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The Metacognitive Functioning of Middle School Students with and without Learning Disabilities During Mathematical Problem Solving

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THE METACOGNITIVE FUNCTIONING OF MIDDLE SCHOOL STUDENTS WITH AND WITHOUT LEARNING DISABILITIES DURING MATHEMATICAL PROBLEM SOLVING

By

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A DISSERTATION

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THE METACOGNITIVE FUNCTIONING OF MIDDLE SCHOOL STUDENTS
WITH AND WITHOUT LEARNING DISABILITIES DURING MATHEMATICAL
PROBLEM SOLVING

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The purpose of this study was to investigate the metacognitive functioning of students with learning disabilities (LD), low-achieving (LA) students, and average-achieving (AA) students within the context of math problem solving. Metacognition, that is, the awareness individuals have regarding their own mental processes and ability to self-regulate performance, is an important predictor of learning. Deficits in metacognition have been attributed to an inability to effectively balance the cognitive and metacognitive strategies necessary for successful problem solving. Students with LD have considerable difficulty with self-regulation. This study investigates three components of metacognition: metacognitive knowledge, metacognitive experience, and metacognitive skills. The differences in these components among students with LD ($n = 15$), LA students ($n = 38$), and AA students ($n = 29$) and their influence on students’ math word problem solving was studied. Furthermore, the relationships among the three components of metacognition were investigated in the context of ability group differences. To assess metacognitive functioning, students were administered a structured interview and a survey and they solved
three math word problems while thinking aloud. Additionally, to assess math problem-solving ability, students were administered a 10-item math word problem-solving test. Results indicated that students with LD demonstrated a different pattern of metacognitive function than AA students and LA students. Students across ability groups look relatively equivalent in the quantity of metacognitive skills. However, when discriminating between the type and quality of the metacognitive skills employed, ability group differences were evident. Ability group differences in metacognitive functioning emerged with respect to problem difficulty. The directions of the relationships among the components of metacognition were the same across ability groups. However, the magnitude and strength of the relationships differed by ability. Additionally, metacognitive knowledge was a significant predictor of math word problem-solving performance for AA students, but not for the other ability groups. Furthermore, there was a significant difference in the relationship between metacognitive experience and math word problem solving for students with LD and AA students. Educational implications are discussed for teaching students to use metacognition during problem solving.
# TABLE OF CONTENTS

LIST OF FIGURES........................................................................................................... v  
LIST OF TABLES............................................................................................................... vi  
ABBREVIATIONS............................................................................................................. vii  

## CHAPTER 1  
INTRODUCTION TO THE STUDY............................................................................... 1  
Flavell’s Tripartite Theory of Metacognition......................................................... 4  
Montague’s Model of Effective Problem Solving in Mathematics…..………………… 6  
Problem Solving and Students with LD................................................................. 8  
Statement of the Problem....................................................................................... 10  

## CHAPTER 2  
LITERATURE REVIEW.................................................................................................. 11  
Metacognition and LD.............................................................................................. 13  
  Metacognitive knowledge............................................................................... 13  
  Metacognitive experience.......................................................................... 15  
  Metacognitive skills.................................................................................... 17  
Metacognition and Measurement........................................................................ 21  
Research Questions.............................................................................................. 26  

## CHAPTER 3  
METHODOLOGY............................................................................................................ 27  
Participants............................................................................................................. 27  
Procedure............................................................................................................... 29  
  Transcription and coding................................................................. 32  
    Think-aloud protocols.......................................................... 32  
  Math Problem Solving Assessment – Short Form….………………… 34  
Measures  
  Metacognitive skills........................................................................... 35  
  Metacognitive experience.................................................................. 35  
  Metacognitive knowledge.................................................................. 37  
  Math word problem solving.................................................. 37  
Design and Analysis  
  ANOVA........................................................................................................... 38  
  Mixed-design ANOVA........................................................................... 39  
  Correlation Coefficients-----------------------------------------……………... 39  
  Regression---------------------------------------------------------------……………... 40
# LIST OF FIGURES

## CHAPTER 1

| Figure 1.1 | Flavell’s Tripartite Theory of Metacognition | 3 |
| Figure 1.2 | Montague’s Model of Successful Math Problem Solving | 3 |

## CHAPTER 3

| Figure 3.1 | Data Collection Procedures | 30 |

## CHAPTER 4

| Figure 4.1 | Ability X Difficulty: Metacognitive Experience | 47 |
| Figure 4.2 | Metacognition Type X Difficulty X Ability | 50 |
| Figure 4.3 | Relationship between Metacognition and MWPS:  
All Abilities | 58 |
| Figure 4.4 | Relationship between Metacognition and MWPS:  
By Ability | 62 |

## CHAPTER 5

| Figure 5.1 | Two Coded Think-Aloud Protocols: LD | 74 |
# LIST OF TABLES

## CHAPTER 3

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 3.1</td>
<td>Participant Information</td>
<td>28</td>
</tr>
<tr>
<td>Table 3.2</td>
<td>Procedures</td>
<td>30</td>
</tr>
</tbody>
</table>

## CHAPTER 4

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 4.1</td>
<td>Means and Standard Deviations: Metacognition</td>
<td>43</td>
</tr>
<tr>
<td>Table 4.2</td>
<td>Mixed-Design Summary Statistics: Metacognitive Experience</td>
<td>44</td>
</tr>
<tr>
<td>Table 4.3</td>
<td>Means and Standard Deviations Ability X Metacognition X Difficulty</td>
<td>45</td>
</tr>
<tr>
<td>Table 4.4</td>
<td>Mixed-Design Summary Statistics: Metacognitive Skills</td>
<td>48</td>
</tr>
<tr>
<td>Table 4.5</td>
<td>Means and Standard Deviations: Metacognitive Skill</td>
<td>49</td>
</tr>
<tr>
<td>Table 4.6</td>
<td>Correlation Matrix</td>
<td>54</td>
</tr>
<tr>
<td>Table 4.7</td>
<td>Regression: Summary Statistics</td>
<td>61</td>
</tr>
</tbody>
</table>
ABBREVIATIONS

AA – Average-Achieving Students
LA – Low-Achieving Students
LD – Learning Disabilities
ME – Metacognitive Experience
MK – Metacognitive Knowledge
MS – Metacognitive Skill
MWPS – Math Word Problem Solving
MWPSP – Math Word Problem Solving Probes
NPMV – Non-Productive Metacognitive Verbalizations
PMV – Productive Metacognitive Verbalizations
CHAPTER 1

Introduction to the Study

The purpose of this study is to investigate three components of metacognition, specifically knowledge, experience and skill, and examine the differences among students with learning disabilities (LD), low-achieving students (LA) and average-achieving (AA) students in the area of mathematical problem solving, as well as the relationship among these components and their influence on academic performance. In this chapter the three components of metacognition are discussed within a theoretical framework that draws upon Flavell’s tripartite theory of metacognition (1979; see Figure 1.1) and Montague’s model of effective math word problem solving (1993; see Figure 1.2). Because the primary focus of this study is the performance of students with LD, an overview of their problem-solving difficulties is provided.

Metacognition is important for academic success (Pintrich, Anderson, & Klobucar, 1994; Trainin & Swanson, 2005), problem solving and ultimately, academic achievement (Decorte, Greer & Verschaffel, 1996; Lucangeli & Cornoldi, 1997; Swanson, 1990). Research suggests that metacognition develops alongside general aptitude and may be more predictive of learning performance than intelligence (Swanson, 1990; Vennman & Spaans, 2005). Intelligence may contribute as little as nine to 25% of the explained variance in performance, whereas metacognition may account for as much as 75% (Van Luit & Kroesbergen, 2006). Metacognition, that is, higher-order thinking strategies that serve to control cognitive processes, consists of three components,
metacognitive knowledge, metacognitive experience and metacognitive skill (Flavell, 1979; Lucangeli & Cabrele, 2006). Metacognition is useful in helping students become active participants in their learning by developing curiosity, persistence, creativity and strategic thinking (Jones & Idol, 1990).

In 2000 the National Council for Teachers of Mathematics (NCTM) released *Principles and Standards for School Mathematics*, its fourth standards document, which advocated a move away from the traditional basic skills approach to the teaching and learning of mathematics to a problem-solving approach that fosters conceptual understanding. These new goals encourage teachers to develop students’ ability to understand and make connections across math concepts as well as actively engage in meaningful communication about math. Specifically, students are required to analyze the relationship among problem parts, represent problems, decide on solution paths, monitor progress, self-question, check solutions, and judge the soundness of answers. Effectively accomplishing the tasks proposed in the *Principles and Standards for School Mathematics* requires students to be active participants in the learning process and aware of their progress as they work towards achieving their goals.

Research in self-regulation has sought to understand the processes intrinsic to student learning that lead to academic success (Zimmerman, 2002). Common to this research is the notion that students who are involved in the attainment of personal objectives through the use of metacognitive strategies (e.g., self-instruction, self-questioning and self-evaluation) are good self-regulators as their behavior is purposeful and strategic.
Figure 1.1. Flavell’s Tripartite Theory of Metacognition. This figure illustrates the relationship among the three components of metacognition.

Figure 1.2. Montague’s cognitive and metacognitive model of mathematical problem solving (MPS).
**Flavell’s Tripartite Theory of Metacognition.** Flavell is a social cognitive developmental psychologist who dedicated his research to children’s cognitive development. Before turning his attention to metacognition, his work focused on communication skills and memory skills, where he noted that children need to understand the concept of memory before they can develop skills to utilize and improve memory. He called this awareness metamemory. Metamemory, now more commonly known as metacognition, is understood as a developmental process that gradually becomes more explicit and under the conscious control of individuals (Kuhn, 2000). Flavell (1976), defined metacognition as follows:

In any kind of cognitive transaction with the human or non-human environment, a variety of information processing activities may go on. Metacognition refers, among other things, to the active monitoring and consequent regulation and orchestration of these processes in relation to the cognitive objects or data on which they bear, usually in service of some concrete goal or objective (p. 232).

In other words, metacognition is an individual’s conscious awareness of the need to monitor and control his/her performance in order to successfully accomplish a task. Flavell suggested that metacognition is an interaction among three subcomponents: (1) metacognitive knowledge, (2) metacognitive experience, and (3) task and strategies. The current literature on metacognition refers to component three as metacognitive skills (e.g., Desoete & Roeyers, 2002; Bannert & Mengelkamp, 2008) and will thus be addressed in the remainder of this paper.
The first component of metacognition, *metacognitive knowledge*, refers to the interaction of the beliefs and knowledge stored in one’s memory regarding personal functioning, task execution and strategy selection. It is considered to be the declarative, procedural and conditional knowledge that has accumulated over many years of experience (Sperling, Howard, Staley, & DuBois, 2004). Furthermore, metacognitive knowledge resides in one’s long-term memory and may be consciously or unconsciously activated during task execution (Flavell, 1985).

*Metacognitive experience* refers to self-motivational beliefs, which include conscious reactions and self-judgments regarding personal performance before, during or after task execution. These beliefs or experiences involve an individual’s self-efficacy, outcome expectancy, intrinsic value and learning-goal orientation and are the result of the self-interpretation or self-evaluation of one’s familiarity with the task, comprehension of the task, perception of difficulty, effort needed to complete the task, and confidence in ability to accomplish the task (Efklides, Kiorpelidou, & Kiosseoglou, 2006). These experiences are comparisons one makes against a standard, such as a personal past performance or peer performance (Zimmerman, 2002). Causal attributions or attribution theory is the rationale that students provide to explain successes and failures. These rationales will likely affect their motivational beliefs and future performance (Weiner, 1992). Metacognitive experiences have the potential to alter one’s knowledge base by assimilating and accommodating new information into stored long-term memory. In this way metacognitive knowledge and
metacognitive experience overlap in that task-specific experiences will likely influence a person’s more stable self-beliefs and knowledge (Flavell, 1979).

The final component, *metacognitive skills*, refers to the authentic procedures and strategies performed during task execution in order to monitor and control one’s cognition (Efklides et al., 2006). These strategies may include self-observation, self-evaluation, self-control, self-monitoring, self-instruction, and self-questioning, and are particularly useful in assessing comprehension and employing knowledge to novel or challenging tasks (Lucangeli & Cabrele, 2006). Differing from metacognitive *knowledge*, which is the declarative knowledge of the interaction between self, task and strategies, metacognitive skillfulness serves to regulate and control cognitive activities as they are happening in real time (Veenman, Van Hout-Wolters & Afflerbach, 2006).

In sum, these three components of metacognition, specifically, knowledge, experience and skill, describe *what* individuals know about what they know and who they are in relation to a task or event, *why* they engage or withdraw from a task or event, and *how* they attempt to accomplish the task or event.

**Montague’s Model of Effective Problem Solving in Mathematics.**

Montague’s model originated from research in general problem solving, mathematical problem solving, self-regulation, and affective variables related to successful problem solving (Montague, 1997). Self-regulation refers to a process in which students actively monitor their performance (metacognitive) while selecting from among strategies (cognitive) in order to attain a goal (Pape, Bell, & Yetkin, 2003). This process is cyclical and encompasses various cognitive and
metacognitive activities, which undergo a series of interactions in order to achieve a predetermined end. During task-execution a good problem solver will pass through the three phases: forethought, performance and self-reflection (Zimmerman, 2002). The cognitive processes and metacognitive strategies presented in Montague’s model reflect the strategies that expert problem solvers know and use effectively (Montague, Applegate, & Marquard, 1993). Her instructional routine, Solve It! was designed to improve mathematical problem solving of students with LD. Montague identified seven cognitive processes necessary for successful problem solving and developed a metacognitive routine to facilitate their use (Montague, Warger, & Morgan, 2000). With the help of a mnemonic, RPV-HECC, students learn the seven cognitive processes to effectively solve math problems and the metacognitive strategies to monitor their application. The seven cognitive processes are: Read (for understanding), Paraphrase (your own words), Visualize (a picture or diagram), Hypothesize (a plan to solve the problem), Estimate (make a prediction), Compute (do the arithmetic), and Check (make sure everything is right). The metacognitive strategies include self-instruction, self-questioning and self-checking in the form of a routine that helps students gain access to processes and monitor their application as they solve math problems.

Understanding the connection between what students say they know (metacognitive knowledge), how they actually behave (metacognitive skills) and the motivating factors behind their performance (metacognitive experiences) are important first steps in trying to remediate their academic performance. Flavell's
metacognition theory and Montague’s math problem-solving model provide a practical theoretical framework for the study of metacognition in the domain of math problem solving.

**Problem Solving and Students with LD**

Problem solving is a complex behavior that is considered one of the most important aspects of cognitive development for adolescents (Swanson & Sachs-Lee, 2001). The United States Department of Education Institute of Educational Sciences (n.d.) defines problem solving as:

An individual’s capacity to use cognitive processes to confront and resolve real, cross-disciplinary situations where the solution is not immediately obvious, and where the literacy domains or curricular areas that might be applicable are not within a single domain of mathematics, science, or reading.

Successful problem solving, however, requires more than the “capacity to use cognitive processes,” as it involves the integration and execution of cognitive, metacognitive and motivational factors. Mayer (1998) termed these three components, skill – domain specific knowledge, metaskill – strategies on how to use and control knowledge, and will – motivation and task interest. Furthermore, he suggested that expertise in any one of these components is not sufficient for successful problem solving and transfer. Rather, the ability to properly execute the appropriate strategies for the appropriate task requires students to be proficient in coordinating what knowledge to apply as well as how and when to apply it.
Traditionally, and in line with the federal definition of problem solving, being an expert means that an individual has received extensive training in a specific domain and is now considered to be highly knowledgeable in that specific area. This approach does not regard a person’s ability to acquire and assimilate new information. What distinguishes expert learners from their less successful peers is not necessarily how much more knowledge they possess, but rather their ability to be aware of the need to adapt and modify learning and strategy selection when faced with challenging tasks (Ertmer & Newby, 1996).

