boldsymbol{\rho}')^2\right] \exp[\psi(\boldsymbol{\rho}, \boldsymbol{\rho}', z, \omega)] \mathrm{d}^2\boldsymbol{\rho}', \tag{2}$$

where $k = 2\pi/\lambda$ is the wave number, λ is the wavelength, $\boldsymbol{\rho}'$ and $\boldsymbol{\rho}$ denote the two-dimensional vectors in the source plane and the output plane, respectively. ψ is the random part of the complex phase of a spherical wave propagating through the turbulent atmosphere.

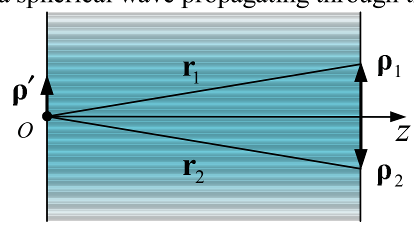

Fig. 1. Illustrating the notation relating to propagation of a stochastic electromagnetic beam through the turbulent atmosphere.

On substituting from Eq. (2) into Eq. (1), the following expression for the elements of the cross-spectral density matrix of the electric field in the output plane is obtained [13]:

$$\begin{aligned} W_{ij}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z, \omega) = \frac{k^2}{4\pi^2 z^2} \int\int\int\int W_{ij}^{(0)}(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2', \omega) \exp\left[-\frac{ik}{2z}(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_1')^2 + \frac{ik}{2z}(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_2')^2\right] \\ \times \langle \exp[\psi^*(\boldsymbol{\rho}_1, \boldsymbol{\rho}_1', z, \omega) + \psi(\boldsymbol{\rho}_2, \boldsymbol{\rho}_2', z, \omega)] \rangle_{\mathrm{m}} \mathrm{d}^2\boldsymbol{\rho}_1' \mathrm{d}^2\boldsymbol{\rho}_2', \end{aligned} \tag{3}$$

where subscript m implies that the average is taken over the ensemble of statistical realizations of the turbulent medium. The term in the sharp brackets with the subscript m in Eq. (3) can be written as [23]

$$\begin{aligned} \langle \exp[\psi^*(\boldsymbol{\rho}_1, \boldsymbol{\rho}_1', z, \omega) + \psi(\boldsymbol{\rho}_2, \boldsymbol{\rho}_2', z, \omega)] \rangle_{\mathrm{m}} &= \exp[-(1/2) D_\psi(\boldsymbol{\rho}_d', \boldsymbol{\rho}_d)] \\ &\cong \exp[-(1/\rho_0^2)(\boldsymbol{\rho}_d'^2 + \boldsymbol{\rho}_d' \cdot \boldsymbol{\rho}_d + \boldsymbol{\rho}_d^2)], \end{aligned} \tag{4}$$

with $\boldsymbol{\rho}_d' = \boldsymbol{\rho}_1' - \boldsymbol{\rho}_2'$, and $\boldsymbol{\rho}_d = \boldsymbol{\rho}_1 - \boldsymbol{\rho}_2$. Here D_ψ is the structure function of the random complex phase in Rytov's representation, $\rho_0 = (0.545 C_n^2 k^2 z)^{-3/5}$ is the coherence length of a spherical wave propagating in the turbulent medium and C_n^2 is the structure parameter of the refractive index. It should be noted that Rytov's phase structure function is accepted to be valid not only in the regime weak fluctuations atmospheric fluctautions, but also in the strong fluctuations regime. We have employed a quadratic approximation [2] for the Rytov's phase structure function in order to obtain simpler and viewable analytical results.

