In Situ Wave Measurements: Sensor Comparison and Data Analysis

Clarence O. Collins III
University of Miami, Tripphysicist@gmail.com

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UNIVERSITY OF MIAMI

IN SITU WAVE MEASUREMENTS: SENSOR COMPARISON AND DATA ANALYSIS

By

Clarence O. Collins III

A THESIS

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IN SITU WAVE MEASUREMENTS: SENSOR COMPARISON AND DATA ANALYSIS

Clarence O. Collins III

Approved:

Hans C. Graber, Sc.D.  M. Brian Blake, Ph.D.
Professor of Applied Marine Physics  Dean of the Graduate School

Brian K. Haus, Ph.D.  Ad J.H.M. Reniers, Ph.D.
Professor of Applied Marine Physics  Professor of Applied Marine Physics

Robert E. Jensen, Ph.D.  C. Linwood Vincent, Ph.D.
Coastal and Hydraulics Laboratory  Professor of Applied Marine Physics
USACE Engineer Research and Development Center
Vicksburg, Mississippi
This study examines new and standard techniques for comparing, contrasting, and describing differences of wave measurements from different in-situ platforms. There are few standard tools which can intuitively illuminate differences as functions of energy and frequency. Therefore these subtle differences, which may have been obscured in previous studies, may be explored with the new Wave sensor Evaluation Tool (WET) intercomparison program. Although this is a step forward, the interpretation of WET graphs can be tricky and uncertain. One chapter is dedicated to learning more about the WET intercomparison tool through applying it to artificially produced data. Three field experiments: Gulf Of Mexico 1999 (GOM99), SHOaling Wave Experiment (SHOWEX), and Shallow Water 2006 (SW06) make up the core of the thesis. Each data set makes up a complete chapter of work. Comparisons are made of wave data using a variety of techniques. Results are mostly in line with previous studies. The Air-Sea Interaction Spar (ASIS) platform is common to all of the field experiments and strengths and weakness of the platform are made evident. 2 other platforms are evaluated relative to ASIS. Case studies from the data sets are explored in greater depth. Case studies include analysis of directional spread as a function of frequency, slanting fetch conditions, sampling variability
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<td>Gulf Of Mexico 1999</td>
</tr>
<tr>
<td>HPR</td>
<td>Heave-Pitch-Roll</td>
</tr>
<tr>
<td>H_s, H_m0</td>
<td>Significant Wave Height</td>
</tr>
<tr>
<td>JONSWAP</td>
<td>JOint North Sea WAve Project</td>
</tr>
<tr>
<td>MAB</td>
<td>Mid-Atlantic Bight</td>
</tr>
<tr>
<td>MLM</td>
<td>Maximum Likelihood Method</td>
</tr>
<tr>
<td>NDBC</td>
<td>National Data Buoy Center</td>
</tr>
<tr>
<td>NOAA</td>
<td>National Oceanic and Atmospheric Association</td>
</tr>
<tr>
<td>ONR</td>
<td>Office of Naval Research</td>
</tr>
<tr>
<td>RSMAS</td>
<td>Rosenstiel School for Marine and Atmospheric Science</td>
</tr>
<tr>
<td>SHOWEX</td>
<td>SHOaling Wave EXperiment</td>
</tr>
<tr>
<td>SW06</td>
<td>Shallow Water 2006</td>
</tr>
<tr>
<td>SWADE</td>
<td>Surface WAve Dynamics Experiment</td>
</tr>
<tr>
<td>SWH</td>
<td>Significant Wave Height</td>
</tr>
<tr>
<td>USACE</td>
<td>US Army Corps of Engineers</td>
</tr>
<tr>
<td>WADIC</td>
<td>WAve DIrection Calibration experiment</td>
</tr>
<tr>
<td>WAFO</td>
<td>Wave Analysis for Fatigue and Oceanography</td>
</tr>
<tr>
<td>WET</td>
<td>Wave (sensor) Evaluation Tool</td>
</tr>
<tr>
<td>WHOI</td>
<td>Woods Hole Oceanographic Institute</td>
</tr>
<tr>
<td>1-D, 2-D</td>
<td>1-Dimensional, 2-Dimensional</td>
</tr>
</tbody>
</table>
List of Symbols

\( a_0 \)  
First Fourier Coefficient/Energy

\( a, a_p \)  
Wave Amplitude, Peak Wave Amplitude

\( a_1, b_1, a_2, b_2 \)  
Fourier Coefficients/Directional Moments (Longuet-Higgins)

\( \text{atan} \)  
Arctangent

\( \text{atan2} \)  
Arctangent Logical Algorithm

\( A \)  
Wave Amplitude

\( A_x \)  
Acceleration in x direction

\( A_1, B_1, A_2, B_2 \)  
Normalized Directional Coefficients

\( A_{1b}, B_{1b} \)  
Bulk Weighted Directional Coefficients

\( A_{1b4}, B_{1b4} \)  
Fourth Power Energy Weighted Directional Coefficients

\( \%b \)  
Percent Bias

\( \cos \)  
Trigonometric Cosine

\( \text{cr}(f) \)  
Check Ratio

\( C_{ij} \)  
Co-Spectral Density of the \( i^{th} \) and \( j^{th} \) component

\( C_p \)  
Phase Speed of Peak Waves

\( C_d \)  
Drag Coefficient

\( C_\theta \)  
Cosine Mean Angle Component

\( d \)  
Water Depth

\( \text{dtheta} \)  
Angular Resolution

\( D(\theta) \)  
Directional Distribution

\( D_1 \)  
Non-Dimensional Depth (\( dk_p \))

\( D_2 \)  
Non-Dimensional Depth (\( gd/U_{10}^2 \))

\( E \)  
Energy

\( E(f, \theta) \)  
Directional Spectrum

\( f \)  
Frequency

\( f_p \)  
Peak Frequency

\( f_p WD \)  
Neighbor Energy Weighted Peak Frequency

\( f_{p4} \)  
Fourth Power Energy Weighted Peak Frequency

\( fs \)  
Sampling Frequency

\( f_z \)  
Mean Zero Crossing Frequency

\( g \)  
Gravitational Acceleration

\( H_{m_0} \)  
Significant Wave Height

\( H_s \)  
Significant Wave Height

\( \text{Hz} \)  
Hertz

\( \Im \{ \} \)  
Imaginary Part

\( k \)  
Wavenumber Vector
\( k, k_x \) Wavenumber Magnitude, Component in x direction

\( L \) Wavelength, Unit Length
\( L_0 \) Obukhov Length
\( L_1, L_2, L_3 \) Vector from Motion Package to the Water Surface at the Wave Wire

\( m \) Meter
\( m_0 \) First Spectral Moment
\( m_2 \) \( m_2 = a_2 \cos(2\theta_{m1}) + b_2 \sin(2\theta_{m1}) \)
\( \max(x) \) Maximum of \( x \)
\( \min(x) \) Minimum of \( x \)
\( \text{MSE} \) Mean-Square-Error
\( \text{ns} \) Number of Samples per Average Data Block
\( n_2 \) \( n_2 = b_2 \cos(2\theta_{m1}) - a_2 \sin(2\theta_{m1}) \)

\( p \) Probability Value
\( P_{xy} \) Cross Correlation

\( Q_{ij} \) Quadrature-Spectral Density of the \( i^{th} \) and \( j^{th} \) component

\( r_1, r_2 \) Euclidean Norm of \( A_1, B_1 \) and \( A_2, B_2 \)
\( r_1, \alpha_1, r_2, \alpha_2 \) NDBC Normalized Directional Coefficients
\( \text{rms} \) Root Mean Square
\( R, R^2 \) Pearson Correlation Coefficient
\( R_b \) Bulk Richardson Number

\( R_\vartheta \) Mean Resultant Vector Length
\( \text{RMSE} \) Root-Mean-Square-Error
\( \Re\{} \) Real Part

\( s \) second
\( s_b \) Bulk Directional Dispersion
\( s_1, s_2 \) Spread of the Directional Distribution from \( r_1, r_2 \)
\( \sin \) Trigonometric Sine
\( \text{std} \) Standard Deviation (Standard Difference)
\( \% \, \text{std} \) Relative Standard Deviation (Relative Standard Difference)
\( S \) Variance Density Spectrum
\( S(f) \) Frequency Spectrum
\( S_{\text{LH}} \) Non-Dimensional Steepness
\( S_{xy} \) Cross Spectrum
\( S_\vartheta \) Sine Mean Angle Component
\( S_S \) Significant Steepness

\( t \) Time
tanh Hyperbolic Tangent
$\tan^{-1}$ Inverse Tangent
T Unit Time
$T_p$ Peak Period
$T_m$ Mean Period
$T_x$ Temperature of x
$T_z$ Mean Zero Crossing Period
TR.UTC Time Difference between Local Time and UTC

$u^*$ Friction Velocity
$U, U_{10}$ Wind Speed, Wind Speed at 10 meters
$U_{10}/C_p$ Inverse Wave Age

$\text{var}, \sigma$ Variance

x 2-D Horizontal Space Vector

z, $z_l$ Sensor Height
$z_m$ Untransformed Wave Wire Measurement of Water Level
$z_0$ Roughness Length

$\alpha$ Charnock Parameter

$\gamma$ Directional Skewness
$\gamma_p$ Probably for Cumulative Standard Normal Distribution
$\gamma_1$ Surface Elevation Skewness
$\gamma_2$ Surface Elevation Kurtosis

$\delta$ Directional Kurtosis
$\delta$ Parameter of Coefficient of Variance
$\delta$ Non-dimensional Depth
$\Delta \theta$ $\Delta \theta = T_{\text{sea}} - T_{\text{air}}$

$\epsilon$ Non-dimensional Energy

$\eta$ Ocean Surface Elevation

$\theta$ Pitch
$\theta_d$ Difference between Wind Direction and Wave Direction
$\theta_{mp}$ Mean Direction from $A_1, B_1$ at the Peak
$\theta_{m1}$ Mean Direction from $A_1, B_1$
$\theta_{m2}$ Mean Direction from $A_2, B_2$
$\theta_p$ Mean Direction at the Peak
$\theta_U$ Wind Direction
$\theta_x$ Direction of x
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>κ</td>
<td>Non-dimensional Wavenumber</td>
</tr>
<tr>
<td>μ</td>
<td>Viscosity</td>
</tr>
<tr>
<td>μ&lt;sub&gt;x&lt;/sub&gt;</td>
<td>Mean of x</td>
</tr>
<tr>
<td>ν</td>
<td>Non-dimensional Frequency</td>
</tr>
<tr>
<td>ν&lt;sub&gt;f&lt;/sub&gt;</td>
<td>Spectral Width Parameter</td>
</tr>
<tr>
<td>ν&lt;sub&gt;k&lt;/sub&gt;</td>
<td>Kinematic Viscosity</td>
</tr>
<tr>
<td>ρ</td>
<td>Water Density</td>
</tr>
<tr>
<td>σ&lt;sub&gt;p&lt;/sub&gt;</td>
<td>Directional Spread at the Peak</td>
</tr>
<tr>
<td>σ&lt;sub&gt;s&lt;/sub&gt;</td>
<td>Directional Spread</td>
</tr>
<tr>
<td>∑&lt;sub&gt;x&lt;/sub&gt;</td>
<td>Sum x</td>
</tr>
<tr>
<td>φ</td>
<td>Phase Angle</td>
</tr>
<tr>
<td>φ</td>
<td>Roll</td>
</tr>
<tr>
<td>Φ(γ&lt;sub&gt;p&lt;/sub&gt;)</td>
<td>Cumulative Standard Normal Distribution</td>
</tr>
<tr>
<td>χ</td>
<td>Non-dimensional Fetch</td>
</tr>
<tr>
<td>ψ</td>
<td>Bow Heading</td>
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<tr>
<td>ω</td>
<td>Angular Frequency</td>
</tr>
<tr>
<td>Ω&lt;sub&gt;x&lt;/sub&gt;</td>
<td>Angular Rate for x</td>
</tr>
<tr>
<td>°</td>
<td>Degree</td>
</tr>
<tr>
<td># pts</td>
<td>Number of Points</td>
</tr>
<tr>
<td>~</td>
<td>About, Approximately Equal To</td>
</tr>
<tr>
<td>∂η&lt;sub&gt;x&lt;/sub&gt;, ∂η&lt;sub&gt;y&lt;/sub&gt;</td>
<td>Surface Slope</td>
</tr>
</tbody>
</table>
1 Chapter: Introduction

1.1 Wave Sensor Inter-Comparisons and Analysis

In-situ measurements of ocean surface waves in the wind-wave frequency band (also referred to as simply wind waves) are most commonly performed by surface buoys. An important part of evaluating buoy performance is wave data intercomparison. Several valuable references for wave sensor comparisons include (Allender, 1989; Drennan et al., 2003; Krogstad, Wolf, Thompson, & Wyatt, 1999; O'Reilly, Herbers, Seymour, & Guza, 1996; Pettersson et al., 2003).

The general consensus is that instruments agree well on 1-Dimensional (1-D) engineering parameters, but there is much poorer agreement on directional parameters. These intercomparisons have proved difficult to interpret because of the complexity in determining sources of measurement variability. Measurements between two systems may vary for following reasons:

- The systems may utilize different measurement principles
- Different system limitations (perhaps due to measurement principles)
- Imperfect calibration causing systematic error
- Different sampling variability of the systems
- Differences in data processing
- Natural variability of the sea surface
- Measurement offsets in time and space
- Differences in mooring forces (which are environmentally dependent) (Krogstad et al., 1999)

The difficulties listed above are not uniform across varying sea-states and varying frequency bands. The newly developed Wave Evaluation Tool (WET) intercomparison program specifically addresses variability across sea-state and frequency (Thomas, O'Reilly, & Jensen, 2008), which traditional inter-comparisons (i.e. scatter plots) do not.
It is typical of inter-comparisons analysis to assume that one of the systems is “true” and the other is experimental or “test”. Any disparities are attributed to the test system. This allows for traditional statistical methods to compare the systems (e.g. regression analysis, mean error, correlation coefficients, bias etc). It is important to keep in mind that no system is able to measure the “true” sea-state and interpretation of differences between systems is a subtle issue (COST Action 714 Working Group 3, 2005).

It is my contention that analysis cannot be meaningfully performed without an understanding of the measurement characteristics and limitations. In this regard, the tool for evaluating measurements is comparison.

1.2 Motivation
Ocean waves are interesting. It is fortunate that information learned from wave studies also find a wide range of important application. A few of the applications include understanding of regional wave climates, extreme wave events, damage from hurricanes, the engineering of structures in the ocean environment, the safety of boats and transport vessels, ocean emergency air rescues, beach erosion, and connected coastal processes depend on our knowledge of ocean wave systems. Aside from practical applications, ocean waves are an important part of the Earth environment system. A more complete understanding our environment is necessary to answer big questions such as those associated with global climate change.

1.3 Thesis Preview
The following sections, chapters 2-5, provide useful background information for this study including sensor comparison methods, calculation of
parameters and mathematical relationships, introduction of pertinent surface buoys, and a literature review and outlook. Chapter 6 is a section dedicated to the technical details of the processing methods used in this thesis. Chapter 7 is an in depth study of the WET inter-comparison program. Chapters 8, 9, and 10 explore data sets from the 3 field experiments GOM99, SHOWEX, and SW06, respectively. Chapter 11 is a general conclusion and a look at lines of investigation which may be drawn out from this body of work. The references are found in the final section.
2 Chapter: Background I: Intercomparison Methods

The following section briefly introduces the inter-comparison methods used in the study. The commonly used methods are time-series plots and scatter plots. More attention is paid to the uncommon WET method in a later section.

2.1 Difference Statistics and Association Measures

2.1.1 Numerical Data

In the following discussion the words error and difference are used interchangeably. Strictly speaking all errors are differences, but not all differences are errors. Furthermore a difference is an error when the difference is in contradiction of a known reference or standard. Although some platforms may make measurements of superior quality, an absolute standard for wave measuring devices has yet to be devised so such a distinction is trivial.

Errors are an innate aspect of any measurement and can be one of 2 kind: random or systematic. There are many metrics for highlighting differences, random or systematic, between two measurements or measurement systems. To describe the difference between just two numbers, the simplest measure is absolute error:

\[ \text{absolute error} = |M_r - M_t| \]

Where \( M_r \) a reference datum and \( M_t \) is a experimental or test datum. One can normalize by the reference to get percent error of the test measurement:

\[ \text{percent error} = 100 \times \left( \frac{M_r - M_t}{M_r} \right) \]
One measurement cannot tell you if the error is random or systematic. To learn more it is necessary to have many measurements, the more the better (statistically speaking). If each measurement set is considered to be a normal random variable then one can treat the difference as a normally distributed variable.

\[ M_r - M_t = D \]

The boldface type signifies a vector or array of measurements. To describe the difference distribution one can use a central value measure and deviation measures such as the mean difference:

\[ \text{mean difference (bias)} = \mu = \frac{\sum D}{n} \]

The value of mean difference can be interpreted as a bias of one instrument relative to another instrument or reference. One expects the average of random error to be zero, so bias is the systematic error (i.e. the error inherent to the system). One could also calculate the average percent error by normalizing by the reference value:

\[ \%b = 100 \times \left( \frac{\sum D}{M_r} \right) \]

The mean-square-error (MSE) and root-MSE (RMSE) are measures of the average deviations from the mean difference:

\[ MSE = \frac{\sum D^2}{n} \quad \text{and} \quad RMSE = \sqrt{\frac{\sum D^2}{n}} \]
RMSE is a measure which includes contributions from both random and systematic errors. To get a better understanding of the random error involved, one can take the bias out of the MSE or RMSE. These “unbiased” measures are known as the variance (var) and standard deviation (also referred to as standard difference) (std):

$$\text{var} = \sigma = \frac{\sum (D - \mu)^2}{n} \text{ and } \text{std} = \sqrt{\sigma}$$

A relative std (%std) is calculated by normalizing the std by the mean reference value and is expressed as a percentage:

$$\%\text{std} = 100 \times \frac{\sqrt{\sigma}}{\mu_r}$$

When this applied to data from a single sensor it is the coefficient of variation (COV):

$$\text{COV}_x = \frac{\sqrt{\sigma_x}}{\mu_x}$$

These are calculated from the time series of parameter $x$. The Pearson correlation coefficient is a measure of association between the data sets:

$$R = \frac{\sum_{i=1}^{n} (M_{r_i} - \bar{M}_r)(M_{t_i} - \bar{M}_t)}{\sqrt{\sum_{i=1}^{n} (M_{r_i} - \bar{M}_r)^2 \sum_{i=1}^{n} (M_{t_i} - \bar{M}_t)^2}}$$

These useful values are descriptive statistics of data and their differences. A much more comprehensive discussion can be found in any statistics textbook and in context to geo-sciences in the following reference (Willmott et al., 1985).
2.1.2 Directional Data

Directional data is a different beast. Directional data lies on a scale which has no zero and “high” and “low” values are merely conventional. For this discussion, directional data are terms of degrees which are modulo 360°. Directional statistics are based on the mean of the angle’s components:

\[ S_\theta = \text{mean}(\sin(\theta)) \quad \text{and} \quad C_\theta = \text{mean}(\cos(\theta)) \]

To find the directional mean take the inverse tangent, using the sign of the components put the angle in the proper quadrant:

\[
\begin{align*}
\text{Mean Direction} = \mu_\theta &= \begin{cases} \\
\tan^{-1} \left( \frac{S_\theta}{C_\theta} \right), & \text{for } S_\theta > 0, C_\theta > 0 \\
\tan^{-1} \left( \frac{S_\theta}{C_\theta} \right) + \pi, & \text{for } C_\theta < 0 \\
\tan^{-1} \left( \frac{S_\theta}{C_\theta} \right) + 2\pi, & \text{for } S_\theta < 0, C_\theta > 0
\end{cases}
\end{align*}
\]

Most computer languages have this logical statement built into an arctangent function and is generally referred to as atan2. If \( \theta \) is the difference between 2 angles, the mean resultant vector length, \( R_\theta \), is an association measure:

\[ R_\theta = \sqrt{S_\theta^2 + C_\theta^2} \]

And the directional standard deviation is:

\[ \text{std}_\theta = \sqrt{2(1 - \text{Coh})} \]

Further discussion can be found here (Berens, 2009; Jones, 2006).

2.2 Graphical Representations

The following sections introduce four methods of showing data. The first section describes plotting time series, the second section describes scatter plots,
the third section describes average parameters as a function of frequency, and the fourth section introduces WET plots.

### 2.2.1 Graphical Representations: Time-Series Plots

Engineering parameters are derived from the frequency spectrum of the surface waves. These engineering parameters, as measured by each respective buoy, are plotted simultaneously to get a qualitative comparative picture of differences. It is useful to plot these along with time series of wind measurements and other meteorological data to get an idea of the environmental conditions during a given experiment. With this method one can look at the chronology of events, and can identify areas of disparity with specific meteorological phenomena.

![Graphical Representations](image)

*Figure 2.1: This example of a time series plot is an ASIS buoy (solid black line) and a Directional Wave Rider made by Datawell (red circles). The comparison comes from (Drennan et al., 2003).*
2.2.2 Graphical Representations: Scatter Plots

Figure 2.2) This is a scatter plot also from (Drennan et al., 2003). This is a comparison between significant wave height and peak frequency as measured by an ASIS buoy and a Directional Wave Rider made by Datawell. The outer dotted lines form a 90% confidence region based on estimated sampling variability.

The engineering parameters may be used to create scatter plots. Scatter plots are the standard quantitative inter-comparison method. First one data set is interpolated to the other so that both data sets are equally sampled. The measurements from one system become the x coordinates and the simultaneous measurements from the other system become y coordinates. These graphs are typically accompanied by several quantities: 1) a regression (best fit) which assigns a relationship between the two measurements, 2) a number which describes the association between the two systems (correlation coefficient), 3) statistics which describe the differences (bias, std, RMSE, etc). When comparing test data against a standard reference, a linear regression is used to fit the test data to the standard. When comparing 2 test sensors, both of which are expected to have errors associated with the data, it is best to use a Maximum
Likelihood (ML) regression (Krogstad et al., 1999). In order to assess the agreement in a scatter plot, confidence regions should be established based on the sampling variability. The sampling variability of wave parameters is discussed in section 4.2.7.

2.2.3 Graphical Representations: Average Spectra

A useful way to see systematic differences between buoys is to plot average parameters as a function of frequency. This can be done variance density (1-D spectra), mean direction, and directional spread. An example of mean direction as a function for several buoys is shown below.

![Average Mean Direction for Year Day 93-97](image)

Figure 2.3) Average mean direction over a 4 day period for ASIS buoy Bravo, ASIS buoy Romeo, ASIS Buoy Yankee, and NDBC 3-meter discus in blue, red, green, and black, respectively.
Figure 2.3 shows mean direction averaged over a 4 day period. The top plot shows the mean direction for 4 buoys which are relatively close to each other. The bottom plot shows the mean direction normalized by the measurements from one particular buoy.

2.2.4 Graphical Representations: WET Program

The WET inter-comparison program was developed by W. O'Reilly et al. (CDIP, 2009). The program shows the relative or absolute std and bias for wave parameters as a function of energy (wave height) and frequency. The advantage of this method is that graphs reveal differences otherwise hidden. The disadvantage is that interpretations of these graphs are difficult. Further details are in a later section.

Figure 2.4) This is the root-mean-square-error between two buoys measurement of the energy. Each block is a frequency-energy component. This was taken off the CDIP website www.cdip.ucsd.edu.
3 Chapter: Background II: Parameters and Mathematical Relations

There are many wave parameters in the literature. They may be derived from the properties of a time series of surface elevation directly by looking at the statistics of apparent waves defined through zero-crossing analysis, or they may be derived from spectral properties. This thesis concentrates on parameters calculated from properties of directional spectrum or frequency spectrum exclusively.

3.1 Omni-directional Spectral Sea-State Parameters

Often spectral comparisons utilize variables which are parameterizations of spectra. Spectral sea-state parameters are defined in the context of the omni-directional (1-D) frequency spectrum, $S(f)$. The spectrum is estimated from the time history of a weakly-stationary, quasi-Gaussian random sea surface. A realization of the surface is defined as:

$$\eta(x, t) = \sum_n A_n \cos(k_n \cdot x_n - \omega_n t + \varphi_n)$$

With $k$, the wavenumber vector, and $\omega$, the angular frequency, connected through the dispersion relation found in small amplitude linear wave theory.

$$\omega = \sqrt{gk \tanh(kh)} \text{ where } k = |k|$$

The phase $\varphi_n$ is an independent random variable. The surface elevation is interpreted as the ensemble of possible surfaces constructed from random linear free waves. The sum of a large number of independent random variables tends towards a normal (Gaussian) distribution. This is a consequence of the
central limit theorem (Bendat & Piersol, 2000). The spectral density, $S(f)$, is the distribution of variance density over frequency:

$$S(f) = \frac{d\langle \eta^2 \rangle}{df}$$

$$\langle \eta^2 \rangle = \sum_n \frac{a_n^2}{2} = \int_0^\infty S(f)df$$

Many parameterizations of the omni-directional spectrum are based on the spectral moments:

$$m_n = \int_0^\infty f^n S(f)df$$

$m_0$ is the first spectral moment, i.e. the total variance of the wave spectrum:

$$m_0 = \int_0^\infty S(f)df = \langle \eta^2 \rangle$$

It is possible to calculate higher order moments of the sea surface elevation (i.e. skewness and kurtosis), and it will be shown in a later section. The significant wave height is defined as:

$$H_{m_0} = 4\sqrt{m_0}$$

$H_{m_0}$ is largely a historic sea-state measure, and in deep water it is essentially the same as the average height of 1/3 highest waves. From significant wave height, the mean wave energy per square meter is:

$$E = \frac{1}{16} \rho g H_{m_0}^2 = \rho g m_0$$
Where \( \rho \) is the density of ocean water, \( g \) is gravitational acceleration.

There are several period parameters that are derived from the spectral moments. \( T_m \) is known as the mean period and \( T_z \) is the mean zero-crossing period (also referred to as \( T_{01} \) and \( T_{02} \), respectively).

\[
T_m = T_{01} = \frac{m_0}{m_1}
\]

\[
T_z = T_{02} = \sqrt{\frac{m_0}{m_2}}
\]

The spectral peak period, is defined as the period at the peak of the frequency spectrum.

\[
T_p = \frac{1}{f_p}
\]

Where \( f_p \) is the frequency at the spectral peak.

\[
f_p = \max f(S(f))
\]

(Dean & Dalrymple, 1991; {COST} Action 714 Working Group 3, 2005)

For completeness, there are variations of the calculation \( f_p \), many of which can be found in (Young, 1995; Young, 1999). The period parameters derived from the spectral moments are more stable (less variation) than peak period measures, and their error characteristics are known from the properties of the spectral estimate. Parameters which describe bandwidth and wave steepness are also in use. The normalized radius of gyration of the spectrum about its mean frequency (also known as the frequency centroid) is a spectral width parameter proposed by Longuet-Higgins:

\[
v_f = \sqrt{\frac{m_0 m_2}{m_1^2} - 1}
\]
Significant steepness, $S_s$, is a steepness measure:

$$S_s = \frac{2\pi H_{m0}}{g T_z^2}$$

The section is not meant to be an exhaustive list of parameters. We have demonstrated only those parameters which may be used in this thesis.