Problem-solving strategies in the domain of mathematics were identified as early as 1945 by George Polya. Polya’s (1957) work on the four basic principles of how to solve a problem (i.e., understand the problem, devise a plan, carry out the plan, and look back) was considered innovative at the time. Modern math reformers regard these strategies as important but also view metacognition as essential for successful math problem solving (DeCourt, Greer & Verschaffel, 1996). Metacognition helps problem solvers actively plan, monitor and reflect on their performance as they apply the processes.

Students with LD are a heterogeneous group who display a myriad of deficits while solving mathematical word problems. Specifically, they exhibit working memory deficits (Fuchs & Fuchs, 2002; Geary, 2004; Gonzalez & Espinel, 2002), impulsiveness and a failure to verify the solution path and evaluate answers (Bryant, Bryant, & Hammill, 2000), difficulty with multi-step problems (Bryant et al., 2000; Fuchs & Fuchs, 2002), a lack of math vocabulary (Bryant et al., 2000), and fact retrieval and computation problems (Fuchs &
Fuchs, 2002; Gonzalez & Espinel, 2002; Montague, 1991). Students with LD are less likely to effectively use strategies and experience difficulty developing their own strategies when compared with students without LD (Butler & Winne, 1995; Swanson & Sachs-Lee, 2001). Due to the complex interaction among cognitive, metacognitive and motivational variables, researchers have established that students with LD are characteristically poor problem solvers with restricted cognitive and metacognitive knowledge (Gonzalez & Espinel, 2002; Montague & Applegate, 1993).

**Statement of the Problem**

It is well established that the components of metacognition (knowledge, experience and skills) are interdependent (Flavell, 1987). Much of the current research in metacognition considers each component in isolation or solely the relationship between knowledge and skills. Neglecting the personal experiences that an individual brings to the task prevents researchers from understanding how knowledge is translated into skill, or examining contributing factors that may explain differences in performance. The purpose of this study is to investigate the three components of metacognition, and examine the differences among students with LD, low-achieving students and average-achieving students in the area of mathematical problem solving, as well as the relationship among these components and their influence on academic performance.
CHAPTER 2

Literature Review

A typical math classroom experience generally includes remembering math facts and rehearsing mathematical concepts and formulas. In an attempt to transform the way math is learned and perceived by students and teachers alike, in 2000 the National Council of Teacher of Mathematics (NCTM) released their *Principles and Standards for School Mathematics*. This document emphasized conceptual understanding and a problem-solving approach to mathematics. NCTM advocated abandoning instruction that focused on having students passively recite learned knowledge and encouraged teachers to actively engage students in meaningful communication about math and develop their ability to understand and make connections across math concepts. Schoenfeld (1987) called this type of classroom “a microcosm of mathematical culture.” He suggested that this approach has the potential to increase the meaningfulness of the classroom experience as students begin to think of mathematics as an integral part of their everyday lives. He emphasized the importance of metacognition on the process and the disconnect between students’ mathematics education and everyday math. Encouraging classroom discussion allows students and teachers to analyze the problem, discuss multiple solution paths and verify the soundness of their plan.

Problem solving is a complex behavior that requires the proper integration and execution of both the cognitive and metacognitive components. Proficiency in any one of these components is not sufficient for problem solving and transfer
Successful problem solving requires students to employ declarative, procedural and conditional knowledge regarding strategy use. In other words, students need to know what strategies to employ, under what conditions to execute them, and how to use them in a flexible manner. Due to the complex interaction among cognitive, metacognitive and motivational variables, problem solving continues to be a challenging area for many students but is particularly difficult for students with learning disabilities (LD) (Gonzalez & Espinel, 2002; Montague & Applegate, 1993).

Many researchers have studied the relationship between metacognition and the learning process from three different perspectives: knowledge, experiences and skills (Flavell, 1979; Lucangeli and Cabrele, 2006; Schoenfeld, 1987). Metacognitive knowledge refers to the interaction of the beliefs and knowledge stored in one’s long-term memory regarding personal functioning, task execution and strategy selection. Metacognitive experience refers to self-motivational beliefs, which include conscious reactions and self-judgments regarding personal performance before, during or after task execution. Metacognitive skills refer to the authentic procedures and strategies performed during task execution in order to monitor and control one’s cognition. There are relatively few studies that examine the relationship among these three sub-domains of metacognition, fewer that address metacognition in the area of mathematics learning and problem solving, and even fewer still that consider the metacognitive functioning of students with LD in the area of math problem solving.
Flavell (1992) suggested that the development of metacognition follows the same trajectory as Piaget’s stages of cognitive development eventually being instantiated during the formal operational thinking stage at about age 12 (Kuhn, 1999; Veenman, Wilhelm & Beishuizen, 2004). Before this stage, he suggested that metacognition is developing and still immature. Children as young as four years may have a feeling that something is wrong but are not developmentally prepared to handle the complexity of analytical thought to understand what is wrong or how to fix it. Mature metacognitive knowledge is dependent upon years of accumulated experience in making thought the object of thinking (Flavell, 1979).

The purpose of this literature review is to present research that examines each of the three components of metacognition in the area of math problem solving while highlighting the metacognitive functioning of students with LD. Furthermore, a discussion of the issues surrounding the measurement of metacognition is presented.

**Metacognition and LD**

**Metacognitive knowledge.** Students differ not only in the amount of knowledge they have, but also in the organization and accessibility of that information (Garner & Alexander, 1989). Research on metacognitive knowledge or awareness has sought to understand what students know about learning and what strategies they use to help them learn. In a particular academic domain students may not have a strong knowledge base to rely on. The manner in which they activate their metacognitive strategies to compensate for this lack of
knowledge may be more related to domain general strategies rather than the knowledge itself. For example, Garner and Alexander (1989) described how novice but successful students cope with their first physics class and their textbook. Although the students had little to no background knowledge of physics to support their learning, they used general strategies such as reading section headings and end-of-chapter summaries as well as visuals and diagrams to help them compensate for this lack of knowledge. This type of student is considered an “intelligent novice” (Brown & Palincsar, 1985). These students clearly activated strategies that helped them acquire knowledge of physics.

Are the strategies used by these “intelligence novices” reflective of general intelligence or metacognitive awareness? To examine the relationship between general academic aptitude and metacognition, Swanson (1990) analyzed the think-aloud protocols of 56 students in grades 4 and 5. His study provided interesting information regarding the functional role of metacognition and its relationship to cognitive ability. Students were separated into high and low cognitive ability groups based on their performance on the Cognitive Abilities Test (Thorndike & Hagen, 1978). Students were then grouped based on performance on a 17-item questionnaire that assessed metacognition in the general domain of problem solving to form high and low metacognitive groups. Thus, four groups were formed: high aptitude-high metacognition (HA/HM), high aptitude-low metacognition (HA/LM), low aptitude-high metacognition (LA/HM), and low aptitude-low metacognition (LA/LM). Students were then audio-recorded solving two tasks, a pendulum task and a combinatorial task. Their think-aloud
protocols were transcribed and coded according to 24 mental components. Results showed that the high metacognitive students outperformed low metacognitive students regardless of aptitude. Furthermore, the LA/HM group performed significantly better than HA/LM students. However, only the HA/HM group used more heuristic and strategy subroutines than the other groups and consistently used hypothetico-deductive reasoning to work through the problem. These findings suggest that metacognition may be independent of general aptitude and have more predictive power for future success than general intelligence (Veenman & Spanns, 2005).

More important than static knowledge, such as that displayed on recall tests, is knowledge for use (Sternberg, 1998). Helping students develop an awareness of the learning process and the metacognitive strategies necessary to achieve academic success will likely benefit them throughout life.

**Metacognitive experience.** As students work through problems they are faced with triumphs and failures. They can forge ahead or surrender to the difficulties they face. Their reactions to these challenges are shaped by previous experiences dealing with similar problems. The more successes these students experience, the more positive their attributions and self-esteem will be and consequently the more likely they are to employ strategies to help them accomplish the task (Pintrich & Schrauben, 1992).

Academic self-perception of students with LD may not be in line with their actual academic performance (Meltzer, Roditi, Houser & Perlman, 1998; Stone, 1997). These inaccuracies may present a problem for appropriately executing
metacognitive skills since the activation of these self-help skills are only triggered by an awareness of a need to use such strategies. Stone and May (2002) investigated the degree of overestimation of academic skills among 52 high school students with LD and 49 students without LD. Their results showed that although students with LD overestimated their performance relative to their average-achieving classmates, their self-perceptions were accurate based on the national standardized mean score of the two administered questionnaires, the Skills Rating Survey (SRS) and the Multidimensional Self-Concept Scale for Children (MSCS). Mediating factors such as differential reference group, poor metacognitive knowledge, and self-protection may account for this differential overestimation of ability by students with LD.

Garrett, Mazzocco and Baker (2006) and Desoete and Roeyers (2002) assessed the prediction and evaluation skills of students with and without LD in math. Both authors found that students with LD were less accurate in their prediction than their average-achieving peers. Garret et al. (2006) found that although they were overconfident in their ability to correctly solve the problem, students were as accurate in predicting those problems they could not solve. Desoete and Roeyers (2002) further examined this overestimation by investigating whether the developmental lag hypothesis explained the poor prediction and evaluation skills. The authors compared the prediction and evaluation skills of students with LD to average-achieving younger students and found that there were significant differences between these groups and therefore
could not conclude that a developmental lag hypothesis explains the self-perspective of students with LD.

Previous research suggests that both cognition and motivation are important components of academic success. However, this literature has predominantly focused on how motivation leads to persistence rather than on performance or strategy choice (Pintrich, Anderson & Klobucar, 1994). According to interest theory (Dewey, 1913), students will engage in deeper more complex thinking and persist longer on a task if they are interested in the subject matter. In other words, cognitive and metacognitive activities are related to the significance and value the students place on the material. Simply knowing about strategies does not ensure application of the strategies, whereas the combination of both strategy knowledge and self-efficacy are required to transfer knowledge into practice (Schoenfeld, 1987). Cognitive engagement and performance are positively correlated with self-efficacy and intrinsic value (Pintrich & De Groot, 1990). Students with LD, however, are not as intrinsically motivated as students without LD (Ellis, 1986). Examining the metacognitive experiences of students allows researchers to not only understand how and if students are applying their reported metacognitive knowledge, but also why and what factors contribute to the activation of metacognitive strategies.

**Metacognitive skill.** Analyzing students’ actual strategy use is best achieved through concurrent verbal reports, such as think-aloud protocols, since researchers have direct access to the ongoing mental processing occurring during task completion. Vygotsky (1986) distinguished between egocentric
speech, essentially overt speech that helps children understand the world around them and is intended for oneself, and inner speech, which is merely egocentric speech that has been internalized and is no longer audible. He believed that egocentric speech, also known as private speech, reflects children's ability to monitor and guide their behaviors during task execution. Supporting Vygotsky's belief that private speech serves a self-regulatory function, Berk and Landau (1993) compared the private speech of 112 third- through sixth-grade students with LD, students with LD symptomatic of attention-deficit hyperactivity disorder (ADHD) and normally achieving students in two different settings, the classroom and the laboratory. Their results showed that differences in private speech depended on context. In laboratory activities, students with LD and normally achieving students did not differ in their type of private speech. Private speech was divided into three types, (1) task-irrelevant private speech (affect and comments), (2) task-relevant externalized private speech (describing one's activities, self-guiding comments, reading aloud), and (3) task-relevant external manifestations of inner speech (self-stimulating behavior, task-facilitating behavior). During classroom activities, however, students with LD used more task-relevant externalized speech than their normally achieving peers. Although both students with LD and normally achieving students produced the same amount of task-relevant internalized speech, the functional relationship with behavior (attention and motor accompaniment to task) was non-significant for the students with LD. The results suggest that the increased use of task-relevant
externalized speech by students with LD show that they are not deficient in their task-relevant speech. Rather, they may be deficient in their ability to internalize.

Further support for the developmental lag hypothesis is offered by Ostad and Sorenson (2007), in which they investigated patterns of private speech and strategy use and their interaction in 134 children with and without mathematical difficulties (MD) while solving number fact problems. In this study, students were each assigned to one of three grade groups (2 – 3, 4 – 5, 6 – 7). Using a cross-sectional design, the researchers observed the participants individually during two laboratory sessions. Their results showed that task-relevant speech is positively correlated with self-control and successful task completion during problem solving. Children with MD consistently used more backup strategies (counting on one’s fingers) than students without MD, whereas students without MD used more retrieval strategies (accessing information from one’s memory). These findings support the developmental lag hypothesis in that the poor metacognitive skills of children with LD are the result of immature, rather than absent, metacognitive skills (Garrett et al., 2006).

Think-Aloud Protocols are integral to the study of verbal thought and, consequently, metacognitive skills, since they reflect the students’ abilities to control, monitor and self-regulate behaviors and activities that occur while solving problems. When thinking out loud students are asked to verbalize everything they are thinking, feeling and doing in order to assess the cognitive and metacognitive processes underlying task performance. For example, Montague and Applegate (1993) used verbal think-aloud protocols to examine the self-
regulation and strategy use of 81 students (LD, \( n = 28 \); average achievers, \( n = 25 \); gifted, \( n = 28 \)). The students were provided with 10 minutes of think-aloud instruction using two verbal reasoning problems. They were then asked to solve a one-step, a two-step and a three-step math word problem while thinking aloud. It was concluded that there were no differences in cognitive or metacognitive verbalizations for problem 1. Gifted students made significantly more cognitive but not metacognitive verbalizations than students with LD for problem 2 and also made significantly more cognitive and metacognitive verbalizations than students with LD and average achievers for problem 3. These findings support the hypothesis that metacognition is activated when individuals are faced with challenging problems. However, a student’s perception of the difficulty of the problem may have an effect on their persistence in solving the problem and decision to activate their self-guiding metacognitive strategies.

Although think-aloud protocols are considered a well suited and valid approach to the study of cognition (Davidson, Vogel & Coffman, 1997), there are two major criticisms of this approach, reactivity and completeness (Bannert & Mengelkamp, 2008). Reactivity questions whether the increased cognitive demand of thinking out loud alters the thinking process, whereas completeness questions whether all cognitive processes are conveyed through the think-aloud method or if some thoughts remain unconscious.

Addressing the question of reactivity, Ericsson and Simon (1993) argued that verbalization of thoughts does not change the thought sequence but merely slows it down. The second concern, completeness, is best addressed in relation
to strategy use and automaticity. Students develop and use strategies to help them accomplish tasks and goal. What begins as explicit strategy use eventually becomes implicit with the more successful experiences these students have using the selected strategy (Crowley, Shrager and Siegler, 1997). In other words, the goal of conscious and deliberate use of metacognitive strategies is to eventually internalize them to the point of automaticity. Shiffrin and Schneider (1977) described automaticity as rapid, involuntary parallel processing, rather than slow, sequential, controlled processing. Students who are proficient problem solvers will not need to activate their metacognitive strategies, whereas struggling students will need to activate these strategies to help monitor their performance throughout task completion. The difficulty lies in determining if, in the absence of verbalized thought, the strategies and processes are deficient, delayed or internalized to the degree of automaticity.

Being an expert problem solver requires more than simply knowing what to do; it involves being able to adapt and apply knowledge in a flexible way to novel situations. Previous research has determined that students with LD have difficulty balancing the cognitive and the metacognitive components when solving problems. This difficulty may be attributed to immature rather than absent metacognitive skills, inaccurate perceptions of one’s ability to solve the problem, or a lack of motivation.

**Metacognition and Measurement**

Since Flavell (1979) introduced the term metacognition, varying methods have been used to collect data on this elusive construct. There is ongoing
debate on the reliability and validity of methods used to assess metacognition and the difficulty in comparing outcomes across studies (Desoete, 2008; Pressley, 2000). The following section describes some of the more frequently used instruments in measuring metacognition and summarizes the strengths and weaknesses of each method. These methods include self-reports, think-aloud protocols and discourse analysis.