On substituting Eq. (4) into Eq. (3) and using a tensor method, we obtain the more general integral formula for the elements of cross-spectral density matrix in the output plane as follows:

$$W_{ij}(\boldsymbol{\rho}_{12}, z, \omega) = \frac{k^2}{4\pi^2}\left[\mathrm{Det}(\overline{\mathbf{B}})\right]^{-1/2} \iiiint W_{ij}^{(0)}(\boldsymbol{\rho}'_{12}, \omega) \\ \times \exp\left[-\frac{ik}{2}\left(\boldsymbol{\rho}'^{T}_{12}\overline{\mathbf{B}}^{-1}\boldsymbol{\rho}'_{12} - 2\boldsymbol{\rho}'^{T}_{12}\overline{\mathbf{B}}^{-1}\boldsymbol{\rho}_{12} + \boldsymbol{\rho}_{12}^{T}\overline{\mathbf{B}}^{-1}\boldsymbol{\rho}_{12}\right)\right], \\ \times \exp\left[-\frac{ik}{2}\left(\boldsymbol{\rho}'^{T}_{12}\overline{\mathbf{P}}\boldsymbol{\rho}'_{12} + \boldsymbol{\rho}'^{T}_{12}\overline{\mathbf{P}}\boldsymbol{\rho}_{12} + \boldsymbol{\rho}_{12}^{T}\overline{\mathbf{P}}\boldsymbol{\rho}_{12}\right)\right]\mathrm{d}^4\boldsymbol{\rho}'_{12} \tag{5}$$

where $\boldsymbol{\rho}'^{T}_{12} = \left(\boldsymbol{\rho}'^{T}_1, \boldsymbol{\rho}'^{T}_2\right) = (x'_1, y'_1, x'_2, y'_2)$ and $\boldsymbol{\rho}_{12}^{T} = \left(\boldsymbol{\rho}_1^{T}, \boldsymbol{\rho}_2^{T}\right) = (x_1, y_1, x_2, y_2)$ are four-dimensional vectors, T stands for the matrix transposition, Det is the determinant,

$$\overline{\mathbf{B}} = \begin{bmatrix} z\mathbf{I} & 0 \\ 0 & -z\mathbf{I} \end{bmatrix}, \quad \overline{\mathbf{P}} = \frac{2}{ik\rho_0^2}\begin{bmatrix} \mathbf{I} & -\mathbf{I} \\ -\mathbf{I} & \mathbf{I} \end{bmatrix}, \tag{6}$$

and $\mathbf{I}$ is a 2×2 unitary matrix.

It is to be noted that our results will reduce to those obtained in Ref. 24, if one lets $\boldsymbol{\rho}_1 = \boldsymbol{\rho}_2$ (i.e. if $\boldsymbol{\rho}_d = 0$).

3. An example

To illustrate the preceding analysis we now consider an anisotropic electromagnetic Schell-model beam. The elements of the cross-spectral density matrix of such a beam in the source plane $z = 0$ are given by the expressions [12]

$$W_{ij}^{(0)}(\boldsymbol{\rho}'_1, \boldsymbol{\rho}'_2, \omega) = \sqrt{S_i^{(0)}(\boldsymbol{\rho}'_1, \omega)}\sqrt{S_j^{(0)}(\boldsymbol{\rho}'_2, \omega)}\,\eta_{ij}^{(0)}(\boldsymbol{\rho}'_2 - \boldsymbol{\rho}'_1, \omega), \tag{7}$$

where $S_i^{(0)}$ is the spectral density of the component E_i of the electric field and $\eta_{ij}^{(0)}$ is the spectral degree of correlation between the components E_i and E_j. Further, if $S_i^{(0)}$ and $\eta_{ij}^{(0)}$ are both Gaussian functions then the source is called an electromagnetic Gaussian Schell-model source and the elements of its cross-spectral density matrix are then given by the expressions [13, 14]

$$W_{ij}^{(0)}(\boldsymbol{\rho}'_1, \boldsymbol{\rho}'_2, \omega) = A_i A_j B_{ij}\exp\left(-\frac{\boldsymbol{\rho}'^2_1}{4\sigma_i^2} - \frac{\boldsymbol{\rho}'^2_2}{4\sigma_j^2}\right)\exp\left(-\frac{\left|\boldsymbol{\rho}'_2 - \boldsymbol{\rho}'_1\right|^2}{2\delta_{ij}^2}\right). \tag{8}$$

Here the coefficients A_i, A_j, B_{ij} and the variances σ_i^2, σ_j^2, δ_{ij}^2 are independent of position but may depend on frequency (see Ref. 21, Sec. 5.3.2).