### 3.2 Directional Spectra and First 5 Fourier Coefficients

Waves on the ocean surface are also distributed in directional space. When a direction distribution is combined with the frequency spectrum it is the directional spectrum, $E(f, \theta)$.

$$E(f, \theta) = S(f) D(\theta)$$

The directional spectrum can be written as a Fourier series:

$$E(f, \theta) = \frac{1}{2} a_0 + (a_1 \cos(\theta) + b_1 \sin(\theta)) + (a_2 \cos(2\theta) + b_2 \sin(2\theta))$$

$$+ (a_3 \cos(3\theta) + b_3 \sin(3\theta)) + \cdots$$

Truncating the series after 5 Fourier coefficients:

$$E(f, \theta) \approx \frac{1}{2} a_0 + (a_1 \cos(\theta) + b_1 \sin(\theta)) + (a_2 \cos(2\theta) + b_2 \sin(2\theta))$$

The coefficients are determined by the following equations:

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} E(f, \theta) d\theta$$

$$a_1 = \frac{1}{\pi} \int_0^{2\pi} E(f, \theta) \cos(\theta) d\theta \text{ and } b_1 = \frac{1}{\pi} \int_0^{2\pi} E(f, \theta) \sin(\theta) d\theta$$

$$a_2 = \frac{1}{\pi} \int_0^{2\pi} E(f, \theta) \cos(2\theta) d\theta \text{ and } b_2 = \frac{1}{\pi} \int_0^{2\pi} E(f, \theta) \sin(2\theta) d\theta$$

These first 5 Fourier coefficients represent the energy ($a_0$) and the low order directional moments ($a_1, b_1, a_2, b_2$).
3.3 Relation to Buoy Measurements

The first to measure the directional spectrum via buoy was (Longuet-Higgins, Cartwright, & Smith, 1963). This section examines the Fourier expansion method which was developed in that pioneering paper. Most directional surface buoys (and other measurement systems) are single point triplet devices which simultaneously record the time series of 3 complimentary properties of the ocean surface. For example, heave-pitch-roll (HPR) buoys typically use an accelerometer (twice integrated) to measure the heave (vertical displacement $\eta$), and angular rate gyroscopes to measure the pitch and roll (which may be east-west slope and north-south slope respectively $\frac{\partial \eta}{\partial x}$ and $\frac{\partial \eta}{\partial y}$).

The three time series for an HPR buoy would then be:

$$\eta, \frac{\partial \eta}{\partial x}, \frac{\partial \eta}{\partial y}$$

Explicitly these three are:

$$\eta(x, t) = \sum_n A_n \cos(l_n x + m_n y - \omega_n t + \varphi_n)$$

Where $l_n$ and $m_n$ are the x and y components of the wavenumber vector:

$$l_n = k_n \cos(\theta) \text{ and } m_n = k_n \sin(\theta)$$

Therefore

$$\frac{\partial \eta(x, t)}{\partial x} = \sum_n -l_n A_n \sin(l_n x + m_n y - \omega_n t + \varphi_n)$$

$$\frac{\partial \eta(x, t)}{\partial y} = \sum_n -m_n A_n \sin(l_n x + m_n y - \omega_n t + \varphi_n)$$

The cross spectral density between two time series, $x(t)$ and $y(t)$, can be found by 1) taking the Fourier transform of the cross correlation function, $P_{xy}$, or
by multiplying the Fourier transforms $\hat{x}(f)$ and $\hat{y}(f)$ of the two time series. The second method is used consistently in this thesis.

$$P_{xy}(t) = \int_0^T \overline{x(\tau)} y(t + \tau) d\tau$$

$$\hat{x}(f) = \int e^{-2\pi ft} x(t) \, dt \text{ and } \hat{y}(f) = \int e^{-2\pi ft} y(t) \, dt$$

$$S_{xy}(f) = \int e^{-2\pi ft} P_{xy}(t) \, d\tau = \overline{\hat{x}(f)} \hat{y}(f)$$

The over bar indicates complex-conjugation. The real part of the cross spectrum is the co-spectrum, $C_{xy}$, and the imaginary part is the quadrature-spectrum (quad-spectrum), $Q_{xy}$:

$$C_{xy} = \Re \{S_{xy}(f)\}$$

$$Q_{xy} = \Im \{S_{xy}(f)\}$$

For simplicity, let us denote $\eta = 1$, $\frac{\partial \eta}{\partial x} = 2$, and $\frac{\partial \eta}{\partial y} = 3$. Then the Co- and Quad-spectra are:

$$C_{11} = \int_0^{2\pi} E(f, \theta) \, d\theta$$

$$C_{22} = \int_0^{2\pi} k^2 \cos^2(\theta) E(f, \theta) \, d\theta$$

$$C_{33} = \int_0^{2\pi} k^2 \sin^2(\theta) E(f, \theta) \, d\theta$$

$$C_{23} = \int_0^{2\pi} k^2 \cos(\theta) \sin(\theta) E(f, \theta) \, d\theta$$

$$Q_{12} = \int_0^{2\pi} k \cos(\theta) E(f, \theta) \, d\theta$$
\[ Q_{13} = \int_{0}^{2\pi} k \sin(\theta) E(f, \theta) d\theta \]

More concisely, the Fourier coefficients of the directional spectrum are:

\[ a_n + ib_n = \frac{1}{\pi} \int_{0}^{2\pi} e^{i\theta} E(f, \theta) d\theta \]

By simple substitution, it can be shown that first five Fourier coefficients are related to the co- and quad-spectra of the time series as measured by the buoy.

\[ a_0(f) = \frac{1}{\pi} C_{11}(f) \]

\[ a_1(f) = \frac{1}{\pi k} Q_{12}(f) \]

\[ b_1(f) = \frac{1}{\pi k} Q_{13}(f) \]

\[ a_2(f) = \frac{1}{\pi k^2} (C_{22}(f) - C_{33}(f)) \]

\[ b_2(f) = \frac{1}{\pi k^2} C_{23}(f) \]

The wavenumber, \( k \), can either be calculated by assuming the linear dispersion relation or measured directly:

\[ k = \sqrt{\frac{(C_{22}(f) + C_{33}(f))}{C_{11}(f)}} \]

Using the above relation, it is convenient to use normalized directional coefficients (Long, 1980; Tucker, 1991). The normalized directional coefficients will be referred to in capitals:
\[ A_1(f) = \frac{Q_{12}(f)}{\sqrt{C_{11}(C_{22}(f) + C_{33}(f))}} \]

\[ B_1(f) = \frac{Q_{13}(f)}{\sqrt{C_{11}(C_{22}(f) + C_{33}(f))}} \]

\[ A_2(f) = \frac{C_{22}(f) - C_{33}(f)}{C_{22}(f) + C_{33}(f)} \]

\[ B_2(f) = \frac{2C_{23}(f)}{C_{22}(f) + C_{33}(f)} \]

\[ \int_0^{2\pi} D(\theta) d\theta = 1 \]

The National Oceanic and Atmospheric’s (NOAA) National Data Buoy Center (NDBC) uses a slightly different set of normalized coefficients. They are set in a different coordinate system, and the NDBC coefficients are related to the directional parameters described in the following section. The functional notation is suppressed for tidiness, but all coefficients are a function of frequency.

\[ r_1 = \frac{\sqrt{a_1^2 + b_1^2}}{a_0} \]

\[ r_2 = \frac{\sqrt{a_2^2 + b_2^2}}{a_0} \]

\[ a_1 = 270^\circ - \tan^{-1}(b_1,a_1) \]

\[ a_2 = 270^\circ - \frac{1}{2}\tan^{-1}(b_2,a_2) \text{ or } \theta_2 = 270^\circ - \frac{1}{2}\tan^{-1}(b_2,a_2) + 180^\circ \]

The calculation of \( \theta_2 \) which gives an angle closer to \( \theta_1 \) is used. For completeness the conversion back to Longuet-Higgins directional coefficients from NDBC coefficients is as follows:
\[ a_1 = r_1 \cos(\alpha_1) \]
\[ b_1 = r_1 \sin(\alpha_1) \]
\[ a_2 = r_2 \cos(2\alpha_2) \]
\[ b_2 = r_2 \sin(2\alpha_2) \]

One may be interested in simplifying the coefficients, which are a function of frequency, into one number. This is a kind of bulk parameterization of the coefficients. One way to do this is to use the values at the peak frequency, \( f_p \). Another way to do this is to weight each frequency band by the energy contained within and integrate over all frequencies. The energy weighted bulk Fourier directional coefficients:

\[
A_{1b} = \frac{\int_0^\infty A_0(f) A_1(f) df}{\int_0^\infty A_0(f) df}
\]
\[
B_{1b} = \frac{\int_0^\infty A_0(f) B_1(f) df}{\int_0^\infty A_0(f) df}
\]
\[
A_{2b} = \frac{\int_0^\infty A_0(f) A_2(f) df}{\int_0^\infty A_0(f) df}
\]
\[
B_{2b} = \frac{\int_0^\infty A_0(f) B_2(f) df}{\int_0^\infty A_0(f) df}
\]

### 3.4 Directional Parameters

Just as there are parameters related to the 1-D frequency spectrum, there are also parameters related to the directional distribution. The most common directional parameters are mean direction, directional spread, directional skewness, directional kurtosis, peak direction, and peak spreading. The first four are functions of frequency (functional notation is suppressed) where the latter
two are not. Mean direction is defined for both $A_1,B_1$ and $A_2,B_2$ and indicates the approximate wave direction at each frequency:

$$\theta_{m1} = \tan^{-1}(\frac{B_1}{A_1})$$

$$\theta_{m2} = \frac{1}{2} \tan^{-1}(\frac{B_2}{A_2}) \text{ or } \theta_{m2} = \frac{1}{2} \tan^{-1}(\frac{B_2}{A_2}) + 180^\circ$$

Whichever calculation of $\theta_{m2}$ gives an angle closer to $\theta_{m1}$ is used. Directional spread, skewness, and kurtosis are based on circular moments of the directional distribution. These can be interpreted similarly to line moments if the distributions are sufficiently narrow (Kuik, van Vledder, & Holthuijsen, 1988).

Directional spreading and the dispersion of the distribution are calculated from the resultants of $A_1, B_1$ or $A_2, B_2$ and is an indication of the width of the directional distribution. Directional spread:

$$\sigma_s = \sqrt{2(1 - r)}$$

Where

$$r_1 = \sqrt{A_1^2 + B_1^2}$$

The dispersion of the directional distribution, $s_1$, is related to $\cos^2$ model function proposed in (Longuet-Higgins, 1962; Tucker, 1991). $s_1$ has a direct relationship to the directional spread (Tucker, 1991):

$$s_1 = \frac{r_1}{1 - r_1} = \frac{2 - \sigma_s^2}{\sigma_s^2}$$

One may also calculate a second dispersion of the directional distribution from $A_2$ and $B_2$: 
\[ s_2 = \frac{1 + 3r_2 + \sqrt{1 + 14r_2 + r_2^2}}{2(1 - r_2)} \]

Where

\[ r_2 = \frac{A_2^2 + B_2^2}{\sqrt{A_2^2 + B_2^2}} \]

The directional skewness is a shape measure of the distribution and together with kurtosis can be used to indicate the modality of a directional distribution (Kuik et al., 1988). Directional skewness:

\[ \gamma = -\frac{n_2}{(2(1 - r_1))^{3/2}} \]

Where

\[ n_2 = b_2 \cos(2\theta_{m1}) - a_2 \sin(2\theta_{m1}) \]

Directional kurtosis:

\[ \delta = \frac{6 - 8r_1 + 2m_2}{(2(1 - r_1))^2} \]

Where

\[ m_2 = a_2 \cos(2\theta_{m1}) + b_2 \sin(2\theta_{m1}) \]

The peak direction is the direction of wave propagation at the peak of the spectrum:

\[ \theta_p = \max_\theta (E(f, \theta)) \]

As with \( f_p \) there are various formulations of \( \theta_p \) in use. One can also calculate the mean direction, spreading, skewness, and kurtosis at the peak frequency or with the bulk coefficients. Example 1) mean direction at the peak:

\[ \theta_{mp} = \tan^{-1}\left(\frac{B_1(f_p)}{A_1(f_p)}\right) \]
Example 2) the bulk directional dispersion:

\[ s_b = \frac{r_b}{1 - r_b} \]

Where

\[ r_p = \sqrt{A_{1b}^2 + B_{1b}^2} \]

More advanced discussions can be found in (Krogstad et al., 1999; Kuik et al., 1988).

### 3.5 Introducing New Bulk Directional Parameters

(Young, 1995) proposed a robust estimation of \( f_p \):

\[ f_p = \frac{\int_{0}^{\infty} f S^4(f) df}{\int_{0}^{\infty} S^4(f) df} \]

This form was recommended in (Young, 1995) for its superior error characteristics, and work done for this research has supported this claim. Here the method of (Young, 1995) is adapted for peak direction and directional spread at the peak. First, fourth power energy weighted Fourier coefficients were calculated:

\[ A_{1b4} = \frac{\int_{0}^{\infty} A_1(f) S^4(f) df}{\int_{0}^{\infty} S^4(f) df} \]

\[ B_{1b4} = \frac{\int_{0}^{\infty} B_1(f) S^4(f) df}{\int_{0}^{\infty} S^4(f) df} \]

These are then used to calculate direction at peak and directional spreading at peak:

\[ \theta_p = \text{atan2}(B_{1b4}, A_{1b4}) \]
\[ \sigma_p = \sqrt{2 \left( 1 - \sqrt{A_{1b4}^2 + B_{1b4}^2} \right)} \]

As far as I am aware, these forms of peak direction and peak directional spread do not exist in the literature. These formulations were tested over the more common peak direction calculations for measurements made between Bravo and Romeo during the GOM99 experiment:

<table>
<thead>
<tr>
<th>Peak Directional Formulation/Statistical Measure</th>
<th># pts</th>
<th>Slope</th>
<th>intercept</th>
<th>( R_\theta )</th>
<th>bias</th>
<th>Std</th>
</tr>
</thead>
<tbody>
<tr>
<td>WD point</td>
<td>1487</td>
<td>0.79</td>
<td>46.1</td>
<td>0.742</td>
<td>3.5</td>
<td>41.2</td>
</tr>
<tr>
<td>WD neighbor energy weighted average</td>
<td>1487</td>
<td>0.84</td>
<td>38.3</td>
<td>0.774</td>
<td>3.5</td>
<td>38.2</td>
</tr>
<tr>
<td>WD fourth power energy weighted average</td>
<td>1487</td>
<td>0.84</td>
<td>38.3</td>
<td>0.812</td>
<td>3.0</td>
<td>35.1</td>
</tr>
</tbody>
</table>

Table 3.1) # pts is the number of points in the comparison, Slope and intercept are the linear regression fit parameters (i.e. Bravo=Slope*Romeo + intercept), \( R_\theta \) is the directional association measure, bias id directional bias, and std is the directional standard deviation.

For this experiment, and these 2 buoys, the agreement for the above formulation is significantly better than the other formulations (see table 1 for statistics). As will be seen in the following section, this method of calculation should be very similar to the estimation directly from the peak with much improved stability. The drawback of the formulation is that it tends to drift away from the peak as the directional distribution become significantly broad. In the future the above formulations should be characterized and more thoroughly tested, but presently they will be used in this study.

3.6 Choose Wisely; Compare Consistently

We have described the calculation of parameters, but I want to make a note about the practical use of these parameters. The point may seem obvious, but when comparing data parameters matter. There are two important points
here: 1) one must know the properties of a chosen parameter and be sure that its application is appropriate, and 2) one must be sure to compare only parameters calculated in a consistent way.

Most parameters are not fit to describe multi-modal (either in frequency of direction) seas. Methods which average over the whole spectrum are especially inappropriate. The spectra should be partitioned and parameters derived for each separate wave system. Some parameters are sensitive not only to the modality, but also the spectral (or directional) width.

For example, let us calculate the frequency parameters using 4 common methods in the literature:

\[
f_p = \max_f (S(f))
\]

\[
f_{p, WD} = \frac{\max_f (S(f)) \cdot S(f) + \max_{f+1} (S(f)) \cdot S(f + 1) + \max_{f-1} (S(f)) \cdot S(f - 1)}{S(f) + S(f + 1) + S(f - 1)}
\]

\[
f_{p, w4} = \frac{\int_0^\infty fS^4(f)df}{\int_0^\infty S^4(f)df}
\]

\[
f_z = \frac{1}{T_z} = (m_0/m_2)^{\frac{1}{2}}
\]

The value of each formulation possesses varying degrees of sensitivity to spectral resolution, spectral averaging, and peakedness of the spectrum (Young, 1995). \(f_p\) is the most sensitive parameter, \(f_{p, WD}\) is the energy weighted average of \(f_p\) and its closest neighbors which makes it slightly less sensitive to the spectral properties, \(f_{p, w4}\) is a fourth power weighted spectral integral and is very stable if the spectrum is somewhat narrow banded, and \(f_z\) uses spectral moments and
does not correspond with the peak in general. A time series calculated for spectra during a storm gives the following values:

![Graph of Frequency Measures](image.png)

**Figure 3.1** The figure displays 4 frequency measures of $f_{pw4}$, $f_{pWD}$, $f_p$, and $1/T_z$ and their 4 point running averages in blue, black, red, and green, respectively. Year Day is on the x-axis and frequency is on the y-axis.

It can be seen in figure 5 that the values of $f_p$ are the most scattered, and the values of $f_{pWD}$ offer marginal improvement. The values of $f_{pw4}$ have much less scatter and are generally consistent with $f_p$ except for the period around mid-day 288 when the seas become broad banded. $f_z$ has the least amount of scatter, but it does not represent a measure of the peak frequency.

Here are two example spectra with the values of the 4 frequency parameters marked. The spectrum on top is from year day 288.6 (broad band) and the spectrum below is from year day 287.4 (narrow band), and $f_p$ is marked in red, $f_{pWD}$ is in black, $f_{pw4}$ is in blue, and $f_z$ is in green:
Figure 3.2) Spectra is plotted in purple with the values of frequency measures $f_p$, $fpw4$, $fpWD$, and $1/Tz$ marked in red, blue, black, and green, respectively. X-axis is frequency [Hz] and y-axis is variance density [m$^2$/Hz]. The top plot is an example of a relatively wide banded spectrum, whereas the bottom plot shows a somewhat narrower spectrum.

If one repeats this plot, changing the spectral resolution and spectral averaging, the values of the parameters would also change. The change would be most marked for $f_p$ and $fpWD$.

Similarly to $f_p$, (Significant Wave Height) SWH may be calculated using 3 different methods, each of which may give you a different value. If one sensor uses one method to calculate SWH and different sensor uses another method, the differences found between sensors will be due to both real differences and differences in the calculation method. The bottom line is that when one is analyzing data; care should be taken in choosing parameters that are appropriate, and when one is making comparisons, the parameter being compared should be calculated in a consistent manner.
3.7 Variability of Parameters

The inherent variability of a parameter is a consequence of the environmental variability, sampling variability associated with estimating a stochastic variable, and noise associated with the sensor. It was also shown above (to a greater or less extent) some parameters have variability which originate from the details of the spectral analysis techniques. By examining a period where the environmental variability is minimized (stationary conditions), one can estimate the coefficient of variance (COV) which are used in producing confidence regions on scatter plots. Following (Krogstad et al., 1999), confidence regions on a scatter plot, where one expects a certain percentage of points to fall within, are defined below:

\[ y = \tan\left(\frac{\pi}{4} \pm \delta\right) x \]

Where

\[ \delta = \sin^{-1}\left(\frac{COV \cdot \gamma_p}{\sqrt{2}}\right) \]

Here \( \gamma_p \) is comes from looking up the desired probability, \( p \), in the cumulative standard normal distribution, \( \Phi(\gamma_p) \).

\[ 2\Phi(\gamma_p) - 1 = p \]

For example for a 90% confidence region the value of \( \gamma_p = 1.27 \). A reasonable COV for \( H_{m0} \) is 0.06 and \( f_p \) estimated from the peak may be 2-3 times this. If say 80% of the data fall within the 90% confidence region, one can say that most of the scatter is explained by sampling variability. The take home point here is that disagreement on a scatter plot doesn’t matter so much, as long as it
can be explained by the inherent sampling variability. The scatter becomes an issue when there is variability in excess of that expected from sampling variability. The COVs of different parameters will be estimated during a stationary period in section 7.3.2.

### 3.8 Measurable Quantities and Non-dimensional Parameters

There are only so many measurable features of a wave field. Some basic measurable quantities and units are in the table below. Many common parameters are not measured directly, but are determined by relationships to these measurable quantities:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>SI units [fundamental units]</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>Water Depth</td>
<td>m [L]</td>
</tr>
<tr>
<td>T</td>
<td>Time</td>
<td>s [T]</td>
</tr>
<tr>
<td>A&lt;sub&gt;surge&lt;/sub&gt;, A&lt;sub&gt;sway&lt;/sub&gt;, A&lt;sub&gt;heave&lt;/sub&gt;</td>
<td>Buoy Acceleration</td>
<td>m/s&lt;sup&gt;2&lt;/sup&gt; [LT&lt;sup&gt;-2&lt;/sup&gt;]</td>
</tr>
<tr>
<td>Ω&lt;sub&gt;pitch&lt;/sub&gt;, Ω&lt;sub&gt;roll&lt;/sub&gt;, Ω&lt;sub&gt;yaw&lt;/sub&gt;</td>
<td>Buoy Angular Rate</td>
<td>°/s [T&lt;sup&gt;-1&lt;/sup&gt;]</td>
</tr>
<tr>
<td>H</td>
<td>Sea Surface Height</td>
<td>m [L]</td>
</tr>
<tr>
<td>∂η/∂x, ∂η/∂y</td>
<td>Sea Surface Slope</td>
<td>[]</td>
</tr>
<tr>
<td>θ&lt;sub&gt;wind&lt;/sub&gt;, θ&lt;sub&gt;compass&lt;/sub&gt;</td>
<td>Direction</td>
<td>° [°]</td>
</tr>
<tr>
<td>U</td>
<td>Wind Speed</td>
<td>m/s [LT&lt;sup&gt;-1&lt;/sup&gt;]</td>
</tr>
<tr>
<td>ρ&lt;sub&gt;water&lt;/sub&gt;, ρ&lt;sub&gt;air&lt;/sub&gt;</td>
<td>Density</td>
<td>kg/m&lt;sup&gt;3&lt;/sup&gt; [ML&lt;sup&gt;-3&lt;/sup&gt;]</td>
</tr>
<tr>
<td>T&lt;sub&gt;water&lt;/sub&gt;, T&lt;sub&gt;air&lt;/sub&gt;</td>
<td>Temperature</td>
<td>°C [K]</td>
</tr>
<tr>
<td>G</td>
<td>Gravity</td>
<td>m/s&lt;sup&gt;2&lt;/sup&gt; [LT&lt;sup&gt;-2&lt;/sup&gt;]</td>
</tr>
<tr>
<td>M</td>
<td>Viscosity</td>
<td>Pa·s [ML&lt;sup&gt;-1&lt;/sup&gt;T&lt;sup&gt;-1&lt;/sup&gt;]</td>
</tr>
<tr>
<td>z&lt;sub&gt;t&lt;/sub&gt;, z&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Height of Sensor (Temp., Wind)</td>
<td>m [L]</td>
</tr>
</tbody>
</table>

Table 3.2) Examples of quantities which are measurable in an air-sea interaction field experiment.

Of these measurable quantities in table 2, gravity, density, and viscosity do not vary (to first order) with time. The mean water level is typically detrended to deal with slowly carrying depths such as tides. The rest are a variable function of time. Wave parameters are related to these basic measurable quantities.

---

<sup>1</sup> Strictly speaking, all modern buoy measurements are "non-direct", digitized electronic signals.
Sometimes the relationships are trivial, and other times assumptions are required. If the assumption made is that these quantities are stationary and ergodic, then one can perform spectral analysis. This is the foundation of wave analysis and for most cases this assumption is trivial (given the period of time for analysis suitable). Other common assumptions are the validity of small amplitude wave theory (linear wave theory) and Monin-Obukhov (MO) similarity theory. Details on these theories can be found in any air-sea interaction or waves text book. The following table presents common wave field parameters, a short description, assumptions involved in calculation, and units. Note that there are variations in the parameters, and in general subscript \( p \) indicates the value of the parameter at the spectral peak. Other variants are covered in the section above on calculating parameters.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Assumption</th>
<th>SI units [fundamental units]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S )</td>
<td>Variance Density</td>
<td>(Stationarity, Ergodicity)=1</td>
<td>( m^2/\text{Hz} [ \text{L}^2 \text{T}^-] )</td>
</tr>
<tr>
<td>( H_{m0} )</td>
<td>Significant Wave Height</td>
<td>1, Raleigh Distributed Wave Heights i.e. Narrow Band Spectrum</td>
<td>( \text{m} [\text{L}] )</td>
</tr>
<tr>
<td>( f, f_p, f_m )</td>
<td>Frequency</td>
<td>1</td>
<td>( \text{Hz} [\text{T}^{-1}] )</td>
</tr>
<tr>
<td>( T, T_p, T_m, T_z )</td>
<td>Period (1/f)</td>
<td>1</td>
<td>( \text{s} [\text{T}] )</td>
</tr>
<tr>
<td>( H, a, H_p, a_p )</td>
<td>Wave Height, Amplitude</td>
<td>1</td>
<td>( \text{m} [\text{L}] )</td>
</tr>
<tr>
<td>( E, E_p )</td>
<td>Energy ( (\rho_w g \int S , df) )</td>
<td>1</td>
<td>( \text{J/m}^2 [\text{MT}^{-2}] )</td>
</tr>
<tr>
<td>( \theta_m, \theta_p )</td>
<td>Wave Direction</td>
<td>1</td>
<td>( \circ [\text{]} )</td>
</tr>
<tr>
<td>( \omega, \omega_p )</td>
<td>Radial frequency ( (2\pi f) )</td>
<td>1</td>
<td>( \text{Hz} [\text{T}^{-1}] )</td>
</tr>
<tr>
<td>( k, k_x, k_p )</td>
<td>Wavenumber</td>
<td>1, Linear waves=2</td>
<td>( \text{m}^{-1} [\text{L}^{-1}] )</td>
</tr>
<tr>
<td>( L ) or ( \lambda, L_p )</td>
<td>Wavelength</td>
<td>1,2</td>
<td>( \text{m} [\text{L}] )</td>
</tr>
<tr>
<td>( C_p )</td>
<td>Phase Speed</td>
<td>1,2</td>
<td>( \text{m/s} [\text{LT}^{-1}] )</td>
</tr>
<tr>
<td>( x )</td>
<td>Fetch Length</td>
<td>Constant wind speed and direction over known area</td>
<td>( \text{m} [\text{L}] )</td>
</tr>
</tbody>
</table>
$$U_{10}, U_{10N}$$  | 10 meter wind speed | MO scaling theory=3 | m/s [LT^{-1}] |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$$z_0$$</td>
<td>Roughness Length</td>
<td>3</td>
<td>m [L]</td>
</tr>
<tr>
<td>$$u_*$$</td>
<td>Friction Velocity</td>
<td>3</td>
<td>m/s [LT^{-1}]</td>
</tr>
<tr>
<td>$$L_0$$</td>
<td>Obukhov Length</td>
<td>3</td>
<td>m [L]</td>
</tr>
<tr>
<td>$$v_k$$</td>
<td>Kinematic Viscosity ($\mu/\rho$)</td>
<td></td>
<td>m^{3/2}/s [L^{1/2}T^{-1}]</td>
</tr>
</tbody>
</table>

Table 3.3) Common wave parameters derivable from measured quantities in table 2.