The self-report is a research method that aims at measuring individual’s attitudes, behaviors, feelings or thoughts. Examples of self-reports include retrospective verbal reports (students recall what they were thinking immediately following a task), concurrent verbal reports (students recall what they are thinking while engaged in a task), self-estimates (students estimate their performance before and after task completion), questionnaires (students record their thinking in response to standardized questions), and interviews (student responses to fixed or open-ended questions regarding thinking are recorded). Self-reports remain among the most popular choice for measuring cognitive phenomena due in part to its practical and information-rich nature (Paulhus & Vazire, 2007). All of these methods, however, are dependent upon the individual’s ability to recall information and are thus subject to memory failure. Additionally, they offer “after-the-fact” descriptions or projections of thinking that can be incomplete and biased (Hacker, Dunlosky, & Graesser, 1998). Since students may adjust their responses to fit what they expect the researcher wants to hear, these methods may be inaccurate or altogether false reports.
Think-aloud protocols are another method of collecting metacognitive data that provide rich verbal data regarding a student’s reasoning abilities during problem-solving activities (Fonteyn, Kuipers, & Grobe, 1993). Think-aloud protocols are primarily used to obtain information regarding information on cognitive processing (Ericsson & Simon, 1980). This can either be done concurrently, while the participant is engaged in problem solving, or retrospectively, once the participant has finished the task. Thinking out loud does not affect cognitive processes or performance speed (Ericsson & Simon, 1980) and may provide a more accurate account of applied metacognitive knowledge or metacognitive skills since they do not require the students to remember what they were thinking. Students are simply asked to verbalize what they are thinking as they are thinking it. This method may be less subject to the students’ interpretation. The difficulty with think-aloud protocols is that not all students are capable of consciously expressing routines and strategies that may have already been internalized to the point of being automated and therefore might provide an incomplete account of metacognitive functioning.

Finally, discourse analysis is a method that includes cross-age peer tutoring and group work. In contrast to think-aloud protocols where students are individually recorded while thinking through the problem, discourse analysis records students working through problems and negotiating solution paths as a group. It is argued that if students know something that will help accomplish the task, they will likely try and relate that information to other students (Garner & Alexander, 1989). One of the major concerns with discourse analysis is group
dynamics. For instance, mentoring programs rely on the success of the relationship between mentor and mentee. Both individuals bring their own unique needs, beliefs, values and concerns to the relationship and effectively matching a mentor with a mentee can be difficult if not altogether impossible (Hale, 2000). This mismatch may potentially inhibit verbalization thus providing an inaccurate account of the student’s actual level of awareness or metacognition.

Despite the shortcoming of these methods for assessing metacognition, they do provide the researcher with a means to access the otherwise inaccessible cognitive processing of individuals. More research is needed that examines the validity of the methods used to measure this construct (Sigler & Tallent-Runnels, 2006). The multidimensionality of metacognition should be considered when selecting the method used to assess an individual's awareness of cognitive processing since some methods may be more appropriate to measure certain dimensions. For example, self-reports may be more conducive to measuring what students think they know about a subject or event and why they think the way they do. On the other hand, think-aloud protocols or discourse analysis would likely provide better information as to how this knowledge is applied in actual contexts.

Undeniably, metacognition is essential for learners to develop good problem-solving skills and achieve mathematical success (Pape et al, 2003; Montague, 2008). However, and perhaps due to different methods of data collection, the results are still mixed regarding the efficacy of metacognition for
improving the math performance of students with LD. Furthermore, the relationship between the cognitive and metacognitive strategies in reading, writing, and mathematics has received little attention for students with LD or their typically developing peers (Vaidya, 1999). Researchers suggest that a multi-method assessment of metacognitive functioning may provide a more accurate picture of students’ metacognitive functioning since varying methods do not share the same source of error (Desoete, 2008; Garner & Alexander, 1989).

The present study is important for three reasons. First, understanding how students differ with respect to each component of metacognition helps to identify at what point in the learning process the content of what is being taught deviates from becoming internalized and useable information. For example, deficits in metacognitive knowledge may imply that the student has not acquired the strategy, whereas deficits in metacognitive skills may suggest a failure to generalize the strategy to a calibration problem. Finally, deficits in metacognitive experiences may indicate a motivational or self-efficacy failure. Second, an examination of the relationship among the three components will provide a more complete description of metacognitive differences among ability groups and increase sensitivity to subtle nuances in metacognitive functioning that may not be apparent when considering each component in isolation. Finally, understanding how these components influence student performance will provide invaluable information to aid in guiding future instruction aiming to improve the academic performance of students with LD.
Research Questions

The three questions that guide this study are as follows:

1. What is the difference in metacognitive functioning among students with LD, low-achieving students and average-achieving students?
2. What is the relationship among the three components of metacognition (knowledge, experience and skills) and does the relationship differ for ability groups?
3. To what extent do metacognitive knowledge, experience and skills predict math problem-solving performance and is there a difference among ability groups?
CHAPTER 3
Methodology

Participants

Eighth-grade middle school students from ten middle schools in a large metropolitan school district in the southeastern United States were recruited to participate in this study ($n = 82$). Participants who met research-criteria (described below) were drawn from a three-year study [2007-2010] entitled “Improving Mathematics Performance of At Risk Students and Students with Learning Disabilities in Urban Middle Schools - Middle School Math Project.” This Middle School Math study was funded by the United States Department of Education Institute of Education Sciences and directed by Marjorie Montague, Ph.D., the Principal Investigator.

Upon receipt of both the signed parent consent and youth assent forms, student academic records were screened for demographic information, academic performance, English Second Language (ESL) placement and special education services. Students receiving special education services for specific learning disabilities, had an active Individualized Education Plan (IEP) and received a level 1 or 2 on the math section of the FCAT were categorized as students with learning disabilities (LD, $n = 15$). The FCAT is administered to students in Grades 3 through 10 and consists of criterion-referenced tests that measure selected benchmarks in mathematics, reading, science, and writing from the Sunshine State Standards (FDOE, 2005). Students are assigned one of five achievement levels based on their scale scores. Levels 1 and 2 indicate below
grade level performance. Level 3 indicates grade level performance and levels 4 and 5 indicate above grade performance. At the time of this study, the school district used the IQ-achievement discrepancy model to identify students with LD, based on average intelligence (>80) and low performance (≥ 1.5 SD difference). Students who received an FCAT level 1 or 2 on the math section were categorized as low-achieving students (LA, n = 38), and students who received an FCAT level of 3 or higher on the math section were categorized as average-achieving students (AA, n = 29). Students with ESOL levels 1, 2, or 3 were excluded from the study. See Table 3.1 for participant information.

<table>
<thead>
<tr>
<th>Table 3.1</th>
<th>Participant Information</th>
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<tbody>
<tr>
<td>Characteristic</td>
<td>LD (n = 15)</td>
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<tr>
<td></td>
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<tr>
<td>Gender</td>
<td></td>
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<tr>
<td>Male</td>
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<tr>
<td>Female</td>
<td>6</td>
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<tr>
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<tr>
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<tr>
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<tr>
<td>Asian</td>
<td>-</td>
</tr>
<tr>
<td>Free or Reduced Lunch</td>
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</tr>
<tr>
<td>Yes</td>
<td>13</td>
</tr>
<tr>
<td>No</td>
<td>2</td>
</tr>
</tbody>
</table>

*Note: LD = Learning Disabilities; LA = Low-Achieving; AA = Average Achieving*
Procedure

An introduction letter explaining the purpose of the study and the level of student involvement as well as a youth assent form (Appendix A) were distributed to all eighth-grade students in participating classrooms. Students who assented to participate in the study were given a parent consent form (Appendix B) to be signed and returned by a parent or legal guardian. In addition to participation in the larger intervention study, all three documents explained that students might be selected to be audio-recorded while problem solving and provided an option for both parent and/or child to decline being audio-recorded. Furthermore, within these documents it was made explicit that participation was voluntary and the student could withdraw at any time without repercussion. Students who returned both youth assent and parent consent with agreement to participate as well as be audio-recorded were selected on a first-come, first-serve basis and assessed until the intervention was implemented in October of 2008.

All participants were individually administered the battery of assessments (see Figure 3.1 for procedures) during an elective class period in a quiet setting that facilitated audio recording as they thought out loud while solving the math problems.
### Procedures

<table>
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<tr>
<th>IV</th>
<th>DV</th>
<th>Description</th>
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<tbody>
<tr>
<td>1</td>
<td>Consenting / Assenting</td>
<td>Obtain both parent consent and student assent.</td>
</tr>
<tr>
<td>2</td>
<td>Collect Demographic data</td>
<td>Obtain student information from registrar and separate into three categories.</td>
</tr>
<tr>
<td>3</td>
<td>Think-Aloud Training</td>
<td>Students receive 20 minutes of modeling and practice using three logical-reasoning problems.</td>
</tr>
<tr>
<td>4</td>
<td>Think-Aloud Protocols (TAPs)</td>
<td>Students subsequently solve three math word problems while thinking out-loud.</td>
</tr>
<tr>
<td>5</td>
<td>Metacognitive Experience Survey</td>
<td>30-item survey eliciting information regarding students' task-specific self-efficacy and motivational beliefs before and after solving the three math word problems.</td>
</tr>
<tr>
<td>6</td>
<td>Math Problem-solving Assessment – Short Form (MPSA-SF)</td>
<td>17 open-ended questions eliciting information on students’ awareness of problem-solving strategies.</td>
</tr>
<tr>
<td>7</td>
<td>10-item Math Word Problem-solving Probe</td>
<td>10 math word problems consisting of 1-step, 2-step and 3-step problems.</td>
</tr>
<tr>
<td>8</td>
<td>Transcription and Coding (TAPs and MPSA-SF)</td>
<td>Transcribe and score protocols, score MPSA-SF and establish inter-rater reliability for both measures.</td>
</tr>
</tbody>
</table>

*Figure 3.1. Data collection procedures.*
Prior to solving the problems, students were trained individually in the think-aloud method by the researchers in the following manner. First, the researcher explained the purpose of the study and the reason why a think aloud is a good way to understand how people solve math problems. The researcher read the following script adapted from Johnstone, Bottsford-Miller, and Thompson (2006): “I am interested in how students solve problems, so I want to ask you to solve three problems for me and let me listen to how you solve them. I am not interested in the answer you come up with as much as how you are thinking about the problem. What you say is really important, so I am going to use a tape recorder to make sure I don’t forget anything.” Second, the researcher modeled thinking out loud using a logical reasoning problem demonstrating processes such as self-questioning, checking back, and monitoring progress as well as affective statements related to the problem. Then the students were given an opportunity to practice thinking out loud while working through two different logical reasoning problems. The researcher encouraged the students to speak with appropriate volume and clarity. Participants were then asked to solve the following three math word problems while thinking out loud.

- Bill and Shirley are arranging the chairs for a class play. They brought 252 chairs from the storeroom to the auditorium. Their teacher told them to make rows of 12 chairs each. How many rows will they have?
- Four friends have decided they want to go to the movies on Saturday. Tickets are $2.75 each for students. Altogether they have $8.40. How much more money do they need?
• Chain sells for $1.23 a foot. How much will Farmer Jones have to spend for chain in order to enclose a 70 foot by 30 foot patch of ground leaving a four 4 entrance in the middle of each of the 30 foot sides?

Before and after solving each of the three problems, the students were asked to complete the metacognitive experience survey. The session was audio-recorded and subsequently transcribed to produce verbal protocols. The students were told to constantly think out loud. If they were silent for longer than five seconds, the researcher reminded them to say everything they were thinking, feeling, and doing while solving the problem. With the exception of the reminders, interactions between the researcher and student were minimal in order to interfere as little as possible with the cognitive activities.

Following problem solving, students were administered the MPSA-SF in order to elicit information regarding the students’ metacognitive knowledge or awareness of math problem-solving strategies. Finally, during their regularly scheduled math class, students were given 50 minutes to complete the 10-Item math word problem-solving probe.

Transcription and coding.

Think-aloud protocols. The audio-recordings were transcribed verbatim by the researchers over a one-month period. Twenty-five percent of the transcriptions were cross-checked for accuracy with the original recording with a 100% transcription accuracy score. Montague’s (2003) model of math problem solving, which includes seven cognitive processes (i.e., reading, paraphrasing, visualizing, hypothesizing, estimating, computing, and checking) and three
metacognitive strategies (i.e., self-instruction, self-questioning, and self-monitoring) served as a base for the researcher-developed coding system. Metacognitive strategies were conceptualized as either domain-general (e.g., “Hmm. What should I do? Oh, okay. I get it.”) or domain-specific (e.g., “I need to subtract $8.40 from $11.00 to find out how much more money they need to go to the movies.”) and included additional metacognitive strategies to the three represented in the model. All verbalizations were first coded as being either metacognitive in nature or as belonging to one of the seven cognitive categories. The metacognitive codes were then entered into Atlas ti (Atlas ti; Muhr, 2004), a computer software program used to systematically code and analyze qualitative data and coded using emergent coding. This produced seven metacognitive codes, which were separated into two groups, productive and nonproductive metacognitive verbalizations. Productive verbalizations included self-monitoring, self-instruction, self-questioning, and self-correction statements/questions directly related to solving the problem such as “I need to re-read the question,” “That’s not possible. It can’t be division,” and “What am I doing?” Nonproductive metacognitive verbalizations were more affective in nature such as “I don’t know what to do,” “I’m confused,” and “I need a calculator.” Overall, the coding system included seven cognitive and seven metacognitive codes. See Appendix C.

The cognitive, productive metacognitive (PM) and nonproductive metacognitive (NPM) verbalizations were tallied to provide three frequency counts within each problem-type (1-step, 2-step, 3-step). These frequency counts were then transformed into percentages. To illustrate, the percentage of
productive metacognitive verbalizations in the 1-step problem was calculated by dividing the frequency count of productive metacognitive verbalizations by the total number of verbalizations across categories. The quotient was multiplied by 100 to produce the percentage of productive metacognitive verbalizations for the 1-step problem. Analyzing the percent of metacognitive verbalizations based on total verbalizations allowed a more accurate representation of the students’ metacognitive functioning. For instance, student A had five verbalizations, one of which was coded as productive. Student B had 10 verbalizations and also had one that was coded as productive. By using the percentage rather than frequency of verbalization type, it is evident that student A produced relatively more productive metacognitive verbalizations than student B.

To determine inter-rater agreement (IRA), two researchers each coded and then compared 10 think-aloud protocols to establish initial agreement. An iterative process of discussing and resolving disagreements was used until agreement was reached regarding the coding. All protocols were returned to the pool and then rated by both researchers. IRA was calculated by dividing the number of agreements by the number of agreements plus disagreements and multiplying by 100. Inter-rater agreement was 92%.

Math Problem-Solving Assessment – Short Form (MPSA-SF). A hierarchical rating scale was developed a priori for each question to reflect the objectives of the Solve It! intervention. See Appendix D. Responses reflecting higher-order thinking processes received a higher score. For example, the question “How do you help yourself remember what the problem says?” is rated
on a 4-point rating scale. A score of 0 is given for the inability to provide an answer to the question; 1 for rereading the problem, 2 for identifying important information, and 3 for putting the problem into their own words.

To determine the IRA on scoring, two researchers each scored and then compared 10 MPSA-SFs to establish initial agreement. An iterative process of discussing and resolving disagreements was used until agreement was reached regarding the scoring. All MPSA-SFs were returned to the pool and then 100% were rated by the primary researcher and 20% were rated by a second scorer. IRA was calculated by dividing the number of agreements by the number of agreements plus disagreements and multiplying by 100. IRA was 85%.

**Measures**

**Metacognitive skills.** Evidence of students’ metacognitive skills was obtained from students’ verbalizations on the three think-aloud protocols. The dependent variable for metacognitive skills is the mean percent of metacognitive verbalizations (productive + nonproductive) across the three problems.

**Metacognitive experience.** A metacognitive experience survey was administered to elicit information regarding students’ metacognitive experience before and after task completion. See Appendix E. Contrary to metacognitive knowledge, which describes students’ general beliefs about mathematics as a whole, metacognitive experience relates to task-specific instances of math performance, specifically self-efficacy and self-motivational beliefs. The survey targets students’ feelings of familiarity, knowing, confidence, satisfaction and difficulty (Efklides et al., 2006) and includes 5 items representing both the
present and past tenses. Combined, these five “feelings” represent students’ disposition towards math problem solving, in other words, their metacognitive experience.