In order to use the method of vector integration, it will be convenient to rewrite Eq. (8) in the form

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$$W_{ij}^{(0)}(\boldsymbol{\rho}'_{12},\omega)=A_iA_jB_{ij}\exp\left(-\frac{ik}{2}\boldsymbol{\rho}'^{T}_{12}\mathbf{M}'^{-1}_{ij}\boldsymbol{\rho}'_{12}\right), \tag{9}$$

where $\boldsymbol{\rho}'_{12}=(\boldsymbol{\rho}'_1,\boldsymbol{\rho}'_2)$ and $\mathbf{M}'^{-1}_{ij}$ is a 4×4 matrix (whose elements are complex numbers, in general) of the form

$$\mathbf{M}'^{-1}_{ij}=\begin{bmatrix} -\frac{i}{2k}\sigma_i^{-2}-\frac{i}{k}\delta_{ij}^{-2} & 0 & \frac{i}{k}\delta_{ij}^{-2} & 0 \\ 0 & -\frac{i}{2k}\sigma_i^{-2}-\frac{i}{k}\delta_{ij}^{-2} & 0 & \frac{i}{k}\delta_{ij}^{-2} \\ \frac{i}{k}\delta_{ij}^{-2} & 0 & -\frac{i}{2k}\sigma_j^{-2}-\frac{i}{k}\delta_{ij}^{-2} & 0 \\ 0 & \frac{i}{k}\delta_{ij}^{-2} & 0 & -\frac{i}{2k}\sigma_j^{-2}-\frac{i}{k}\delta_{ij}^{-2} \end{bmatrix}. \tag{10}$$

On substituting from Eq. (9) into Eq. (5), and after performing tedious vector integration, we obtain for the elements of the cross-spectral density matrix of a random electromagnetic beam propagating through the turbulent atmosphere the formula

$$\begin{aligned} W_{ij}(\boldsymbol{\rho}_{12},z,\omega)=A_iA_jB_{ij}\left[\mathrm{Det}\left(\bar{\mathbf{I}}+\bar{\mathbf{B}}\bar{\mathbf{P}}+\bar{\mathbf{B}}\mathbf{M}'^{-1}_{ij}\right)\right]^{-1/2}\exp\Bigg\{-\frac{ik}{2}\boldsymbol{\rho}_{12}^{T}\Bigg[\left(\bar{\mathbf{B}}^{-1}+\bar{\mathbf{P}}\right) \\ -\left(\bar{\mathbf{B}}^{-1}-\frac{1}{2}\bar{\mathbf{P}}\right)^{T}\left(\bar{\mathbf{B}}^{-1}+\bar{\mathbf{P}}+\mathbf{M}'^{-1}_{ij}\right)^{-1}\left(\bar{\mathbf{B}}^{-1}-\frac{1}{2}\bar{\mathbf{P}}\right)\Bigg]\boldsymbol{\rho}_{12}\Bigg\}\end{aligned}, \tag{11}$$

where $\bar{\mathbf{I}}$ is a the 4×4 unitary matrix. Equation (11) is the main result of this paper. From this formula we can now derive the formulas for the spectral density, the spectral degree of coherence and the spectral degree of polarization of an anisotropic electromagnetic Gaussian-Schell-model beam (see Ref. 10). The spectral density of the beam at the point $(\boldsymbol{\rho},z)$ with $\boldsymbol{\rho}_{12}^{T}=(\boldsymbol{\rho}^{T},\boldsymbol{\rho}^{T})$ is given by the formula

$$\mathrm{S}(\boldsymbol{\rho},z,\omega)=\mathrm{Tr}\,\overset{\leftrightarrow}{W}(\boldsymbol{\rho},z,\omega), \tag{12}$$

where Tr stands for the trace.

The spectral degree of coherence of the electric field at a pair of points $\mathbf{r}_1\equiv(\boldsymbol{\rho}_1,z)$ and $\mathbf{r}_2\equiv(\boldsymbol{\rho}_2,z)$ is given by the formula

$$\mu(\boldsymbol{\rho}_{12},z,\omega)=\frac{\mathrm{Tr}\,\overset{\leftrightarrow}{W}(\boldsymbol{\rho}_{12},z,\omega)}{\sqrt{\mathrm{Tr}\,\overset{\leftrightarrow}{W}(\boldsymbol{\rho}_{11},z,\omega)}\sqrt{\mathrm{Tr}\,\overset{\leftrightarrow}{W}(\boldsymbol{\rho}_{22},z,\omega)}}, \tag{13}$$

where $\boldsymbol{\rho}_{11}^{T}=(\boldsymbol{\rho}_1^{T},\boldsymbol{\rho}_1^{T})$, and $\boldsymbol{\rho}_{22}^{T}=(\boldsymbol{\rho}_2^{T},\boldsymbol{\rho}_2^{T})$.