Analysis in wave science relies heavily on non-dimensional parameters which characterize dynamic ranges in an experiment. Non-dimensional parameters are formed through suitable combinations of the above parameters. Dimensional analysis is the method by which these combinations are determined. In the following table some of the more common non-dimensional parameters are listed with a description and a calculation (when it is not obvious).

<table>
<thead>
<tr>
<th>Non-dimensional Parameter/Symbol</th>
<th>Description</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>kh, kph, H_m0/L_p, ka, (S_s)</td>
<td>Non-dimensional Wave (Significant) Steepness</td>
<td>$$S_s = \frac{2\pi H_{m0}}{gT_z^2}$$</td>
</tr>
<tr>
<td>kd, δ</td>
<td>Non-dimensional depth</td>
<td>$$\delta = \frac{gd}{U_{10}^2}$$</td>
</tr>
<tr>
<td>χ</td>
<td>Non-dimensional fetch</td>
<td>$$\chi = \frac{gx}{U_{10}^2}$$</td>
</tr>
<tr>
<td>ε</td>
<td>Non-dimensional energy</td>
<td>$$\varepsilon = \frac{g^2 E}{U_{10}^4}$$</td>
</tr>
<tr>
<td>ν</td>
<td>Non-dimensional frequency</td>
<td>$$\nu = \frac{f_p U_{10}}{g}$$</td>
</tr>
<tr>
<td>κ</td>
<td>Non-dimensional wavenumber</td>
<td>$$\kappa = \frac{U_{10} k_p}{g}$$</td>
</tr>
<tr>
<td>ζ</td>
<td>Non-dimensional duration</td>
<td>$$\zeta = \frac{gt}{U_{10}}$$</td>
</tr>
<tr>
<td>f/f_p</td>
<td>Normalized frequency</td>
<td></td>
</tr>
<tr>
<td>C_p/U_{10}, C_p/(U_{10} \cos(\theta_d))</td>
<td>Wave Age</td>
<td></td>
</tr>
<tr>
<td>U_{10}/C_p, (U_{10} \cos(\theta_d))/C_p</td>
<td>Inverse Wave Age</td>
<td></td>
</tr>
<tr>
<td>α</td>
<td>Charnock Parameter</td>
<td>$$\alpha = \frac{z_0 g}{U_{10}^2}$$</td>
</tr>
<tr>
<td>Z_0/H_{m0}</td>
<td>Non-dimensional roughness</td>
<td></td>
</tr>
<tr>
<td>ν_l</td>
<td>The normalized radius of gyration of the spectrum</td>
<td>$$\nu_l = \sqrt{\frac{m_0 m_2}{m_1^2} - 1}$$</td>
</tr>
</tbody>
</table>
about its mean frequency

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u \cdot z_0/v_k$</td>
<td>Roughness Reynolds Number</td>
<td>$C_d = \frac{u^*}{U_{10}} = \frac{\kappa^2}{\ln^2(10/z_0)}$</td>
</tr>
<tr>
<td>$C_d$</td>
<td>Drag Coefficient</td>
<td>$R_b = \frac{g(T_{air} - T_{sea})}{z \cdot T_{air} (U/z)^2}$</td>
</tr>
<tr>
<td>$R_b$</td>
<td>Bulk Richardson Number</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.4) Table of common non-dimensional parameters.

Note that there is not a consensus in the literature for all of these parameters. For instance, different scaling velocities may be found throughout the literature ($U_{10}$, $u^*$, etc), I have chosen $U_{10}$ because of its simple relation to a measureable quantity. Some of these non-dimensional parameters will featured in later analysis.
4 Chapter: Background III: Measurement Platforms
The study focuses on measurements of 3 wave sensors: the ASIS buoy, the NDBC 3-meter discus, and the Datawell Directional Waverider. Here we give a short description of each focused on the measurement principals. Further details on performance will be given in the literature review.

4.1 Air-Sea Interaction Spar
Scientist at the University of Miami’s Rosenstiel School for Marine and Atmospheric Science (RSMAS) and Woods Hole Oceanographic Institute (WHOI) developed the Air-Sea Interaction Spar (ASIS) buoy to be a configurable, multipurpose, near surface instrument platform. A spar buoy is constructed to have a particular heave resonance. The ASIS’s heave resonance is typically in the wind-wave spectrum near the lower wind-wave frequencies. It typically has a unity response with waves with a period of ~8s or more depending on the exact configuration. As a result, ASIS rides the lower frequency swell and acts as a stable platform relative to the higher frequencies of the wind wave spectrum. As a semi-stable platform, ASIS is an ideal host for many air-sea instruments and a buoy typically accommodates several varieties of anemometers, radiometers, thermo-coupling devices, pressures sensors, satellite telemetry, and a wave measurement package. The ASIS buoy may function as a drifting platform or tethered to a moored buoy to isolate the sensors from mooring forces (Graber et al., 2000). For the experiments featured in this study, ASIS is typically tethered to a buoy dedicated to this purpose so that the “mooring buoy” absorbs the mooring forces and creates little drag on ASIS.
4.2 ASIS Wave Measurement

To measure waves, ASIS utilizes a combination of systems. One system is a full motion package which records all 6 independent degrees of freedom. The motion package is made up of a 3-axis accelerometer in conjunction with angular rate gyroscopes and a compass. The main wave sensor is an array of 8 capacitance wave wires. The outer array forms a pentagon, the sides of which are about 1 meter long. The inner array forms a triangle with each side about 3cm. The sea surface elevation is recorded at each wire. By incorporating the motion of the buoy, the local elevations at each wire are transformed into sea surface heights. Certain combinations of wave wires may be used to generate directional wave spectra via the Maximum Likelihood Method (MLM) (Capon, 1983; Drennan, Graber, Donelan, & Terray, 1998).
Each wave wire on an ASIS buoy is labeled as follows:

![Diagram of ASIS buoy setup](image)

**Figure 4.2** a) This is a top view diagram of an ASIS buoy setup (not to scale). Each number labels a specific wave wire. b) These are 4 different 3 wire configurations. A minimum of three wires is necessary but not always sufficient to resolve the directional spectrum. On the top are two configurations (green) which would resolve the directional spectrum. On the left is a 2-5-7 configuration and on the right is a 3-4-8. On the bottom are two configurations (red) which would not have directional resolving power. On the left is a 1-3-4 configuration and on the right is a 3-4-6.

It is possible for specific wave wires die or malfunction. When this happens, the remaining wave wires make up a specific configuration. There are many possible wave wire configurations, but only certain configurations which allow for the directional spectrum to be accurately resolved (refer to figure 6b).

### 4.2.1 Coordinate Transformation

Since the wave wires measure in the frame of reference of the buoy, the buoy motion needs to be considered to set the measurements in a stationary frame of reference. This is done through a coordinate transformation. To apply the appropriate coordinate transformation for a wave wire measurement, one must take into account the instantaneous sea surface elevation measured by a wave wire, $z_m$, the vector from the motion package to the surface at the wave wire, $(L_1, L_2, L_3)$, the tilt of the platform, the vertical displacement of the motion...
package, and the vertical displacement between the motion package and the surface due to relative rotation ([COST] Action 714 Working Group 3, 2005). The following equation is used to perform the transformation:

\[
\eta = z_m \cos \theta \cos \varphi + \int \int [a \cdot (-\sin \theta, \cos \theta \sin \varphi, \cos \theta \cos \varphi) - g] \, dt \, dt
\]

\[+ \int \left[ L_2 (-\theta_t \sin \psi + \varphi_t \cos \theta \cos \psi) - L_2 (\theta_t \cos \psi + \varphi_t \cos \theta \sin \psi) \right] \, dt \]

The main components of transformed (corrected) sea surfaces elevation signal are the heave motion of the buoy, the wave wire signal, and rotational motion. The heave signal is dominant for long/low frequency waves and the wave wire signal is dominant for short/high frequency waves (Drennan et al., 1998).

For completeness, any vector can be transformed into an earth-fixed coordinate system. This is done using the a rotation matrix:

\[\mathbf{x}_e = \mathbf{T}_e \mathbf{x}_s\]

\[\mathbf{T}_e = \begin{bmatrix}
\cos \theta \cos \psi & \sin \psi \sin \theta \cos \psi - \cos \phi \sin \psi & \cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi \\
\cos \theta \sin \psi & \sin \psi \sin \theta \sin \psi + \cos \phi \cos \psi & \cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi \\
-sin \theta & \sin \phi \cos \theta & \cos \phi \cos \theta
\end{bmatrix}\]

Where \(\mathbf{x}_s\) is a vector in the buoy reference frame, \(\mathbf{x}_e\) is a the rotated vector in an earth-fixed reference frame, \(\theta\) is pitch, \(\varphi\) is roll, and \(\psi\) is bow heading. All accelerations should be reported in an Earth fixed reference frame to prevent spurious heave signals (Bender & Guinasso, 2010). For more information refer to (Anctil, Donelan, & Drennan, 1994).
4.3 NDBC 3-meter Discus Buoy

Figure 4.3) NDBC 3-meter discus buoy operating in the ocean. Photo credit: www.noaa.ndbc.gov.

The National Oceanic and Atmospheric Association’s (NOAA) National Data Buoy Center (NDBC) operates a network of buoys which collect oceanic and meteorological data. Stations 42036 and 44014 are 3-meter discus buoy which operates as Heave-Pitch-Roll buoys. NDBC has demonstrated a great deal of thought in the development of these buoys and the associated data systems including corrections for electronic, sensor, and hull effects to the hull-measured cross-spectra (Steele, Lau, & Hsu, 1985; Steele, Teng, & Wang, 1992).

4.4 Directional Wave Rider

The Directional Wave Rider (DWR) is a commercial buoy produced by Datawell. These buoys are considered a “standard” because of their wide spread use and validation against other wave measuring instruments. These are
spherical buoys with a radius of ~0.9m. In contrast to discus HPR buoys, DWRs operate principally by recording displacement and are not a reliable platform for meteorological measurements. The north-south displacement, east-west displacement, and vertical displacement are used to calculate directional spectra. They are considered to be very accurate and perhaps the best commercial choice when the experimental objectives are primarily focused on measuring waves (O'Reilly et al., 1996).

Figure 4.4) Datawell directional Waverider. Photo Credit: Ocean Waves Laboratory at the Naval Postgraduate School retrieved from Google images <http://www.oc.nps.edu/wavelab/loc_wave/more_inf.html>.
5 Chapter: Background IV: Literature Review

There are countless papers in the field of wave science. Here the review is restricted to a large comprehensive sensor evaluation (Allender, 1989), evaluations of instruments relevant to this study (Anctil, 1993; Drennan et al., 2003; Graber et al., 2000; O’Reilly et al., 1996; Pettersson et al., 2003), and two papers which form the basis of wave comparisons (Krogstad et al., 1999; Kuik et al., 1988).

5.1 Sensor Comparison Studies

The most comprehensive directional wave sensor resource is from the WAve DIrection Calibration (WADIC) project (Allender, 1989). WADIC took place in the North Sea in the winter of 1985-1986. There was a wide variety of wave conditions up to 10m $H_m\theta$. They compared all of the commercially available directional wave systems to a reference data set. The reference data, referred to as Best Estimate Data Set (BEDS), was compiled by combining the measurements of 5 sub-surface pressure transducer/current meter triplets, a wave wire, and a pentagonal laser array. All of the BEDS sensors were based on the Edda oil platform. BEDS included estimation of the first 5 Fourier coefficients the range of 0.03-0.45Hz in with 0.01Hz bandwidths. The frequency spectra ($A_0$) were based on averaging together the elements of the laser array which passed quality control tests. The directional coefficients from 0.03-0.25 were a combination of the 5 pressure cell/current meter triples, and from 0.11-0.45 the directional coefficients were provided by the laser array (data from the
latter is incomplete due to issues with the lasers). Record lengths were 3 hours and 1 hour during storms, with a sampling interval of 1Hz.

The result was that $H_{m0}$ was estimated robustly for most instruments, except for very high sea states when the stability of the some buoys was compromised. Most sensors produced noisy variance estimates from 0.04-0.1Hz but the BEDS (averaged laser elements) data set was biased high compared to the wave wire and pressure cell. The variance estimates of the most energetic range (from 0.1-0.2Hz) were close to BEDS, and there was general fall off of the variance ratio of the test over BEDS in the high frequencies due to the physical size of the buoys (diameter filtering). In mean wave direction, most sensors fall within $\pm 10^\circ$, but they found a 3-5° bias in the BEDS direction due flow distortion around the platform. At low frequencies, estimate of mean direction from the current meters was less variable than buoy’s estimation. At frequencies above 0.4Hz the estimate direction from BEDS (laser array) seemed to be of a lesser quality than the buoys, using wind direction as a reference. All sensors (including the laser array) greatly overestimated directional spread at all frequencies compared to current meters. For buoys, the bias was said to be due to noise in the pitch-roll signals and reduced directional resolution inherent in the system. For the laser array the bias was perhaps due to slight errors in the position coordinates. The overall conclusion was that most systems reliably report the engineering parameters ($H_{m0}$, $T_z$, and $\theta_p$). There is general poor agreement outside the 0.1-0.2Hz range, and disagreement during extreme sea states. My impression is that the quality of the BEDS was questionable for
frequency dependant measures and nothing can be definitely stated about performance outside of integral parameters. It was noted in (Rademakers, 1993) that stationary sensors (i.e. anything mounted on a stationary platform) would tend to overestimate high frequency energy. In a stationary frame of reference, the high frequency waves riding on low frequency waves would be Doppler shifted by the orbital velocity of the long waves, in effect spreading out the energy to surrounding frequencies. This would not happen for a quasi-Lagrangian surface follower. He disputes the claim in (Allender, 1989) that surface following buoys underestimate energy at high frequencies and attributes the discrepancy to wave elevation time history distortion by the aforementioned effect. The argument is not altogether convincing.

In the summer of 1989 as part of Surface WAve Dynamics Experiment (SWADE), a NDBC 3-meter discus buoy was compared to a reference sensor (Anctil, 1993). Again the reference was a combination of a wave wire and orthogonal current meters at 6-meter depth attached to fixed platform in the Gulf of Mexico. The wave conditions were not as varied as the WADIC experiment. The results were reported for 4 non-directional parameters ($H_{m0}$, $T_p$, $T_z$, and $\nu_f$) and 4 directional parameters ($\theta_p$, $\sigma_p$, $\gamma_p$, $\delta_p$). $H_{m0}$ and $T_p$ were estimated with little or no bias but with increased scatter for the latter parameter. $T_z$ and $\nu_f$ were well correlated but biased high for the former and biased low for the latter. The directional parameters were averaged over the 5 bands adjacent to the peak. The scatter was high in $\theta_p$ and $\sigma_p$. $\gamma_p$ was uncorrelated and $\delta_p$ was often way too high. When the comparison was limited to periods with $H_{m0} > 1$ m, the results
were greatly improved and scatter seen in the parameters of $\theta_p$ and $\sigma_p$ was probably within the sampling variability. No rigorous quantitative comparisons were made of parameters as a function of frequency, but a couple of average plots were shown for variance, mean direction, and directional spread with good qualitative agreement. This adds little to the findings of (Allender, 1989), but showed that the NDBC 3-meter discus buoy performed at least as well as the other buoys in the WADIC study. It also brought up the point that single-point triplets probably do not resolve enough of the directional distribution to accurately report $\gamma_p$ and $\delta_p$.

(O’Reilly et al., 1996) compared a NDBC 3-meter discus buoy and a Datawell Directional Waverider (DWR) to a reference sensor. The paper noted the pitfalls of the previous studies; specifically that previous reference sensors were unreliable or untested, and previous comparisons focused only on peak parameters. The reference sensor in this case was an array of 6 pressure gauges at a 14 meter depth, again on a deep sea oil platform. This array undersampled high frequency waves, so the comparison was restricted to the 0.06-0.14Hz frequency band. They averaged about two and a half hours of data together on the premise that long swell in this band are approximately stationary for this analysis length. They showed that the measurements of the platform array adhered very closely to the linear dispersion relation in this range, confirming the quality of the reference. They compared parameters of variance, mean direction, directional spread, directional skewness, and directional kurtosis calculated with bulk, energy-weighted Fourier coefficients. Both buoys compared
well against the reference in variance and mean direction. The DWR showed significant improvement over the NDBC in directional spread, the NDBC buoy was biased high by 6°. The DWR was also superior in measuring directional skewness and kurtosis where the NDBC showed decreased correlations. There are several things to note. No attempt to look at frequency dependant parameters was made. The comparison for each buoy was performed at different times. Finally, although the estimation of directional skewness and kurtosis was improved over other studies, the parameters compared were bulk, energy-weighted and only for a limited range (0.06-0.14Hz).

(Graber et al., 2000) introduces the ASIS buoy, and compares some trial data with a nearby NDBC 3-meter discus buoy. Only data during a storm was considered; about 4 days of data in April of 1997. After matching the sampling scheme of the NDBC buoy, they compared the wave parameters of $H_{m0}$, $T_p$, and $T_z$. There was good agreement on $H_{m0}$ and $T_p$ with $R^2$ values greater than 0.95, low bias, and low RMSE. They noted that $T_z$ compared well, but the NDBC was biased high over the ASIS buoy which they attributed to the NDBC buoy’s 0.35Hz cut-off frequency. Directional spectra were qualitatively compared, and the estimates looked reasonably close.

The Flux, Etat de la mer et Te´ le´ de´tection en Condition de fetchh variable (FETCH) experiment took place in the Gulf of Lion in the Mediterranean Sea (Pettersson et al., 2003). An ASIS buoy and a Datawell DWR were separated by 2km for about 1 week in March of 1998. They forego the typical MLM processing for ASIS and derive the sea surface slopes directly from the
elevations. Parameters chosen for comparison are $H_{m0}$, $f_p$, $\theta_p$, and $\sigma_p$ where the peak parameters were determined from the values of three frequencies along the peak (presumably an energy weighted average). Again we see excellent agreement in $H_{m0}$ and $f_p$. There is agreement on $\sigma_p$ ($R^2=0.87$, ASIS bias=+3°) with disagreement confined to a singular, unexplained period. Better agreement was found by including spreading from all frequencies above $f_p$ and variance greater than 2% of the peak variance. Little discussion of the agreement on $\theta_p$ was provided except to say it was good and the average difference was 5.8°. Examples of 1- and 2-dimensional spectra were shown along with mean direction and directional spread as a function of frequency. The analysis was qualitative, and the agreement was acceptable.

The volume of work that makes up ([COST] Action 714 Working Group 3, 2005) contains many pertinent studies. Of particular interest was (Drennan et al., 2003). This was essentially an extension to previously described paper. In analysis of the same data, they found no relationship between the two buoys' measurements of surface skewness. There was evidence that the surface elevation skewness of the ASIS, in contrast to the DRW, was correlated with wind speed and wave steepness and supported the relation with steepness proposed by (Srokosz & Longuet-Higgins, 1986). There was an extended discussion on the disagreement of directional spread which will be returned to in a later section. Again there was some qualitative looks at frequency dependant measures (1-D spectra, directional spread).
In the same volume (Hauser et al., 2005) reported a qualitative comparison for directional spread (as a function of frequency) and 1-D spectra between airborne radar and an ASIS buoy. The results were mixed for different case studies, but good agreement meant directional spread was of the same magnitude and trended similarly. They claimed that non-homogeneous and/or non-stationary conditions at ASIS's location could explain some of the differences observed.

5.2 Comparison Methods

There are 2 papers which have set the guidelines for analysis and comparison of wave data. (Kuik et al., 1988) made recommendations on comparing single-point triplet data based on (Longuet-Higgins et al., 1963) style analysis. They derived model free parameters (i.e. do not require further assumptions) and it avoids calculation of directional spectra. (Krogstad et al., 1999) makes recommendations for the comparison and interpretation of directional wave data. The following bulk parameters are considered most central in describing sea-state: $H_{m0}$, $T_p$, $T_m$, $T_z$, $\theta_p$, and $\sigma_p$ and these should be calculated separately for wave systems separated in frequency and/or direction. Comparisons should include an estimate of the sampling variability, so that variability in excess of the sampling variability may be interpreted as real differences between sensors. They recommend a Maximum Likelihood (ML) regression to establish a relationship between data, assuming there are errors in both sensors. They also recommend assessing frequency dependant measures by looking at mean spectral ratios over fixed data ranges, but recommend this be
limited to data above the peak of the spectrum. Much of the details of these two papers were discussed in earlier sections on comparisons and deriving parameters.

5.3 Outlook

Papers which claim to have a “reference sensor” use a sensor(s) mounted on an oil platform. There was little discussion about the platforms’ effect on the measurements (i.e. reflection, flow distortion, etc.) It is even questionable whether or not Eularian and Lagrangian (or quasi-Lagrangian) measurements of the wave field are the same. In (Longuet-Higgins, 1986; Srokosz & Longuet-Higgins, 1986) it was claimed that for narrow band spectra the measurements should be equivalent to second order, but (Magnusson, 1999; Rademakers, 1993) offer different perspectives. Only one study verified the quality of the reference, but then only compared bulk data in a very limited range of frequencies.

Bulk parameters derived from 1-D spectra (i.e. $H_m$, $T_p$, $T_z$) compare well within the available range of sea-states (up to 10m) when sampling variability is accounted for. There is decent qualitative agreement in 1-D spectra for the range 0.10-0.20Hz. Agreement demonstrated outside this range is generally poor, and mean spectral ratios show disagreement in excess of sampling variability. The mean direction at the peak usually agrees within 10°, sensors do not agree on mean direction across the entire frequency range. There is even less agreement with directional spread, and noisy data seems to produce high bias. Disagreement on spread is exacerbated at the peak of the spectrum.
There is little or no agreement between sensors on directional skewness and kurtosis, although the DRW was correlated with a reference for the small range of frequencies considered.

This is a satisfactory state of affairs if you are an engineer who needs reliable design parameters. This is a pitiful state of affairs for an oceanographer/wave scientist interested in understanding the ocean. We do not know with certainty that any wave measuring platform gives the true frequency spectrum over the entire wind-wave\(^2\) frequency range (i.e. 0.03-0.50 Hz). Very little can be said about measurements in sea-states greater than 10m, except for buoys may underestimate extreme waves by dodging waves or getting dragged under. Higher order moments of the surface elevation and directional parameters (especially as a function of frequency) are not well agreed upon, with the exception of \(\theta_p\). There is little information available on higher moments of the sea surface elevation (e.g. skewness, kurtosis).

There are studies which have derived theoretical/empirical spectra and directional distributions. Studies which focused on 1-D studies have proven somewhat robust, but there are disagreements about the exact shape of the spectrum and detailed differences with measurements are difficult to explain. There is more disagreement about the form of the directional distribution, specifically the directional spread. Some authors claim directional spread has a dependence of fetch or inverse wave age, while others do not. Who is correct? Modern wave models use the full spectrum with sources and sinks based on

\(^2\) As an aside, there are scant field measurements of high frequency wind-/gravity-capillary (also called ultra-gravity) waves (> 0.50 Hz) which are important for air-sea coupling and remote sensing.
physical mechanism (or parameterizations). Do the sources and sinks match reality? Remote sensing techniques use direct non-linear transforms of backscatter modulation to produce wavenumber spectra. Are these complicated remote sensing platforms getting it right? The answers to the questions will be based on comparison and evaluation against \textit{in situ} sensors. Until there is a better understanding of wave measurements and differences among in situ sensors, progress with these questions will be difficult.

One step in the right direction is more thorough analysis of the sensors differences as a function of frequency and energy. To better understand inter-frequency differences as well as inter-energy differences an inter-comparison tool is being developed and will be discussed in the following sections.
6 Chapter: Methods

6.1 Data Processing

Each data set, in fact each buoy, required various levels of processing. The archived ASIS data from each experiment/buoy were at seemingly random stages of processing. The archived data from the NDBC and DWR buoy data were standardized (although a different standard for each). The following section outlines the data processing from raw data for an ASIS buoy. The last section explains the processing of NDBC, and DWR.

6.2 ASIS Processing

Several distinct processing steps are required to generate the results from raw data. The first processing step transformed the raw binary data into engineering units. The second step was to quality control each wave wire. The third step calculated wave spectra from the engineering units; the calculation was dependant on the wave wire configuration. The third step derived the first 5 Fourier coefficients and other wave parameters from wave spectra. The wave parameters were then compared using traditional methods. The first 5 Fourier coefficients were printed in the format that WET program requires. The last step was to run the WET program and produce the inter-comparison graphs.
6.2.1 Raw Data

The data was recorded using a custom system built by J. Gabriele from Environment Canada. All data is recorded at a 20 Hz sampling interval. The ASIS_Flux program, coded for MATLAB, was written, improved, and modified over the years by M. Donelan, W. Drennan, N. Williams, and R. Ramos from RSMAS. The program transforms raw binary data from each instrument into voltage 20 Hz time series by taking into account instrumental calibrations and experimental setup. The WET program uses 1 hour bins, so if processing from raw data was necessary, hourly time series files were produced at this stage. Some of the data had previously been processed in 20 or 30 minute increments.
6.2.2 Time Series and Quality Control

The ASIS buoys are not used for operational purposes, therefore there are no statistical quality control measures built into the ASIS_Flux program. The time series of wave wire voltage were used to assess the wave wire individual wave wire quality. Basically, it was noted when the voltage of a wave wire didn’t make physical sense. For example, the variance of the wave wire voltage may have been much larger or smaller compared to other wave wires, there may have been spikes, or the voltage may have flat lined or sky rocketed. The following is the voltages time series of all of the wave wires on Romeo during the SW06 experiment:

![Graph 1](image1)

![Graph 2](image2)

![Graph 3](image3)

![Graph 4](image4)

![Graph 5](image5)

![Graph 6](image6)

![Graph 7](image7)

![Graph 8](image8)

Figure 6.2) These 8 graphs are of the voltages (y-axis) and year-day (x-axis) for the 8 wave wires on the Romeo buoy. Wave wire 1 and 3 remain working for the duration of the experiment, but all others fail at some point (see wave wire timeline).

The quality control was conservative in nature; if at any point a wave wire looked suspicious, the data went unused from that point on. There was one exception: during SW06, there was one hour where a spike occurs in the
voltages of every wave wire on the Yankee buoy. This hour was removed, but the data after the spike was deemed usable. The wave wire time-line sets the wave wire configuration regimes for each buoy. Therefore separately customized runs of the wave spectra calculating program are required for each buoy configuration.

6.2.3 Wave Spectra

The ASIS_Wave program was also written and updated by M. Donelan, W. Drennan, N. Williams, and R. Ramos from RSMAS. There are several inputs to the program that must be determined. The water depth (d), the original sampling frequency (fs), the bin averaging sampling frequency (fa), the angular resolution of the direction spectrum (dtheta), and the number of samples per data block for spectral averaging (ns). For example the settings for SW06 were:

\[ d = 78\text{m}, fs = 20\text{Hz}, fa = 2\text{Hz}, d\text{theta} = 5^\circ, ns = 128 \]

The program first identifies the optimum wave wire configuration from the given possible configurations. Then ASIS_Wave uses a MLM to compute the directional spectrum from the transformed sea surface elevations.