The five items are: (1) I have seen this type of problem before, (2) I understand (understood) what the problem asks me to do, (3) The problem is going to be (was) difficult to solve, (4) I will need (needed) to use a lot of effort to solve this problem, and (5) I am confident that I will solve (solved) this problem correctly. The students respond to each of these items by placing an “X” in the box that best describes how each statement applies to them. The selection choices included (1) not at all true, (2) hardly true, (3) mostly true, and (4) absolutely true. Each choice was given a score of 1 through 4. Reverse coding was used for two of the items to ensure that scoring was monotonically increasing. Therefore, the more positive the student’s disposition towards math problem solving, the higher the score will be. Students were administered the 5 items before and after solving each of the three math word problems for which they were required to think aloud. Therefore, the students responded to the 5 items a total of 6 times, for a total of 30 items. In the regression analysis the dependent variable for metacognitive experience is the sum of the scores across the 30 items and has a range from 30 to 120. For the participants in this study, the reliability coefficient for the scores obtained from the 30-item survey was .82. In the mixed-design ANOVA the dependent variables for metacognitive experience is the sum of the scores for the five items administered before and after solving each problem. This results in a separate ten-item survey for each
level of problem difficulty with scores that range from 10 to 40.

**Metacognitive knowledge.** A modified version of the Math Problem-Solving Assessment – Short Form (MPSA-SF; Montague, 1992) was used to measure students' metacognitive knowledge. Items that specifically addressed metacognitive knowledge were preserved. The assessment used in this study consists of 17 open-ended questions administered in a structured interview format, which probed students for information regarding their metacognitive knowledge, use and control of cognitive processes and metacognitive strategies during problem solving. The MPSA-SF was developed as an informal measure of the processes and strategies students use during math word problem solving. Therefore, the interview questions were categorized to provide information regarding students’ behaviors for each of the seven steps of the Solve It! intervention (i.e., Reading, Paraphrasing, Visualizing, Hypothesizing, Estimating, Computing, Checking). For the participants in this study, the reliability coefficient for the scores obtained from the 17-item questionnaire was .78. The dependent variable for metacognitive knowledge is the individual’s summated score of the 17-item questionnaire and can range from 0 to 48.

**Math word problem-solving probes (MWPSP).** To measure students’ math word problem-solving performance, 10 textbook-style mathematical word problems were selected from the Solve It! manual (Montague, 2003). These word problems represent typical textbook 1-step, 2-step and 3-step problems and only require knowledge of basic math skills such as adding, subtracting, multiplying and dividing. Furthermore, these problems contain only whole
numbers and decimals, therefore eliminating differences in prior knowledge. For the participants in this study, the reliability coefficient for the 10 items was .68.

**Design and Analysis**

A univariate Analysis of Variance (ANOVA) was conducted to determine whether metacognitive knowledge differs as a function of ability level. Follow-up analyses were conducted for significant main effects. Additionally, since metacognition is said to function as a consequence of problem difficulty, a 3 (group: LD, LA, AA) by 3 (difficulty: 1-step, 2-step, 3-step) mixed-design ANOVA was performed for metacognitive experience, whereas a 3 (group: LD, LA, AA) by 2 (metacognition: productive, non-productive) by 3 (problem difficulty: 1-step, 2-step, 3-step) mixed-design ANOVA was performed for metacognitive skills. Pairwise comparisons were performed for significant effects. Next, bivariate correlations were conducted using the Pearson-moment correlation coefficient (r) to determine whether the three components of metacognition (knowledge, experience and skills) as well as math problem-solving performance are linearly related. Finally, in order to determine if these three components of metacognition predict math problem-solving ability, regression analyses were conducted. Furthermore, dummy codes for the ability group variable and cross-product variables were created to examine whether ability moderates the effect of metacognition on academic performance.

**ANOVA.** ANOVA is a hypothesis testing procedure used to evaluate mean differences between two or more populations. This study examines the metacognitive knowledge difference among three different ability groups,
specifically, students with LD, LA students and AA students. The null hypothesis states that there is no difference in metacognitive knowledge among the groups. That is, the population means for the three ability groups are the same.

**Mixed-design.** A mixed-design ANOVA is one in which individuals are measured more than once on the same dependent variable (Gravetter & Wallnau, 2005). A primary advantage of the mixed-design is that it requires fewer participants than the independent-measures design. This reduces the risk of participant characteristics being considerably different at each measurement phase. Mixed-design is well suited to examine changes in participants' behaviors from one situation to another or from one time to another.

The term mixed-design implies that there is at least one between-subjects variable and one within-subjects variable. In the present study, a 3 (group) by 3 (problem difficulty) mixed-design ANOVA was used. Students' metacognitive experience and metacognitive skills were measured across three math word problems (1-step, 2-step, 3-step). The between-group independent variable is ability group membership, whereas the within-group independent variable, also the repeated-measures variable, is the level of problem difficulty. The dependent variable is the students' metacognitive score. Follow-up post-hoc analyses were performed for statistically significant effects.

**Correlation coefficients.** Correlation coefficients were examined in order to establish theory verification, in other words, to make predictions about the relationship between two variables. The most common correlation is the Pearson correlation, which measures the magnitude and the direction of the
linear relationship between two variables. The direction of the relationship is expressed as either positive or negative, whereas the strength of the relationship is indicated by the numerical value of the correlation. Correlation coefficients range from -1 to 1. A perfect positive correlation is identified as 1; a perfect negative correlation is identified as -1, whereas no correlation is identified as 0.

**Regression.** The primary purpose of the regression analyses was to test whether the slope of the regression lines differed by ability group. The secondary purpose was to determine the unique effects of each independent variable on the dependent variable, that is, to understand the effect of each component of metacognition (i.e., metacognitive knowledge, metacognitive experience and metacognitive skill) as well as their interactions with ability on MWPS performance, when all other variables are controlled.

Dummy coding was used to transform the original categorical variable (i.e., ability) into as many dummy variables as there are group categories, minus one (g – 1). Thus, in this study, since there are three group categories (i.e., LD, LA, AA), two dummy variables were needed to capture the same information contained in the original categorical variable. The first dummy variable, LD_Dum, (meaning belonging to the learning disabilities group) was given the value of 1 for group membership, while all other participants were coded 0. The second dummy variable, LA_Dum (meaning belonging to the low-achieving group) was coded so that members of the low-achieving group were coded 1, while all other participants were coded 0. The average-achieving group was held constant and thus used as the control group. The result is that in the multiple regression the
first dummy variable contrasts students with learning disabilities with average achieving students, and the second dummy variable contrasts low-achieving students with average-achieving students.

In order to maintain consistency across scales, three new centered versions for metacognitive knowledge, metacognitive experience and metacognitive skill were created; MK_Cent, ME_Cent and MS_Cent, respectively. This was achieved by subtracting the mean of each variable from an individual’s score thereby making the mean of each predictor variable zero. From these new centered variables, six cross-product terms were created to test the interaction between ability and the predictor variables. This was done by multiplying each dummy code with one of the components of metacognition. These interaction terms are represented as LD_MK, LD_ME, LD_MS, LA_MK, LA_ME, and LA_MS, respectively.

In order to determine the effect of metacognition on student’s math problem performance, math problem-solving performance, as measured by the 10 math word problem-solving probe, was regressed on metacognitive knowledge, as measured by the MPSA-SF, metacognitive experience, as measured by the Metacognitive Experience Survey, and metacognitive skills, as measured by the think-aloud protocols.
CHAPTER 4

Results

The primary purpose of this study was to provide a comprehensive investigation of the differences among students with learning disabilities (LD), low-achieving (LA) students and average-achieving (AA) students for three components of metacognition, namely, metacognitive knowledge (MK), metacognitive experience (ME) and metacognitive skills (MS). The secondary purpose was to consider the impact of ability group differences on the relationship among these components and math word problem solving (MWPS) performance.

This chapter is organized according to the research questions under investigation. First, a univariate Analysis of Variance (ANOVA) was used to determine mean differences in MK among three ability groups. Since metacognition is activated as a self-help tool to work through challenging problems, two mixed-design ANOVAs were used to further probe mean differences for the ME and MS components of the metacognitive framework. In particular, a 3 (ability group) X 3 (problem difficulty) mixed-design ANOVA was used to examine how students’ ME levels differed by ability group and problem difficulty. Also, a 3 (ability group) X 2 (type of metacognitive verbalizations) X 3 (problem difficulty) mixed-design ANOVA was used to examine if type of metacognitive verbalizations, ability group, and problem difficulty yield differences in students’ MS. Second, bivariate correlations were conducted for
each ability group to investigate differences in the relationships among the three
components of metacognition and MWPS. Next, a regression analysis was used
to determine the differential effects of each component of metacognition by ability
group on MWPS performance.

Question 1: Ability Group Differences in Metacognitive Functioning

**Metacognitive knowledge.** Table 4.1 displays the means and standard
deviations of MK scores by ability group. Students with LD had the lowest score
followed by LA students and then AA students. Interestingly, students with LD
had the lowest score with the highest standard deviation, indicating considerable
variability in the group with respect to reported metacognitive knowledge.

A one-way ANOVA was used to compare the metacognitive knowledge of
students with LD, LA students, and AA students. ANOVA shows that mean
differences were not statistically significant ($F [2, 79] = 2.410, p = .096, \eta^2 =
.057$).

<table>
<thead>
<tr>
<th></th>
<th>$M$</th>
<th>$SD$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LD</td>
<td>19.27</td>
<td>9.03</td>
</tr>
<tr>
<td>LA</td>
<td>21.49</td>
<td>6.97</td>
</tr>
<tr>
<td>AA</td>
<td>24.34</td>
<td>7.38</td>
</tr>
</tbody>
</table>

*Note: LD = Learning Disabilities; LA = Low-Achieving; AA = Average Achieving*
Metacognitive experience. In order to investigate how ability groups differed by problem difficulty, a 3 (group: LD, LA, AA) X 3 (difficulty: 1-step, 2-step, 3-step) mixed-design ANOVA was performed. Tables 4.2 display results.

Table 4.2
Mixed-design ANOVA Summary Statistics: Metacognitive Experience

<table>
<thead>
<tr>
<th></th>
<th>F</th>
<th>df</th>
<th>$\eta^2$</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ability</td>
<td>6.93</td>
<td>2</td>
<td>.15</td>
<td>.002**</td>
</tr>
<tr>
<td>Problem Difficulty (PD)</td>
<td>88.00</td>
<td>2</td>
<td>.527</td>
<td>&lt; .001***</td>
</tr>
<tr>
<td>Ability*PD</td>
<td>2.70</td>
<td>4</td>
<td>.064</td>
<td>.033*</td>
</tr>
</tbody>
</table>

Note. *$p < .05$, **$p < .01$, ***$p < .001$

There was a significant main effect for problem difficulty and ability. The interaction between ability and problem difficulty was also significant. Post-hoc analysis using LSD adjustments were conducted to examine the significant interactions between ability and problem difficulty.

Table 4.3 displays the means and standard deviations for ability by problem difficulty. First, the ability interaction was explored across problem difficulties, in other words, changes in ME from the 1-step to the 2-step problem, 2-step to the 3-step problem, and overall from the 1-step to the 3-step problem. Next, the ability interaction was examined within each level of problem difficulty. Therefore, mean differences between ability groups for problem 1, problem 2 and problem 3 were explored.
### Table 4.3

*Means and Standard Deviations: Ability Group by Problem Difficulty: Metacognitive Experience*

<table>
<thead>
<tr>
<th></th>
<th>1 - Step Problem</th>
<th>2 - Step Problem</th>
<th>3 - Step Problem</th>
<th>Mean Across Three Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LD</td>
<td>LA</td>
<td>AA</td>
<td>Total</td>
</tr>
<tr>
<td>n</td>
<td>15</td>
<td>38</td>
<td>29</td>
<td>82</td>
</tr>
<tr>
<td>M</td>
<td>28.92</td>
<td>27.52</td>
<td>32.59</td>
<td>29.68</td>
</tr>
<tr>
<td>SD</td>
<td>4.35</td>
<td>5.00</td>
<td>4.20</td>
<td>4.52</td>
</tr>
</tbody>
</table>

*Note.* LD = learning disabilities; LA = low-achieving; AA = average-achieving
Post-hoc analysis indicates that ME differences by ability level was significant depending on the problem difficulty ($F_{[6,154]} = 3.953$, $p = .001$, $\eta^2 = .133$). Specifically, students with LD had significantly lower ME scores than AA students at 1-step problem ($M_{\text{dif}} = -4.40$, $p = .003$, $d = -0.86$), and LA students had significantly lower ME than AA students at 1-step problem ($M_{\text{dif}} = -4.86$, $p < .001$, $d = -1.10$).

Figure 4.1 displays the interaction effect between ability group and problem difficulty. For students with LD, there was a significant overall decrease in ME from the 1-step to the 3-step problem ($M_{\text{dif}} = 4.92$, $p = .032$, $d = 1.08$), and from the 2-step to the 3-step problem ($M_{\text{dif}} = 6.85$, $p < .001$, $d = 1.84$). For LA students, there was an overall significant decrease in ME from the 1-step to the 3-step problem, ($M_{\text{dif}} = 6.30$, $p < .001$, $d = 1.30$). However, the ME of LA students significantly increased from the 1-step to the 2-step problem, ($M_{\text{dif}} = 3.2$, $p = .003$, $d = -0.65$), yet decreased from the 2-step to the 3-step problem, ($M_{\text{dif}} = 9.5$, $p < .001$, $d = 2.01$) For AA students, there was an overall significant decrease in ME from the 1-step to the 3-step problem, ($M_{\text{dif}} = 9.49$, $p < .001$, $d = 2.06$), and from the 2-step to the 3-step problem, ($M_{\text{dif}} = 9.45$, $p < .001$, $d = 2.07$). There was no significant difference between the 1-step and the 2-step problem.

Overall, results indicate that as the problems increased in difficulty the ME of students significantly decreased. The LA students, however, significantly increased their ME, $p = .003$, from the 1-step to the 2-step problem, while the
other two groups’ ME remained unchanged. Furthermore, the ME of students with LD and AA students interact at the 2-step to the 3-step level, meaning the ME of AA students was higher for the 2-step and then decreased below that of students with LD for the 3-step problem.

Figure 4.1. Metacognitive Experience. Interaction between ability and problem difficulty.