Finally, the spectral degree of polarization at the point $(\boldsymbol{\rho},z)$ with $\boldsymbol{\rho}_{12}^{T}=(\boldsymbol{\rho}^{T},\boldsymbol{\rho}^{T})$ is given by the formula

$$P(\boldsymbol{\rho},z,\omega)=\sqrt{1-\frac{4\mathrm{Det}\,\overset{\leftrightarrow}{W}(\boldsymbol{\rho},z,\omega)}{\left[\mathrm{Tr}\,\overset{\leftrightarrow}{W}(\boldsymbol{\rho},z,\omega)\right]^{2}}}. \tag{14}$$

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In Fig. 2 the changes in the normalized spectral density along the *z*-axis are shown in the limiting cases when atmospheric turbulence is locally extremely strong ($C_n^2 = 10^{-12}$ m$^{-2/3}$) and locally very weak ($C_n^2 = 10^{-16}$ m$^{-2/3}$). S_0 is the spectral density in the source plane. From Fig. 2 one can see that the spectral density tends to zero after the beams propagate at sufficiently long distances from the source due to diffraction of the beam propagating through the turbulent atmosphere. We also can see that the drop in the spectral density is influenced more by the source coherence than by the values of C_n^2.

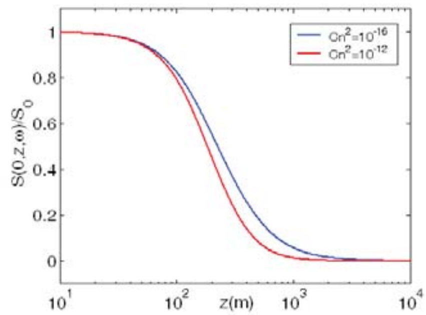


Fig. 2. Changes in the normalized spectral density S/S_0 along the *z*-axis of anisotropic electromagnetic beams through the turbulent atmosphere with different C_n^2. The source is assumed to be electromagnetic Gaussian Schell-model source with the parameters: $\lambda = 632.8\,\text{nm}$, $A_x = 2$, $A_y = 1$, $B_{xy} = 0.2\exp(i\pi/3)$, $\sigma_x = 1\,\text{cm}$, $\sigma_y = 2\,\text{cm}$, $\delta_{xx} = \delta_{yy} = 2\,\text{mm}$, $\delta_{xy} = 3\,\text{mm}$. The unit of C_n^2 is $\text{m}^{-2/3}$.

In Fig. 3 the changes in the spectral degree of coherence of random electromagnetic beams propagating through the turbulent atmosphere are shown for the case when the pair of field points are located symmetrically with respect to the *z*-axis (i. e. $\boldsymbol{\rho}_2 = -\boldsymbol{\rho}_1$; see Fig. 1). The initial spectral degrees of coherence are different in Figs. 3(a) and 3(b), but both of them tend to zero with increasing distance of propagation through the turbulent atmosphere. On the contrary, if the beam propagates in free space (i.e. $C_n^2 = 0$), the degree of coherence at a pair of field points tends to 1 in the far-zone. The result implies that the atmospheric turbulence destroys spatial coherence of stochastic electromagnetic beams and it is so at smaller distances from the source for larger values of C_n^2.