6.3 Other Data Systems

6.3.1 NDBC Data

Data for the NDBC buoys are archived on the NDBC website. The data are available in terms of the NDBC normalized coefficients. Besides a strange factor of 100 (noted on the NDBC website) it is trivial to calculate the normalized Fourier coefficients. Dr. R. Jensen kindly provided quality controlled data for NDBC 42036 during Gom99 and NDBC 41044 during SHOWEX.
6.3.2 DRW Data

The data from DRWs (X1, X2, X3, X4) was provided by Dr. Hans Graber, while the field research facility (FRF) DWR was provided by Dr. R. Jensen. The X buoy data included, 1D spectra, the mean direction and spreading from a1, b1, and the a2 and b2 coefficients. The WET source code was modified to use mean direction and directional spread since the a1, b1 coefficients cannot be recovered from these two parameters alone. The FRF buoy was processed similarly to the NDBC buoys.

6.3.3 The Artificial Data Set

The artificial data is a combination of 26 different JONSWAP spectrums with $H_s$ that vary from .1-16 m and $T_p$ from 2 to 20 s. Each particular spectrum last for two weeks. Gamma is a peak enhancement factor. The MATLAB toolbox Wave Analysis for Fatigue and Oceanography (WAFO) was used to create the data set (more details in the next section) and gamma is chosen automatically. The following table provides WAFO parameters which used to produce each of the spectra:

<table>
<thead>
<tr>
<th>Name</th>
<th>Spectral Shape</th>
<th>Directional Function Cos²</th>
<th>Function-</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>JONSWAP, $H_m0 = 1$, $T_p = 2$, gamma = 1.0000</td>
<td>[15 15 0.52 5 -2.5 0 1 inf]</td>
<td>2 weeks</td>
<td></td>
</tr>
<tr>
<td>S2</td>
<td>JONSWAP, $H_m0 = .25$, $T_p = 3$, gamma = 1.0000</td>
<td>[15 15 0.52 5 -2.5 0 1 inf]</td>
<td>2 weeks</td>
<td></td>
</tr>
<tr>
<td>S3</td>
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<td>[15 15 0.52 5 -2.5 0 1 inf]</td>
<td>2 weeks</td>
<td></td>
</tr>
<tr>
<td>S4</td>
<td>JONSWAP, $H_m0 = .75$, $T_p = 4.5$, gamma = 1.0000</td>
<td>[15 15 0.52 5 -2.5 0 1 inf]</td>
<td>2 weeks</td>
<td></td>
</tr>
<tr>
<td>S5</td>
<td>JONSWAP, $H_m0 = 1$, $T_p = 5$, gamma = 1.0445</td>
<td>[15 15 0.52 5 -2.5 0 1 inf]</td>
<td>2 weeks</td>
<td></td>
</tr>
<tr>
<td>S6</td>
<td>JONSWAP, $H_m0 = 1.5$, $T_p = 5.5$, gamma = 1.5639</td>
<td>[15 15 0.52 5 -2.5 0 1 inf]</td>
<td>2 weeks</td>
<td></td>
</tr>
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<td>[15 15 0.52 5 -2.5 0 1 inf]</td>
<td>2 weeks</td>
<td></td>
</tr>
<tr>
<td></td>
<td>JONSWAP, Hm0 = 2.5, Tp = 6.5, gamma = 2.5431</td>
<td>[15 15 0.52 5 -2.5 0 1] 2 weeks inf</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>-----------------------------------------------</td>
<td>-----------------------------------</td>
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<td></td>
</tr>
<tr>
<td>S9</td>
<td>JONSWAP, Hm0 = 3, Tp = 7, gamma = 2.8013</td>
<td>[15 15 0.52 5 -2.5 0 1] 2 weeks inf</td>
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<td></td>
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<tr>
<td>S10</td>
<td>JONSWAP, Hm0 = 3.5, Tp = 7.5, gamma = 2.9323</td>
<td>[15 15 0.52 5 -2.5 0 1] 2 weeks inf</td>
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<tr>
<td>S11</td>
<td>JONSWAP, Hm0 = 4, Tp = 8, gamma = 2.9694</td>
<td>[15 15 0.52 5 -2.5 0 1] 2 weeks inf</td>
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<td></td>
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<tr>
<td>S12</td>
<td>JONSWAP, Hm0 = 4.5, Tp = 8.5, gamma = 2.9405</td>
<td>[15 15 0.52 5 -2.5 0 1] 2 weeks inf</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S13</td>
<td>JONSWAP, Hm0 = 5, Tp = 9, gamma = 2.867</td>
<td>[15 15 0.52 5 -2.5 0 1] 2 weeks inf</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S14</td>
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<td></td>
</tr>
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<td>S15</td>
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<td>[15 15 0.52 5 -2.5 0 1] 2 weeks inf</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S16</td>
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<td>[15 15 0.52 5 -2.5 0 1] 2 weeks inf</td>
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<td></td>
</tr>
<tr>
<td>S17</td>
<td>JONSWAP, Hm0 = 7, Tp = 10, gamma = 3.0589</td>
<td>[15 15 0.52 5 -2.5 0 1] 2 weeks inf</td>
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<td></td>
</tr>
<tr>
<td>S18</td>
<td>JONSWAP, Hm0 = 7.5, Tp = 10.5, gamma = 3.757</td>
<td>[15 15 0.52 5 -2.5 0 1] 2 weeks inf</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S19</td>
<td>JONSWAP, Hm0 = 8, Tp = 11, gamma = 3.4744</td>
<td>[15 15 0.52 5 -2.5 0 1] 2 weeks inf</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S20</td>
<td>JONSWAP, Hm0 = 8.5, Tp = 12, gamma = 2.5256</td>
<td>[15 15 0.52 5 -2.5 0 1] 2 weeks inf</td>
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<td></td>
</tr>
<tr>
<td>S21</td>
<td>JONSWAP, Hm0 = 9, Tp = 13, gamma = 1.8912</td>
<td>[15 15 0.52 5 -2.5 0 1] 2 weeks inf</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S22</td>
<td>JONSWAP, Hm0 = 9.5, Tp = 14, gamma = 1.4769</td>
<td>[15 15 0.52 5 -2.5 0 1] 2 weeks inf</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S23</td>
<td>JONSWAP, Hm0 = 10, Tp = 15, gamma = 1.2174</td>
<td>[15 15 0.52 5 -2.5 0 1] 2 weeks inf</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S24</td>
<td>JONSWAP, Hm0 = 12, Tp = 17, gamma = 1.0881</td>
<td>[15 15 0.52 5 -2.5 0 1] 2 weeks inf</td>
<td></td>
<td></td>
</tr>
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<td>S25</td>
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<td>[15 15 0.52 5 -2.5 0 1] 2 weeks inf</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S26</td>
<td>JONSWAP, Hm0 = 16, Tp = 20, gamma = 1.0445</td>
<td>[15 15 0.52 5 -2.5 0 1] 2 weeks inf</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.1) Table describing wave parameters for producing the synthetic data set in WAFO.

6.4 Common Processing: First 5 Fourier Coefficients

2-D spectra are functions of frequency and direction. The first 5 Fourier coefficients were determined per frequency band per time interval. These were used to calculate parameters for traditional comparison techniques and also for input into the WET program. The WET conversion codes were written by the author. The code(s) essentially take spectra from a particular source, derives the
first 5 Fourier coefficients using equations from section 4.2.3. The coefficients are then compiled into the proper file input for the WET program.

6.5 WET File Format

The WET program uses a specially formatted file called a .vsp file. It is structured in the following way:

- line 1: Year Month Day Hour Minute TR.UTC n
- line 2: \( f_1 \) bandwidth\((f_1)\) \( A_0(f_1) \) \( A_1(f_1) \) \( B_1(f_1) \) \( A_2(f_1) \) \( B_2(f_1) \) \( \text{cr}(f_1) \)
- line 3: \( f_2 \) bandwidth\((f_2)\) \( A_0(f_2) \) \( A_1(f_2) \) \( B_1(f_2) \) \( A_2(f_2) \) \( B_2(f_2) \) \( \text{cr}(f_2) \)
- ...
- line \( n+1 \): \( f_n \) bandwidth\((f_n)\) \( A_0(f_n) \) \( A_1(f_n) \) \( B_1(f_n) \) \( A_2(f_n) \) \( B_2(f_n) \) \( \text{cr}(f_n) \)
- line \( n+2 \): Year Month Day Hour Minute TR.UTC n
- ...

Where \( A_0(f_i) \) is the wave energy density \((m^2/Hz)\) at frequency \((f_i)\), \( A_1(f_i), B_1(f_i) \), \( A_2(f_i), B_2(f_i) \) are the normalized directional Fourier coefficients at frequency \((f_i)\). \( \text{cr}(f) \) is a check ratio which is not currently used so it is always set to 0. TR.UTC is the time difference between UTC and the local time on the wave record. Since all data is recorded in UTC this is also 0. \( n \) is total number of frequency bands.
Chapter: WET Intercomparison Program

This chapter details what has been learned about the WET program. There is scarce literature involving this technique, and none explaining the details. The motivation of this chapter is to better understand the technique so that it may be more skillfully applied, discover any errors or weaknesses, and make recommendations for improvement. The code for this program is still evolving. Since the start of this thesis, the programs available have been changed and updated and most likely will continue to do so. This represents the author’s understanding of the currently available technology.

The first section is an introduction to the WET program. The second section discusses the different graphs which can be produced. The third section explains the details of the calculations, and the building of the graphs. The fourth section discusses the artificial data set, and the tests performed with the data. The fifth section discusses interpretation of the graphs and provides concluding remarks. Some of the material in this chapter has been informed by the WET manual. For more information please refer to the original document which is available online (CDIP, 2009). Specific formats for input files into the program can be found in the previous section. How to install and run the program can be found in the manual.

7.1 Introduction to WET plots

The wave evaluation tool is a computer program which was developed by W.C. O’Reilly et al at CDIP. This program essentially allows one to intercompare wave spectral parameters derived from two different platforms (i.e
instruments, sensors) assuming the platforms are in roughly the same place at the same time. WET differs from traditional comparison methods in that it looks at differences as a function of frequency and energy. The comparison method is based on the first five Fourier coefficients, also known as the low order directional moments. These are the 5 coefficients which are derived from measurements using the Fourier expansion method (FEM); see the sections 4.2.2 and 4.2.3 for a more detailed discussion. The program uses coefficients in frequency range of 0.03-0.5 Hz with 0.01 Hz bandwidths and averaged over 1 hour durations, reshaping input data when necessary so that both data sets have similar sampling schemes.

There are 2 main plots that one can produce in WET. There are time series and energy-frequency component plots. The time series plots use a variety of bulk parameters. Although there is nothing new about plotting time series, the ability to change the frequency band from which the parameters are derived is a convenient extension. The component plots are a novel presentation. Each component plot applies 1 of 2 difference measures to any one of seven parameters (14 possible component plots). For the analysis, one chooses a “reference” and “test” buoy. If a reference sensor is available the differences may be interpreted as errors. If both buoys are test sensors, then the results are interpreted as relative differences. The design of the plot is such it is visually intuitive to find regions of disagreement. Further details are in the following sections.
7.2 WET Plots

7.2.1 Time Series

There are 5 time series graphs (referred to as ts1-ts5) that can be produced by WET. Each graph is a stack of 3 plots. The reference sensor is plotted in blue and the test sensor is plotted in red. The parameters are either derived from the peak or calculated from the energy weighted bulk directional Fourier coefficients. Each graph shows parameter values for the frequency band of interest. A description of the parameters plotted (from top to bottom) follows.

Ts1 presents wave height, peak period, and mean direction at the peak frequency. Ts2 shows wave energy at the peak frequency, peak frequency, and frequency centroid. Ts3 shows total wave energy with bulk mean wave direction and bulk directional spread calculated from A₁, B₁. Ts4 shows total wave energy with bulk mean wave direction and bulk directional spread calculated from A₂, B₂. Ts5 shows total wave energy with bulk directional skewness and bulk directional kurtosis. It is possible to produce time series for isolated frequency bands (i.e. long swell or wind chop) for all the aforementioned plots.

7.2.2 Number of Concurrent Hourly Observations Component Plot

This graph shows the number of hours of concurrent data in each energy-frequency bin. This plot is based on the data from the reference sensor. For an hour to be recorded in an energy-frequency bin 2 requirements must be met. First requirement: the energy spectrum as measured by the “reference” sensor for a particular hour must lie in the energy-frequency bin. Second requirement: the “test” sensor must be simultaneously operational. It is not required that the two sensors agree on the energy-frequency bins encountered during that hour,
and they typically do not. The total number of hours is added up and reported for each component. This number is used as the "degrees of freedom" for the bias and RSME calculation in each respective bin. A high number of observations should improve the quality of the statistic. A minimum of threshold of 10 hours of observation is set (although this is adjustable).

7.2.3 Component Plots

The relative or absolute bias or standard difference (std) is calculated for average values of parameters in each energy-frequency bin. The seven parameters used in WET component plots are:

1) wave height
2) energy
3) mean wave direction calculated from $A_1$ and $B_1$
4) mean wave direction calculated from $A_2$ and $B_2$
5) directional spread calculated from $A_1$ and $B_1$
6) directional spread calculated from $A_2$ and $B_2$
7) directional skewness

The calculations involved in producing the component plots are detailed in the next section.

7.3 Calculation and Plotting Details

Refer to section 4.2 for descriptions of the calculation of the parameters from coefficients, and the calculation of coefficients themselves. In the original Fortan/C code, the component plots are produced in the following way:

Step 1) Bin average data in time to a 1 hour temporal resolution and bin average in frequency into to .01 Hz bandwidths. Discard data for frequencies lower than .03 Hz and higher than .5 Hz.
Step 2) Synchronize the data from each sensor by looking at the time stamps and build a matrix of energy values. The energy value matrix comes from converting the energy density (i.e. a0) to energy by multiplying by the bandwidth. The energy matrix is a function of time and frequency, i.e. E(hour, Hz). The energy matrix for each sensor has a row for each hour of concurrent observation 48 columns (one for each frequency). The parameter matrices may be calculated at this point.

Step 3) Energy is mapped to a log scale. Making the total matrix energy-frequency bins 17 X 48 with 17 energy bins on a log scale and 48 frequency bins on a linear scale. The energies are generally different for the sensors, so the mapping is unique for each sensor (see number of concurrent hourly observations component plots).

There is an “if statement” about minimum number of hourly observations. This statement requires that the minimum threshold (10 hours) is met for each particular energy-frequency bin according to the reference sensor.

Step 4) If the minimum number of hourly observation is met then bias or std is calculated. Relative measures are used for energy and height (i.e. %b and %std), and absolute measures (i.e. bias and std) are calculated for directional parameters. The bias and std matrices are calculated in following manner where bold letters indicate matrices, n is the number of hourly observations, subscript r is reference data and subscript t is test data. First calculate the differences:

$M_t - M_r = D$
For absolute measures just take the average and standard deviation of $D$ to get bias and std. For relative measures the bias and std need be divided by the reference value:

$$\%b = 100 \times \left( \frac{\sum D}{M_r} \right)$$

and

$$\%std = 100 \times \frac{\sqrt{\sigma}}{\mu_r}$$

where $\sigma = \frac{\sum (D-\mu_D)^2}{n}$ and $\mu_x = \frac{\sum x}{n}$.

The bias and std of mean direction (calculated from $a1,b1$ and from $a2,b2$) should be calculated using the appropriate angular mean and std. This was a change to the source which was implemented in this study.

Step 5) Threshold values of $\%b$ and $\%std$ set the colors of the components. The particular threshold values are different for each parameter, but in general there is a 4 color range. Dark blue marking best agreement, light blue marking good agreement, yellow marking less good agreement, and red marking poor agreement.

### 7.4 Demonstration

This section made up of a few visual demonstrations of the WET program. For an example plot, a wave with 1 meter $H_{\text{rms}}$ and 0.1 Hz (or equivalently a 1 m$^2$, 10 second) component wave would be plotted in this bin:
Figure 7.1) A WET inter-comparison plot with one energy-frequency component highlighted.

The program maps frequency spectra from the reference data set onto the spectral component plot. The following figure is an example of the mapping 1 reference frequency spectrum into spectral component bins.

Figure 7.2) The energy-frequency binning of one particular hourly spectrum.
Once the bins are established from the reference data set, the program calculates the statistic of interest for one of seven parameters. The more observations that occur in a particular energy-frequency band, the better the statistical estimate. For example, here is a plot of the number of observations in each energy-frequency over a two month period:

![Figure 7.3](image)

This figure is the number of coincident hours of observation between two buoys.

Let us now take a look of a real example. The example may be found in (CDIP, 2009). Two buoys, one that uses global positioning satellites (GPS) to obtain displacement and the other is a datawell directional wavewrde, were located outside of Waimea bay, Hawaii during a test deployment. The following figure is a bias component plot for the directional spread as calculated from the first directional moment, $A_1$ and $B_1$:
Figure 7.4) An average directional spread bias component plot.

The numbers in each energy-frequency bin are the calculated bias percentage. The color scale is intended to help the eye identify problem areas. The areas in blue represent small bias (i.e. areas with good agreement between sensors) and the red areas represent high bias. The color scale is arbitrary, but based on typical ranges seen at CDIP. The above graph shows that the bias is generally small at low frequencies and large at high frequencies. This case is interesting because it related to a specific mounting problem on the “test” sensor. Adjusting the mounting on the buoy, or accounting for it during the data processing would eliminate this directional spreading bias in the high frequencies (CDIP, 2009).
7.5 Artificial Data Set and Program Test Results

To better understand and improve interpretation of WET graphs, an artificial data set was built. The data set was varied in controlled ways to try and bring out the peculiarities of the WET program. The artificial data set was designed to cover most of the energy-frequency ranges on the component plot. The data set was built using the Wave Analysis for Fatigue and Oceanography (WAFO) MATLAB toolbox (Brodtkorb et al., 2000). The WAFO control data set was based on JONSWAP spectra (K. Hasselmann et al., 1973) with various $H_s$ and $T_p$. Details about the control data can be found in sections 5.3. Once a control was built, it was varied in the following ways:

- Time offset (0 hours, 2 hours, 20 hours)
- Energy bias (0%, 5%, 10%, 25%, %50, %75, %100)
- Offset of direction distribution (120°)
- Different directional distributions ($\cos^2$, boxcar, poisson, sech)
- Gaussian white noise coefficients (5%, 10%, 20%, %35)

There are plots for every variation, rendering it prohibitive to reproduce all the plots here. There is an appendix at the end which provides all of the plots which may be referred to.

7.5.1 Control

Below the control data graphs are shown for the number of concurrent hourly observations and %b. The control data is referred to as SynthC2 in the graph. These are the baseline (no variation) graphs which represent what you would expect for perfect agreement.
**Figure 7.5** Number of concurrent hourly observations component plot for control data.

**Figure 7.6** Energy bias component plot for control data.
7.5.2 Time Offset

The greatest differences occur as a result of combining two factors. The first is low number of hours in the energy-frequency bin. The second factor, and perhaps more important, is a large difference between measurements placed in adjacent bins. One can see how this combination affects the low-energy low frequency block. As expected, the agreement decays as function of off-set time.

Here is the 20 hour offset %b plot:

![Figure 7.7) Energy percent bias component plot for a time offset of 20 hours.](image)

7.5.3 Energy Bias

The bias component plots essentially give what one would expect, but it becomes less predictable as the bias increases. Giving the most extreme example, a bias of 100% gives:
There are two observations of note. First, nearly half the components are missing. Second, all of the ones that are present report a bias 75%. This was a bizarre finding. It turns out that in the newer MATLAB iteration of the program (which was used for these tests) some calculation details were changed. Specifically some additions were made in steps 3 and 4 listed above.

The %b and %std are also calculated as described above, but then a separate %b and %std are calculated with the reference and test switched. Then the two numbers (i.e. calculated by normalizing by reference and calculated by normalizing with test) are averaged together. This is an averaging of two relative measures, and in the end it represents a measure with a mixed normalization.

For the example above it meant that it was getting 2 numbers:

\[
\% b_r = 100 \times \left( \frac{\sum D_{M_r}}{n} \right) = 100\% \quad \text{and} \quad \% b_t = 100 \times \left( \frac{\sum D_{M_t}}{n} \right) = 50\%
\]
So their average is 75%. These are plotted in the energy/frequency bins that meet the minimum threshold as before with the added requirement that both sensors meet the minimum. This explains the missing components.

In the author’s view, this is nonsensical because it is a combination of numbers normalized by different values. Therefore the author recommends using the MATLAB programs retro-fitted with the original versions of the calculations described above.

7.5.4 Directional Offset

This is the most straightforward of the artificial data comparisons. The comparison using $A_1$, $B_1$ is exactly what one would expect for the given a $210^\circ$ ($-150^\circ$) directional offset. The comparison using $A_2$, $B_2$ gives $30^\circ$, which is exactly $180^\circ$ different than the actual offset. This is because there is a $180^\circ$ ambiguity when calculating mean direction from $A_2$, $B_2$ (see section 4.2.4). Apparently the program does not resolve the ambiguity, and this may confuse the interpretation of mean direction plots calculated from $A_2$, $B_2$.

7.5.5 Directional Distributions

The directional distributions made are constant for every sea-state. Therefore all differences are a function of frequency only. Such consistency is never found over a range of sea-states in a field experiment. These plots reveal that very different comparisons can be expected for spread calculated from $A_1$, $B_1$ and spread calculated from $A_2$, $B_2$.

7.5.6 Noise

Noise in the coefficients seems to have different effects on the parameters. The noise in $A_0$ only affects the energies (and wave heights) that
are similar in magnitude to the noise. It was the author's hypothesis that the noise would not change the bias, but this is not the case. For example %b and noise with 5% of the total variance:

Figure 7.9) Energy percent bias component plot for 5% noise in the A₀.

The bias remains unchanged for only some higher energy components and there is a negative bias stripe that runs diagonally through the graph. The %std of energy exhibits a similar color pattern with slightly larger numbers. Noise in the directional coefficients seems to affect A₁, B₁ mean wave direction only at the high and low frequencies, where the A₂, B₂ mean direction is affected at low frequencies and frequencies around .25 Hz. The A₁, B₁ directional spread is affected at all frequencies but more so in the range of .05-.23 Hz. A₂, B₂ directional spread shows little effect of noise. For all graphs, the agreement decayed with increasing noise, and the %std was more strongly effected than
%b. All of these patterns are strange, and a reasonable explanation does not come to mind. Noise in coefficients is not straightforward issue in the program.

### 7.5.7 General Remarks and Recommendations

Evidently, it is crucially important which file is chosen as the reference and which is chosen as the test. The structure of the comparison is based on the unique mapping of the reference sensor data. A vs B has no simple relation to B vs A. Here is an example of the synthetic data set hourly observations. The first has the control as a reference and the second uses the noisy data set (35%) as the reference.

![Figure 7.10) Number of concurrent hourly observations component plot with control data as reference sensor.](image-url)
Therefore it is important that there is some confidence in one of the sensors involved in the comparison, otherwise the relative differences have no context. If both sensors are considered test (or have measurements of equivalent quality), it may be beneficial to check both A vs B and B vs A.

As seen in the case of time offset, differences may occur as a result of low number of hours in the energy-frequency bin and a large difference between measurements placed in adjacent bins (or misplaced energy bins). This artifact turns up in many plots as poor agreement along the high-energy high-frequency edge and should be interpreted with caution.
The following additions/changes, some of which have been implemented for this study, are recommended for improvement of the program:

- The new version of the program uses calculations of bias and std which are not correct. Use the bias and std calculations from the old version of the program.

- Change the calculation of directional bias and std to the appropriate angular calculations.

- Many plots show high std, but is this in excess of sampling variability? There should be a way to use estimated sampling variability so that variability outside of the sampling variability should be reported.

- The colors representing good and bad agreement are useful, but should be fine tuned. Perhaps a gradient of colors should be implemented.

- Resolve the ambiguity in mean wave direction from $A_2$, $B_2$
Chapter: GOM99

As was mentioned in the background chapter, differences between sensors can stem from multiple sources. Differences that exceed the sampling variability of the parameter indicate either natural variability of the ocean or errors in the sensors. So, it is important to estimate the sampling variability of the parameters you compare. The first section introduces the experiment. A detailed sensor comparison is described in section 2. A 2 day period where the wave parameters are nearly stationary is identified. The third section details the observation of a slanting fetch and the estimation of sampling variability of wave parameters during the stationary period. The last section concludes the chapter.

8.1 Introduction to GOM99

The Gulf of Mexico experiment in April and May of 1999 (GOM99) was an Office of Naval Research (ONR) initiative for "Measurements of Atmospheric and Oceanic Parameters Affecting Brightness Temperature in Passive Microwave Radiometry". The experiment took place off the west coast of Florida. Three ASIS buoys surrounded the three meter discus NDBC station 42036. NDBC Station 42036 was a fully operational at the time so the data is available on the NOAA/NDBC website. Spacing between ASIS buoys was about 15km and the mooring depth was around 55m for all buoys. The following figure (Figure 8.1) shows the location of the four buoys involved in the experiment as well as other NOAA stations in the area. One can see that the gulf shelters the four buoys from open Atlantic swells so that one would expect to observe primarily small wind-seas.
In the figure above the ASIS buoy labeled A1 will be referred to as Yankee and was located at 28.33.44 °N and 84.27.25 °W, the ASIS buoy labeled A2 will be referred to as Bravo and was located at 28.25.04 °N and 84.30.28 °W, and the ASIS buoy labeled A3 will be known as Romeo and was located at 28.31.80 °N and 84.34.10 °W. The buoy in the center is NDBC station 42036 and was located at 28.500 °N and 84.517 °W. The approximate distances between each buoy and their respective locations are listed in the table below.

![Figure 8.1) Map of GOM99 experiment area. The ASIS buoys are labeled A1, A2, and A3. The ASIS buoys surround NDBC station 42036.](image)

<table>
<thead>
<tr>
<th>Buoy</th>
<th>Lat. [° N]</th>
<th>Long. [° W]</th>
<th>Distance [km]</th>
</tr>
</thead>
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<td>Bravo</td>
<td>28.4178</td>
<td>84.5078</td>
<td>0</td>
</tr>
<tr>
<td>Yankee</td>
<td>28.5622</td>
<td>84.4569</td>
<td>16.81</td>
</tr>
<tr>
<td>Romeo</td>
<td>28.5389</td>
<td>84.5694</td>
<td>14.75</td>
</tr>
<tr>
<td>42036</td>
<td>28.500</td>
<td>84.517</td>
<td>9.18</td>
</tr>
</tbody>
</table>

Table 8.1) The location of each buoy and their relative distances to one another.
The NDBC buoy collected data continuously throughout the experiment. The ASIS buoys were plagued with large data gaps due to battery assembly problems. The conditions were calm (Hs<~2m) for the entire experiment duration. Although the conditions do not allow evaluation over a large range of sea-state conditions it is of interest to compare the wave measurements. The close proximity of the buoys provides a unique opportunity to evaluate the buoy performances relative to one another. Also there was a short period where the environmental conditions were relatively stationary. During this time the sampling variability of wave parameters can be assessed.