Metacognitive skill. To investigate differences in MS among the ability groups, difficulty and metacognition, a 3 (group: LD, LA, AA) x 2 (metacognitive verbalization: productive, nonproductive) x 3 (problem difficulty: 1-step, 2-step, 3-step) mixed-design ANOVA was used. Thus, differences in type of metacognitive verbalizations among ability groups as a function of problem difficulty were investigated. See Table 4.4 for the summary statistics of the three main effects and four interaction effects of the analysis.
Table 4.4

Mixed-design ANOVA Summary Statistics: Metacognitive Experience

<table>
<thead>
<tr>
<th></th>
<th>F</th>
<th>df</th>
<th>η²</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ability</td>
<td>2.97</td>
<td>2.70</td>
<td>.078</td>
<td>.058</td>
</tr>
<tr>
<td>Problem Difficulty (PD)</td>
<td>14.18</td>
<td>2, 140</td>
<td>.168</td>
<td>&lt;.001***</td>
</tr>
<tr>
<td>Metacognition Type (MT)</td>
<td>4.11</td>
<td>1, 70</td>
<td>.056</td>
<td>.046*</td>
</tr>
<tr>
<td>Ability*PD</td>
<td>3.32</td>
<td>4,140</td>
<td>.087</td>
<td>.013*</td>
</tr>
<tr>
<td>Ability*MT</td>
<td>1.79</td>
<td>2, 70</td>
<td>.049</td>
<td>.175</td>
</tr>
<tr>
<td>PD*MT</td>
<td>9.12</td>
<td>2, 140</td>
<td>.115</td>
<td>&lt;.001***</td>
</tr>
<tr>
<td>Ability<em>PD</em>MT</td>
<td>3.27</td>
<td>2, 140</td>
<td>.085</td>
<td>.013*</td>
</tr>
</tbody>
</table>

Note. *p < .05, **p < .01, ***p < .001

The main effect for ability was marginally significant. The main effects for problem difficulty and metacognitive verbalization were significant. The two-way interactions between ability and problem difficulty and metacognitive verbalization type and problem difficulty were also significant. Finally, a significant three-way interaction was found among ability, metacognitive verbalization type and problem difficulty. (See Table 4.5 for means and standard deviations).
<table>
<thead>
<tr>
<th></th>
<th>One-Step Problem</th>
<th>Two-Step Problem</th>
<th>Three-Step Problem</th>
<th>Mean Across Three Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LD</td>
<td>LA</td>
<td>AA</td>
<td>Total</td>
</tr>
<tr>
<td>n</td>
<td>14</td>
<td>34</td>
<td>25</td>
<td>73</td>
</tr>
<tr>
<td>Percent Total Metacognitive Verbalizations</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>19.07</td>
<td>21.81</td>
<td>9.16</td>
<td>16.95</td>
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<tr>
<td>Percent Productive Metacognitive Metacognition</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percent Non-Productive Metacognitive</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>8.21</td>
<td>9.10</td>
<td>5.44</td>
<td>7.68</td>
</tr>
<tr>
<td>SD</td>
<td>13.78</td>
<td>12.77</td>
<td>9.67</td>
<td>12.00</td>
</tr>
</tbody>
</table>

**Note.**
LD = learning disabilities; LA = low-achieving; AA = average achieving
Post-hoc analyses using LSD adjustments were conducted to further probe the three-way interaction effect. Using post-hoc analyses, the interaction can be examined in several ways depending on which one is considered to be focal variable. In this study, the three-way interaction was examined by looking at whether the problem difficulty and type of metacognition interaction was the same for all three ability levels. See Figure 4.2 for the three-way interaction. In other words, the differential patterns of PM and NPM verbalizations as the level of problem difficulty changed were compared across ability levels.

![Figure 4.2. Three-way interaction: ability, metacognition type and problem difficulty.](image)

Students with LD significantly increased their NPM verbalizations from the 1-step to the 3-step problem \((M_{\text{dif}} = 23.74, p = .003, d = -0.91)\), and from the 2-step to the 3-step problem \((M_{\text{dif}} = 20.87, p = .003, d = -1.29)\), indicating that these students’ NPM verbalizations increased as the problems increased in
difficulty. Furthermore, there were no significant differences in PM verbalizations across the difficulty levels, suggesting that they did not modify or adapt their problem-solving strategies to meet the demands of the increasing difficulty levels of the problem.

There were significant differences between their PM verbalizations and NPM verbalizations for the 2-step problem ($M_{\text{dif}} = 8.8, p = .01, d = 1.07$), and the 3-step problem ($M_{\text{dif}} = -15.20, p = .010, d = -0.92$). In other words, students with LD produced significantly more PM verbalizations than NPM verbalizations in the 2-step problem, whereas on the 3-step problem they produced significantly more NPM verbalizations than PM verbalizations. This indicates that they may have perceived the 2-step problem to be within their problem-solving ability, whereas the 3-step problem may have been perceived as too difficult to solve.

LA students produced significantly fewer NPM verbalizations from the 1-step to the 2-step problem ($M_{\text{dif}} = 6.8, p = .008, d = 0.69$), and significantly more NPM verbalizations from the 2-step to the 3-step problem ($M_{\text{dif}} = 6.42, p = .029, d = -0.57$), indicating that they may have perceived the 1-step problem to be more difficult than the 2-step problem, and the 2-step problem to be easier than the 3-step problem. There was no significant difference in PM verbalizations across the three levels of problem difficulty, suggesting that these students did not adapt their problem solving strategies to meet the changing demands of the problems.

However, these students produced significantly more PM verbalizations than NPM verbalizations on the 2-step problem ($M_{\text{dif}} = 10.29, p = .003, d = 1.12$),
indicating that they may have perceived the 2-step problem to be within their problem-solving ability.

For AA students there was an overall significant increase in PM verbalizations from the 1-step to the 3-step problem ($M_{\text{diff}} = 10.62$, $p < .001$, $d = -1.17$), and the 1-step to the 2-step problem ($M_{\text{diff}} = 8.19$, $p < .001$, $d = -0.83$). They also produced significantly more NPM verbalizations from the 2-step to the 3-step problem ($M_{\text{diff}} = 8.35$, $p = .001$, $d = -0.29$). This suggests that they may have perceived the 3-step problem as more difficult than the other two problems.

Furthermore, AA students produced significantly more PM verbalizations than NPM verbalizations on the 2-step problem ($M_{\text{diff}} = 10.04$, $p < .001$, $d = 1.08$). Of the three ability groups only the AA students increased both their PM verbalizations and NPM verbalizations for the 3-step problem while students with LD and LA students did not adjust their PM verbalizations to meet the increased demands of the problem.

In sum, all students increased their NPM verbalizations from the 2-step to the 3-step problem, which suggests that they all perceived the 3-step problem to be more difficult than the other two problems. Only students with LD, however had a significant difference between their PM verbalizations and NPM verbalization for the 3-step problem, indicating that these students either lacked the necessary strategies to solve the problem or were unable to apply the strategies they did possess since they may have perceived this problem to be beyond their problem-solving abilities and gave up.
Based on the lack of significant differences between NPM verbalizations and PM verbalizations for the 3-step problem it may appear that the LA students and AA students seemed to use the PM verbalizations to solve the problem, but examination of their metacognitive strategy use across the three problems indicate that only AA students adjusted their PMV to meet the increased demands of the problem.

All students had significantly more PM verbalizations than NPM verbalizations for the 2-step problem, as well as fewer NPM verbalizations from the 1-step to the 2-step problem. This suggests that students may have perceived the 2-step problem to be easier than the 1-step problem despite the monotonically increasing difficulty levels of the problems.

**Question 2: Relationship among Metacognitive Components and Math Word Problem Solving.**

The current study investigated bivariate correlations between each of the three components of the metacognitive framework and math problem-solving ability for a total of four correlation coefficients. Additionally, separate correlations were analyzed by ability group to determine if the relationships between these variables vary as a function of ability group membership. See Table 4.6 for the correlation matrix across ability groups.

Correlations were computed using the total scores from each of the metacognitive measures and the 10-item math word problem-solving probes.
(MWPSP). In addition to the $p$-values, effect sizes, which are not affected by sample size, were examined to determine the practical significance of the bivariate relationships. In this study, Keith's (1999) criteria was used for judging the magnitude of effects in psychology.

Table 4.6

Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>$n$</th>
<th>MK</th>
<th>ME</th>
<th>MS</th>
<th>MWPS</th>
</tr>
</thead>
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<td>MK</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>82</td>
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<td></td>
</tr>
<tr>
<td>LD</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LA</td>
<td>38</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AA</td>
<td>29</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ME</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>82</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>LD</td>
<td>15</td>
<td>.07</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>LA</td>
<td>38</td>
<td>.04</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>MS</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>All</td>
<td>75</td>
<td>-.37**</td>
<td>-.17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LD</td>
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<td>-.64*</td>
<td>-.26</td>
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<tr>
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<td>-.20</td>
<td>-.11</td>
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<tr>
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<td>MWPS</td>
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<td>.13</td>
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<td>-.49</td>
<td>-.02</td>
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<tr>
<td>LA</td>
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<td>-.01</td>
<td>.07</td>
<td>-.12$^a$</td>
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<tr>
<td>AA</td>
<td>28</td>
<td>.38*</td>
<td>.29</td>
<td>-.26$^b$</td>
<td></td>
</tr>
</tbody>
</table>

Note.
MK = Metacognitive Knowledge; ME = Metacognitive Experience; MS = Metacognitive Skill; MWPSP = Math Word Problem-solving Probes

*p < .05. **p < .01. ***p < .001

$^a$ $n = 34$

$^b$ $n = 24$

All abilities. First, correlations were computed for all students irrespective of ability group membership. Although not significant, there was a small, positive correlation between MK and ME and a moderate, negative correlation between
ME and MS. This suggests that the more knowledge students possess, the more likely they are to feel efficacious about solving the problem correctly. Furthermore, the more efficacious students feel, the less likely they are to use their MS to help solve the problem.

There was a significantly large, negative correlation between MK and MS, indicating that as students’ MK increased the use of their MS decreased. MK accounted for 12% of the variance in students’ MS. Additionally, there was a significantly moderate, positive correlation between MK and MWPS performance, indicating that as MK increased, so did MWPS performance. MK accounted for 5% of the variance in math word problem-solving performance. Furthermore, there was a moderate negative correlation between MS and MWPS performance that was marginally significant, indicating that as MS increased, MWPS performance decreased. MS accounted for 5% of the variance in MWPS performance.

**Students with learning disabilities.** When considering the bivariate relationships between these variables for students with LD there was a large, significant, negative correlation between MK and MS, indicating that as students’ MK increased, the use of the MS skills decreased. Stated otherwise, as their MK decreased, the use of their MS increased. For students with LD, MK accounted for 41% of the variance in MS. Although not significant, there was a very small, positive correlation between MK and ME, indicating that as students' knowledge increased, so did their ME. Furthermore, there was a large, negative correlation
between ME and MS, indicating that as students’ ME increased, their MS decreased. In relation to MWPS performance, there was a small to moderate negative correlation for MK, a large, negative correlation for ME and a very small, negative correlation for MS, none of which was statistically significant.

**Low-achieving students.** There were no statistically significant bivariate correlations among the three components of metacognition and the MWPS for LA students. However, the magnitude of the bivariate effects is of interest. There was a small, positive correlation between MK and ME, and a moderate, negative correlation between ME and MS. This indicates that as students’ MK increased, so did their ME. Furthermore, the more efficacious these students felt, the less likely they were to employ MS to help solve the problem. There was also a moderate, negative correlation between MK and MS, indicating that as students’ MK increased, their MS decreased. In relation to MWPS, there was a very small, negative correlation for MK, a small, positive correlation for ME and a moderate, negative correlation for MS.

**Average-achieving students.** There was a very small, positive correlation between MK and ME, whereas there was a moderate to large negative correlation between ME and MS. This suggests that for AA students, there was no meaningful relationship between the amount of MK students’ possess and their ME. However, the more efficacious an AA student felt about their ability to solve the problem, the less likely they were to use MS to solve the problem. Additionally, there was a moderate, negative correlation between MK
and MS, indicating that as students’ MK increased, their MS decreased. MK and MWPS performance was significantly, positively correlated, indicating that as MK increased so did MWPS performance. For average achieving students, MK accounted for 14% of the variance in MWPS performance. Although not significant, there was a large, positive correlation between ME and MWPS and a large, negative correlation between MS and MWPS.

In sum, the directions of the bivariate correlations among the three components of metacognition were the same for all three ability groups (i.e., a positive correlation between MK and ME, a negative correlation between ME and MS, a negative correlation between MK and MS). However, the magnitude, or strength of these relationships, differed as a function of ability group membership. For example, the relationship between MK and MS was small for students with LD, moderate for LA students and very small for AA students. The relationship between ME and MS was large for students with LD and moderate to large for LA students and AA students. Finally, the relationship between MK and MS was very large for students with LD, large for LA students and moderate for AA students. With respect to MWPS, there was a significantly large, positive correlation between MK and MWPS performance only for AA students. This relationship was small to moderate for students with LD and very small for LA students. Interestingly, the relationship between ME and MWPS was large and negative for students with LD, but large and positive for AA students. This may indicate differences in underlying cognitive processing.
**Question 3: Effect of Metacognition on Math Word Problem Solving**

To test the possible differential effects of ability and metacognitive functioning on math word problem-solving (MWPS) performance, a regression analysis was performed. Differing from bivariate correlations, which do not make a distinction between the independent and dependent variables, the linear regression does. Figure 4.3 shows the relationship, specifically the direction and magnitude, between each component of metacognition and MWPS performance irrespective of ability group membership.

*Figure 4.3. All Abilities. Relationship between Metacognition and Performance.*
Examination of each correlation between metacognition and MWPS across ability groups revealed to be differences. For example, when looking at the bivariate correlation between ME and MWPS performance for each ability group, LD = - .49, LA = +.07, and AA = +.29 both the direction and magnitude of the relationships show divergent trends. Therefore, the purpose of the regression was first to determine whether slope differences existed across ability groups. In other words, the null hypothesis (i.e., no differences existed among the ability groups’ regression lines was tested). Second, the extent each variable influenced MWPS performance when all other variables were controlled for was determined.

The following regression includes all independent variables and the interaction terms in a single model. MWPS performance was regressed on ability, MK, ME, MS, MK_LD, MK_LA, ME_LD, ME_LA, MS_LD and MS_LA. Together, ability, MK, ME, MS, and the interaction terms accounted for 40.2% of the variance in MWPS performance. The omnibus test was statistically significant, $F (11, 59) = 3.605, p = .001$, indicating that the independent variables are related to the dependent variable. In other words, a common regression line fits the data and a relationship is observed between ability, the three components of metacognition, the interaction terms and MWPS performance.

The subtests of the estimated coefficients indicate that there was a significant ability group difference on MWPS performance, $F (2,59) = 4.826, p = .011$. As was expected, AA students significantly outperformed both students
with LD \( (M_{\text{dif}} = -.324, p = .013) \), and LA students \( (M_{\text{dif}} = -.359, p = .006) \). Furthermore, the slopes of metacognition significantly differed by ability groups, \( F(3,59) = 3.066, p = .035 \). Statistical testing of the slope difference for ME between students with LD and AA was significant \( (M_{\text{dif}} = -.309, t = -2.62, p = .011) \), indicating that students with LD had significantly lower ME than AA students. Although the comparison of slopes of MK by ability groups was not statistically significant, \( F(2,59) = 2.370, p = .102 \), MK was a significant predictor of MWPS performance for AA students \( (M_{\text{dif}} = .439, t = 3.232, p = .024) \). This indicates a large, positive effect, which means that the more knowledge these students’ possess, the more likely their performance will increase.

Furthermore, the slope difference between students with LD and AA students approached significance, \( (M_{\text{dif}} = -.332, t = -1.96, p = .055) \), indicating that the relationship between MK and MWPS performance is different for students with LD and AA students. The regression coefficients show the extent of the influence of each IV on the DV.
Table 4.7

Summary of Regression Analysis for Variables Predicting Math Word Problem Solving Performance

<table>
<thead>
<tr>
<th></th>
<th>B</th>
<th>SE B</th>
<th>β</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>5.72</td>
<td>0.48</td>
<td>-0.32</td>
</tr>
<tr>
<td>LD_Dum</td>
<td>-1.60</td>
<td>0.76</td>
<td>-0.36*</td>
</tr>
<tr>
<td>LA_Dum</td>
<td>-1.68</td>
<td>0.59</td>
<td>-0.36**</td>
</tr>
<tr>
<td>MK</td>
<td>0.13</td>
<td>0.06</td>
<td>0.44*</td>
</tr>
<tr>
<td>ME</td>
<td>0.06</td>
<td>0.04</td>
<td>0.25</td>
</tr>
<tr>
<td>MS</td>
<td>-0.05</td>
<td>0.05</td>
<td>-0.23</td>
</tr>
<tr>
<td>LD_MK</td>
<td>-0.19</td>
<td>0.10</td>
<td>-0.33</td>
</tr>
<tr>
<td>LD_ME</td>
<td>-0.20</td>
<td>0.08</td>
<td>-0.31</td>
</tr>
<tr>
<td>LD_MS</td>
<td>0.00</td>
<td>0.08</td>
<td>0.00</td>
</tr>
<tr>
<td>LA_MK</td>
<td>-0.13</td>
<td>0.08</td>
<td>-0.27</td>
</tr>
<tr>
<td>LA_ME</td>
<td>-0.04</td>
<td>0.05</td>
<td>-0.12</td>
</tr>
<tr>
<td>LA_MS</td>
<td>0.03</td>
<td>0.06</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Note. *p < .05, **p < .01, ***p < .001

Figure 4.4 shows the separate regression lines for each metacognitive component across ability groups. The graph illustrates the similarities and differences among ability groups’ metacognitive functioning. Students with LD and LA students appear to have similar slopes for both MK and MS. However, for ME, students with LD have a large, negative slope, whereas LA students have a small to moderate positive slope. AA students appear to behave differently from the other two ability groups. For example, AA students have large, positive slopes for MK and ME, and a large, negative slope for MS. However, with the exception of the large, negative ME slope of students with LD,
the strength of the relationships between the metacognitive components and MWPS performance is not as strong for students with LD and LA students as for AA students.