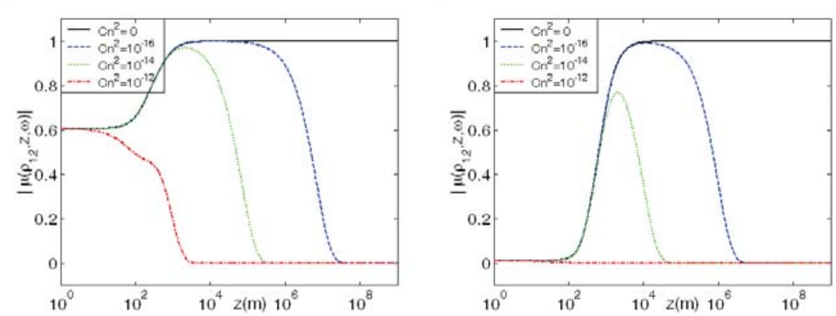
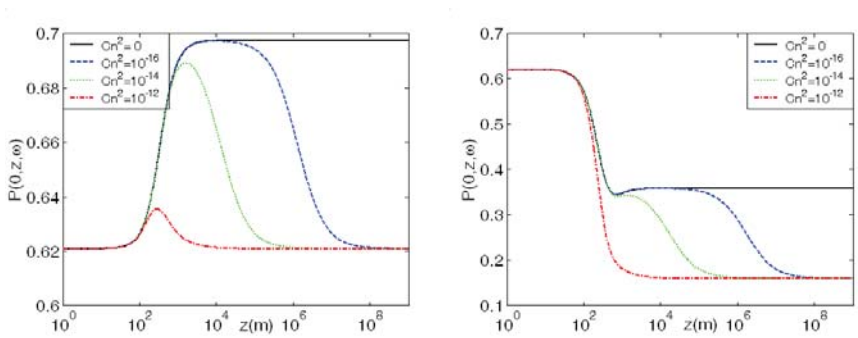


Fig. 3. Changes in the spectral degree of coherence μ along the *z*-axis of anisotropic electromagnetic Gaussian Schell-model beams propagating through the turbulent atmosphere with different C_n^2. The source parameters are the same as Fig. 2. Pairs of field points: (a) $\boldsymbol{\rho}_{12}{}^T = (1\,\text{mm},\,0,\,-1\,\text{mm},\,0)$, (b) $\boldsymbol{\rho}_{12}{}^T = (3\,\text{mm},\,0,\,-3\,\text{mm},\,0)$. The unit of C_n^2 is $\text{m}^{-2/3}$.

In Fig. 4 we illustrate the changes in the spectral degree of polarization along the *z*-axis ($\boldsymbol{\rho}_{12} = 0$) of electromagnetic anisotropic Gaussian Schell-model beams passing through the turbulent atmosphere with different structure parameters. In Fig. 4(a) σ_i and σ_j are assumed to be equal, the spectral degree of polarization of the beam returns to its value in the source plane after propagating a long distance that is in agreement with the previous results [13-17]. In Fig. 4(b) we show that the spectral degree of polarization of a beam generated by an anisotropic source, i.e. the source with $\sigma_i \neq \sigma_j$, does not, in general, tend to the same value as it has in the source plane. This result is in striking difference with the one obtained previously for isotropic beams (Compare with Fig. 4(a)) and is independent of the local strength of atmospheric turbulence, i.e. of parameter C_n^2.



(a) (b)

Fig. 4. Changes in the spectral degree of polarization P along the *z*-axis of anisotropic electromagnetic Gaussian Schell-model beams propagating through the turbulent atmosphere with different C_n^2. The source parameters are the same as in Fig. 2, but (a) $\sigma_x = \sigma_y = 1\,\text{cm}$, (b) $\sigma_x = 1\,\text{cm}$, $\sigma_y = 2\,\text{cm}$. The unit of C_n^2 is $\text{m}^{-2/3}$.

4. Concluding remarks

In summary, general analytic expressions for the elements of the cross-spectral density matrix of an anisotropic stochastic electromagnetic beam propagating in the homogeneous and isotropic turbulent atmosphere have been derived. With the help of numerical calculations we have then studied the changes in the statistical properties of an anisotropic electromagnetic beam of the Gaussian Schell-model type. In particular, we found that, unlike an isotropic beam, an anisotropic stochastic electromagnetic beam may not, in general, recover its polarization properties in the source after propagation in the turbulent atmosphere over sufficiently long propagation distance.

Of course, the method of vector integration can also be applied to study the propagation of other classes of partially coherent beams (such as pulsed beams, flattened beams, vortex beams, etc.) passing through the turbulent atmosphere or other linear isotropic and homogeneous media.

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