8.2 Experimental Conditions and Qualitative Comparisons

Figure 8.2) The figure above shows the conditions during the GOM99 field experiment.
In figure 8.2, $U_{10}$ is the wind speed converted from wind speed measured at 5 meters and $U_{10}/C_p$ is the inverse wave age as measured by NDBC 42036. The inverse wave age has been scaled by 5 for a visual aid, and the dotted line represents a division between wind sea and swell. Wind Dir. is the direction of the wind (meteorological convention), $H_{m0}$ is the significant wave height, $\theta_p$ is the wave direction at the spectral peak (meteorological convention), and Temperature shows the air and sea surface temperature.

The wind and waves were relatively mild throughout the experiment. The wave parameters are shown as measured by each buoy: Bravo in blue, Romeo in red, Yankee in green, and NDBC 42036 in black. These color conventions will remain consistent throughout the study.

Figure 8.3) The from top to bottom the spectral evolution is plotted for Bravo, Romeo, Yankee, and NDBC station 42036.
The plot above shows the evolution of the frequency spectra for each buoy. The plots have been smoothed in time and frequency space as a visual aid. Energy, represented by color, is on a natural log scale so that the number on the color scale is the exponent. Where the buoys have simultaneous measurements the buoys agree quite well. The NDBC buoy has a high frequency cutoff at .35 Hz.

![GOM99 Directional Evolution](image)

**Figure 8.4** From top to bottom directional distribution evolution is plotted for Bravo, Romeo, Yankee, and NDBC station 42036. Direction in radians is on the y-axis, year day on the x-axis.

The plot above is the directional evolution of the wave spectrum. This plot best elucidates the data gaps for the ASIS buoys. The directional spectrum for the NDBC buoy was produced with a simple Longuet-Higgins weighted Fourier addition. It is known that this method artificially broadens the spectral width. The ASIS directional spectra are produced using a Maximum Likelihood Estimation
method which was designed for processing these buoys. The directions at the peak seem to agree well.

8.3 Spectral Parameters and Quantitative Comparison

8.3.1 Calculation of Parameters

Four spectral parameters were chosen for an in depth comparison: significant wave height ($H_{m0}$), frequency at the spectral peak ($f_p$), direction at the spectral peak ($\theta_p$), and directional spread at the peak ($\sigma_p$). $H_{m0}$ was calculated in the usual way from the first spectral moment:

$$H_{m0} = 4\sqrt{m_0} = 4 \sqrt{\int_0^\infty S(f) df}$$

$f_p$ is calculated using a robust estimation:

$$f_p = \frac{\int_0^\infty f S^4(f) df}{\int_0^\infty S^4(f) df}$$

Direction at the peak and directional spread at the peak are calculated with the fourth power energy weighted Fourier coefficients as described in section 4.2.1:

$$\theta_p = \text{atan2}(B_{1b4}, A_{1b4})$$

$$\sigma_p = \sqrt{2 \left(1 - \sqrt{A_{1b4}^2 + B_{1b4}^2}\right)}$$
8.3.2 Parameter Time Series:

The four parameters chosen for a closer examination are \( H_{m0} \), \( f_p \), \( \theta_p \), and \( \sigma_p \). \( H_{m0} \) and \( f_p \) compare well for all buoys, and there is relatively little scatter. There is much more scatter in the wave directions at the peak although there is still good agreement. Furthermore, the ASIS buoys were processed in 20 minute blocks, whereas the NDBC was processed in 40 minute blocks reported hourly. The reduced scatter in the NDBC \( \theta_p \) may be due to the longer averaging and as a result the NDBC signal appears to be a smoother version of the ASIS signals. The ASIS buoys qualitatively agree on the directional spread, but they do not agree with the NDBC buoy. Possible explanations will be discussed below.
8.3.3 Parameter Scatter Plots and Statistics

The following scatter plots show one-on-one comparisons between each buoy. A Maximum Likelihood (ML) regression is used to produce a fit because all buoys are assumed to have error in their measurements (Krogstad et al., 1999).

8.3.3.1 Significant Wave Height

![Scatter plots for Hm0 during GOM99.](image)

<table>
<thead>
<tr>
<th>Comparison/Stat Hm0</th>
<th># pts</th>
<th>R²</th>
<th>Slope</th>
<th>Interc.[m]</th>
<th>Bias</th>
<th>Std [m]</th>
<th>RMSE[m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bravo/Yankee</td>
<td>288</td>
<td>0.943</td>
<td>0.982</td>
<td>-0.025</td>
<td>-0.015</td>
<td>0.059</td>
<td>0.061</td>
</tr>
<tr>
<td>Bravo/Romeo</td>
<td>1487</td>
<td>0.982</td>
<td>1.118</td>
<td>-0.061</td>
<td>-0.021</td>
<td>0.099</td>
<td>0.101</td>
</tr>
<tr>
<td>Yankee/Romeo</td>
<td>576</td>
<td>0.953</td>
<td>0.982</td>
<td>-0.022</td>
<td>0.033</td>
<td>0.059</td>
<td>0.067</td>
</tr>
<tr>
<td>Bravo/NDBC 42036</td>
<td>1909</td>
<td>0.982</td>
<td>1.101</td>
<td>-0.040</td>
<td>-0.026</td>
<td>0.090</td>
<td>0.093</td>
</tr>
<tr>
<td>Yankee/NDBC 42036</td>
<td>691</td>
<td>0.940</td>
<td>0.955</td>
<td>-0.005</td>
<td>0.032</td>
<td>0.067</td>
<td>0.074</td>
</tr>
<tr>
<td>Romeo/NDBC 42036</td>
<td>2025</td>
<td>0.984</td>
<td>0.985</td>
<td>0.019</td>
<td>-0.007</td>
<td>0.084</td>
<td>0.084</td>
</tr>
</tbody>
</table>

Table 8.2 The table above describes the goodness of fit parameters as well as the statistical difference measures for Hm0. The statistical difference measures are bias, standard deviation (std), and root-mean-square-error (RMSE). Calculation of various statistics is described in section 4.1.1. The remaining column headers above are the following: # pts is the number of data points used in
the comparison, $R^2$ is the correlation coefficient, slope is the slope of the ML regression fit, and interc. is the intercept of the ML regression fit.

The consensus of previous studies is that $H_{m0}$ tends to agree well among different sensors, so, as expected, all buoys compare well on this parameter. There is some scatter, but it is not known quantitatively if this scatter is within what is expected due to the sampling variability of the parameter. The data from this experiment will be used to define sampling variability in later experiments. High $R^2$ values and slope fit parameters not significantly different from 1 are sufficient criteria for good agreement.

**8.3.3.2 Peak Frequency**

![Figure 8.7] Scatter plots for $f_p$ during GOM99.

<table>
<thead>
<tr>
<th>Comparison/Stat for $f_p$</th>
<th># pts</th>
<th>$R^2$</th>
<th>slope</th>
<th>Inter.[Hz]</th>
<th>Bias[Hz]</th>
<th>Std [Hz]</th>
<th>RMSE[Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bravo/Yankee</td>
<td>288</td>
<td>0.839</td>
<td>0.983</td>
<td>0.003</td>
<td>0.000</td>
<td>0.020</td>
<td>0.020</td>
</tr>
<tr>
<td>Bravo/Romeo</td>
<td>1487</td>
<td>0.940</td>
<td>0.968</td>
<td>0.007</td>
<td>0.000</td>
<td>0.018</td>
<td>0.018</td>
</tr>
<tr>
<td>Yankee/Romeo</td>
<td>576</td>
<td>0.882</td>
<td>0.949</td>
<td>0.012</td>
<td>-0.002</td>
<td>0.017</td>
<td>0.017</td>
</tr>
</tbody>
</table>
Table 8.3) The table above describes the goodness of fit parameters as well as the statistical difference measures for \( f_p \). The statistical difference measures are bias, standard deviation (std), and root-mean-square-error (RMSE). Calculation of various statistics is described in section 4.1.1. The other column headers above are the following: # pts is the number of data points used in the comparison, \( R^2 \) is the correlation coefficient, slope is the slope of the ML regression fit, and interc. is the intercept of the ML regression fit.

Given previous studies, agreement among buoys on peak frequency is expected to be high. The agreement is sufficiently high to meet this expectation. The \( R^2 \) values are not quite as high as those for \( H_m0 \), but the slopes of the ML fit are not sufficiently different from 1. The decreased correlation is due to increased sampling variability inherent to the peak frequency parameter as well as other factors which will be discussed below.

8.3.3.3 Direction at the Peak

Figure 8.8) Scatter plots for \( \theta_p \) during GOM99.

<table>
<thead>
<tr>
<th>Comparison/Stat for WD</th>
<th># pts</th>
<th>( R_0 )</th>
<th>Slope</th>
<th>Interc. [°]</th>
<th>Bias [°]</th>
<th>Std [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bravo/NDBC 42036</td>
<td>1909</td>
<td>0.899</td>
<td>0.935</td>
<td>0.014</td>
<td>-0.001</td>
<td>0.024</td>
</tr>
<tr>
<td>Yankee/NDBC 42036</td>
<td>691</td>
<td>0.826</td>
<td>0.923</td>
<td>0.015</td>
<td>0.000</td>
<td>0.021</td>
</tr>
<tr>
<td>Romeo/NDBC 42036</td>
<td>2025</td>
<td>0.893</td>
<td>0.944</td>
<td>0.011</td>
<td>0.001</td>
<td>0.022</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Comparison/Stat for WD</th>
<th># pts</th>
<th>( R_0 )</th>
<th>Slope</th>
<th>Interc. [°]</th>
<th>Bias [°]</th>
<th>Std [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bravo/NDBC 42036</td>
<td>1909</td>
<td>0.899</td>
<td>0.935</td>
<td>0.014</td>
<td>-0.001</td>
<td>0.024</td>
</tr>
<tr>
<td>Yankee/NDBC 42036</td>
<td>691</td>
<td>0.826</td>
<td>0.923</td>
<td>0.015</td>
<td>0.000</td>
<td>0.021</td>
</tr>
<tr>
<td>Romeo/NDBC 42036</td>
<td>2025</td>
<td>0.893</td>
<td>0.944</td>
<td>0.011</td>
<td>0.001</td>
<td>0.022</td>
</tr>
<tr>
<td>Combination</td>
<td># pts</td>
<td>Rθ</td>
<td>slope</td>
<td>interc.</td>
<td>Rθ</td>
<td>48.84</td>
</tr>
<tr>
<td>----------------------</td>
<td>-------</td>
<td>----</td>
<td>---------</td>
<td>---------</td>
<td>------</td>
<td>-------</td>
</tr>
<tr>
<td>Bravo/Yankee</td>
<td>288</td>
<td>0.637</td>
<td>0.193</td>
<td>172.30</td>
<td>0.97</td>
<td></td>
</tr>
<tr>
<td>Bravo/Romeo</td>
<td>1487</td>
<td>0.812</td>
<td>0.843</td>
<td>38.30</td>
<td>2.99</td>
<td>35.12</td>
</tr>
<tr>
<td>Yankee/Romeo</td>
<td>576</td>
<td>0.853</td>
<td>1.926</td>
<td>-206.58</td>
<td>6.43</td>
<td>30.96</td>
</tr>
<tr>
<td>Bravo/NDBC 42036</td>
<td>1909</td>
<td>0.814</td>
<td>0.887</td>
<td>21.99</td>
<td>4.18</td>
<td>34.90</td>
</tr>
<tr>
<td>Yankee/NDBC 42036</td>
<td>691</td>
<td>0.959</td>
<td>0.980</td>
<td>-0.02</td>
<td>6.09</td>
<td>16.39</td>
</tr>
<tr>
<td>Romeo/NDBC 42036</td>
<td>2025</td>
<td>0.883</td>
<td>0.912</td>
<td>5.35</td>
<td>1.61</td>
<td>27.76</td>
</tr>
</tbody>
</table>

Table 8.4): The table above describes the goodness of fit parameters as well as the statistical difference measures for $\theta_p$. The statistical difference measures are directional bias and directional standard deviation. The column headers above are the following: # pts is the number of data points used in the comparison, $R_\theta$ is the directional association, slope is the slope of the ML regression fit, interc. is the intercept of the ML regression fit.

Given previous studies the agreement for mean direction at the peak is expected to be fairly consistent, but with significant scatter. Most studies find a directional bias of ± 5°, and our findings are in agreement with the consensus from previous studies. The overall agreement is quite good, indicated by high $R_\theta$ values, but there are some slopes of the ML fit which are significantly different than 1. The poorest agreement is between Bravo and Yankee. This is probably due to 2 major reasons: 1) there is a relatively small number of data points for the comparison meaning a relatively limited number of environmental conditions, 2) there may have been multimodal directional distribution during that limited time. All ASIS buoys show multiple peaks in the directional distribution around Year Day 95, and for reasons unknown Bravo is especially sensitive to multiple peaks (not shown).

Multi-modal seas in direction space (which the NDBC cannot resolve) and/or frequency space with similar energies in each mode are a possibility throughout the experiment. The multi-modality seas cause parameters derived from the peak of the spectrum to jump back and forth between peaks. The jumping causes comparisons to be poor and muddles the interpretation of
results. It is suspected that multi-modal seas occur sporadically throughout the experiment, but this is speculation until it can be investigated further. To properly deal with multi-modal seas the spectra should be partitioned, but this is beyond the scope of this study. Furthermore, it suffices to note that some of the variability in the comparisons of peak frequency, mean direction at the peak, and directional spread at the peak is due to multi-modality.

8.3.3.4 Directional Spread at the Peak

![Scatter plots for \( \sigma_p \) during GOM99](image)

<table>
<thead>
<tr>
<th>Comparison/Stat for WD</th>
<th># pts</th>
<th>( R_0 )</th>
<th>( R^2 )</th>
<th>Slope</th>
<th>Interc. [°]</th>
<th>Bias [°]</th>
<th>Std [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bravo/Yankee</td>
<td>288</td>
<td>0.988</td>
<td>0.355</td>
<td>0.361</td>
<td>32.72</td>
<td>8.50</td>
<td>8.89</td>
</tr>
<tr>
<td>Bravo/Romeo</td>
<td>1487</td>
<td>0.990</td>
<td>0.623</td>
<td>0.976</td>
<td>-1.27</td>
<td>2.63</td>
<td>8.26</td>
</tr>
<tr>
<td>Yankee/Romeo</td>
<td>576</td>
<td>0.992</td>
<td>0.356</td>
<td>2.122</td>
<td>-58.87</td>
<td>-2.59</td>
<td>7.30</td>
</tr>
<tr>
<td>Bravo/NDBC 42036</td>
<td>1909</td>
<td>0.981</td>
<td>0.386</td>
<td>1.308</td>
<td>-43.84</td>
<td>25.97</td>
<td>11.30</td>
</tr>
<tr>
<td>Yankee/NDBC 42036</td>
<td>691</td>
<td>0.987</td>
<td>0.015</td>
<td>7.100</td>
<td>-357.81</td>
<td>24.79</td>
<td>9.06</td>
</tr>
<tr>
<td>Romeo/NDBC 42036</td>
<td>2025</td>
<td>0.985</td>
<td>0.452</td>
<td>1.157</td>
<td>-31.74</td>
<td>23.04</td>
<td>9.95</td>
</tr>
</tbody>
</table>

Table 8.5) The table above describes the goodness of fit parameters as well as the statistical difference measures for \( \sigma_p \). The statistical difference measures are directional bias and directional standard deviation. The column headers above are the following: # pts is the number of data points.
used in the comparison, $R_\theta$ is the directional association, $R^2$ is the correlation coefficient, slope is the slope of the ML regression fit, interc. is the intercept of the ML regression fit.

Between the two association measures, the correlation coefficient better represents the agreement of this data. Given this, the agreement was poor. Among ASIS buoys the agreement was somewhat improved, but all ASIS/NDBC comparisons reported a consistent $25^\circ$ bias. The reasons for these discrepancies are unclear and will be investigated further.

8.3.4 WET Plots

8.3.4.1 Wave Height

![Wave Height Percent Bias Component Plot](image)

Figure 8.10) Wave height percent bias component plot for Bravo relative to NDBC 42036 during GOM99.

The numbers above are normalized by the measurements so that they represent average relative error or percent bias (%b). Although the buoys’ measurement of significant wave height is consistent for the bulk parameterization of wave height, $H_{m0}$, the WET wave height %b component plot reveals that there were frequency dependent differences. Relative to Bravo, the
NDBC buoy overestimated in the high frequencies and underestimated in the low frequencies. The graph of Romeo vs NDBC 42036 exhibited the same feature. The wave height relative standard difference (%std) component plot is shown directly below. The %std was low over all frequencies but increased slightly as wave height decreased (probably due to low signal to noise ratio).

Figure 8.11) Wave height %std component plot for Bravo relative to NDBC 42036 during GOM99.

Romeo vs Bravo showed a more consistent wave height measure across frequency:

Figure 8.12) Wave height percent bias component plot for Bravo relative to Romeo during GOM99.
Higher bias was limited to the region along the high-frequency, high-energy edge. This may have been an artifact of low number of observations and misplaced energy bins. The %std between Romeo and Bravo (not shown) was low across the board.

8.3.4.2 Mean Direction and Directional Spread

The graphs for mean direction bias and standard deviation (std) (calculated from A₁, B₁) were remarkably similar for all buoy comparisons. The numbers for directional bias were quite low for all buoys, but the relative standard deviation was quite high especially in the low energy, low frequency bins:

![Figure 8.13](image)

**Figure 8.13** Mean direction from A₁, B₁ bias component plot for NDBC 42036 relative to Romeo during GOM99.

Higher directional std in the lower frequencies was probably due to noise.

The directional spread bias is low for Bravo vs Romeo and did not show a pattern over frequency or energy space, but the std did show an interesting pattern. It seems that the variance in that parameter increased in the mid energy – mid
frequency ranges. This may be because outside of this range the buoys reported consistently high numbers.

Predictably, the bias was high for all ASIS-NDBC directional spread plots. For example:
The agreement was good at either end of the frequency extremes, where presumably all buoys report high directional spread. A similar pattern was found with std of directional spread between these two buoys.

8.3.5 Case Study 1: Simultaneous Measurements Comparison

To better understand parameter differences among buoys as a function of frequency, a case study is performed when all 4 buoys were operational. This happened during a 4 day period, Year Day 93-97, near the beginning of the experiment. The plot below summarizes the conditions during this time.

![Environmental and Conditions](image)

**Figure 8.16** Air-sea parameters from year day 93 to year day 97. From top to bottom $U_{10}$, $U_{10}/C_p$, and swell-wind sea division from NDBC 42036, wind direction from NDBC 42036, significant wave height, direction at the peak, and spreading at the peak from all buoys, and air and sea temperature from NDBC 42036.

The mean and std of wave related parameters during that period, as recorded by each buoy, is in the table below:
<table>
<thead>
<tr>
<th>Param./Buoy</th>
<th>Bravo (mean±std)</th>
<th>Romeo (mean±std)</th>
<th>Yankee (mean±std)</th>
<th>NDBC 42036 (mean±std)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H_m0 [m]</td>
<td>0.543±0.160</td>
<td>0.5270±0.1522</td>
<td>0.5547±0.1496</td>
<td>0.5412±0.1644</td>
</tr>
<tr>
<td>f_p [Hz]</td>
<td>0.1733±0.0351</td>
<td>0.1769±0.0319</td>
<td>0.1715±0.0338</td>
<td>0.1725±0.0324</td>
</tr>
<tr>
<td>θ_p [°]</td>
<td>210.9±52.0</td>
<td>204.1±34.2</td>
<td>209.5±21.0</td>
<td>200.2±18.3</td>
</tr>
<tr>
<td>σ_p [°]</td>
<td>64.4±9.5</td>
<td>59.4±7.9</td>
<td>55.3±6.3</td>
<td>26.8±6.0</td>
</tr>
<tr>
<td>C_p [m/s]</td>
<td>9.26±1.30</td>
<td>9.04±1.22</td>
<td>9.35±1.31</td>
<td>9.28±1.24</td>
</tr>
<tr>
<td>U_10/C_p</td>
<td>0.349±0.238</td>
<td>0.359±0.236</td>
<td>0.348±0.244</td>
<td>0.351±0.238</td>
</tr>
</tbody>
</table>

Table 8.6) The mean and std of 6 parameters from each buoy for year day 93 to year day 97.

Below we plot frequency dependent parameters of variance, mean direction, and directional spread averaged for this period:

![Average Frequency Spectra for Julian Day 93-97](image)

Figure 8.17) Average frequency spectra for each buoy for year day 93 to year day 97. Bravo, Romeo, Yankee, and NDBC station 42036 are in blue, red, green, and black, respectively. From top to bottom: linear scale, log scale, and linear scale normalized by average spectrum from Bravo.

Each instrument gives a slightly different impression of the average spectra for this period. The bottom plot shows the spectra normalized by Bravo’s average spectra. The bottom plot also shows that in the frequency range from .05-.1 Hz the NDBC buoy consistently reported lower energies, and for the frequency range above .15 Hz the NDBC buoy consistently reported higher energies. This graph reinforces what was shown in the WET wave height bias
The mean variance (and Hm0) for the frequency range 0.03-0.35 was calculated for each buoy during this time period and is listed in the table below:

<table>
<thead>
<tr>
<th>Buoy/Data Header</th>
<th>Number (Length) of Spectra</th>
<th>Mean Variance (Hm0) [0.03-0.35 Hz]</th>
<th>% Diff. Mean Var (Hm0) [rel. to Bravo]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bravo</td>
<td>236 (20 min)</td>
<td>0.0180 (0.537)</td>
<td>0.0% (0.0%)</td>
</tr>
<tr>
<td>Romeo</td>
<td>246 (20 min)</td>
<td>0.0170 (0.522)</td>
<td>-5.6% (-0.5%)</td>
</tr>
<tr>
<td>Yankee</td>
<td>240 (20 min)</td>
<td>0.0180 (0.537)</td>
<td>-0.1% (0.0%)</td>
</tr>
<tr>
<td>S42036</td>
<td>84 (40 min)</td>
<td>0.0200 (0.565)</td>
<td>10.7% (0.9%)</td>
</tr>
</tbody>
</table>

Table 8.7) Number of spectra averaged, mean variance (significant wave height) from 0.03-0.35 Hz, and the percent difference relative to Bravo of mean variance (significant wave height).

The NDBC shows there was significant bias in energy in the frequency range considered. This shows 1) that NDBC’s relative underestimation of the energy at low frequencies was not consequential due to the lack of energy in these bands, and 2) that the NDBC’s relative overestimation at higher frequencies was just enough to compensate for the underestimation at both lower frequencies and (perhaps more importantly) the cut off at 0.35 Hz.

![Average Mean Direction for Year Day 93-97](image)

Figure 8.18) Top plot: average mean direction for all buoys year day 93 to year 97. Bravo, Romeo, Yankee and NDBC station 42036 are in blue, red, green, and black, respectively. Bottom plot is mean direction relative to Bravo.
The agreement in direction was quite good above 0.13 Hz, although NDBC had tendency to report a slightly more east direction. Below 0.13 Hz there was a notable discrepancy between the ASIS buoys and the NDBC buoy. As can be seen in the average variance plot, these frequencies were below the peak, and there was not a lot of energy in these bands. It is known that NDBC routinely applies low frequency noise corrections (Earle, 1996), but it is unclear exactly how this may manifest in wave parameters. The directional differences reported in this range may be due to different methods of handling low frequency noise. This would explain why the ASIS buoys tend to agree. One last note is that even if these are assumed to be real, it is difficult to believe that swell (even with very little energy) would be coming from the south (as reported by the ASIS buoys) let alone the east (as reported by the NDBC buoy).

Figure 8.19) Top plot: Average directional spread for all buoys from year day 93 to year day 97. Bravo, Romeo, Yankee, and NDBC station 42036 are in blue, red, green, and black, respectively. Bottom plot is directional spread normalized by directional spread from Bravo.
The plot above shows the average directional spread as a function of frequency. The NDBC buoy consistently showed a lower directional spread than the ASIS buoys. The difference was especially pronounced around and just below the spectral peak (~0.15 Hz). It is unsettling that the difference is most significant where there is energy, and the differences are not reconcilable, i.e. one platform is clearly wrong. Also, it is common to compare parameters at the peak, and looking at only the peak spreading would give an especially poor impression of the directional spread comparison. To investigate the matter further, plotted below is directional spread for each data point (20 min for ASIS and 1 hour for NDBC) as a function of non dimensional frequency along with the respective averages.

Figure 8.20) Directional spread as a function of frequency normalized by the peak frequency. From top to bottom is Bravo, Romeo, Yankee, and NDBC station 42036. The solid black line is the average, the dashed line is Mitsuyasu’s empirical relationship, and the dash-dotted is Hasselman’s empirical relationship. The magenta triangle marks the frequency of the average lowest spread.
The solid black line is the average of all the directional spreads. The dashed and dash-dotted line is the empirical directional spreads given by (Mitsuyasu et al., 1975) and (D. E. Hasselmann, Dunckel, & Ewing, 1980), respectively. Both of these forms have a dependence on inverse wave age although Mitsuyasu’s relation has a much stronger dependence. The inverse wave age was about 0.35 on the average. Mitsuyasu used data from a cloverleaf buoy and Hasselmann from a HPR buoy. This may partly explain the exceptionally good fit of the Hasselmann relation to the data from the NDBC buoy. Using an array of wave wires, the directional spread of (Donelan, Hamilton, & Hui, 1985) (not shown) is even more narrow than the 2 relations shown above, but all 3 studies found the minimum directional spread near the peak frequency (more specifically just before it). The minimum spread (magenta triangle) was near the peak frequency for the NDBC buoy and well above it for the ASIS buoys. The evidence suggests that the spread reported by the NDBC buoy should be trusted over ASIS. It should be noted that the empirical relations were derived during fetch-limited conditions with inverse wave ages of 1-2, and these conditions are quite different in this study.

The previous note notwithstanding, it is clear that the spread reported by ASIS buoys is too high. There are 2 possible explanations: 1) it was shown in (Pettersson et al., 2003) that a slow change of the orientation of an ASIS buoy, perhaps due to changing winds, during an average time series (20 mins.) results in an artificial increase in directional spread, 2) MLM processing for constructing directional spectra from Fourier coefficients has been shown to artificially
increase the directional spread (Earle, 1999; {COST} Action 714 Working Group 3, 2005), and it is suspected to similarly do so in constructing directional spectra from wave wire array data.

In (Drennan et al., 2003), the MLM processing was by-passed by deriving the sea surface slope signals directly from the surface elevation from wave wires. The slopes and elevation are used to calculate the co- and quad-spectra and the corresponding directional Fourier coefficients and parameters in the manner of Longuet-Higgins. In a comparison with a DWR, they found much better agreement in the directional spread, but there were also periods of disagreement with ASIS measuring higher directional spread. Similar to our findings, the highest areas of disagreement were near the peak of the spectrum. They found some of the increased spread was due to explanation 1) above, and clearly showed a reduction in directional spread with a shorter analysis time (~2 min) but a large difference in spread still remained at low frequencies. At other times they argued that the DWR was reporting artificially low spread due to a signal associated with mooring forces on the buoy. It has been shown that directional spread is sensitive to noise (Kuik et al., 1988; Tucker, 1991), and was stated in (Drennan et al., 2003) that the directional spread should only be reported at frequencies with wave energy clearly greater than noise.

Since ASIS was isolated from mooring forces, it is unclear if and how noise may be enter the system. Clearly the MLM processing, in combination with other factors such as non-stationarity and perhaps noise, artificially increase the
spread. A more satisfying explanation and potential correction await further analysis.