Figure 4.4. Regression: MWPS on metacognition across ability groups.
CHAPTER 5

Discussion

Research has established the importance of metacognition to academic success (e.g., Pintrich, Anderson, & Klobucar, 1994; Trainin & Swanson, 2005; Wong, Harris, Graham, & Butler, 2006) and its particular importance to successful problem solving (De Corte, Greer, & Verschaffel, 1996; Lucangeli & Cornoldi, 1997; Montague, 2008; Swanson, 1990). Research also suggests that metacognition develops alongside general aptitude and may be even more influential than general aptitude in predicting learning performance (Swanson, 1990; Veenman & Spaans, 2005). Metacognitive strategies (e.g., self-instruction, self-questioning, and self-evaluation) are used to monitor and evaluate cognitive progress during task execution (Montague, 2008; Zimmerman, 2002). It has been widely substantiated in reading research that students with LD demonstrate inadequate metacognitive awareness, possess inefficient strategies, and exhibit a lack of control over their academic behavior (e.g., Mason, Meadan, Hedin, & Corso, 2006; Wong et al., 2006). In contrast, there is limited research on the metacognitive functioning of students with LD focusing on math problem solving (e.g., Carr, Alexander, & Folds-Bennet, 1994; Case, Harris, & Graham, 1992; Montague & Applegate, 1993).

It is agreed upon that the components of metacognition (knowledge, experience and skills) are interdependent (Flavell, 1987). Much of the current research in metacognition considers each component in isolation or considers the relationship between knowledge and skills, while neglecting the personal
experiences that an individual brings to the task. This disregard of personal experience prevents researchers from understanding how knowledge is translated into skill or why some knowledge transfers while other knowledge remains dormant when students are engaged in task execution. Examining personal factors, such as the self-motivational beliefs and self-efficacy of students may help to explain differences in math word problem-solving (MWPS) performance.

The purpose of this study was to investigate three components of metacognition, and examine differences in metacognitive functioning among students with learning disabilities (LD), low-achieving (LA) students and average-achieving (AA) students in the area of MWPS. Additionally, the relationship among these components and their influence on academic performance were investigated. This chapter is organized into four sections. First, ability group differences with respect to each of the components of metacognition (i.e., metacognitive knowledge, metacognitive experience and metacognitive skill) are discussed. Second, the relationship among these three components and MWPS performance is discussed. Next, the effects of each metacognitive component as well as the interaction between ability and metacognition components on problem-solving performance are discussed. Finally, the limitations of the study, implications for instruction and directions for future research are considered.
**Ability Group Differences and Metacognitive Functioning.**

**Metacognitive knowledge.** Based on the results of the ANOVA, there were no significant differences among the three ability groups in their metacognitive knowledge, that is, the declarative, procedural and conditional awareness regarding personal functioning, task execution and strategy selection. Considering students with LD and LA students are lower performers than AA students, it is interesting that their reported knowledge concerning the strategies used to solve math word problems was not different. This may be because AA students have internalized their beliefs and knowledge and are unable to consciously recall them. Previous research suggests that students differ in the amount of knowledge they have as well as the organization and accessibility of that information (Garner & Alexander, 1989). Findings from the current study do not support differences in amount of knowledge. However, as will be discussed next, differences in the organization and accessibility of that knowledge may be more indicative of ability group differences.

**Metacognitive experience.** The mixed-design repeated measures ANOVA explored differences in metacognitive experience among students with learning disabilities, low-achieving students, and average-achieving students as a function of problem difficulty and also investigated the interactions among metacognitive experience and ability group membership. Metacognitive experience was operationalized as task-specific self-efficacy and self-motivational beliefs. Students responded to statements pertaining to their
affective disposition on three increasingly difficult math word problems. Statements included, “I am confident I can solve this problem correctly” and “this problem is going to be difficult to solve.” Research suggests a positive correlation between self-efficacy and cognitive engagement (Hoffman & Spatariu, 2007; Pintrich & DrGroot, 1990). In other words, the more efficacious and confident a person feels towards a given task, the more likely he or she will persist in accomplishing the task. Results of this analysis suggested that when one does not consider the difficulty level of the problem, students with LD look similar to AA students on this measure of metacognitive experience, as indicated significant decreases in the ME for both groups as the problems became more difficult. However, different patterns of metacognitive activity were evident for the ability groups when level of problem difficulty is considered. Several of the salient findings are described below.

Overall, as expected, students were significantly less confident in their self-assessment on the 3-step problem than on the 1-step. That is, on the most challenging problem, all students perceived the problem as being more difficult. There was an unexpected significant increase in metacognitive experience from the 1-step to the 2-step problem for low-achieving students, despite the problem being more challenging. This increase may be explained by the nature of the problem and the mathematical operations necessary to solve the problems. The context of the 1-step problem, for example, involved arranging chairs in a school auditorium, whereas the 2-step problem, involved friends going to the movies.
Students are more likely to have experienced the second scenario rather than the first. Furthermore, although the 1-step problem required only one operation (i.e., division), the 2-step problem could have been solved using only addition and subtraction, operations which may have been perceived as easier by the students. Some students, for example, knew that the problem required division. However, they were unable to divide 252 by 21 without the use of a calculator. These students attempted to solve the problem by adding 21 multiple times until they reached 252 or they simply guessed. These two reasons may explain the unexpected increase in metacognitive experience for low-achieving students despite the increased difficulty of the problem.

The metacognitive experience of students with LD and AA students, on the other hand did not significantly change from the 1-step to the 2-step problem. The similar metacognitive experience between these two groups is surprising given that only 7% of students with LD solved the problem correctly, whereas 41% of AA students were successful in solving the problem. Previous research (Hoffman, Spatariu, 2007; Pintrich & DeGroot, 1990) suggests that increased self-efficacy leads to greater problem-solving accuracy. However, this is clearly not the case for students with LD. The findings from the current study are comparable with research on the metacognitive experience of students with LD (Alvarez & Adelman, 1986). This research suggests that students with LD overestimate their abilities and place more attention on the cognitive demands of the problem rather than on monitoring their comprehension and performance.
Consequently, this allocation of attention to cognitive demands makes them less metacognitively aware (Butler, 1996).

Interestingly, students with LD had the highest ME on the 3-step problem of the three groups, but not significantly higher. This suggests that they may have perceived the problem to be within their problem-solving capabilities. However, given that none of the students solved the 3-step problem correctly, they were the most inaccurate in their ME. Furthermore, on the 3-step problem, the ME of LA students and AA students was well below that of the students with LD. These findings suggest that the other two ability groups were more aware of the difficulty of the problem and consequently more accurate in their ME. In contrast to previous research, which suggests that LD students are more accurate in predicting difficult tasks than those that are within their range (Alvarez & Adelman, 1986), the current findings show that students with LD overestimated their ability even on the most difficult task. These students have been referred to as “self-enhancers,” attributing their successes to extrinsic abilities while dismissing failures as consequences of intrinsic factors (Stolp & Zabrucky, 2009). These self-enhancing behaviors are said to benefit students’ short term task engagement. However, these inaccurate self-motivational beliefs may have negative consequences for long-term goals.

Understanding differences in students’ ME provides useful information regarding how students come to use their cognitive strategies and become metacognitively engaged in tasks. Students’ ME, although non-cognitive in
nature, has the potential to enhance or interfere with the allocation of cognitive and metacognitive resources and knowledge (Goos and Galbraith, 1996). Schunk (1985) suggested that although cognitive strategies are more highly associated with academic performance, instructing students how to use these strategies and increasing their ME may lead to increased application of these cognitive strategies. The results of the present study suggest that this was not the case for students with LD since their increased ME was not reflective of an increase in application of strategy use. It may, therefore, not be a matter of increased ME that contributes to strategy use. Rather, it may be students’ ability to calibrate their ME when confronted with challenging tasks that explains increased performance. The following example may help clarify the relationship between ME and MS. Although students may have strong feelings of confidence regarding their ability to solve a problem, in other words, they find the task easy it is their awareness of miscomprehension that helps them activate their MS to work through the problem. If, on the other hand, students are unaware of the difficulty of the problem and misjudge their ability to solve the problem, they will be less likely to call upon the necessary strategies to help them correctly solve the problem.

**Metacognitive skill.** The mixed-design ANOVA explored differences in metacognitive verbalizations among students with LD, LA students, and AA students as a function of problem difficulty as well as examined the interactions among ability group, metacognitive verbalizations and problem difficulty.
Metacognitive verbalizations were operationalized as either productive or nonproductive. Productive metacognitive (PM) verbalizations include strategies such as self-instruction, self-questioning, and self-monitoring, which help students navigate the word problem as they solve it. Nonproductive metacognitive (NPM) verbalizations include affective responses and reactions (e.g., negative self-talk and expressions of confusion or frustration) to the problem that do not directly move the student toward a solution. Results of this analysis suggested that different ability groups engage in different patterns of metacognitive functioning when type of metacognitive verbalization and problem difficulty are considered.

It appears that students with LD, despite making significantly more metacognitive verbalizations than AA students, were making more nonproductive verbalizations, indicating that these students may have perceived the problems to be more challenging than the AA students. Initially, students with LD increased their PM verbalizations from the 1-step to the 2-step problem, but then decreased these more useful verbalizations on the 3-step problem. Theoretically, they should have used more PM due to the increased demands of the problem. Nonetheless, this decrease in PM strategies may be due to either the students’ perception that the problem was too difficult and they simply shut down or that they had exhausted their metacognitive resources.

Generally, students with LD and the LA students demonstrated similarities in their patterns of metacognitive behavior. However, differences were
evidenced on the most difficult problem, the 3-step problem. Both groups significantly increased their NPM verbalizations from the 2-step to the 3-step problem, which suggests that, when faced with a very difficult problem, these students may not have or may not use appropriate resources for solving the problem. Furthermore, both students with LD and LA students demonstrated almost no variation in PM verbalizations as problems became more difficult suggesting that they do not discriminate and use the same resources regardless of problem difficulty. However, the students with LD had significantly more nonproductive statements than both the LA and AA students on the 3-step problem indicating their increased frustration with the problem. It should be noted that all ability groups had a significant increase in NPM verbalizations from the 2-step to the 3-step problem, but only the AA students significantly increased their PM verbalizations as problems became more difficult. Additionally, none of the students from any of the ability groups correctly solved the 3-step problem. As reflected by the greater number of PM verbalizations made by the AA students on the 3-step problem, it seems that these students were more productively engaged in solving the problem. For the 1-step problem, 42% of students with LD, 41% of LA students and 76% of AA students solved the problem correctly. For the 2-step problem, 7% of students with LD, 32% of LA students and 52% of AA students solved the problem correctly. Based on these findings, it appears that more metacognition does not necessarily mean better metacognitive activity or better MWPS performance.
Metacognitive strategies need to be anchored in developmentally appropriate cognitive skills in order to be useful during problem solving. The literature suggests that in the absence of the cognitive processes and skills necessary for successful problem completion, students rely on their metacognitive skillfulness to work through the problem (Veenman & Spaans, 2005). Expert problem solvers generally have the cognitive processes and skills required to solve problems and do not necessarily need to activate their metacognitive skills unless the problem presents some difficulty (Crowley, Shrager & Siegler, 1997). This illustrates the important role that automaticity plays in problem solving. The goal of conscious and deliberate use of metacognitive strategies is to help students apply their cognitive knowledge until it eventually becomes internalized to the point of automaticity. Expert problem solvers will not need to activate their metacognitive skills to solve math word problems and will therefore produce fewer metacognitive verbalizations than students who have not yet mastered the problem-solving processes and skills. In the present study, students with LD and LA students, who presumably were not as skilled in math problem solving as the AA students, used metacognition but their verbalizations were primarily nonproductive. In other words, their metacognitive behavior did not aid in providing the necessary support to successfully work through the problem. These students may not have developed the self-regulation strategies or, if they had, they may not have had the requisite problem-solving processes and skills in place. To illustrate, Figure 5.1 presents two think-aloud protocols of students with
LD. The first student did not make any metacognitive verbalizations and answered the problem correctly, whereas 33% of the second student’s verbalizations were metacognitive and that student answered the problem incorrectly. It is evident that the first student was operating at the automatic level and had the necessary cognitive skills in place to solve the problem, whereas the second student did not. There is obviously variability within the ability groups with respect to problem-solving ability and metacognitive functioning. Both need to be addressed to ensure that students acquire the necessary math problem-solving processes, skills, and strategies for successful problem solving.

**Relationships among Metacognitive Components and MWPS performance.**

Correlational analyses explored relationships among the metacognitive components and MWPS performance for students with LD, LA students, and AA students in order to determine whether these relationships varied as a function of ability group membership. Overall, results showed that the directions of the bivariate correlations were the same for all three ability groups. However, the ability groups did not share the same magnitude and strength of these relationships. Metacognitive skills (MS) are self-help tools that aid in understanding and working through challenging or novel problems. The negative correlation between metacognitive knowledge (MK) and MS suggests that the more awareness students have about problem-solving strategies, the less they will need to activate their MS to help them work through the problem.
### Figure 5.1: Two Coded Think-Aloud Protocols of Students with Learning Disabilities

<table>
<thead>
<tr>
<th>Participant #</th>
<th>Ability</th>
<th>Time</th>
<th>Solved</th>
</tr>
</thead>
<tbody>
<tr>
<td>20443</td>
<td>LD</td>
<td>45s</td>
<td>Correct</td>
</tr>
<tr>
<td>R</td>
<td>Bob and Shirley are arranging the chairs for a class play. They brought two fifty-two chairs from the Storeroom to the auditorium. Their teacher told them to make rows of twelve each. How many rows Will they have? So you twelve divided by two hundred and fifty-two. And then you start with two, so you can’t. Twenty-five, so that’s like two and then twelve divided by twelve is one...twenty-one. Cognitive: 100% Meta: 0% Prometa: 0% Nonprometa: 0%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th>Participant #</th>
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</tr>
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<tbody>
<tr>
<td>20501</td>
<td>LD</td>
<td>171s</td>
<td>Incorrect</td>
</tr>
<tr>
<td>R</td>
<td>Bill and Shirley are arranging the chairs for a class play. They brought two hundred and fifty-two chairs from the Storeroom to the auditorium. Their teacher told them to make rows of twelve chairs each. How many rows will they have? First, I’m gonna underline the important details, which is Billy and Shirley are arranging the chairs for a class play. They brought two hundred and fifty-two chairs from the Storeroom...and their teacher told them to make rows of twelve chairs each. And the question is how many rows will they have. So I’m going to divide two hundred and fifty-two by twelve. [7sec]. So first I’m gonna do two goes into two. Wait [student erases work]. Remember to say everything out loud. So Twelve divided by two hundred and fifty-two. First I’m going to see how many times can twelve...two hundred and fifty-two times...which is zero. And twelve times zero is zero. And I subtract that from Twenty-five, which gives me twenty-five. And then I add the two. So then I see how many times does Twelve go into twenty-five; two hundred and twenty-five... so two goes into two once. Two divided by Five is two. Two goes into two one. One divided by two is too. One divided by five is five, and one divided by two is two. Then I add them...one, two plus two is four, five plus one is six. Let me see. And my answer is two, and my answer is two um wait. So I would need to have two hundred, two thousand six hundred and forty-one rows. Cognitive: 67% Meta: 33% Prometa: 33% Nonprometa: 0%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Stated differently, the less MK students have pertaining to problem solving, the more likely they will need to apply their MS to help accomplish the task. Furthermore, the negative correlation between MS and MWPS performance indicates that the employment of MS does not ensure a sound solution. Activation of MS suggests that students may be uncertain about how to solve the problem or may not have the cognitive resources available to effectively and efficiently solve the problem. That is to say, the presence of MS could be an indication of students’ level of mastery. The more students know about problem solving (i.e., their MK), the less likely they will need to use self-help tools (i.e., their MS), and, consequently, the more likely they will solve the problem correctly.