8.3.6 Comparison Discussion and Conclusion

The time series reveal the large data gaps for the two ASIS buoys, although Romeo retains most of data with Hs greater than 1 meter. It is notable that the ASIS buoys compare similarly against the NDBC buoy, which indicates consistency among these buoys. ASIS vs ASIS comparisons confirm the consistency of their measurements; the agreement is better among ASIS buoys than against the NDBC buoy. The differences seen in the bulk parameters are reasonable and probably within the sampling variability of each parameter, but with the notable exception of directional spread at the peak.

The real differences between buoys are more apparent for parameters which are functions of energy and frequency. Considering the proximity of the buoys, it is unlikely that the differences between the ASIS buoys and the NDBC buoy are due to spatial off-set alone. Differences in the measurement platforms and the analysis method may play a roll. It is known that MLM artificially broadens the directional distribution (Earle, 1999; {COST} Action 714 Working Group 3, 2005). Another note: the highest frequency wave recorded by the NDBC buoy is .35 Hz, so any energy-frequency blocks beyond this were meaningless and it was assumed that the prevalence of these blocks is an indication of a glitch in the WET program.

It is likely that the NDBC buoy slightly over estimates the energy at most frequencies in such a way to compensate for the high-frequency cutoff. There
was good agreement in mean direction as a function of frequency in the energy
containing part of the spectrum. Some detailed analysis was done on the
directional spread of each buoy, and explanations were offered for the high
directional spread reported by ASIS buoys. It was pointed out in (Krogstad et al.,
1999) that one should only compare those frequencies where the energy level is
clearly above the noise level. Here the wave conditions are very small, and it
was seen in (Anctil, 1993) that the agreement improved by comparing
measurements made in sea-states regimes larger than the ones compared here.
Therefore some disagreement, especially in the low frequencies, is to be
expected.

8.4 Case Study 2: Stationary Period

8.4.1 Part 1: Observation of a Slanting Fetch

Figure 8.21) On the left from top to bottom: Wind speed at 10 meters, inverse wave age, and swell-
wind sea separation, significant wave height, and directional spread at the peak. On the right from
top to bottom: Mean wave direction at the peak and wind direction, peak frequency, and air and sea
temperature. Wind measurements are from NDBC station 42036. Bravo, Romeo and NDBC station
42036 are shown in blue, red, and black, respectively.
By considering the wind speed and direction only, one can isolate this section from experimental time series as relatively stationary. The mean wind speed is trending down from ~10 to ~6, with diurnal variations. The wind direction remains relatively stable and comes from out of the southeast. With these forcing conditions, one would also expect the wave parameters to be somewhat stable, but something unexpected happens around year day 125. At about the same time that the inverse wave age indicates full development, the wave direction turns west of the wind direction, the peak frequency drops, the NDBC buoy indicates a decrease in directional spread, and the wave height increases. After year day 125, the wind speed is on average 6.25 m/s from 155° and the wave direction is from around 210°.

What can explain this discrepancy in wind and wave directions? Since the wave age is at or near full development, and the wind speed and direction is relatively stable, a plausible explanation is that the dominant wave conditions are being advected instead of being locally generated. It may be that a slanted fetch (i.e. the fetch in the look direction of the peak waves) becomes dominant over the direct fetch (i.e. the fetch in the wind direction). To answer this question we assume that the wind speed and direction is the same over the gulf. Then following (Donelan et al., 1985) we calculate the expected non-dimensional energy and frequency with a given non-dimensional fetch. The calculation of non-dimensional parameters (NDP) is as follows, first non-dimensional fetch:

$$\chi^*(\theta) = \frac{X(\theta)g\cos(\theta - \theta_w)}{U_{10}^2}$$

Non-dimensional energy:
\[ \varepsilon = \frac{g^2 m_0}{U_{10}^4} \]

And non-dimensional frequency:
\[ \nu = \frac{f_p U_{10}}{g} \]

To characterize the period before and after the switch let us calculate the mean of variables in look direction of the peak waves. A distance tool on Google maps was used to measure the approximate distance from the buoy until land in the up wave direction:

<table>
<thead>
<tr>
<th>Time/Variable</th>
<th>( f_p ) [Hz]</th>
<th>( U_{10} ) [m/s]</th>
<th>( \theta_u ) [°]</th>
<th>( \theta_p ) [°]</th>
<th>( m_0 ) [m²]</th>
<th>( X ) [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before</td>
<td>0.170</td>
<td>9.39</td>
<td>149.4</td>
<td>150.0</td>
<td>0.166</td>
<td>520</td>
</tr>
<tr>
<td>After</td>
<td>0.136</td>
<td>6.25</td>
<td>155.4</td>
<td>214.6</td>
<td>0.137</td>
<td>1500</td>
</tr>
</tbody>
</table>

Table 8.8) From left to right: peak frequency, wind speed at 10 meters, wind direction, wave direction at the peak, total variance, and fetch length. These parameters are listed for before (top) and after (bottom) the switch.

If we now consider those 2 different look directions, at 150° and 215°, we can calculate the input from each fetch in terms of NDPs:

<table>
<thead>
<tr>
<th>Look direction/NDP</th>
<th>Observed ( \chi^* ) (difference)</th>
<th>( \varepsilon )</th>
<th>( \nu )</th>
<th>( \varepsilon )</th>
<th>( \nu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>150° (Before)</td>
<td>57.9 (15.3)</td>
<td>0.0021</td>
<td>0.1627</td>
<td>.000018</td>
<td>0.7275</td>
</tr>
<tr>
<td>214° (Before)</td>
<td>73.2</td>
<td></td>
<td></td>
<td>.000022</td>
<td>0.6893</td>
</tr>
<tr>
<td>150° (After)</td>
<td>130.6 (34.5)</td>
<td>0.0087</td>
<td>0.0867</td>
<td>.000034</td>
<td>0.6033</td>
</tr>
<tr>
<td>214° (After)</td>
<td>165.1</td>
<td></td>
<td></td>
<td>.000041</td>
<td>0.5716</td>
</tr>
</tbody>
</table>

Table 8.9) Non-dimensional fetch calculated in the wind and off wind directions before and after the switch. Non-dimensional energy and frequency calculated from observations and calculated with Donelan’s empirical relationship.

The second column above shows that the slanted fetch was always greater than the direct fetch, but the difference is more pronounced after the switch. Non-dimensional fetch goes with \( 1/U_{10}^2 \), so it is only in low wind speed conditions that the differences in fetch lengths are important. This supports the
case of slanting fetch. In columns 3 and 4 are the observed values for $\epsilon$ and $\nu$, respectively, and in last to columns are the values of $\epsilon$ and $\nu$ calculated using the empirical growth relations of (Donelan et al., 1985; Young, 1999). The values are not close to what was observed, but this is not a great concern. There are many different empirical growth relations and all are associated with large scatter. To find one that best fits the data, a full statistical analysis of the NDPs during fetch limited conditions would need to be considered, but this is will not be explored further.

8.4.2 Part 2: Wave Parameter Variation

We turn our attention to the time series of data immediately after the shift due to slanting fetch. During this time period the wave parameters of interest are essentially stationary. The geophysical variability is minimized for stationary data, so that variation in the data can be attributed to sampling variability. Once the sampling variability of a parameter is estimated, it may be used to give context to variability in a scatter plot. Furthermore a confidence region may be established on a scatter plot by applying the method of (Krogstad et al., 1999). Once a confidence region is established, one may determine the percentage of points that lay inside or outside the confidence region. This procedure is useful for defining agreement criteria, such that if 90% of the data fall within the 90% confidence region the scatter is due solely to sampling variability and the agreement is perfect. True stationary conditions do not exist, so some variation due to geophysical forcing is expected and can sometimes be dealt with. A time series for the conditions is shown directly below:
The time series above shows the nearly stationary period between year day 125 and 127. The wave height is fairly stable with a downward trend reflecting a slightly dropping wind speed. The wave and wind direction are remarkably stable. There is some unexplained variation in the peak frequency, but it remains relatively stationary. The table below gives the mean and std for the time series shown above.

<table>
<thead>
<tr>
<th>Param./Buoy</th>
<th>NDBC 42036 (mean±std)</th>
<th>Bravo (mean±std)</th>
<th>Romeo (mean±std)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{m0}$ [m]</td>
<td>1.47±0.19</td>
<td>1.38±0.18</td>
<td>1.50±0.22</td>
</tr>
<tr>
<td>$f_p$ [Hz]</td>
<td>0.136±0.006</td>
<td>0.139±0.007</td>
<td>0.138±0.007</td>
</tr>
<tr>
<td>$\theta_p$ [°]</td>
<td>214.6±6.9</td>
<td>202.8±11.9</td>
<td>208.8±9.1</td>
</tr>
<tr>
<td>$\sigma_p$ [°]</td>
<td>26.7±3.9</td>
<td>48.0±4.8</td>
<td>47.3±4.7</td>
</tr>
<tr>
<td>$U_{10}$ [m/s]</td>
<td>6.23±1.45</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>U Dir. [°]</td>
<td>155.4±11.6</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>$C_p$ [m/s]</td>
<td>11.5±0.5</td>
<td>11.3±0.6</td>
<td>11.3±0.5</td>
</tr>
<tr>
<td>$U_{10}/C_p$</td>
<td>0.54±0.12</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>$H_{m0}^D$ [m]</td>
<td>1.47±0.12</td>
<td>1.38±0.11</td>
<td>1.50±0.15</td>
</tr>
</tbody>
</table>

Table 8.10) For each buoy from top to bottom is the mean and std of significant wave height, peak frequency, direction at the peak, directional spread at the peak, wind speed at 10 meters, wind direction, peak phase speed, inverse wave age, and detrended significant wave height.
The mean and std of parameters for each buoys are listed above. All parameters above, with the exception of $H_{m0}^D$, have previously been defined in this document. $H_{m0}^D$ is $H_{m0}$ with the downward trend removed. As can be seen in the coefficient of variance (COV) table below, removing the trend reduces the variability considerably.

<table>
<thead>
<tr>
<th>Param./Buoy</th>
<th>NDBC 42036</th>
<th>Bravo</th>
<th>Romeo</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{m0}$</td>
<td>0.13</td>
<td>0.13</td>
<td>0.15</td>
</tr>
<tr>
<td>$f_p$</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>$\theta_p$ [°]</td>
<td>6.9*</td>
<td>11.9*</td>
<td>9.1*</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>0.15</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>$U_{10}$</td>
<td>0.23</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>U Dir. [°]</td>
<td>11.6*</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>$C_p$</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>$U_{10}/C_p$</td>
<td>0.22</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>$H_{m0}^D$ [m]</td>
<td>0.08</td>
<td>0.08</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Table 8.11) COVs for NDBC 42036, Bravo, and Romeo. From top to bottom: significant wave height, peak frequency, peak wave direction, directional spread at the peak, wind speed at 10 meters, wind direction, peak phase speed, inverse wave age, and detrended significant wave height. * indicates the std instead of COV because the mean is arbitrary for directional variables.

Following (Drennan & Shay, 2006) we try to identify geophysical forcing of parameters by calculating the correlation between wave parameters and forcing parameters. Significant correlations, defined as $R^2>0.30$, would indicate that variation is somehow coupled between parameters. If theoretical or empirical relationships exist which describe the coupling, it may be possible to eliminate the variation of the wave parameter due to the forcing parameter. At this point attention is restricted to NDBC 42036, although future analysis should be extended to all buoys. To supplement the forcing parameters of wind speed ($U_{10}$), change in wind speed ($\Delta U$), change in wind direction ($\Delta U$ Dir), significant
wave height, peak period, and inverse wave age we introduce significant steepness ($S_S$) and the difference in air-sea temperature ($\Delta \Theta$):

$$S_S = \frac{2\pi H_{m0}}{gT_{z}^{2}}$$

$$\Delta \Theta = T_{Sea} - T_{Air}$$

<table>
<thead>
<tr>
<th>Buoy</th>
<th>$R^2$</th>
<th>NDRC 42036</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_{m0}$</td>
<td>0.33</td>
<td>0.00, 0.06</td>
</tr>
<tr>
<td>$f_p$</td>
<td>-0.41</td>
<td>0.14, -0.06</td>
</tr>
<tr>
<td>$\Delta \Theta_p$</td>
<td>-0.12</td>
<td>-0.10, -0.21</td>
</tr>
<tr>
<td>$\alpha_p$</td>
<td>-0.17</td>
<td>-0.03, -0.12</td>
</tr>
<tr>
<td>$H_{m0}^D$</td>
<td>-0.23</td>
<td>0.05, -0.07</td>
</tr>
</tbody>
</table>

Table 8.12) Correlations between wave parameters and forcing variables.

All of the typical parameters are featured in the analysis above, with the added addition of some difference (gradient) parameters indicated by a $\Delta$. Let us address each significant correlation. $H_{m0}$ (and $H_{m0}^D$) had a spuriously high correlation with $S_S$ since it was used in the calculation of $S_S$. $\Delta \Theta$ is an indicator of atmospheric stability. If the sea temperature was higher than the air temperature the atmosphere was unstable, and if the sea temperature was lower than the air temperature the atmosphere was stable. The high correlation between $H_{m0}$ (and $H_{m0}^D$) and $\Delta \Theta$ must be a meaningless coincidence because one would expect a high negative correlation (if any at all) given atmospheric stability, i.e. the wave height is expected to increase given decreasing atmospheric stability. $H_{m0}$ also showed a mild correlation to $U_{10}$, and this correlation was reduced after removing the trend. $f_p$ showed a strong inverse relation to wind speed, this is predicted by empirical fetch limited growth curves. Since the stationary time period may be thought of as dominated by a slanting
fetch and an appropriate growth curve was not identified, no attempt to remove
this variation was made. $\sigma_p$ showed a significant correlation with both $H_m0$ and $f_p$,
but there were no known relationships between these parameters so the
variation could not be removed. We proceed now with some confidence that all
possible measures were taken to minimize geophysical variation in the data. It
may be of note that since these seas are most likely advected in and not
generated locally, the correlations are expected to be low with the local
gephysical forcing parameters. The COV could now be used to determine
confidence regions in scatter plots (Krogstad et al., 1999). First, assuming equal
sampling variability in both NDBC 42036 and Bravo, let us look at a 90%
confidence region on a scatter plot defined by the COVs determined during the
stationary period by both $H_m0$ and $H_m0^D$. We expect 90% of the data points to fall
within the given confidence region, the actual percentage is listed in the table
below.

![Figure 8.23](image)

**Figure 8.23** Significant wave height scatter plot between NDBC station 42036 and Bravo. The
confidence regions for COV = 0.08 is within the dash-dotted lines and the confidence region for COV
= 0.13 is within the dashed lines. The red line is a ML fit, and the solid black line is 1:1.
The table below shows the amount of data points within each confidence region.

<table>
<thead>
<tr>
<th>COV</th>
<th>% of Points inside 90% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.08</td>
<td>76.2</td>
</tr>
<tr>
<td>0.13</td>
<td>93.1</td>
</tr>
</tbody>
</table>

Table 8.13) COVs which define a 90% confidence region and the percentage of data points within the region.

The COV of 0.13 gives us 93.1% of the data points within the confidence region meaning that the sampling variability is over estimated. This supports the case for the reduced COV derived from the detrended $H_{m0}$. Since 76.2% of the data points lie within the 90% confidence region, we may conclude that most of the variability is accounted for by the sampling variability. To end this section we offer one more example. Below we plot directional spread at the peak between ASIS buoys Romeo and Bravo:

![Figure 8.24) Scatter plot for directional spread at the peak between Romeo and Bravo.](image)

Notice that the variability of the parameter is quite high, but the percentage of points inside the 90% CI is 77.8%. Because the sampling
variability of the parameter is also quite high, we may again conclude the majority of scatter is due to sampling variability of the parameter. The COVs derived above will be used in sensor comparisons during other experiments.

### 8.5 Discussion and Conclusion

The first section is a detailed comparison of between the 4 sensors deployed for the experiment. Not all the sensors were functional simultaneously, so each comparison is representative of slightly different wave climates. The comparison follows in line with the consensus from previous results, the 1-D parameters of $H_{m0}$ and $f_p$ compared very well. There was increased scatter in $f_p$ due in part to multi-modal seas. The direction at the peak compared favorably but suffered from increased scatter most likely due to the inherently high sampling variability of the parameter. The directional spreading at the peak was consistently high for the ASIS buoys vs the NDBC. A time period was chosen when all sensors were functional. The average variance density, mean direction, and directional spread as a function of frequency were calculated and compared. A large discrepancy in directional spread was observed again. This was studied and detail and possible explanations were offered.

A short period of nearly stationary conditions was examined, and slanting fetch was observed. The coefficient of variance (COV) was estimated for NDBC 42036 during this time. These COVs will be used in future comparisons to define confidence regions on scatter plots.

Future work will derive the COVs from the other 3 buoys, and observe how these change spatially from buoy to buoy. This should be examined in the
context of the assumptions of stationarity, homogeneity and the ergodic hypothesis. A question that comes to mind is how far out can we assume conditions are similar in space and in time? The answers to these questions may have significance to remote sensing. For example a scatterometer measurement has certain size footprint, but only gives a “point” measurement which assumes homogeneity over the footprint. How much variability might there be within the scatterometer footprint?
9 Chapter: SHOWEX

This chapter is broken into 4 main sections. The first section is an introduction which describes the experiment including sensors, environmental conditions, and a literature review. The second section is a comprehensive comparison between an ASIS buoy and two adjacent DWRs. The third section is an exploratory study on the skewness (and kurtosis) of surface elevation and possible relationship to wave steepness. The last section is a conclusion which summarizes the work.

9.1 Introduction to SHOWEX

The SHOaling Waves EXperiment (SHOWEX) occurred off the coast of Duck, North Carolina at the Field Research Facility (FRF) operated by the United States Army Corps of Engineers (USACE). Just off the heels of the GOM99 experiment, SHOWEX took place from August to December in 1999. The experimental objective was to investigate the properties and evolution of surface gravity waves in intermediate and shallow water depths. This included exploring many different wave processes and interactions as evident in the literature review section to follow.

To facilitate the experimental objectives there were many institutions and even more sensors involved including buoys, airborne sensors, and a research vessel. Our consideration will be with three ASIS buoys, a series of DWRs, and NDBC station 44014. NDBC station 44014 is an operational 3-meter discus buoy. The following two figures (figure 9.1 and figure 9.2) map out the placement
of the buoys and show the depth. The locations and depths of the buoys were
strategically chosen to compliment the experimental objectives.

Figure 9.1) Positions of buoys during SHOWEX.

Figure 9.2) Map of buoy moorings and depth chart during SHOWEX. Source: (Ardhuin et al., 2007).
The FRF operates various instrument platforms at their location off the coast of Duck, NC. Starting with the shallowest buoy, FRF operated a Directional Waverider (FRFWR), then there were two transects of buoys going offshore just North and South of East. The southern transect was a series of Datawell directional waveriders operated by the Naval Postgraduate School and the Scripps Institute of Oceanography Center for Coastal Studies. The northern transect was a series of ASIS buoys and an NDBC 3-meter discus buoy, station 44014. The coordinates and water depths at each buoy are recorded in the table below. Note that Romeo and Yankee are mistakenly switched in the map directly above.

<table>
<thead>
<tr>
<th>Instrument/Location</th>
<th>Platform</th>
<th>Latitude [°N]</th>
<th>Longitude [°W]</th>
<th>Approx. Depth [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRFDWR</td>
<td>DWR</td>
<td>36.199883</td>
<td>-75.714050</td>
<td>17.4</td>
</tr>
<tr>
<td>Bravo</td>
<td>ASIS</td>
<td>36.232500</td>
<td>-75.643867</td>
<td>20</td>
</tr>
<tr>
<td>Yankee</td>
<td>ASIS</td>
<td>36.408333</td>
<td>-75.508383</td>
<td>25</td>
</tr>
<tr>
<td>Romeo</td>
<td>ASIS</td>
<td>36.473617</td>
<td>-75.255550</td>
<td>35</td>
</tr>
<tr>
<td>X1</td>
<td>DWR</td>
<td>36.228833</td>
<td>-75.704667</td>
<td>21</td>
</tr>
<tr>
<td>X2</td>
<td>DWR</td>
<td>36.227000</td>
<td>-75.611167</td>
<td>24</td>
</tr>
<tr>
<td>X3</td>
<td>DWR</td>
<td>36.204500</td>
<td>-75.498500</td>
<td>26</td>
</tr>
<tr>
<td>X4</td>
<td>DWR</td>
<td>36.158333</td>
<td>-75.325500</td>
<td>33</td>
</tr>
<tr>
<td>NDBC 44014</td>
<td>3-m discus</td>
<td>36.583056</td>
<td>-74.833611</td>
<td>47.5</td>
</tr>
</tbody>
</table>

Table 9.1) From left to right: platform type, latitude, longitude, and approximate depth for each sensor.

The depths listed are from what could be gathered from available information and should be considered approximate. The buoys were able to move about due to some slack in the mooring lines, this play allowed the buoys to travel over slightly variable topography. One discrepancy that is obvious from the maps is that X1 is probably somewhat shallower than Bravo. This will be discussed in the comparison below.
The DWRs did not record wind speed or wind direction. The buoys which did measure wind speed and direction agree well on wind speed but not direction of the wind. When calculating DWR parameters that require the wind speed, measurements from NDBC 44014 are used. Parameters which require wind direction will not be calculated for the DWRs.

9.1.1 Environmental Conditions

Figure 9.3) Environmental conditions during SHOWEX.

Shown are the measurements from Bravo and NDBC 44014 where data from Bravo is given by blue markers and NDBC 44014 black markers. In the top box wind speed and the inverse wave age (times 10 for a visual aid) is plotted. In the
calculation of inverse wave age $\theta_d$ is the difference between the wind direction and the peak wave direction ($\theta_d = \theta_p - \theta_U$). The second box is the wind direction and the direction of the peak waves. The third box is the significant wave height. The fourth box is the directional spreading at the peak, and the fifth box is the air and water temperature (air Temperature is from NDBC 44014 and water temperature is from DWR X2).

Due to the spatial offset, difference in depths, and proximities to shore, these two buoys offer different but complimentary accounts of the SHOWEX story. The conditions were relatively calm during the experimental period ($H_s = \sim 1m$) marked by short intervals of increased wave heights ($H_s = 2m - 4m$) due to the passage of Nor’easters. There were especially strong low pressure systems which moved through around year days 307 and 337. There was a swell event around the year day 323 which can be traced back to Hurricane Lenny in the Caribbean. From the $H_m0$ time series there was evidence of attenuation of shoaling waves as they approach shore.

**9.1.2 Review of Previous SHOWEX Studies**

Quite a few studies have been published which use the data from SHOWEX, including a special issue of the Journal of Atmospheric and Oceanic Technology (Graber, 2005). Here, only the publications which used wave data from in situ buoys are considered.

In a 2 part paper, (Ardhuin, Herbers, Jessen, & O'Reilly, 2003; Ardhuin, O'Reilly, Herbers, & Jessen, 2003) looked at the attenuation of swell cross the continental shelf. The idea was to better understand the attenuation mechanism
and improve source and sink terms in wave models. They determined that the attenuation mechanism was dominated by interaction with the bottom. Increased directional spread across the shelf, in spite of refraction (which predicts decreased spread), was due to Bragg scattering off bottom irregularities. Part 2 tested various models and model parameterizations against the observations.

(Ardhuin et al., 2007) looked at fetch limited growth in the presence of swell. They observed wind seas where the equilibrium face of the spectra traveled in line with the wind direction, but around the peak of the spectrum waves traveled alongshore (20°-30° different than the wind direction). This phenomenon was attributed to slanting fetch (Donelan et al., 1985). By including the slanting fetch seas they showed that the spectral wave growth matched well with (Kahma, 1981), and then only considering the higher frequency wind seas in-line with the wind (without the seas due to slanting fetch) they showed wave growth more in line with (K. Hasselmann et al., 1973). Since swell was not present in the studies which matched their data, they concluded swell was not important to growth of wind sea. They identified an especially pronounced case of multi-directional wind seas (presumably due to slanting fetch) and tested models and model parameterizations against observations.

(Zhang, Drennan, Haus, & Graber, 2009) considered the interaction of waves, winds, and currents during SHOWEX in the context of wind stress. They showed that the current shear at the edge of a current refracted waves at wind sea peak away from the mean direction, and hence turned the wind stress vector away from the wind direction. They cited the model of (Alpers & Hennings, 1984)
to explain why the current refracted only the peak waves. This model states that the wind forcing time scale of the short waves is small compared to the time it takes the slow moving waves to cross the current shear, this is in contrast to the longer waves. Hence shorter waves are being actively forced on time scales equal to or less than that of the current shear and they keep their direction in line with the wind. Long waves are also actively forced but on a longer time scale then refraction due to current shear and hence are turned away from the wind. They considered (Ardhuin et al., 2007) and concluded that current shear, not slanting fetch, was the dominant mechanism shifting the peak wind waves away from the wind direction. In closing they discuss the implications of these findings to remote sensing.

9.2 ASIS-DWR Comparison

Here we make a detailed comparison of the measurements made by ASIS buoy Bravo and the surrounding DRWs X1 and X2. The three buoys made up an eastward transect with Bravo in the middle, X1 was about 5.5km West of Bravo and X2 was about 3km East of Bravo (see figure above). The depth at Bravo was about 20m, with X1 a few meters shallower and X2 a few meters deeper. The wave parameters of interest are $H_{m0}$, $f_p$, $\theta_p$, and $\sigma_p$, all calculated using method of (Young, 1995) and the directional method proposed in this document which was inspired by (Young, 1995). The COV for the parameters were derived from the GOM99 data set, and it is assumed that the instruments have equal sampling variability.
9.2.1 Time Series

Shown are the measurements from X1, Bravo, and X2 where data from X1 is shown with red dots, Bravo is given by blue markers and X2 with green dots. In the first box we have wind speed and 10 times the inverse wave age (Bravo only). The second box is the wind direction (Bravo only) and the mean direction of the peak waves. The third box is the significant wave height, the fourth box is the peak frequency, and the fifth box is directional spreading at the peak.

Here there is very good agreement for all wave parameters, save $\sigma_p$, over a variety of conditions. The 2 DWRs seem to agree very well on $\sigma_p$, but Bravo consistently reports higher directional spread. This was a pattern already found in the GOM99 data with comparisons against a NDBC 3-meter discuss buoy, and was also reported in (Drennan et al., 2003; Pettersson et al., 2003). In the
discussion it was shown that a shorter analysis time may improve the estimate of directional spread (at least in the high frequency ranges), but a full explanation remains elusive. Also, as expected the $H_{m0}$ of $X2$ is slightly greater than that of $X1$. This is a result of the depth difference. There are intermittent periods of disagreement in the parameters of $f_p$ and $\theta_p$ occurring around year days 308, 310-312, and 314-316. As will be demonstrated in the figure below, these times are associated with real differences in the wave field. There is local development of waves in all cases occurring in times with little or no background wave energy.

![Spectral Evolution](image)

**Figure 9.5) Spectral evolution plots.**

In the figure above, energy (indicated by color) is on a natural log scale. You can clearly see fetch limited development of waves during the time periods of increased disagreement among the buoys. The peak frequency decreases, and the energy increases as a function of fetch length (distance from the coast...
when the wind is blowing offshore). To better understand differences due to error, these time periods have been excluded from the following comparisons. Removing the obvious cases of building seas does not remove all the cases; some variability due to fetch limited development is expected in the comparison.