This pattern is evident when considering the relationship among the three components and MWPS performance for each ability group. Findings from this study support previous research (i.e., Stolp & Zabrucky, 2009) that suggests that possessing high levels of MK may lead to academic success. That knowledge, however, may not always transfer into practice. For example, knowing that re-reading a problem is useful in aiding comprehension does not imply that students will make use of that knowledge. In this study, AA students exhibit a significantly strong, positive correlation between MK and MWPS success, whereas students with LD reveal a significantly large, negative correlation between MK and MS. One rationalization explaining these different patterns in metacognition is that AA students may have internalized their MK to the point of automaticity and do not
need to activate their MS to correctly solve the problems. Students with LD, on the other hand, who, according to these measures, are not significantly different in MK from AA students need to consciously activate and manipulate their MS to help solve the problem. However, this activation did not lead to problem solving success.

Another explanation may be that students' ME plays a role in explaining why certain knowledge transfers, while other knowledge does not. Previous research (Glenburg & Epsteing, 1987) suggests a positive correlation between MK and ME. In other words, the more knowledge students possess in a given domain, the more likely they are to feel efficacious about their abilities. It is expected that this increase in problem-solving confidence will likely encourage students to persist longer and apply more strategies while attempting to solve the problem. Examinations of the bivariate correlations revealed no significant relationship between MK and ME for any of the ability groups. Furthermore, examination of the effect sizes support a positive correlation between MK and ME for all ability groups. Interestingly, however, the magnitude of the relationship for each ability group was different. For example, the relationship between MK and ME was small for students with LD, small to moderate for LA students and very small for AA students. This indicates that the amount of knowledge students’ possess does not have a very strong influence on their self-efficacy and confidence in solving problems.
Although the relationship between MK and ME was not significant and the strength of the relationship was generally small, there was a moderate to large, negative correlation between ME and MS, which also was not significant. Contrary to expectation, this suggests that the greater the students ME, the less likely they apply MS to help them solve the problem. These findings are not surprising given that MS are activated in light of uncertainty and confusion. Future research will benefit from an examination of ability group differences in students’ accuracy of ME and calibration of their ME before and after problem solving.

Finally, no significant correlations were found among the components of metacognition and MWPS performance for LA students. It is important to consider these findings in comparison to students with LD. For example, both ability groups are considered low performers and in need of additional instruction. When examining the bivariate correlations among the metacognitive components for each of these ability groups (See Table 4.6) different relationships are observed. LA students seem to be a heterogeneous group that do not exhibit any defining relationships. In other words, these students are representative of a “garden-variety” type of low performers. Students with LD, on the other hand, had a strong, negative correlation between MK and MS, indicating that these students are more homogeneous and share a similar relationship explaining their poor performance (i.e., lack of internalized or automated knowledge). It would be unwarranted to consider these two groups of students as identical, since there
appears to be a difference in their underlying cognitive processing. Consequently, instruction that does not address these cognitive differences risks failing to provide appropriate remediation for these students.

**Effects of Metacognition on Math Word Problem solving**

The purpose of conducting the regression analyses was two-fold. Given the different correlational trends among ability groups, the first purpose was to test slope differences across ability groups. The second purpose was to determine which component of metacognition best predicts MWPS performance. Results of these analyses indicated that MK was a significant predictor of MWPS performance for AA students. Examination of slope differences show that for AA students, the more metacognitive knowledge students possess, the more likely they will experience MWPS success. The slope difference between students with LD and AA students approached significance. This suggests that the relationship between MK and MWPS performance is different for students with LD. These students were lower in MK than AA students. Since the difference was not significant this may indicate that students with LD are able to report the declarative knowledge, but are unable to apply the procedural or conditional knowledge that allows them to effectively solve math word problems.

Students with LD also had significantly lower ME than AA students. For AA students there was a positive but not significant relationship between ME and MWPS performance. That is, the more efficacious students’ feel about their abilities to solve problems, the more likely they are to solve them correctly.
Interestingly, the relationship is the opposite for students with LD, that is, the more confident they are, the less likely they are to solve the problem correctly. This could be an indication of “illusions of understanding” whereby students with LD are overconfident in their abilities and therefore do not apply the appropriate strategies or allocate enough resources to solve the problem correctly. Their boldness in solving the problem prohibits them from fully understanding the problem requirements. These students may impulsively arrive at an answer without assessing their comprehension or checking the soundness of their solution.

**Implications for Instruction**

Instruction in metacognition is important because it affects the acquisition, comprehension, retention and application of learned material (Hartman, 1998). Metacognition enables self-control over thinking and learning as students consciously and actively work through a problem become aware of their comprehension, and monitor their progress. Instruction in metacognition entails helping students become active participants in their learning and guiding students through the appropriate cognitive processes that expert problem solvers use when confronted with difficult or challenging tasks. This is best accomplished through teacher modeling. By thinking out loud as they solve the problem, teachers provide students with direct access to the cognitive and metacognitive strategies used when problem solving as well as teach students how effective and efficient problem solvers attend to the components of the task.
and work through “road-blocks” using these strategies. The goal of metacognitive instruction is to first increase awareness of the necessary strategies required to successfully solve the problem and then to have the students internalize these strategies to the point of automaticity in order to alleviate the cognitive load and free-up working memory.

Evidently, the relationships among the metacognitive knowledge students possess, the metacognitive skills that students use, and the motivational beliefs influencing the interaction among these components are complicated. Cognitive strategy instruction (CSI) is a popular instructional technique for enhancing the self-regulation of students. During CSI students are first provided with a model of how to effectively and efficiently solve a problem by an experienced problem solver. They are then provided with guided practice, corrective feedback and positive reinforcement and finally encouraged to accomplish the task independently. Furthermore, throughout instruction students are taught to externalize their inner speech by thinking out loud, which allows not only the students to become aware of their thought processes, but also provides teachers with a window into their thinking. The increased support, explicit instructional routine and positive reinforcement and guidance will likely increase students’ MK, enhance students’ ME and help them select and apply appropriate MS when faced with difficult or challenging tasks.
Limitations and Directions for Future Research

There are several limitations of the present study that should be considered when interpreting the results and in conducting future research. First, much of the research using think-aloud methodology utilizes small sample sizes (e.g. Bannert & Mengelkamp, 2008; Swanson, 1990). Thus, while the groups in this study \( n = 15, 38, 29 \) were comparable to or larger than sample sizes in similar studies, by traditional quantitative standards the groups were small. A larger sample would help to describe characteristics of students within groups to generate a more comprehensive profile of students with differing abilities, particularly those with LD. Furthermore, a consequence of small sample size is the likelihood of being underpowered, which, in some cases, may explain the lack of significant results. The regression analysis, for example, included 12 variables. However, since effect sizes are standardized units independent of sample sizes, effect sizes were examined in addition to \( p \)-values to determine the practical significance of the non-significant results. Therefore, it would be beneficial for future research to replicate these findings with a larger sample size.

Second, the participants in this study were primarily Hispanic and African American students. Therefore, findings from the current study may not be generalizable to non-minority or non-urban populations. Third, there was wide variability within groups on all measures of metacognitive activity. Future research might consider more extensive participant selection in order to reduce this heterogeneity within groups and provide a better opportunity to detect
differences. Lastly, the nature of the construct under investigation requires measures that may be subject to participant or researcher bias, such as self-reports and think-aloud measures, and thus the validity of the measures are questionable. Using think-aloud protocols as a means of data collection, for example, rests on the assumption that student are capable of thinking out loud while engaged in task completion. However, it was still possible that students were engaged in metacognitive activity that was not verbalized and therefore was not accessible to the researcher. Future research would benefit from assessing the reliability and validity of the various methods of collecting data on metacognition.

Despite these limitations, this study made valuable contributions to the literature in several ways. First, much of the research on the metacognitive functioning of students with LD has focused on reading comprehension. However, this study contributed to the relatively small body of research concerning the metacognitive functioning of students with LD in the area of mathematical problem solving. Second, investigating each component in isolation fails to capture the interdependent nature of metacognition. This study not only provided an in-depth examination of ability group differences in metacognitive functioning, it also investigated possible differential patterns of the relationship among the three components of metacognition. Understanding how certain knowledge is translated into practice and why some knowledge transfers while other knowledge does not is important to providing students with the necessary
remediation to be successful. Third, since metacognition is activated during challenging and novel tasks, this study investigated how students adjust their metacognition when faced with increasingly difficult math word problems.

Examining metacognition at a single moment on a single task fails to capture the functional nature of metacognition. Finally, metacognitive research has focused predominantly on differences between students with LD and average or high achieving students largely ignoring more subtle differences between students with LD and LA students. This study provided a description of similarities and differences between these two groups, revealing subtle processing differences that would not be apparent from performance-based measures of ability.

In sum, there are several salient findings to this study: First, students with LD demonstrated a different pattern of metacognitive functioning than AA students and LA students. The findings suggest that AA students have internalized their MK to the point of being automatic, which helped them to effectively and efficiently solve math word problems. Students with LD, on the other hand, needed to employ MS to help them work through the problem. This indicates a lack of comprehension or cognitive failure. Additionally, categorizing LA students and students with LD into the same low-performing group fails to capture their different underlying cognitive profiles. Second, when one does not discriminate between the type and quality of the metacognitive skills, students across ability groups look relatively equivalent in the quantity of strategies used.

Third, reducing the study of metacognition to a single “snapshot” or an isolated
event fails to capture the functional nature of metacognition as a self-help tool that is activated when individuals encounter difficult or novel tasks.
Appendix A: Youth Assent

YOUTH ASSENT FORM

Purpose: You are invited to participate in a research study that will help us learn more about how to improve middle school students’ mathematical problem solving (Middle School Math Project: MSM Project). The study was awarded to the University of Miami and is funded by the U.S. Department of Education, Institute for Education Sciences, 2007-2010. The purpose of the study is to improve the teaching and learning of mathematics. You will receive instruction that is typically provided by your regular math teacher.

Procedures for control group:

- You will practice math problems during approximately 3 weeks in the fall semester. Practice sessions or tests will be given every two weeks.
- You will be assessed twice during the year as a pretest and posttest during class time. One informal measure will be administered individually twice yearly to a randomly selected group of students.
- As part of data collection, we will check your student records for both ability and achievement data, including previous psychoeducational assessments, individual education plans, report card grades, and FCAT scores.
- In order for us to understand what you are thinking as you solve math word problems, you will be audio recorded as you think out loud during the individually administered informal math measure.

Benefits: While we cannot promise direct benefits to you, some students benefit from talking through how they solve problems because it helps them to understand better.

Risks: We do not anticipate any risks to participants. Confidentiality will be protected. All records are locked in the Principal Investigator’s (Marjorie Montague) University of Miami designated research office.

Alternatives: You may refuse to participate in the study. You can quit the study at any time. Nothing will happen to you if you drop out of the study.

Confidentiality: All data collected are confidential which means your answers are kept private. Only student subject codes will be used to connect audio data to student data files. Student names and other personal identifiers will not be recorded. You may request that the tape recorder be turned off at any time during test administration. Audiotapes will be stored in the locked office of the P.I. After the information on the audiotapes is transcribed, all tapes will be destroyed. When we are finished with this study we will write a report about what was learned. This report will not include your name or that you were in the study.
Name of Student (Please print) __________________ Signature of Student __________________ Date __________

Principal Investigator: Marjorie Montague, Ph.D. Phone: 305-284-2891

<table>
<thead>
<tr>
<th>YES. I want to participate in this study.</th>
<th>YES   ______</th>
</tr>
</thead>
<tbody>
<tr>
<td>NO. I DO NOT want to participate in this study.</td>
<td>NO  ______</td>
</tr>
</tbody>
</table>

**AUDIO RECORDING**

<table>
<thead>
<tr>
<th>YES. I agree to be audio recorded in this study.</th>
<th>YES   ______</th>
</tr>
</thead>
<tbody>
<tr>
<td>NO. I DO NOT agree to be audio recorded in this study.</td>
<td>NO  ______</td>
</tr>
</tbody>
</table>
Appendix B: Parent Consent

Improving Mathematics Performance of At Risk Students and Students with Learning Disabilities (LD) in Urban Middle Schools
(Middle School Math: MSM Project)

PARENT INFORMED CONSENT FORM

Purpose: Your son/daughter is invited to participate in a research study to evaluate methods to improve middle school students’ mathematical problem solving (Middle School Math Project: MSM Project). The study was awarded to the University of Miami and is funded by the U.S. Department of Education, Institute for Education Sciences, 2007-2010. The purpose of the study is to improve the teaching and learning of mathematics. Teachers will use the research-based instructional program during your son/daughter’s regular mathematics classes as an add-on to the general curriculum. Both teachers and students in this study will be randomly assigned to either an intervention or control group based on their school. This means that they were assigned by chance, like flipping a coin. The intervention group of teachers and students will participate in learning a special math problem solving program. The control group of teachers and students will practice the regular math curriculum for problem solving offered by the Miami-Dade County Public Schools.

Procedures for the intervention group:

- Your son/daughter’s teacher will attend a summer workshop to learn how to implement the intervention, Solve It!, a program designed by the researcher, Dr. Marjorie Montague, to teach students how to solve math problems.
- Your son/daughter’s teacher will implement the instructional program during approximately 3 weeks in the fall semester. Practice sessions will be given every two weeks, and progress will be monitored during the school year.
- Your child will be assessed to measure performance over time using both standardized and informal measures of mathematics achievement. The standardized instruments will be administered twice yearly as a pretest and posttest during class time. One informal measure will be administered individually twice yearly to a randomly selected group of students of which your child may be a part. Curriculum-based measures (CBM) developed for Solve It! consist of tests of 10 one-, two-, and three-step problems administered for pretest/posttest purposes and tests three times following intervention at regular intervals across the school year to measure progress and performance maintenance.
- About 12 students and 12 parents will participate in a focus group to get opinions of the program. If you and your child agree to participate in the focus group, you will not need to reveal your name, or you may use a false name. You must agree not to tell others who are not in the focus group anything you learn from group discussions or other activities because this information is private.
- As part of data collection, we will access your child’s records for both ability and achievement data, including previous psychoeducational assessments, individual education plans, report card grades, and FCAT scores.
In order for us to understand what your child is thinking as he/she solves math word problems, your child will be audio recorded as he/she thinks out loud during the individually administered informal math measure.

**Risks:** We do not anticipate any risks to your child. Informed consent will be obtained by project staff from your child. Teachers will complete consent forms. Teachers will distribute and collect consent forms signed by you from your son/daughter, and your child will then sign a consent form if they are willing to participate. All information relating to the proposed study will be provided to your child before he/she consents to participate. Risk will be minimal, and every effort to protect against risk will be maintained. Confidentiality will be protected. All records are locked in the Principal Investigator’s (Marjorie Montague) University of Miami designated research office.

**Benefits:** While we cannot promise direct benefits to your child, some students benefit from talking through how they solve problems because it helps them to understand better.

**Alternatives:** You may refuse to allow your child to participate in the study. Also, while your child is being tested or interviewed, he or she can quit the study at any time. Nothing will happen to your child if he/she drops out of the study. Your child will continue to receive teacher instruction but will not participate in any data collection activities.

**Confidentiality:** With the exception of agency specific auditing purposes (see below), only the research team will have access to your child’s identity. Data are collected with your child as a group or individually by trained project staff. Data are collected specifically for the proposed research project. There will be no way to link information collected with your child in the final publication of results. All of the answers will be coded by a special identifying number rather than by your child’s name. All of the papers pertaining to the study will be kept in a locked file cabinet, and all electronic data will be stored in computer files. Only people who are directly involved with the project will have access to those records. When the project is finished and results are reported, no individual will be identified in any way.

By signing this consent, you authorize the investigator and her staff to access your child’s study information as may be necessary for purposes of this study.