### 9.2.2 Scatter Plots and Data Tables

![Figure 9.6](image_url) Significant wave height scatter plots.

<table>
<thead>
<tr>
<th>Comp/Stat</th>
<th># of pts</th>
<th>slope</th>
<th>Interce. [m]</th>
<th>$R^2$</th>
<th>bias [m]</th>
<th>std [m]</th>
<th>RMS E [m]</th>
<th>COV</th>
<th>P90 [%]</th>
<th>P90b [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bravo/X2</td>
<td>369</td>
<td>0.928</td>
<td>0.005</td>
<td>0.99</td>
<td>-0.085</td>
<td>0.102</td>
<td>0.133</td>
<td>0.08</td>
<td>85.6</td>
<td>92.7</td>
</tr>
<tr>
<td>X1/X2</td>
<td>773</td>
<td>0.931</td>
<td>-0.007</td>
<td>0.99</td>
<td>-0.090</td>
<td>0.100</td>
<td>0.135</td>
<td>0.08</td>
<td>82.1</td>
<td>92.8</td>
</tr>
<tr>
<td>X1/Bravo</td>
<td>438</td>
<td>1.000</td>
<td>-0.010</td>
<td>0.99</td>
<td>-0.010</td>
<td>0.079</td>
<td>0.079</td>
<td>0.08</td>
<td>95.2</td>
<td>96.3</td>
</tr>
</tbody>
</table>

Table 9.2 The table above describes the goodness of fit parameters as well as the statistical difference measures for $H_{m0}$. The statistical difference measures are bias, standard deviation (std), and root-mean-square-error (RMSE). Calculations of various statistics are described in section 4.1.1. The remaining column headers above are the following: # pts is the number of data points used in the comparison, slope and interc. are the fit parameters for the ML regression, $R^2$ is the correlation coefficient, coefficient of variance (COV) derived from the GOM99 data set, the percentage of points that lie within the 90% confidence region (P90), and the percentage of points that lie within the 90% confidence region with bias taken into account (P90b).

Since the sensors were at different depths, a slight difference in wave height was expected. Specifically, the sensor X2, at ~24m depth, was expected to report somewhat higher wave heights than both X1 and bravo both at ~20m
depth. The difference in height was mostly due to the attenuation of incoming low frequency waves and somewhat due to build up of high frequency waves as the distance from shore increases. This type of difference can be seen in the data. The overall agreement between buoys is exceptional.

![Figure 9.7](image)

**Figure 9.7** Peak frequency scatter plots.

<table>
<thead>
<tr>
<th>Comp/Stat</th>
<th># of pts</th>
<th>slope</th>
<th>intercept</th>
<th>$R^2$</th>
<th>bias [Hz]</th>
<th>std [Hz]</th>
<th>RMSE [Hz]</th>
<th>COV</th>
<th>P90 [%]</th>
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<tbody>
<tr>
<td>Bravo/X2</td>
<td>369</td>
<td>0.89</td>
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<td>0.91</td>
<td>-0.0030</td>
<td>0.0107</td>
<td>0.0111</td>
<td>0.05</td>
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<td>87</td>
</tr>
<tr>
<td>X1/X2</td>
<td>773</td>
<td>0.97</td>
<td>0.0042</td>
<td>0.90</td>
<td>-0.0001</td>
<td>0.0119</td>
<td>0.0119</td>
<td>0.05</td>
<td>86</td>
<td>87</td>
</tr>
<tr>
<td>X1/Bravo</td>
<td>438</td>
<td>1.09</td>
<td>-0.0084</td>
<td>0.94</td>
<td>0.0026</td>
<td>0.0096</td>
<td>0.0100</td>
<td>0.05</td>
<td>87</td>
<td>88</td>
</tr>
</tbody>
</table>

Table 9.3 The table above describes the goodness of fit parameters as well as the statistical difference measures for $f_p$. The statistical difference measures are bias, standard deviation (std), and root-mean-square-error (RMSE). Calculations of various statistics are described in section 4.1.1. The remaining column headers above are the following: # pts is the number of data points used in the comparison, slope and intercept are the fit parameters for the ML regression, $R^2$ is the correlation coefficient, coefficient of variance (COV) derived from the GOM99 data set, the percentage of points that lie within the 90% confidence region (P90), and the percentage of points that lie within the 90% confidence region with bias taken into account (P90b).

It can be seen in the P90 statistic that nearly all of the variability in these plots is due to the sampling variability of the parameter. What little bit falls outside is most likely due to periods where seas are building offshore. At the distance from shore of X2, the wind seas are built enough to become the peak
seas so X2 reports a higher frequency $f_p$ than the inshore buoys. Peak parameters are inadequate in describing multi-modal seas. So this peak parameter comparison and the two that follow suffer somewhat from the presence of these multi-modal seas.

![Figure 9.8](image.png)

**Figure 9.8** Direction at the peak scatter plots.

<table>
<thead>
<tr>
<th>Comp/Stat</th>
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<th>intercept [°]</th>
<th>$R^2$</th>
<th>bias [°]</th>
<th>std [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bravo/X2</td>
<td>369</td>
<td>1.307</td>
<td>-23.3</td>
<td>0.963</td>
<td>2.2</td>
<td>15.5</td>
</tr>
<tr>
<td>X1/X2</td>
<td>773</td>
<td>0.896</td>
<td>7.8</td>
<td>0.989</td>
<td>-1.3</td>
<td>8.5</td>
</tr>
<tr>
<td>X1/Bravo</td>
<td>438</td>
<td>0.673</td>
<td>25.7</td>
<td>0.953</td>
<td>-4.3</td>
<td>17.6</td>
</tr>
</tbody>
</table>

Table 9.4: The table above describes the goodness of fit parameters as well as the statistical difference measures for $\theta_p$. The statistical difference measures are directional bias and directional standard deviation. Calculations of directional statistics are demonstrated in section 4.1.1. The column headers above are the following: # of pts is the number of data points used in the comparison, $R^2$ is the directional association measure, slope and intercept are the fit parameters of the ML regression.

The agreement of the two DWRs is somewhat better than with ASIS and either of the DWRs. As evidence of this, directional std of the comparisons involving Bravo are much higher than those of the comparison between DWRs. There is no significant directional bias between any of the buoys on the average, but there are some problematic areas. The ASIS and DWRs agreement
deteriorates outward from east, and bias in the wave direction appears to be significant in these areas.

![Directional spread at the peak scatter plots.](image)

Table 9.5) The table above describes the goodness of fit parameters as well as the statistical difference measures for $\sigma_p$. The statistical difference measures are bias, standard deviation (std), and root-mean-square-error (RMSE). Calculations of various statistics are described in section 4.1.1. The remaining column headers above are the following: # pts is the number of data points used in the comparison, slope and intercept are the fit parameters for the ML regression, $R^2$ is the correlation coefficient, coefficient of variance (COV) derived from the GOM99 data set, the percentage of points that lie within the 90% confidence region (P90), and the percentage of points that lie within the 90% confidence region with bias taken into account (P90b).

<table>
<thead>
<tr>
<th>Comp/Stat</th>
<th># of pts</th>
<th>slope</th>
<th>intercept</th>
<th>$R^2$</th>
<th>bias [°]</th>
<th>std [°]</th>
<th>RMSE [°]</th>
<th>COV [%]</th>
<th>P90 [%]</th>
<th>P90b [%]</th>
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<tbody>
<tr>
<td>Bravo/X2</td>
<td>369</td>
<td>2.552</td>
<td>-2.54</td>
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<td>33.98</td>
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<td>34.68</td>
<td>0.10</td>
<td>0</td>
<td>40.7</td>
</tr>
<tr>
<td>X1/X2</td>
<td>773</td>
<td>1.085</td>
<td>-1.54</td>
<td>0.779</td>
<td>0.50</td>
<td>3.48</td>
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<td>0.10</td>
<td>82</td>
<td>81.1</td>
</tr>
<tr>
<td>X1/Bravo</td>
<td>438</td>
<td>0.490</td>
<td>-4.45</td>
<td>0.611</td>
<td>34.06</td>
<td>6.78</td>
<td>34.73</td>
<td>0.10</td>
<td>0</td>
<td>87.0</td>
</tr>
</tbody>
</table>

There is high bias associated with Bravo measurement of spread vs. either of the DWRs, double the standard deviation, and the correlation coefficients are significantly smaller. As was seen in previous studies the directional spreading at the peak does not compare well among different sensors. The ASIS buoy is probably over estimating for a variety of reasons.
suggested in the GOM99 study, but it is also not clear that the DWRs or any other sensor is accurate. It has been suggested that directional spread may be most useful as a relative parameter, indicating increasing or decreasing spread, and it is not very well suited for describing an absolute value (W.M. Drennan 2012 personal communication).

The comparison between the DWRs is much improved, and the P90 confirms that most of the variability is due to sampling variability. The extreme bias distorts the projection of the data, so P90b for Bravo/X2 is somewhat underestimated and the P90b for X1/Bravo is somewhat overestimated. A better estimate is an average of those, somewhere around 65% of the points lie within the 90% confidence region when bias and the geometry of the data projection are taken into account. So there is significant amount of variability outside of the sampling variability for comparisons involving Bravo.

9.2.3 WET Plots
In the GOM99 dataset, it was possible to analyze the shapes of the spectra, mean direction, and directional spread as a function of frequency. This kind of analysis is more complicated for this data set. The spectral shapes are complex due to the sometimes simultaneous presence of swell, locally generated, advected, refracted, and crossing wind seas. The spectral shapes were variable, and changed rapidly during storm passages; hence it was difficult to define average spectra, average mean direction, and average directional spread. These difficulties are present even if the data are partitioned by wave height, or if the time period for analysis was severely restricted. The WET
comparison program is well suited to deal with this kind of data. Here we use the WET comparison tool to better understand the sensor differences.

Another note on difference between the use of WET here and other uses in the thesis: previously this was used indiscriminately with all available data. Here I have compared only same periods that were compared in the scatter plots and for the same reasons listed above. The aim is to minimize the variability due to geophysical forcing to better understand sensor differences. One can imagine a circumstance, where there was a lot of confidence in the agreement of two sensors, the WET program could be used to study geophysical variability, but this will not be pursued further here.

9.2.3.1 X1 relative to Bravo

Figure 9.10) Energy percent bias component plot for X1 relative to Bravo.
Here the WET comparisons for X1 relative to Bravo is made. Recall that X1 – Bravo – X2 make up a linear transect. The following two sections will present X2 relative to X1 and Bravo relative to X2.

In figure 9.10 we have the percent bias of total energy (variance). A pattern is immediately discernible: X1 is underestimating the low frequency waves and overestimating the high frequency waves relative to Bravo. The pattern is similar to what was seen in GOM99 between ASIS buoys and the NDBC s-meter discus buoy, except that the overestimation is also enhanced in low energy regions. The region of relative good agreement is from 0.1 Hz to 0.3 Hz in the higher energy range of the plot. Disagreement of this kind may arise from different frequency responses of the respective systems. It is not possible to say which system is correct.

Figure 9.11) Energy percent std component plot for X1 relative to Bravo.
This is the percent standard deviation. The pattern shows that the variability has an inverse relationship to energy.

Figure 9.12) Mean direction bias component plot for X1 relative to Bravo.

The component plot showing average bias in mean direction is shown above. There is disagreement in the low energy, low frequency range and this is probably due to noise. There is relative low bias in the central region. There are intermittent blocks of high bias. Negatively and positively biased are somewhat clumped together. It is not clear what more these patterns can say except for they probably indicate some kind of systematic difference between the two buoys.
This is a very interesting pattern of higher and lower directional std of mean direction. The root of the pattern is not clear. The zone of minimum directional std is high energy waves with frequencies between 0.15 and 0.22 Hz. One might suppose that the directional std of mean direction increases away from the peak of the spectrum, but there is not enough information in the plot to confirm this theory.
The spread bias is predictably high in all frequency-energy ranges, especially in the central frequencies. The agreement improves at either extreme end of the frequency range, this is probably because all buoys report high directional spread for these frequencies, not because of an improved estimate from ASIS.

Despite the egregious bias, the std of directional spread is relatively tame across the board. This supports the conjecture that directional spread may be best thought of as a relative measure.
9.2.3.2 X2 relative to X1

Here we have X2 relative to X1. The plot shown above is the percent bias component plot for wave energy. Keep in mind that these buoys are the most far apart and have the greatest depth difference. X2 is seeing higher wave energy nearly across the board, and the percent bias is increasing as energy blocks decrease. It was expected that X2 would see more wave energy, and the hypothesis was that difference in energy was due to attenuation of longer waves and the buildup of short waves (fetch limited development). This would appear as high positive bias along the low frequencies and high frequency-energy border, but contrary to this hypothesis there is actually some negative bias along the high frequency-energy border. This is difficult to explain, but it is known that this area is prone to spurious placement of frequency-energy blocks. The discrepancy seen here may be an artifact of the program.
Figure 9.17) Energy percent std component plot for X2 relative to X1.

It is surprising to see such high %std in the above wave energy component plot. The highest %stds are confined to a region around lowest energy blocks at 0.25 Hz. The %std improves radially from there. This may be related to fetch-limited development of waves.

Figure 9.18) Mean direction bias component plot for X2 relative to X1.
The mean direction bias is near zero for the frequency range from 0.05-0.25 Hz and consistently low for higher frequencies.

The directional std of mean direction is very low in the same frequency range that low directional bias was seen to be in. There is an abrupt change in the directional std in high energy blocks above 0.2 Hz. Overall there is excellent agreement in mean direction in the central frequency ranges.
There is very little bias in directional spread across the board.

The std of directional spread increases as energy decreases, but it is exceptionally low in all energy-frequency component blocks.

### 9.2.3.3 X2 Relative to Bravo

The average relative difference in energy percent bias component plot for X2 relative to Bravo.
The component plot above shows variance bias of X2 relative to Bravo. The pattern is similar to the variance bias plot of X1 relative to Bravo with negative bias in the low frequencies and positive bias in the high frequencies. This plot has a much stronger positive bias, and less strong negative bias which is probably a result of the depth difference.

Figure 9.23) Energy percent std component plot for X2 relative to Bravo.

The %std of variance increases as a energy decreases as can be seen in the component plot above. Overall the %std is higher than the comparison of X1 relative to Bravo, and it may be a result of increased spatial separation and depth difference.
Figure 9.24) Mean direction bias component plot for X2 relative to Bravo.

There is very little bias in the mean direction except for the low energy low frequency blocks where noise tends to dominate.

Figure 9.25) Mean direction std component plot for X2 relative to Bravo.
The patterns seen in the component plot above for directional std in mean direction are similar to the comparison of X1 relative to Bravo. The following two component plots of bias of directional spread bias and std of directional spread, respectively, are also very similar to the previous DWR/ASIS comparison in section 8.2.3.1. Please refer back to the comments in the section on the X1 relative to Bravo comparison which also applicable here.

Figure 9.26) Directional spread bias component plot for X2 relative to Bravo.
9.2.4 Depth Transformed Spectra

Let us further investigate transforming deep water spectra unto shallow water. (Bouws, Günther, Rosenthal, & Vincent, 1985) extended the deep water similarity theory of (Kitaigorodskii, Krasitskii, & Zaslavskii, 1975) into finite water depth to come up with spectral form as a function of depth. We can use this theory to transform the spectra measured by X2 to the 20 m water depth of Bravo and X1. This assumes that the spectra measured at X2 is in relative deep water (this is not strictly true, it is intermediate water with an average kd=1.6), and that we are observing active generation of waves (this is also not strictly true, there are some periods with significant swell energy advecting in). Nevertheless, it allows us to compare the theoretically transformed spectra with the measured spectra. Analysis takes advantage of scripts written for the WAFO matlab toolbox (Brodtkorb et al., 2000).
Figure 9.28) Scatter plot for significant wave height and peak frequency for Bravo and X1 vs depth transformed X2.

<table>
<thead>
<tr>
<th>Comp/Stat</th>
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<th>Interc</th>
<th>$R^2$</th>
<th>bias</th>
<th>std [m]</th>
<th>RMSE [m]</th>
<th>COV</th>
<th>P90</th>
<th>P90b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bravo/TX2</td>
<td>369</td>
<td>1.028</td>
<td>0.055</td>
<td>0.98</td>
<td>0.086</td>
<td>0.117</td>
<td>0.145</td>
<td>0.08</td>
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<td>84.6</td>
</tr>
<tr>
<td>X1/TX2</td>
<td>773</td>
<td>1.033</td>
<td>0.043</td>
<td>0.98</td>
<td>0.078</td>
<td>0.104</td>
<td>0.130</td>
<td>0.08</td>
<td>74.5</td>
<td>85.9</td>
</tr>
</tbody>
</table>

Table 9.6) The table above describes the goodness of fit parameters as well as the statistical difference measures for $H_{m0}$ and $f_p$. The statistical difference measures are bias, standard deviation (std), and root-mean-square-error (RMSE). Calculations of various statistics are described in section 4.1.1. The remaining column headers above are the following: # pts is the number of data points used in the comparison, slope and intercept are the fit parameters for the ML regression, $R^2$ is the correlation coefficient, coefficient of variance (COV) derived from the GOM99 data set, the percentage of points that lie within the 90% confidence region (P90), and the percentage of points that lie within the 90% confidence region with bias taken into account (P90b).

The theoretical transformation over estimates the amount of wave height attenuation, despite this, the comparison is relatively good. The transformation dramatically over estimates an increase in $f_p$, and the overall comparison for this parameter is not very good. We will not say that the poor comparison is an indication of an inadequate theory because the assumptions involved in the theory were not strictly met. It is possible that overestimation in $H_{m0}$ attenuation
is due starting with intermediate depth spectra vs. assumed deep water spectra. Likewise, the poor agreement in $f_p$ may be related to active generation conditions not being met in general.

Figure 9.29) Energy percent bias component plots for X2 relative to Bravo.

Here we have a side by side comparison of the WET energy percent bias component plot before and after transformation of X2’s spectra. The comparison of the untransformed X2 is on the left and the transformed X2 is on the right. The actual numbers on the plots are unimportant; the differences between the plots can be seen through the different color components. The frequency range of agreement is shifted up and narrowed, and the overall agreement is not improved.

Figure 9.30) Energy percent std component plots for X2 relative to Bravo.
Here we have a side by side comparison of the WET variance %std plot before and after transformation of X2’s spectra. Again, the comparison of the untransformed X2 is on the left and the transformed X2 is on the right, and again the numbers are inconsequential and the colors are important. The %std in the low frequency end is actually improved, while the high frequencies are unchanged.

9.2.5 Comparison Conclusion
The wave height comparison showed bias where it was expected, and there was excellent overall agreement in $H_{m0}$. The component plot showed that relative to Bravo, the DWRs underestimated energy in the low frequencies and overestimated energy in the high frequencies. This was a pattern previously discovered with ASIS relative to a 3-meter discus, and it is not clear at this point which buoys are more accurate. The X2 relative to X2 variance comparison showed surprisingly high %std confined to a small area which may be related to local wave growth. The peak frequency comparison was plagued by bi-modal seas, but the agreement was good overall. The direction at the peak compared well for all buoys, the X1 vs X2 comparison was especially consistent. The WET plots for mean direction showed consistent patterns of agreement for ASIS vs DWRs, so the disagreement was most likely systematic. The X2 relative to X1 mean direction plots showed high agreement for the frequency range of 0.05-0.2 Hz with little bias everywhere but increasing std especially for high energy waves. The directional spread of ASIS was high biased over the DWRs. The variability of directional spread at the peak for X1 vs X2 was within the sampling
variability. The WET plots revealed consistent, across the board agreement on directional spread from X2 relative to X1. Transforming X2 spectra into shallow water via (Bouws et al., 1985) does not improve agreement, but the conditions were not strictly met for a proper transformation.

The study is mostly consistent (Drennan et al., 2003) with good agreement of 1-D spectral parameters. The strongest inconsistency was that this study found much poorer agreement in directional spread. This is perhaps due to the differences in analysis methods (MLM vs co- and quad-spectra). This study is also consistent with the comprehensive comparison for GOM99.

### 9.3 Surface Skewness and Kurtosis

Not to be confused with directional skewness and kurtosis, surface elevation skewness and kurtosis are the 3rd and 4th central moments of the surface elevation, respectively. There is scarce literature on surface skewness, so perhaps there is an opportunity to learn more. Here we present some analysis of surface skewness. Skewness is associated with asymmetry in the wave profile. Asymmetry in the wave profile can be either horizontal or vertical (see figure). Vertical skewness may be associated by non-linear wave forms (e.g. stokes wave), the extent of which may be indicated by wave steepness. Horizontal skewness may be associated with active wind force which is responsible for changing the shape of the wave. There is also increased asymmetry of both varieties as waves enter very shallow water.
Figure 9.31) Illustration of vertical and horizontal asymmetry which contribute to skewness.

The figure above illustrates vertical and horizontal asymmetry with wind direction indicated by the blue arrow. If the sea surface elevation was well represented normal distribution then the skewness and kurtosis of the surface elevation would be 0 and 3, respectively. It is assumed that the ocean surface is a linear quasi-Gaussian weakly-stationary process, but it is known that the assumption is only approximate. In fact it is well known that stokes second order non-linear theory is a more accurate description of the surface. Therefore higher order moments of surface elevation exist and may be an indication of the extent of non-linearity present. One may calculate the skewness, $\gamma_1$, directly from the surface elevation time series as follows:

$$\gamma_1 = \mathbb{E} \left[ \left( \frac{\eta - \mu}{\sigma} \right)^3 \right]$$

Spectral analysis has been central to this thesis, so instead an alternative method of calculating the skewness and kurtosis is used (Sroksz & Longuet-Higgins, 1986). This method convolves the integral of the spectrum [script available in (Brodtkorb et al., 2000)].

$$\gamma_1 = 3 \int_0^\infty \int_0^\infty \min(k, k') S(f)S(f')dfdf'$$
\[ \gamma_2 = \frac{4}{3} \gamma_1^2 \]

Above \( \gamma_2 \) is surface kurtosis. Due to the trivial relationship between skewness and kurtosis, any interesting pattern found in skewness will automatically be present in kurtosis. For this reason, we continue concentrating discussion on skewness only. (Srokosz & Longuet-Higgins, 1986) proposed a theoretical relationship between non-dimensional steepness and skewness for very narrow band spectra:

\[ \gamma_{\text{Narrow}} = 6\pi S_{\text{LH}} \]

Where

\[ S_{\text{LH}} = \frac{\sqrt{m_0}}{L_p} \]

For the special forms of Phillips and Toba spectra, the forward faces of which are \( S(f) \sim \alpha f^{-5} \) and \( S(f) \sim \alpha f^{-4} \), respectively, it was shown (Massel, 1996):

\[ \gamma_{\text{Phillips}} = 9\pi S_{\text{LH}} \]
\[ \gamma_{\text{Toba}} = 8\pi S_{\text{LH}} \]

In analyzing field data, (Babanin & Polnikov, 1995) showed that skewness was non-zero. Interestingly, they found no correlation with slope (steepness) or wave age as one might expect. (Drennan et al., 2003) showed that the skewness calculated from an ASIS buoy (in contrast with an adjacent DWR) was correlated to the significant steepness and wind speed.

In this section, we follow up on the previous work by looking for relationships between skewness and steepness and wind measures (e.g. speed, inverse wave age). First this is done for the ASIS buoy Bravo and then extended...
to other platforms. To calculate skewness we convolve integrals of the spectrum following (Babanin & Polnikov, 1995; Srokosz & Longuet-Higgins, 1986). In the following plots, we do not discriminate different data cases (i.e. fetch limited, fully developed), so each plot includes all available data. Here is skewness vs. non-dimensional steepness with the aforementioned theoretical relationships plotted on top.

![Bravo Skewness Steepness and Theoretical Relationships](image)

*Figure 9.32) Scatter plot of skewness vs. steepness.*

In the figure above, there is a clear relationship between skewness and significant steepness. There is quite a bit of scatter in the data, but the theoretical skewness of Phillips spectra seem to represent a lower limit steepness for skewness greater than 0.1. By looking at the same data color coded by inverse wave age, it is shown that the scatter is dependent on the inverse wave age (defined as $\cos(\theta_d)U_{10}$ where $\theta_d = \theta_{U10} - \theta_p$):
Figure 9.33) Scatter plot of skewness vs. steepness with inverse wave age shown by color.

The figure above shows that when the waves and winds are in line, the skewness dependence on steepness collapses onto a line. When the waves and winds are opposing, there is increased steepness relative to skewness for strong opposing wind forcing. Physically, the winds are holding the forward faces of the waves; increasing the steepness and also increasing horizontal skewness but to a lesser extent. This implies that horizontal skewness is the dominant form of asymmetry, though it cannot be proven. Steepness may be represented by another measure, significant steepness which we will define again here:

$$S_s = \frac{2\pi H_{m0}}{T_Z^2}$$

For this measure, there is decreased scatter in the relationship with skewness and a clear dependence on wind speed:
When the wind speed is greater than about 6 m/s, there is a strong linear relationship between Steepness and Skewness.

The fit above is $\gamma_1 = 3.57S_2 - 0.050$ and the data has an $R^2$ value of 0.985. This is interesting because one is able to determine skewness from two
simple spectral parameters namely \( H_{m0} \) and \( T_z \), but is the relationship general? A better question is what else might skewness depend on? One might contend that the reference frame or the relative depth of the sensor may affect skewness. To check this we may look at the correlation between skewness and 2 non-dimensional depth measures:

\[
\begin{align*}
D_1 &= d k_p \\
D_2 &= \frac{g d}{U_{10}^2}
\end{align*}
\]

For \( D_1 \), \( R^2 = 0.254 \) and for \( D_2 \), \( R^2 = -0.754 \) (scatter plots not shown). It is suspected that \( D_2 \) correlation is spuriously related to a formulation involving \( U_{10} \), and that there is actually no correlation between skewness and non-dimensional depth. One does expect increasing vertical asymmetry to increase with decreasing depth, but the depth range at this buoy was not shallow enough to enhance vertical asymmetry greatly with \( \min(D_1) \equiv \min(D_2) \equiv 0.7 \). We may also look at data from the other buoys to check if the fit is independent of depth:

![Figure 9.36](image) Scatter plot of skewness vs. steepness for Bravo, Romeo, Yankee, and NDBC station 44014 in blue, red, green, and black, respectively. Linear fits are also shown.
In the plot above, the fits have very similar slope, but the intercept is slightly different for the ASIS buoys and greatly different for the NDBC buoy. It is unclear, but the intercept may be related to systematic difference in the buoys. The fit parameters for each buoy are listed in the table below.

<table>
<thead>
<tr>
<th>Buoy/Fit Parameter</th>
<th>Slope</th>
<th>Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bravo</td>
<td>3.4688</td>
<td>-0.0406</td>
</tr>
<tr>
<td>Yankee</td>
<td>3.4025</td>
<td>-0.0400</td>
</tr>
<tr>
<td>Romeo</td>
<td>3.3913</td>
<td>-0.0471</td>
</tr>
<tr>
<td>NDBC Station 44014</td>
<td>3.4791</td>
<td>-0.0154</td>
</tr>
</tbody>
</table>

Table 9.7) Linear fit parameters for each buoy.

This section will conclude with a few notes. First, this special relationship was only produced when the data could be divided up for cases with wind speeds higher than 6 m/s. So the buoy has to simultaneously record wind and waves locally for this relationship to be apparent. For example even when the wind data from NDBC 44014 was matched up to a DWR the relationship was consistent. In this analysis, a strong correlation between skewness and steepness and skewness and wind speed for all buoys was found in contrast to (Drennan et al., 2003). Also, the shallowest buoy (FRFDWR d=17m) showed some relationship between non-dimensional depth and skewness for the low end of skewness. When all data from all buoys are taken together, the slope of the fit appears to be dependent on the Longuett-Higgins measure of spectral bandwidth

\[ \nu_f = \sqrt{\frac{m_0 m_2}{m_1^2}} - 1 \] as seen in the following plot:
In addition to spectral bandwidth, skewness should also be dependant on directional spreading (A.J.H.M. Reniers 2012, personal communication). In conclusion, a strong relationship between steepness and skewness in wind regimes higher than 6 m/s has been discovered. This relationship may be useful for determining skewness with only 2 spectral parameters. The skewness measured by other platforms, the dependence on depth, and the dependence on spectral width were explored during this analysis, and the door was left open for future work.

9.4 SHOWEX General Conclusion

The comparison was performed for three buoys including the ASIS buoy Bravo and the two surrounding DWRs. The comparison was good for spectral parameters except for directional spread at the peak where the ASIS buoy was very high biased. WET component plots showed interesting patterns of bias and
std for different parameters. The study was mostly consistent with previous studies.

A case study looking at surface skewness was performed which was inspired by (Drennan et al., 2003). Very strong relationships were found between skewness and steepness. This was found to be dependent on other measures such as wind speed, spectral width, and inverse wave age. Contrary to (Drennan et al., 2003), all buoys showed these relationships.

A good case study for this experiment would be to establish the dynamic ranges and identify cases of fetch limited, swell dominated, mixed seas. One could fit the measurements to non dimensional growth curves for high sea state periods or over the whole experiment. This line of thought was abandoned due to inconsistencies in wind direction between the ASIS buoys and NDBC station 44014, but should be investigated further.
10 Chapter: SW06

This chapter describes work that has been done analyzing data from the Shallow Water experiment that took place in 2006 (SW06). The first section is an introduction to the data set. The second section is a detailed comparison of the measurements from two ASIS buoys. The final section is a case study of wave properties related to tropical storm Ernesto.

10.1 Introduction to SW06

SW06 was a multi-institutional experiment that took place from July to September in 2006 on the Mid-Atlantic Bight (MAB) off the coast of New Jersey. The experiment was a joint investigation of acoustic propagation and non-linear internal wave physics. There were arrays of acoustics sensors (see the blue dots in figure 10.1) and a scientific research vessel involved. The measurement of surface waves was a primary objective for SW06. Therefore there were only two buoys which provided wave data. ASIS buoys, Romeo and Yankee, were deployed and moored approximately 11 kilometers apart. Romeo was moored at depth of approximately 71 meters and Yankee at depth of approximately 78 meters. Because wave measurements were not considered important to the scientific objectives, no papers have been published utilizing the wave data taken during this experiment. This provides a twofold unique opportunity. First all work this section is new because the dataset remains unexplored. Secondly, some of the data was collected during some very high sea states as a result of the passage of tropical storm Ernesto and may augment the very few high sea-state measurements.
The ASIS buoy Romeo was located at 39 01.1483 °N, 73 03.2127 °W and ASIS buoy Yankee was located at 39 04.434 °N, 73 09.846 °W. The two Romeo and Yankee collected data continuously for approximately 40 days, and they recorded a variety of meteorological and oceanographic data. For most of the experiment duration, the weather and sea state were relatively calm (on the order of 1 meter Hs). There was an exceptional increase in the Hs and wind speed when tropical storm Ernesto traveled near the SW06 experiment area in the beginning of September 2006.
In the figure above, Yankee data is represented by green markers and Romeo data by dark blue markers. The entire experiment duration is shown. The top box in the figure above shows the neutral wind speed and direction at 10m height. The second box is wind direction. The third and fourth boxes are Pressure and friction velocity, respectively. Air and sea temperature as measured by both Romeo and Yankee buoys are in the bottom box. The most notable event was centered on year day 244. That was the time period associated with the passage of tropical storm Ernesto. The wind speed reached over 20 m/s and the $H_{m0}$ was nearly 8m.
10.1.1 Wave Wire Timeline and Wave Parameters

As described in the detail in the background section, the directional resolving ability of an ASIS buoy hinges on the functionality of the pentagonal array of wave wires. There was a mass failure of wave wires during this experiment, and therefore the directional resolution was compromised. The wave wire timeline for each buoy is as follows:

<table>
<thead>
<tr>
<th>Romeo Spectra</th>
<th>Working Wave Wires</th>
<th>Time Period (Year Days)</th>
<th>Duration (Days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-D</td>
<td>1 2 3 4</td>
<td>213.4847-222.2125</td>
<td>8.7278</td>
</tr>
<tr>
<td>1-D</td>
<td>1 2 3</td>
<td>222.2125-224.6424</td>
<td>2.4299</td>
</tr>
<tr>
<td>1-D</td>
<td>1 3</td>
<td>224.6424-251.5438</td>
<td>26.9014</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>38.0591</td>
</tr>
</tbody>
</table>

Table 10.1) Directional data was available for the first 8 days from Romeo. At which point only the 1-D spectra can reliably determined. The wires not listed were not functioning during that period.

<table>
<thead>
<tr>
<th>Yankee Spectra</th>
<th>Working Wave Wires</th>
<th>Time Period (Year Days)</th>
<th>Duration (Days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-D</td>
<td>1 3 4 5 7</td>
<td>210.6924-223.4083</td>
<td>12.7159</td>
</tr>
<tr>
<td>1-D</td>
<td>3 4 5 7</td>
<td>223.4083-245.7188</td>
<td>22.3105</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td>245.7188-245.8049</td>
<td>0.0861</td>
</tr>
<tr>
<td>1-D</td>
<td>3 4 5 7</td>
<td>245.8049-249.1271</td>
<td>3.3222</td>
</tr>
<tr>
<td>1-D</td>
<td>3 4 7</td>
<td>249.1271-251.3819</td>
<td>2.2548</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>40.6895</td>
</tr>
</tbody>
</table>

Table 10.2) Sometimes it happens that the wires which are compromised leave the buoy array unable to resolve the directional spectrum. Sometimes this happens right from the get-go as with the Yankee buoy. There is also one hour where there is a voltage spike in all of the wave wires. This hour is deleted from the data set for the inter-comparison. The wires not listed were not functioning during that period.

The loss of key wave wires rendered the buoys unable to resolve directional properties of the sea surface. Despite the loss of directional wave information, the 1-D spectra were reliably measured by each buoy. The following figures show the wave parameters and spectral evolution plots.
Figure 10.3) Wave parameters during SW06.

The figure above shows the time-series of the significant wave height, peak frequency, mean direction at the peak (Romeo only), and the spectral evolution from each buoy. The significant wave height compares well although it appears that Romeo consistently measured a slightly higher wave height than Yankee. The peak frequency also compares well between the buoys, but there were times when Romeo’s peak frequency increased relative to Yankee, most notably around year day 222 and 224. It may be seen from the spectral evolution plots that these time periods were associated with bimodal seas. There was an increase in energy in the high frequencies in the Romeo data which was also present in the Yankee data but in a less pronounced fashion. Consequently, Romeo was more susceptible to choosing a frequency in wind sea band as the
peak. The spectral evolutions are plotted in log space where the color indicates the exponent, and are qualitatively very similar between the two buoys.

10.2 Quantitative Comparison

The two ASIS buoys rely on the same measurement principles, were relatively close to each other, took simultaneous measurements for over a month, and were identically processed. By looking at two similar buoys in the same place at the same time many of the sources of discrepancy mentioned in section 3.1 are eliminated. Any variability (greater than sampling variability) in the measurements made by the two systems must either be error (i.e. set-up, calibration, noise) or due to the natural variations on the ocean surface. By analyzing the differences between ASIS buoys the hope is to 1) validate the ASIS buoy as a consistent and reliable wave measurement system, and 2) gain insight into how buoy design or set-up may be improved.

10.2.1 Scatter Plots and Data Tables

The two spectral parameters chosen for detailed comparison of significant wave height, $H_s$, and peak frequency, $f_p$. $H_s$ was calculated in the usual way:

$$H_s = H_{m0} = 4\sqrt{m_0}$$

$f_p$ was calculated following (Young, 1995) (see section 4.2 for a detailed explanation):

$$f_p = \frac{\int_0^\infty f S^4(f) df}{\int_0^\infty S^4(f) df}$$

The plot (figure 10.4) shows the $H_s$ scatter plot.
The figure is the scatter plot for $H_s$. The correlation between the two was very high, but there was some significant bias present in the data. This was by far the highest bias (~2.5X) seen in this thesis. The bias will undergo further analysis in a later section. The std is also high, but this is to be expected from because of larger range of wave heights (as compared to the previous experiments). The P90b shows that nearly 70% of the data lie within the confidence region when bias is taken into account. We may conclude that most of the bias is due to the sampling variability of the parameter, but there is still some variability that goes unexplained.
Table 10.4) The table above describes the goodness of fit parameters as well as the statistical difference measures for $f_p$. The statistical difference measures are bias, standard deviation (std), and root-mean-square-error (RMSE). Calculation of various statistics is described in section 4.1.1. The remaining column headers above are the following: # pts is the number of data points used in the comparison, $R^2$ is the correlation coefficient, and slope and intercept refer to the fit parameters of the ML regression.

The plot above shows the $f_p$ scatter plot. This comparison is acceptable, but the agreement is clearly not as strong as the agreement observed in the previous experiments. As was mentioned earlier, some of the disagreement is associated with bi-modal seas where Romeo is obviously choosing the higher frequency seas and Yankee lower frequency seas. To get 80% of the data within the 90% probability region the COV must increase from 0.05 to 0.08. We conclude that most of the variance was accounted for in the sampling variability of the parameter, but clearly not all of it.
10.2.2 WET Component Plots

Here we use the WET program to look at difference in the data as a function of frequency and energy.

This is a plot of the concurrent observations of the two buoys. Each number represents the number of hours that Romeo sensed waves in that particular energy-frequency component. The number is essentially the degrees of freedom when calculating statistical comparison (i.e. std or bias). The threshold value to make a comparison was set to 10 concurrent hourly observations.
Figure 10.7) Wave height percent bias component plot for Yankee relative to Romeo during SW06.

The above component plot shows the percent bias of wave height of Yankee relative to Romeo. The bias is negative, meaning Romeo measures higher energy and wave height in almost all energy-frequency components. The bias seems to drop off in the frequency range of .07-.11 Hz. This was around the frequency range that surface elevation signal is dominated by the motion of the buoy. The bias increased outside of this range. The percent bias increases as frequency increases. The surface elevation signal is dominated by the wave wire signal as the frequency bands increase. This suggests that the wave wires (perhaps calibration) are responsible for the bias in higher frequency range.
10.2.3 Comparison Discussion

Overall, the comparison between buoys was good but somewhat less than previous agreement seen between ASIS buoys. Although when the results examined closely there were a few relative std and bias issues. %std increased in the low frequency/low energy components. The waves in these components are very long and not very high. The signal to noise ratio for any wave sensing device is decreased in this measurement domain, so an increase in RMSE is to be expected (personal communication with W.C. O'Reilly 2010).

The average bias of Yankee in $H_b$ is -0.243 m when compared to Romeo. What could explain the observed bias? The buoys were separated by
approximately 11 km. Nominally inter-comparisons should be between co-
located buoys, so it is possible that the spatial offset between the buoys may
explain the presence of bias in the measured wave heights. There were similar
distances between buoys in previous experiments but such strong biases were
not observed. This suggests the possibility of systematic error, but we will first
consider other plausible explanations. As a result of the spatial offset, there are
some natural processes which may explain the measured differences. The
processes are shoaling, wave growth, and an inhomogeneous wave field. There
was also the possibility that some of the data in one or both of the buoys was
outside of the dynamic range of the wave wires, which effectively reduces the
calculated significant wave height. If the Yankee buoy had more out of range
data, then it would explain some of the negative bias. Besides bias due to spatial
offsets and out of range data, the third possibility is that calibration of either the
motion package and/or the wave wires was not done perfectly. Let's explore
each possibility in detail.

Is it possible that shoaling might explain some of the bias? The Romeo
and Yankee were at a depth 71 and 78 meters respectively. But, at this depth
only waves longer than \( \sim L = 150 \text{m} \) (\( \sim T = 10 \text{s} \)) would actually “feel” the bottom. In
fact, the difference in the shoaling coefficients for \( \sim L = 450 \text{m} \) (\( \sim T = 20 \text{s} \)) wave in 78
m depth and 71 m depth is only 0.005. For a 10m wave this would be a height
difference of 5 cm. Shoaling cannot explain the bias. Other shelf processes
(e.g. interaction with the bottom, Bragg scattering off of irregular topography)
would attenuate waves not increase them.
Is it possible that wave growth might explain some of the bias? This question is more difficult to answer. If there were strong, continuous, and consistent South-East winds then 11 km difference in fetch may show up as increased wave height for Romeo. Of course, if this set of circumstances happened during the experiment, it did not happen in any consistent way. The wave growth conditions (i.e. wind speed, direction, fetch, and duration) were so varied throughout the experiment that difference would show up as random error, not a consistent bias. As Tropical Storm Ernesto approached, it sent strong winds in an east direction, and the difference in wave heights are made worse during this time, but this accounts for very little of the total bias in the data.

The sea-state is naturally inhomogeneous (spatially varying) across the ocean. The amount of natural variability depends on the spatial coherency or decorrelation length of the random wave field and the various properties of the co-existing wave systems, but there is no evidence that this would show up as bias in data. I am inclined to rule out a natural explanation for the bias. It is possible that exceeding the dynamic range (which can happen for a number of reasons) of a buoys wave wire would result in a reduction of wave height. The following graphs show the percentage of out of range data for Romeo and Yankee, respectively:
Figure 10.9) Percentage of data that exceeded the dynamic range of the Yankee wave wires. The figure above shows the out of range data for the Yankee buoy during SW06.

The spike in Wave Wire 3 in between year day 210 and year 215 is due to the deployment of the buoy. As can clearly be seen, the out of range data are comparable and occur at similar times. If anything, Romeo has slightly more out of range data, so this cannot explain the bias.

Figure 10.10) Percentage of data that exceeded the dynamic range of the Romeo wave wires. The figure above shows the out of range data for the Romeo buoy during SW06.
Just looking at the average bias does not tell us much about the possible explanations. This is where the results from WET become advantageous. Looking at the wave height percent bias component graph, the bias is relatively small where the heave motion of the buoy would be dominant and increases as a function of frequency. This would lead me to rule out the motion package as a source of bias. It was observed that as the signal from the wave wires became the dominant signal in computing surface elevation the bias increased. This leads to the conclusion that wave wires are at fault, and supports the argument that the bias is from imperfect wave wire calibration.

Although the above argument is compelling, it is not straight forward to check by isolating the influence of wave wire calibration. A major complication is that sea surface elevations from each wire are averaged to produce spectra, and the sea surface elevation at each wire is a combination of buoy heave, wave wire elevation, and rotation. To explore the issue, the voltage signal from each working wave wire was averaged together and then averaged over the entire experiment. The mean voltage for Romeo was 0.067 and the mean voltage for Yankee was -0.067. This supports the argument above, but after converting the voltage to meters the mean difference between the wave wire voltage amounts to about 4 cm. Citing the complications above, it is not possible to say quantitatively how this difference influences the spectra in the end.

An explanation that was not discussed is wave-current interaction. The current measurements during the experiment were considered unreliable. Therefore there were no observations to disclose the influence of currents. Both
buoys were located on the mid Atlantic bight, presumably strong currents are not
common, and if present the currents would probably affect the buoys similarly.

To conclude the section, wave data from two ASIS buoys were compared. A
variety of approaches were used to explore the data, and the agreement was
reasonable but not perfect. Strong biases in the wave significant wave heights
were observed. Using the WET program, one could see that the bias increased
as a function of frequency. After ruling out many possible natural and
mechanical explanations, it was found the systematic error was probably due to
wave wire calibration. It is not possible to know which buoy (if either) more
accurately reflected the sea-surface.

10.3 Case Study: Tropical Storm Ernesto
Hurricane Ernesto developed from a tropical depression in the Caribbean
and tracked northward through south-east Florida. Due to the storm track and
the position of the buoys, distant swell did not make an impact on the buoys. It
wasn’t until “direct contact” with the winds of the storm, which was weakening
over land as a tropical storm, that the buoys sensed any effect from Ernesto. I
will typically refer to Ernesto as a tropical storm since the data relevant to this
study was taken while Ernesto was classified this way. Tropical cyclones in the
northern hemisphere rotate counter-clockwise and the storm was located to the
south of the buoys. Therefore the winds were steady from the east and then took
a sudden drop as combined effect of the storm advecting away to the West and
simultaneously dissipating over land.
The pressure dropped, but not as precipitously as one might expect with a strong low pressure system because the center of the low pressure did not pass directly over the buoys. The wind picked up in the beginning of year day 242, but it wasn’t until year day 243 that there was change in the ambient wave height. This was due to residual sea state associated with 10 m/s winds only days before the arrival of the tropical storm. The previously mentioned sudden drop in wind speed can be seen around year day 245. It is of interest to note that the seas did not dissipate as suddenly as the wind and remained quite high after winds had dropped considerable. This sudden change of sea-state from highly wind forced to mature seas may be of interest in a future study.
It was stated in (Ochi, 2003) that higher ambient wave conditions contribute to increased wave growth. He developed parametric relationships between wind speed and $H_{m0}$ depending on the ambient sea-state over the previous weeks. For a calm sea-state:

$$H_{m0} = 0.24U_{10}$$

Or for a disturbed sea-state:

$$H_{m0} = 0.078U_{10}^{1.57}$$
Since the wave conditions were quite high prior to the arrival of the tropical storm, I would expect the disturbed sea-state relationship to better fit the data from Ernesto. The two relationships stated above are plotted in the following figure with the data during Ernesto:

![Figure 10.13) Wave height vs. 10 meter wind speed during tropical storm Ernesto. Ochi’s relationships for calm and disturbed sea-states are shown in dashed and solid lines, respectively.](image)

The plot above shows the observed relationship between $H_{m0}$ and $U_{10}$ for Romeo and Yankee during the growing stages of tropical storm Ernesto (Year Day 243-244.2). Also shown are the parametric relationships from (Ochi, 2003). Neither form matches our observations exactly. The calm sea-state form intersects the observations but the slope is significantly different, and the slope from the disturbed sea-state looks better but the numbers do not agree with observations. (Young, 2003) points out that Ochi’s calm sea-state relationship “may be less reliable in regions distant from the maximum wind areas of storm” which is almost certainly the case here.
We may also look at the appropriateness of non-dimensional growth curves compared to our data. Seas due to tropical storms are complex due to the natural variability and structure of tropical storm winds. The winds decrease with distance from the center of low pressure and rotate in a counter-clockwise fashion around this center. The center of low pressure itself is translating in space. This makes for very dynamic wind conditions and it is difficult in most circumstances to find conditions in which waves may be described as purely fetch or duration limited. This being said, (Young, 2003) finds that the fetch-limited relationships proposed by (K. Hasselmann et al., 1973) and (Donelan et al., 1985) apply remarkably well to active generation of waves by tropical cyclones. To see if our data supports this claim we calculate the appropriate non-dimensional parameters and plot them with the proposed relationships.

The JONSWAP relation derived from their extensive field experiment in the North Sea is $\varepsilon = 7.13 \times 10^{-6} \nu^{-3.03}$. The Donelan relationship derived from what is considered very high quality data on lake Ontario is $\varepsilon = 7.13 \times 10^{-6} \nu^{-3.03}$. Where $\varepsilon$ is non-dimensional energy and $\nu$ is non-dimensional peak frequency:

$$\varepsilon = \frac{g^2 E}{U_{10}^4}$$
$$\nu = \frac{f_p U_{10}}{g}$$

Our data can be seen in the following plot:
In the plot above we show non-dimensional energy vs non-dimensional peak frequency. The dark and light grey dots represent data for the whole experiment and the red and green dots highlight data associated with tropical storm Ernesto for Romeo and Yankee, respectively. In the discussion we only consider the data from Ernesto, but the rest of the data is presented for completeness. The JONSWAP relationship fits Yankee’s data from quite well and is somewhat low for Romeo. The Donelan relationship finds itself riding the upper portion of Yankee and the lower portion of Romeo. In general our data supports the findings of (Young, 2003) and others which find growth relationships during tropical cyclone conditions similar to fetch-limited conditions.

Lastly, let us take a look at the skewness data from this experiment. Does the skewness relationship found in SHOWEX change due to the high wind speeds related to Ernesto?
Figure 10.15) Steepness vs. skewness during tropical storm Ernesto.

It was found that the slope was closer to 3.1 for SW06 data (it was ~3.4 during SHOWEX) for all cases with $U_{10} > 6$ m/s. For Romeo the fit was essentially the same when only the subset of data from Ernesto was considered. The slope of the fit was slightly less steep for Yankee for the same Ernesto subset. See the table below:

<table>
<thead>
<tr>
<th>Data</th>
<th>Slope</th>
<th>Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>Romeo ($U_{10} &gt; 6$ m/s)</td>
<td>3.0644</td>
<td>-0.0304</td>
</tr>
<tr>
<td>Yankee ($U_{10} &gt; 6$ m/s)</td>
<td>3.1098</td>
<td>-0.0231</td>
</tr>
<tr>
<td>Romeo (Ernesto)</td>
<td>3.0412</td>
<td>-0.0304</td>
</tr>
<tr>
<td>Yankee (Ernesto)</td>
<td>2.7710</td>
<td>-0.0011</td>
</tr>
</tbody>
</table>

Table 10.5) The table shows the linear fit parameters for data from SW06.

For now, the discrepancy between SW06 and SHOWEX goes unexplained.

10.4 Discussion and Conclusion

A comparison between 2 ASIS buoys during the SW06 experiment was performed. There was significant bias in the wave heights and higher std, but the range of wave heights (up to 8 m) was also significantly higher than ranges from
other experimental data sets found in this thesis. After many explanations were explored, the bias was found to most likely be due to imperfect calibration of wave wires. One obvious recommendation is that very careful wave wire calibration should be performed on each ASIS buoy before and after future experiments.

Data associated with the passage of tropical storm Ernesto was explored. The fetch-limited relationship of JONSWAP (1973) matched the data from Yankee well during the storm. The fetch-limited relationship from Donelan (1985) was too high for Yankee. Both relations were too low for Romeo. This hints that perhaps Romeo is overestimating the energy. If this is true then the bias seen in the comparison is due to Romeo, but the evidence is not conclusive. The Ernesto case study supports evidence that fetch-limited growth relations are well suited to tropical storm data. Skewness as related to steepness was shown. The relationships were close but significantly different from the data fits from SHOWEX.

In the future, it would be interesting to do some time domain analysis and look at the statistical distributions of waves. Are the distributions significantly different in storm conditions? Is there an increase in extreme waves (are they present at all)? If there are extreme waves, what is their form (e.g. height relative to $H_s$, steepness)? Since 20 Hz time series of surface elevation is available from both ASIS buoys during this experiment this kind of analysis can be done, but is a matter of future research.
11 Chapter: General Discussion and Conclusion

The general objective of this study was to perform comprehensive sensor comparisons and increase knowledge of wind waves. This was done by looking at the available body of literature and trying to extend current findings.

This thesis was based on data from 3 field experiments: GOM99, SHOWEX, and SW06. The core of the thesis was the sensor inter-comparisons. For GOM99, 3 ASIS buoys surround a NDBC 3-meter discus station 44014. During SHOWEX, an ASIS buoy was compared to 2 surrounding DWRs, and during SW06 two ASIS buoys were compared. Many traditional comparison techniques were utilized including simultaneous plots of time series, spectral and directional evolution plots, parameter scatter plots, statistics, correlations, and average spectra, average mean direction, and average directional spread plots. This thesis also benefited from a new inter-comparison tool, WET. Chapter 5 was dedicated to testing the WET program using a synthetic data set and recommendations were made to improve the program.

The consensus of previous wave sensor comparison studies is that when sampling variability is accounted for there is high agreement for 1-D bulk spectral parameters for the range of sea-states (up to 10m) studied. For 1-D spectra in the range 0.10-0.20Hz there is relative good agreement, and there is little agreement outside this frequency range. The mean direction at the peak of the spectrum agrees within ±5°. There is not good agreement on the directional spread at the peak between different platforms, but there is good agreement for
inter-platform agreement. There has been little study of mean direction and directional spread as a function of frequency.

The findings from three experiments in this study support the previous findings in general. The agreement in $H_s$ was very good with exception of some bias between ASIS buoys during SW06 which was probably due to imperfect wave wire calibration. Peak parameter comparisons were affected by multimodal seas, but overall agreement was good. When directional data was available, buoys agreed on mean direction at peak within the range found in previous studies. The ASIS buoys were seen to have significant bias when estimating directional spread at the peak of the spectrum and possible explanations were explored in section 7.2.5. The WET program showed that the NDBC buoy and DWR buoys overestimated the energy of the high frequency waves and under estimated the energy of low frequency waves relative to ASIS buoys. Other interesting patterns were seen in WET component plots.

Some short data studies were performed during each experiment. During GOM99, a slanting fetch was observed during stationary conditions. The sampling variability was estimated for different wave parameters during the same stationary period. This sampling variability was used in SHOWEX and SW06 for establishing confidence regions on scatter plots.

During SHOWEX the skewness of surface elevation was investigated for an ASIS buoy. Skewness and steepness displayed a strong relationship. Other relationships were also examined, and the analysis was extended to other platforms. During SW06 the buoys recorded data associated with tropical storm
Ernesto. The observations compared well with fetch-limited growth curves. A relationship between skewness and steepness found in SW06 data was similar to but different from the relationship established in SHOWEX. The data from each experiment was explored in detail and there is room for much future work.

11.1 Work Beyond the Scope of this Thesis

It may be cliché, but one always discovers more questions than answers during an investigation. This section points out some lines of research that could be continued from this body of work. Suggestions which were documented in the bulk of the thesis will be summarized and some new ones will also be made here.

The suggested changes to the WET program, made in Chapter 5, should be implemented and documented. The most important of which is figuring out a way to include sampling variability in the std component plots.

There is some work to be done on GOM99 data set. The COVs were derived for one buoy, but the same analysis should be extended to the other 3 buoys. A study could take a critical look at the common assumptions made in wave analysis specifically homogenous, stationary, and ergodic conditions.

There is an opportunity to take a second look at the fetch-limited conditions during SHOWEX and compare to findings in previous studies. It would also be interesting to take a detailed look at shoaling wave conditions during the experiment.
Time domain analysis for each of the data sets would compliment this work. Do the different platforms give slightly different wave height or wave period distributions? How do they match theoretical distributions?

It may be interesting to eliminate the influence of the MLM on the ASIS data by obtaining the Fourier coefficients directly from cross-spectra. This would reveal more about the differences of the sea-states and measurement systems, and perhaps reveal something about the influence of MLM.

In general, there is very little known about very large waves. There is a tremendous opportunity to learn about large waves generated by tropical cyclones especially the directional properties of these waves. Fortunately there is a high quality data set collected during the Impact of Typhoons On the Pacific (ITOP) experiment in 2010. I intend to spend much of my future effort exploring this data.
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