The investigator and her assistants will consider your child’s records confidential to the extent permitted by law. The U.S. Department of Health and Human Services (DHHS) may request to review and obtain copies of your child’s records. Your child’s records may also be reviewed for audit purposes by authorized University or other agents who will be bound by the same provisions of confidentiality.

Only student subject codes will be used to connect audio data to student data files. Student names and other personal identifiers will not be recorded. Your child may request that the tape recorder be turned off at any time during test administration. Audiotapes will be stored in the locked office of the P.I. After the information on the audiotapes is transcribed, all tapes will be destroyed.
The results of this research study may be presented at meetings or in publications. Your child’s identity will not be disclosed in those presentations.

**Right to Withdraw:** Your child’s participation is voluntary. Your child has the right to withdraw from the study, and your child’s future will not be affected by withdrawal or lack of participation. The researcher can remove your child from the study without your consent for administrative reasons.

**Other Pertinent Information:** Dr. Montague will answer any questions you may have regarding the study. You may call her at 305-284-2891. You will receive a copy of this consent form. If you have any questions about your child’s rights as a research participant, please contact the Human Subjects Research Office at the University of Miami (305-243-3195).

Name of Parent/Guardian (Please print)  Signature of Parent/Guardian  Date

Name of Student (Please print)

Principal Investigator: Marjorie Montague, Ph.D. Phone: 305-284-2891

YES. I want my child to participate in this study.  YES ______

NO. I DO NOT want my child to participate in this study.  NO ______

**AUDIO RECORDING**

YES. I agree for my child to be audio recorded in this study.  YES ______

NO. I DO NOT agree for my child to be audio recorded in this study.  NO ______
## Appendix C: Think-Aloud Protocol Coding / Scoring Sheet

**SUBCODE __________-____-____- PRE / POST**

<table>
<thead>
<tr>
<th>Answered correctly:</th>
<th>Yes (2) - No (1) - DNS (0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error Type:</td>
<td>Correct (3) - Product (2) - Process (1)</td>
</tr>
<tr>
<td>Time</td>
<td></td>
</tr>
</tbody>
</table>

### Cognitive

<table>
<thead>
<tr>
<th>Category</th>
<th>Operational Definition</th>
<th>Code</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
</tr>
</thead>
<tbody>
<tr>
<td>F % F % F %</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reading</td>
<td>Reads the problem in its entirety</td>
<td>R</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Paraphrasing</td>
<td>Restates the problem in own words</td>
<td>P</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Visualize</td>
<td>Use of images (diagrams, pictures, mental imagery) to understanding task</td>
<td>V</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hypothesizing</td>
<td>Sets up a plan, decides on a solution path, sets up a goal identifying operations to use</td>
<td>H</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Estimating</td>
<td>Predicts an answer</td>
<td>E</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Computing</td>
<td>Verbalizes computation</td>
<td>C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Checking</td>
<td>Checks steps are completed, information is used, computations are accurate</td>
<td>CH</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Metacognitive

<table>
<thead>
<tr>
<th>Category</th>
<th>Operational Definition</th>
<th>Code</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
</tr>
</thead>
<tbody>
<tr>
<td>F % F % F %</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-productive</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculator</td>
<td>Requests the use of a calculator</td>
<td>Cal</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Comment</td>
<td>Statements of personal functioning during task execution</td>
<td>Com</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Affect</td>
<td>Statements concerning emotional disposition</td>
<td>NF</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Productive</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self-correct</td>
<td>Corrects products of process errors</td>
<td>SC</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self-Instruct</td>
<td>Statements regarding procedural control</td>
<td>SI</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self-Monitor</td>
<td>Observes performance and progress</td>
<td>SM</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self-Question</td>
<td>Considers problem and solution path</td>
<td>SQ</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grand Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Appendix D: MPSA-SF Rating Scale

**Interview Questions and Scoring Guide**

**TARGET:** Read (for understanding)

Say: Read the problem. If I don’t understand, read it again.

Ask: Have I read and understood the problem?

Check: For understanding as I solve the problem.

<table>
<thead>
<tr>
<th>Question</th>
<th>Scoring Key</th>
<th>Examples</th>
</tr>
</thead>
</table>
| 11. As you read, how do you help yourself understand math story problems? What else do you do when you read math story problems? | 0. Does not provide a response or I don’t know  
1. Mentions 1 Strategy  
2. Mentions 2 Strategies  
3. Mentions 3+ Strategies | Strategies (not inclusive): Reread; underline; write down numbers take out the irrelevant information; look for key words; figure out what the question is asking; break it down; summarize; analyze information; read out loud. (Note: Initial reading, computing and think about the problems are not strategies) |
| 13. If you don’t understand something about the problem, what do you do?  | 0. Does not provide a response or I don’t know  
1. Skips it and come back to it or try my best; or elicit help  
2. Reread it or look for clues  
3. Paraphrase or break it down | Guess; skip it  
Elicit help: teachers or peers |
| 14. When you are finished reading a math word problem, what questions do you ask yourself before, during, and after you solve the problem? | 0. Does not provide a response or I don’t know  
1. Provides questions relating only to product outcome or difficulty level  
2. Provides 1 question relating to procedure  
3. Provides 2+ questions relating to procedure | Product Questions:  
"Am I going to get this right? Did I get the answer right?"  
Procedure Questions:  
"What do I have to do? What is the question asking me?"  
"Do I have all the information necessary to solve the problem?",  
"Is this hard or easy?" |

Total: 9 points
**TARGET:**  **Paraphrase** (your own words)

Say: Underline the important information. Put the problem in my own words

Ask: Have I underlined the important information? What is the question? What am I looking for?

Check: That the information goes with the problem.

<table>
<thead>
<tr>
<th>Question</th>
<th>Scoring Key</th>
<th>Examples</th>
</tr>
</thead>
</table>
| 15. How do you help yourself remember what the problem says? | 0. Does not provide a response or I don’t know or incorrect  
1. Rereads the problem.  
2. Identifies important information.  
3. Puts the problem in own words. | “It’s there in my head.” “write it down”  
Memorize it  
Underlining; Pulling out key words and numbers; Look for the question; notes; clues  
Writing the problem in numbers, paraphrasing, summarizing |
| 16. Do you put what you read into your own words? If so, how do you do this? | 0. Does not provide a response or I don’t know or answers no.  
1. Yes  
2. Sometimes | |
| 17. When you put the problem into your own words, how do you know that what you said is correct? | 0. Does not provide a response or I don’t know  
1. Expresses understanding.  
2. Rereads the problem or look at the question  
3. Makes sure that all the information (numbers and important information) is the same as the problem. | “If I get the right answer it’s correct.”  
“I understand it”; “it makes sense to me”, “I change the names, not the problem”  
“go over it”  
“Check with the real problem” |

Total: 8 points
TARGET: **Visualize** (a picture or diagram)

Say: Make a drawing or a diagram.
Ask: Does the picture fit the problem?
Check: The picture against the problem information.

<table>
<thead>
<tr>
<th>Question</th>
<th>Scoring Key</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>18. What do you do to make a picture in your mind? Is there anything else you do when you visualize?</td>
<td>0. Does not provide a response or I don’t know or incorrect</td>
<td>“I just picture it”; “get the numbers”</td>
</tr>
<tr>
<td></td>
<td>1. Imagine self or others in problem or reread</td>
<td>Think of the problem</td>
</tr>
<tr>
<td></td>
<td>2. Visualize the components of the problem.</td>
<td>Diagrams; graphs; key parts of the problem; “imagine what they are doing”</td>
</tr>
<tr>
<td></td>
<td>3. Considers relationship among problem parts.</td>
<td>“Determine which operation to use”; “analyze important information”; “think about the question”; “try to interpret”; “do the problem in my head”;</td>
</tr>
<tr>
<td>21. How do your pictures help you solve math word problems?</td>
<td>0. Does not provide a response or I don’t know or they don’t or incorrect</td>
<td>“draw what it’s talking about”; “think about it”</td>
</tr>
<tr>
<td></td>
<td>1. Less confusing than words or helps to visualize problem.</td>
<td>“Gives clues”; “it’s easier”; “put it in own way”; “see how it looks”</td>
</tr>
<tr>
<td></td>
<td>2. Focuses on the relevant information.</td>
<td>“Breaks down the question”; “understand the question /problem”</td>
</tr>
<tr>
<td></td>
<td>3. Sees what needs to be done to solve the problem.</td>
<td>“You see where all the numbers go, like a cheat sheet”; “make sure all the information is used”</td>
</tr>
</tbody>
</table>

**Total: 6 points**
**TARGET:** **Hypothesize** (a plan to solve the problem)

Say: Decide how many steps and operations are needed. Write the operation symbols (+, -, x, /)

Ask: If I do _____, what will I get? If I do _____, then what do I need to do next?

How many steps are needed?

Check: That the plan makes sense.

<table>
<thead>
<tr>
<th>Question</th>
<th>Scoring Key</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>22. How do you make a plan to solve math word problems?</td>
<td>0. Does not provide a response or I don’t know or I don’t make a plan.</td>
<td>PEMDAS; “do it in my head”</td>
</tr>
<tr>
<td>(Many students did not know how to respond to this question)</td>
<td>1. Reads the problem or identify important information.</td>
<td>“I use the numbers in the problem.”</td>
</tr>
<tr>
<td></td>
<td>2. Decides on the operations.</td>
<td>“Look at what the question is asking”</td>
</tr>
<tr>
<td></td>
<td>3. Decides on the operations and order of steps.</td>
<td>“Figure out what to do next”</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Examples</td>
</tr>
<tr>
<td>23. How do you use your plan to help you solve math word problems?</td>
<td>0. Does not provide a response or I don’t know or I don’t make a plan or incorrect</td>
<td>“I just use it”; “I understand it”; “It helps me”; “using my head”</td>
</tr>
<tr>
<td>(Many students did not know how to respond to this question)</td>
<td>1. Follow the steps in the plan.</td>
<td>“Break it into parts”; “helps to concentrate on relevant information”</td>
</tr>
<tr>
<td></td>
<td>2. Considers the soundness of the plan while solving or adjusts plan according to the problem</td>
<td>Student understands that the plan can be modified while solving.</td>
</tr>
</tbody>
</table>

**Total: 5 points**
**TARGET:** Estimate (Predict the answer)
Say: Round the numbers, do the problem in my head, and write the estimate.
Ask: Did I round up or down?
Check: That I used the important information.

<table>
<thead>
<tr>
<th>Question</th>
<th>Scoring Key</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>27. Estimation is making a prediction about the answer using the information in the problem. How does estimation help in solving math word problems?</td>
<td>0. Does not provide a response or I don’t know or it doesn’t or incorrect. 1. It’s quicker. 2. Gets you close to the answer. 3. Compares the soundness of the answer to the estimate.</td>
<td>“Helps to understand the problem”; “When you don’t know what to do”; “rounding” “easy numbers to add”; “clearer” This suggests that the estimate will help get the answer “Rounds a number close to what I should get”; “Shows if you’re on the right path”; “It will be around the answer”;</td>
</tr>
<tr>
<td>28. How do you estimate imagine or predict the answer before you complete the operations for a math word problem?</td>
<td>0. Does not provide a response or I don’t know or I don’t or incorrect 1. Round the numbers. 2. Do the math in my head (but does not round the numbers) 3. Rounds the numbers and completes the operations and steps (either mentally or on paper).</td>
<td>“I read it”; “guess”; “redo it” “simplify the numbers” “Visualize in my head”; “look at the numbers and the key words” “Quick math in my head, probably not the right numbers.”</td>
</tr>
<tr>
<td>29. How do you compare your estimate with your answer?</td>
<td>0. Does not provide a response or I don’t know or I don’t or incorrect 1. See how close or far they are from each other 2. Mentions something in relation to soundness of answer.</td>
<td>“rounding around the answer”; “think about it” “If my answer is close to my estimate than I am probably right”</td>
</tr>
</tbody>
</table>

Total: 8 points
**TARGET:** **Compute** (do the arithmetic)

Say: Do the operations in the right order.

Ask: How does my answer compare with my estimate? Does my answer make sense? Are the decimals and money signs in the right places?

Check: That all the operations were done in the right order.

<table>
<thead>
<tr>
<th>Question</th>
<th>Scoring Key</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>30. <strong>What do you do when you compute answers to word problems?</strong> What goes on in your head while you are computing?</td>
<td>0. Does not provide a response or I don’t know or incorrect&lt;br&gt;1. Do the math or Reread or look at important information&lt;br&gt;2. Focuses on answer outcome&lt;br&gt;3. Focuses on relationship among problem parts or how to solve the problem.</td>
<td>“pull numbers aside”; “just do it”&lt;br&gt;“use a calculator”; “pull numbers aside”; identify important information&lt;br&gt;“Am I going to get this problem right?”; “what is the answer?”&lt;br&gt;“What do I have to do?”, “What is the question?”; “what should I do first?”; “take out or leave the important details”; “trying to get the right answer”;</td>
</tr>
<tr>
<td>31. <strong>How do you know that your computation is correct?</strong></td>
<td>0. Does not provide a response or I don’t know or incorrect&lt;br&gt;1. Redoing the math or checks it&lt;br&gt;2. Checks that the information was used or opposite operation or Reread&lt;br&gt;3. Compares answer to estimate.</td>
<td>“Ask the teacher”; “It depends on how I feel”; “do it slowly”; “it makes sense”&lt;br&gt;“Check all the steps”; “do it a different way”;</td>
</tr>
</tbody>
</table>

Total: 6 points
**TARGET:** Check (Make sure everything is right)
Say: Check the computation.
Ask: Have I checked every step? Have I checked the computation? Is my answer right?
Check: That everything is right. If not, go back. Then ask for help if I need it.

<table>
<thead>
<tr>
<th>Question</th>
<th>Scoring Key</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>33. How do you check that you have correctly completed a math word problem?</td>
<td>0. Does not provide a response or I don’t know or incorrect. 1. Redo the math</td>
<td>“look at work”, key words; “have someone check it”; “just do it”</td>
</tr>
<tr>
<td></td>
<td>2. Reread the problem or opposite operation</td>
<td>“use a calculator”</td>
</tr>
<tr>
<td></td>
<td>3. Checks that all the necessary information in the problem was used or compare to estimate</td>
<td>“Redo it a different way”</td>
</tr>
<tr>
<td></td>
<td></td>
<td>“see if I missed anything”</td>
</tr>
<tr>
<td>34. Tell me about the problem-solving strategies you use when you solve math word problems.</td>
<td>0. Does not provide a response or I don’t know or incorrect 1. Mentions 1 or 2 strategies 2. Mentions 3 – 4 strategies 3. Mentions 5 or more strategies</td>
<td>PEMDAS; lists operations</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Strategies (not inclusive list): Rereading; Paraphrasing; Hypothesizing; Estimating; Checking; key words; how many steps; break it down; look for important information; draw a picture; look for the question (Note: Initial reading and computing are not strategies).</td>
</tr>
</tbody>
</table>

Total: 9 points
Appendix E: Metacognitive Experience Survey

**Problem 1, 2, 3**

**Prospective**

Directions: Read the following questions carefully and then place an (X) in the box that best describes how each statement below applies to you.

<table>
<thead>
<tr>
<th></th>
<th>Not at all True</th>
<th>Hardly True</th>
<th>Mostly True</th>
<th>Absolutely True</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>1. I have seen this type of problem before.</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>C</td>
<td>2. I understand what the problem asks me to do.</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>D</td>
<td>3. The problem is going to be difficult to solve.</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>E</td>
<td>4. I will need to use a lot of effort to solve the problem.</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>5. I am confident that I will solve this problem correctly.</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

**Retrospective**

<table>
<thead>
<tr>
<th></th>
<th>Not at all True</th>
<th>Hardly True</th>
<th>Mostly True</th>
<th>Absolutely True</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>1. I have seen this type of problem before.</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>C</td>
<td>2. I understand what the problem asks me to do.</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>D</td>
<td>3. The problem is going to be difficult to solve.</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>E</td>
<td>4. I will need to use a lot of effort to solve the problem.</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>5. I am confident that I will solve this problem correctly.</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Total: ____ /40
References


